An illustration of the light-front coupled-cluster method in quantum electrodynamics

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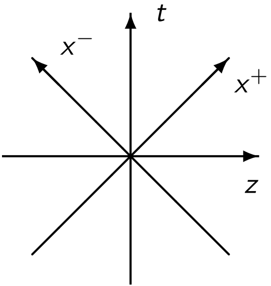
Light-cone coordinates

Dirac, RMP **21**, 392 (1949).
Brodsky, Pauli, and Pinsky, Phys. Rep. **301**, 299 (1997).

- *Time:* $x^+ = t + z$
- *Space:* $\underline{x} = (x^-, \vec{x}_\perp)$, $\; x^- \equiv t - z$, $\; \vec{x}_\perp = (x, y)$
- *Energy:* $p^- = E - p_z$
- *Momentum:* $\underline{p} = (p^+, \vec{p}_\perp)$, $\; p^+ \equiv E + p_z$, $\; \vec{p}_\perp = (p_x, p_y)$
- Mass-shell condition: $p^2 = m^2 \;\Rightarrow\; p^- = \frac{m^2 + p_\perp^2}{p^+}$



LFCC method

To solve $\mathcal{P}^-|\psi\rangle = \frac{M^2+P_\perp^2}{P^+}|\psi\rangle$ without truncation,
build eigenstate as $|\psi\rangle = \sqrt{Z}e^T|\phi\rangle$ from valence state $|\phi\rangle$
and operator T that increases particle number:

$$e^{-T}\mathcal{P}^-e^T|\phi\rangle = e^{-T}\frac{M^2+P_\perp^2}{P^+}e^T|\phi\rangle,$$

New effective Hamiltonian $\overline{\mathcal{P}^-} = e^{-T}\mathcal{P}^-e^T$,
using a Baker–Hausdorff expansion

$$\overline{\mathcal{P}^-} = \mathcal{P}^- + [\mathcal{P}^-, T] + \frac{1}{2}[[\mathcal{P}^-, T], T] + \dots$$

Eigenvalue problem becomes $\overline{\mathcal{P}^-}|\phi\rangle = \frac{M^2+P_\perp^2}{P^+}|\phi\rangle$
Project it onto the valence and orthogonal sectors

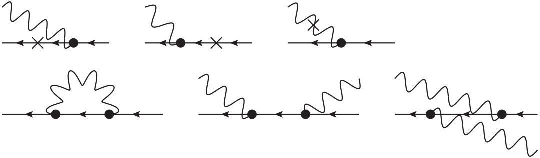
$$P_v\overline{\mathcal{P}^-}|\phi\rangle = \frac{M^2+P_\perp^2}{P^+}|\phi\rangle, \qquad (1-P_v)\overline{\mathcal{P}^-}|\phi\rangle = 0.$$

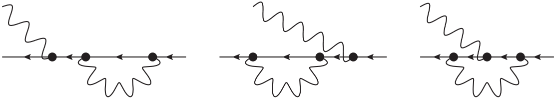
No spectator dependence and no uncanceled divergences!
SC and J.R. Hiller, PLB **711**, 417 (2012).

Graphical representation

$T =$  (truncated but conserves $\underline{P}$, J_z, charge, …)

$\mathcal{P}^- =$ 

$[\mathcal{P}^-, T] \to$ 

$[[\mathcal{P}^-, T], T] \to$ 

PV-regulated QED Lagrangian

$$\mathcal{L} = \sum_{i=0}^{2}(-1)^i\left[-\frac{1}{4}F_i^{\mu\nu}F_{i,\mu\nu} + \frac{1}{2}\mu_i^2 A_i^\mu A_{i\mu} - \frac{1}{2}\zeta\left(\partial^\mu A_{i\mu}\right)^2\right] + \sum_{i=0}^{2}(-1)^i\bar{\psi}_i(i\gamma^\mu\partial_\mu - m_i)\psi_i - e\bar{\psi}\gamma^\mu\psi A_\mu ,$$

$$\psi = \sum_{i=0}^{2}\sqrt{\beta_i}\psi_i, \quad A_\mu = \sum_{i=0}^{2}\sqrt{\xi_i}A_{i\mu}, \quad F_{i\mu\nu} = \partial_\mu A_{i\nu} - \partial_\nu A_{i\mu}.$$

$$\xi_0 = 1, \quad \sum_{i=0}^{2}(-1)^i\xi_i = 0, \quad \beta_0 = 1, \quad \sum_{i=0}^{2}(-1)^i\beta_i = 0,$$

and require chiral symmetry restoration and zero photon mass, to fix ξ_2 and β_2.

SC and J.R. Hiller, PRD **84**, 034001 (2011); arXiv:1203.0250.

Electron in QED w/o positrons

Right and left-hand valence states ($\overline{\mathcal{P}^-}$ not Hermitian!):

$|\phi_a^{\pm}\rangle = \sum_i z_{ai} b_{i\pm}^{\dagger}(\underline{P})|0\rangle$ and $\langle\tilde{\phi}_a^{\pm}| = \langle 0|\sum_i \tilde{z}_{ai} b_{i\pm}(\underline{P})$

$$T = \sum_{ijls\sigma\lambda} \int dy d\vec{k}_\perp \int \frac{d\underline{p}}{\sqrt{16\pi^3}} \sqrt{p^+} t_{ijl}^{\sigma s\lambda}(y,\vec{k}_\perp) \times a_{l\lambda}^{\dagger}(yp^+, y\vec{p}_\perp + \vec{k}_\perp) b_{js}^{\dagger}((1-y)p^+, (1-y)\vec{p}_\perp - \vec{k}_\perp) b_{i\sigma}(\underline{p})$$

$$\begin{aligned}\mathcal{P}^- = \sum_{i,s} \int d\underline{p} \frac{m_i^2 + p_\perp^2}{p^+} (-1)^i b_{is}^{\dagger}(\underline{p}) b_{is}(\underline{p}) + \sum_{l,\lambda} \int d\underline{k} \frac{\mu_{l\lambda}^2 + k_\perp^2}{k^+} (-1)^l \epsilon^\lambda a_{l\lambda}^{\dagger}(\underline{k}) a_{l\lambda}(\underline{k}) \\ + \sum_{ijls\sigma\lambda} \int dy d\vec{k}_\perp \int \frac{d\underline{p}}{\sqrt{16\pi^3 p^+}} \Big\{ h_{ijl}^{\sigma s\lambda}(y,\vec{k}_\perp) \times a_{l\lambda}^{\dagger}(yp^+, y\vec{p}_\perp + \vec{k}_\perp) b_{js}^{\dagger}((1-y)p^+, (1-y)\vec{p}_\perp - \vec{k}_\perp) b_{i\sigma}(\underline{p}) + \text{H.c.} \Big\}\end{aligned}$$

Effective Hamiltonian

$$\overline{\mathcal{P}^-} = \sum_{ijs} \int d\underline{p}(-1)^i \left[\delta_{ij}\frac{m_i^2+p_\perp^2}{p^+} + \frac{I_{ji}}{p^+}\right] b_{js}^\dagger(\underline{p}) b_{is}(\underline{p})$$
$$+\sum_{ijls\sigma\lambda}\int dy d\vec{k}_\perp \int \frac{d\underline{p}}{\sqrt{16\pi^3 p^+}} \left\{ h_{ijl}^{\sigma s\lambda}(y,\vec{k}_\perp) + \frac{1}{2} V_{ijl}^{\sigma s\lambda}(y,\vec{k}_\perp)\right.$$
$$+\left[\frac{m_j^2+k_\perp^2}{1-y} + \frac{\mu_{l\lambda}^2+k_\perp^2}{y} - m_i^2\right] t_{ijl}^{\sigma s\lambda}(y,\vec{k}_\perp)$$
$$\left. +\frac{1}{2}\sum_{i'} \frac{I_{ji'}}{1-y} t_{ii'l}^{\sigma s\lambda}(y,\vec{k}_\perp) - \sum_{j'}(-1)^{i+j'} t_{j'jl}^{\sigma s\lambda}(y,\vec{k}_\perp) I_{j'i}\right\}$$
$$\times a_{l\lambda}^\dagger(yp^+, y\vec{p}_\perp + \vec{k}_\perp) b_{js}^\dagger((1-y)p^+,(1-y)\vec{p}_\perp - \vec{k}_\perp) b_{i\sigma}(\underline{p}),$$

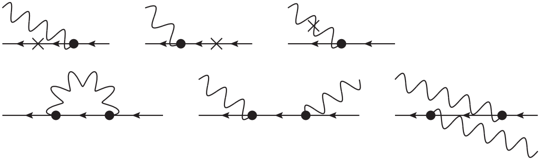
$$I_{ji} = (-1)^i \sum_{i'ls\lambda}(-1)^{i'+l}\epsilon^\lambda \int \frac{dy d\vec{k}'_\perp}{16\pi^3} h_{ji'l}^{\sigma s\lambda *}(y,\vec{k}_\perp) t_{ii'l}^{\sigma s\lambda}(y,\vec{k}_\perp),$$

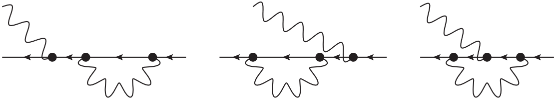
$$V_{ijl}^{\sigma s\lambda}(y,\vec{k}_\perp) = \sum_{i'j'l's'\sigma'\lambda'}(-1)^{i'+l'+j'}\epsilon^{\lambda'} \int \frac{dy' d\vec{k}'_\perp}{16\pi^3} \frac{\theta(1-y-y')}{(1-y')^{1/2}(1-y)^{3/2}}$$
$$\times h_{jj'l'}^{ss'\lambda' *}\left(\frac{y'}{1-y}, \vec{k}'_\perp + \frac{y'}{1-y}\vec{k}_\perp\right) t_{i'j'l}^{\sigma' s'\lambda}\left(\frac{y}{1-y'}, \vec{k}_\perp + \frac{y}{1-y'}\vec{k}'_\perp\right) t_{ii'l'}^{\sigma\sigma'\lambda'}(y',\vec{k}'_\perp)$$

Graphical representation

$T =$ 

$\mathcal{P}^- =$ 

$[\mathcal{P}^-, T] \to$ 

$[[\mathcal{P}^-, T], T] \to$ 

Valence-sector equations

The projections

$P_v\overline{\mathcal{P}^-}P_v|\phi_a^\pm\rangle = \frac{M_a^2+P_\perp^2}{P^+}|\phi_a^\pm\rangle$ and $(P_v\overline{\mathcal{P}^-}P_v)^\dagger|\tilde{\phi}_a^\pm\rangle = \frac{M_a^2+P_\perp^2}{P^+}|\tilde{\phi}_a^\pm\rangle$

yield

$m_i^2 z_{ai}^\pm + \sum_j I_{ij} z_{aj}^\pm = M_a^2 z_{ai}^\pm$

and

$m_i^2 \tilde{z}_{ai}^\pm + \sum_j (-1)^{i+j} I_{ji} \tilde{z}_{aj}^\pm = M_a^2 \tilde{z}_{ai}^\pm$,

with $a = 0, 1$ and M_a the ath eigenmass.

The valence eigenvectors are orthonormal and complete in the following sense:

$\sum_i (-1)^i \tilde{z}_{ai}^\pm z_{bi}^\pm = (-1)^a \delta_{ab}$ and $\sum_a (-1)^a z_{ia}^\pm \tilde{z}_{ja}^\pm = (-1)^i \delta_{ij}$

Equations for t functions

Projection onto $|e\gamma\rangle$, orthogonal to $|\phi\rangle$,gives

$$
\begin{aligned}
0 &= \sum_i (-1)^i z_{ai}^{\pm} \left\{ h_{ijl}^{\pm s\lambda}(y,\vec{k}_\perp) + \frac{1}{2} V_{ijl}^{\pm s\lambda}(y,\vec{k}_\perp) \right. \\
&\quad + \left[\frac{m_j^2 + k_\perp^2}{1-y} + \frac{\mu_{l\lambda}^2 + k_\perp^2}{y} - m_i^2 \right] t_{ijl}^{\pm s\lambda}(y,\vec{k}_\perp) \\
&\quad \left. + \frac{1}{2} \sum_{i'} \frac{I_{ji'}}{1-y} t_{ii'l}^{\pm s\lambda}(y,\vec{k}_\perp) - \sum_{j'} (-1)^{i+j'} t_{j'jl}^{\pm s\lambda}(y,\vec{k}_\perp) I_{j'i} \right\}
\end{aligned}
$$

To partially diagonalize in flavor, we define

$$
C_{abl}^{\pm s\lambda}(y,\vec{k}_\perp) = \sum_{ij} (-1)^{i+j} z_{ai}^{\pm} \tilde{z}_{bj}^{\pm} t_{ijl}^{\pm s\lambda}(y,\vec{k}_\perp)
$$

Flavor-diagonal equation

With analogous definitions for H, I, and V :

$$\left[M_a^2 - \frac{M_b^2 + k_\perp^2}{1-y} - \frac{\mu_{l\lambda}^2 + k_\perp^2}{y}\right] C_{abl}^{\pm s\lambda}(y, \vec{k}_\perp)$$

$$= H_{abl}^{\pm s\lambda}(y, \vec{k}_\perp) + \frac{1}{2}\left[V_{abl}^{\pm s\lambda}(y, \vec{k}_\perp) - \sum_{b'} \frac{I_{bb'}}{1-y} C_{ab'l}^{\pm s\lambda}(y, \vec{k}_\perp)\right]$$

to be solved simultaneously with valence sector equations, which depend on C/t through the self-energy matrix I.

The physical mass M_b has replaced the bare mass in the kinetic energy term, without use of sector-dependent renormalization!

Left-hand eigenstates

The dual to $\langle\tilde\psi| = \sqrt{Z}\langle\psi|e^T$ is a right eigenstate of $\overline{\mathcal{P}^-}^\dagger$

$$\overline{\mathcal{P}^-}^\dagger|\tilde\psi(\underline{P})\rangle = e^{T^\dagger}\mathcal{P}^- e^{-T^\dagger}\sqrt{Z}e^{T^\dagger}|\psi\rangle = \frac{M^2+P_\perp^2}{P^+}|\tilde\psi(\underline{P})\rangle$$

normalized such that

$$\langle\tilde\psi(\underline{P}')|\phi(\underline{P})\rangle = \sqrt{Z}\langle\psi(\underline{P}')|e^T|\phi(\underline{P})\rangle = \langle\psi(\underline{P}')|\psi(\underline{P})\rangle$$
$$= \delta(\underline{P}' - \underline{P})$$

For the dressed electron, construct as

$$|\tilde\psi_a^\pm(\underline{P})\rangle = |\tilde\phi_a^\pm(\underline{P})\rangle + \sum_{jls\lambda}\int dy d\vec{k}_\perp \sqrt{\frac{P^+}{16\pi^3}} l_{ajl}^{\pm s\lambda}(y,\vec{k}_\perp)$$
$$\times a_{l\lambda}^\dagger(yP^+, y\vec{P}_\perp + \vec{k}_\perp) b_{js}^\dagger((1-y)P^+, (1-y)\vec{P}_\perp - \vec{k}_\perp)|0\rangle$$

Left-hand eigenvalue problem

Flavor-diagonalize left-hand wave functions

$$D_{abl}^{\pm s\lambda}(y,\vec{k}_\perp) \equiv \sum_j (-1)^j z_{bj}^s l_{ajl}^{\pm s\lambda}(y,\vec{k}_\perp),$$

to obtain

$$\left[M_a^2 - \frac{M_b^2 + k_\perp^2}{1-y} - \frac{\mu_{l\lambda}^2 + k_\perp^2}{y}\right] D_{abl}^{\sigma s\lambda}(y,\vec{k}_\perp)$$
$$= \tilde{H}_{abl}^{\sigma s\lambda}(y,\vec{k}_\perp) + W_{abl}^{\sigma s\lambda}(y,\vec{k}_\perp) - \sum_{b'} J_{b'a}^\sigma \tilde{H}_{b'bl}^{\sigma s\lambda *}(y,\vec{k}_\perp).$$

where $W_{abl}^{\sigma s\lambda}$ is a vertex-correction analog of $V_{abl}^{\sigma s\lambda}$, though linear in D, and J_{ba}^σ is a self-energy analog of I_{ba}.
Solutions for M_a, z_{ai}^σ, $\tilde{z}_{ai}^\sigma$, and $C_{abl}^{\sigma s\lambda}$ are used as input.

Dirac and Pauli form factors

Compute anomalous moment a_e from spin-flip matrix element of current $J^+ = \overline{\psi}\gamma^+\psi$ coupled to photon of momentum q in the Drell–Yan $(q^+ = 0)$frame

$$16\pi^3\langle\psi_a^\sigma(\underline{P}+\underline{q})|J^+(0)|\psi_a^\pm(\underline{P})\rangle = 2\delta_{\sigma\pm}F_1(q^2) \pm \frac{q^1 \pm iq^2}{M_a}\delta_{\sigma\mp}F_2(q^2).$$

In limit of infinite PV masses,
and with $M_0 = m_e$ the electron mass,

$$F_1(q^2) = 1 + \sum_{s\lambda}\epsilon^\lambda \int \frac{dyd\vec{k}_\perp}{16\pi^3}\left\{l_{000}^{\pm s\lambda *}(y, \vec{k}_\perp - y\vec{q}_\perp)t_{000}^{\pm s\lambda}(y,\vec{k}_\perp)\, . \right.$$
$$\left. -l_{000}^{\pm s\lambda *}(y,\vec{k}_\perp)t_{000}^{\pm s\lambda}(y,\vec{k}_\perp)\right\}$$

$$F_2(q^2) = \pm\frac{2m_e}{q^1 \pm iq^2}\sum_{s\lambda}\epsilon^\lambda \int \frac{dyd\vec{k}_\perp}{16\pi^3} l_{000}^{\mp s\lambda *}(y,\vec{k}_\perp - y\vec{q}_\perp)t_{000}^{\pm s\lambda}(y,\vec{k}_\perp)$$

A second term is absent in F_2 because l and t are orthogonal for opposite spins.

Anomalous moment

$q^2 \to 0$ limit can be taken, to find $F_1(0) = 1$ and

$$a_e = F_2(0) = \pm m_e \sum_{s\lambda} \epsilon^\lambda \int \frac{dy d\vec{k}_\perp}{16\pi^3} y l_{000}^{\mp s\lambda *}(y, \vec{k}_\perp) \times \left(\frac{\partial}{\partial k^1} \mp i \frac{\partial}{\partial k^2}\right) t_{000}^{\pm s\lambda}(y, \vec{k}_\perp).$$

As a check, consider a perturbative solution

$$t_{000}^{\sigma s\lambda} = l_{000}^{\sigma s\lambda} = h_{000}^{\sigma s\lambda} / \left[m_e^2 - \frac{m_e^2 + k_\perp^2}{1-y} - \frac{\mu_{l\lambda}^2 + k_\perp^2}{y} \right].$$

Substitution into expression for a_e gives immediately the Schwinger result $\alpha/2\pi$, in the limit of zero photon mass, for any covariant gauge.

The full (numerical) solution will include all α^2 contributions without electron-positron pairs and partial summation of higher orders.

Summary

- advantages of LFCC approach:
  - no Fock-space truncation.
  - no sector dependence or spectator dependence.
  - systematically improvable.
- future work:
  - finish dressed-electron-state calculation:
    - solve coupled systems numerically for t and l.
    - compute anomalous moment.
    - test gauge independence.
  - dressed photon state, extend dressed-electron state to include electron-positron pair.
  - muonium, positronium.
  - symmetry breaking in scalar theories.
  - adapt methods to (holographic) QCD.