Subleading order Heavy-quark potential from AdS/CFT and meson melting

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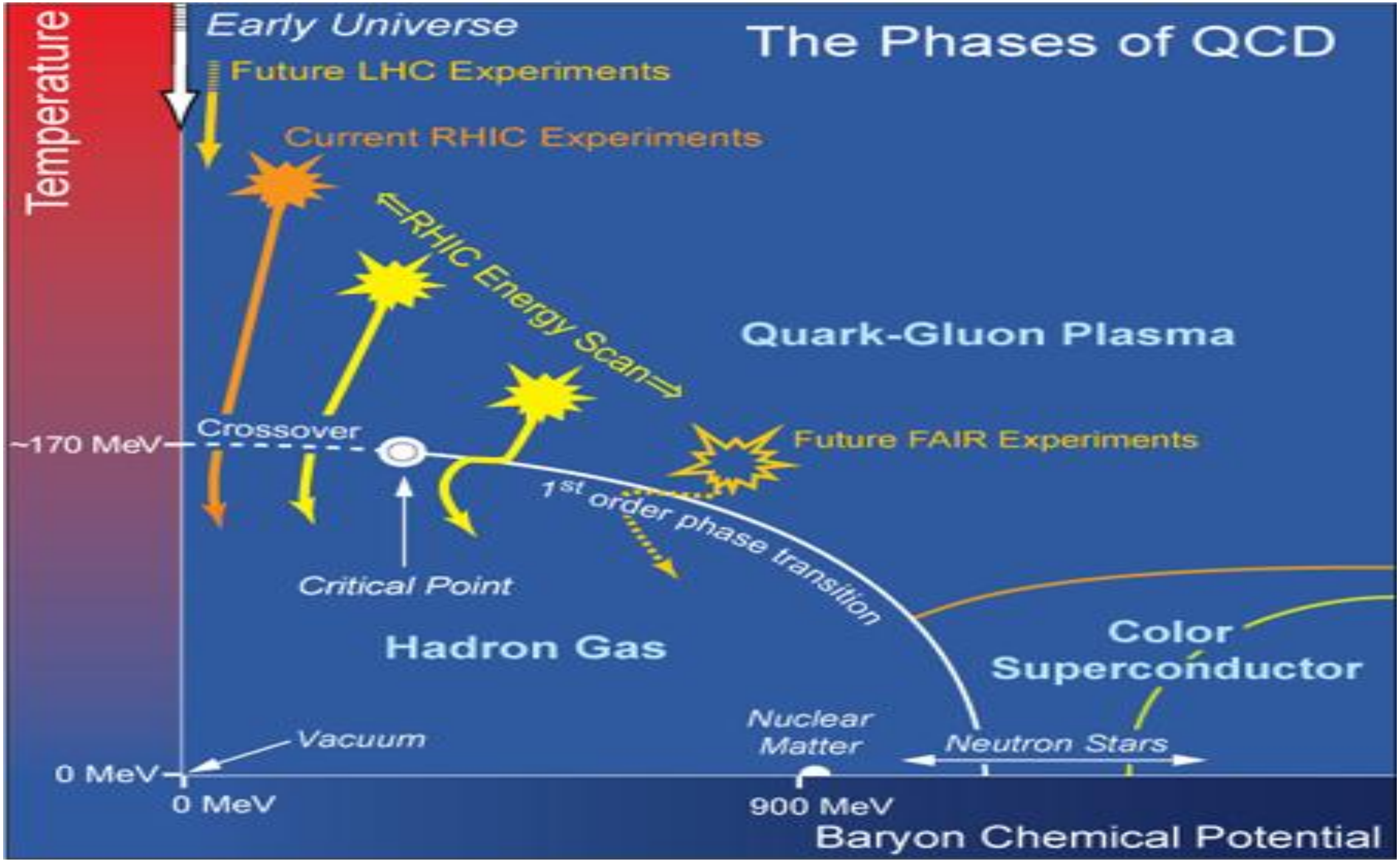
OUTLINES

- **Introduction**
- **Holographic heavy quark potential**
- **Higher order correction**
- **Meson melting in QGP**

Zhang, Hou, Ren,Yin JHEP07:035（2011）
Hou, Liu, Li, Ren, JHEP 1007:042 (2010)
Hou, Ren JHEP01:029（2008）
Chu, Hou，Ren JHEP08: 004 (2009)

QCD Phase Structure



Many interesting phenomena in QCD lie in the strongly-coupled region

AdS/CFT corerspondence

4dim. Large-Nc strongly coupled
SU(Nc) N=4 SYM (finite T).

Maldacena '97 ↕ ***conjecture*** Witten '98

Type II B Super String theory
on AdS-BH $\times$S5

$$N=4\ \text{SUSY YM on the boundary} \quad \Leftrightarrow \quad \text{Type IIB string theory in the bulk}$$

'‘t Hooft coupling $\lambda \equiv N_c g_{YM}^2 = \frac{1}{\alpha'^2}$ $\quad$ (string tension $= \frac{1}{2\pi\alpha'}$)

$$\frac{\lambda}{N_c} = 4\pi g_s$$

$$< e^{\int d^4x \phi_0(x) O(x)} > = Z_{\text{string}}[\phi(x,0)=\phi_0(\text{x})]$$

In the limit $N_c \to \infty$ and $\lambda \to \infty$

$$Z_{\text{string}}[\phi(x,0)=\phi_0(x)] = e^{-I_{\text{sugra}}[\phi]} \big|_{\phi(x,0)=\phi_0(x)}$$

$$I_{\text{sugra}}[\phi] = \text{classical supergravity action}$$

AdS/CFT applied to RHIC physics

- **Viscosity, η/s** $$\frac{\eta}{s} = \frac{1}{4\pi}$$ *Policastro, Son and Starinet*
- **Thermodynamics** $$s = \frac{3}{4} s^{(0)}$$ *Witten*
- **Jet quenching** $$\hat{q} = \pi^{\frac{3}{2}} \frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)} \sqrt{\lambda} T^3$$ *Liu, Rajagopal and Wiederman*
- **Heavy quarkonium (hard probe)**
- **…..**

Heavy quark potential probes confinement hadronic phase and meson melting in plasma

Correlation function from AdS/CFT

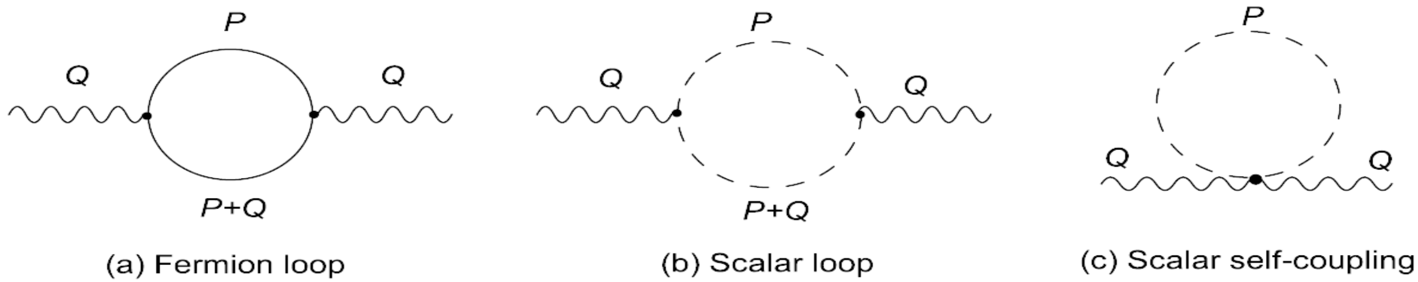
- **Solving the Maxwell equation and the linearized Einstein equation subject to the boundary conditions**

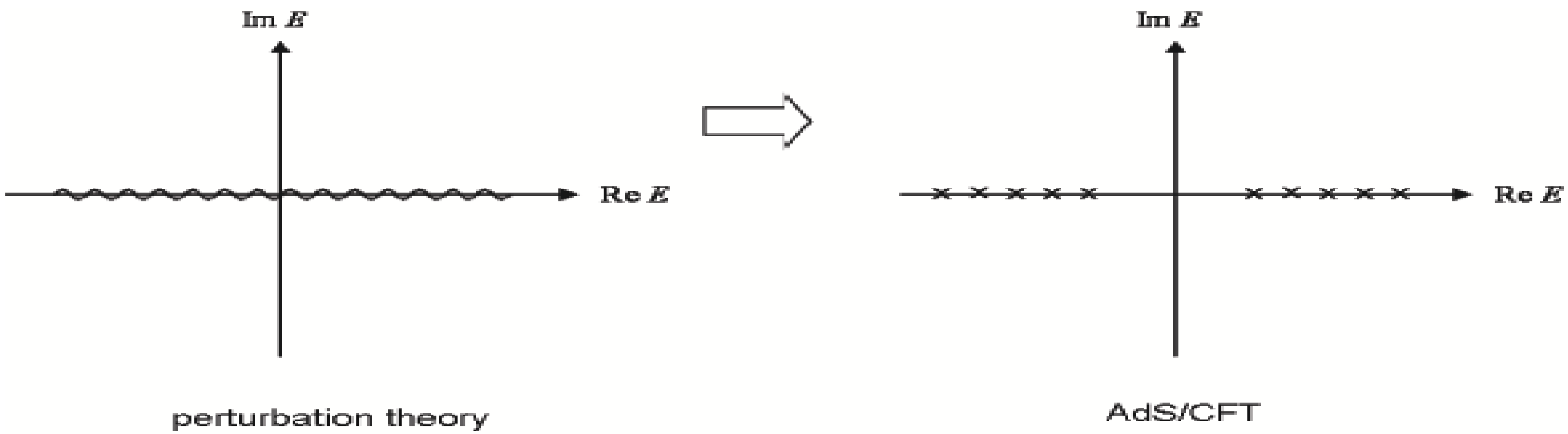
$$S_{\rm sugr} = S^{(0)}_{\rm sugr} + \frac{1}{2}\int_{u=0} d^4x \int_{u=0} d^4y \Big[\mathcal{C}_{\mu\nu}(x-y)\bar{A}^{\mu}(x)\bar{A}^{\nu}(y) + \frac{1}{4}\mathcal{C}_{\mu\nu,\rho\lambda}(x-y)\bar{h}^{\mu\nu}(x)\bar{h}^{\rho\lambda}(y)\Big]$$
$$= \frac{1}{2}\int \frac{d^4\vec{Q}}{(2\pi)^4}\Big[\mathcal{C}_{\mu\nu}(Q)\bar{A}^{\mu *}(Q)\bar{A}^{\nu}(Q) + \frac{1}{4}\mathcal{C}_{\mu\nu,\rho\lambda}(Q)\bar{h}^{\mu\nu *}(Q)\bar{h}^{\rho\lambda}(Q)\Big]$$

- **The coefficients $C_{\mu\nu}$, give rise to the R-photon self-energy tensor**

$$F(q) \equiv \mathcal{C}_{00}(0,q) = -\frac{N_c^2 T^2}{8}\frac{A_0'(\varepsilon|q)}{A_0(\varepsilon|q)}$$

Policastro, Son & Starinets, JHEP0209(02)043





Continuum spectrum *Bound sates*

JHEP 1007:042,2010, Hou, Liu , Li , Ren

Heavy quark Potential from AdS/CFT

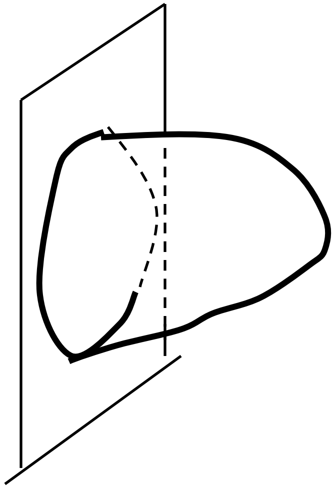
The gravity dual of a Wilson loop at large λ and large N_c

$$\mathrm{tr} < W(C) >= e^{-\sqrt{\lambda} S_{\min}[C]}$$

the minu.area of string world sheet in the AdS_5

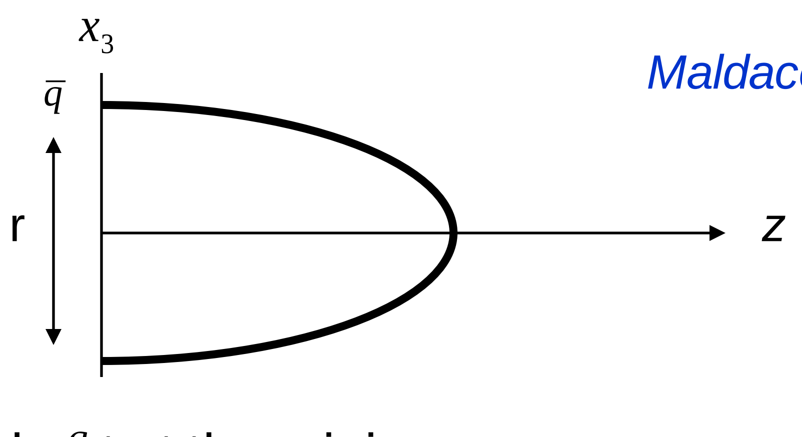
$$W(C) = P e^{-i \oint_C dx^\mu A_\mu(x)}$$

$$F(r,T) = T\left(S_{\min}[\text{parallel lines}] - 2S_{\min}[\text{single line}]\right)$$



Heavy quark potential at zero temperature

Maldacena



The world sheet at the minimum

$$x_3 = \pm \int_z^{z_0} d\varsigma \frac{\varsigma^2}{\sqrt{z_0^4 - \varsigma^4}} \qquad \text{with} \qquad z_0 = \frac{\Gamma^2\left(\frac{1}{4}\right)}{(2\pi)^{\frac{3}{2}}} r$$

The potential

$$V(r,0) = F(r,0) = -\frac{4\pi^2\sqrt{2N_c g_{YM}^2}}{\Gamma^4\left(\frac{1}{4}\right) r}$$

No confinement in N=4 SYM!

Heavy quark potential at a nonzero temperature

Rey, Theisen and Yee

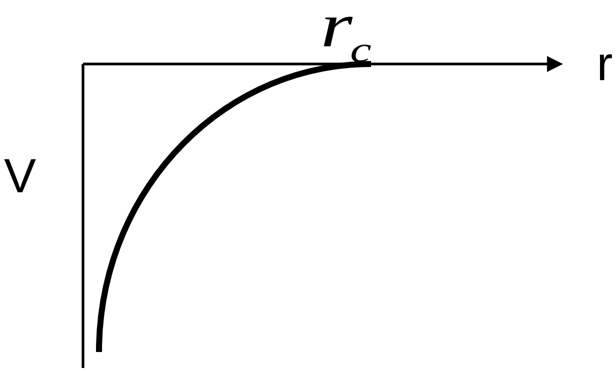
$$F(r,T) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma^4\left(\frac{1}{4}\right)r}\phi(\pi T r)\theta(r_c - r)$$

$$\phi(\pi T r_c) = 0 \qquad r_c \cong \frac{0.7541}{\pi T}$$

Potential:

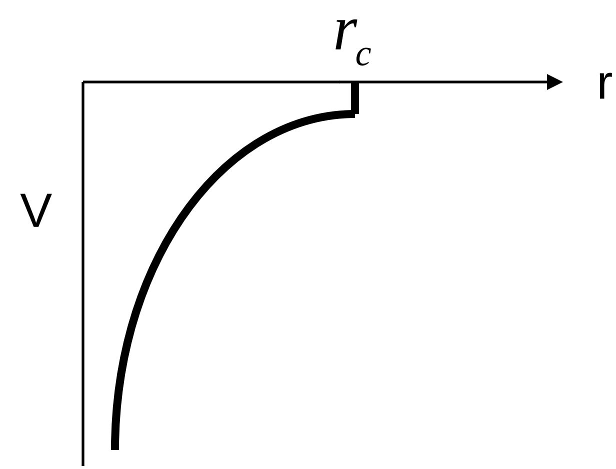
F-ansatz

$$V(r,T) = F(r,T)$$



U-ansatz

$$V(r,T) = F(r,T) + TS(r,T) = -T^2\frac{\partial}{\partial T}\left(\frac{F}{T}\right)$$



Higher order corrections

- The t'Hooft coupling is not infinity

$$5.5 < \lambda < 6\pi.$$

- The super gravity correction to the AdS-Schwarschild metric is of order $O(\lambda^{-\frac{3}{2}})$

,

- The fluctuation around the minimum world sheet presents at all T, and is of order $O(\lambda^{-\frac{1}{2}})$ (more important)

Gravity dual of a Wilson loop at finite coupling

$$W[C] \equiv \left\langle \exp\left(i\oint_C dx^\mu A_\mu \right) \right\rangle = \int [dX][d\theta] \exp\left[\frac{i}{2\pi\alpha'} S(X,\theta) \right]$$

Strong coupling expansion ⇄ Semi-classical expansion

$$\ln W[C] = i\sqrt{\lambda}\left[S(\bar{X},0) + \frac{b[C]}{\sqrt{\lambda}} + \ldots \right]$$

$\bar{X}$ = the solution of the classical equation of motion;

$b[C]$ comes from the fluctuation of the string world sheet around $\bar{X}$

more significant than α'^3 -correction for Wilson loops.

$$\frac{1}{2\pi\alpha'} S(X,\theta) = \text{the superstring action in } AdS_5 \times S^5$$

Metsaev and Tseytlin

With fluctuations:

$$X^{\mu} = \bar{X}^{\mu} + \delta X^{\mu}, \qquad \theta \neq 0 \qquad g_{ij} = \bar{g}_{ij} + \delta g_{ij}$$

$$S(X,\theta) = S(\bar{X},0) + S_B^{(2)}(\delta X) + S_F^{(2)}(\theta) + \ldots$$

Bosonic and fermionic fluctuations decouple.

$$W[C] = e^{iS(\bar{X},0)} Z \qquad Z = Z_B Z_F$$

The partition functions of fluctuations at zero T

Forste, Ghoshal and Theisen
Drukker, Gross and Tseytlin

Single line:

$$W[C_1] = -\ln \frac{\det^4(-i\gamma^\alpha \nabla_\alpha + \tau_3)}{\det^{\frac{3}{2}}(-\nabla^2 + 2)\det^{\frac{5}{2}}(-\nabla^2)}$$

Parallel lines:

$$W[C_2] = -\ln \frac{\det^4(-i\gamma^\alpha \nabla_\alpha + \tau_3)}{\det(-\nabla^2 + 2)\det^{\frac{1}{2}}(-\nabla^2 + 4 + R)\det^{\frac{5}{2}}(-\nabla^2)}$$

Partition function at finite T with fluct.

Hou, Liu, Ren, PRD80, 2009

Straight line:

$$Z = Z_B Z_F = \frac{\det^2\left(-\nabla_+^2 + 1 + \frac{1}{4}R^{(2)}\right)\det^2\left(-\nabla_-^2 + 1 + \frac{1}{4}R^{(2)}\right)}{\det^{\frac{3}{2}}\left(-\nabla^2 + \frac{8}{3} + \frac{1}{2}R^{(2)}\right)\det^{\frac{5}{2}}(-\nabla^2)}$$

Parallel lines:

$$Z = \frac{\det^2\left(-\nabla_+^2 + 1 + \frac{1}{4}R^{(2)}\right)\det^2\left(-\nabla_-^2 + 1 + \frac{1}{4}R^{(2)}\right)}{\det^{\frac{1}{2}}\left(-\nabla^2 + 4 + R^{(2)} - 2\delta\right)\det(-\nabla^2 + 2 + \delta)\det^{\frac{5}{2}}(-\nabla^2)}$$

Gelfand-Yaglom's method for determinant ratios

$$\frac{\det H_2}{\det H_1} = \frac{\Lambda[u_2, v_2]}{\Lambda[u_1, v_1]}$$

J.Math.Phys.,1,48(1960)
J.Math.Phys.,40,6044(1999)[physics/9712048]

(u_i, v_i) Are 2 independent solutions 。

$$\Lambda[u_i, v_i] = \frac{u_i(a)v_i(b) - u_i(b)v_i(a)}{W[u_i, v_i]}$$

Wronskian determinant

Reduce evaluating functional determinants to a set of 2nd order ordinary differential equations, which are solved numerically

Next leading order Results

$$V(r) \approx -\frac{4\pi^2}{\Gamma^4(\frac{1}{4})}\frac{\sqrt{\lambda}}{r}[1-\frac{1.33460}{\sqrt{\lambda}}+O(\frac{1}{\lambda})] \qquad \text{for } \lambda \gg 1$$

Chu, Hou, Ren, JHEP0908, (2009)

$$V_{\text{ladder}}(r) = -\frac{\sqrt{\lambda}}{\pi r}\left(1-\frac{\pi}{\sqrt{\lambda}}\right).$$

Erickson etc. NPB582, 2000

$$-\frac{\lambda}{4\pi r}\left[1-\frac{\lambda}{2\pi^2}\left(\ln\frac{2\pi}{\lambda}-\gamma_E+1\right)+O(\lambda^2)\right] \quad \text{for } \lambda \ll 1$$

Erickson etc. NPB582, 2000;

Confirmed by Forini JHEP 1011 (2010) 079 arXiv:1009.3939

$$V_{q\bar{q}}(\lambda, L) = -\frac{c(\lambda)}{L} \;,$$

$$c(\lambda) = \begin{cases} \frac{\lambda}{4\pi}\left[1 - \frac{\lambda}{2\pi^2}\left(\ln\frac{2\pi}{\lambda} - \gamma_E + 1\right) + \mathcal{O}(\lambda^2)\right] & \lambda \ll 1 \;, \\ \frac{\sqrt{\lambda}\,\pi}{4\,\mathbb{K}^2}\left[1 + \frac{a_1}{\sqrt{\lambda}} + \mathcal{O}\left(\frac{1}{(\sqrt{\lambda})^2}\right)\right] & \lambda \gg 1 \;. \end{cases}$$

$$\begin{aligned} a_1 &= \frac{5\pi}{12} - 3\ln 2 + \frac{2\mathbb{K}}{\pi}\left(\mathbb{K} - \sqrt{2}\,(\pi + \ln 2) + \mathcal{I}^{\mathrm{num}}\right) \\ &= -1.334595305280600773 64\ldots \;, \end{aligned}$$

-1.33460

NL potential at finite T

$$V(r) \simeq -\frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)}\frac{\sqrt{\lambda}}{r}\left[g_0(rT) - \frac{1.33460 g_1(rT)}{\sqrt{\lambda}} + O\left(\frac{1}{\lambda}\right)\right]$$

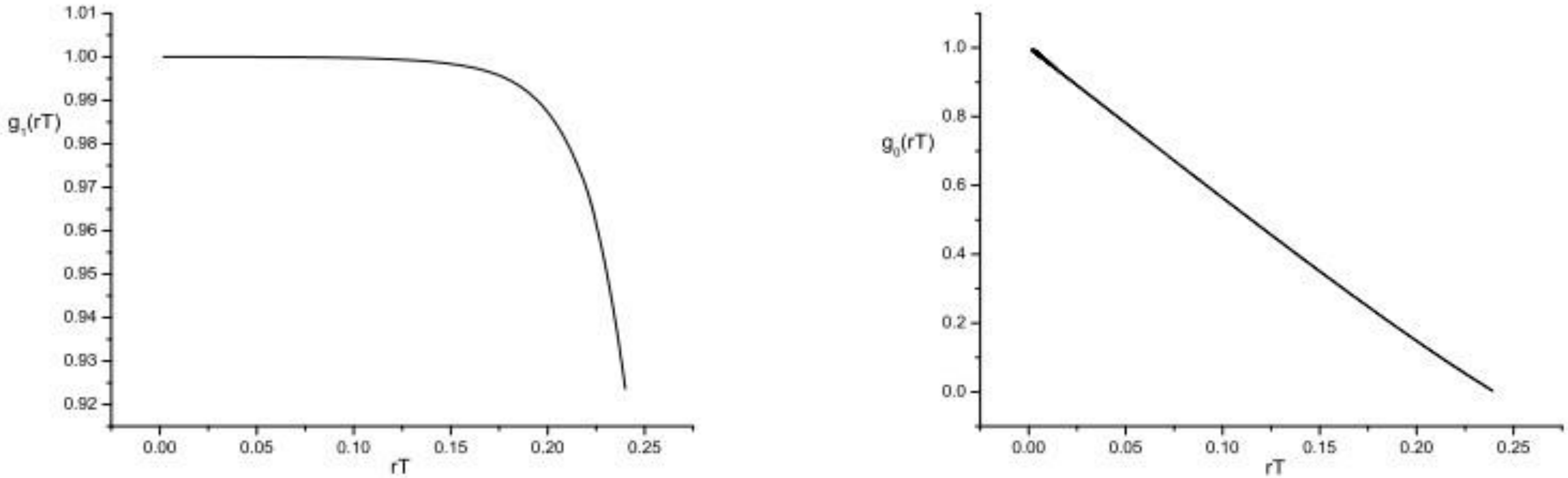


Figure 3. The left curve represents $g_1(rT)$, while the right represents $g_0(rT)$.

Zhang,Hou, Ren,Yin JHEP07:035（2011）

Meson melting in QGP

Hou and Ren, JHEP0801:029

Formulation:

- NR potential model with the holographic potential

The Schroedinger equation for $q\bar{q}$

$$-\frac{1}{M_q}\nabla^2\psi + V(r,T)\psi = (2M_q + E)\psi$$

- Order the bound state energy according to

$$E_0(T) \le E_1(T) \le E_2(T) \le \ldots \le 0$$

Each level increases with T

- The melting temperature of the n-th state is determined by

$$E_n(T_d) = 0$$

Parameters:

$$5.5 \le \lambda \le 6\pi$$

Mc=1.65Gev, Mb=4.85Gev, Tc=186MeV

The melting temperature

ansatz	$J/\psi(1S)$	$J/\psi(2S)$	$J/\psi(1P)$	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(1P)$
F	67-124	15-28	13-25	197-364	44-81	40-73
U	143-265	27-50	31-58	421-780	80-148	92-171

$$T_d(1S) \cong \begin{cases} 0.0174\sqrt{\lambda}M_q & \text{for F-ansatz} \\ 0.0368\sqrt{\lambda}M_q & \text{for U-ansatz} \end{cases}$$

Nonrelatovistic approximation

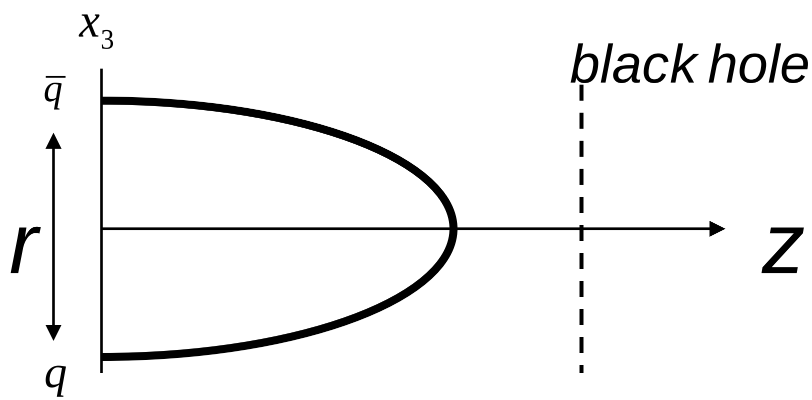
$$0.109 \le v^2 \le 0.408$$

Holographic spectral function approach

$$T_d(1s) \cong \frac{2.17}{\sqrt{\lambda}}M_q$$

Hoyos et.al.

Meson melting with a deformed metric



The soft wall AdS/QCD

$$F = -\frac{T}{16\pi G_5}\int d^4x \int dz e^{-cz^2}\sqrt{g}\left(R-12\right)$$

Karch et. al.

c is determined by the rho-meson mass

The deformed metric

$$ds^2 = \frac{e^{bz^2}}{z^2}\left(-fdt^2 + d\mathbf{x}^2 + \frac{dz^2}{f}\right)$$

b is determined by the lattice res

Td with deformed metric

Hou and Ren, JHEP0801:029

ansatz	J/ψ	Υ
F	NA	235-385
U	219-322	459-780

ansatz	J/ψ(holographic)	J/ψ(lattice)	Υ(holographic)	Υ(lattice)
F	NA	1.1	1.3-2.1	2.3
U	1.2-1.7	2.0	2.5-4.2	4.5

$$T_d/T_c$$

Summary and outlook

AdS/CFT may provide a useful method to address RHIC physics at strong coupling

We computed the heavy-quark potential up to sub-leading order, confirmed by analytic calculation by V. Forini. The strength of the potential is weakend as the 't Hooft coupling is reduced from infinity as is expected intuitively.

We calculated dissociation temperatures Td of heavy quarkonium states from AdS/CFT

The applicability of these results demand phenomenological work to explain them in a way which can be translated to QCD

Relativistic correction to Td, next order correction to jet quenching parameter will appear soon