In medium effects on vector mesons through holographic QCD

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Why are spectral functions interesting?

- behaviour of vector mesons in medium
- melting
- transport coefficients and hydrodynamical quantities
- ...

How?

Soft wall model: holographic bottom-up approach to QCD

AdS/QCD correspondence

AdS/CFT correspondence [Maldacena, '97]

Type IIB string theory in $AdS_5 \times S^5$	$\Longleftrightarrow$	$\mathcal{N}$=4 SYM theory on 4d Minkowski
SUGRA limit	$g_s = g^2_{\text{YM}}$	large N + NP limit
$g_s \to 0$	$R^4 = 4\pi g_s N \alpha'$	$N \to \infty$
$R \to \infty$		$\lambda = g^2_{\text{YM}} N \to \infty$

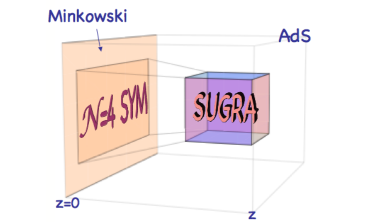
How can the theories be linked?

→ Holographic description [Witten '98, Gubser *et al.* '98]

Dictionary:

1. field $\phi(x,z)$ $\leftrightarrow$ operator $\mathcal{O}(x)$
2. m^2_{d+1} $\leftrightarrow$ Δ
3. $\langle \mathrm{e}^{\int_{\partial AdS_{d+1}} \phi_0(x)\mathcal{O}(x)} \rangle = Z_S[\phi_0(x)] \approx \mathrm{e}^{-S}$
4. $\phi(x,z) = \int_{\partial AdS_{d+1}} d^dx' \; K(x-x',z) \; \phi_0(x')$
5. $K(x-x',z) \xrightarrow[\partial AdS_{d+1}]{} z^\lambda \; \delta^d(x-x')$

$$\lambda = \frac{d}{2} - \sqrt{\frac{d^2}{4} + m^2_{d+1}R^2}$$



$$ds^2 = \frac{R^2}{z^2}(dt^2 - d\bar{x}^2 - dz^2) \quad z > 0$$

apply to QCD, but ...

QCD is

1. not supersymmetric
2. not conformal (running coupling constant)

break!

$\Rightarrow$

possible solutions:

1. introduce independent bosonic and fermionic fields
2. introduce a mass scale

How? We focus on soft wall model: "dilaton" profile in the metric or action

$$\mathrm{e}^{-\varphi(z)} \qquad \varphi(z) = c^2 z^2$$

Karch *et al.* '06

$\rightarrow$ Regge trajectories:

Vector mesons	$m_n^2 = c^2(4n+4)$
Scalar mesons	$m_n^2 = c^2(4n+6)$
Scalar glueballs	$m_n^2 = c^2(4n+8)$

$m_\rho = 0.776$ GeV $\quad\rightarrow\quad c = 0.388$ GeV

Finite temperature and density effects: charged Black-Hole

QCD	SW
add to generating functional $\mu\bar{q}\gamma^0 q$	$5d$ U(1) gauge field A_0
Periodic Euclidean time	Black Hole

$$\text{BH} + A_0 \longrightarrow \text{AdS/RN metric}$$

$$ds^2 = \frac{R^2}{z^2}\left(f(z)dt^2 - d\bar{x}^2 - \frac{dz^2}{f(z)}\right) \qquad 0 < z < z_h = \text{outer horizon of BH} \qquad (f(z_h) = 0)$$

$$f(z) = 1 - (1+Q^2)\left(\frac{z}{z_h}\right)^4 + Q^2\left(\frac{z}{z_h}\right)^6 \qquad 0 \leqslant Q \leqslant \sqrt{2} \qquad \text{prop. to BH charge}$$

$$A_0(z) = \mu - k\frac{Q^2}{z_h^3}z^2$$

Temperature and density are linked to BH parameters by:

$$A_0(z_h) = 0 \quad \rightarrow \quad \mu = k\frac{Q}{z_h} \qquad (k\text{=1 will be set})$$

$$T = \frac{1}{4\pi}\left|\frac{df}{dz}\right|_{z_h} \quad \rightarrow \quad T = \frac{1}{\pi z_h}\left(1 - \frac{Q^2}{2}\right) \qquad \text{Hawking temperature}$$

Vector mesons

operator	gauge field
$\bar{q}\gamma_\mu T^a q$	$V^a_M(x,z)$

$$S = -\frac{1}{2\,k_V\,g_5^2}\int d^5x\sqrt{g}\,e^{-\varphi(z)}\,\text{Tr}\left[F_V^{MN}\,F_{V\,MN}\right]$$

$F_V^{MN} = \partial^M V^N - \partial^N V^M \qquad k_V g_5^2 = 12\pi^2/N_c$

we fix the gauge $V_z = 0$, and c=1 (mass unit).

Eq. of motion in Fourier space for $V_i(z,\omega^2)$ in meson rest frame $\bar{p} = 0$

$$\partial_z\left(\frac{e^{-\phi(z)}}{z}f(z)\partial_z V_i(z,\omega^2)\right) + \frac{e^{-\phi(z)}}{z\,f(z)}\omega^2\,V_i(z,\omega^2) = 0$$

1. High z_h

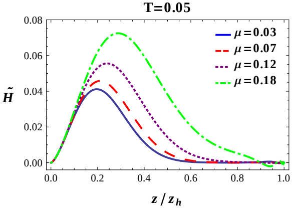
BH horizon does not affect meson wave functions

wave functions as eigenfunctions of the Schrödinger equation:

$$V_i(z,\omega^2) = e^{B(z)/2} H(z,\omega^2) \qquad B(z) = z^2 + \log z - \log f(z)$$

$$-\partial_z^2 H(z,\omega^2) + U(z)\,H(z,\omega^2) = \frac{\omega^2}{f(z)^2} H(z,\omega^2) \qquad U(z) = \frac{B'^2}{4} - \frac{B''}{2}$$

$$\begin{cases} H(0,\omega^2) = 0 \\ H(z,\omega^2) \text{ normalizable} \end{cases} \Longrightarrow \quad \text{eigenfunctions, eigenvalues}$$



2. Low z_h

masses: positions of the peaks of the spectral function

Boundary conditions for the bulk-to-boundary propagator $V(z,\omega^2)$

$$V_i(z,\omega^2) = V(z,\omega^2)\, V_i^0(\omega^2)$$

$$\begin{cases} V(0,\omega^2) = 1 \\ V(z,\omega^2) \xrightarrow{z\to z_h} (1-z/z_h)^{-i\frac{\sqrt{\omega^2}\, z_h}{2(2-Q^2)}} \left(1+\mathcal{O}(1-z/z_h)\right) \end{cases}$$

falling in solution

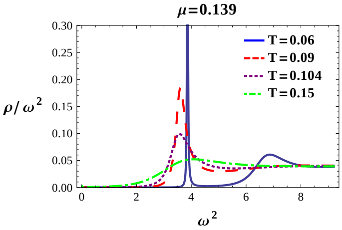
Retarded Green's function:

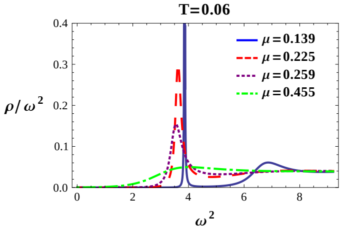
$$G^R_{ij}(\omega^2) = \frac{\delta^2 S}{\delta V_i^0(-\omega)\delta V_j^0(\omega)} = \delta_{ij}\, \frac{e^{-\phi(z)}\, f(z)}{g_5^2\, k_V} V(z,\omega^2)\frac{\partial_z\, V(z,\omega^2)}{z}\Bigg|_{z=0}$$

spectral function:

$$\rho(\omega^2) = \mathrm{Im}\left(G^R(\omega^2)\right)$$

Results

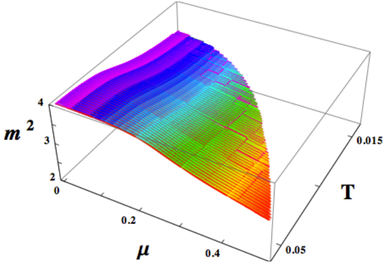


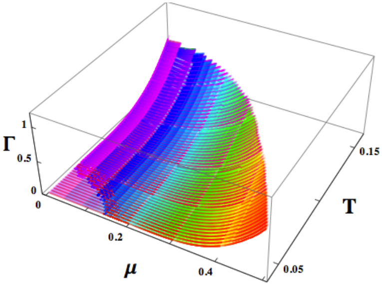


- ✓ $\rho \underset{\omega\to\infty}{\sim} \omega^2$
- ? peaks moving towards lower masses at increasing T and μ
- ✓ broadening of peaks at increasing T and μ
- ✓ melting (at lower T and μ for excited states)

Fit with modified Breit-Wigner function to extract masses and width

$$\rho_{\mathrm{BW}}(x) = \frac{a\, m\, \Gamma\, x^b}{(x - m^2)^2 + m^2\Gamma^2}$$





mass decrease $\frac{m|_{\mu=0} - m|_{\mu=\mu_c}}{m|_{\mu=0}}$:

- 13% effect at $T \sim 0$ (25% on squared mass)
- 8% effect at $T \sim 0.1c$ (16% on squared mass)

Comparison with other models and experiments:

✓ Width

width increases at increasing T and μ, in agreement with other models and experiments

? Mass

- $\longrightarrow$ in SW mass decreases at increasing T and μ
- $\longrightarrow$ models suggest a mass decrease or increase
- $\longrightarrow$ in experiments found a small decrease or no effect

In particular

- Brown-Rho dropping: mass decrease related to chiral symmetry breaking parameters
- Hatsuda-Lee:
  $m(\rho)/m(0) = 1 - \alpha\,\rho/\rho_0 \qquad \alpha = 0.16 \pm 0.06$
  $T = 0$, $\rho_0 =$ nuclear matter density
  Assuming this scaling, we find at $T = 0.023c$
  $\alpha = 0.012$ with $\mu_0 = 0.209c$

Conclusions

- Little analytical and numerical effort to compute spectral functions
- Broadening of peaks in the spectral functions, as in many models and experiments
- Downward mass shift, debated issue!
- Very small mass decrease at nuclear density, in agreement with experiments

ground state			
T/c	T (MeV)	μ/c	μ (MeV)
0	0	$0.5k$	$194k$
0.162	63	0	0