The Complete $B \to K^*(\to K\pi)\ell^+\ell^-$ and its Symmetries

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S. Descotes-Genon, M.Ramon, J. Matias, JV In Preparation

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Abstract:

The angular distribution of this 4-body decay provides a wealth of angular observables that can be studied to unravel the short distance dynamics of BSM physics. Many observables have been studied, some better than others.

We show that:

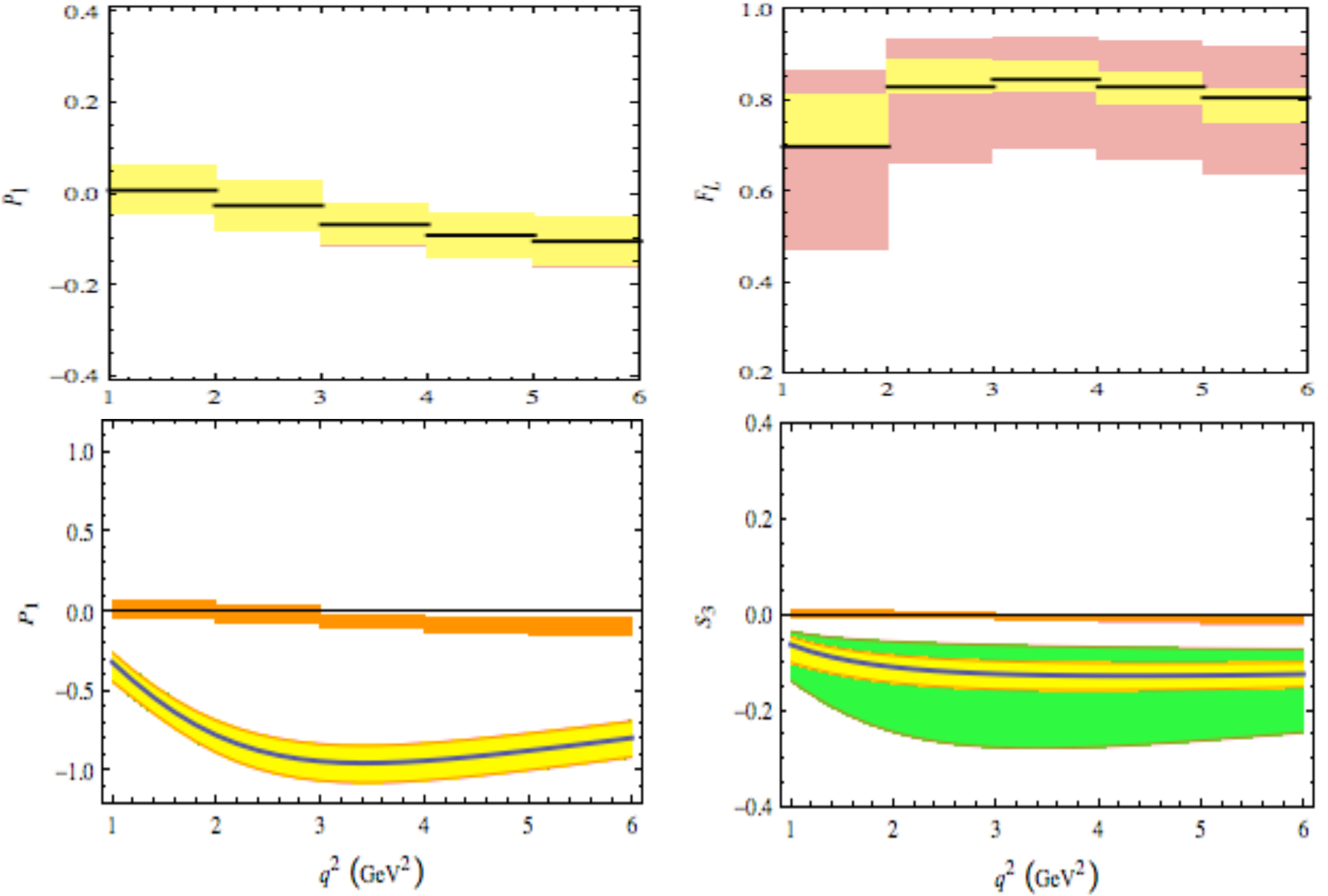
There is a MINIMAL basis of OBSERVABLES, that encode ALL the INFORMATION present in the angular distribution in the MOST EFFICIENT WAY.

This basis is EXHAUSTIVE: All known and unknown observables can be written as a function of this basis. The *theoretically clean observable sector* is singled out.

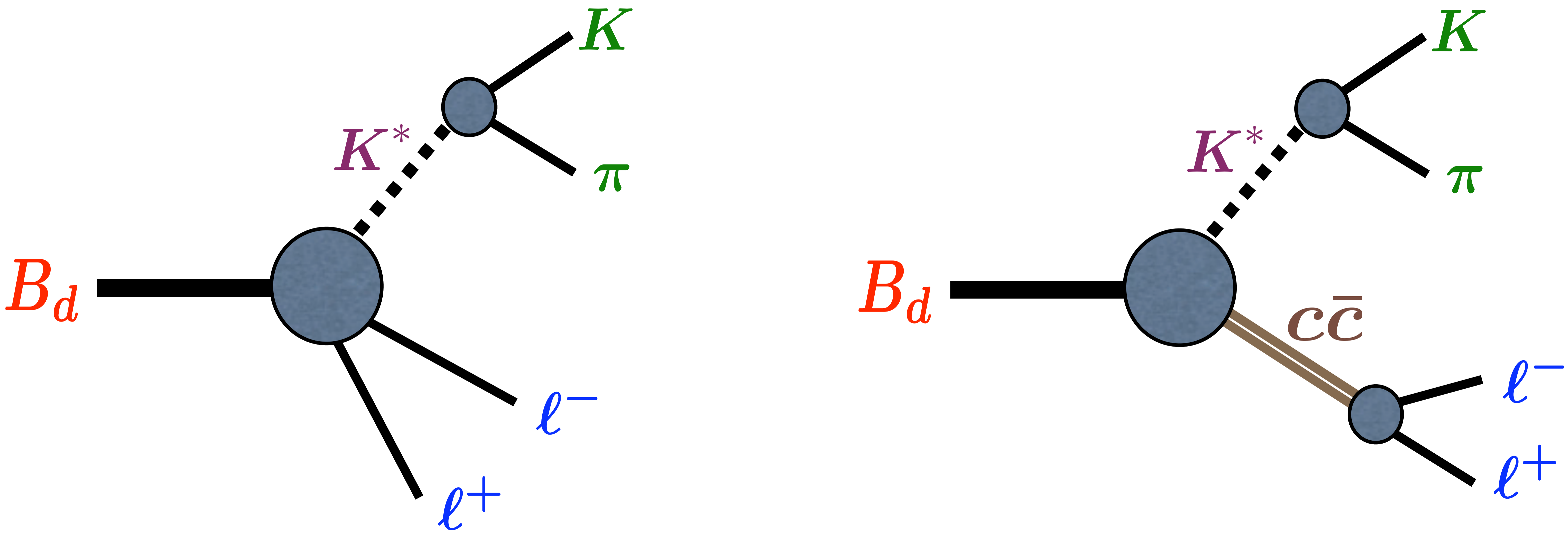
The Complete $B \to K^*(\to K\pi)\ell^+\ell^-$ and its Symmetries

+ Overview
  - Kinematics & Angular Distribution
  - Experiment: Past, Present, Future
  - Theory: Heff, Matrix Elements & Form Factors, Spin Amplitudes
+ Angular Observables
  - Properties of *good* observables
  - Symmetries of the Angular Distribution
  - Construction of an OPTIMAL BASIS OF (angular) OBSERVABLES
  - Sensitivity study of the *Primary Observables*
+ Summary & Outlook: Massive case & Scalars
+ EXTRA: MODEL INDEPENDENT CONSTRAINTS ON NP

A BIG MOTIVATION



OUR FRIEND, THE $\mathrm{B} \to \mathrm{K}^*(\to \mathrm{K}\pi)\ell^+\ell^-$ DECAY



It is a $b \to s$ *penguin process.*

LARGE number of angular observables available experimentally.

Leptons can be electrons, muons or taus. Each has its own pheno.

Also: CP Violation, Isospin asymmetry,... lepton polarizations (future?)

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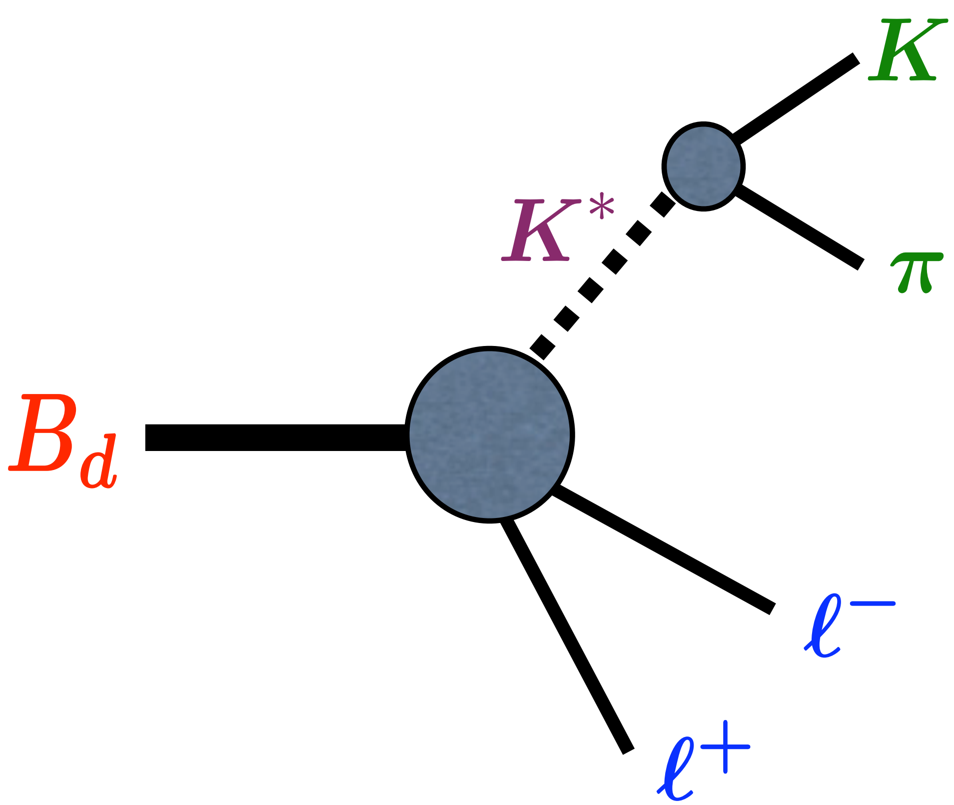

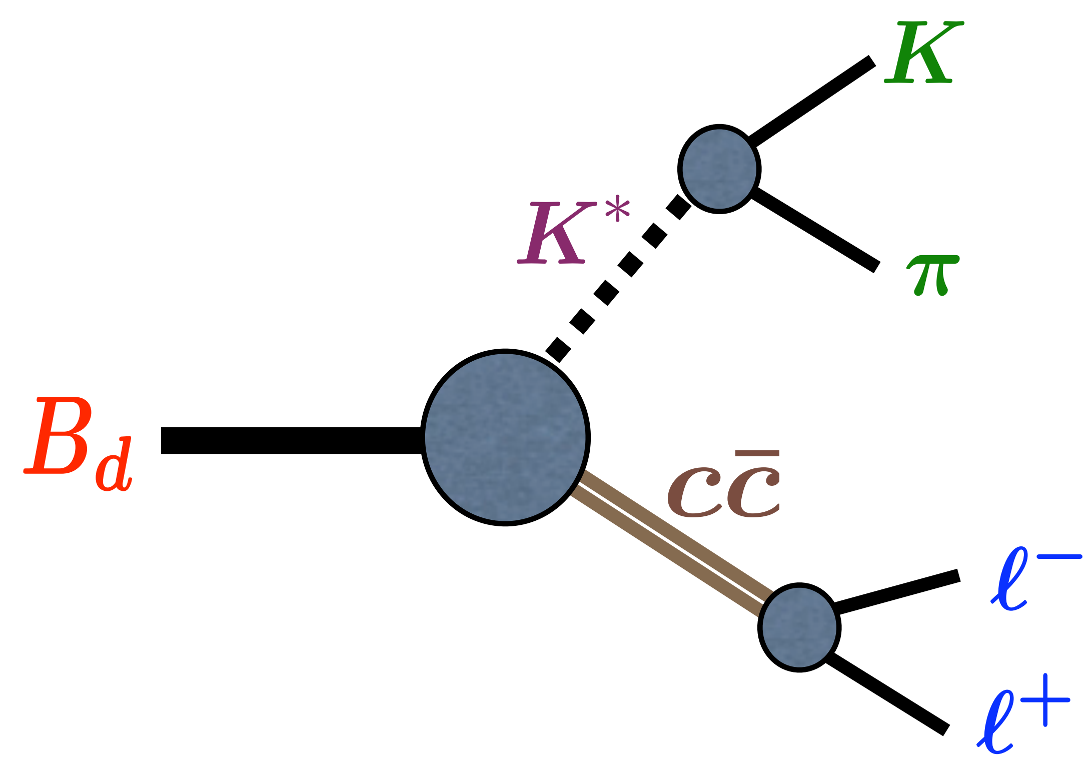

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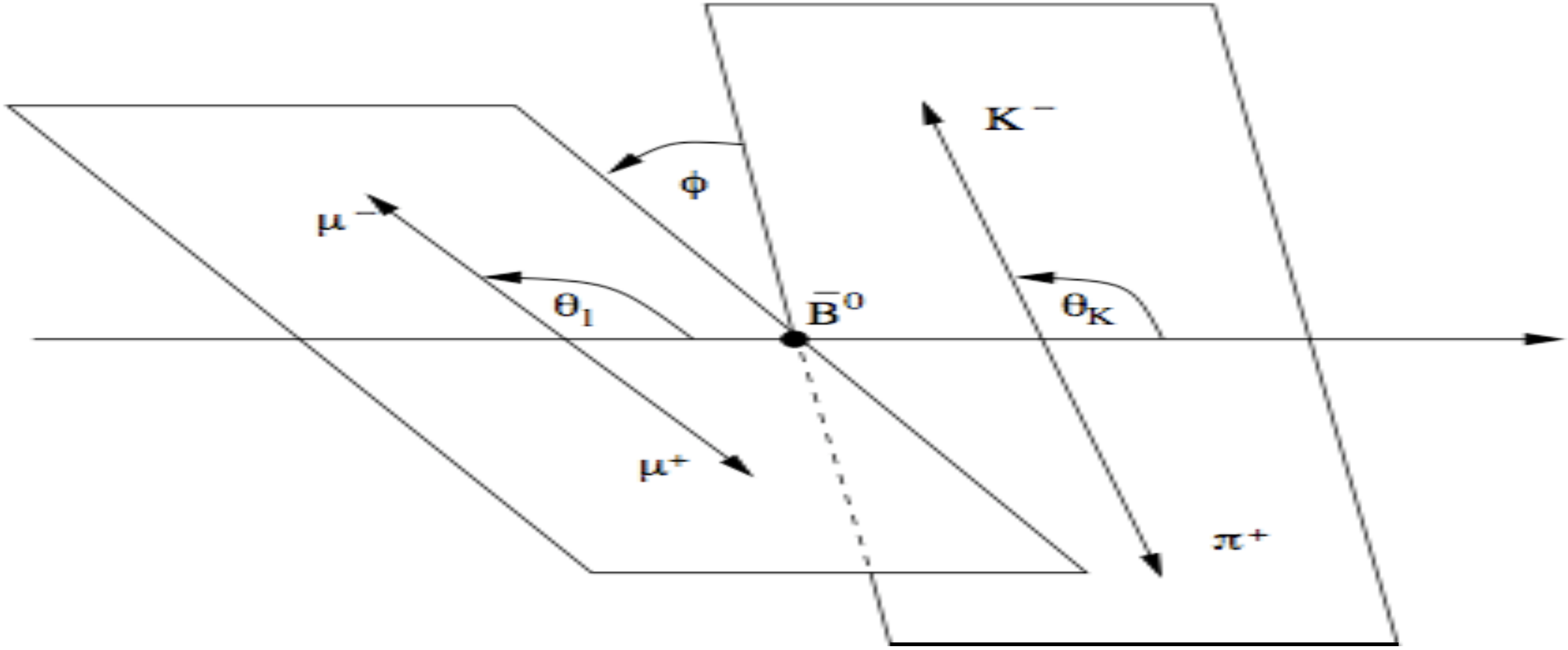
This talk ⟶ Angular Observables

KINEMATICS

The kinematics of the 4 body decay $\mathrm{B} \to \mathrm{K}^*(\to \mathrm{K}\pi)\ell^+\ell^-$ *is descibed by 4 kinematic variables:* q^2, θ_ℓ, θ_K *and* ϕ



The variables are constrained in:

- $0 < \theta_\ell, \theta_K < \pi$
- $0 < \phi < 2\pi$
- $4m_\ell^2 < q^2 < (M_B - M_{K^*})^2$

$q^2 \lesssim 7\,\mathrm{GeV}^2 \longrightarrow$ *Large Recoil*

$14\,\mathrm{GeV}^2 \lesssim q^2 \lesssim 20\,\mathrm{GeV}^2 \longrightarrow$ *Low Recoil*

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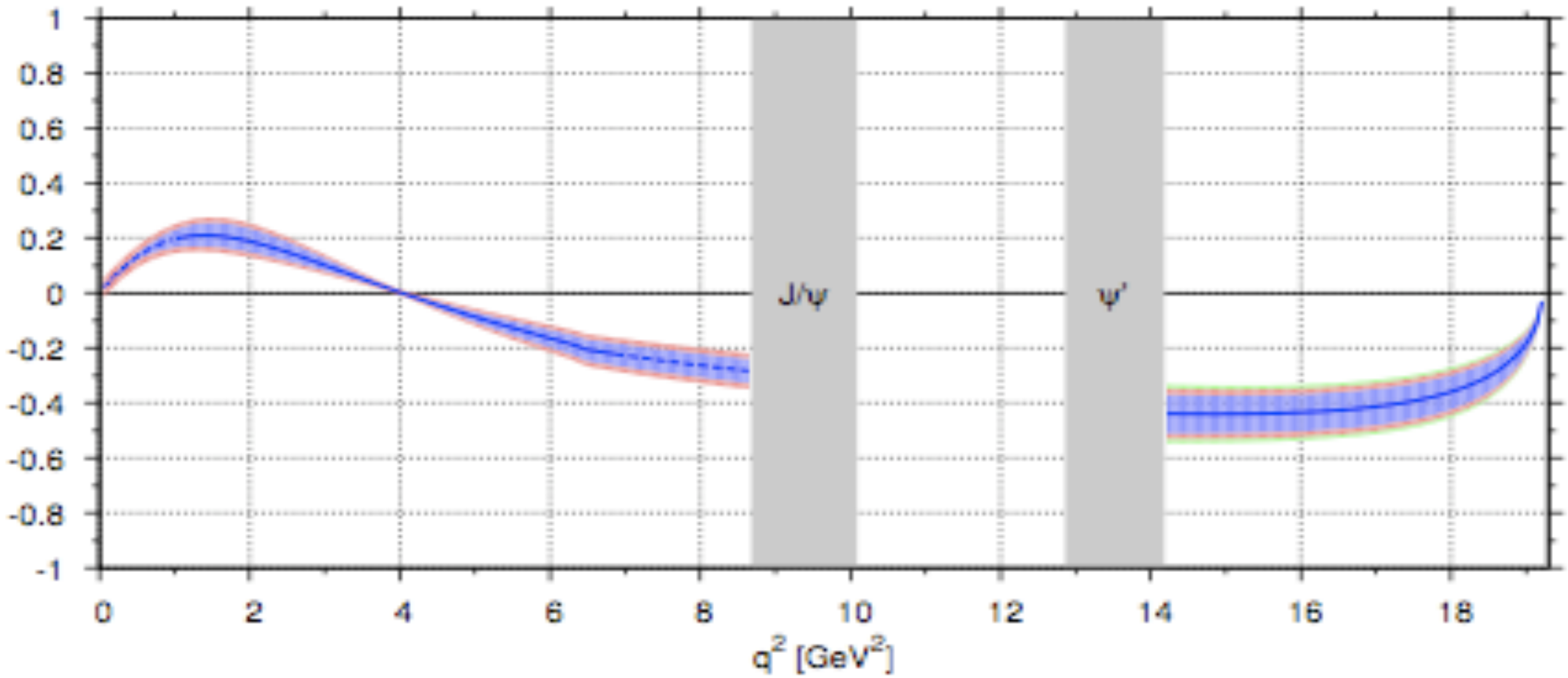

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ANGULAR DISTRIBUTION

The angular distribution of the differential decay rate is

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_K\, d\cos\theta_l\, d\phi} = \frac{9}{32\pi}\Big[J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_l$$

$$+J_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi + J_4\sin 2\theta_K\sin 2\theta_l\cos\phi + J_5\sin 2\theta_K\sin\theta_l\cos\phi$$

$$+(J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_l + J_7\sin 2\theta_K\sin\theta_l\sin\phi + J_8\sin 2\theta_K\sin 2\theta_l\sin\phi$$

$$+J_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\Big]$$

[Kruger et.al. 2000]

The coefficients $J_i(q^2)$ *are functions of the* $\ell^+\ell^-$ *invariant mass squared* q^2

The J's contain all the information available from the angular distribution.

Partial information can also be obtain (better statistics). Examples:

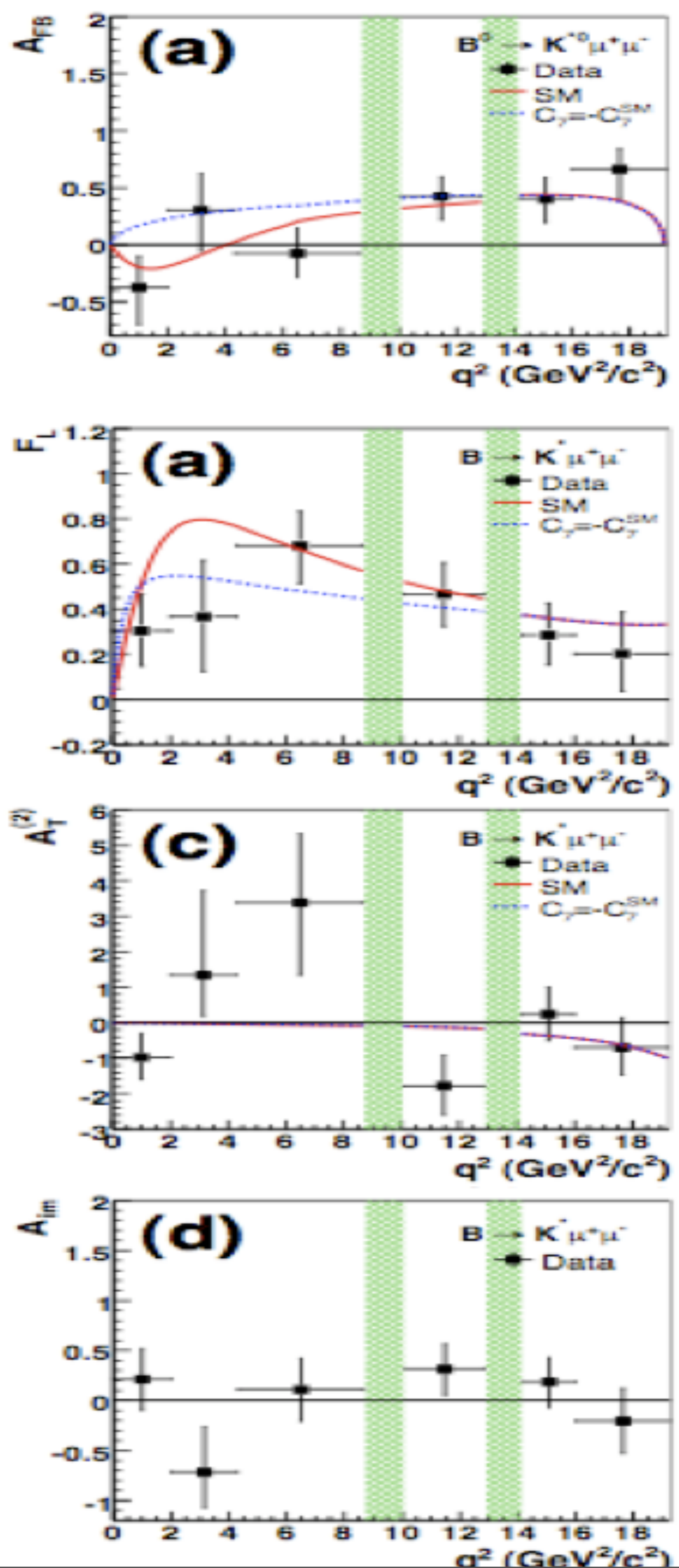
+ q^2 integrated results.

+ partial angular integration: $A_{FB} = \frac{1}{d\Gamma/dq^2}\left[\int_{-1}^{0} - \int_{0}^{1}\right] d\cos\theta_l \frac{d^2\Gamma}{dq^2 d\cos\theta_l} = -\frac{3J_{6s}}{3J_{1c}+6J_{1s}-J_{2c}-2J_{2s}}$.

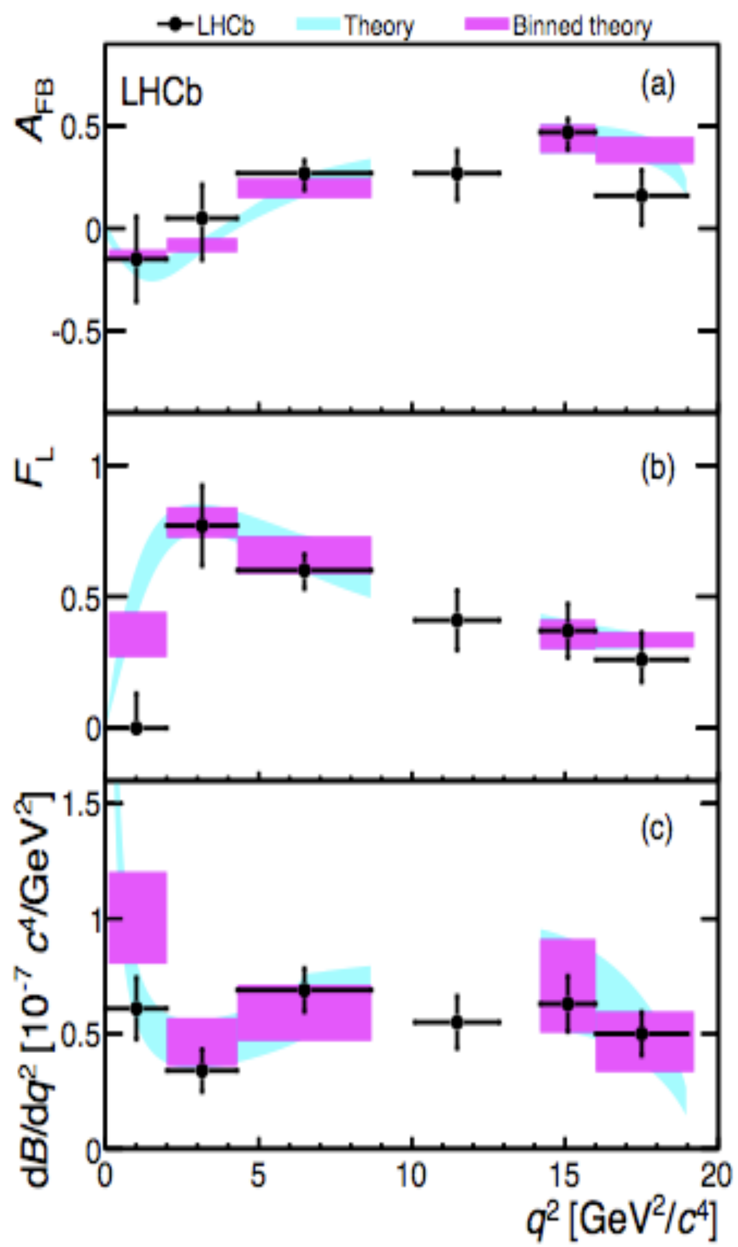
EXPERIMENT $(\mathbf{B} \to \mathbf{K}^* \mu^+ \mu^-)$

 # Up to now, some q^2- dependent observables have been measured (from uni-angular distributions)

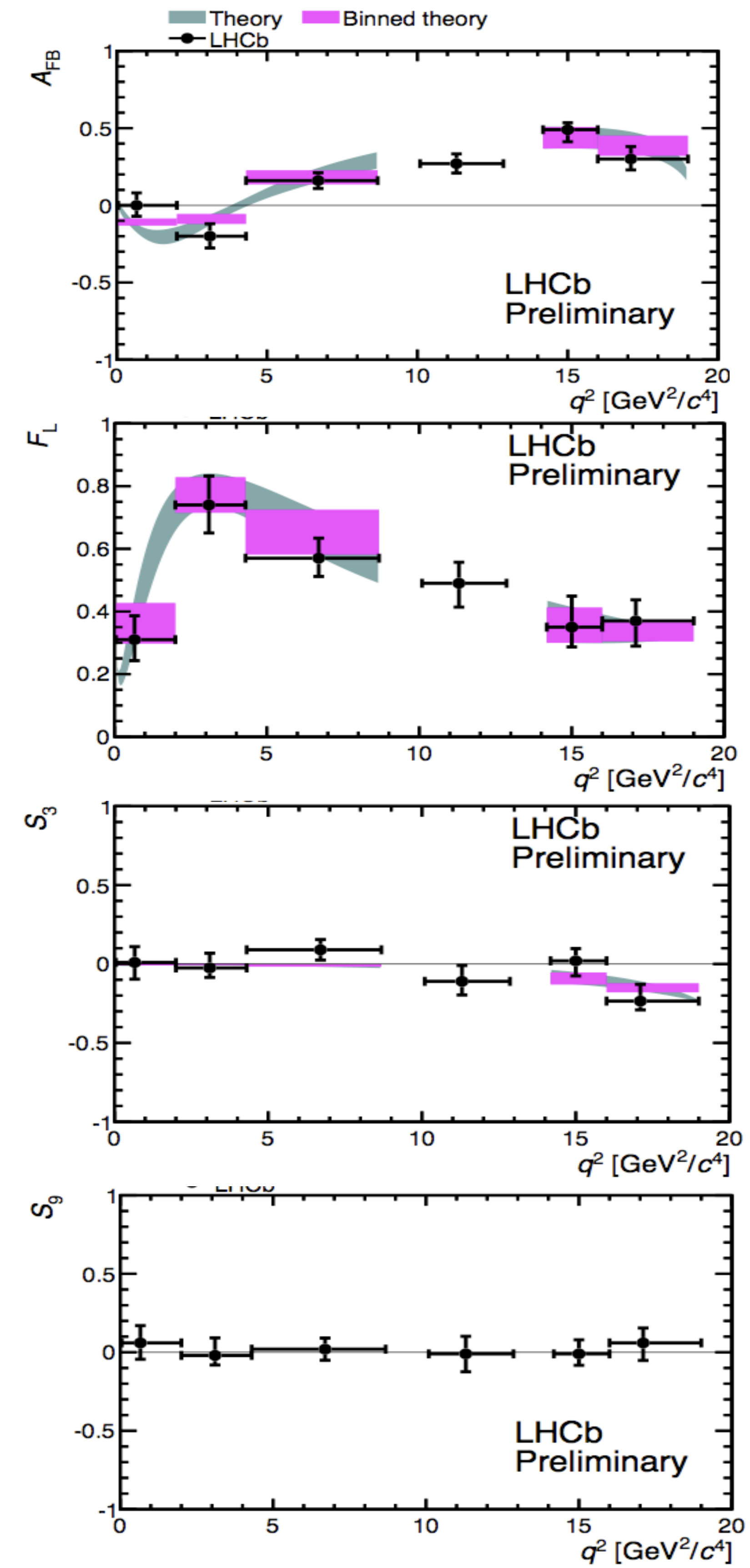
CDF (Aug. 2011)



LHCb (Dec. 2011)



LHCb (Mar. 2012)



EXPERIMENT $(\mathbf{B} \to \mathbf{K}^* \mu^+ \mu^-)$

Up to now, some q^2- dependent observables have been measured (from uni-angular distributions)

Future: Full angular analysis with small q^2 binning

THEORY

The effective Hamiltonian describing the $b \to s l^+ l^-$ transition

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} [\mathbf{C}_\mathbf{i}(\mu)\mathcal{O}_\mathbf{i}(\mu) + \mathbf{C}'_\mathbf{i}(\mu)\mathcal{O}'_\mathbf{i}(\mu)],$$

$\mathbf{C}_\mathbf{i}^{(\prime)}(\mu)$ are Wilson coefficients and $\mathcal{O}_\mathbf{i}^{(\prime)}(\mu)$ are local operators.

We concentrate on *Electromagnetic dipole+ semileptonic operators:*

$$\mathcal{O}_\mathbf{7} = \frac{e}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_R b) F^{\mu\nu}, \quad \mathcal{O}_\mathbf{9} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l),$$

$$\mathcal{O}_\mathbf{10} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu \gamma_5 l),$$

where $P_{L,R} = (1 \mp \gamma_5)/2$ and **primed counterpart operators**

$$\mathcal{O}'_\mathbf{7} = \frac{e}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu}, \quad \mathcal{O}'_\mathbf{9} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_R b)(\bar{l}\gamma^\mu l),$$

$$\mathcal{O}'_\mathbf{10} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_R b)(\bar{l}\gamma^\mu \gamma_5 l),$$

Matrix Elements and Form Factors

$$\langle \bar{K}^*(k)|\bar{s}\gamma_\mu(1-\gamma_5)b|\bar{B}(p)\rangle = \varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho k^\sigma \frac{2V(q^2)}{m_B+m_{K^*}} - i\varepsilon^*_\mu(m_B+m_{K^*})A_1(q^2)$$
$$+i(p+k)_\mu(\varepsilon^*\cdot q)\frac{A_2(q^2)}{m_B+m_{K^*}} + iq_\mu(\varepsilon^*\cdot q)\frac{2m_{K^*}}{q^2}\left[A_3(q^2)-A_0(q^2)\right],$$

$$\langle \bar{K}^*(k)|\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|\bar{B}(p)\rangle = 2i\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho k^\sigma T_1(q^2)$$
$$+\left[\varepsilon^*_\mu(m_B^2-m_{K^*}^2)-(\varepsilon^*\cdot q)\,(2p-q)_\mu\right]T_2(q^2)$$
$$+(\varepsilon^*\cdot q)\left[q_\mu - \frac{q^2}{m_B^2-m_{K^*}^2}(p+k)_\mu\right]T_3(q^2),$$

As: $\mathbf{m_b \to \infty}$ **and** $\mathbf{E_{K^*} \to \infty}$ *(Heavy Quark limit + Large recoil)*

$$\mathbf{FF's \longrightarrow \xi_\parallel(q^2),\ \xi_\perp(q^2)}$$

[Charles et.al. 1999]

+ pertubative corrections in SCET

[Beneke,Feldmann + Bauer et. al. 2001]

THEORY

Spin Amplitudes: *(Heavy Quark limit + Large recoil)*

$$A_\perp^{L,R} = \sqrt{2} N m_B (1-\hat{s}) \left[(C_9^{(\text{eff})} + C_9'^{(\text{eff})}) \mp (C_{10}^{(\text{eff})} + C_{10}'^{(\text{eff})}) + \frac{2\hat{m}_b}{\hat{s}} (C_7^{(\text{eff})} + C_7'^{(\text{eff})}) \right] \xi_\perp(E_{K^*})$$

$$A_\parallel^{L,R} = -\sqrt{2} N m_B (1-\hat{s}) \left[(C_9^{(\text{eff})} - C_9'^{(\text{eff})}) \mp (C_{10}^{(\text{eff})} - C_{10}'^{(\text{eff})}) + \frac{2\hat{m}_b}{\hat{s}} (C_7^{(\text{eff})} - C_7'^{(\text{eff})}) \right] \xi_\perp(E_{K^*})$$

$$A_0^{L,R} = -\frac{N m_B (1-\hat{s})^2}{2\hat{m}_{K^*}} \left[(C_9^{(\text{eff})} - C_9'^{(\text{eff})}) \mp (C_{10}^{(\text{eff})} - C_{10}'^{(\text{eff})}) + \frac{2\hat{m}_b}{\hat{s}} (C_7^{(\text{eff})} - C_7'^{(\text{eff})}) \right] \xi_\parallel(E_{K^*})$$

+ *massive terms:*

$$A_t = \frac{N m_B (1-\hat{s})^2}{2\hat{m}_{K^*}\sqrt{\hat{s}}} \left[2(C_{10}^{(\text{eff})} - C_{10}'^{(\text{eff})}) + \frac{q^2}{m_\ell} (C_P - C_P') \right] \xi_\parallel(E_{K^*})$$

+ *scalar operators:*

$$A_S = -\frac{N m_B^2 (1-\hat{s})^2}{\hat{m}_{K^*}\sqrt{\hat{s}}} \left[C_S - C_S' \right] \xi_\parallel(E_{K^*})$$

$$\hat{s} \equiv q^2/m_B^2$$
$$\hat{m}_i \equiv m_i/m_B$$
$$N = \text{Normalization}$$

THEORY

The coefficients J_i *can be written in terms of the Spin Amplitudes:*

$$J_{1s} = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\perp^R|^2 + |A_\parallel^R|^2\right] + \frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R^*} + A_\parallel^L A_\parallel^{R^*}\right) ,$$

$$J_{1c} = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_\ell^2}{q^2}\left[|A_t|^2 + 2\mathrm{Re}(A_0^L A_0^{R^*})\right] + \beta_\ell^2\, |A_S|^2 ,$$

$$J_{2s} = \frac{\beta_\ell^2}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\perp^R|^2 + |A_\parallel^R|^2\right] , \qquad J_{2c} = -\beta_\ell^2\left[|A_0^L|^2 + |A_0^R|^2\right] ,$$

$$J_3 = \frac{1}{2}\beta_\ell^2\left[|A_\perp^L|^2 - |A_\parallel^L|^2 + |A_\perp^R|^2 - |A_\parallel^R|^2\right] , \qquad J_4 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Re}(A_0^L A_\parallel^{L^*} + A_0^R A_\parallel^{R^*})\right] ,$$

$$J_5 = \sqrt{2}\beta_\ell\left[\mathrm{Re}(A_0^L A_\perp^{L^*} - A_0^R A_\perp^{R^*}) - \frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_\parallel^L A_S^* + A_\parallel^{R^*} A_S)\right] ,$$

$$J_{6s} = 2\beta_\ell\left[\mathrm{Re}(A_\parallel^L A_\perp^{L^*} - A_\parallel^R A_\perp^{R^*})\right] , \qquad J_{6c} = 4\beta_\ell\frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_0^L A_S^* + A_0^{R^*} A_S) ,$$

$$J_7 = \sqrt{2}\beta_\ell\left[\mathrm{Im}(A_0^L A_\parallel^{L^*} - A_0^R A_\parallel^{R^*}) + \frac{m_\ell}{\sqrt{q^2}}\mathrm{Im}(A_\perp^L A_S^* - A_\perp^{R^*} A_S)\right] ,$$

$$J_8 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Im}(A_0^L A_\perp^{L^*} + A_0^R A_\perp^{R^*})\right] , \qquad J_9 = \beta_\ell^2\left[\mathrm{Im}(A_\parallel^{L^*} A_\perp^L + A_\parallel^{R^*} A_\perp^R)\right] ,$$

OBSERVABLES

From the Theory point of view we look for observables that:

1. Do not suffer from large hadronic uncertainties.

2. Have an enhanced sensitivity to the presence of physics BSM.

3. Can be extracted from the angular distribution.

OBSERVABLES

1. Do not suffer from large hadronic uncertainties.

Build Observables that do NOT depend on the Soft Form Factors $\xi_\parallel, \xi_\perp$

Examples:

- $$\frac{d\Gamma}{dq^2} = \int d\cos\theta_l \, d\cos\theta_K d\phi \frac{d^4\Gamma}{dq^2 \, d\cos\theta_K \, d\cos\theta_l \, d\phi} = \frac{1}{4}(3J_{1c} + 6J_{1s} - J_{2c} - 2J_{2s}) \quad \propto \xi_\parallel, \xi_\perp$$ ✗

- $$A_{\rm FB} = \frac{1}{d\Gamma/dq^2}\left[\int_{-1}^{0} - \int_{0}^{1}\right] d\cos\theta_l \frac{d^2\Gamma}{dq^2 d\cos\theta_l} = -\frac{3J_{6s}}{3J_{1c} + 6J_{1s} - J_{2c} - 2J_{2s}} \quad \propto \xi_\perp/\xi_\parallel$$ ✗

- $$A_T^{(1)} = \frac{\Gamma_- - \Gamma_+}{\Gamma_- + \Gamma_+} = \frac{-2{\rm Re}(A_\parallel A_\perp^*)}{|A_\perp|^2 + |A_\parallel|^2}. \quad \propto \xi_\perp/\xi_\perp = 1$$ ✓ [Melikhov et.al. 1998]

- $$A_T^{(2)} = \frac{|A_\perp|^2 - |A_\parallel|^2}{|A_\perp|^2 + |A_\parallel|^2}. \quad \propto \xi_\perp/\xi_\perp = 1$$ ✓ [Kruger, Matias 2005]

OBSERVABLES

1. Do not suffer from large hadronic uncertainties.

Build Observables that do NOT depend on the Soft Form Factors $\xi_\parallel, \xi_\perp$

Examples:

- $$\frac{d\Gamma}{dq^2} = \int d\cos\theta_l \, d\cos\theta_K d\phi \, \frac{d^4\Gamma}{dq^2 \, d\cos\theta_K \, d\cos\theta_l \, d\phi} = \frac{1}{4}(3J_{1c} + 6J_{1s} - J_{2c} - 2J_{2s}) \propto \xi_\parallel, \xi_\perp$$

- $$A_{FB} = \frac{1}{d\Gamma/dq^2}\left[\int_{-1}^{0} - \int_{0}^{1}\right] d\cos\theta_l \frac{d^2\Gamma}{dq^2 d\cos\theta_l} = -\frac{3J_{6s}}{3J_{1c} + 6J_{1s} - J_{2c} - 2J_{2s}} \propto \xi_\perp / \xi_\parallel$$

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  [Melikhov et.al. 1998]

- $$A_T^{(2)} = \frac{|A_\perp|^2 - |A_\parallel|^2}{|A_\perp|^2 + |A_\parallel|^2}. \propto \xi_\perp / \xi_\perp = 1$$

  [Kruger, Matias 2005]

2. Enhanced sensitivity to RH currents....

OBSERVABLES

3. Can be extracted from the angular distribution.

- $$A_T^{(2)} = \frac{|A_\perp|^2 - |A_\parallel|^2}{|A_\perp|^2 + |A_\parallel|^2}.$$ $\rightarrow$

$$J_3 = \frac{1}{2}\beta_\ell(|A_\perp|^2 - |A_\parallel|^2)$$

$$J_{2s} = \frac{1}{4}\beta_\ell(|A_\perp|^2 + |A_\parallel|^2)$$

$\rightarrow$ $$A_T^{(2)} = \frac{J_3}{2J_{2s}}$$

- $$A_T^{(1)} = \frac{\Gamma_- - \Gamma_+}{\Gamma_- + \Gamma_+} = \frac{-2\mathrm{Re}(A_\parallel A_\perp^*)}{|A_\perp|^2 + |A_\parallel|^2}.$$

OBSERVABLES

3. Can be extracted from the angular distribution.

- $A_T^{(2)} = \dfrac{|A_\perp|^2 - |A_\parallel|^2}{|A_\perp|^2 + |A_\parallel|^2}$. → $J_3 = \dfrac{1}{2}\beta_\ell(|A_\perp|^2 - |A_\parallel|^2)$, $J_{2s} = \dfrac{1}{4}\beta_\ell(|A_\perp|^2 + |A_\parallel|^2)$ → $A_T^{(2)} = \dfrac{J_3}{2J_{2s}}$ ✓

- $A_T^{(1)} = \dfrac{\Gamma_- - \Gamma_+}{\Gamma_- + \Gamma_+} = \dfrac{-2\mathrm{Re}(A_\parallel A_\perp^*)}{|A_\perp|^2 + |A_\parallel|^2}$. ✗

How can we know ?

Can't we just invert the equations J(A) and write this obs. in terms of the J's ?

OBSERVABLES

3. Can be extracted from the angular distribution.

- $$A_T^{(2)} = \frac{|A_\perp|^2 - |A_\parallel|^2}{|A_\perp|^2 + |A_\parallel|^2}. \rightarrow \begin{array}{l} J_3 = \frac{1}{2}\beta_\ell(|A_\perp|^2 - |A_\parallel|^2) \\ J_{2s} = \frac{1}{4}\beta_\ell(|A_\perp|^2 + |A_\parallel|^2) \end{array} \rightarrow A_T^{(2)} = \frac{J_3}{2J_{2s}}$$ ✓

- $$A_T^{(1)} = \frac{\Gamma_- - \Gamma_+}{\Gamma_- + \Gamma_+} = \frac{-2\mathrm{Re}(A_\parallel A_\perp^*)}{|A_\perp|^2 + |A_\parallel|^2}.$$ ✗

How can we know ?

Can't we just invert the equations J(A) and write this obs. in terms of the J's ?

NO ⇨ ***There are transformations (SYMMETRIES) among the A's that leave invariant the angular distribution***

[Egede, Hurth, Matias, Ramón, Reece 2008]

OBSERVABLES

For example the tranformation:

$$
\begin{aligned}
A'_{\perp L} &= +\cos\theta A_{\perp L} + \sin\theta A^*_{\perp R} \\
A'_{\perp R} &= -\sin\theta A^*_{\perp L} + \cos\theta A_{\perp R} \\
A'_{0L} &= +\cos\theta A_{0L} - \sin\theta A^*_{0R} \\
A'_{0R} &= +\sin\theta A^*_{0L} + \cos\theta A_{0R} \\
A'_{\parallel L} &= +\cos\theta A_{\parallel L} - \sin\theta A^*_{\parallel R} \\
A'_{\parallel R} &= +\sin\theta A^*_{\parallel L} + \cos\theta A_{\parallel R} \,.
\end{aligned}
$$

Leaves invariant the angular distribution: $J'_i = J_i$

BUT: $A_T^{(1)} \to A_T^{(1)\prime} = \frac{-2\mathrm{Re}(A'_\parallel A'^*_\perp)}{|A'_\perp|^2 + |A'_\parallel|^2} = \frac{-2\mathrm{Re}(A_\parallel A^*_\perp)}{|A_\perp|^2 + |A_\parallel|^2} - \frac{4\mathrm{Re}(A^L_\parallel A^R_\perp - A^R_\parallel A^L_\perp)}{|A_\perp|^2 + |A_\parallel|^2}\theta + \cdots$

$A_T^{(1)}$ cannot be extracted from the angular distribution ✗

On the other hand, $A_T^{(2)} = \frac{J_3}{2J_{2s}}$ CAN be extracted from the A.D. ✓

SYMMETRIES and EXPERIMENTAL d.o.f.

Suppose there are:

n_A complex spin amplitudes ($2n_A$ real theoretical parameters)

n_s transformations among the A's (symmetries) that leave invariant the J's.

The number of independent experimental degrees of freedom is:

$$n_{\text{exp}} = 2n_A - n_s$$

n_{exp} *independent observables*

<u>*Corollary:*</u> If the number n_{J} of coefficients $J_i(q^2)$ is greater than n_{exp} then the coefficients are NOT independent:

$$n_{\text{exp}} = 2n_A - n_s = n_{\text{J}} - n_r$$

where: n_r - number of relationships between the J's

SYMMETRIES (massless case)

There are $n_s = 4$ *symmetries, which can be written as:*

- complex vectors: $n_\parallel = \begin{pmatrix} A_\parallel^L \\ A_\parallel^{R*} \end{pmatrix}, \quad n_\perp = \begin{pmatrix} A_\perp^L \\ -A_\perp^{R*} \end{pmatrix}, \quad n_0 = \begin{pmatrix} A_0^L \\ A_0^{R*} \end{pmatrix}$

- Symmetries: $$n_i' = \begin{bmatrix} e^{i\phi_L} & 0 \\ 0 & e^{-i\phi_R} \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cosh i\tilde{\theta} & -\sinh i\tilde{\theta} \\ -\sinh i\tilde{\theta} & \cosh i\tilde{\theta} \end{bmatrix} n_i$$

There are $n_A = 6$ *complex amplitudes.*

$$n_{\exp} = 2n_A - n_s = n_J - n_r \quad \Rightarrow \quad n_{\exp} = 12 - 4 = 12 - n_r$$

Conclusions:

- There are $n_{\exp} = 8$ independent angular observables.
- There are $n_r = 4$ relationships between the $J_i(q^2)$

Questions:

What are the 4 relationships between the J's ?

What is the best choice for the 8 independent observables ?

SYMMETRIES (massless case)

4 Relationships between the J's (massless: $\beta_\ell \to 1$)

$$J_{1s} = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\perp^R|^2 + |A_\parallel^R|^2\right] + \cancel{\frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R*} + A_\parallel^L A_\parallel^{R*}\right)}\ ,$$

$$J_{1c} = |A_0^L|^2 + |A_0^R|^2 + \cancel{\frac{4m_\ell^2}{q^2}\left[|A_t|^2 + 2\mathrm{Re}(A_0^L A_0^{R*})\right] + \beta_\ell^2 |A_S|^2}\ ,$$

$$J_{2s} = \frac{\beta_\ell^2}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\perp^R|^2 + |A_\parallel^R|^2\right], \qquad J_{2c} = -\beta_\ell^2\left[|A_0^L|^2 + |A_0^R|^2\right],$$

$$J_3 = \frac{1}{2}\beta_\ell^2\left[|A_\perp^L|^2 - |A_\parallel^L|^2 + |A_\perp^R|^2 - |A_\parallel^R|^2\right], \qquad J_4 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Re}(A_0^L A_\parallel^{L*} + A_0^R A_\parallel^{R*})\right],$$

$$J_5 = \sqrt{2}\beta_\ell\left[\mathrm{Re}(A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*}) - \cancel{\frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_\parallel^L A_S^* + A_\parallel^{R*} A_S)}\right],$$

$$J_{6s} = 2\beta_\ell\left[\mathrm{Re}(A_\parallel^L A_\perp^{L*} - A_\parallel^R A_\perp^{R*})\right], \qquad J_{6c} = \cancel{4\beta_\ell\frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_0^L A_S^* + A_0^{R*} A_S)}\ ,$$

$$J_7 = \sqrt{2}\beta_\ell\left[\mathrm{Im}(A_0^L A_\parallel^{L*} - A_0^R A_\parallel^{R*}) + \cancel{\frac{m_\ell}{\sqrt{q^2}}\mathrm{Im}(A_\perp^L A_S^* - A_\perp^{R*} A_S)}\right],$$

$$J_8 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Im}(A_0^L A_\perp^{L*} + A_0^R A_\perp^{R*})\right], \qquad J_9 = \beta_\ell^2\left[\mathrm{Im}(A_\parallel^{L*} A_\perp^L + A_\parallel^{R*} A_\perp^R)\right],$$

SYMMETRIES (massless case)

4 Relationships between the J's (massless: $\beta_\ell \to 1$)

$$J_{1s} = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\perp^R|^2 + |A_\parallel^R|^2\right] + \cancel{\frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R*} + A_\parallel^L A_\parallel^{R*}\right)} \,,$$

$$J_{1c} = |A_0^L|^2 + |A_0^R|^2 + \cancel{\frac{4m_\ell^2}{q^2}\left[|A_t|^2 + 2\mathrm{Re}(A_0^L A_0^{R*})\right] + \beta_\ell^2 |A_S|^2} \,,$$

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Where's the 4th relationship ??

SYMMETRIES (massless case)

We construct the following products:

$$|n_{\|}|^2 = |A_{\|}^L|^2 + |A_{\|}^R|^2 = \frac{2J_{2s} - J_3}{\beta_\ell^2}\,, \quad n_{\perp}^{\dagger}\, n_{\|} = A_{\perp}^{L*} A_{\|}^L - A_{\perp}^R A_{\|}^{R*} = \frac{\beta_\ell J_{6s} - 2iJ_9}{2\beta_\ell^2}\,,$$

$$|n_{\perp}|^2 = |A_{\perp}^L|^2 + |A_{\perp}^R|^2 = \frac{2J_{2s} + J_3}{\beta_\ell^2}\,, \quad n_0^{\dagger}\, n_{\|} = A_0^{L*} A_{\|}^L + A_0^R A_{\|}^{R*} = \frac{2J_4 - i\beta_\ell J_7}{\sqrt{2}\beta_\ell^2}\,,$$

$$|n_0|^2 = |A_0^L|^2 + |A_0^R|^2 = -\frac{J_{2c}}{\beta_\ell^2}\,, \quad n_0^{\dagger}\, n_{\perp} = A_0^{L*} A_{\perp}^L - A_0^R A_{\perp}^{R*} = \frac{\beta_\ell J_5 - 2iJ_8}{\sqrt{2}\beta_\ell^2}\,.$$

These 9 quantities respect the symmetries of the angular distribution.

These relations can be inverted [easy to find J(n.n)]

Three complex vectors $n_0, n_{\|}, n_{\perp}$ *<u>ALWAYS</u> satisfy the equation:*

$$\left|(n_{\|}^{\dagger}\, n_{\perp})|n_0|^2 - (n_{\|}^{\dagger}\, n_0)(n_0^{\dagger}\, n_{\perp})\right|^2 = (|n_0|^2|n_{\|}|^2 - |n_0^{\dagger}\, n_{\|}|^2)(|n_0|^2|n_{\perp}|^2 - |n_0^{\dagger}\, n_{\perp}|^2)$$

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Translated to the J's, we find the ***<u>4th RELATIONSHIP</u>****:*

$$J_{2c} = -6\,\frac{(2J_{2s}+J_3)\left(4J_4^2+\beta_\ell^2 J_7^2\right)+(2J_{2s}-J_3)\left(\beta_\ell^2 J_5^2+4J_8^2\right)}{16J_{2s}^2-3\left(4J_3^2+\beta_\ell^2 J_{6s}^2+4J_9^2\right)} + 12\,\frac{\beta_\ell^2 J_{6s}(J_4 J_5+J_7 J_8)+J_9(\beta_\ell^2 J_5 J_7-4J_4 J_8)}{16J_{2s}^2-3\left(4J_3^2+\beta_\ell^2 J_{6s}^2+4J_9^2\right)}$$

This relationship is <u>PRESERVED</u> in the massive case.

[Egede, Hurth, Matias, Ramón, Reece 2010]

OPTIMAL SET of OBSERVABLES (massless case)

Recapitulation:

1. A complete set of independent observables contains EXACTLY 8 observables (a basis).

2. These observables should be extracted from the angular distribution.

   ⟶ Expressible in terms of the J's (respect the symmetries)

3. These observables should be THEORETICALLY CLEAN.

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$$n_0 \propto \xi_{\|} \quad \textbf{and} \quad n_{\|}, n_{\perp} \propto \xi_{\perp}$$

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- These primary observables satisfy all the requirements for *good observables*

- This set is COMPLETE: Any *good observable is a function of the P's*

REDISCOVERING KNOWN OBSERVABLES

1. Many *theoretically unpleasant* observables have been studied:

$$A_{FB},\ d\Gamma/dq^2,\ F_L,\ F_T,\ J_i,\ \ldots$$

2. All these can be expressed in terms of the 8 observables $O = \left\{ \frac{d\Gamma}{dq^2}, A_{FB}, P_1, P_2, P_3, P_4, P_5, P_6 \right\}$

$$F_T = 1 - F_L = -\frac{2\beta_\ell}{3} \frac{A_{\rm FB}}{P_2}$$

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3. Many ***excellent*** observables have been studied:

[Kruger, Matias 2005, Egede et.al. 2008,2010, Becirevic,Schneider 2011, Bobeth, et.al, 2010]

$$A_T^{(2,3,4,5)},\ A_T^{(\mathrm{re,im})},\ H_T^{(1,2,3)},\ \ldots$$

4. All these can be expressed in terms of the 6 primary observables $P_1, P_2, P_3, P_4, P_5, P_6$

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$$A_T^{(2)} = P_1\,, \qquad A_T^{(5)} = \tfrac{1}{2}\sqrt{1 - P_1^2 - 4P_2^2 - 4P_3^2}\,,$$

$$A_T^{(\rm re)} = 2P_2 \qquad A_T^{(\rm im)} = 2P_3$$

$$H_T^{(1)} = P_4 \qquad H_T^{(2)} = P_5$$

$$A_T^{(3)} = \sqrt{\frac{4J_4^2 + J_7^2}{-2J_{2c}(2J_{2s} + J_3)}} \qquad A_T^{(4)} = \sqrt{\frac{4J_8^2 + J_5^2}{4J_4^2 + J_7^2}}$$

THE FULL DISTRIBUTION FROM THE *BASIS*

$$J_{1s} = \frac{3}{4} F_T \frac{d\Gamma}{dq^2} \qquad J_{2s} = \frac{1}{4} F_T \frac{d\Gamma}{dq^2} \qquad J_{6s} = 2 P_2 F_T \frac{d\Gamma}{dq^2}$$

$$J_{1c} = F_L \frac{d\Gamma}{dq^2} \qquad J_{2c} = -F_L \frac{d\Gamma}{dq^2} \qquad J_9 = -P_3 F_T \frac{d\Gamma}{dq^2}$$

$$J_3 = \frac{1}{2} P_1 F_T \frac{d\Gamma}{dq^2}$$

$$J_4 = \frac{1}{2} P_4' \sqrt{F_T F_L} \frac{d\Gamma}{dq^2}$$

$$J_5 = P_5' \sqrt{F_T F_L} \frac{d\Gamma}{dq^2}$$

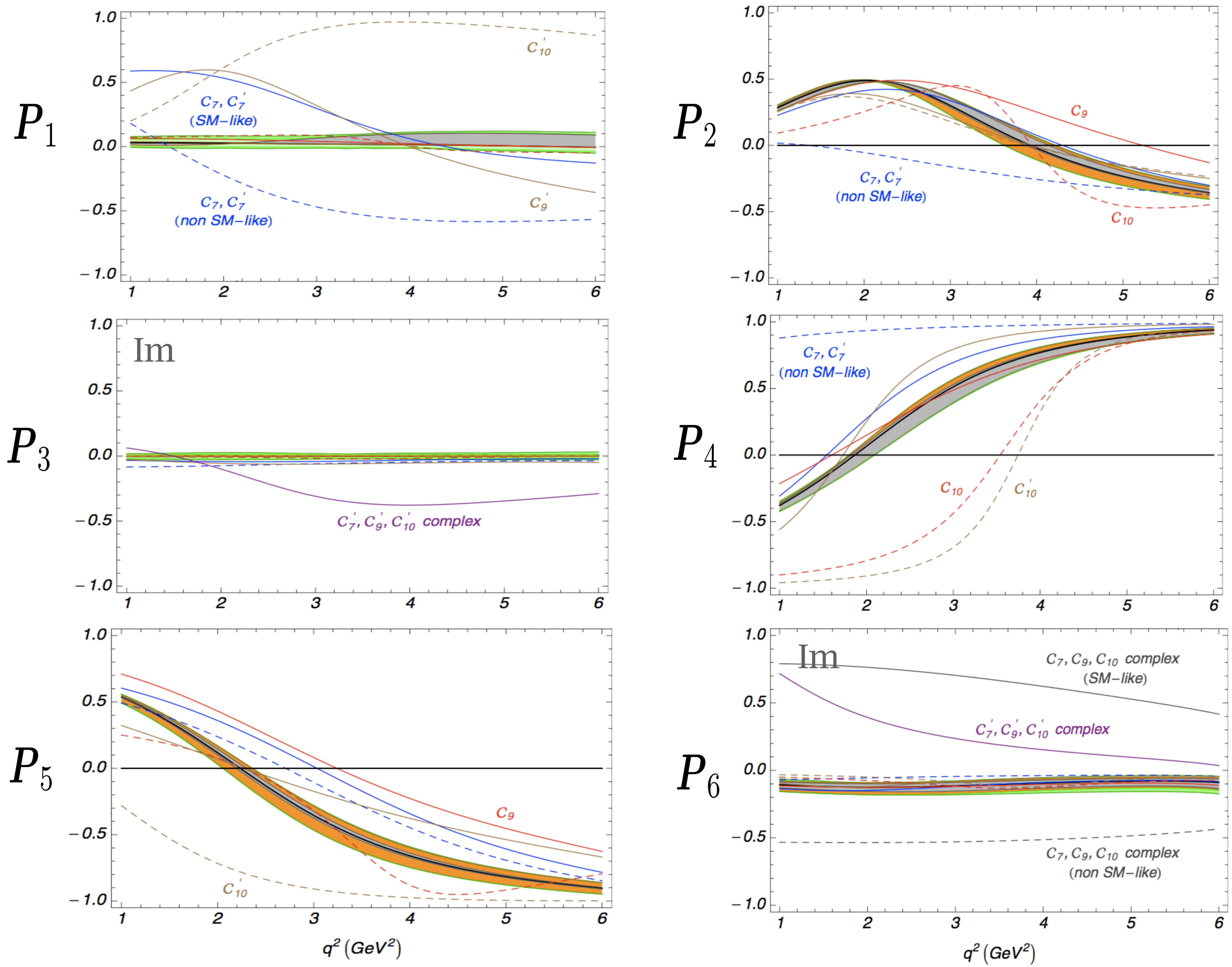
$$J_7 = -P_6' \sqrt{F_T F_L} \frac{d\Gamma}{dq^2}$$

4th rel.

↓

J_8

NP SENSITIVITY of the PRIMARY OBSERVABLES



SUMMARY (Symmetries)

1. The angular distribution of the 4-body decay $\mathrm{B} \to \mathrm{K}^*(\to \mathrm{K}\pi)\ell^+\ell^-$ is described by a number $n_J = 12$ of coefficients $J_i(q^2)$

2. The theoretical and experimental degrees of freedom have to match:

$$n_{\rm exp} = 2n_A - n_s = n_{\rm J} - n_r$$

3. This equation specifies the exact number of independent observables that can be extracted from the angular distribution.

4. *Good* observables must satisfy 3 requirements: 1) Small hadronic uncertainties 2) Sensitivity to NP and 3) Respect the symmetries of the angular distribution.

5. The best one can do is $n_{\rm exp} - 2$ theoretically clean observables.

6. There is a prescription to *construct* such observables:

Massless case: $$O = \left\{ \frac{d\Gamma}{dq^2}, A_{FB}, P_1, P_2, P_3, P_4, P_5, P_6 \right\}$$

7. The same can be done to include masses and scalars.

MODEL-INDEPENDENT CONSTRAINTS

ON: **C7, C7', C9, C10, C9', C10'**

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_R b) F^{\mu\nu}, \quad \mathcal{O}_9 = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l),$$

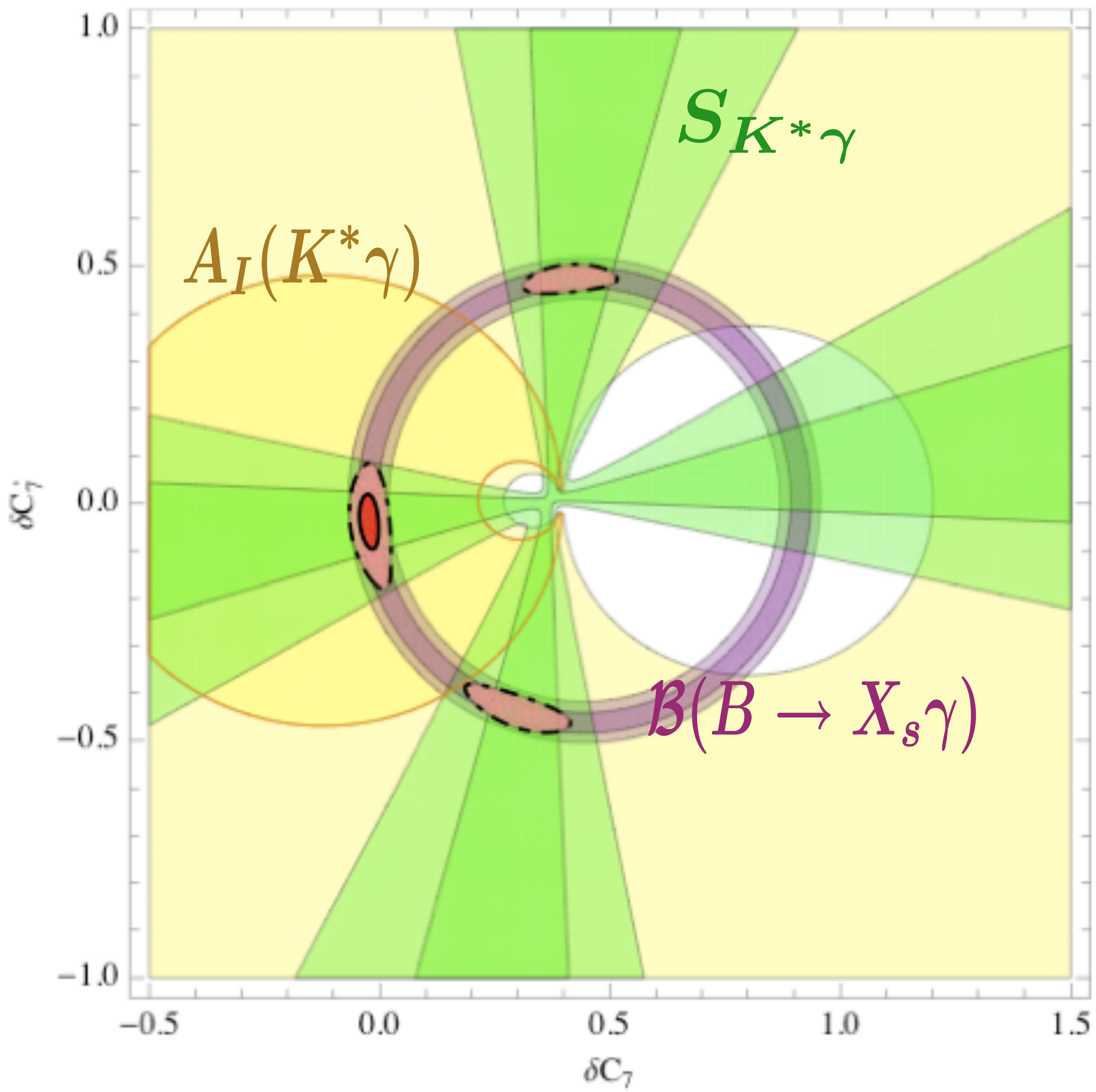
$$\mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu \gamma_5 l),$$

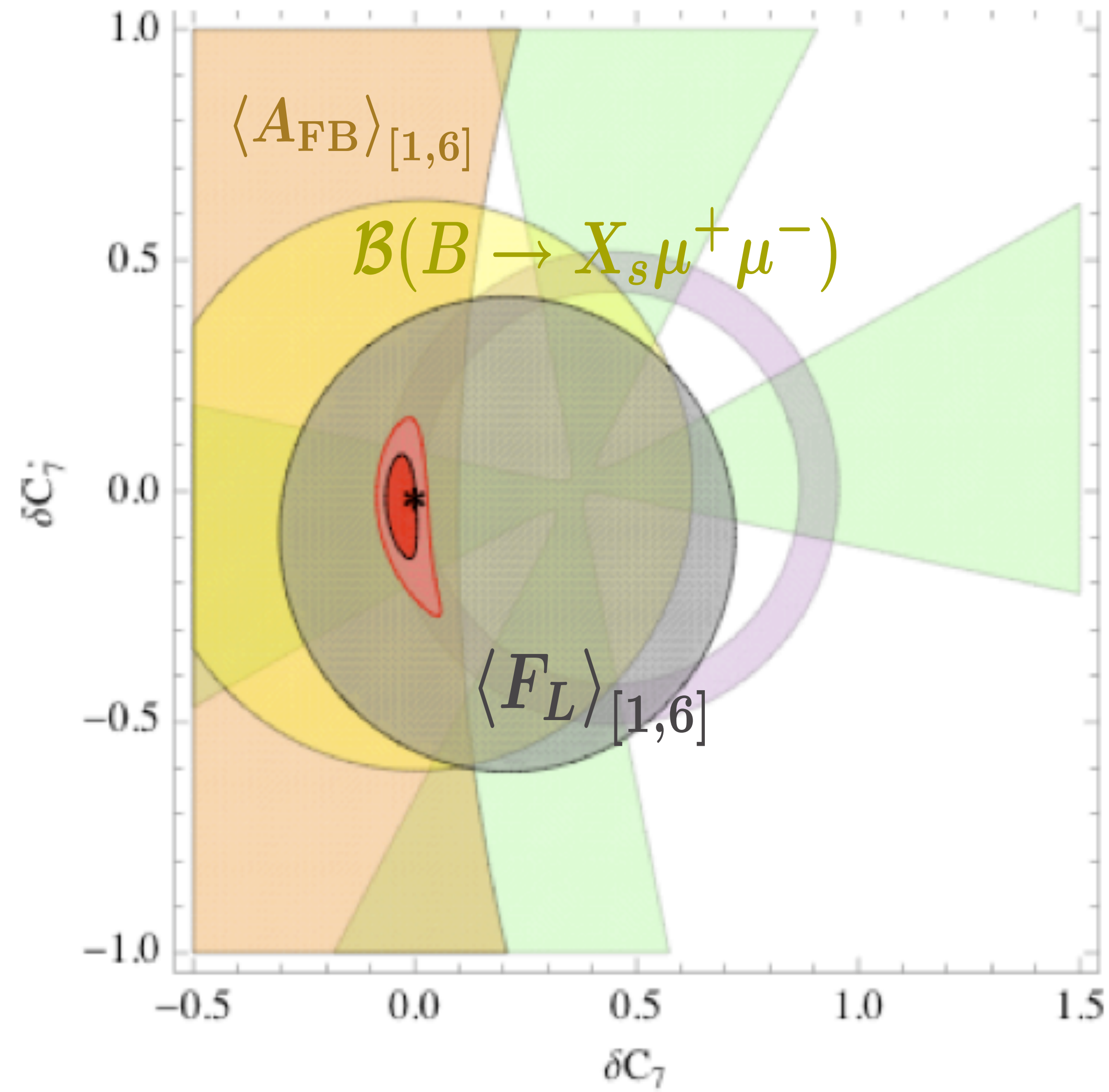
$$\mathcal{O}'_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu}, \quad \mathcal{O}'_9 = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_R b)(\bar{l}\gamma^\mu l),$$

$$\mathcal{O}'_{10} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_R b)(\bar{l}\gamma^\mu \gamma_5 l),$$

MODEL-INDEPENDENT CONSTRAINTS

1. Constraints on **C7, C7'** (all other NP to zero).

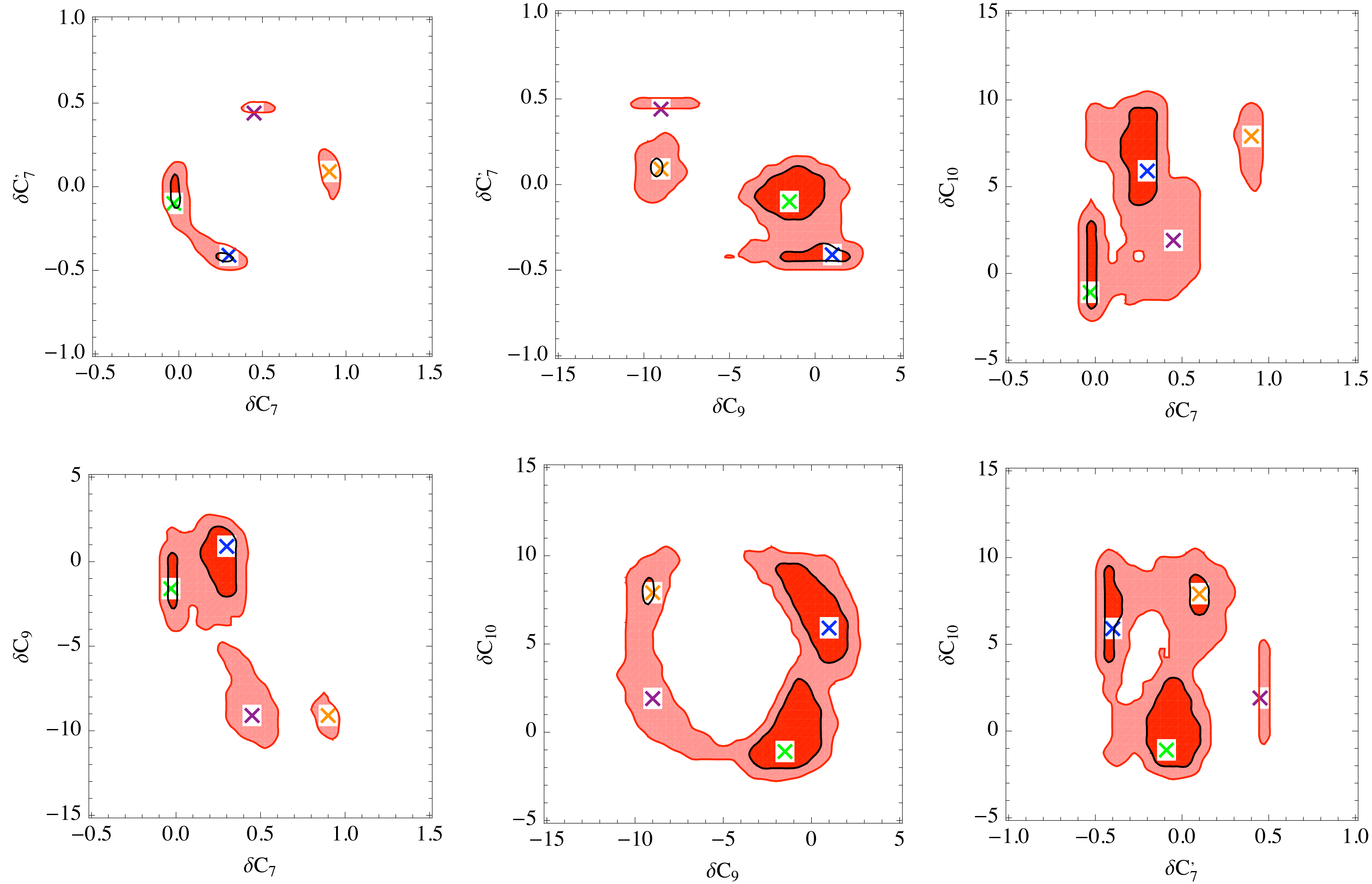




$b \to s\,\ell^+\ell^-$ decays are **COMPLEMENTARY** to *radiative B decays*

MODEL-INDEPENDENT CONSTRAINTS

2. Constraints on **C7, C7', C9, C10** (all other NP to zero).

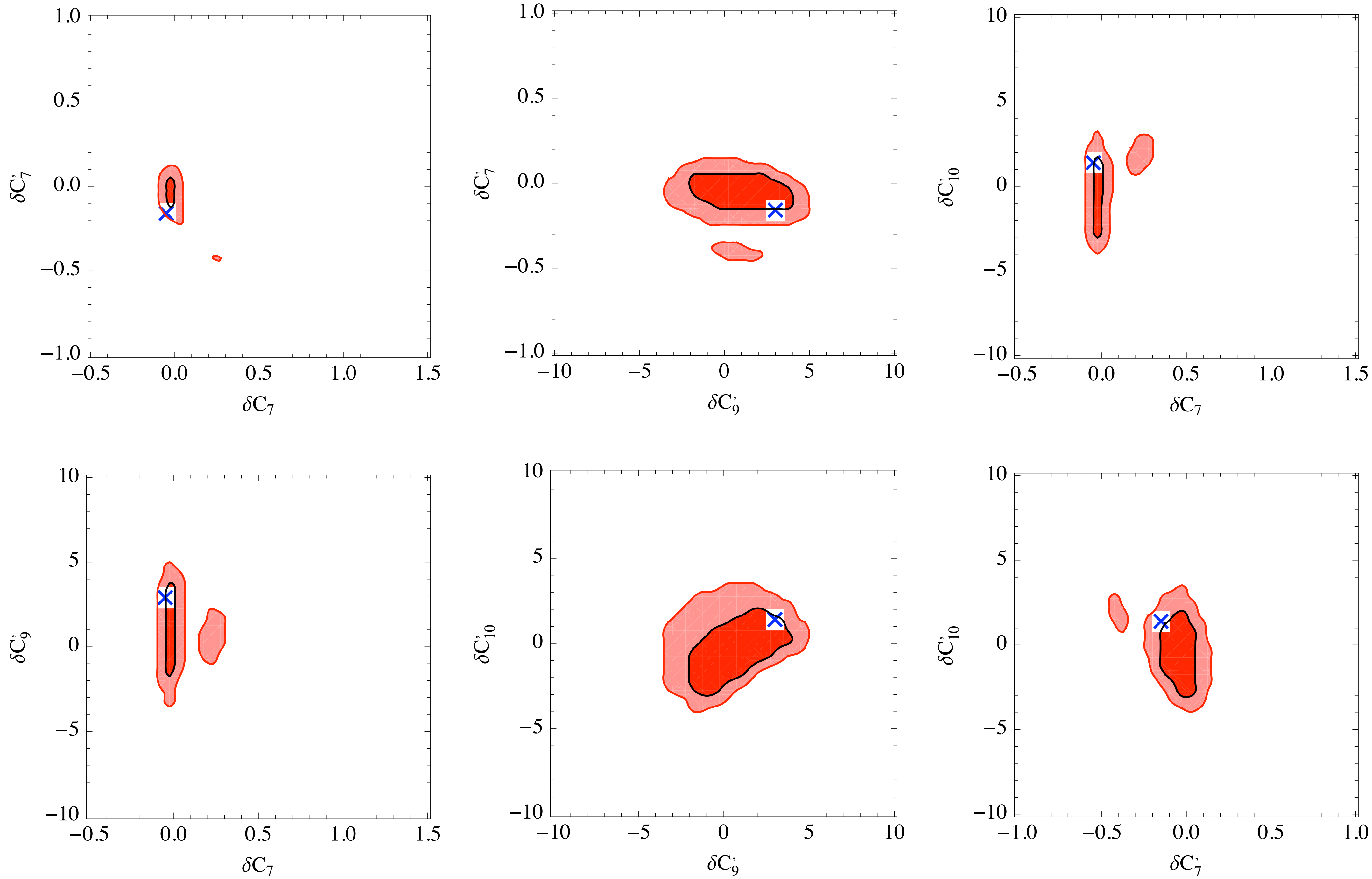


Flipped-sign solution for **C7** recovers statistical significance.

Four islands in the space of WCs (study "benchmark points").

MODEL-INDEPENDENT CONSTRAINTS

3. Constraints on **C7, C7’, C9’, C10’** (all other NP to zero).

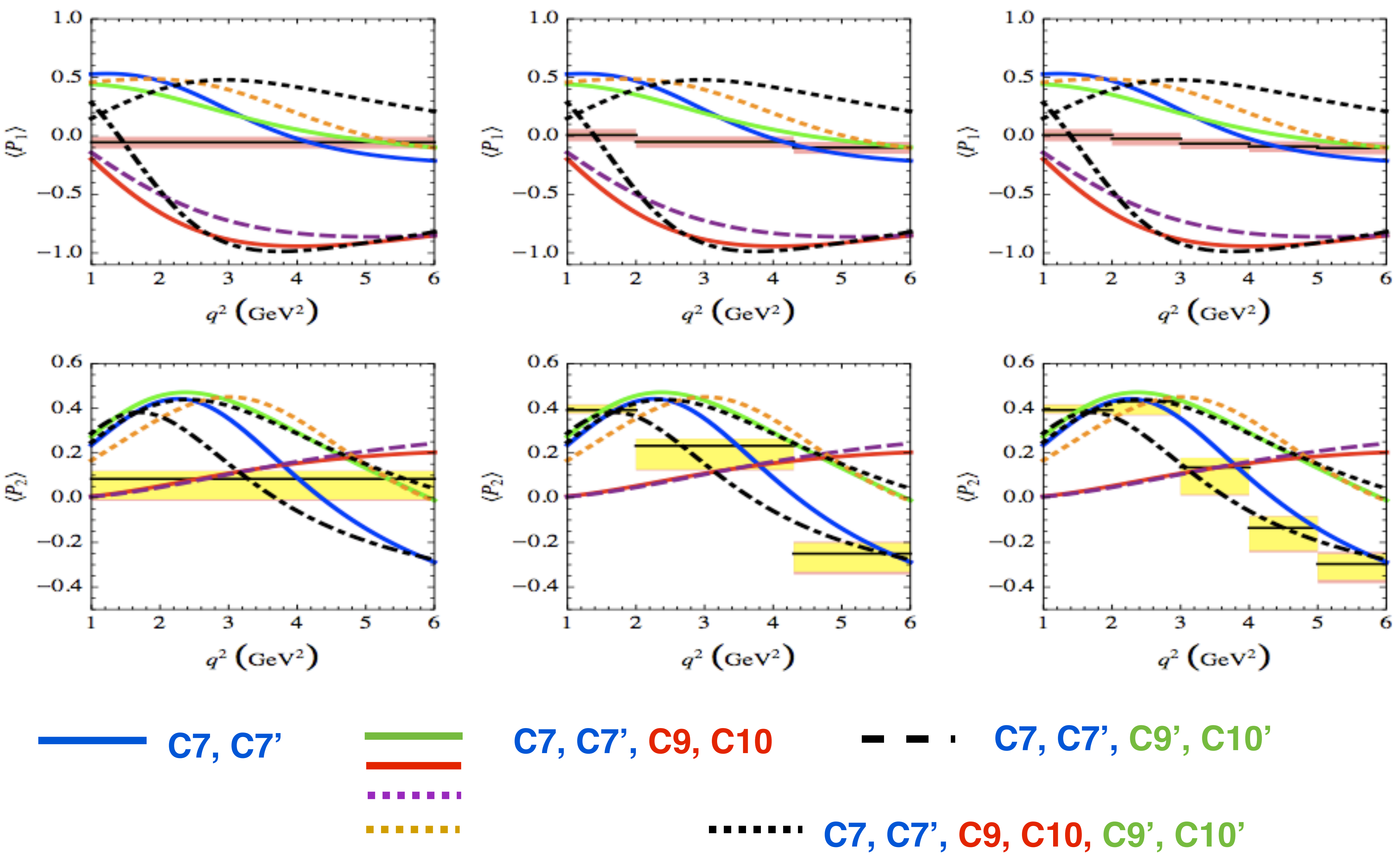


Flipped-sign solution for **C7** low statistical significance.

One benchmark point.

MODEL-INDEPENDENT CONSTRAINTS

Examples: benchmark points (C7, C7’, C9, C10, C9’, C10’)



MODEL-INDEPENDENT CONSTRAINTS

To be continued.....