Pion-photon transition form factor in light-cone sum rules

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based on 1205.3770 [hep-ph]

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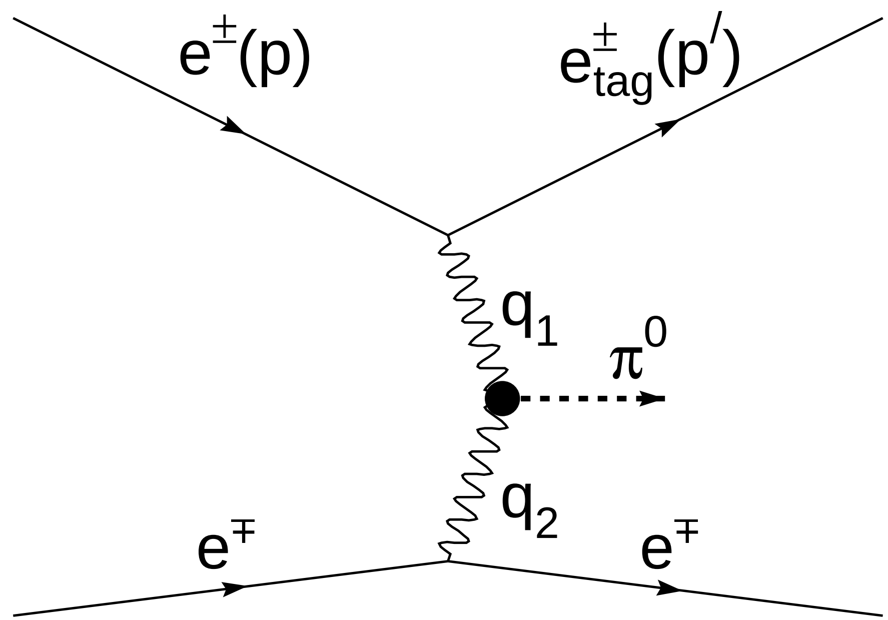
Outline:

- Pion-photon transition FF in LC SR
- Pion DA and its evolution.
- Light Cone Sum Rules
- Pion DA from experiment
- Conclusions

Feynman diagram for $e^+e^- \to e^+e^-\pi^0$

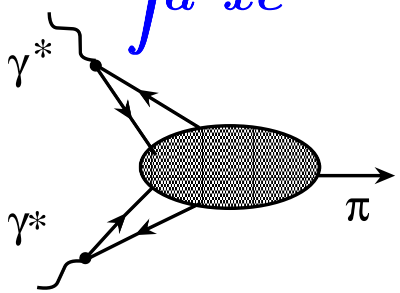
One of the most accurate data on exclusive reactions is data on transition FF $F^{\gamma^*\gamma^*\pi^0}(q_1^2, q_2^2)$ provided by series of experiments $e^+e^- \to e^+e^-\pi^0$ with $q_2^2 \approx 0$.

CELLO (1991) $0.7 - 2.2$ GeV^2 ,
CLEO (1998) $1.6 - 8.0$ GeV^2 ,
BaBar (2009) $4 - 40$ GeV^2 ,
Belle (2012) $4 - 40$ GeV^2 ,
BESIII (????) < 10 GeV^2 .



"Factorization" $\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(P)$ in pQCD

$$\int d^4x e^{-iq_1\cdot z}\langle \pi^0(P)|T\{j_\mu(z)j_\nu(0)\}|0\rangle = i\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta \cdot F^{\gamma^*\gamma^*\pi}(Q^2,q^2)\,,$$

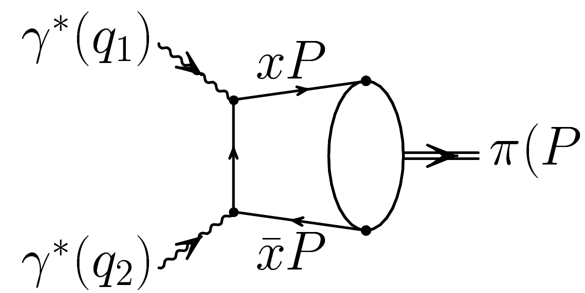


where $-q_1^2 = Q^2 > 0, \;\; -q_2^2 = q^2 \geq 0$

Collinear factorization at $Q^2, q^2 \gg$ (hadron scale $\sim m_\rho^2$)

$$F^{\gamma^*\gamma^*\pi}(Q^2,q^2) = T(Q^2,q^2,\mu_F^2;x) \otimes \varphi_\pi(x;\mu_F^2) + O(\frac{1}{Q^4})\,,$$

where μ_F^2 – boundary between large scale and hadronic one.



$$F^{\gamma^*\gamma^*\pi}(Q^2,q^2) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 dx \frac{1}{Q^2 x + q^2 \bar{x}}\, \varphi_\pi(x)$$

$$Q^2 F^{\gamma^*\gamma\pi}(Q^2,q^2 \to 0) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 \frac{dx}{x}\varphi_\pi(x) \equiv \frac{\sqrt{2}}{3} f_\pi \langle x^{-1}\rangle_\pi$$

Pion distribution amplitude $\varphi_\pi(x,\mu^2)$

- The pion DA parameterizes this matrix element:

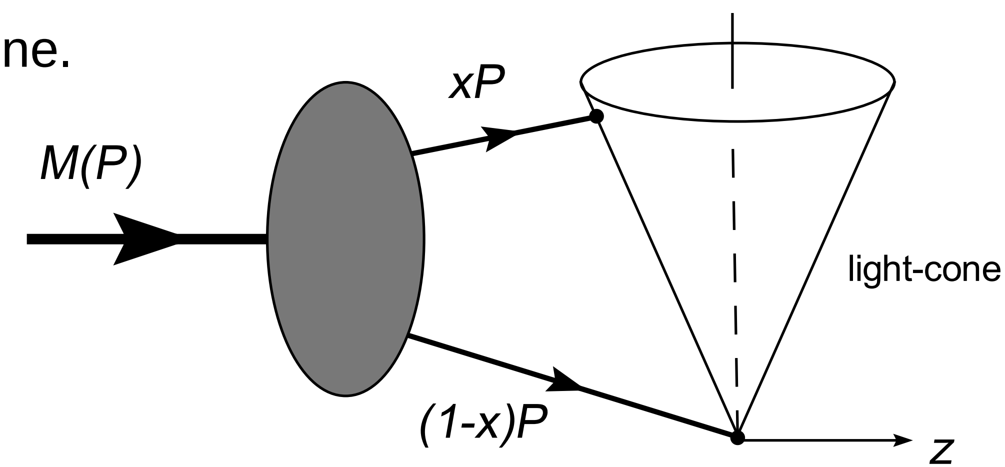
$$\langle 0|\,\bar{d}(z)\gamma_\nu\gamma_5[z,0]u(0)\,|\,\pi(P)\rangle\Big|_{z^2=0} = i f_\pi P_\nu \int_0^1 dx\; e^{ix(zP)}\varphi_\pi(x,\mu^2)\,.$$

where the path-ordered exponential

$$[z,0] = \mathcal{P}\,exp\left[ig\int_0^z t^a A_\mu^a(y)dy^\mu\right],$$

i.e., the light-like gauge link, ensures the gauge invariance.

- Pion DA describes the transition of a physical pion into two valence quarks, separated at light cone.



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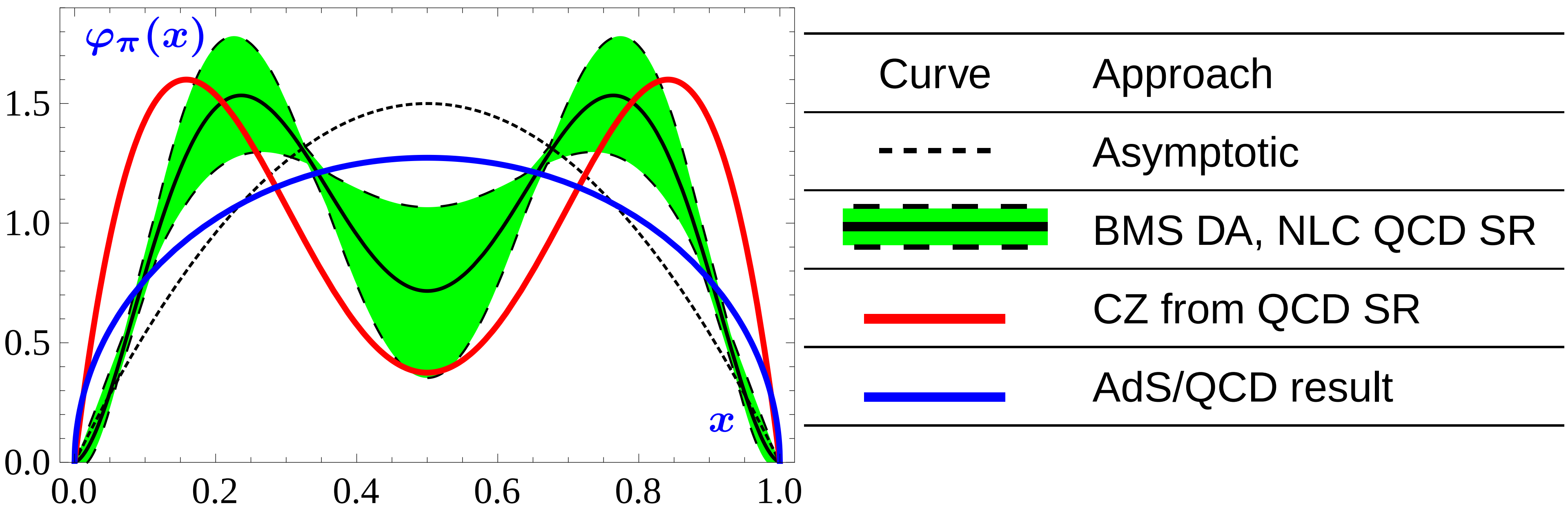


- DA evolution with μ^2, ERBL [79-80]. Gegenbauer expansion:

$$\varphi_\pi(x,\mu^2) = 6x\bar{x}(1 + a_2(\mu^2)C_2^{3/2}(x-\bar{x}) + a_4(\mu^2)C_4^{3/2}(x-\bar{x}) + \ldots)$$

$\gamma^*\gamma \to \pi$: Light-Cone Sum Rules!

LCSR effectively accounts for long-distances effects of real photon using quark-hadron duality in vector channel and dispersion relation in q^2 (Balitsky et. al.-89, Khodjamirian **[EJPC (1999)]**)

$$F_{\gamma\gamma^*\pi}(Q^2, q^2) \;=\; \int_0^{s_0} \frac{\rho^{\text{PT}}(Q^2, s)}{m_\rho^2 + q^2} e^{(m_\rho^2 - s)/M^2} ds + \int_{s_0}^{\infty} \frac{\rho^{\text{PT}}(Q^2, s)}{s + q^2} ds \,,$$

where $s_0 \simeq 1.5$ **GeV**^{2} – effective threshold in vector channel,
M^2 – Borel parameter ($0.5 - 0.9$ GeV^2).
Real-photon limit $q^2 \to 0$ can be easily done.

Spectral density was calculated in QCD:

$$\rho^{\text{PT}}(Q^2, s) = \textbf{Im} F^{\text{PT}}_{\gamma^*\gamma^*\pi}(Q^2, -s - \imath\varepsilon) = \text{Tw-2} + \text{Tw-4} + \text{Tw-6} + \ldots \,,$$

where twists contributions given in a form of convolution with pion DA:

$$\text{Tw-2} \sim (T_{\text{LO}} + T_{\text{NLO}} + T_{\text{NNLO}_{\beta_0}} + \ldots) \otimes \varphi_\pi^{\text{Tw2}}(x, \mu) \,.$$

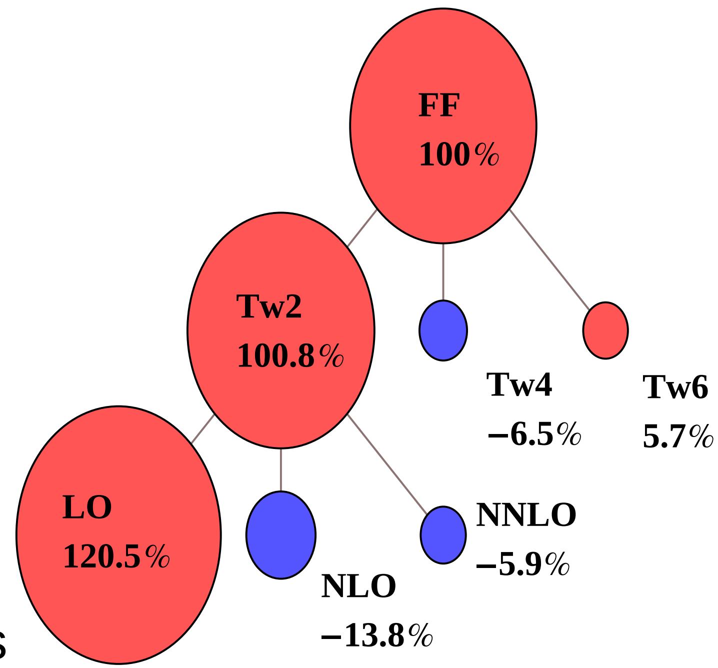
Main Ingredients of Spectral Density

- **LO** Spectral Density, **Tw-4** term — **Khodjamirian[EJPC (1999)]**
- **NLO** Spectral Density — in **[Mikhailov&Stefanis(2009)]**
- **NNLO$_{\beta_0}$** Spectral Density — in **[M&S(2009)]**
- **Tw-6** contribution — in **[Agaev et.al.–PRD83(2011)0540020]**

- **NLO** evolution of pion DA **[Kadantseva&M&R – Sov.J.NP.86 (1986)]**

Terms of Pion-Photon FF at $Q^2 = 8\,\text{GeV}^2$

- Result is dominated by Hard Part of Twist-2 LO contribution.
- Twist-6 contribution is taken into account together with **NNLO$_{\beta_0}$** one — they has close absolute values and opposite signs.



Blue - negative terms

Red - positive terms

Parameters of LC SR

From QCD SR:

- Borel parameter $M^2_{\text{LCSR}} \in [0.7, 1]$ GeV^2
- Vector Chan. Threshold s_0
- Twist-4 $\delta^2 \pm 20\% = \lambda_q^2/2$
- Twist-6 $(\alpha_S \langle \bar{q} q \rangle)$

From PDG:

- $\alpha_s(m_Z^2)$
- Masses m_ρ, m_ω
- Decay Widths Γ_ρ, Γ_ω

Light-Cone Sum Rules:

$$\text{FF} = (\text{LO} + \text{NLO}) \otimes (\pi\text{-DA}_{\text{NLO}}) + \text{Tw-4} \pm \Delta\text{FF}$$

$$\Delta\text{FF} = \pi\text{-}\Delta\text{DA} + \Delta\text{Tw-4} + (\text{NNLO}_{\beta_0} \otimes (\pi\text{-DA}) + \text{Tw-6})$$

π-DA model

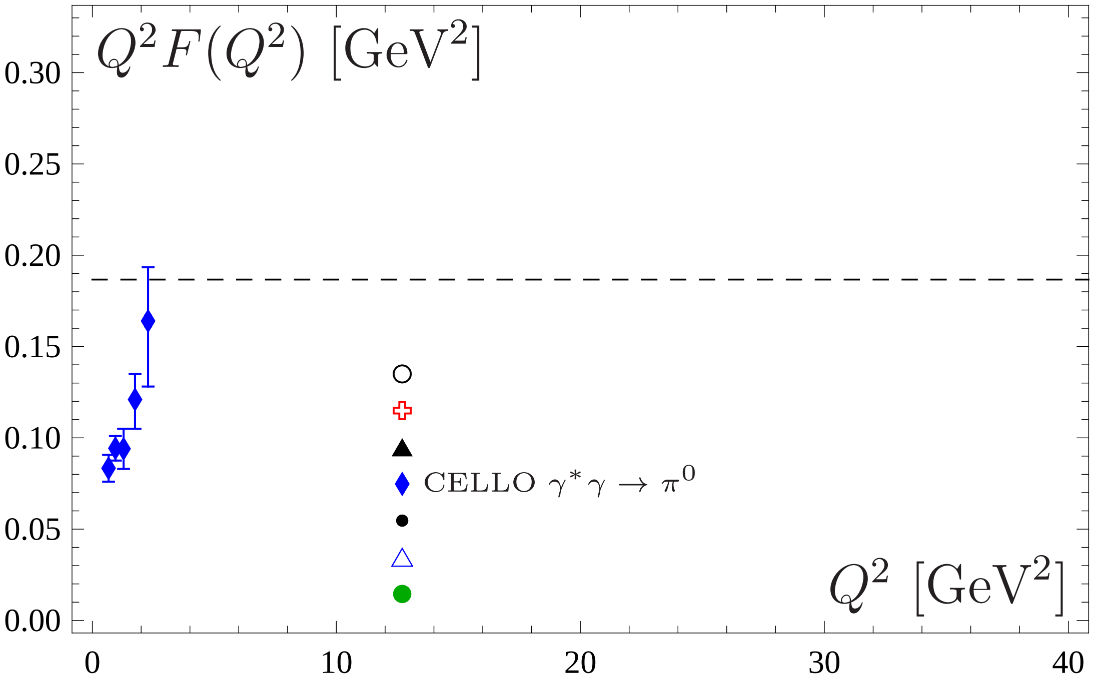
Data on FF

FF Prediction

Fitting π-DA (a_n)

Pion-gamma FF data

Experimental Data on $F_{\gamma\gamma^*\pi}$: **CELLO**, CLEO, **BaBar** and **Belle [1205.3249[hep-ex]]**



Data	Collab.
◆	**CELLO (1991)**

dashed line = $\sqrt{2}\, f_\pi$

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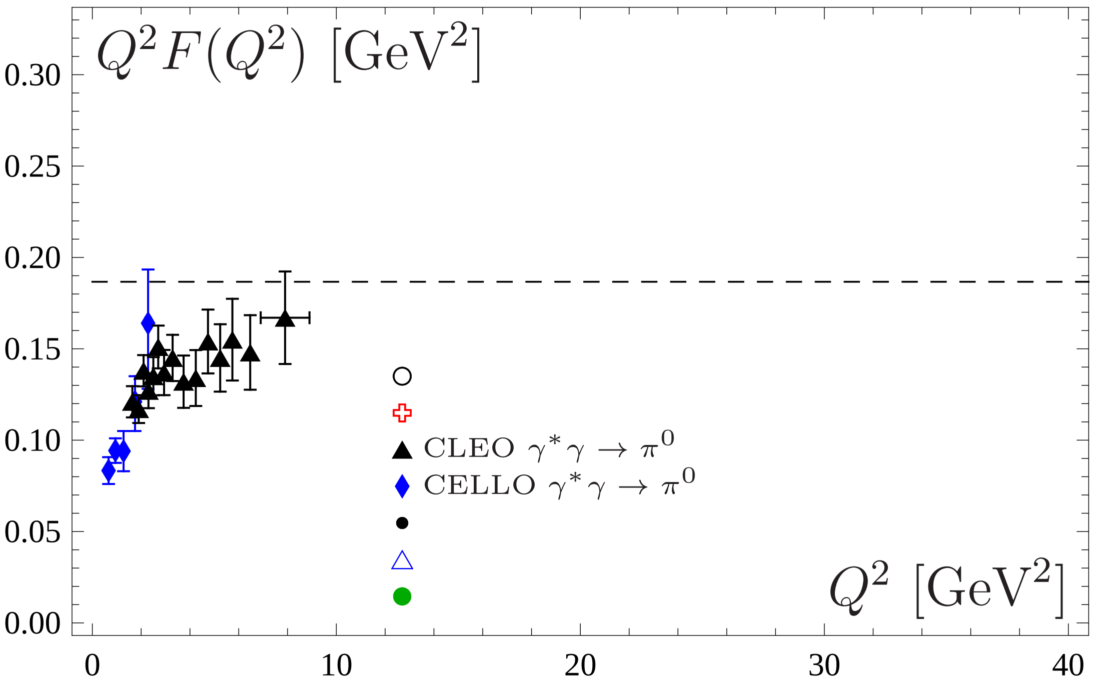

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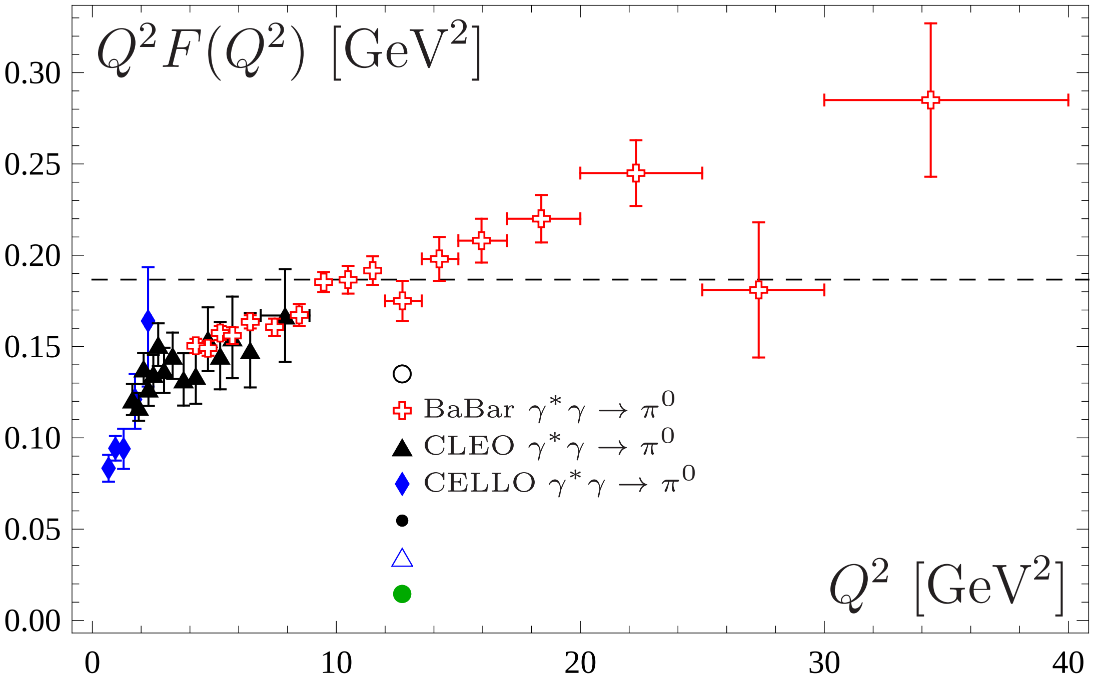

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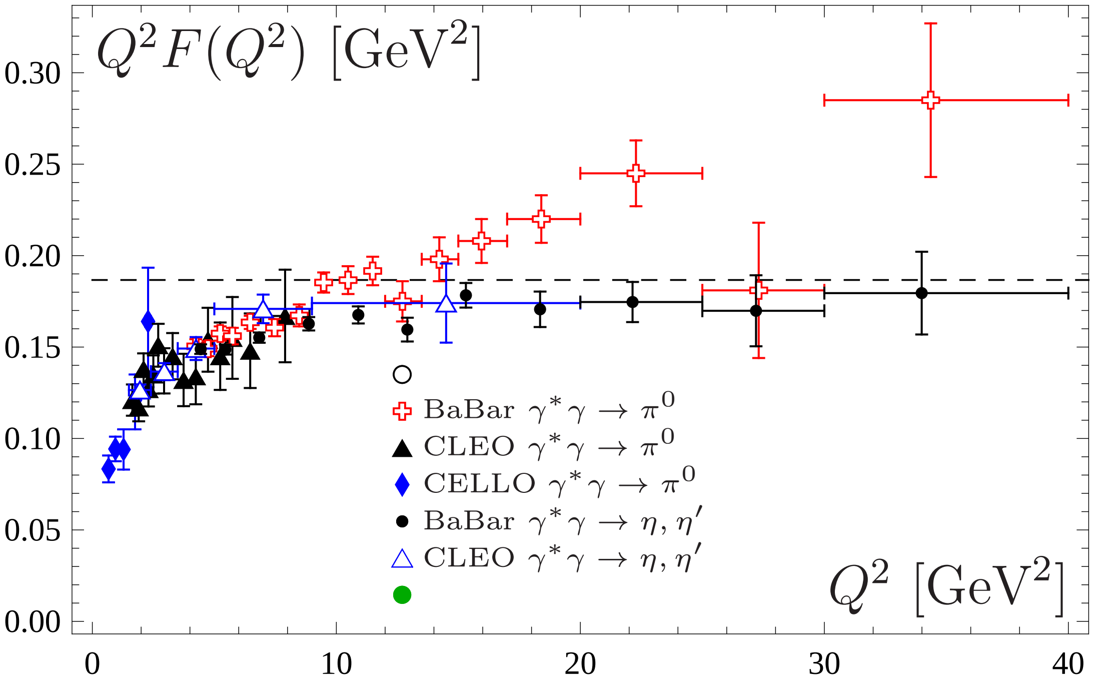

Data	Collab.
◆	**CELLO (1991)**
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✚	**BaBar (2009)**
●	**BaBar$^{\eta}_{\eta'}$ (2011)**

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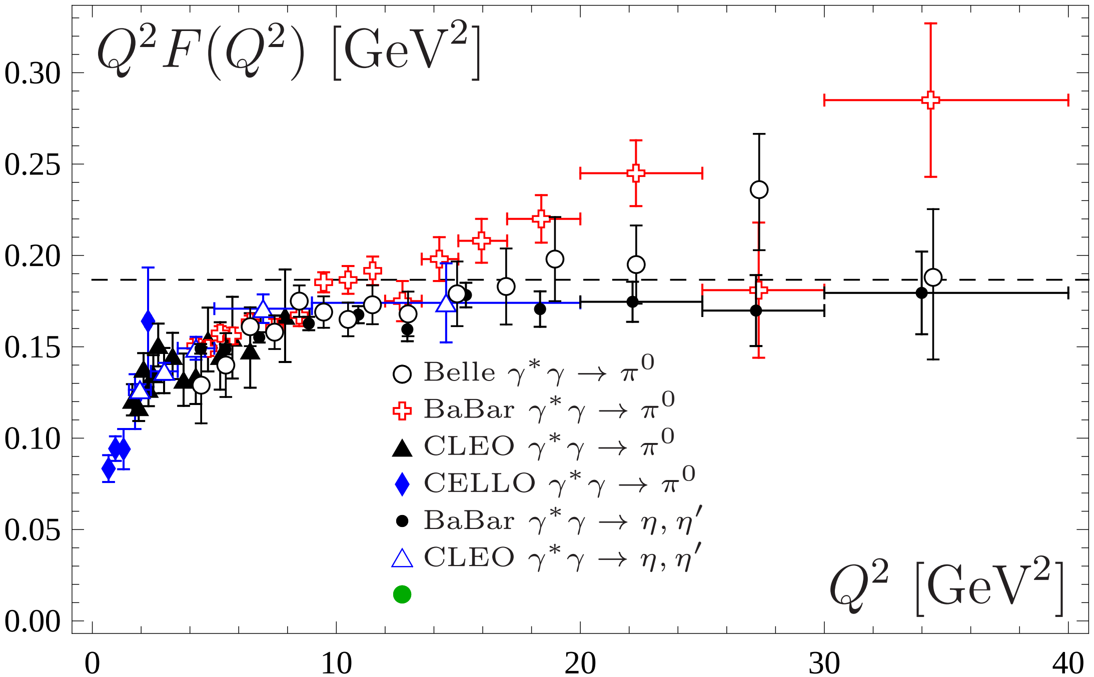

Data	Collab.
◆	**CELLO (1991)**
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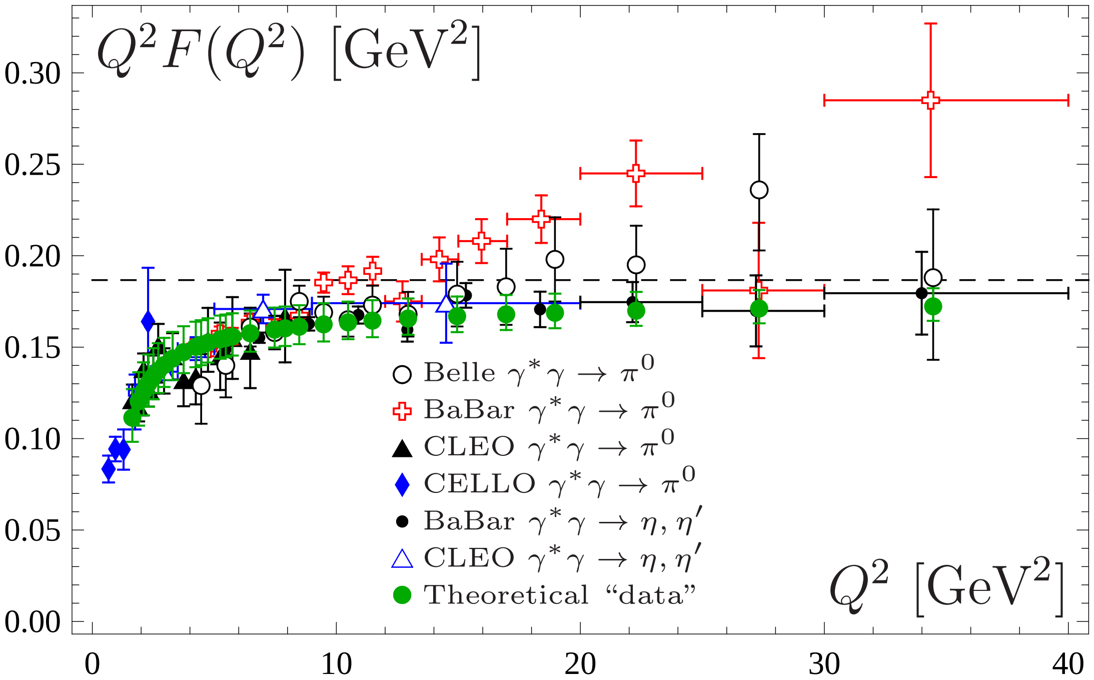
✚	**BaBar (2009)**
●	**BaBar$^{\eta}_{\eta'}$, (2011)**
❍	Belle (2012)
dashed line = $\sqrt{2}\, f_\pi$	

Belle data do not confirm auxetic form factor behavior above 10 GeV^2 (except outlier at $Q^2 = 27.33$ GeV^2).

Pion-gamma FF data

Experimental Data on $F_{\gamma\gamma^*\pi}$: **CELLO**, CLEO, **BaBar** and **Belle [1205.3249[hep-ex]]**

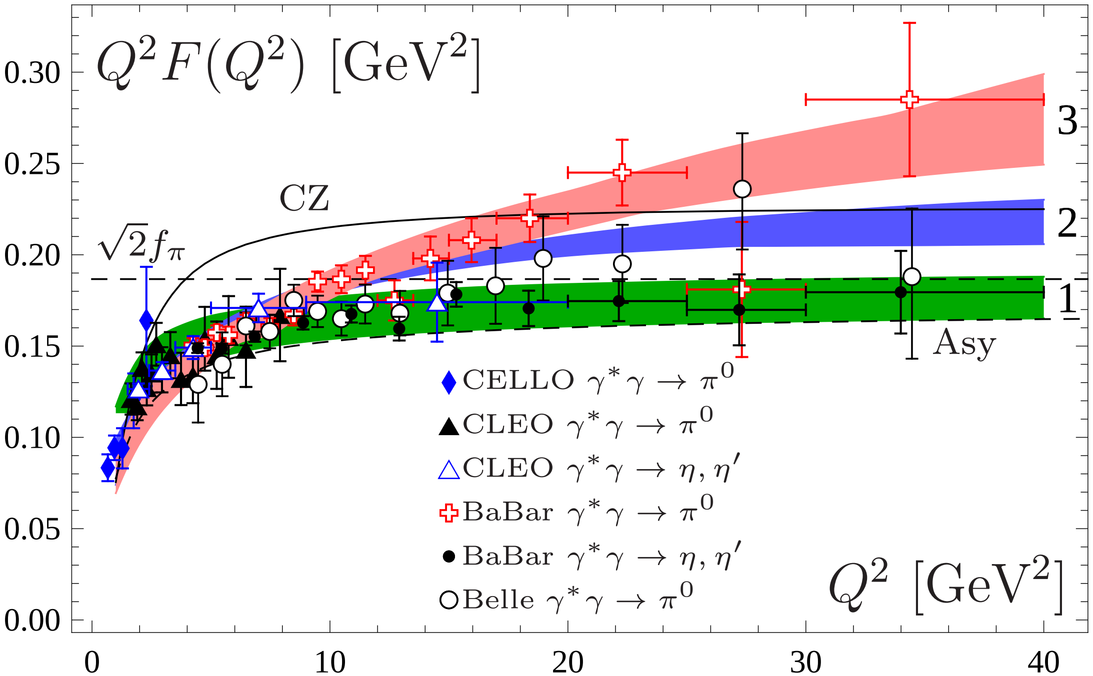


Data	Collab.
◆	**CELLO (1991)**
▲	CLEO (1998)
✚ (open)	**BaBar (2009)**
●	**BaBar$^{\eta}_{\eta'}$ (2011)**
○	Belle (2012)
● (green)	**BMPS** (1202.1781) in revision

Belle data do not confirm auxetic form factor behavior above 10 GeV^2 (except outlier at $Q^2 = 27.33$ GeV^2).

BMPS predicted "data" agree well with CELLO, CLEO, BaBar$_{Q^2<9\text{ GeV}^2}$ (2009), BaBar$^{\eta}_{\eta'}$ (2011), and most Belle (2012).

Pion TFF Data and Models

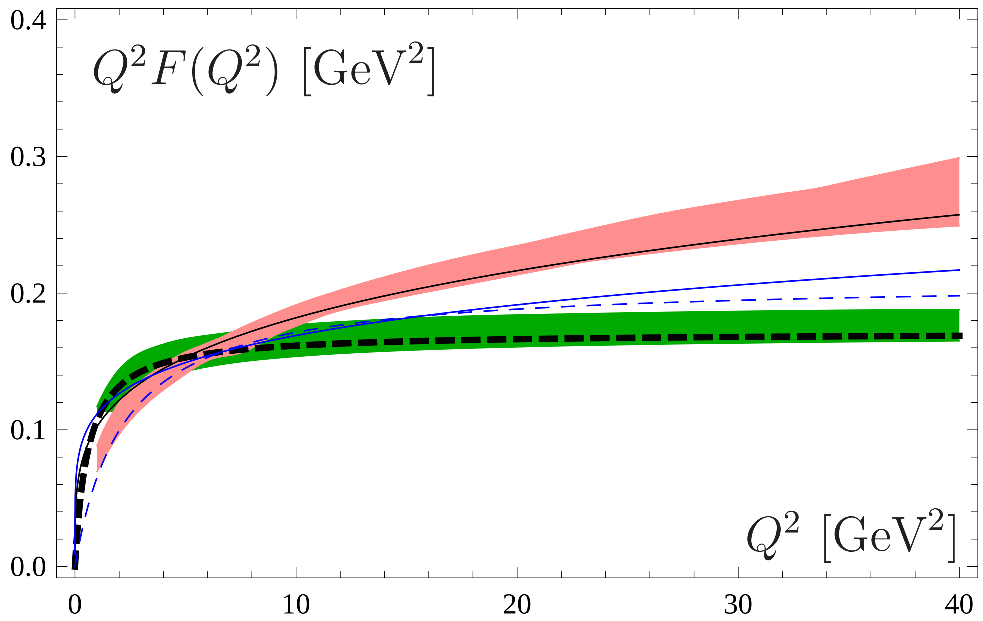


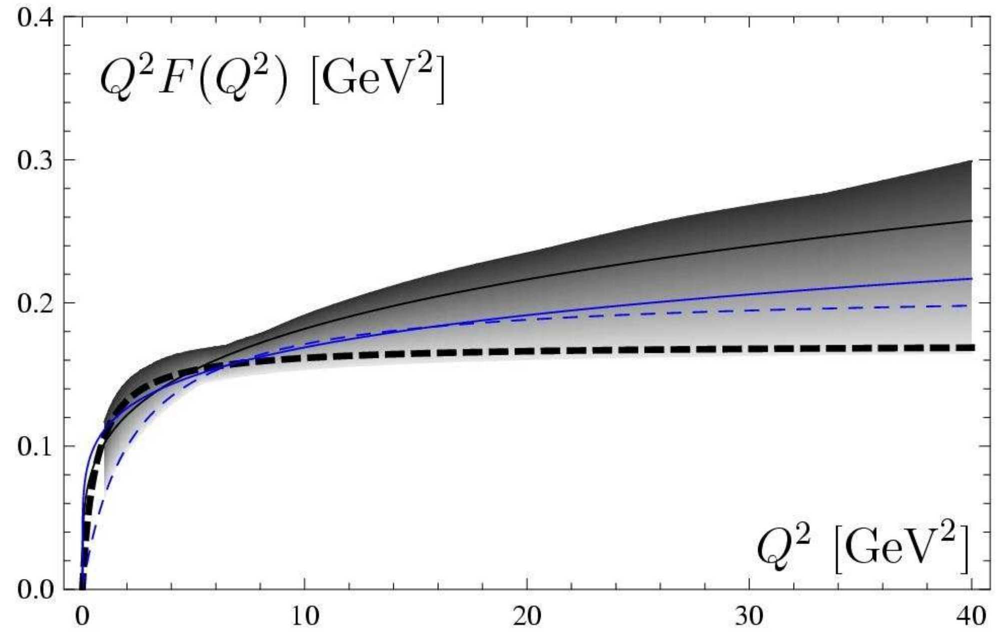
Data	Collab.
■■ (red)	**BaBar, 8 models**
■■ (blue)	**Intermediate, 5 models**
■■ (green)	**BMPS & Holography, 4 models**

- Most data points either inside **green "Belle" strip** (**scaling**) or within **red "BaBar" strip** (**auxesis**).
- BaBar η, η' data are **within green strip**
- **Blue strip** mostly theoretical.

Alternatives for Pion-Gamma FF Analysis

Alternative: To consider data as forming **two independent data strips** (left) or **one single data strip** (right) **[1205.3770]**





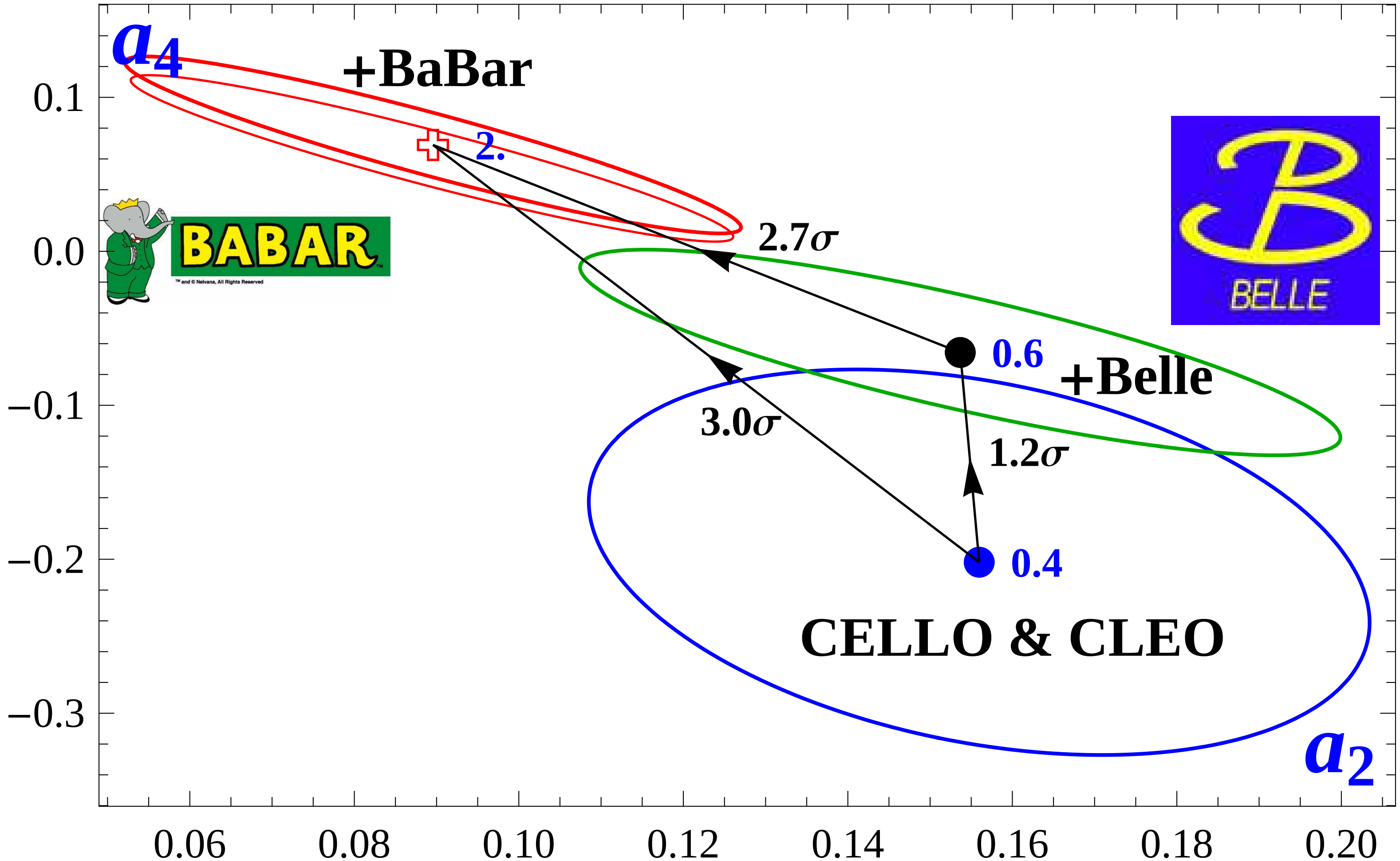
We suggest to explore the first **Alternative**:

To consider all data as forming

two independent data strips,

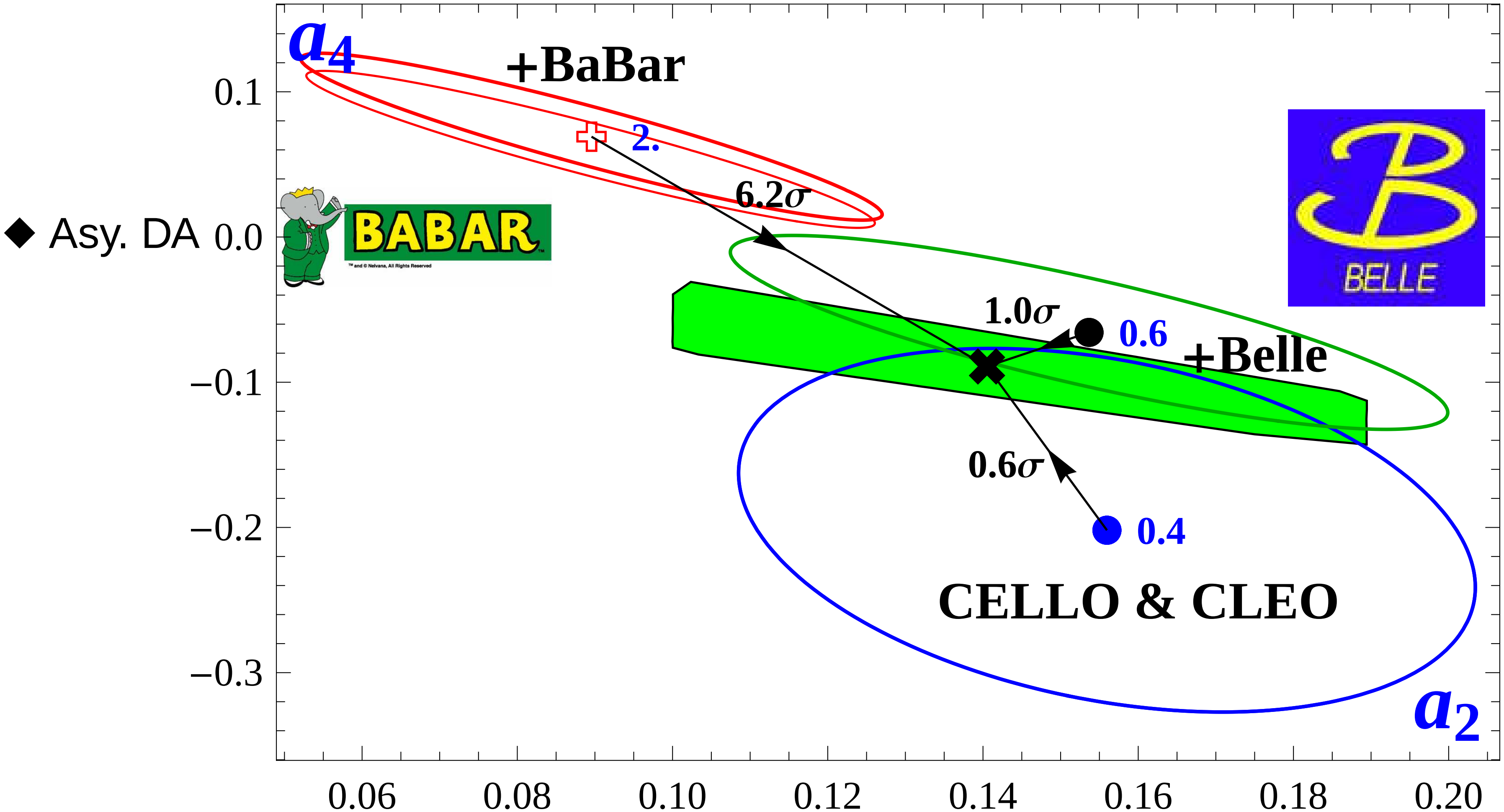
namely, **CELLO&CLEO&Belle** and **CELLO&CLEO&BaBar**

Confidential regions in 2D (a_2, a_4)



- In vertexes of a triangle - χ^2/ndf
- On sides of triangle: discrepancy in terms of std. deviation ($1\sigma \approx 68\%$)

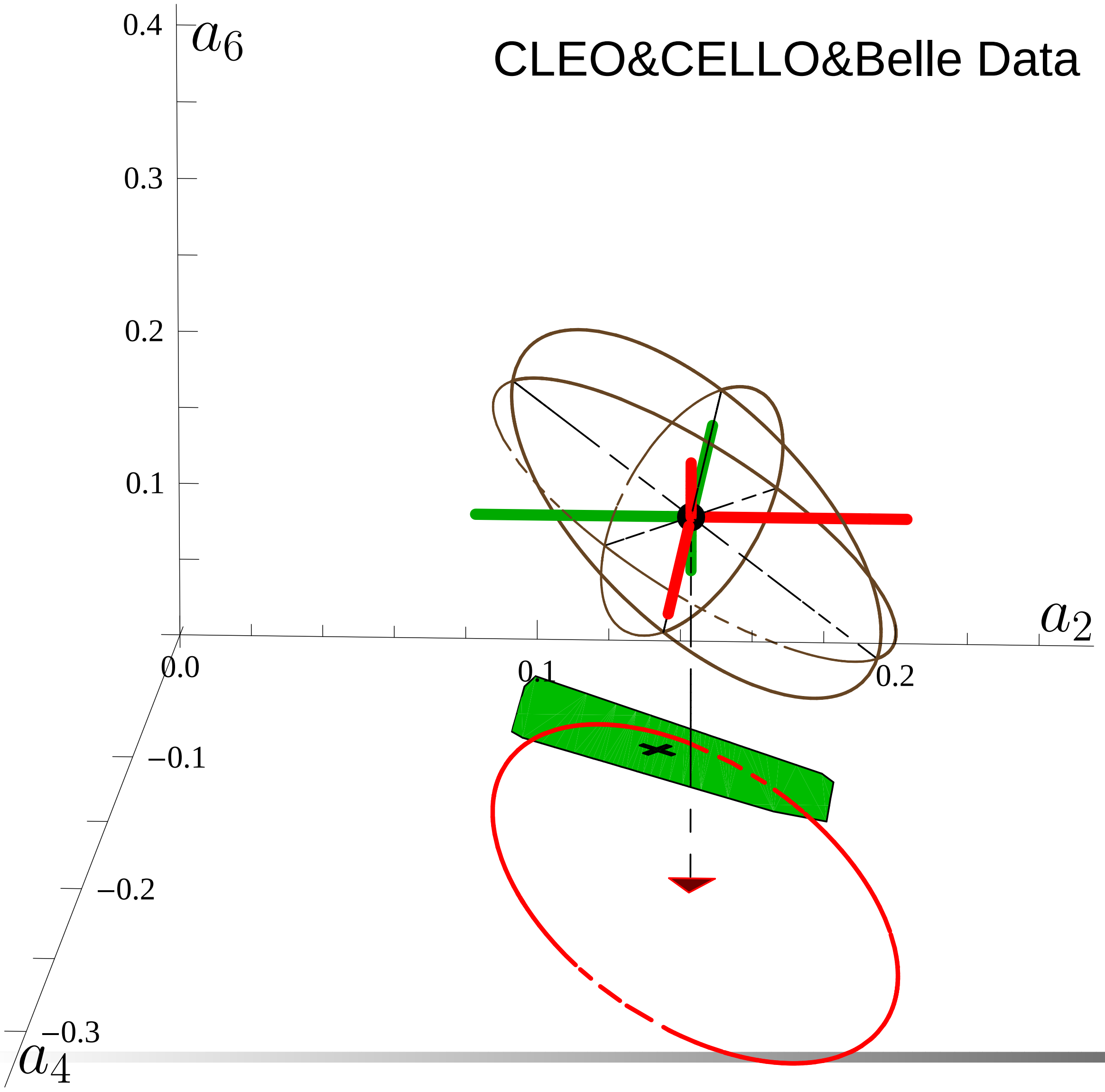
Confidential regions in 2D (a_2, a_4)



- BMS DA (✖&green bunch) from QCD Sum Rules with nonlocal condensates: $\lesssim 1\sigma$.
- Asymptotic DA, CZ DA: $> 6\sigma$.

NLC SR Results vs 3D Constraints

3D $\mathbf{1\sigma}$-error ellipsoid for (a_2, a_4, a_6) at $\mu_{\rm SY} = 2.4$ GeV scale with theoretical $\Delta\delta^2_{\rm tw4}$-error shown by green and red lines.



(red line)	2D proj. of 1σ-ellipsoid
●	$\chi^2_{\rm ndf} \approx 0.4$

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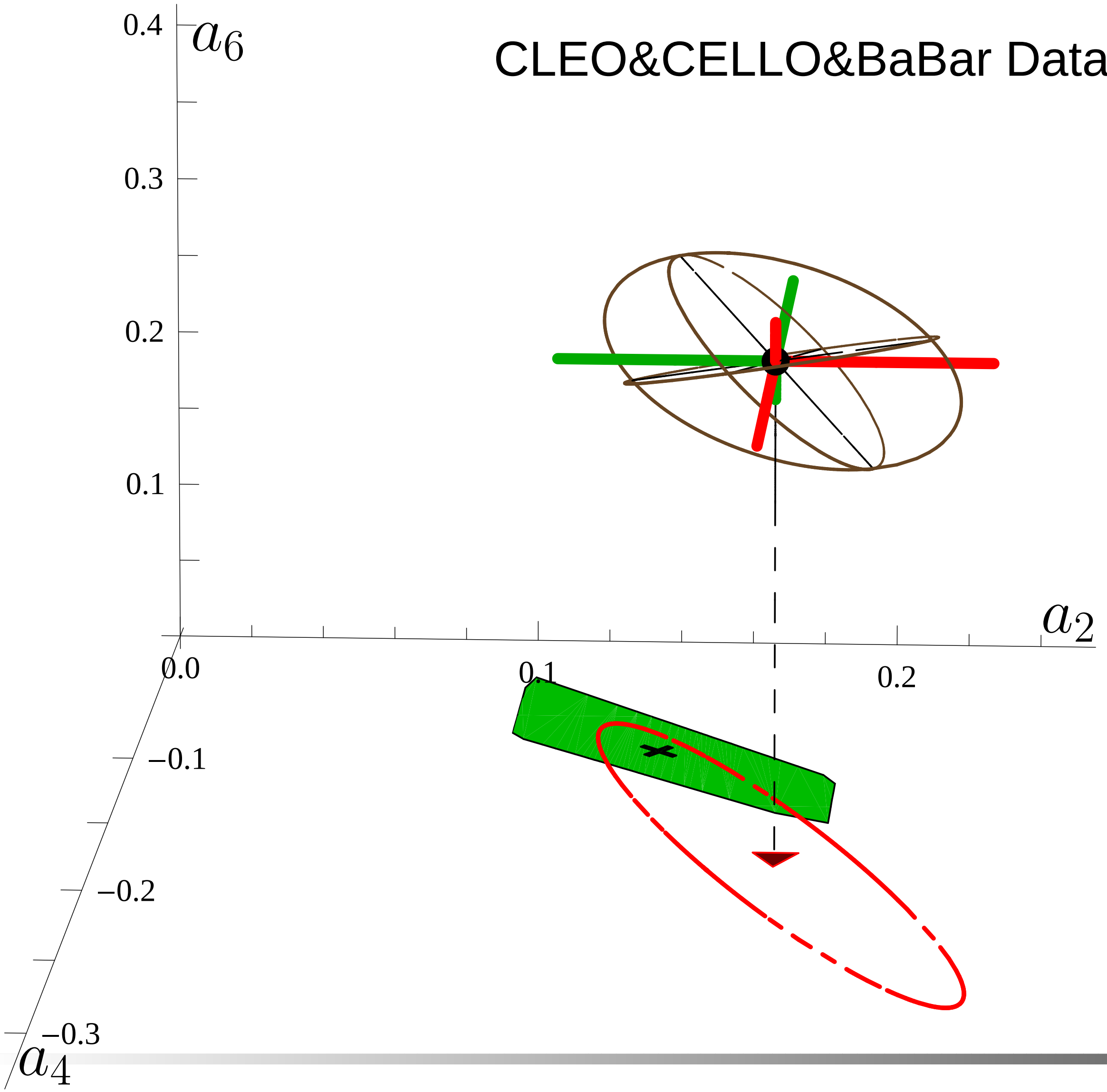
3D $\mathbf{1\sigma}$-error ellipsoid for $(\boldsymbol{a_2}, \boldsymbol{a_4}, \boldsymbol{a_6})$ at $\mu_{\text{SY}} = \mathbf{2.4}$ GeV scale with theoretical $\boldsymbol{\Delta\delta^2_{\text{tw4}}}$-error shown by green and red lines.



(red line)	2D proj. of $\mathbf{1\sigma}$-ellipsoid
●	$\chi^2_{\text{ndf}} \approx$ **1.0**

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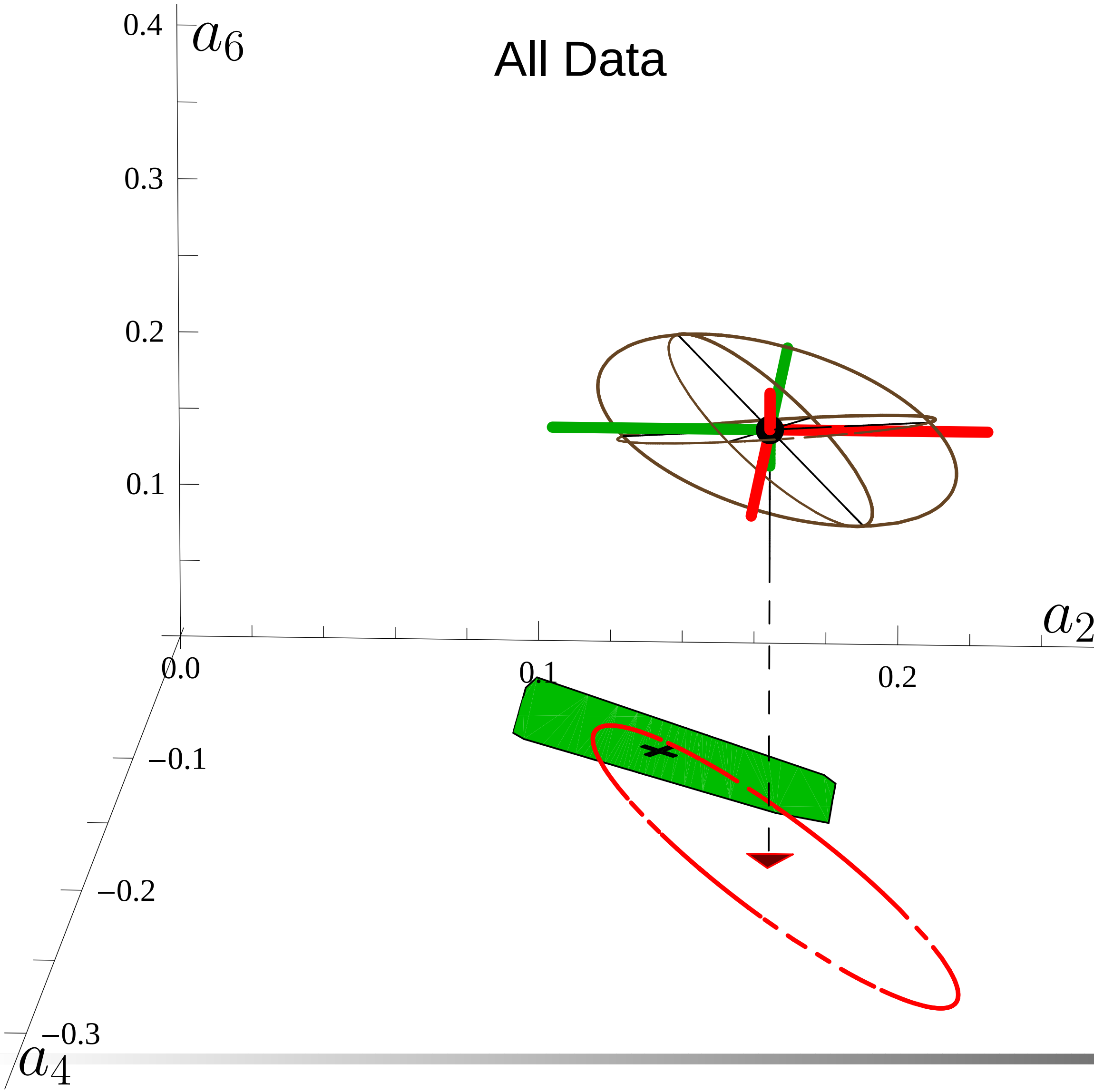
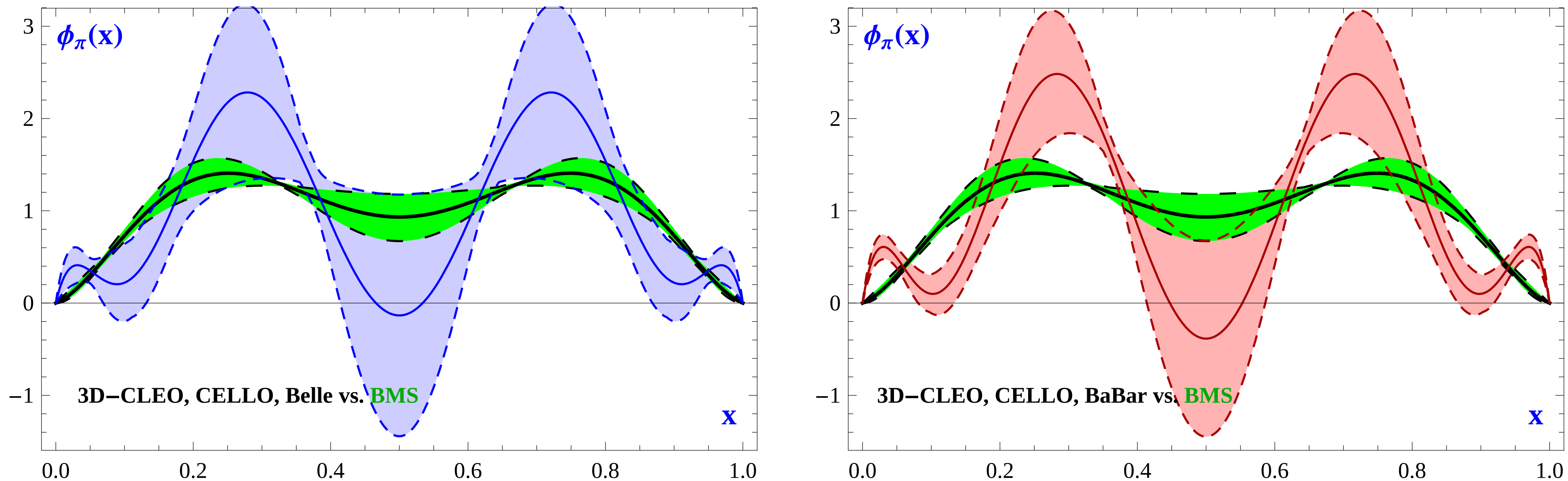


(red line)	2D proj. of $\mathbf{1\sigma}$-ellipsoid
●	$\chi^2_{\rm ndf} \approx$ **1.0**

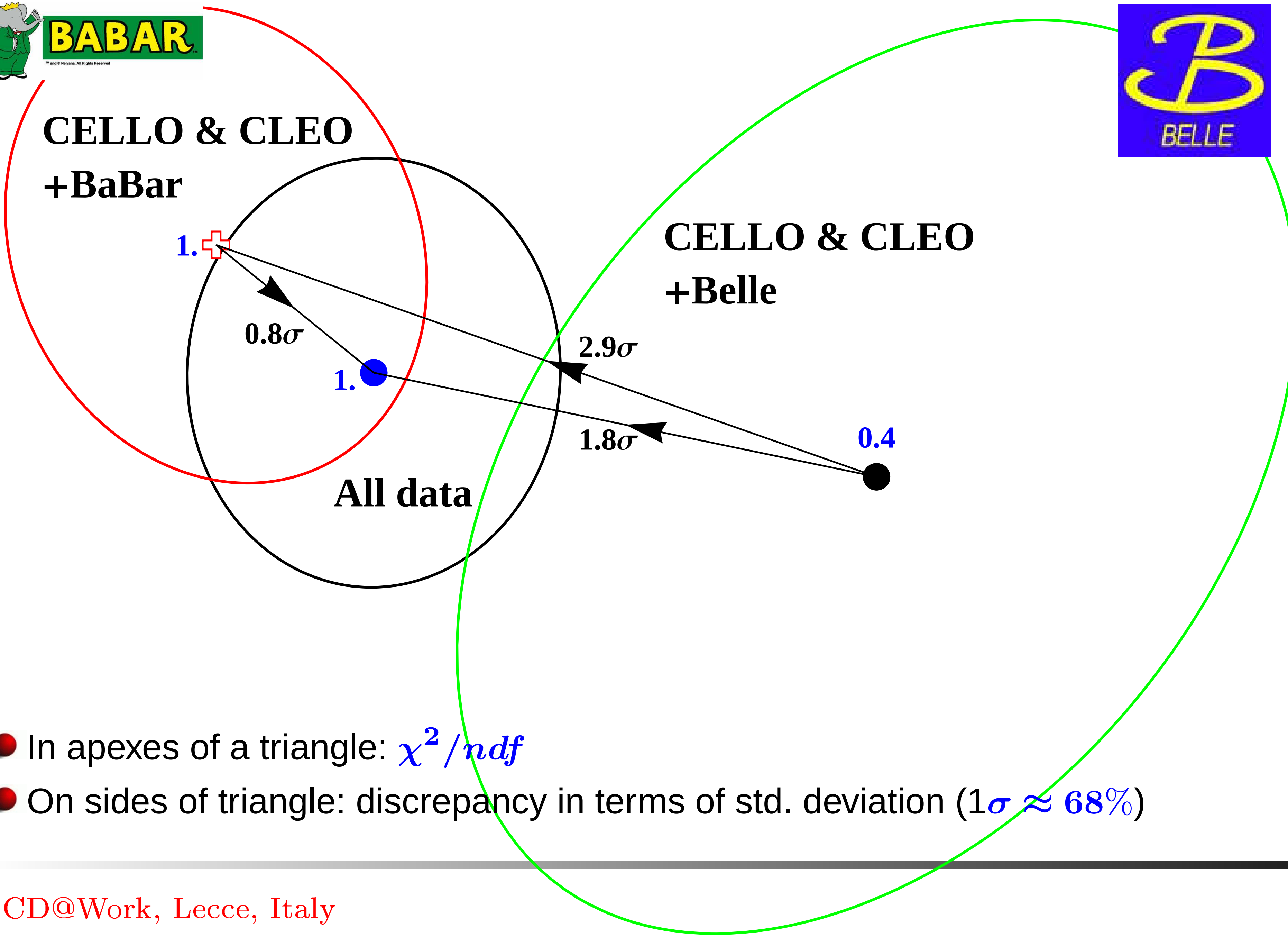
Data fit of pion DA vs QCD SR

→ BMS, → Belle, → BaBar $\mathbf{1-40}$ GeV^2 at $\mu_{\mathrm{SY}} = \mathbf{2.4}$ GeV scale



- CLEO, CELLO, Belle data agrees with BMS bunch based on NLC QCD SR
- BaBar data above $\mathbf{10}$ GeV^2 does not support BMS bunch.
- Confidence bunch based on data subset (CLEO, CELLO, Belle) includes bunch based on all data or subset (CLEO, CELLO, BaBar).

2D cut of 3D confidential regions (a_2, a_4, a_4)



- In apexes of a triangle: χ^2/ndf
- On sides of triangle: discrepancy in terms of std. deviation ($1\sigma \approx 68\%$)

Conclusions

- Performed 2-D and 3-D analysis of CELLO, CLEO, BaBar, Belle data using LCSRs at NLO and Tw-4 term.
- We showed that the data from **CELLO**, **CLEO**, **BaBar**, and **Belle** at $Q^2 = 1 - 9$ **GeV^2** in 2D analysis favor a pion DA with endpoint suppression, like **BMS** model;
- Beyond $Q^2 = 10$ **GeV^2**, the best fit to data including BaBar on $F_{\gamma^*\gamma\to\pi}$ requires a sizeable coefficient a_6;
- Beyond $Q^2 = 10$ **GeV^2**, only CLEO-CELLO-Belle data agrees well with **BMS** bunch;
- Though we consider the **Belle** data to be more preferable for the QCD framework, the **BaBar** data should not be neglected.