Theoretical inputs and discussion of the errors in the new hadronic currents in TAUOLA

Pablo Roig Garcés (IFAE and UAB, Barcelona)

In collaboration with T. Przedzinski, O. Shekhovtsova and Z. Was

e-Print: **arXiv:1203.3955** [hep-ph]

and work in progress (also in collaboration with Ian Nugent)

QCD@Work 18-21 June 2012 Lecce

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See Olga's talk

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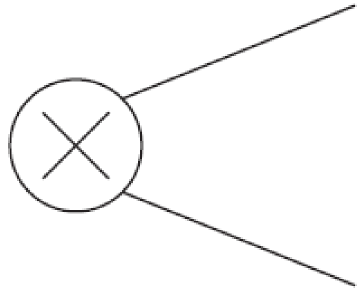
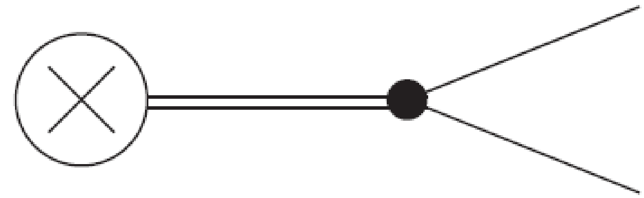
- Form factors in two meson τ decays
- Form factors in three meson τ decays
- Discussion on the errors
- Ongoing/Future improvements

Form factors in two meson τ decays

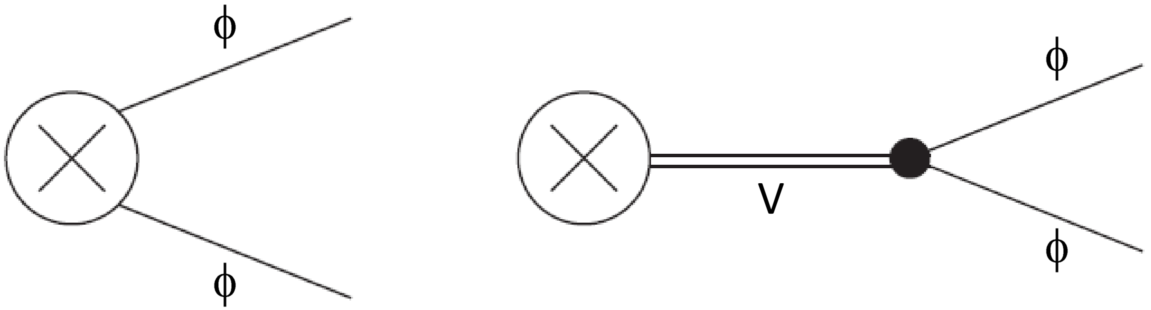




Form factors in two meson τ decays

$\phi = \pi, K$

$V = \rho, K^*$



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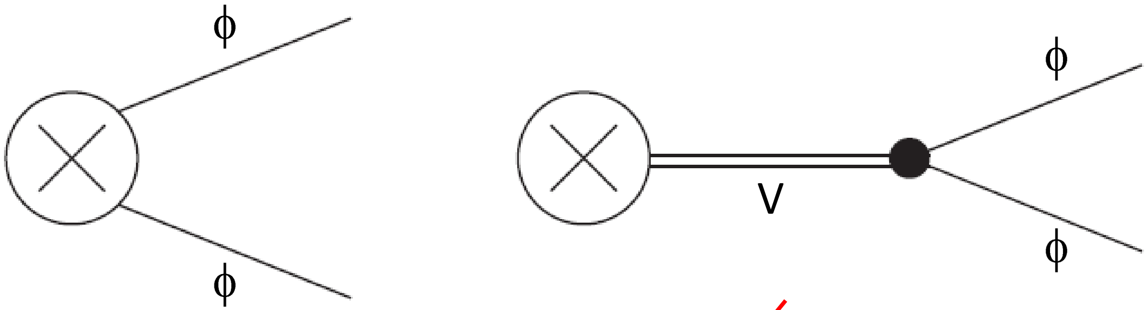
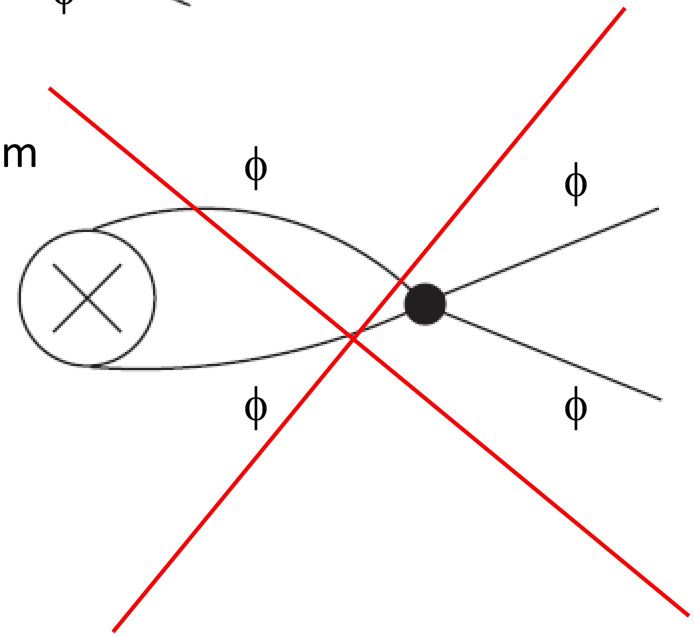


Antisymmetric tensor formalism for spin-one resonances

Ecker *et al.*

Phys.Lett.B223:425,1989

Nucl.Phys.B321:311,1989



To avoid double counting

Form factors in two meson τ decays

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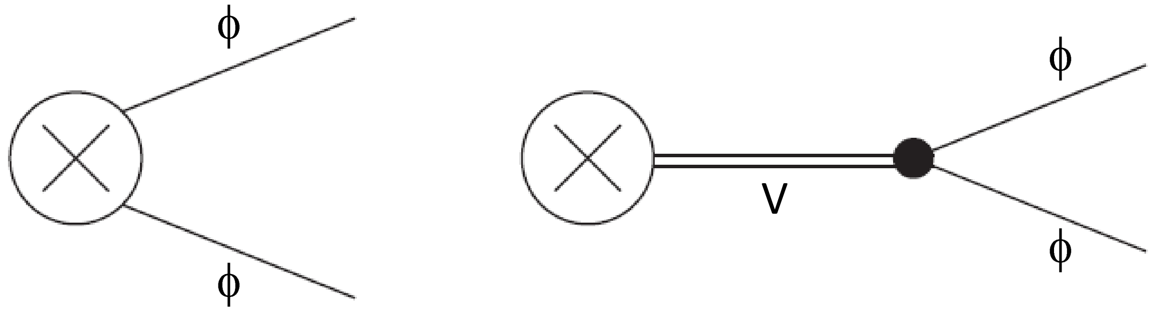

Ecker *et al.*

Phys.Lett.B223:425,1989

Nucl.Phys.B321:311,1989

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{i G_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle$$

$$u_\mu = i \left\{ u^\dagger (\partial_\mu - i r_\mu) u - u (\partial_\mu - i l_\mu) u^\dagger \right\}$$

$$f_\pm^{\mu\nu} = u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u$$

$$u(\varphi) = \exp\left\{ i \frac{\Phi}{\sqrt{2} F} \right\} \qquad \Phi(x) \equiv \frac{1}{\sqrt{2}} \sum_{a=1}^{8} \lambda_a \varphi_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

Form factors in two meson τ decays

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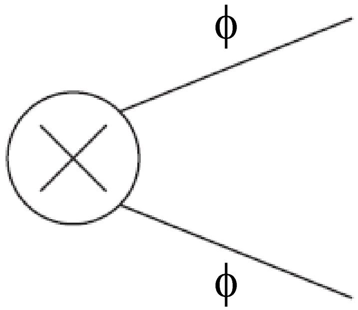

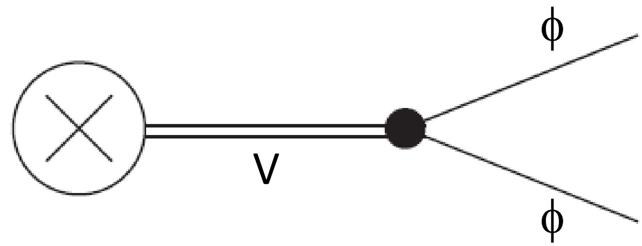

Ecker *et al.*

Phys.Lett.B223:425,1989

Nucl.Phys.B321:311,1989

$$F(s)^V = 1 + \frac{F_V G_V}{F^2} \frac{s}{M_V^2 - s}$$

Short-distance constraints

F(s) $\to$0, for s$\to\infty$ $\Rightarrow$ $F_V\, G_V = F^2$

$$F(s)^{VMD} = \frac{M_V^2}{M_V^2 - s}$$

Form factors in two meson τ decays

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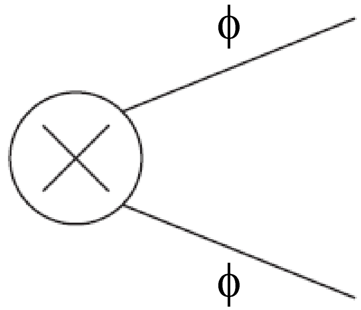

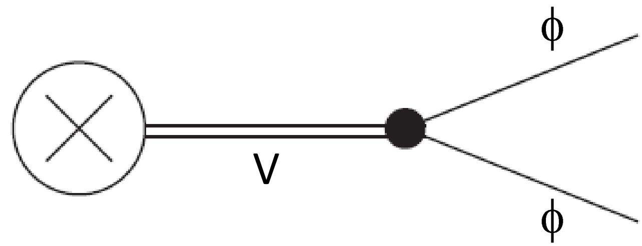

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$$\frac{1}{M_V^2 - s} \to \frac{1}{M_V^2 \left[1 + \sum_{P,Q} N_{loop}^{PQ} \frac{s}{96\pi^2 F^2} A_{PQ}(s)\right] - s}$$

$$A(m_P^2/s, m_P^2/\mu^2) = \ln\left(m_P^2/\mu^2\right) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P + 1}{\sigma_P - 1}\right) \qquad \sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

Form factors in two meson τ decays

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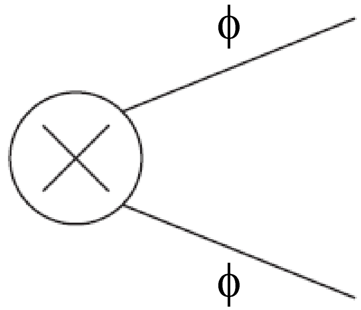

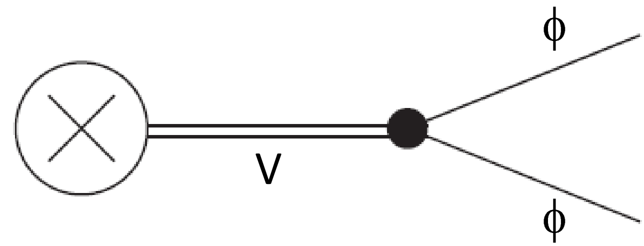

Short-distance constraints

$$F(s)^V = 1 + \frac{F_V G_V}{F^2}\frac{s}{M_V^2 - s}$$

F(s) →0, for s→∞ ⇒ $F_V\, G_V = F^2$

$$F(s)^{VMD} = \frac{M_V^2}{M_V^2 - s}$$

$$\frac{1}{M_V^2 - s} \to \frac{1}{M_V^2\left[1 + \sum_{P,Q} N_{loop}^{PQ}\frac{s}{96\pi^2 F^2} A_{PQ}(s)\right] - s}$$

$$A(m_P^2/s, m_P^2/\mu^2) = \ln\left(m_P^2/\mu^2\right) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P + 1}{\sigma_P - 1}\right)$$

$$\sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

Phys.Lett.B412:382-388,1997

Guerrero, Pich

→ There are several ways of resumming FSI: Omnès exponentiation of Re $A_{PQ}(s)$, dispersion relations, etc...

Nuovo Cim.8:316-326,1958

Form factors in two meson τ decays

$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-}-p_{\pi^0})^\mu F_V^\pi(s)$$

$$F(s)^{\mathrm{VMD}} = \frac{M_\rho^2}{M_\rho^2 - s}$$

Guerrero, Pich '97

$$F(s)^{\mathrm{ChPT}}_{O(p^4)} = 1 + \frac{2L_9^r(\mu)}{f_\pi^2}\, s - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/\mu^2) + \frac{1}{2}A(m_K^2/s, m_K^2/\mu^2)\right]$$

$$A(m_P^2/s, m_P^2/\mu^2) = \ln\left(m_P^2/\mu^2\right) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P+1}{\sigma_P-1}\right) \qquad \sigma_P \equiv \sqrt{1-4m_P^2/s}$$

ChPT+VMD

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

Form factors in two meson τ decays

ChPT+VMD Guerrero, Pich '97

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

Unitarity+Analiticity Omnés, '58

Form factors in two meson τ decays

ChPT+VMD Guerrero, Pich '97

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Unitarity+Analiticity Omnés, '58

O(p^2) result for $\delta^1{}_1$ (s)

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s}\exp\left\{\frac{-s}{96\pi^2 f^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

Form factors in two meson τ decays

ChPT+VMD Guerrero, Pich '97

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Guerrero, Pich '97

$$\Gamma_\rho(s) = \frac{M_\rho\, s}{96\pi f_\pi^2}\left\{\theta(s-4m_\pi^2)\,\sigma_\pi^3 + \frac{1}{2}\theta(s-4m_K^2)\,\sigma_K^3\right\}$$

Gómez-Dumm, Pich, Portolés '00

Phys.Rev.D62:054014,2000

$$= -\frac{M_\rho\, s}{96\pi^2 f_\pi^2}\,\mathrm{Im}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)}\exp\left\{\frac{-s}{96\pi^2 f_\pi^2}\left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

Form factors in two meson τ decays

Starting point

Guerrero, Pich '97

Match χPT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{\frac{-s}{96\pi^2 f_\pi^2}\left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

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Phys.Rev.D57:4136-4141,1998

- χPT up to $O(p^4)$ and leading $O(p^6)$ contributions Guerrero '98
- Right fall-off at high energies
- SU(2)
- Analiticity and unitarity constraints (NNLO)

Phys.Lett.B640:176-181,2006

Idea: Follow the approach of Jamin, Pich, Portolés '06 including excited resonances while retaining (some of) these nice properties

Form factors in two meson τ decays

Starting point

Guerrero, Pich '97

Match χPT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

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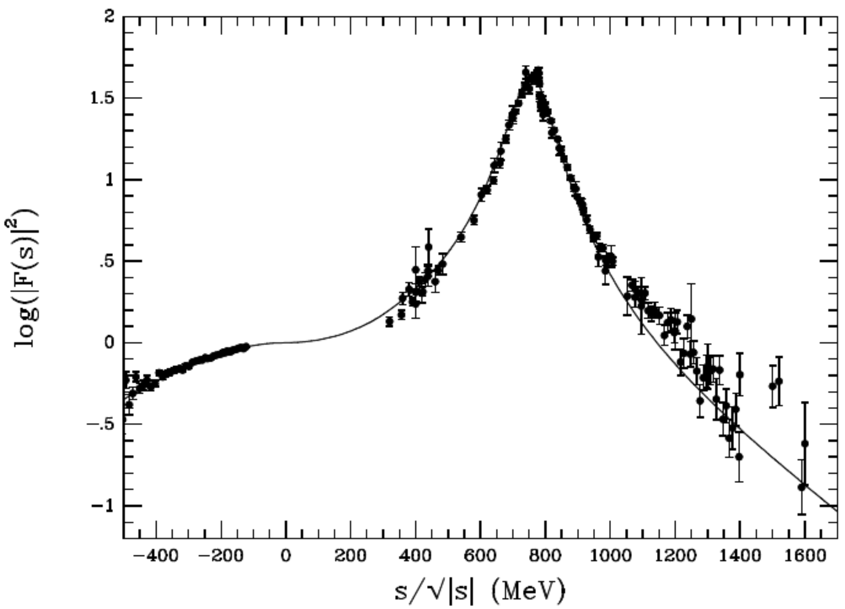

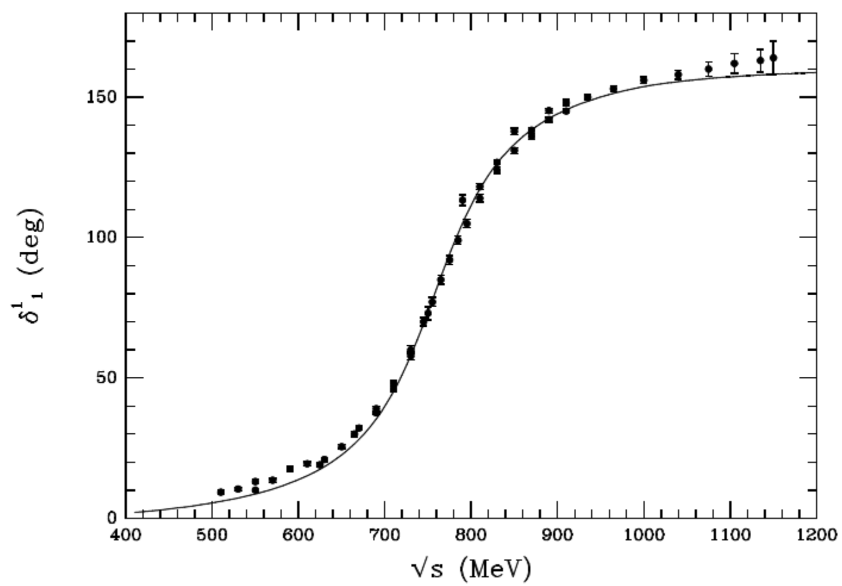

Form factors in two meson τ decays

Starting point

Guerrero, Pich '97

Match χPT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{\frac{-s}{96\pi^2 f_\pi^2}\left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

Our formula

Roig '11

e-Print: **arXiv:1112.0962**

$$F_V^-(s) = \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)}\exp\left[\frac{-s}{96\pi^2 F_\pi^2}\left(\Re e A_\pi(s) + \Re e A_K(s)/2\right)\right] - \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)}\exp\left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3\sigma_\pi^3(M_{\rho'}^2)}\Re e A_\pi(s)\right] - \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)}\exp\left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3\sigma_\pi^3(M_{\rho''}^2)}\Re e A_\pi(s)\right].$$

- χPT up to $O(p^4)$ and leading $O(p^6)$ contributions Guerrero '98
- Right fall-off at high energies
- SU(2)

- Analiticity and unitarity constraints (NNLO)
- (Phenomenological) contribution of ρ' + ρ''

Form factors in two meson τ decays

Starting point

Guerrero, Pich '97

Match χPT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{\frac{-s}{96\pi^2 f_\pi^2}\left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

Our formula

$$F_V^-(s) = \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left[\frac{-s}{96\pi^2 F_\pi^2}\left(\Re e A_\pi(s) + \Re e A_K(s)/2\right)\right]$$
$$- \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re e A_\pi(s)\right]$$
$$- \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re e A_\pi(s)\right] .$$

Roig '11

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- χPT up to $O(p^4)$ and leading $O(p^6)$ contributions Guerrero '98
- Right fall-off at high energies
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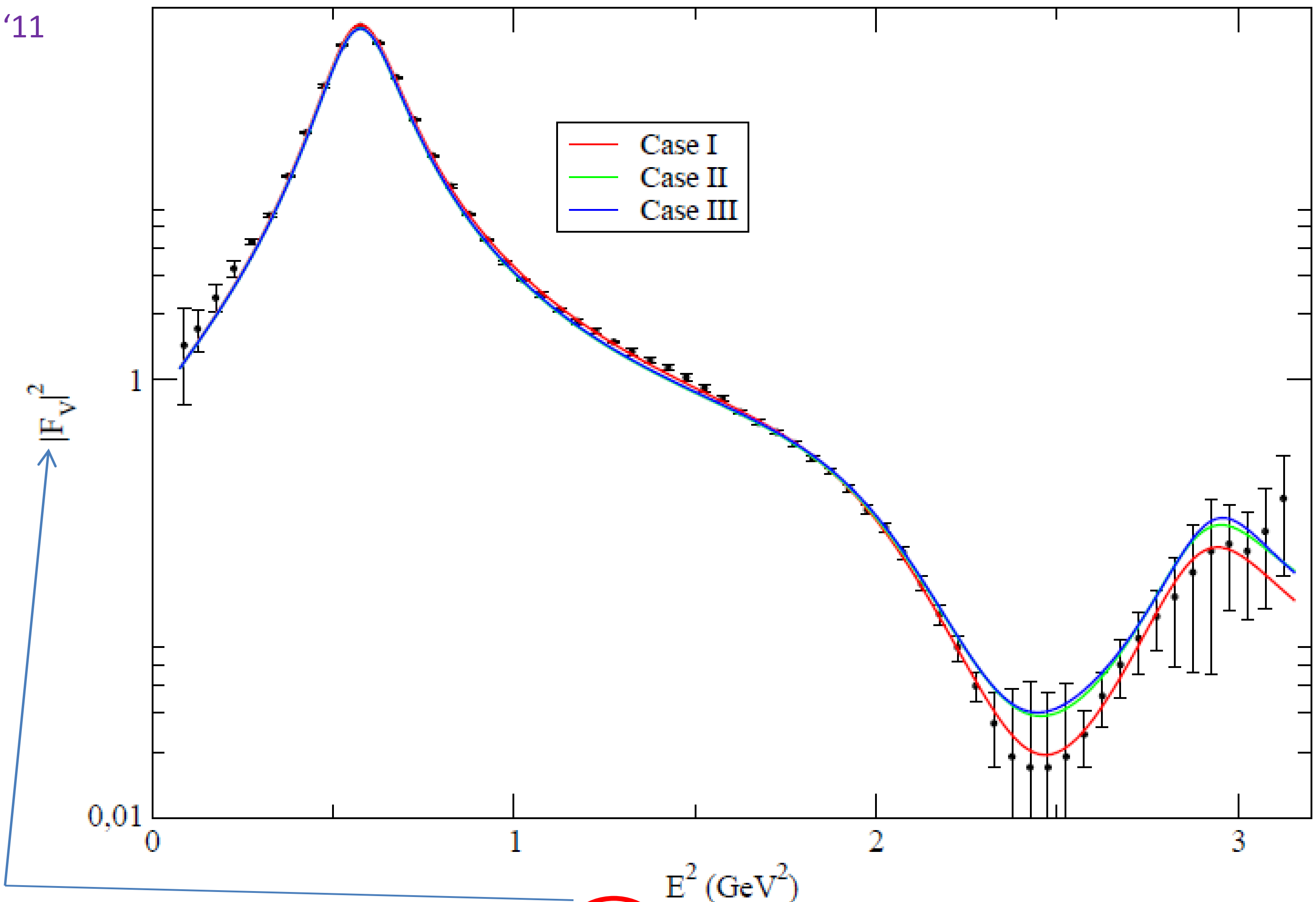
- (Phenomenological) contribution of ρ' + ρ''

This is what is included in TAUOLA right now

Form factors in two meson τ decays

Roig '11



$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\overline{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

Form factors in two meson τ decays

On the inclusion of excited resonances

Our formula

Roig '11

$$F_V^-(s) = \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left[\frac{-s}{96\pi^2 F_\pi^2}\left(\Re e A_\pi(s) + \Re e A_K(s)/2\right)\right] - \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re e A_\pi(s)\right] - \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re e A_\pi(s)\right] .$$

$$\gamma \equiv -F_V' G_V' / F^2 \qquad \delta \equiv -F_V'' G_V'' / F^2 \qquad F_V G_V + F_V' G_V' + F_V'' G_V'' + \ldots = F^2$$

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle$$

Form factors in two meson τ decays

On the inclusion of excited resonances

Our formula

Roig '11

$$F_V^-(s) = \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left[\frac{-s}{96\pi^2 F_\pi^2}\left(\Re e A_\pi(s) + \Re e A_K(s)/2\right)\right]$$
$$- \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re e A_\pi(s)\right]$$
$$- \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re e A_\pi(s)\right] .$$

$$\gamma \equiv -F'_V G'_V / F^2 \qquad \delta \equiv -F''_V G''_V / F^2 \qquad F_V G_V + F'_V G'_V + F''_V G''_V + \ldots = F^2$$

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{i G_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle$$

→ Easy to implement for two meson modes. For three meson modes a number of new couplings (involving new operator structures) appear. At which stage shall we include them?

Form factors in two meson τ decays

• The procedure for other two meson decay channels is analogous (Although SFF relevant in some decay channels).

•An alternative procedure for resummation is described at 'Ongoing/Future improvements'

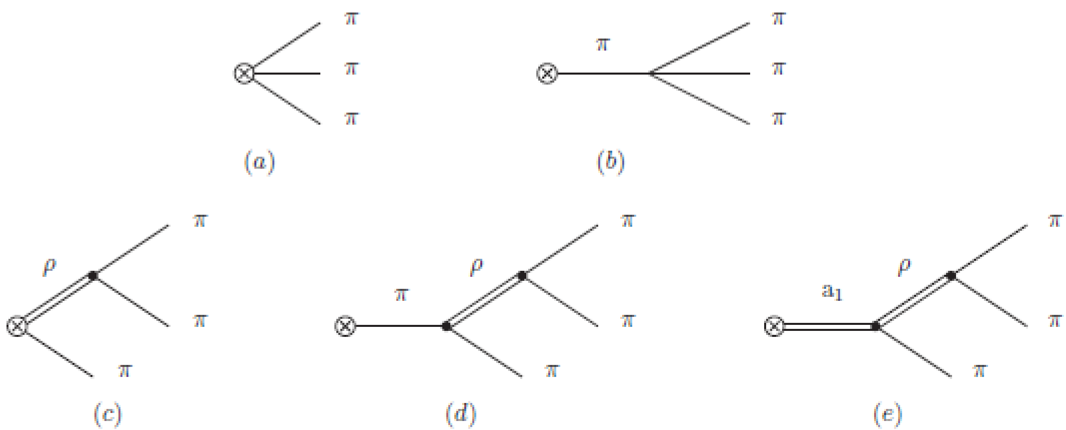
$$F(s)^{VMD} = \frac{M_V^2}{M_V^2 - s - iM_V\Gamma_V(s)}$$

$$F_{PQ}^{V}(s) = F^{VMD}(s)\,\exp\left[\sum_{P,Q} N_{loop}^{PQ}\frac{-s}{96\pi^2F^2}ReA_{PQ}(s)\right]$$

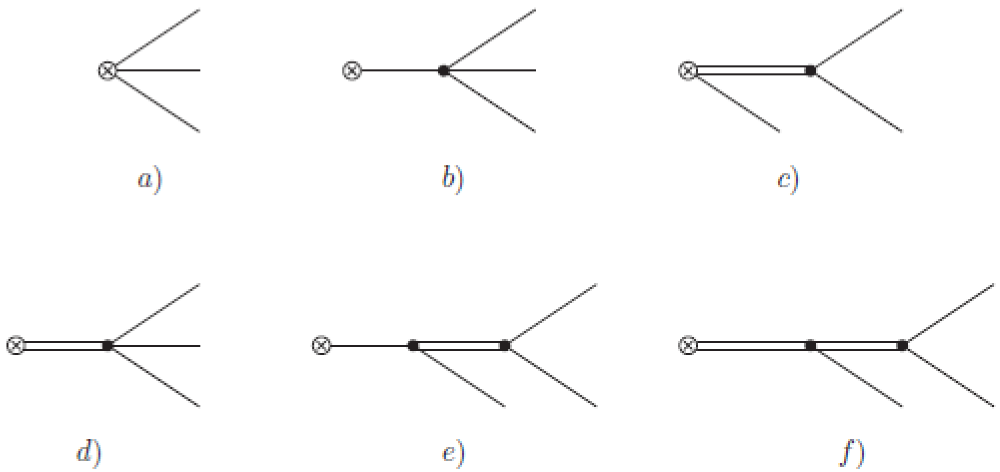
$$A(m_P^2/s, m_P^2/\mu^2) = \ln\left(m_P^2/\mu^2\right) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3\,\ln\left(\frac{\sigma_P+1}{\sigma_P-1}\right) \qquad \sigma_P \equiv \sqrt{1-4m_P^2/s}$$

Form factors in three meson τ decays

$\tau \longrightarrow \pi\pi\pi\nu_\tau$



$\tau \longrightarrow KK\pi\nu_\tau$

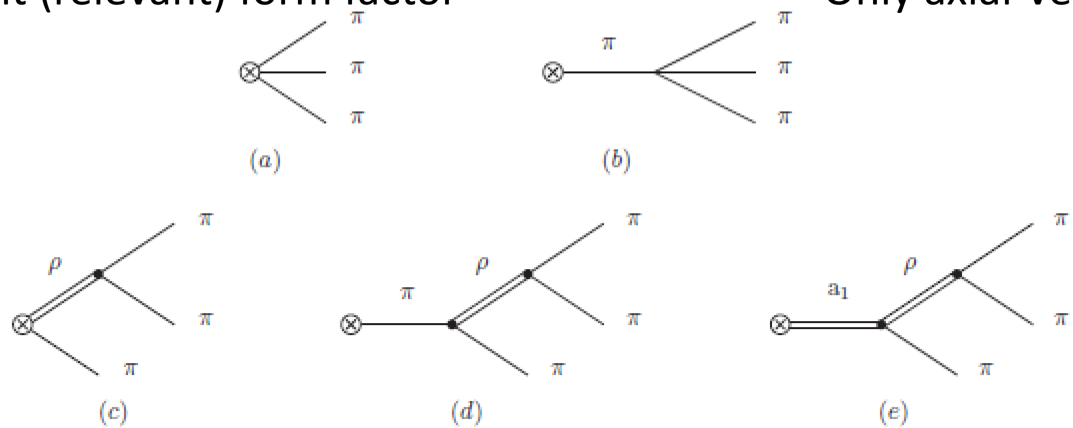


Form factors in three meson τ decays

$\tau \longrightarrow \pi\pi\pi\nu_\tau$

Only one independent (relevant) form factor

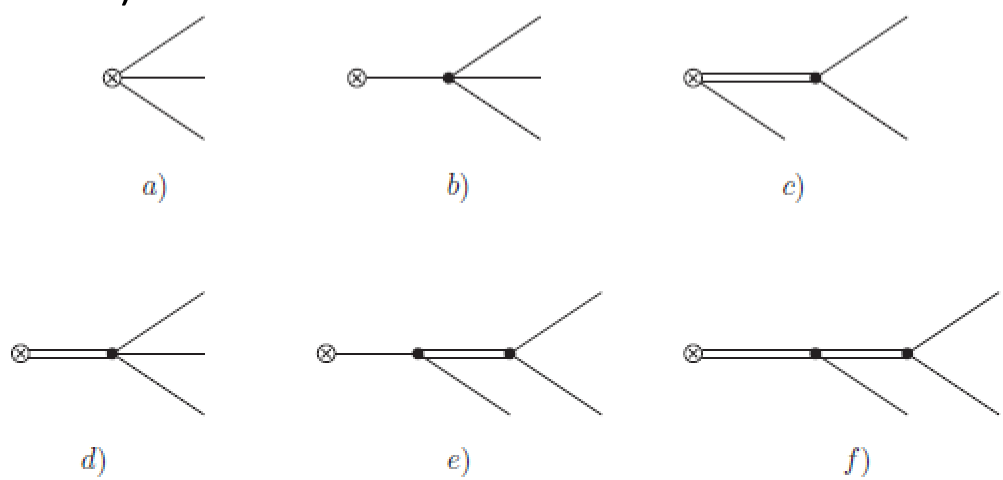
Only axial-vector current



$\tau \longrightarrow KK\pi\nu_\tau$

Three independent (relevant) FFs

Both axial-vector and vector current



Form factors in three meson τ decays

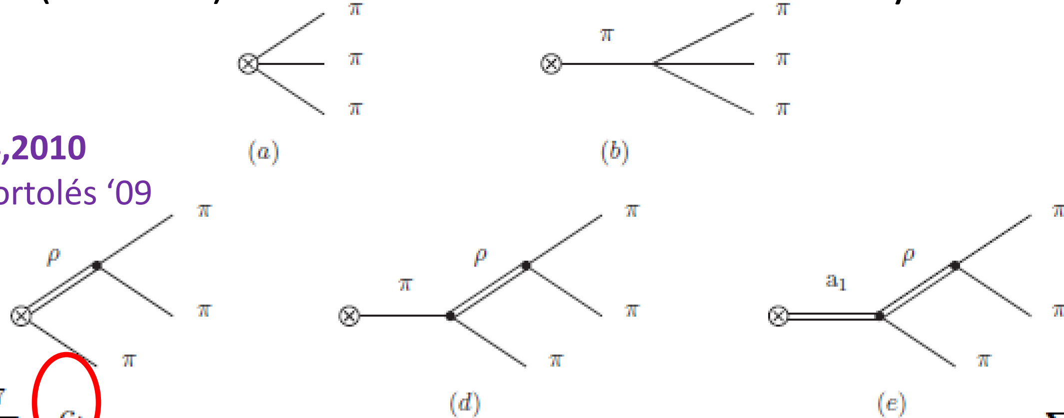
$\tau \longrightarrow \pi\pi\pi\nu_\tau$

Only one independent (relevant) form factor

Only axial-vector current

Phys.Lett.B685:158-164,2010

Gómez-Dumm, Roig, Pich, Portolés ‘09



$$\mathcal{L}_4^{\rm V} = \sum_{i=1}^{5} \frac{g_i}{M_V}\mathcal{O}^i_{\rm VPPP} + \sum_{i=1}^{7} \frac{c_i}{M_V}\mathcal{O}^i_{\rm VJP}$$

$$\mathcal{L}_2^{\rm RR} = \sum_{i=1}^{5} \lambda_i \mathcal{O}^i_{\rm VAP} + \sum_{i=1}^{4} d_i \mathcal{O}^i_{\rm VVP}$$

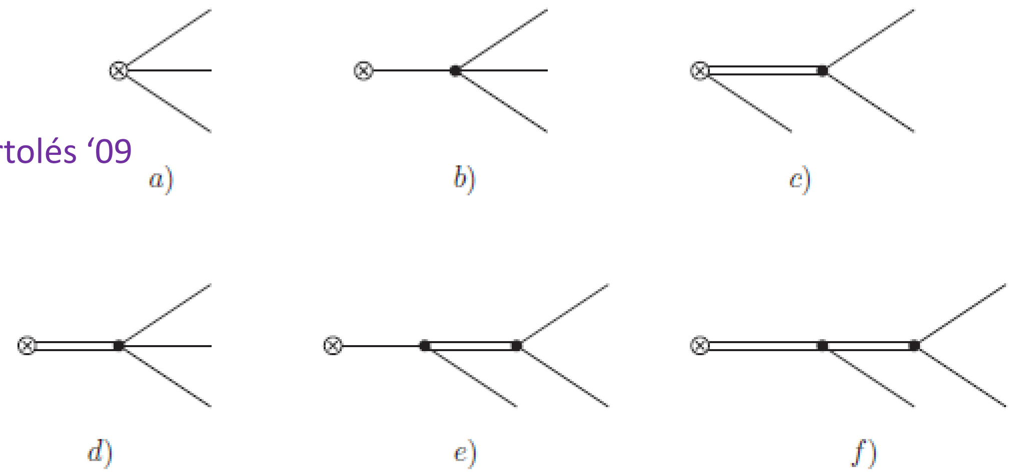
$\tau \longrightarrow K K \pi\nu_\tau$

Three independent (relevant) FFs

Both axial-vector and vector current

Gómez-Dumm, Roig, Pich, Portolés ‘09

Phys.Rev.D81:034031,2010



Pablo Roig (IFAE, Barcelona)

Errors in TAUOLA’s new hadronic currents

Form factors in three meson τ decays

Phys.Rev.D69:073002,2004
Gómez-Dumm, Pich, Portolés '03

Phys.Lett.B685:158-164,2010
Gómez-Dumm, Roig, Pich, Portolés '09

$$F_{\pm i} = \pm(F_i^{\chi} + F_i^{\mathrm{R}} + F_i^{\mathrm{RR}}) \ , \qquad i = 1,2 \qquad F_2(Q^2,s,t) = F_1(Q^2,t,s)$$

$$F_1^{\chi}(Q^2,s,t) = -\frac{2\sqrt{2}}{3F}$$

$$F_1^{\mathrm{R}}(Q^2,s,t) = \frac{\sqrt{2}\,F_V\,G_V}{3\,F^3}\left[\frac{3\,s}{s-M_V^2} - \left(\frac{2G_V}{F_V}-1\right)\left(\frac{2Q^2-2s-u}{s-M_V^2}+\frac{u-s}{t-M_V^2}\right)\right]$$

$$F_1^{\mathrm{RR}}(Q^2,s,t) = \frac{4\,F_A\,G_V}{3\,F^3}\,\frac{Q^2}{Q^2-M_A^2}\left[-(\lambda'+\lambda'')\frac{3\,s}{s-M_V^2} + H(Q^2,s)\,\frac{2Q^2+s-u}{s-M_V^2} + H(Q^2,t)\,\frac{u-s}{t-M_V^2}\right]$$

$$H(Q^2,x) = -\lambda_0\,\frac{m_\pi^2}{Q^2} + \lambda'\,\frac{x}{Q^2} + \lambda''$$

Relations from short-distance QCD:

$$\begin{cases} F_V\,G_V = F^2 \\ F_V^2 - F_A^2 = F^2 \\ F_V^2\,M_V^2 = F_A^2\,M_A^2 \end{cases}$$

$$\lambda' = \frac{F^2}{2\sqrt{2}\,F_A\,G_V} = \frac{M_A}{2\sqrt{2}\,M_V} \ ,$$

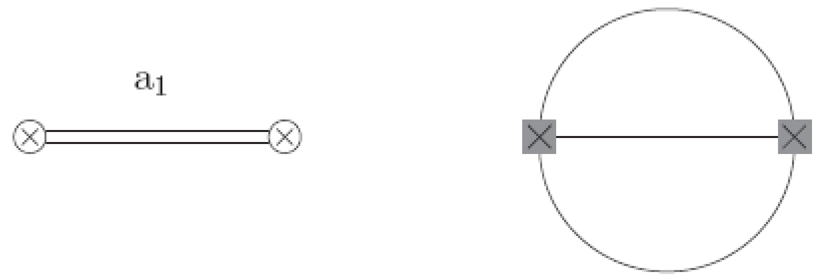
$$\lambda'' = \frac{2\,G_V - F_V}{2\sqrt{2}\,F_A} = \frac{M_A^2 - 2M_V^2}{2\sqrt{2}\,M_V\,M_A} \ ,$$

$$4\,\lambda_0 = \lambda' + \lambda'' = \frac{M_A^2 - M_V^2}{\sqrt{2}\,M_V\,M_A} \ ,$$

Form factors in three meson τ decays

Gómez-Dumm, Roig, Pich, Portolés ‘09

$\Gamma a_1(Q^2)$:



$$\Gamma_{a_1}(Q^2) \;=\; \Gamma^{\pi}_{a_1}(Q^2)\,\theta(Q^2-9m_\pi^2) \;+\; \Gamma^{K}_{a_1}(Q^2)\,\theta(Q^2-(2m_K+m_\pi)^2)\;,$$

$$\Gamma^{\pi,K}_{a_1}(Q^2) \;=\; \frac{-S}{192(2\pi)^3F_A^2M_{a_1}}\left(\frac{M_{a_1}^2}{Q^2}-1\right)^2\int \mathrm{d}s\,\mathrm{d}t\;T^{\pi,K\mu}_{1^+}\,T^{\pi,K*}_{1^+\mu}\;.$$

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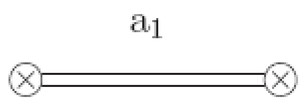

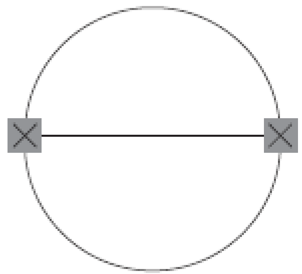

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$$\frac{1}{M_\rho^2-q^2-iM_\rho\Gamma_\rho(q^2)}\longrightarrow\frac{1}{1+\beta_{\rho'}}\left[\frac{1}{M_\rho^2-q^2-iM_\rho\Gamma_\rho(q^2)}+\frac{\beta_{\rho'}}{M_{\rho'}^2-q^2-iM_{\rho'}\Gamma_{\rho'}(q^2)}\right]$$

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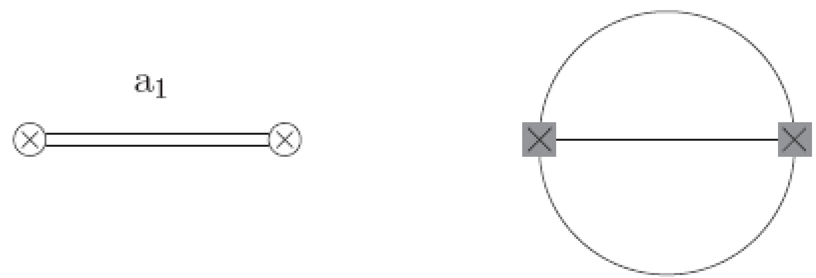

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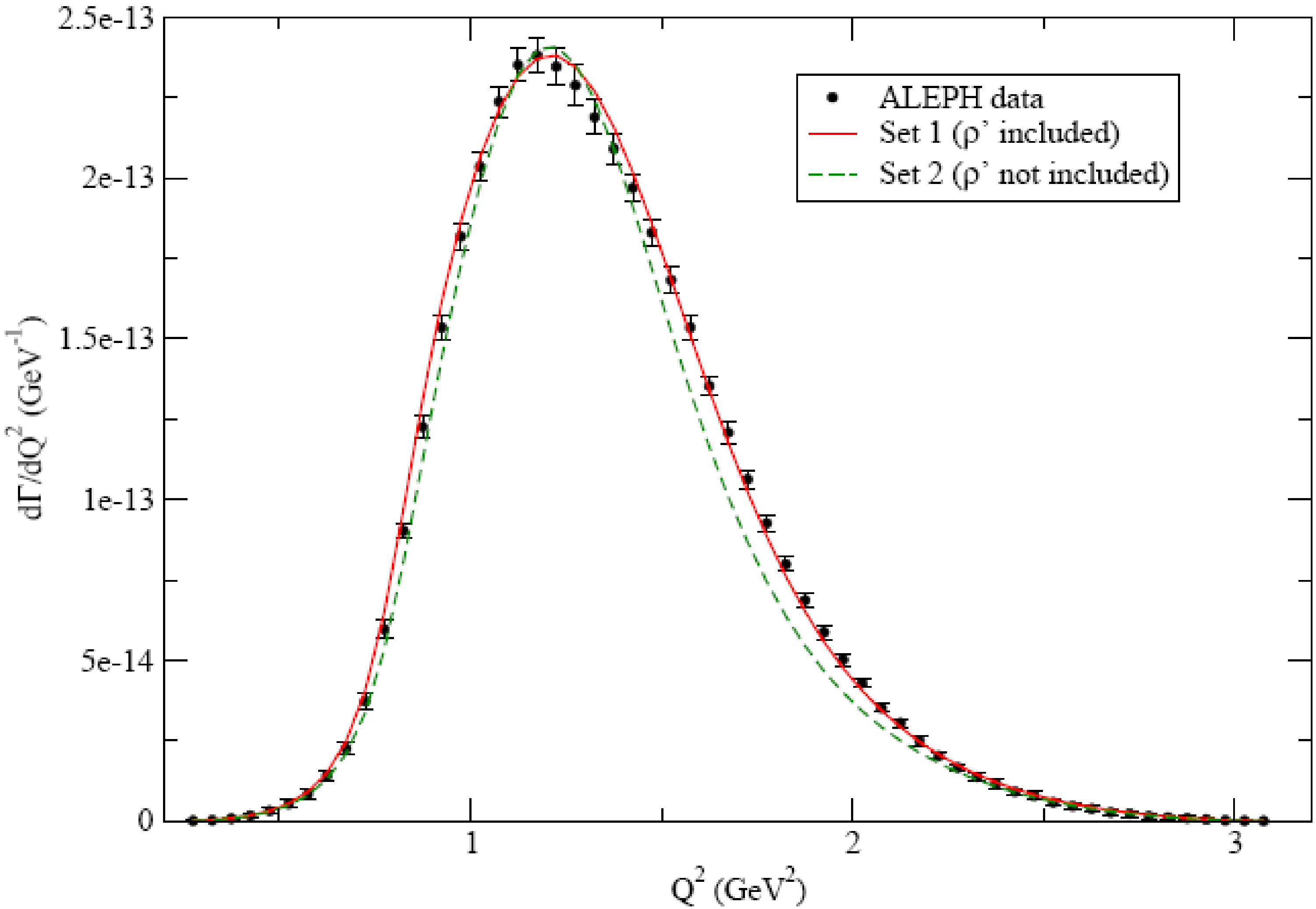
→ Inclusion from a Lagrangian would imply 3 coups. instead of $\beta_{\rho'}$
F_V', G_V', F_A'

Form factors in three meson τ decays

Gómez-Dumm, Roig, Pich, Portolés '09



Discussion on the errors

- $\varepsilon(1/N_c)\sim 1/3$?

't Hooft '74, Witten '79

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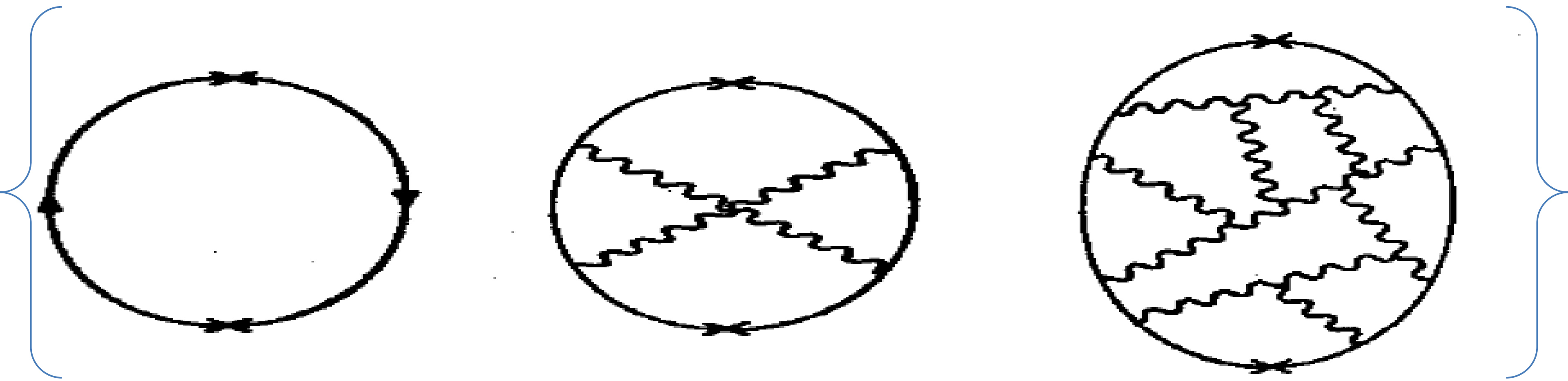

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At NLO in $1/N_C$ non-planar diagrams are suppressed as $1/N_C^2$ and diagrams with internal quark loops as $1/N_C$.

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The latter would be responsible of mixing between $q\bar{q}$ & $q\bar{q}q\bar{q}$ states, which is known to be negligible phenomenologically, from the success of quark meson spectroscopy.

⇒ We can expect leading corrections of order $1/N_C^2$.

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- Resummation in the two meson modes

$$F_{PQ}^{V}(s) = F^{VMD}(s)\,\exp\left[\sum_{P,Q} N_{loop}^{PQ}\frac{-s}{96\pi^2F^2}Re A_{PQ}(s)\right]$$

$$F(s)^{VMD} = \frac{M_V^2}{M_V^2 - s - iM_V\Gamma_V(s)}$$

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Alternatively:

Exact Unitarity

$$F_V(s) = \frac{M_V^2}{M_V^2\left[1+\sum_{P,Q} N_{loop}^{PQ}\frac{s}{96\pi^2F^2}A_{PQ}(s)\right] - s}$$

$$\delta^{PQ}(s) = Im\left[F_V^{PQ}(s)\right]/Re\left[F_V^{PQ}(s)\right]$$

$$F_V^{PQ}(s) = \exp\left\{\alpha_1 s + \alpha_2 s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{s_{cut}} ds'\frac{\delta^{PQ}(s')}{s'^3(s'-s-i\epsilon)}\right\}$$

Tiny differences in observables between both approaches in $\tau\to(K\,\pi)\,\nu_\tau$

Jamin, Pich, Portolés '06, '08 Boito, Escribano, Jamin '07

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• $\varepsilon(1/N_c)<1/3$

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$$F_I^{\rm scal}(x) = F_I^{\rm resonant}(x) + R_I^{\rm scal}(x)$$

$$\begin{aligned} F_+ &= F_+^\chi + F_+^R + F_+^{RR} + \sqrt{2}\left[R_0^{\rm scal}(s) + R_0^{\rm scal}(t)\right] + R_2^{\rm scal}(s) + R_2^{\rm scal}(t)\,, \\ F_- &= -(F_+^\chi + F_+^R + F_+^{RR}) - \left[R_0^{\rm scal}(s) + R_0^{\rm scal}(t)\right] + \sqrt{2}\left[R_2^{\rm scal}(s) + R_2^{\rm scal}(t)\right] \end{aligned}$$

Isidori, Maiani, Nicolacci, Pacetti

JHEP 0605 (2006) 049

$$R_0(x) = \left\{\frac{\alpha_0}{Q^2} + \frac{\alpha_1}{Q^4}(x - M_{f_0}^2) + \mathcal{O}\left[(x - M_{f_0}^2)^2\right]\right\} e^{i\delta_0(x)}$$

Schenk **Nucl.Phys. B363 (1991) 97-116**

Colangelo, Gasser, Leutywler

Nucl.Phys. B603 (2001) 125-179

$$\tan\delta_I(x) = \sigma_\pi(x)\left(A_0^I + B_0^I q^2 + C_0^I q^4 + D_0^I q^6\right)\frac{4m_\pi^2 - x_0^I}{x - x_0^I}$$

We have to check if this approach is enough to confront the data successfully.

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- SU(3) breaking terms in the Lagrangian.
- Spin zero resonance contributions.
- Excited resonance contribution in $KK\pi$ channels.
- Complete $O(p^6)$ χPT, i.e. NNLO, in $\pi\pi$ channels.
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- Some important remaining modes: $\pi\pi\pi\pi$, $K\pi\pi$, …

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Ongoing/Future improvements

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Moussallam **Eur.Phys.J.C53:401-412,2008**

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•SU(2) breaking in $\pi\pi$ channels. Cirigliano, Ecker, Neufeld **Phys.Lett.B513:361-370,2001** **JHEP 0208:002,2002**

•Some important remaining modes: $\pi\pi\pi\pi$, $K\pi\pi$, SFF in $K\pi$ … Jamin, Oller, Pich '01,'06

•Remaining two meson modes: $\pi\eta^{(\prime)}, K\eta^{(\prime)}$ Escribano, Jamin, Roig in progress

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THANK YOU!

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