Gauge invariant definition of jet quenching parameter

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Outline

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2. The jet quenching parameter $\hat{q}$
3. Jet broadening in covariant and light-cone gauge
4. Comparison with the literature
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original work based on

M. Benzke, N. Brambilla, M.A. Escobedo and A. Vairo

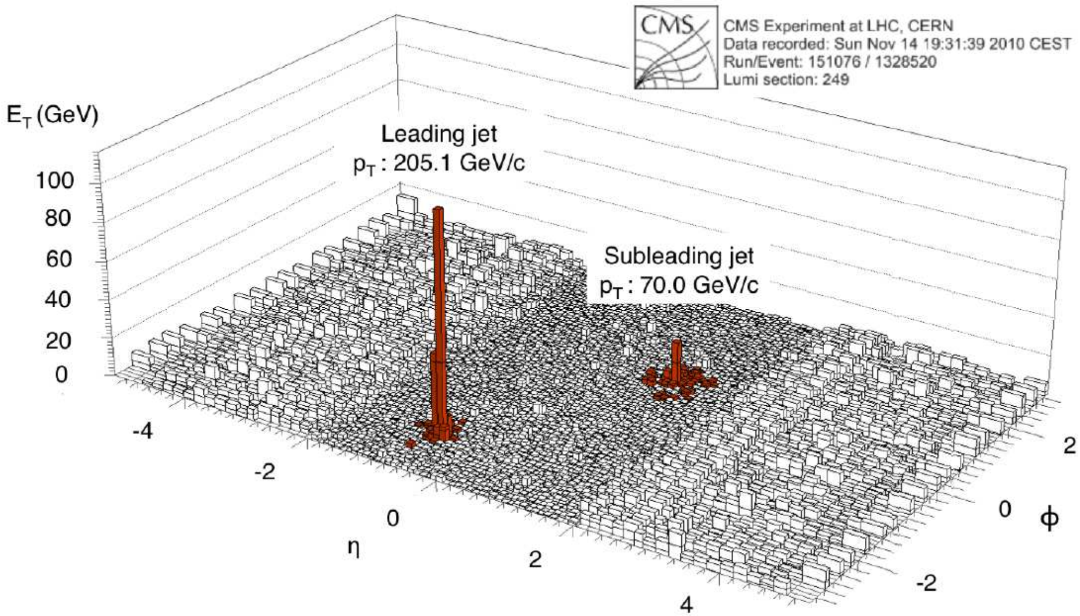
Gauge invariant definition of jet quenching parameter

TUM-EFT 28/11

Jet quenching

Jet quenching was first observed at RHIC and then confirmed at LHC.

This phenomenon happens when a very energetic quark or gluon, $Q \gg T$, which in vacuum would manifest itself as a jet, going through a strongly coupled plasma loses sufficient energy that few high momentum hadrons are seen in the final state.

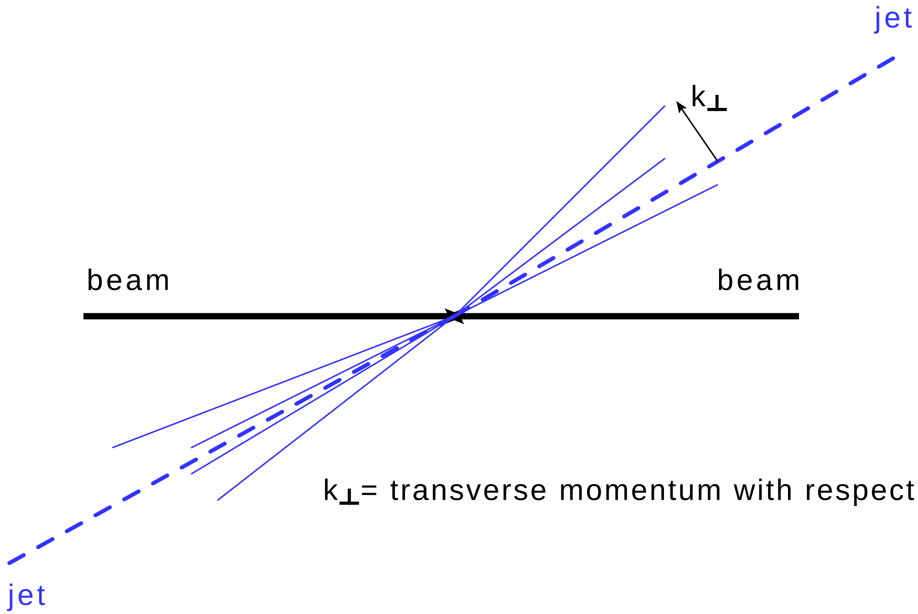


Jet quenching

Jet quenching manifests itself in many observables.

In particular, the hard partons produced in the collision

- lose energy;
- change direction of their momenta: transverse momentum broadening.



$k_\perp$= transverse momentum with respect to the jet direction

Jet quenching parameter $\hat{q}$

$P(k_\perp)$ is the probability that after propagating through the medium for a distance L the hard parton acquires transverse momentum $k_\perp$,

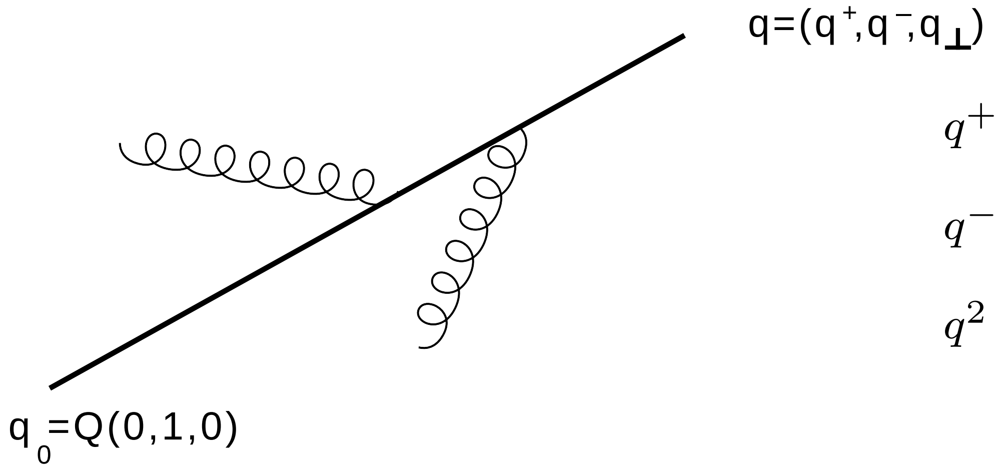
$$\int \frac{d^2 k_\perp}{(2\pi)^2} P(k_\perp) = 1$$

$\hat{q}$ is the jet quenching parameter, i.e. the mean transverse momentum picked up by the hard parton per unit distance travelled,

$$\hat{q} = \frac{\langle k_\perp^2 \rangle}{L} = \frac{1}{L} \int \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 \, P(k_\perp)$$

Energy scales: kinematics and medium

A highly energetic parton, $Q > 100$ GeV, propagates along the light-cone direction $\bar{n}$.



$$q^+ = \bar{n} \cdot q \qquad \bar{n} = (1,0,0,-1)/\sqrt{2}$$

$$q^- = n \cdot q \qquad n = (1,0,0,1)/\sqrt{2}$$

$$q^2 = 2q^- q^+ - q_\perp^2$$

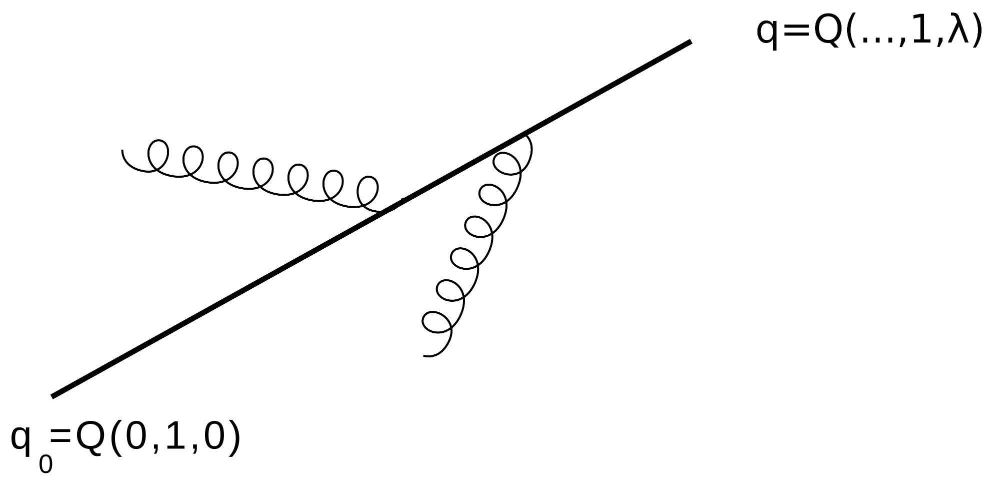
In a medium characterized by a temperature $T \ll Q$:

$$\lambda \equiv \frac{T}{Q} \ll 1$$

λ will be the relevant expansion parameter.

Energy scales: collinear partons

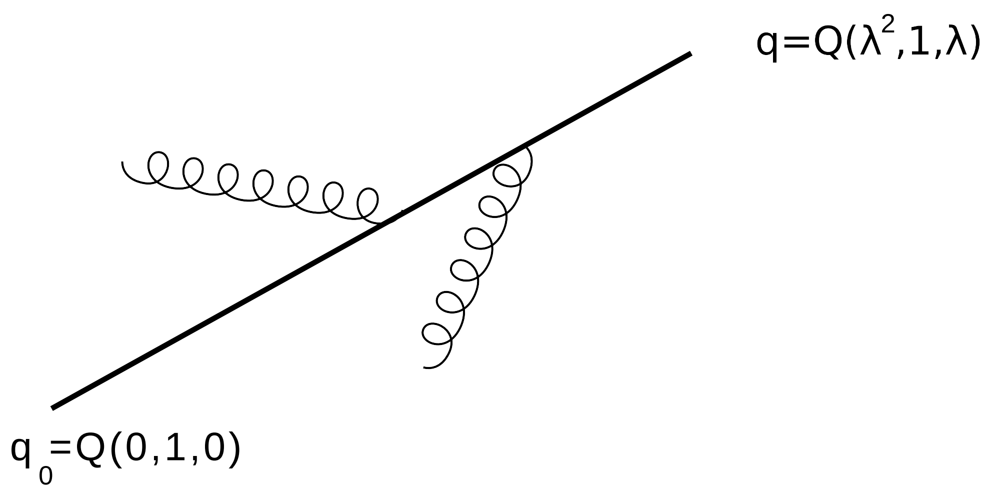
We consider partons that undergo a transverse momentum broadening of order $Q\lambda$:



- if $q = Q(\lambda, 1, \lambda)$, the parton is off shell by $\sim Q^2\lambda$: the process is suppressed by $\alpha_{\rm s}(\sqrt{TQ})$;
- if $q = Q(\lambda^2, 1, \lambda)$, the parton is off shell by $\sim Q^2\lambda^2$: the parton is collinear.

Energy scales: Glauber gluons

We consider the transverse momentum broadening of a collinear parton:



It may happen through

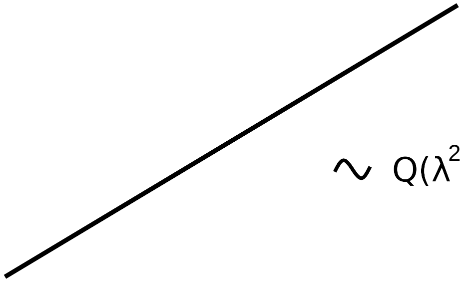
- fragmentation into collinear partons (gluons, quarks) of momentum $q = Q(\lambda^2, 1, \lambda)$;
- scattering by Glauber gluons of momentum $q = Q(\lambda^2, \lambda, \lambda)$ or $q = Q(\lambda^2, \lambda^2, \lambda)$.

○ Idilbi Majumder PR D80 (2009) 054022

Degrees of freedom

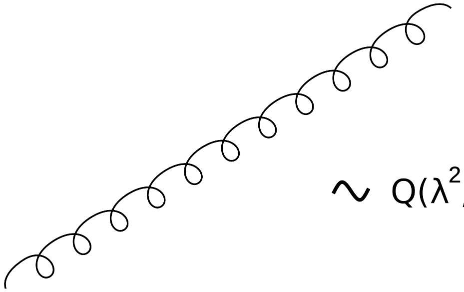
The relevant degrees of freedom are:

- Collinear partons ($\xi_{\bar{n}}$, A_μ^c)



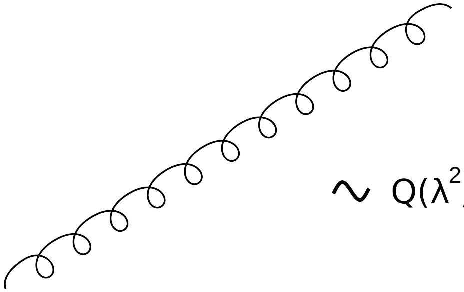
$\sim Q(\lambda^2, 1, \lambda)$

- Glauber gluons (A_μ)



$\sim Q(\lambda^2, \lambda^2, \lambda)$ or $Q(\lambda^2, \lambda, \lambda)$

- Ultrasoft gluons (A_μ)



$\sim Q(\lambda^2, \lambda^2, \lambda^2)$

An EFT for transverse momentum broadening

The EFT that describes the propagation of collinear partons in the $\bar{n}$-direction is SCET coupled to collinear, Glauber and ultrasoft gluons:

$$\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}}\, i\not{n}\bar{n}\cdot D\,\xi_{\bar{n}} + \bar{\xi}_{\bar{n}}\, i\not{D}_\perp \frac{1}{2in\cdot D}\, i\not{D}_\perp \not{n}\,\xi_{\bar{n}}$$

We will consider now the contribution of Glauber gluons only and rescale $\bar{\xi}_{\bar{n}} \to e^{-iQx^+}\bar{\xi}_{\bar{n}}$. The Lagrangian, organized as an expansion in λ, is then

$$\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}}\, i\not{n}\bar{n}\cdot D\,\xi_{\bar{n}} + \bar{\xi}_{\bar{n}}\,\frac{D_\perp^2}{2Q}\not{n}\,\xi_{\bar{n}} - \bar{\xi}_{\bar{n}}\,\frac{G_\perp^{\mu\nu}}{4Q}\gamma_\mu\gamma_\nu\not{n}\,\xi_{\bar{n}} + \dots$$

- Bauer Fleming Pirjol Stewart PR D63 (2001) 114020
  Idilbi Majumder PR D80 (2009) 054022

Power-counting in covariant gauges

In a covariant gauge, the scaling of the fields appearing in the Lagrangian goes like

$$A^+ \sim A_\perp \sim Q\lambda^2$$

which follows from the collinear nature of the propagating parton and Lorentz symmetry.

- Note, however, that the fields are not homogeneous in the energy scales.

Because $\partial_\perp \sim \lambda$, the leading order Lagrangian in λ is

$$\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}}\, i\not{n}\bar{n}\cdot D\, \xi_{\bar{n}} + \bar{\xi}_{\bar{n}}\, \frac{\partial_\perp^2}{2Q} \not{n}\, \xi_{\bar{n}}$$

Power-counting in light-cone gauge

Consider the light-cone gauge $A^+ = 0$:

$$D_{\mu\nu}(k) = D(k^2)\left(g_{\mu\nu} - \frac{k_\mu \bar{n}_\nu + k_\nu \bar{n}_\mu}{[k^+]}\right)$$

for Glauber gluons $k_\perp/[k^+] \sim 1/\lambda$, which leads to on enhancement of order λ in the singular part of the propagator.

We write

$$A_\perp(x^+, x^-, x_\perp) = A_\perp^{\mathrm{cov}}(x^+, x^-, x_\perp) + \theta(x^-) A_\perp(x^+, \infty^-, x_\perp) + \theta(-x^-) A_\perp(x^+, -\infty^-, x_\perp)$$

where $A_\perp(x^+, \infty^-, x_\perp) = \partial_\perp \phi(x^+, \infty^-, x_\perp)$:

- $A_\perp$ does not vanish at infinity where it becomes pure gauge,
- but the field tensor does (because the energy of the gauge field is finite).

○ Belitsky Yuan NP B656 (2003) 165
Garcia-Echevarria Idilbi Scimemi PR D84 (2011) 011502

Power-counting in light-cone gauge

In the $A^+ = 0$ light-cone gauge, the scaling of the fields appearing in the Lagragian goes like

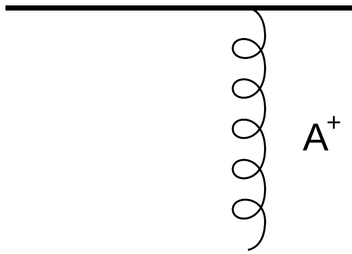
$$A^+ = 0 \qquad A_\perp^{\rm cov} \sim Q\lambda^2 \qquad \partial_\perp \phi \sim Q\lambda$$

The leading order Lagrangian in λ is

$$\mathcal{L}_{\bar{n}} = \bar{\xi}_{\bar{n}}\, i\not{n}\bar{n} \cdot \partial_-\, \xi_{\bar{n}} + \bar{\xi}_{\bar{n}}\, \frac{(\partial_\perp - ig\partial_\perp \phi)^2}{2Q} \not{n}\, \xi_{\bar{n}}$$

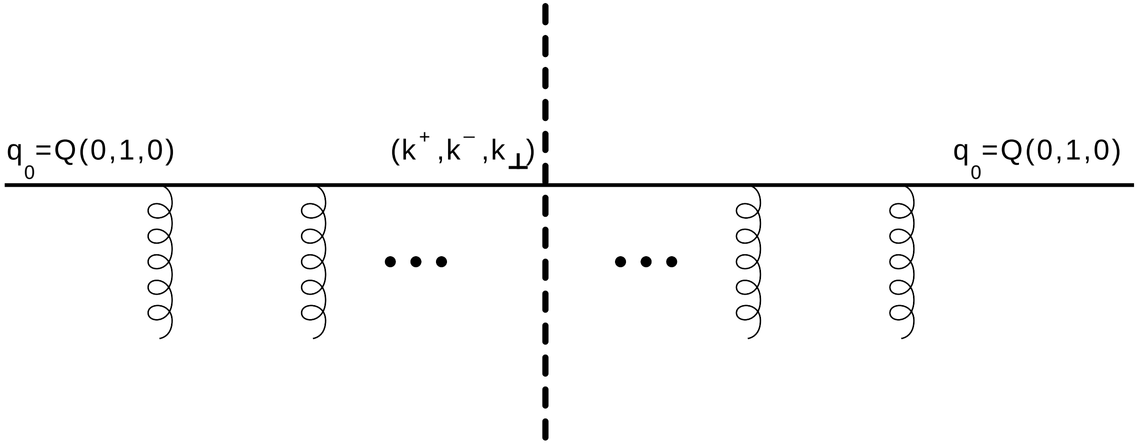
Jet broadening in covariant gauges

Only one relevant vertex



$$= igT^a \bar{n}^\mu \not{n}$$

for the scattering amplitude



$$= P(k_\perp)$$

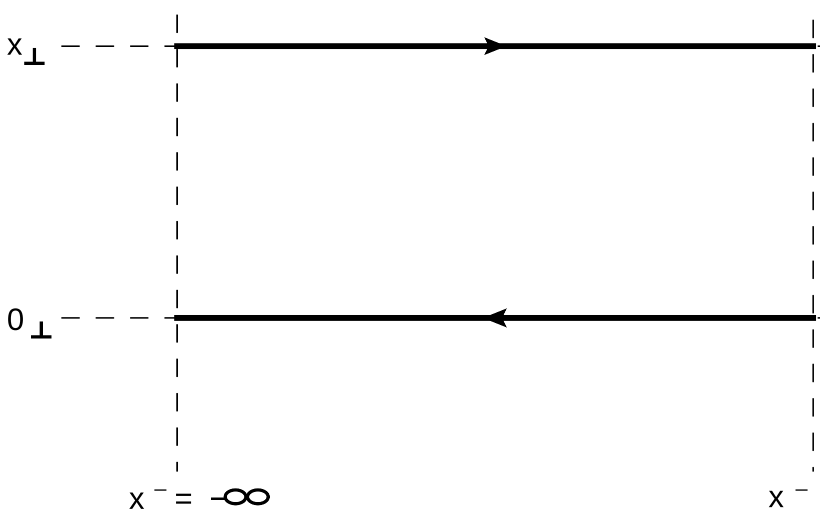
for $k_\perp \neq 0$ and normalizing by the number of particles in the medium ($= \Delta t/L$).

Jet broadening in covariant gauges

$$P(k_\perp) = \frac{1}{d_R} \int d^2x_\perp e^{ik_\perp \cdot x_\perp} \left\langle \mathrm{Tr} \left\{ W_R^\dagger[0^+, x_\perp] W_R[0^+, 0_\perp] \right\} \right\rangle_T$$

where $\langle \ldots \rangle_T$ is a thermal average, $d_F = N_c$, $d_A = N_c^2 - 1$

and $W_R[0^+, x_\perp] = \mathrm{P}\exp\left\{ ig \int_{-L/\sqrt{2}}^{L/\sqrt{2}} dx^- \, A^+(0^+, x^-, x_\perp) \right\}$; for $L \to \infty$:

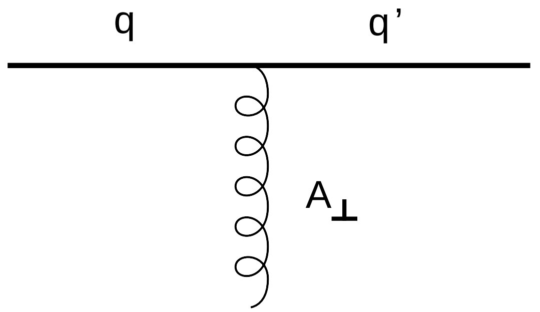


- The fields are path ordered (P) but not time ordered;
- the expression is not gauge invariant (e.g. for $A^+ = 0$, $W_R = 1$).

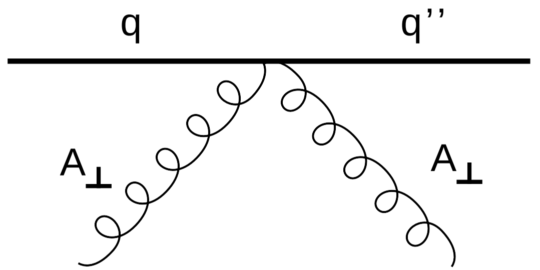
∘ Baier et al NP B483 (1997) 291, Zakharov JETPL 63 (1996) 952
Casalderrey-Solana Salgado APP B38 (2007) 3731
D'Eramo Liu Rajagopal PR D84 (2011) 065015

Jet broadening in light-cone gauge

The relevant vertices are two



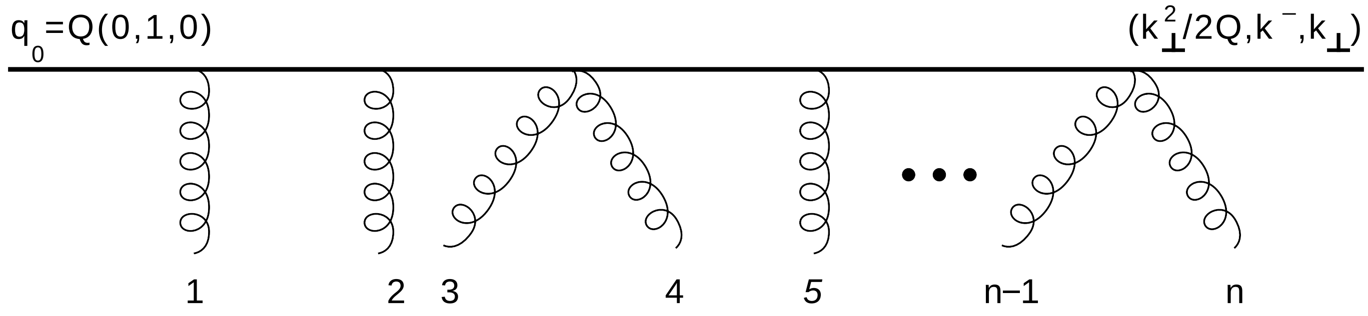
$$= -ig\,\frac{q'_\perp \cdot A_\perp(q'-q) + A_\perp(q'-q)\cdot q_\perp}{2Q}\,\not{n}$$



$$= -\frac{ig^2}{2Q}\int \frac{d^4q'}{(2\pi)^4}\,A^i_\perp(q''-q')A^i_\perp(q'-q)\not{n}$$

Jet broadening in light-cone gauge

From the vertices one constructs the amplitude (on the right of the cut)



$= G_n(k^-, k_\perp)$

G_n is a convolution of G^+_{n-j}, which involves only fields at $x^- = \infty$ and G^-_j, which involves only fields at $x^- = -\infty$:

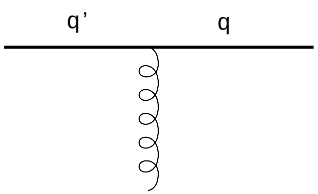
$$G_n(k^-, k_\perp) = \sum_{j=0}^{n} \int \frac{d^4q}{(2\pi)^4} \, G^+_{n-j}(k^-, k_\perp, q) \, \frac{iQ\,\not{n}}{2Qq^+ - q_\perp^2 + i\epsilon} \, G^-_j(q)$$

Jet broadening in light-cone gauge

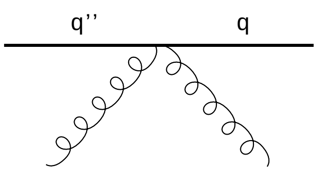
The computation is done through the following steps:

- solve recursively (analogously for $G_n^+(q)$)

$$G_n^-(q) = \int \frac{d^4q'}{(2\pi)^4}\, G_{n-1}^-(q')$$



$$+\int \frac{d^4q''}{(2\pi)^4}\, G_{n-2}^-(q'')$$



- write the differential amplitude as

$$\frac{1}{L^3\sqrt{2}Q}\int\frac{dk^+}{2\pi}\int\frac{dk^-}{2\pi}\,2\pi\,Q\,\delta(2Qk_+ - k_\perp^2)\;\bar{\xi}_{\bar{n}}(q_0)\,G_m^\dagger(k^-,k_\perp)\bar{\not{n}}G_n(k^-,k_\perp)\,\xi_{\bar{n}}(q_0)$$

- eventually sum over all m and n.

Jet broadening in light-cone gauge

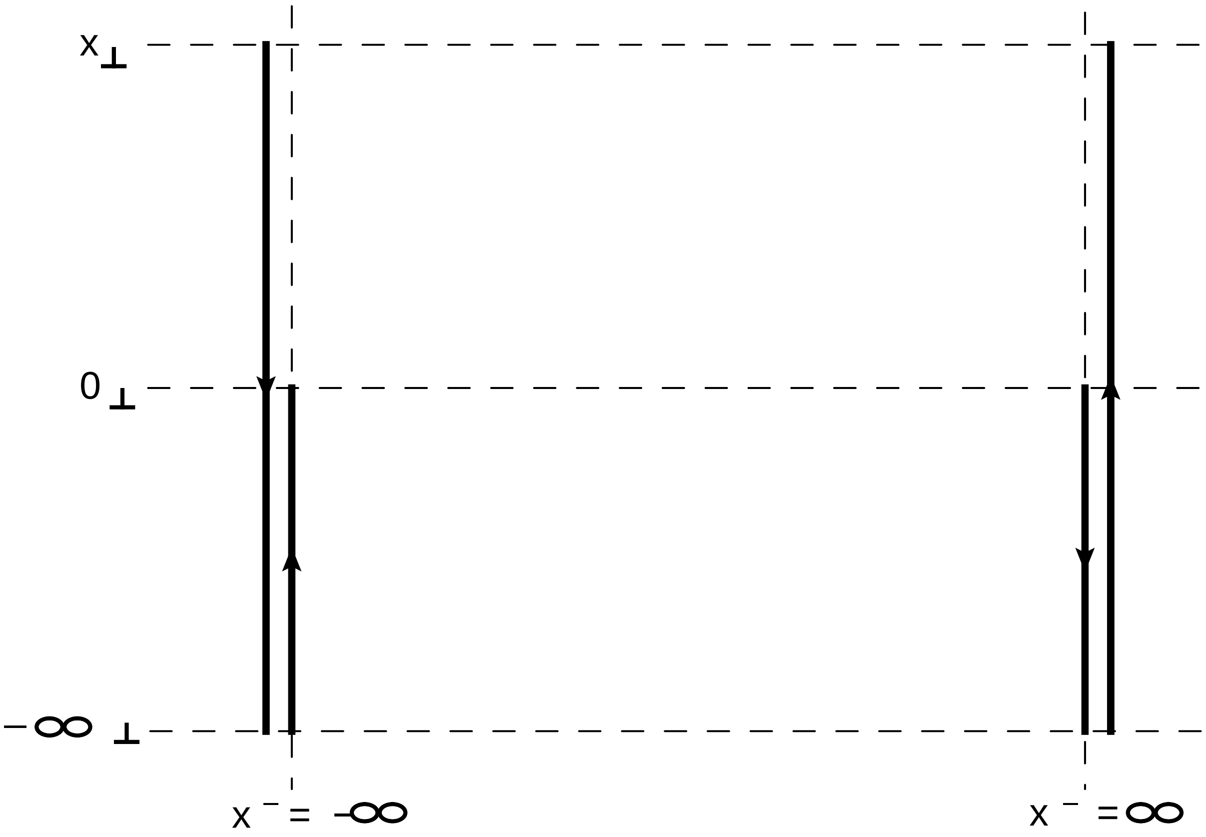
$$P(k_\perp) = \frac{1}{d_R}\int d^2x_\perp e^{ik_\perp\cdot x_\perp}\left\langle \mathrm{Tr}\left\{T^\dagger[0^+,-\infty^-,x_\perp]T[0^+,\infty^-,x_\perp]\right.\right.$$
$$\left.\left.\times\; T^\dagger[0^+,\infty^-,0_\perp]T[0^+,-\infty^-,0_\perp]\right\}\right\rangle_T$$

where $T[0^+,\pm\infty,x_\perp] = \mathrm{P}\exp\left\{-ig\int_{-L/\sqrt{2}}^{0} ds\, l_\perp\cdot A_\perp(0^+,\pm\infty^-,x_\perp+sl_\perp)\right\}$

The relevance of the Wilson line T in light-cone gauge SCET has been discussed in

- `Idilbi Scimemi PL B695 (2011) 463`

Jet broadening in light-cone gauge



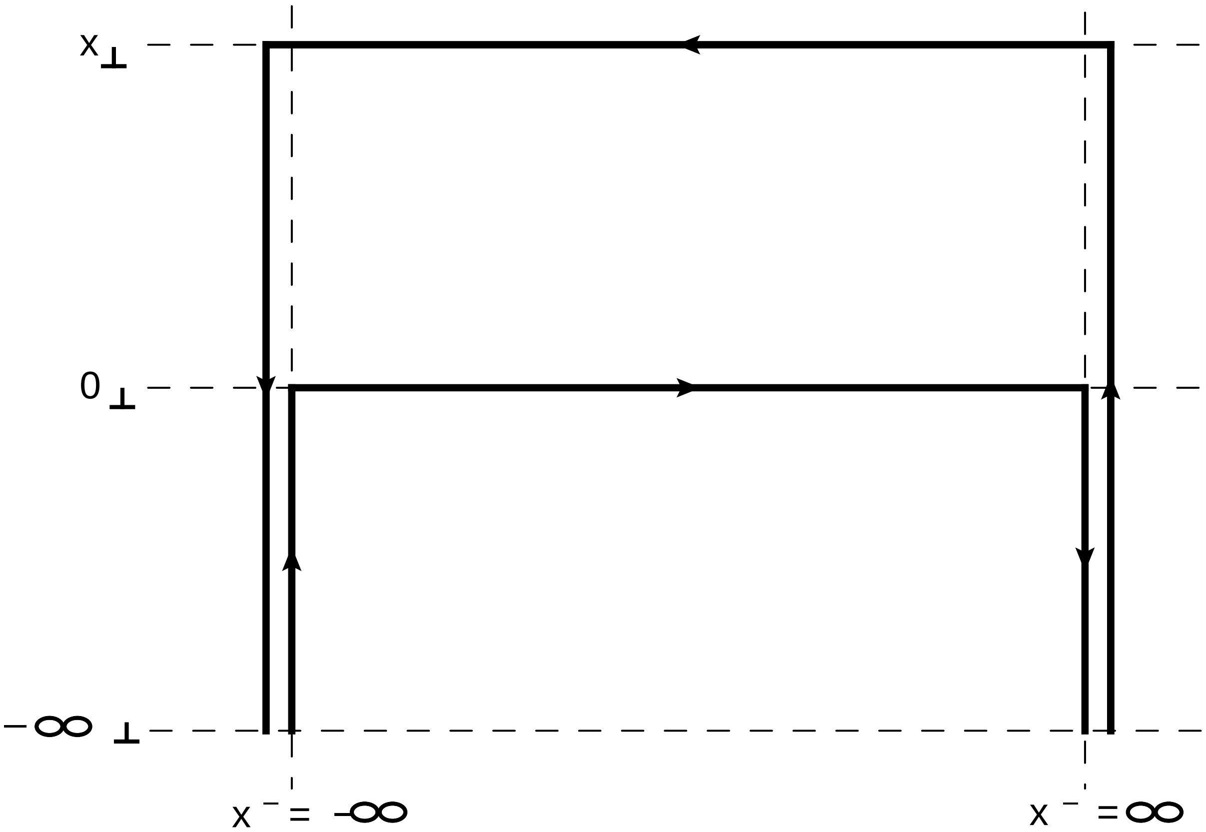
The gauge-invariant expression of $P(k_\perp)$

The gauge invariant expression of $P(k_\perp)$ then reads

$$
\begin{aligned}
P(k_\perp) \;\; = \;\; & \frac{1}{d_R} \int d^2 x_\perp e^{ik_\perp \cdot x_\perp} \Big\langle \mathrm{Tr} \Big\{ T^\dagger[0^+, -\infty^-, x_\perp] W_R^\dagger[0^+, x_\perp] T[0^+, \infty^-, x_\perp] \\
& \times \; T^\dagger[0^+, \infty^-, 0_\perp] W_R[0^+, 0_\perp] T[0^+, -\infty^-, 0_\perp] \Big\} \Big\rangle_T
\end{aligned}
$$

The path ordering prescription implies that fields in the first line are anti-time ordered while fields in the second line are time ordered.

The gauge-invariant expression of $P(k_\perp)$

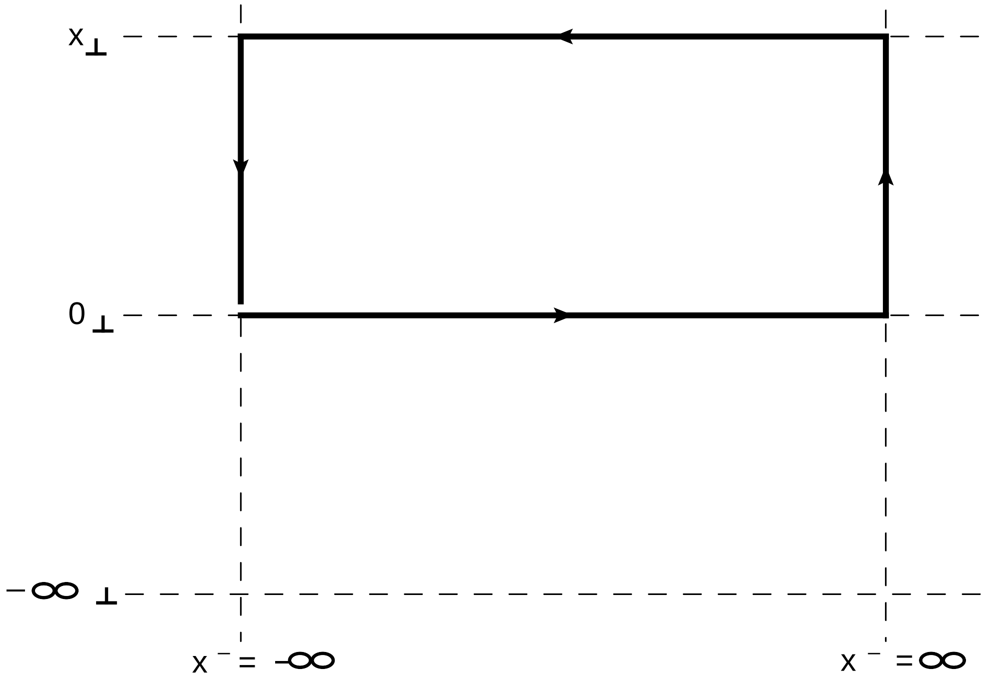


- Note that the Wilson lines at $x^- = \infty$ are contiguous while those at $x^- = -\infty$ are not. This is because the fields are not time ordered.

The gauge-invariant expression of $P(k_\perp)$

The expression of $P(k_\perp)$ may be simplified because:

- contiguous adjoint (unitary) lines cancel;
- fields separated by space-like intervals commute;
- the cyclicity of the trace.



- Note that the fields in $(0^+, -\infty^-, 0_\perp)$ are not contiguous.

Gauge invariance

Under a gauge transformation Ω

$$\mathrm{Tr}\left\{T^{\dagger}[0^{+},-\infty^{-},x_{\perp}]W_{R}^{\dagger}[0^{+},x_{\perp}]\cdots T[0^{+},-\infty^{-},0_{\perp}]\right\}$$

$$\longrightarrow \mathrm{Tr}\left\{\Omega(0^{+},-\infty^{-},-\infty\, l_{\perp})T^{\dagger}[0^{+},-\infty^{-},x_{\perp}]W_{R}^{\dagger}[0^{+},x_{\perp}]\cdots \right.$$
$$\left. \times T[0^{+},-\infty^{-},0_{\perp}]\Omega^{\dagger}(0^{+},-\infty^{-},-\infty\, l_{\perp})\right\}$$

$$= \mathrm{Tr}\left\{T^{\dagger}[0^{+},-\infty^{-},x_{\perp}]W_{R}^{\dagger}[0^{+},x_{\perp}]\cdots T[0^{+},-\infty^{-},0_{\perp}]\right\}$$

for the fields in $\Omega(0^{+},-\infty^{-},-\infty\, l_{\perp})$ commute with all the others (space-like separations) and the cyclicity of the trace.

Literature

- One can express $P(k_\perp)$ as a two particle (time ordered) Wilson loop by modifying the partition function according to the Keldysh–Schwinger contour and by identifying the anti-time ordered fields with fields on the imaginary time line of the Keldysh–Schwinger contour.
  - D'Eramo Liu Rajagopal PR D84 (2011) 065015
- $\hat{q}$ may be written in terms of field correlators as

$$
\begin{aligned}
\hat{q} = \frac{\sqrt{2}}{d_R} \int^{k_{\max}} \frac{d^2 k_\perp}{(2\pi)^2} \int d^2 x_\perp e^{i k_\perp \cdot x_\perp} \int dx^- \\
\Big\langle \mathrm{Tr}\, \Big\{ T^\dagger[0^+, -\infty^-, x_\perp] W_R^\dagger[0^+, x_\perp] T[0^+, \infty^-, x_\perp] \\
\times \left[g F^i_{+\perp}(0^+, x^-, x_\perp) g F^i_{+\perp}(0^+, 0^-, x_\perp) - \frac{i}{\sqrt{2}L} D_{\perp,i} g F^i_{+\perp}(0^+, x^-, x_\perp) \right] \\
\times\ T^\dagger[0^+, \infty^-, 0_\perp] W_R[0^+, 0_\perp] T[0^+, -\infty^-, 0_\perp] \Big\} \Big\rangle_T
\end{aligned}
$$

where the fields are inserted in $W_R^\dagger[0^+, x_\perp]$ in a path ordered fashion and $Q\lambda \lesssim k_{\max} \ll Q$.

Literature

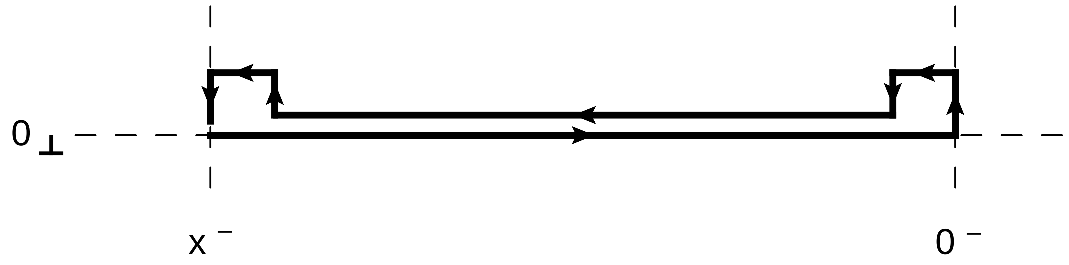
- Similar (but not gauge invariant) expressions can be found in the literature.

  E.g. ○ `Majumder arXiv:1202.5295`

  They seem to require $k_{\max} \to \infty$, which is only justified, under some circumstances, in dimensional regularization. In this case, the gauge invariant expression reads

$$\hat{q} = \frac{\sqrt{2}}{d_R} \int dx^- \left\langle \mathrm{Tr} \left\{ g F^i_{+\perp}(0^+, x^-, 0_\perp) W_R^\dagger[0^+, (x^-, 0^-), 0_\perp] \right.\right.$$
$$\left.\left. \times g F^i_{+\perp}(0^+, 0^-, 0_\perp) W_R[0^+, (0^-, x^-), 0_\perp] \right\} \right\rangle_T$$

  where $W_R[0^+, (0^-, x^-), 0_\perp]$ joins $(0^+, 0^-, 0_\perp)$ with $(0^+, x^-, 0_\perp)$.



The contribution from the scale g^2T

The gauge invariant expression of $\hat{q}$ allows for the use of lattice data.

E.g. Suppose a weakly coupled plasma characterized by the thermodynamical scales:

$$T \gg gT \gg g^2T \quad \text{(non-perturbative)}$$

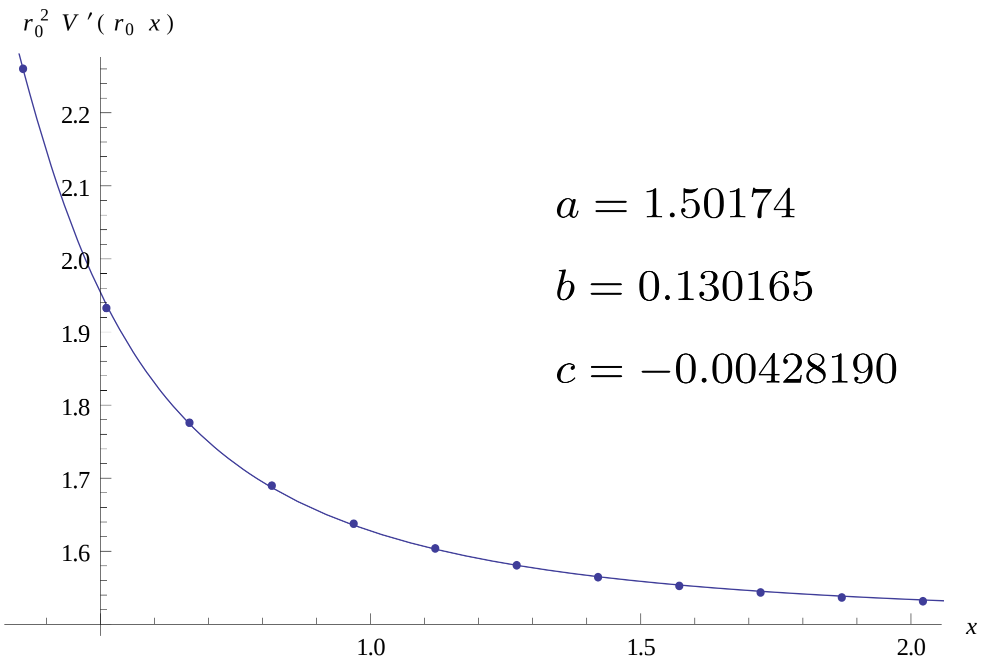
At relative order g^2

- one can analytically continue from Minkowski to Euclidean space-time;
- time ordering is irrelevant;
- the partition function at the energy scale g^2T is described by an EFT (MQCD) that is three-dimensional SU(3) (with coupling g^2T).

○ `Caron-Huot PR D79 (2009) 065039`

The contribution from the scale g^2T may be extracted from the behaviour of the Wilson loop, $\sim e^{-LV}$, in three-dimensional QCD.

The contribution from the scale g^2T



The fit implies

$$\hat{q}\Big|_{\text{from } g^2T} = \left(\frac{aq^*}{r_0^2} + \frac{b(q^*)^3}{3} - \frac{c(q^*)^5 r_0^2}{15}\right)$$

where $k_{\max} \gg gT \gg q^* \gg g^2T$ is the cut off in the $k_\perp$ integration.

- Lüscher Weisz JHEP 0207 (2002) 049 (for the lattice data)

Conclusions and outlook

A systematic treatment of a complex phenomenon like jet quenching is possible in an EFT framework owning to the hierarchy of scales that characterize the system. These are the typical SCET scales, Q, $Q\lambda$, $Q\lambda^2$, with $\lambda = T/Q$, which characterize the propagation of a very energetic parton in the medium and the thermal scales that characterize the medium itself, T, m_D, magnetic mass.

Many contributions need still to be computed both on the SCET side and on the thermal side of the theory. Work in progress includes:

- Inclusion of collinear gluons.
  - D'Eramo Liu Rajagopal JP G38 (2011) 124162
- NNLL contribution from the thermal scales gT and g^2T.
  - Benzke Brambilla Escobedo Vairo TUM-EFT 32/12