A chiral quark-soliton model with broken scale invariance for nuclear matter

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Outline

Theoretical introduction

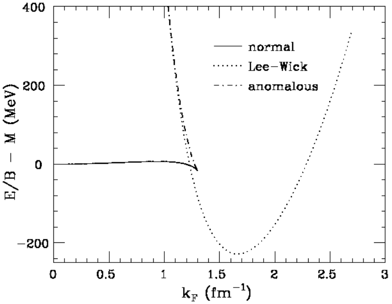
- Review of Chiral Lagrangians for nuclear matter
  - Linear σ- model failure at finite density(R.J.Furnstahl, B.D.Serot,H.-B. Tang,Nucl.Phys.A 598 (1996))
- Chiral-Dilaton Model (Carter,Ellis,Rudaz,Heide, PLB 282 (1992) 271,PLB 293 (1992) 870,NPA 571 (1994),NPA 603 (1996),NPA 618 (1997),NPA 628 (1998))
  - Scale invariance in QCD and the Dilaton Potential
  - the Lagrangian from hadronic to quarks degrees of freedom

Results at finite density:

- Going to finite density
  - the Wigner-Seitz approximation to nuclear matter
  - Single soliton at finite density $\rightarrow$ the effect of the logarithmic term and the role of vector mesons $\rightarrow$ getting saturation
- Conclusions and Outlooks

Failure of Linear-σ model at finite density

- The ground state at high densities is not the normal solution, but the Lee-Wick one, having effective nucleon mass $M^* = 0$
- restauration of chiral symmetry already at $\rho \approx \rho_0$

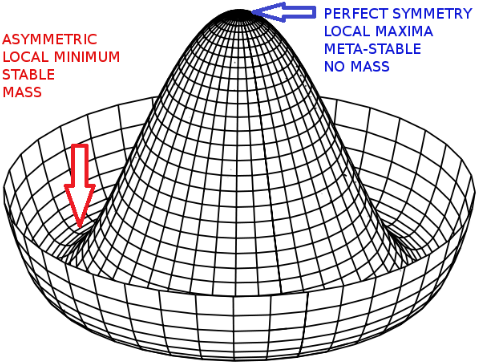


(R.J.Furnstahl, B.D.Serot,H.-B. Tang,Nucl.Phys.A 598 (1996))

HOW CAN WE REACH HIGHER DENSITIES AND STILL INCLUDE CHIRAL SYMMETRY?

Linear realization of chiral symmetry with scale invariance

PROBLEM: the linear sigma model fails to yield saturation. It provides chiral symmetry restoration ($m_N = 0$) already at low density due to the form of the meson self-interaction



Linear realization of chiral symmetry with scale invariance

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SOME PHYSICAL INGREDIENT IS MISSING

So, keeping in mind the idea of including in a dynamic way the chiral symmetry in our model, what can we do to get a better description of nuclear matter at finite density?

CHANGE THE POTENTIAL IN THE LAGRANGIAN DENSITY

Breaking of Scale Invariance in QCD

- In QCD, scale symmetry is broken by trace anomaly. This mechanism is responsible for the existence of Λ_{QCD} parameter, which sets the scale of hadron masses and radii
- Formally the non conservation of the dilatation current is strictly connected to a non vanishing gluon condensate

$$\langle \partial_\mu j^\mu_{QCD} \rangle = \frac{\beta(g)}{2g} \langle F^a_{\mu\nu}(x) F^{a\mu\nu}(x) \rangle$$

- In an effective model, the dynamics of the gluon condensate at mean-field level, is obtained by introducing a scalar field ϕ, the dilaton field ϕ (Schechter (1980), Migdal, Shifman (1982)), so that the potential is determined by:

$$\Theta^\mu_\mu = 4V(\phi) - \phi \frac{\partial V}{\partial \phi} = 4\epsilon_{vac} \left(\frac{\phi}{\phi_0} \right)^4$$

The dilatonic potential

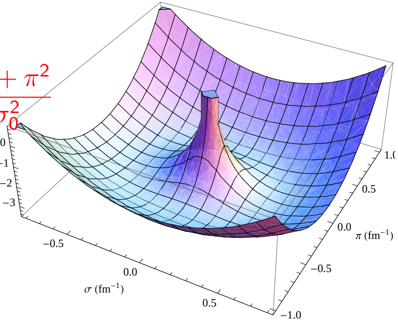
The dilaton field potential:

$$V(\phi,\sigma,\pi) =$$

$$B\phi^4\left(ln\frac{\phi}{\phi_0}-\frac{1}{4}\right)-\frac{1}{2}B\delta\phi^4 ln\frac{\sigma^2+\pi^2}{\sigma_0^2}$$

$$+\frac{1}{2}B\delta\zeta^2\phi^2\left(\sigma^2+\pi^2-\frac{1}{2}\frac{\phi^2}{\zeta^2}\right)$$

$$-\frac{3}{4}\epsilon_1-\frac{1}{4}\epsilon_1\left(\frac{\phi}{\phi_0}\right)^2\left[\frac{4\sigma}{\sigma_0}-2\left(\frac{\sigma^2+\pi^2}{\sigma_0^2}\right)-\left(\frac{\phi}{\phi_0}\right)^2\right]$$



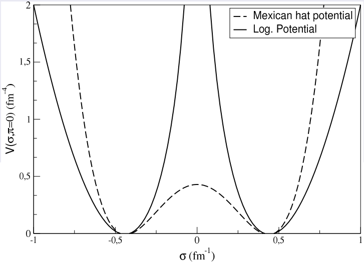
The dilatonic potential: comparison with the Mexican hat

Keeping the dilaton frozen at its vacuum value ϕ_0, the potential reads:

$$V(\sigma,\pi) = \lambda_1^2(\sigma^2+\pi^2) - \lambda_2^2 \ln(\sigma^2+\pi^2) - \sigma_0 m_\pi^2 \sigma$$

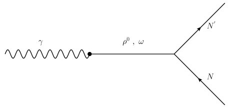
$$\lambda_1^2 = \frac{1}{4}(m_\sigma^2 + m_\pi^2)$$

$$\lambda_2^2 = \frac{\sigma_0^2}{4}(m_\sigma^2 - m_\pi^2)$$



The introduction of vector mesons

- Vector mesons dominance $\rightarrow$ better description of nucleon properties



- $N - N$ interaction $\rightarrow$ provide the necessary repulsion at short distances (OBE model)

How to introduce the vector mesons in an effective Lagrangian

- VM as massive Yang-Mills fields of $SU(2)_L \otimes SU(2)_R$ symmetry group

- principle of universality $\rightarrow$ ρ meson couples to isospin current and ω meson couples to the baryonic current:

$$g_{\rho NN} = g_{\rho qq} = g_{\rho\pi\pi} = g_{\rho\rho\rho} \quad , g_{\omega qq} = \frac{1}{3} g_{\omega NN}(q^2 = 0)$$

The Lagrangian of the Chiral Dilaton Model

- in the hadronic sector $\rightarrow$ fermionic fields are *nucleons*;
- chiral fields $(\sigma, \boldsymbol{\pi}) \rightarrow$ nuclear physics at low densities (Heide, Rudaz, Ellis, Nucl.Phys.A571, 713 (1994)), restoration of chiral symmetry at quite high densities (Drago, Bonanno, Phys.Rev.C79:045801,2009);

MAIN IDEA: use the same nucleon Lagrangian, but now introducing quarks degrees of freedom $\rightarrow$ fermionic fields are quarks

The Lagrangian density becomes:

$$\begin{aligned}\mathcal{L} \quad = \quad & \bar{\psi}\left(i\gamma^\mu\partial_\mu - g_\pi(\sigma + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\gamma_5) + g_\rho\gamma^\mu\frac{\boldsymbol{\tau}}{2}\cdot(\boldsymbol{\rho}_\mu + \gamma_5\mathbf{A}_\mu) - \frac{g_\omega}{3}\gamma^\mu\omega_\mu\right)\psi \\ & + \frac{\beta}{2}(D_\mu\sigma D^\mu\sigma + D_\mu\boldsymbol{\pi}\cdot D^\mu\boldsymbol{\pi}) - \frac{1}{4}(\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \mathbf{A}_{\mu\nu}\cdot\mathbf{A}^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu}) \\ & + \frac{1}{2}m_\rho^2(\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu + \mathbf{A}_\mu\cdot\mathbf{A}^\mu) + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - V(\phi,\sigma,\pi)\end{aligned}$$

Soliton at finite density: the Hedgehog configuration and set parameter

- Mean Field Approximation → there are an infinite set of degenerate configurations that minimize the energy of the system
- HEDGEHOG CONFIGURATION → coupling between the ordinary space and the isospin space

Set of parameters

- model with/without VM: $m_\sigma = 550$ MeV, $g = 5$
- model with VM:
  1. getting saturation: $g = 3.9$, $g_\omega = 12$, $g_\rho = 4$ and $m_\sigma = 1200$ (SET II)

Going to finite density: the Wigner-Seitz approximation to nuclear matter

- Approximating nuclear matter by a lattice of solitons $\rightarrow$ we consider the meson fields configuration centered at each lattice point, generating a periodic potential in which the quarks move
- Wigner-Seitz approximation: replace the cubic lattice by a spherical symmetric one $\rightarrow$ each soliton sits on a spherical cell of radius R with specific boundary conditions on the surface of the sphere

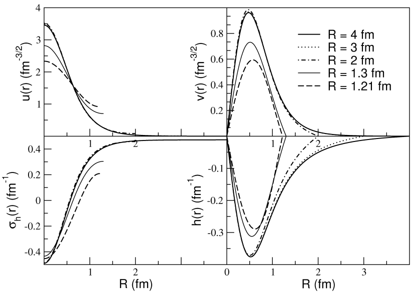
The Hamiltonian for a periodic system must obey Bloch's theorem, so the quark spinor must be of the form:

$$\psi_{\mathbf{k}}(r) = e^{i\mathbf{k}\cdot\mathbf{r}}\Phi_{\mathbf{k}}(r), \quad (\mathbf{k} = 0 \text{ for the ground state})$$

The bottom of the band is defined as the state satisfying the following periodic boundary conditions, dictated by symmetry arguments (parity):

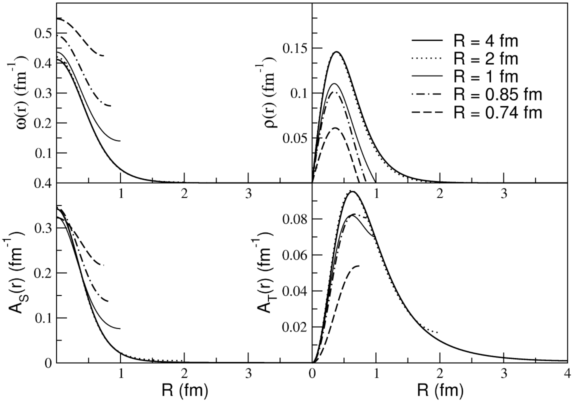
$$v(R) = h(R) = \rho(R) = 0,$$
$$u'(R) = \sigma_h'(R) = \omega'(R) = A_S'(R) = A_T'(R) = 0.$$

Fields at finite density: model without VM



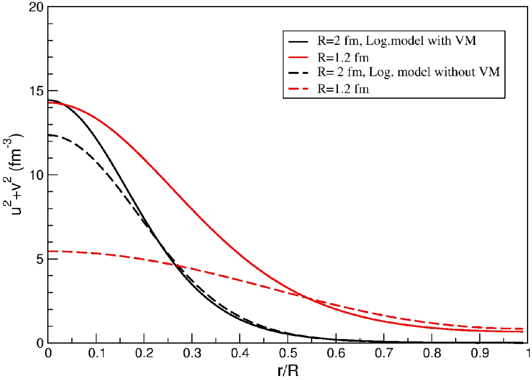
- down to $R \approx 2$ fm the fields do not change significantly, at lower values the finite density effects deeply modify the behaviour of fields.

Fields at finite density: model with VM



- more stable solutions → fields start to get deformed at $R \approx 1$

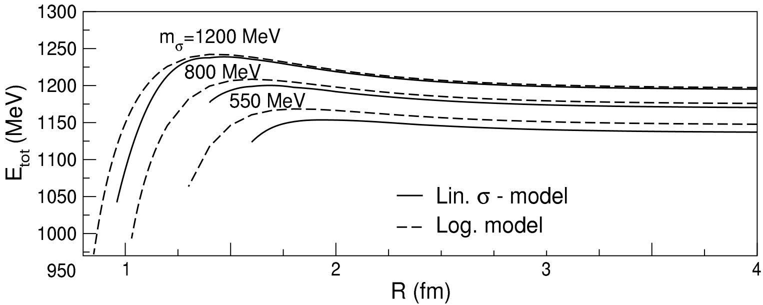
Fields at finite density: model with VM



- repulsion prevents the baryon density to become large in the inter-nucleon region

Results at finite density: the effect of the dilaton potential

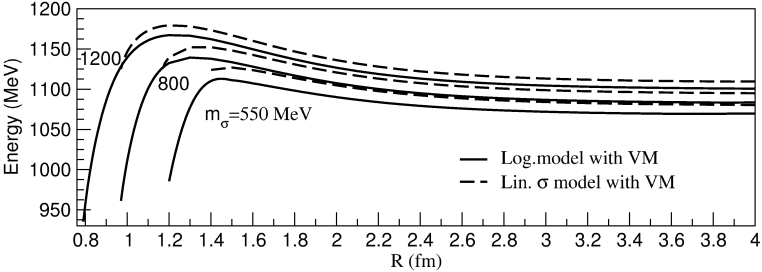
For the model without VM:



- For a fixed value of m_σ, the CDM allows the system to reach higher densities
- as m_σ raises, the system remains stable to lower R

Results at finite density: the effect of vector mesons (I)

For the model with VM:



- the introduction of VM stabilizes the solution at high densities → reach even higher densities in comparison to the model with only chiral fields

Going to finite density: how to define the band width

In our work we use two different methods to estimate the band width:

- A (rather crude) approximation to the width of a band can be obtained by using (Glendenning, Banerjee PRC 34(1986)):

$$\begin{aligned} \Delta &= \sqrt{\epsilon_0^2 + \left(\frac{\pi}{2R}\right)^2} - |\epsilon_0|, \\ \epsilon_{top} &= \epsilon_0 + \Delta. \end{aligned}$$

- An alternative approximation is obtained by imposing that the upper Dirac component vanishes at the boundary(Birse,Rehr,Wilets PRC38 (1988)):

$$u(R) = 0$$

- the eigenvalue obtained imposing this boundary condition represents an upper limit to the top and the true top would be about half way between this upper limit and the bottom of the band

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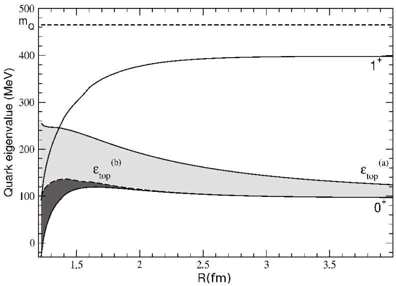
$$u(R) = 0$$

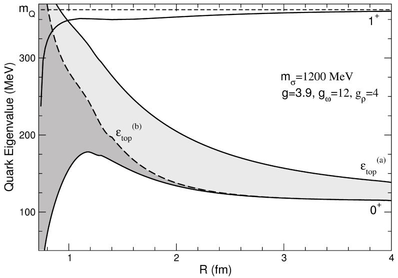
- uniform filling of the band $\rightarrow$ lower band has $G = 0$, color is the only degeneracy left $\rightarrow$ 3 quarks per soliton completely fill the band

Going to finite density: band structure

Model without VM

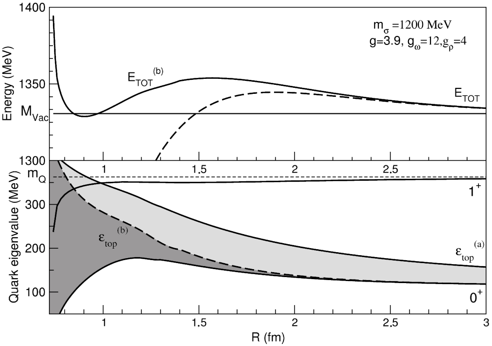


Model with VM



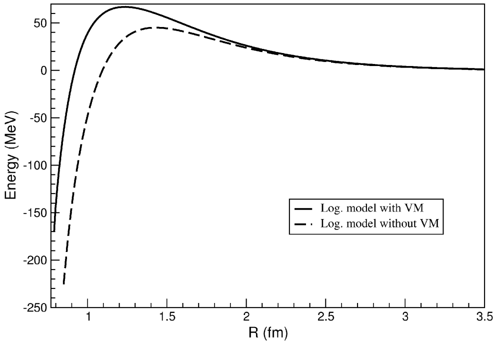
- in absence of VM → saturation can never be obtained
- with VM → significant increase of the top of the band at high densities → saturation

Going to finite density: getting saturation



WHAT IS ACTUALLY PROVIDING REPULSION AT HIGH DENSITIES?

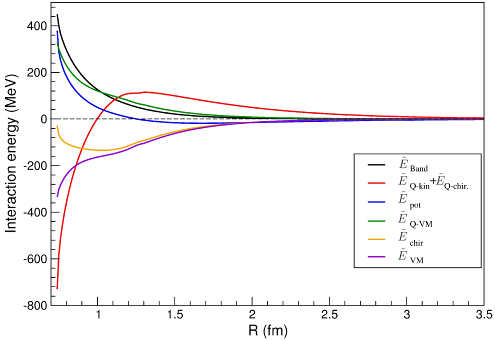
Going to finite density: the effect of vector mesons (II)



- the effect of VM gives a contribution of ~ 100 MeV at $R = 1$ fm

NOT SUFFICIENT TO OBTAIN SATURATION

Going to finite density: repulsion and band effect



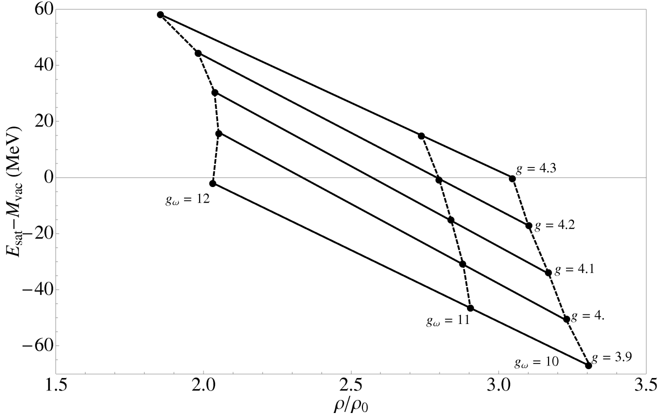
- band effect is the largest contribution to repulsion at $\rho > \rho_0 \rightarrow$ sharing of quarks between nucleons ($N - N$ potential)

Going to finite density: how do we obtain saturation?

- interplay between attraction from chiral fields and repulsion from vector mesons, dominant up to ρ_0
- the logarithmic potential is fundamental to keep the soliton stable at densities large enough that the vector mesons start to provide repulsion
- at densities $\rho > \rho_0$ the band effect provides the necessary repulsion to obtain saturation

THE SATURATION MECHANISM IS STABLE RESPECT TO THE CHOICE OF PARAMETERS

Going to finite density: saturation and parameters



- the model admits "saturation" for different sets of parameters → partial overlap with parameters for the single nucleon (Broniowski, Banerjee PRD34 (1986))

Conclusions and Outlooks

We used a Lagrangian with quarks degrees of freedom based on chiral and scale invariance to study:

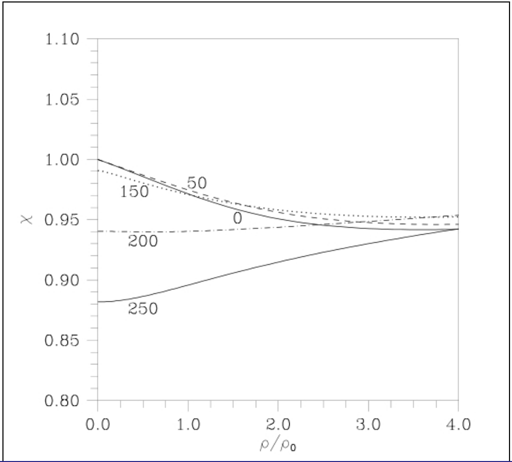
- the single soliton at finite density using the Wigner-Seitz approximation:
    - we showed that the new potential, including the scale invariance, provides more stable solitonic solutions than the σ model at higher densities
    - the introduction of VM provides a more stable solution at finite density and moreover, together with the band effect, it provides repulsion at high densities
    - the model provides saturation for a quite wide range of parameters

Conclusions and Outlooks

In the future:

- improve the calculation of the band effects by using a more sophisticated approach (U.Weber, J.A.McGovern,PRC 57 (1998))and see how this could affect the saturation density.
- go beyond WS approximation (collaboration with Prof. V.Vento and Prof.B.Y.Park), construction of a soliton crystal for the CDM and study the soliton matter at finite density $\rightarrow$ at the moment we are building our $B = 2$ system
- include the dynamics of the dilaton field and study the model also at finite temperature in order to provide a phase diagram but starting from fundamental ingredients

The dilaton dynamics at finite temperature (Carter, Ellis NPA 628 (1998), Bonanno, Drago PRC79 (2009))



- the ratio χ at low temperatures remains close to unity even at large densities
- at the moment the dynamic of the dilaton is not included in the model

Scale invariance in QCD

- scale transformations of fields and coordinates:

$$x \to \lambda^{-1}x \ , \ \lambda > 0,$$
$$\phi(x) \to U(\lambda)\phi(\lambda x)$$

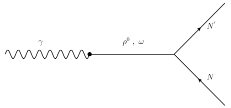
- from Noether's theorem: $\delta\mathcal{S}_{scale} = 0 \to$ conservation of *dilatation* current D_μ
- for a classical massive theory it can be proved (Callan, Curtis, Coleman, Jackiw, Ann.Phys 59 (1970)):

$$\partial_\mu D^\mu = \widetilde{T}^\mu_\mu$$

- quantum anomaly $\to$ symmetry of the lagrangian is broken at quantum level

The introduction of vector mesons

- Vector mesons dominance $\rightarrow$ better description of nucleon properties



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Soliton in vacuum: the Hedgehog configuration

- Mean Field Approximation $\rightarrow$ there are an infinite set of degenerate configurations that minimize the energy of the system
- HEDGEHOG CONFIGURATION $\rightarrow$ coupling between the ordinary space and the isospin space

$$\psi = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} u(r) \\ iv(r)\boldsymbol{\sigma}\cdot\hat{\mathbf{r}} \end{pmatrix} \frac{1}{\sqrt{2}}(|u_\downarrow\rangle - |d_\uparrow\rangle),$$
$$\langle\sigma\rangle = \sigma_h(r),\ \langle\boldsymbol{\pi}_a\rangle = \widehat{r}_a h(r)$$

Soliton in vacuum: projecting the Hedgehog

- the Hedgehog is not an eigenstate of spin and isospin $\rightarrow$ projection on physical states with good quantum numbers J and I(Ruiz-Arriola et al.NPA591, Birse PRD33)
- it also breaks translational invariance $\rightarrow$ projection on linear momentum
  - easier approach to estimate the center-of-mass corrections(Dethier et al. PRD27,(1983)): $M_J = (E_J - \mathbf{P}^2)^{1/2}$

Set of parameters

- model without VM: $m_\sigma = 550$ MeV, $g = 5$
- model with VM:
  1. better fit to nucleon properties: $g = 3.6$, $g_\omega = 13$, $g_\rho = 4$ and $m_\sigma = 1200$ (SET I)

Soliton in vacuum

Contributions to the total soliton energy at MFL:

Quantity	Log. Model	Linear σ-Model
Quark eigenvalue	114.5	112.9
Quark kinetic energy	1075.8	1080.6
E_σ (mass+kin.)	213.8	212.2
E_π (mass+kin.)	393.2	397.3
Potential energy $\sigma - \pi$	81.2	80.4
E_ω (mass+kin.)	-194.4	-196.5
E_ρ (mass+kin.)	162.6	165.4
E_A (mass+kin.)	329.5	334.1
Total energy	1329.5	1331.7

- chiral and vector mesons contributions are comparable $\rightarrow$ VMs play a fundamental role in building up the soliton
- results with the logarithmic model and the linear-σ model are very similar

Soliton in vacuum: projected observables

Model without VM :

Quantity	Log. Model	σ-Model	Exp.
$E_{1/2}\,(MeV)$	1075	1002	
$M_N\,(MeV)$	960	894	938
$E_{3/2}\,(MeV)$	1140	1075	
$M_\Delta\,(MeV)$	1032	975	1232
$\langle r_E^2\rangle_p\,(fm^2)$	0.55	0.61	0.74
$\langle r_E^2\rangle_n\,(fm^2)$	-0.02	-0.02	-0.12
$\langle r_M^2\rangle_p\,(fm^2)$	0.7	0.72	0.74
$\langle r_M^2\rangle_n\,(fm^2)$	0.72	0.75	0.77
$\mu_p\ (\mu_N)$	2.25	2.27	2.79
$\mu_n\ (\mu_N)$	-1.97	-1.92	-1.91
g_a	1.52	1.10	1.26

Model with VM, SET I:

Quantity	Log. Model	σ-Model	Exp.
$E_{1/2}\,(MeV)$	1020	1008	
$M_N\,(MeV)$	926	912	938
$E_{3/2}\,(MeV)$	1148	1147	
$M_\Delta\,(MeV)$	1066	1063	1232
$\langle r_E^2\rangle_p\,(fm^2)$	0.67	0.66	0.74
$\langle r_E^2\rangle_n\,(fm^2)$	-0.05	-0.05	-0.12
$\langle r_M^2\rangle_p\,(fm^2)$	0.77	0.76	0.74
$\langle r_M^2\rangle_n\,(fm^2)$	0.78	0.77	0.77
$\mu_p\ (\mu_N)$	2.63	2.64	2.79
$\mu_n\ (\mu_N)$	-2.37	-2.38	-1.91
g_a	1.58	1.46	1.26