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Massless scalar degrees of freedom in QCD and in the EW sector with gravity

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Outline

- Conformal anomalies in the context of EFT of gravity
- Evidence of new massless degrees of freedom not contained in the Einstein-Hilbert action
- Explicit analytic perturbative computations on anomalous correlators in QCD and in the Electroweak Theory

The conformal anomaly

The trace of the stress tensor obeys at the classical level

$$g_{\mu\nu} T^{\mu\nu}_{(cl)} \sim m\Phi^2$$

At the quantum level the symmetry under global scale transformations

cannot be maintained

We end up with a well defined conformal anomaly

$$g_{\mu\nu} \left\langle T^{\mu\nu} \right\rangle \Big|_{m=0} \neq 0$$

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Gravitational Effective Action

The conformal anomaly provides additional terms to be included in the effective theory, beside the classical Einstein-Hilbert action

$$S_{\text{eff}}[g] = S_{\text{EH}}[g] + S_{\text{anom}}[g]$$

The conformal anomaly implies the existence of

new massless scalar degrees of freedom

Antoniadis, Mazur, Mottola, New J.Phys.9:11,2007
Giannotti, Mottola, PRD79 (2009),
Armillis, Corianò, L. D. R., Phys.Lett.B682:322-327,2009,
Armillis, Corianò, L. D. R., PRD81 (2010) 085001
Corianò, Luigi Delle Rose, M.S. , PRD83, (2011) 125028

The non local effective action

The trace anomaly in 4 space-time dimensions generated by quantum effects in a classical gravitational and gauge background is given by

$$T^{\mu}_{\mu\, anom} = -\frac{1}{8}\left[2bC^2 + 2b'\left(E - \frac{2}{3}\Box R\right) + 2cF^2\right]$$

Weyl and Euler tensors

$$\begin{aligned} C^2 &= C_{\lambda\mu\nu\rho}C^{\lambda\mu\nu\rho} = R_{\lambda\mu\nu\rho}R^{\lambda\mu\nu\rho} - 2R_{\mu\nu}R^{\mu\nu} + \frac{R^2}{3} \\ E &= {}^*R_{\lambda\mu\nu\rho}\,{}^*R^{\lambda\mu\nu\rho} = R_{\lambda\mu\nu\rho}R^{\lambda\mu\nu\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2. \end{aligned}$$

The anomalous effective action is identified by solving the variational equation

$$g^{\mu\nu}\frac{2}{\sqrt{-g}}\frac{\delta S_{anom}}{\delta g^{\mu\nu}} = T^{\mu}_{\mu\, anom}$$

Non local Riegert's solution

The solution of the anomalous effective action was given by Riegert in a non local formulation

$$S_{anom}[g, A] = \frac{1}{8}\int d^4x\sqrt{-g}\int d^4x'\sqrt{-g'}\left(E - \frac{2}{3}\Box R\right)_x \Delta_4^{-1}(x,x')\left[2b\,F + b'\left(E - \frac{2}{3}\Box R\right) + 2\,c\,F_{\mu\nu}F^{\mu\nu}\right]_{x'}$$

Riegert, PLB 134 (1984)

Where $\Delta_4^{-1}(x, x')$ denotes the Green's function of the fourth order differential operator

$$\Delta_4 \equiv \nabla_\mu(\nabla^\mu\nabla^\nu + 2R^{\mu\nu} - \tfrac{2}{3}Rg^{\mu\nu})\nabla_\nu$$
$$= \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu + \tfrac{1}{3}(\nabla^\mu R)\nabla_\mu - \tfrac{2}{3}R\Box$$

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The local formulation

The non local effective action can be recast in a local form after the introduction of two scalar auxiliary fields

To first order in metric variations around flat space

$$S_{anom}[g, A; \varphi, \psi'] = \int d^4x \sqrt{-g} \left[-\psi' \Box \varphi - \frac{R}{3} \psi' + \frac{c}{2} F_{\alpha\beta} F^{\alpha\beta} \varphi \right]$$

with the two fields satisfying the equations

$$\psi' \equiv b \Box \psi \,,$$

$$\Box \psi' = \frac{c}{2} F_{\alpha\beta} F^{\alpha\beta}$$

$$\Box \varphi = -\frac{R}{3} \,.$$

[Antoniadis, Mazur, Mottola, New J.Phys.9:11 (2007)

Giannotti, Mottola, PRD79 (2009)]

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Coming back to the equivalent non local form

$$S_{anom}[g, A] \to -\frac{c}{6} \int d^4x \sqrt{-g} \int d^4x' \sqrt{-g'}\, R_x \Box^{-1}_{x,x'} [F_{\alpha\beta} F^{\alpha\beta}]_{x'}$$

- The anomaly-induced action will be missing homogeneous contributions
- There is currently a disagreement over the correctness of this result

The <TVV> correlator (QCD)

The <TVV> correlator is the first (leading) order contribution in the expansion of the anomalous effective action

From the standard QCD Lagrangian we extract the energy-momentum tensor (EMT)

$$\begin{aligned} T_{\mu\nu} &= -g_{\mu\nu}\mathcal{L}_{QCD} - F^a_{\mu\rho}F^{a\rho}_{\nu} - \frac{1}{\xi}g_{\mu\nu}\partial^\rho(A^a_\rho\partial^\sigma A^a_\sigma) + \frac{1}{\xi}(A^a_\nu\partial_\mu(\partial^\sigma A^a_\sigma) + A^a_\mu\partial_\nu(\partial^\sigma A^a_\sigma)) \\ &+ \frac{i}{4}\Big[\overline{\psi}\gamma_\mu(\overrightarrow{\partial}_\nu - igT^aA^a_\nu)\psi - \overline{\psi}(\overleftarrow{\partial}_\nu + igT^aA^a_\nu)\gamma_\mu\psi + \overline{\psi}\gamma_\nu(\overrightarrow{\partial}_\mu - igT^aA^a_\mu)\psi \\ &- \overline{\psi}(\overleftarrow{\partial}_\mu + igT^aA^a_\mu)\gamma_\nu\psi\Big] + \partial_\mu\overline{\omega}^a(\partial_\nu\omega^a - gf^{abc}A^c_\nu\omega^b) + \partial_\nu\overline{\omega}^a(\partial_\mu\omega^a - gf^{abc}A^c_\mu\omega^b). \end{aligned}$$

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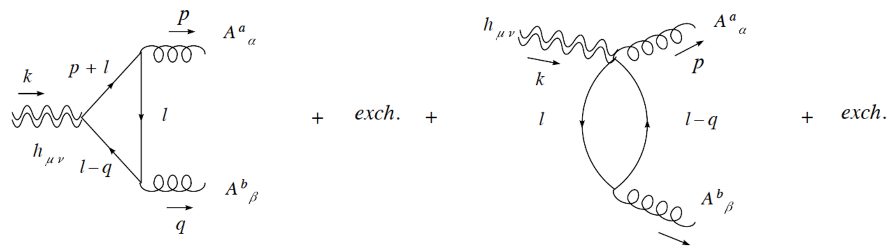
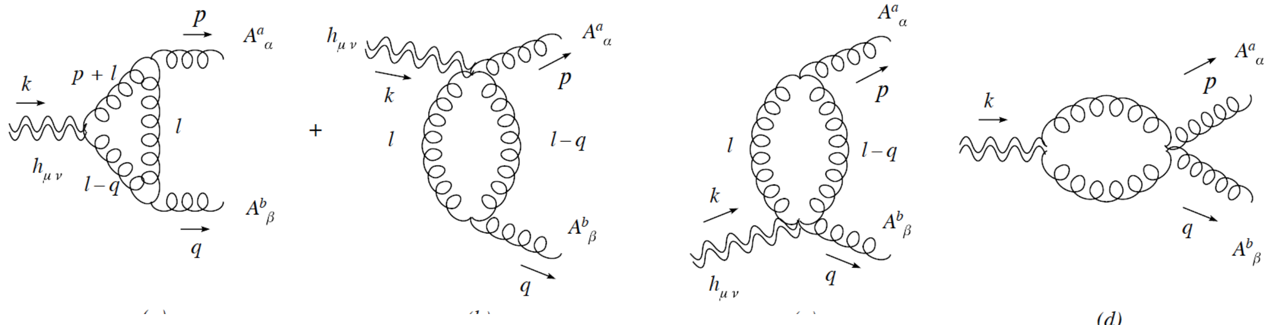
For an exact non-abelian gauge theory the perturbative expansion of the <TVV> correlator is more involved

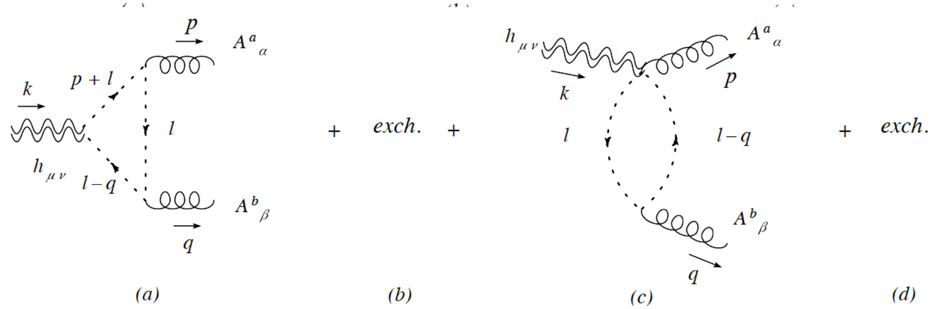
Quark sector



Gauge sector



Ghost sector



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Diffeomorphisms invariance

Invariance under diffeomorphisms implies constraints on the generating functional of 1PI graphs (Γ)

$$\partial_\alpha \frac{\delta\Gamma}{\delta h_{\alpha\beta}} = -\frac{\kappa}{2}\Big\{ -\frac{\delta\Gamma}{\delta\phi_c}\partial^\beta\phi_c - \partial^\beta\phi_c^\dagger\frac{\delta\Gamma}{\delta\phi_c^\dagger} - \frac{\delta\Gamma}{\delta A_{c\,\alpha}}\partial^\beta A_{c\,\alpha} - \partial_\alpha\Big(\frac{\delta\Gamma}{\delta A_{c\,\alpha}}A_c^\beta\Big)$$

$$- \partial^\beta\bar{\psi}_c\frac{\delta\Gamma}{\delta\bar{\psi}_c} + \frac{\delta\Gamma}{\delta\psi_c}\psi_c\partial^\beta\psi_c - \frac{1}{2}\partial_\alpha\Big(\frac{\delta\Gamma}{\delta\psi_c}\sigma^{\alpha\beta}\psi_c - \bar{\psi}_c\sigma^{\alpha\beta}\frac{\delta\Gamma}{\delta\bar{\psi}_c}\Big)\Big\},$$

For the <TVV'> correlators (V,V' = gluons) the EMT conservation gives

$$-i\frac{\kappa}{2}\partial^\mu\langle T_{\mu\nu}(x)V_\alpha(x_1)V'_\beta(x_2)\rangle_{amp} = -\frac{\kappa}{2}\Big\{-\partial_\nu\delta^{(4)}(x_1-x)P^{-1\,VV'}_{\alpha\beta}(x_2,x)$$

$$-\partial_\nu\delta^{(4)}(x_2-x)P^{-1\,VV'}_{\alpha\beta}(x_1,x) + \partial^\mu[\eta_{\alpha\nu}\delta^{(4)}(x_1-x)P^{-1\,VV'}_{\beta\mu}(x_2,x) + \eta_{\beta\nu}\delta^{(4)}(x_2-x)P^{-1\,VV'}_{\alpha\mu}(x_1,x)]\Big\}$$

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BRST symmetry

BRST invariance of the theory implements relations between correlation functions in the form of Slavnov-Taylor identities (STI)

From the BRST variations of the fields

$$\delta A^a_\mu = \lambda D^{ab}_\mu \omega^b \qquad \begin{aligned} \delta\psi &= i g \lambda \omega^a t^a \psi \\ \delta\bar{\psi} &= -i g \bar{\psi} t^a \lambda \omega^a \end{aligned} \qquad \begin{aligned} \delta\omega^a &= -\frac{1}{2} g \lambda f^{abc} \omega^b \omega^c \\ \delta\bar{\omega}^a &= -\frac{1}{\xi} (\partial^\mu A^a_\mu) \lambda \end{aligned}$$

the corresponding variation of the EMT is obtained, i.e.

$$\delta T_{\mu\nu} = \delta(T^{g.f.}_{\mu\nu} + T^{gh}_{\mu\nu}) = \frac{\lambda}{\xi}\left[A^a_\mu \partial_\nu \partial^\rho D^{ab}_\rho \omega^b + A^a_\nu \partial_\mu \partial^\rho D^{ab}_\rho \omega^b - g_{\mu\nu} \partial^\sigma (A^a_\sigma \partial^\rho D^{ab}_\rho \omega^b)\right]$$

which in turn allows to derive the BRTS constraint on the amplitude, which is

$$p^\alpha q^\beta \langle T_{\mu\nu}(k) A^a_\alpha(p) A^b_\beta(q) \rangle_{trunc} = 0$$

Expansion of the amplitude

The sum of the quark and gluon/ghost contributions is written as

$$\Gamma^{\mu\nu\alpha\beta}(p,q) = \Gamma_g^{\mu\nu\alpha\beta}(p,q) + \Gamma_q^{\mu\nu\alpha\beta}(p,q) = \sum_{i=1}^{3} \Phi_i(s,0,0)\,\delta^{ab}\,\phi_i^{\mu\nu\alpha\beta}(p,q)$$

Form factors

Tensor basis elements

$$\phi_1^{\mu\nu\alpha\beta}(p,q) = (s\,g^{\mu\nu} - k^\mu k^\nu)\,u^{\alpha\beta}(p,q),$$

Only traceful tensor in the basis

$$\begin{aligned}
\phi_2^{\mu\nu\alpha\beta}(p,q) &= -2\,u^{\alpha\beta}(p,q)\left[s\,g^{\mu\nu} + 2(p^\mu p^\nu + q^\mu q^\nu) - 4\,(p^\mu q^\nu + q^\mu p^\nu)\right], \\
\phi_3^{\mu\nu\alpha\beta}(p,q) &= (p^\mu q^\nu + p^\nu q^\mu) g^{\alpha\beta} + \frac{s}{2}\left(g^{\alpha\nu} g^{\beta\mu} + g^{\alpha\mu} g^{\beta\nu}\right) \\
&\quad - g^{\mu\nu}\left(\frac{s}{2} g^{\alpha\beta} - q^\alpha p^\beta\right) - \left(g^{\beta\nu} p^\mu + g^{\beta\mu} p^\nu\right) q^\alpha - (g^{\alpha\nu} q^\mu + g^{\alpha\mu} q^\nu) p^\beta,
\end{aligned}$$

The contribution of the trace of the energy momentum tensor is given by

$$g_{\mu\nu}\,\Gamma^{\mu\nu\alpha\beta}(p,q) = 3\,s\,\Phi_1(s;0,0,0)\,u^{\alpha\beta}(p,q) = -2\,\frac{\beta(g)}{g}\,u^{\alpha\beta}(p,q)$$

$$\beta(g) = \frac{g^3}{16\pi^2}\left(-\frac{11}{3}\,C_A + \frac{2}{3}\,n_f\right)$$

$$u^{\alpha\beta}(p,q) = -\frac{1}{4}\int d^4x \int d^4y e^{ip\cdot x+iq\cdot y}\,\frac{\delta^2\{F_{\mu\nu}F^{\mu\nu}(0)\}}{\delta A_\alpha(x)A_\beta(y)}$$

$$\Phi_1(s,0,0) = -\frac{g^2}{72\pi^2\,s}(2n_f - 11C_A)$$

The form factor multiplying the traceful tensor shows the massless pole searched for !

The <TVV> correlators in EWSM

Effective action analysis extended to the neutral gauge boson sector of the electroweak theory

The symmetric and conserved EMT may be obtained by coupling the SM Lagrangian to the gravitational field

$$S = S_G + S_{SM} + S_I = -\frac{1}{\kappa^2}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}\mathcal{L}_{SM} + \frac{1}{6}\int d^4x\sqrt{-g}\,R\,\mathcal{H}^\dagger\mathcal{H}$$

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The full EMT is given by a minimal tensor and a term of improvement generated by the conformal coupling of scalars

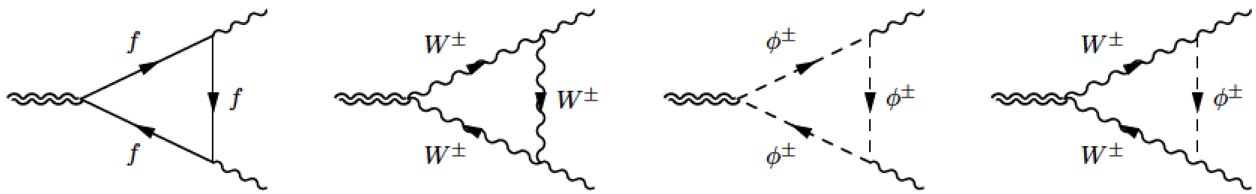
$$T_{\mu\nu} = T^{Min}_{\mu\nu} + T^{I}_{\mu\nu}$$

$$T^{Min}_{\mu\nu} = T^{f.s.}_{\mu\nu} + T^{ferm.}_{\mu\nu} + T^{Higgs}_{\mu\nu} + T^{Yukawa}_{\mu\nu} + T^{g.fix.}_{\mu\nu} + T^{ghost}_{\mu\nu}$$

Diagrammatic expansion (1)

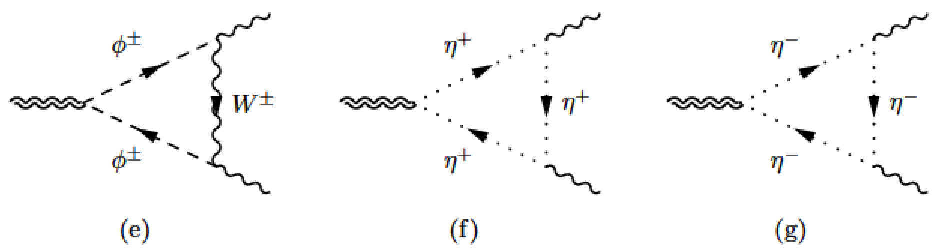
One loop corrections are computed in DR using on-shell renormalization scheme and the 't Hooft-Veltman prescription for ❧

A fully automated methodology is implemented in the symbolic manipulation program *Mathematica* to manage hundreds of diagrams

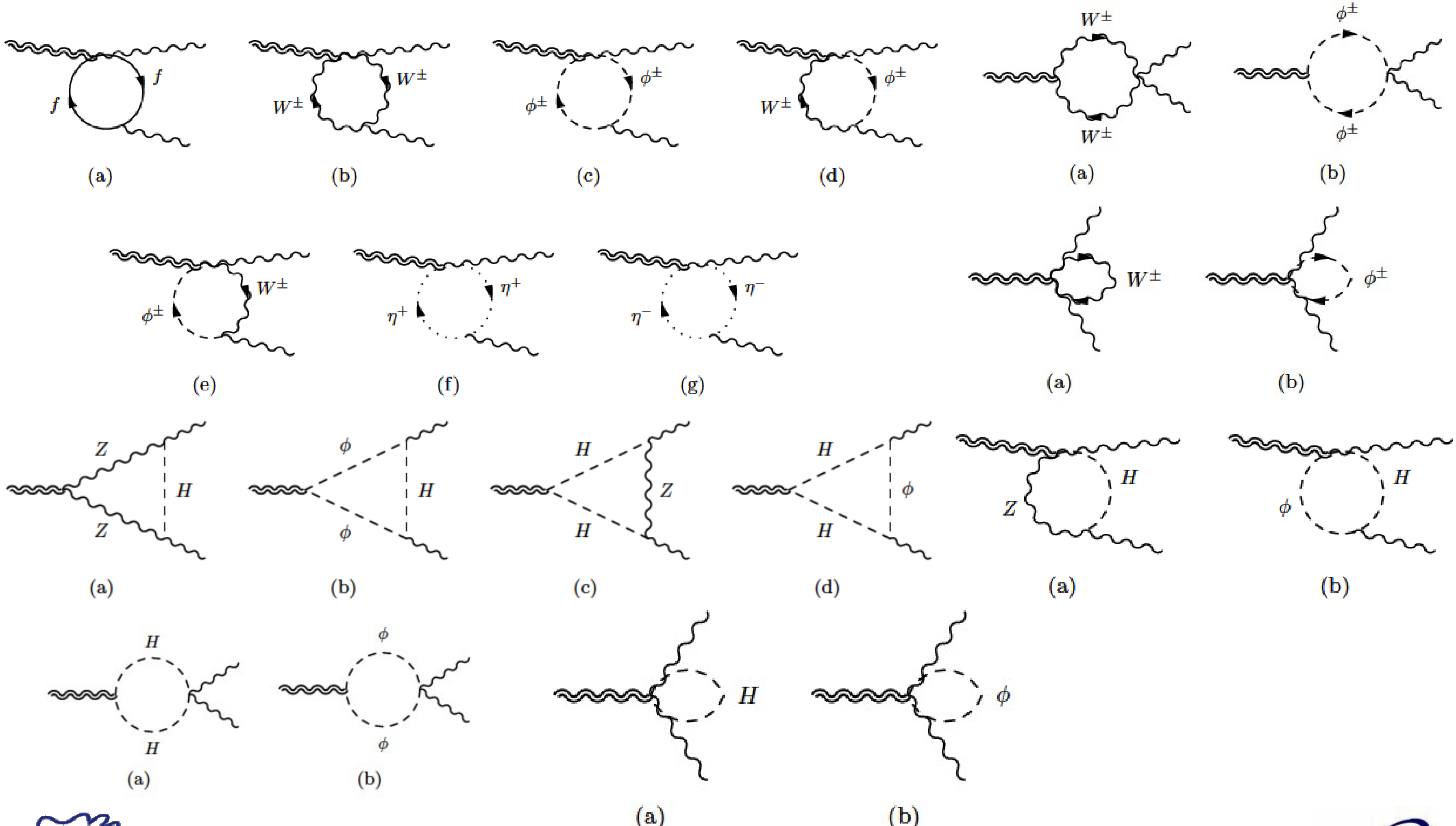


(a) (b) (c) (d)

We show some of them with three, two and one point topology



Diagrammatic expansion (2)



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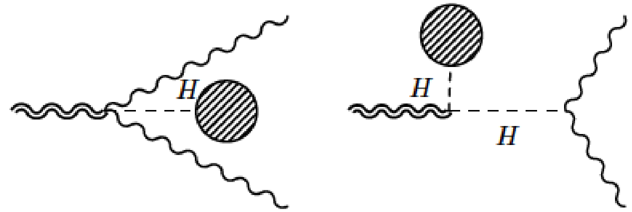
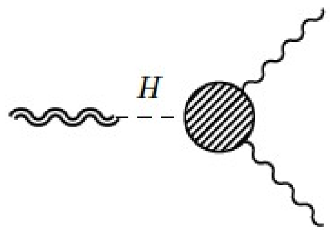
Diagrammatic expansion (3)

Amplitudes with Higgs tadpole

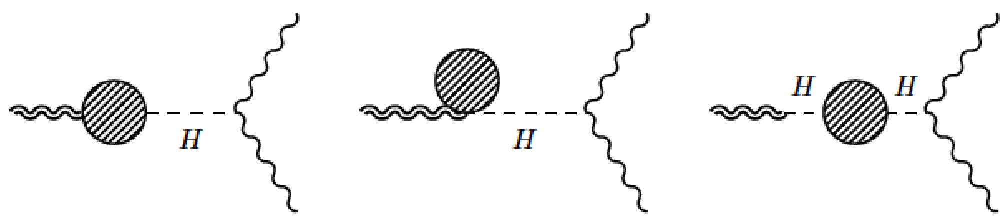


(a) (b)

Amplitude with the graviton-Higgs mixing vertex generated by the improvement term



Leg corrections to the external graviton



(a) (b) (c)

The improvement term

Conformally coupled scalars introduce an improvement term in the EMT

$$T^{I}_{\mu\nu} = -\frac{1}{3}\Big[\partial_\mu \partial_\nu - \eta_{\mu\nu}\,\Box\Big]\mathcal{H}^\dagger\mathcal{H} = -\frac{1}{3}\Big[\partial_\mu \partial_\nu - \eta_{\mu\nu}\,\Box\Big]\left(\frac{H^2}{2} + \frac{\phi^2}{2} + \phi^+\phi^- + v\,H\right)$$

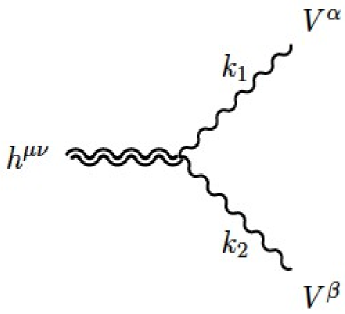
This turns out to be necessary for two reasons

- Ensures the renormalizability of the <TVV'> correlators
- Ensures that the anomaly pole term is the only non-massive contribution to the trace of the correlators

Renormalization

Renormalization is performed in the on-shell scheme

Renormalization of L_{SM} is sufficient to cancel all the singularities from the <TVV'> correlators



$$
\begin{aligned}
\delta[TAA]^{\mu\nu\alpha\beta}(k_1,k_2) &= -i\frac{\kappa}{2}\left\{k_1\cdot k_2\, C^{\mu\nu\alpha\beta} + D^{\mu\nu\alpha\beta}(k_1,k_2)\right\}\delta Z_{AA}\,, \\
\delta[TAZ]^{\mu\nu\alpha\beta}(k_1,k_2) &= -i\frac{\kappa}{2}\left\{\left(\delta c_1^{AZ}\, k_1\cdot k_2 + \delta c_2^{AZ}\, M_Z^2\right) C^{\mu\nu\alpha\beta} + \delta c_1^{AZ}\, D^{\mu\nu\alpha\beta}(k_1,k_2)\right\} \\
\delta[TZZ]^{\mu\nu\alpha\beta}(k_1,k_2) &= -i\frac{\kappa}{2}\left\{\left(\delta c_1^{ZZ}\, k_1\cdot k_2 + \delta c_2^{ZZ}\, M_Z^2\right) C^{\mu\nu\alpha\beta} + \delta c_1^{ZZ}\, D^{\mu\nu\alpha\beta}(k_1,k_2)\right\}
\end{aligned}
$$

$$
\delta c_1^{AZ} = \frac{1}{2}\left(\delta Z_{AZ} + \delta Z_{ZA}\right), \quad \delta c_2^{AZ} = \frac{1}{2}\delta Z_{ZA}\,, \quad \delta c_1^{ZZ} = \delta Z_{ZZ}\,, \quad \delta c_2^{ZZ} = M_Z^2\,\delta Z_{ZZ} + \delta M_Z^2
$$

Conformal anomaly pole in EWSM

As in the other cases the traceful part of the <TVV'> amplitudes contains conformal anomaly poles

<TAA>

$$\Phi_{1,pole} = \Phi^{(F)}_{1,pole} + \Phi^{(W)}_{1,pole} + \Phi^{(I)}_{1,pole} = i\frac{\kappa}{3s}\frac{\beta_e}{e}$$

<TZZ>

$$\Phi_{1,pole} = \Phi^{(F)}_{1,pole} + \Phi^{(W)}_{1,pole} + \Phi^{(Z,H)}_{1,pole} + \Phi^{(I)}_{1,pole} = i\frac{\kappa}{3s}\left[s_w^2\frac{\beta_1}{g_1} + c_w^2\frac{\beta_2}{g_2}\right]$$

The coefficients of the poles depend on the beta functions of the SU(2)W x U(1)Y couplings

$$\beta_1 = \frac{g_1^3}{16\pi^2}\left[\frac{20}{9}n_g + \frac{1}{6}\right]$$

$$\beta_2 = \frac{g_2^3}{16\pi^2}\left[\frac{4}{3}n_g - \frac{22}{3} + \frac{1}{6}\right]$$

$$\beta_e = c_w^2\,\beta_1 + s_w^2\,\beta_2 = \frac{e^3}{16\pi^2}\left[\frac{32}{9}n_g - 7\right]$$

Perturbative computations confirm Riegert's effective action even in a spontaneously broken gauge theory

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Comments

- The non-local gravitational effective action can be rewritten in a local form in terms of two auxiliary scalar fields
- The flat metric expansion of this effective action exhibits a massless pole
- Perturbative computations confirm Riegert's effective action even in a spontaneously broken gauge theory

The graviton correlators computed allow to investigate the interactions between a dilaton and the QCD and the e.w. neutral sector of the Standard Model. The dilatation current of QCD develops an anomaly pole both in the gluon and in the quark sectors

A complete perturbative computation of the <TTT> correlation function has been performed recently (M.S., C. Corianò, L. Delle Rose and E. Mottola). An expansion of the Riegert action in its purely gravitational sector is needed in order to check wether anomaly poles appear there too

Conclusions

Correlation functions involving one graviton and two gauge currents exhibit anomaly poles in each gauge invariant sector.
These correspond to effective degrees of freedom playing an important role in the physics of the early universe.

In a related work we have shown that these are inherited by the QCD dilatation current and can be studied at the LHC.
In this case the current is an interpolating field which corresponds to a dilaton, as for the pion in the chiral case.