Massless scalar degrees of freedom in QCD and in the EW sector with gravity

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Outline

• Conformal anomalies in the context of EFT of gravity

• Evidence of new massless degrees of freedom not contained in the Einstein-Hilbert action

• Explicit analytic perturbative computations on anomalous correlators in QCD and in the Electroweak Theory
The conformal anomaly

The trace of the stress tensor obeys at the classical level

\[ g_{\mu\nu} T_{(cl)}^{\mu\nu} \sim m \Phi^2 \]

At the quantum level the symmetry under global scale transformations cannot be maintained

We end up with a well defined conformal anomaly

\[ g_{\mu\nu} \left< T^{\mu\nu} \right>_{m=0} \neq 0 \]

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The conformal anomaly provides additional terms to be included in the effective theory, beside the classical Einstein-Hilbert action

$$S_{\text{eff}}[g] = S_{\text{EH}}[g] + S_{\text{anom}}[g]$$

The conformal anomaly implies the existence of new massless scalar degrees of freedom

Giannotti, Mottola, PRD79 (2009),
Corianò, Luigi Delle Rose, M.S., PRD83, (2011) 125028
The non local effective action

The trace anomaly in 4 space-time dimensions generated by quantum effects in a classical gravitational and gauge background is given by

\[ T_{\mu}^{\text{anom}} = -\frac{1}{8} \left[ 2bC^2 + 2b' \left( E - \frac{2}{3} \Box R \right) + 2cF^2 \right] \]

Weyl and Euler tensors

\[ C^2 = C_{\lambda\mu\nu\rho}C_{\lambda\mu\nu\rho} = R_{\lambda\mu\nu\rho}R_{\lambda\mu\nu\rho} - 2R_{\mu\nu}R^{\mu\nu} + \frac{R^2}{3} \]

\[ E = \ast R_{\lambda\mu\nu\rho} \ast R_{\lambda\mu\nu\rho} = R_{\lambda\mu\nu\rho}R_{\lambda\mu\nu\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2. \]

The anomalous effective action is identified by solving the variational equation

\[ g^{\mu\nu} \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{anom}}}{\delta g^{\mu\nu}} = T_{\mu}^{\text{anom}} \]
The solution of the anomalous effective action was given by Riegert in a non local formulation

\[ S_{\text{anom}}[g, A] = \frac{1}{8} \int d^4x \sqrt{-g} \int d^4x' \sqrt{-g'} \left( E - \frac{2}{3} \Box R \right)_x \Delta^{-1}_4(x, x') \left[ 2bF + b' \left( E - \frac{2}{3} \Box R \right) + 2cF_{\mu\nu}F^{\mu\nu} \right]_{x'} \]

Riegert, PLB 134 (1984)

Where \( \Delta^{-1}_4(x, x') \) denotes the Green’s function of the fourth order differential operator

\[ \Delta_4 \equiv \nabla_\mu (\nabla^\mu \nabla^\nu + 2R^{\mu\nu} - \frac{2}{3} R g^{\mu\nu}) \nabla_\nu \]

\[ = \Box^2 + 2R^{\mu\nu} \nabla_\mu \nabla_\nu + \frac{1}{3} (\nabla^\mu R) \nabla_\mu - \frac{2}{3} R \Box \]
The non local effective action can be recast in a local form after the introduction of two scalar auxiliary fields.

To first order in metric variations around flat space

\[
S_{\text{anom}}[g, A; \varphi, \psi'] = \int d^4 x \sqrt{-g} \left[ -\psi' \Box \varphi - \frac{R}{3} \psi' + \frac{c}{2} F_{\alpha\beta} F^{\alpha\beta} \varphi \right]
\]

with the two fields satisfying the equations

\[
\psi' \equiv b \Box \psi,
\]

\[
\Box \psi' = \frac{c}{2} F_{\alpha\beta} F^{\alpha\beta}
\]

\[
\Box \varphi = -\frac{R}{3}.
\]


Giannotti, Mottola, PRD79 (2009)
The local formulation

The non local effective action can be recast in a local form after the introduction of two scalar auxiliary fields.

\[ S_{\text{anom}}[g, A; \varphi, \psi'] = \int d^4x \sqrt{-g} \left[ -\psi' \Box \varphi - \frac{R}{3} \psi' + \frac{c}{2} F_{\alpha\beta} F^{\alpha\beta} \varphi \right] \]

Coming back to the equivalent non local form

\[ S_{\text{anom}}[g, A] \rightarrow -\frac{c}{6} \int d^4x \sqrt{-g} \int d^4x' \sqrt{-g'} R_x \Box_{x,x'}^{-1} [F_{\alpha\beta} F^{\alpha\beta}]_{x'} \]

- The anomaly-induced action will be missing homogeneous contributions
- There is currently a disagreement over the correctness of this result.
The $\langle TVV \rangle$ correlator is the first (leading) order contribution in the expansion of the anomalous effective action.

From the standard QCD Lagrangian we extract the energy-momentum tensor (EMT)

$$T_{\mu \nu} = -g_{\mu \nu} \mathcal{L}_{QCD} - F_{\mu \rho} F_{\nu}^{\rho} - \frac{1}{\xi} g_{\mu \nu} \partial^\rho (A^a_{\rho} \partial^\sigma A^a_{\sigma}) + \frac{1}{\xi} (A^a_{\nu} \partial_{\mu} (\partial^\sigma A^a_{\sigma}) + A^a_{\mu} \partial_{\nu} (\partial^\sigma A^a_{\sigma}))$$

$$+ \frac{i}{4} \left[ \bar{\psi} \gamma_\mu (\partial_\nu - ig T^a A^a_{\nu}) \psi - \bar{\psi} (\partial_\nu + ig T^a A^a_{\nu}) \gamma_\mu \psi + \bar{\psi} \gamma_\nu (\partial_\mu - ig T^a A^a_{\mu}) \psi \right]$$

$$- \bar{\psi} (\partial_\mu + ig T^a A^a_{\mu}) \gamma_\nu \psi \right] + \partial_\mu \bar{\omega}^a (\partial_\nu \omega^a - gf^{abc} A^c_{\nu} \omega^b) + \partial_\nu \bar{\omega}^a (\partial_\mu \omega^a - gf^{abc} A^c_{\mu} \omega^b).$$
For an exact non-abelian gauge theory the perturbative expansion of the $<TVV>$ correlator is more involved.

**Quark sector**

**Gauge sector**

**Ghost sector**
Diffeomorphisms invariance

Invariance under diffeomorphisms implies constraints on the generating functional of 1PI graphs ($\tilde{\Pi}$)

\[
\frac{\partial}{\partial h_{\alpha\beta}} \frac{\delta \Gamma}{\delta h_{\alpha\beta}} = -\frac{\kappa}{2} \left\{ -\frac{\delta \Gamma}{\delta \phi} \partial^\beta \phi_c - \frac{\delta \Gamma}{\delta \phi^\dagger} \partial^\beta \phi^\dagger_c - \frac{\delta \Gamma}{\delta A_{c\alpha}} \partial^\beta A_{c\alpha} - \partial^\alpha \left( \frac{\delta \Gamma}{\delta A_{c\alpha}} A^\beta_c \right) 
- \partial^\beta \bar{\psi}_c \frac{\delta \Gamma}{\delta \psi^\dagger_c} + \frac{\delta \Gamma}{\delta \psi_c} \psi_c \partial^\beta \psi_c - \frac{1}{2} \partial^\alpha \left( \frac{\delta \Gamma}{\delta \psi_c} \sigma^{\alpha\beta} \psi_c - \bar{\psi}_c \sigma^{\alpha\beta} \frac{\delta \Gamma}{\delta \psi_c} \right) \right\},
\]

For the $<TVV'>$ correlators ($V, V' = \text{gluons}$) the EMT conservation gives

\[
-i \frac{\kappa}{2} \partial^\mu \langle T_{\mu\nu}(x)V_\alpha(x_1)V^\prime_{\beta}(x_2) \rangle_{\text{amp}} = -\frac{\kappa}{2} \left\{ -\partial_{\nu} \delta^{(4)}(x_1 - x) P^{-1}_{\alpha\beta}(x_2, x) 
- \partial_{\nu} \delta^{(4)}(x_2 - x) P^{-1}_{\alpha\beta}(x_1, x) + \partial^\mu [\eta_{\alpha\nu} \delta^{(4)}(x_1 - x) P^{-1}_{\beta\mu}(x_2, x) + \eta_{\beta\nu} \delta^{(4)}(x_2 - x) P^{-1}_{\alpha\mu}(x_1, x)] \right\}
\]
BRST symmetry

BRST invariance of the theory implements relations between correlation functions in the form of Slavnov-Taylor identities (STI)

From the BRST variations of the fields

\[
\begin{align*}
\delta A_{\mu}^a &= \lambda D_{\mu}^{ab} \omega^b \\
\delta \psi &= ig\lambda \omega^a t^a \psi \\
\delta \psi &= -ig\bar{\psi} t^a \lambda \omega^a \\
\delta \omega^a &= -\frac{1}{2} g\lambda f^{abc} \omega^b \omega^c \\
\delta \bar{\omega}^a &= -\frac{1}{\xi} (\partial^\mu A_{\mu}^a) \lambda
\end{align*}
\]

the corresponding variation of the EMT is obtained, i.e.

\[
\delta T_{\mu\nu} = \delta(T_{\mu\nu}^{g.f.} + T_{\mu\nu}^{gh}) = \frac{\lambda}{\xi} \left[ A_{\mu}^a \partial_{\nu} \partial^\rho D_{\rho}^{ab} \omega^b + A_{\nu}^a \partial_{\mu} \partial^\rho D_{\rho}^{ab} \omega^b - g_{\mu\nu} \partial^\sigma (A_{\sigma}^a \partial^\rho D_{\rho}^{ab} \omega^b) \right]
\]

which in turn allows to derive the BRST constraint on the amplitude, which is

\[
p^\alpha q^\beta \langle T_{\mu\nu}(k) A^a_\alpha(p) A^b_\beta(q) \rangle_{\text{trunc}} = 0
\]
Expansion of the amplitude

The sum of the quark and gluon/ghost contributions is written as

\[ \Gamma_{\mu\nu\alpha\beta}(p, q) = \Gamma_{g}^{\mu\nu\alpha\beta}(p, q) + \Gamma_{q}^{\mu\nu\alpha\beta}(p, q) = \sum_{i=1}^{3} \Phi_{i}(s, 0, 0) \delta^{ab} \phi_{i}^{\mu\nu\alpha\beta}(p, q) \]

\[ \phi_{1}^{\mu\nu\alpha\beta}(p, q) = (s g^{\mu\nu} - k^{\mu} k^{\nu}) u^{\alpha\beta}(p, q), \]
\[ \phi_{2}^{\mu\nu\alpha\beta}(p, q) = -2 u^{\alpha\beta}(p, q) [s g^{\mu\nu} + 2(p^{\mu} p^{\nu} + q^{\mu} q^{\nu}) - 4(p^{\mu} q^{\nu} + q^{\mu} p^{\nu})], \]
\[ \phi_{3}^{\mu\nu\alpha\beta}(p, q) = (p^{\mu} q^{\nu} + p^{\nu} q^{\mu}) g^{\alpha\beta} + \frac{s}{2} \left( g^{\alpha\nu} g^{\beta\mu} + g^{\alpha\mu} g^{\beta\nu} \right) \]
\[ -g^{\mu\nu} \left( \frac{s}{2} g^{\alpha\beta} - q^{\alpha} p^{\beta} \right) - \left( g^{\beta\nu} p^{\mu} + g^{\beta\mu} p^{\nu} \right) q^{\alpha} - (g^{\alpha\nu} q^{\mu} + g^{\alpha\mu} q^{\nu}) p^{\beta}, \]

Form factors

Tensor basis elements

Only traceful tensor in the basis
The contribution of the trace of the energy momentum tensor is given by

\[ g_{\mu\nu} \Gamma_{\mu\nu}^{\alpha\beta}(p, q) = 3\, s\, \Phi_1(s; 0, 0, 0)\, u^{\alpha\beta}(p, q) = -2\, \frac{\beta(g)}{g}\, u^{\alpha\beta}(p, q) \]

\[ \beta(g) = \frac{g^3}{16\pi^2} \left( -\frac{11}{3}\, C_A + \frac{2}{3}\, n_f \right) \]

\[ u^{\alpha\beta}(p, q) = -\frac{1}{4} \int d^4x \int d^4y e^{ip\cdot x + iq\cdot y} \frac{\delta^2\{F_{\mu\nu}F^{\mu\nu}(0)\}}{\delta A_\alpha(x)A_\beta(y)} \]

\[ \Phi_1(s, 0, 0) = -\frac{g^2}{72\pi^2\, s}\, (2n_f - 11C_A) \]

The form factor multiplying the traceful tensor shows the massless pole searched for!

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The $<TVV>$ correlators in EWSM

Effective action analysis extended to the neutral gauge boson sector of the electroweak theory

The symmetric and conserved EMT may be obtained by coupling the SM Lagrangian to the gravitational field

$$S = S_G + S_{SM} + S_I = -\frac{1}{\kappa^2} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \mathcal{L}_{SM} + \frac{1}{6} \int d^4x \sqrt{-g} R \mathcal{H}^\dagger \mathcal{H}$$
The \( <TVV> \) correlators in \textbf{EWSM}

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\[
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\]

The full EMT is given by a minimal tensor and a term of improvement generated by the conformal coupling of scalars

\[
T_{\mu\nu} = T_{\mu\nu}^{\text{Min}} + T_{\mu\nu}^{I}
\]

\[
T_{\mu\nu}^{\text{Min}} = T_{\mu\nu}^{\text{f.s.}} + T_{\mu\nu}^{\text{ferm.}} + T_{\mu\nu}^{\text{Higgs}} + T_{\mu\nu}^{\text{Yukawa}} + T_{\mu\nu}^{\text{g.fix.}} + T_{\mu\nu}^{\text{ghost}}
\]
One loop corrections are computed in DR using on-shell renormalization scheme and the ‘t Hooft-Veltman prescription for $\phi$

A fully automated methodology is implemented in the symbolic manipulation program Mathematica to manage hundreds of diagrams

We show some of them with three, two and one point topology

**Diagrammatic expansion (1)**

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Diagrammatic expansion (2)
Diagrammatic expansion (3)

Amplitudes with Higgs tadpole

Amplitude with the graviton-Higgs mixing vertex generated by the improvement term

Leg corrections to the external graviton
The improvement term

Conformally coupled scalars introduce an improvement term in the EMT

\[ T_{\mu\nu}^I = -\frac{1}{3} \left[ \partial_\mu \partial_\nu - \eta_{\mu\nu} \Box \right] \mathcal{H}^\dagger \mathcal{H} = -\frac{1}{3} \left[ \partial_\mu \partial_\nu - \eta_{\mu\nu} \Box \right] \left( \frac{H^2}{2} + \frac{\phi^2}{2} + \phi^+ \phi^- + v H \right) \]

This turns out to be necessary for two reasons

- Ensures the renormalizability of the \( <TVV'> \) correlators
- Ensures that the anomaly pole term is the only non-massive contribution to the trace of the correlators
Renormalization is performed in the on-shell scheme.

Renormalization of $\mathcal{L}_{\text{SM}}$ is sufficient to cancel all the singularities from the $\langle TVV' \rangle$ correlators.

\[
\delta[T_{AA}]^{\mu\nu\alpha\beta}(k_1, k_2) = -i \frac{\kappa}{2} \left\{ k_1 \cdot k_2 C^{\mu\nu\alpha\beta} + D^{\mu\nu\alpha\beta}(k_1, k_2) \right\} \delta Z_{AA},
\]

\[
\delta[T_{AZ}]^{\mu\nu\alpha\beta}(k_1, k_2) = -i \frac{\kappa}{2} \left\{ (\delta c_1^{AZ} k_1 \cdot k_2 + \delta c_2^{AZ} M_Z^2) C^{\mu\nu\alpha\beta} + \delta c_1^{AZ} D^{\mu\nu\alpha\beta}(k_1, k_2) \right\}
\]

\[
\delta[T_{ZZ}]^{\mu\nu\alpha\beta}(k_1, k_2) = -i \frac{\kappa}{2} \left\{ (\delta c_1^{ZZ} k_1 \cdot k_2 + \delta c_2^{ZZ} M_Z^2) C^{\mu\nu\alpha\beta} + \delta c_1^{ZZ} D^{\mu\nu\alpha\beta}(k_1, k_2) \right\}
\]

\[
\delta c_1^{AZ} = \frac{1}{2} (\delta Z_{AZ} + \delta Z_{ZA}), \quad \delta c_2^{AZ} = \frac{1}{2} \delta Z_{ZA}, \quad \delta c_1^{ZZ} = \delta Z_{ZZ}, \quad \delta c_2^{ZZ} = M_Z^2 \delta Z_{ZZ} + \delta M_Z^2
\]
Conformal anomaly pole in EWSM

As in the other cases the traceful part of the $<TVV'>$ amplitudes contains conformal anomaly poles

$$\Phi_{1,pole} = \Phi_{1,pole}^{(F)} + \Phi_{1,pole}^{(W)} + \Phi_{1,pole}^{(I)} = i \frac{\kappa \beta_e}{3s e}$$

$$\Phi_{1,pole} = \Phi_{1,pole}^{(F)} + \Phi_{1,pole}^{(W)} + \Phi_{1,pole}^{(Z,H)} + \Phi_{1,pole}^{(I)} = i \frac{\kappa}{3s} \left[ s_w^2 \frac{\beta_1}{g_1} + c_w^2 \frac{\beta_2}{g_2} \right]$$

The coefficients of the poles depend on the beta functions of the $SU(2)_W \times U(1)_Y$ couplings

$$\beta_1 = \frac{g_1^3}{16\pi^2} \left[ \frac{20}{9} n_g + \frac{7}{6} \right]$$

$$\beta_2 = \frac{g_2^3}{16\pi^2} \left[ \frac{4}{3} n_g - \frac{22}{3} + \frac{1}{6} \right]$$

$$\beta_e = c_w^2 \beta_1 + s_w^2 \beta_2 = \frac{\epsilon^3}{16\pi^2} \left[ \frac{32}{9} n_g - 7 \right]$$

Perturbative computations confirm Riegert’s effective action even in a spontaneously broken gauge theory

\[ \text{QCD @ Work 2012} \]
The non-local gravitational effective action can be rewritten in a local form in terms of two auxiliary scalar fields.

The flat metric expansion of this effective action exhibits a massless pole.

Perturbative computations confirm Riegert’s effective action even in a spontaneously broken gauge theory.
The graviton correlators computed allow to investigate the interactions between a dilaton and the QCD and the e.w. neutral sector of the Standard Model. The dilatation current of QCD develops an anomaly pole both in the gluon and in the quark sectors.

A complete perturbative computation of the $\langle TTTT \rangle$ correlation function has been performed recently (M.S., C. Corianò, L. Delle Rose and E. Mottola). An expansion of the Riegert action in its purely gravitational sector is needed in order to check whether anomaly poles appear there too.
Correlation functions involving one graviton and two gauge currents exhibit anomaly poles in each gauge invariant sector. These correspond to effective degrees of freedom playing an important role in the physics of the early universe.

In a related work we have shown that these are inherited by the QCD dilatation current and can be studied at the LHC. In this case the current is an interpolating field which corresponds to a dilaton, as for the pion in the chiral case.