Confinement and de-confinement from dynamical holographic model

Mei Huang

Institute of High Energy Physics, CAS

Theoretical Physics Center
for Science Facilities, CAS

QCD@Work, Lecce, June 18-21,2012

Content

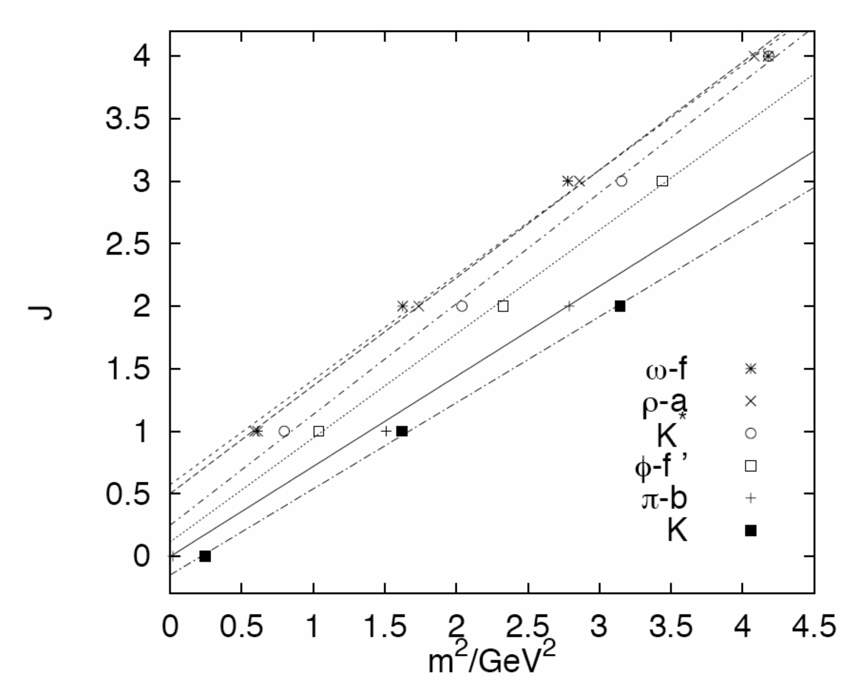
I. Introduction
II. Confinement and deconfinement in graviton-dilaton system
III. Linear Regge behavior and linear quark potential in graviton-dilaton-scalar system
IV. Conclusion and Discussion

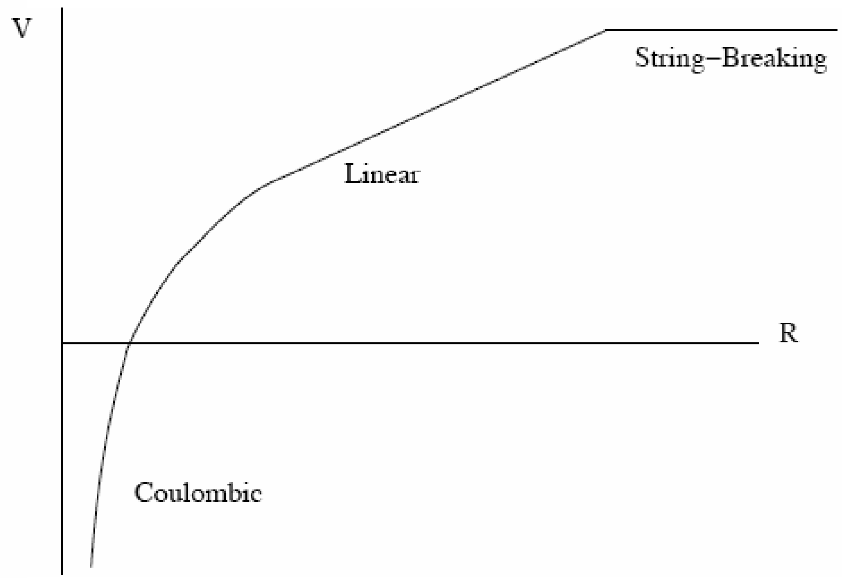
Based on:

S. He, M. H., Q. S. Yan, arXiv:1004.1880, PRD2011
D.N, Li, S. He, M. H., Q. S. Yan, arXiv:1103.5389, JHEP2011
D.N, Li, M. H., Q. S. Yan, arXiv:1206.2824

I. Introduction

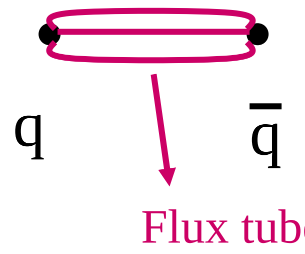
Confinement: Regge behavior and linear quark potential





QCD and string theory I:

String model & confinement



$$V \sim \sigma_s R$$

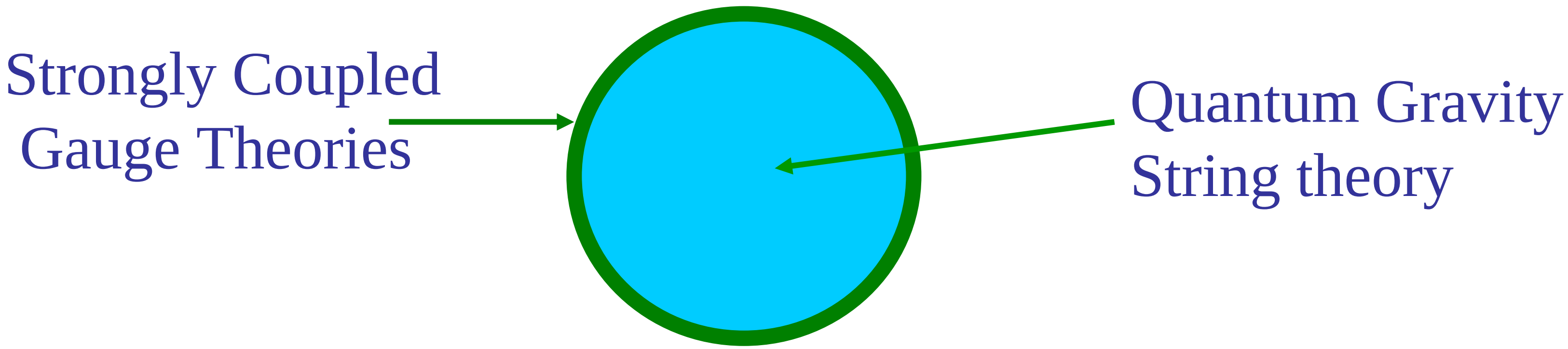
QCD and string theory: AdS/CFT

J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998)

The precise duality relationship is

$$\left\langle e^{\int d^4x \phi_0(\vec{x}) \mathcal{O}(\vec{x})} \right\rangle_{\mathrm{CFT}} = \mathcal{Z}_{\mathrm{string}} \left[\phi\left(\vec{x}, z\right)|_{z=0} \equiv \phi_0\left(\vec{x}\right) \right].$$

Is there a dual string theory for any strongly coupled gauge theory?



QCD and string theory: holographic QCD

Dp-Dq system in type-II superstring theory (10D)

Not my task!

holographic QCD (5D)

Build the connection between QCD dynamics and geometry

3rd step: gravity dual
systematic framework

2nd step: deformed AdS_5
intelligent guess

1st step: just AdS_5
naïve try

Real QCD world:

Rich experimental data and lattice data

II. Confinement and deconfinement in graviton-dilaton system

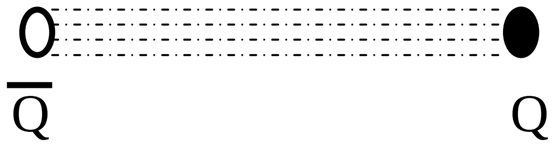
For pure gluon system

S. He, M. H., Q. S. Yan, arXiv:1004.1880, PRD2011
D.N, Li, S. He, M. H., Q. S. Yan, arXiv:1103.5389, JHEP2011

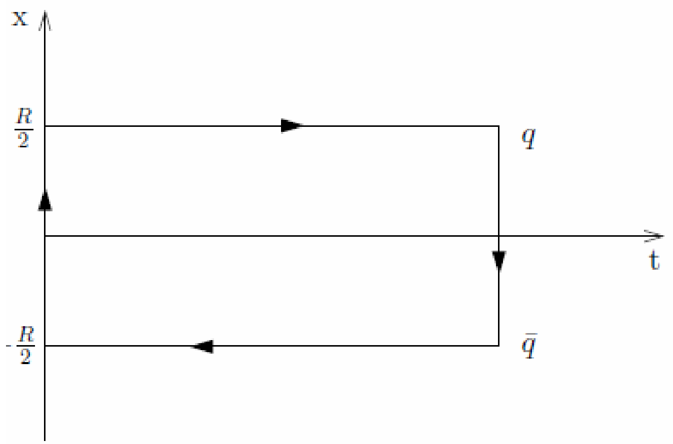
1, Confinement for pure glue system

Confinement potential

$$V_{Q\bar{Q}}(R) = -\frac{\kappa}{R} + \sigma_{str} R + V_0$$



$$W[C] = \frac{1}{N} Tr P \exp[i \oint_C A_\mu dx^\mu]$$



$$\langle W(C) \rangle \propto e^{-TV_{Q\bar{Q}}}$$

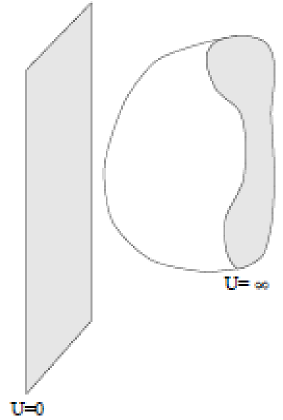
$$T \to \infty$$

Holographic dictionary:

J. M. Maldacena, Phys. Rev. Lett. 80, 4859 (1998), hep-th/9803002.

$$\langle W^{4d}[C]\rangle = Z^{5d}_{string}[C] \simeq e^{-S_{NG}[C]}$$

$$V_{Q\overline{Q}}(r) = \lim_{T\to\infty}\frac{1}{T}S_{NG}[C]$$



Metric structure determines the quark potential !

1, AdS$_5$ only gives Coulomb potential !

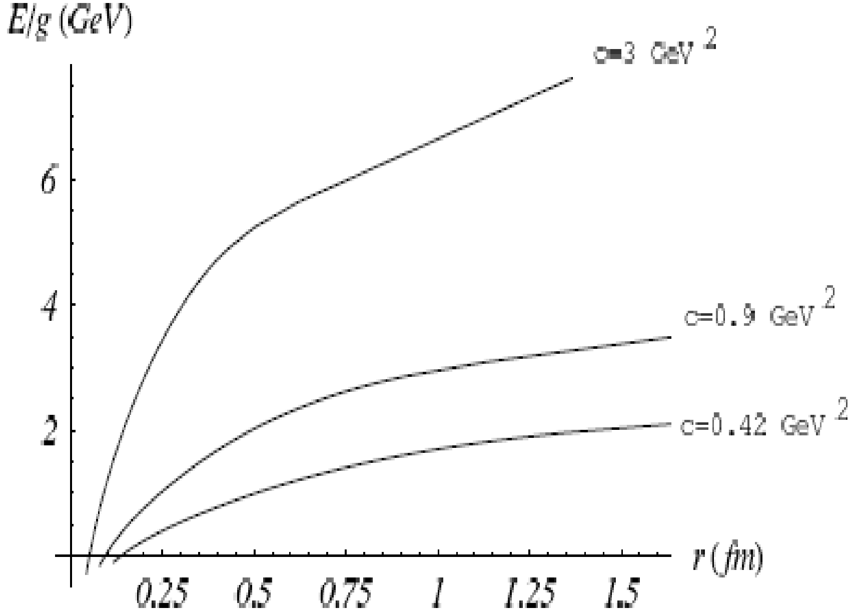
2, Deformed metric structure is needed to produce the linear potential!

Deformed AdS$_5$ models I:

Andreev-Zakharov model: quadratic correction

O. Andreev, V.Zakharov, hep-ph/0604204

$$ds^2 = G_{nm} dX^n dX^m = R^2 \frac{h}{z^2}\left(dx^i dx^i + dz^2\right) \qquad h = \mathrm{e}^{\frac{1}{2}cz^2}$$



Deformed AdS$_5$ models II:

Pirner-Galow model: resemble QCD running coupling

H.J.Pirner, B.Galow, arXiv:0903.2701

$$ds^2_{\rm QCD} = h(z) \cdot ds^2 = h(z) \frac{L^2}{z^2}(-dt^2 + d\vec{x}^2 + dz^2),$$

$$\alpha_s(p) = \frac{1}{4\pi\beta_0 \log(p^2/\Lambda^2_{\rm QCD})} = \frac{4\pi}{(11 - \frac{2}{3}n_f)\log(p^2/\Lambda^2_{\rm QCD})}$$

$$\boxed{h(z) = \frac{c_2}{\log\left[\frac{1}{z^2 + l_s^2}\frac{1}{\Lambda^2}\right]}}$$

Deformed AdS_5 models III:

Our holographic model

S. He, M. H., Q. S. Yan, arXiv:1004.1880, PRD2011

String Frame:

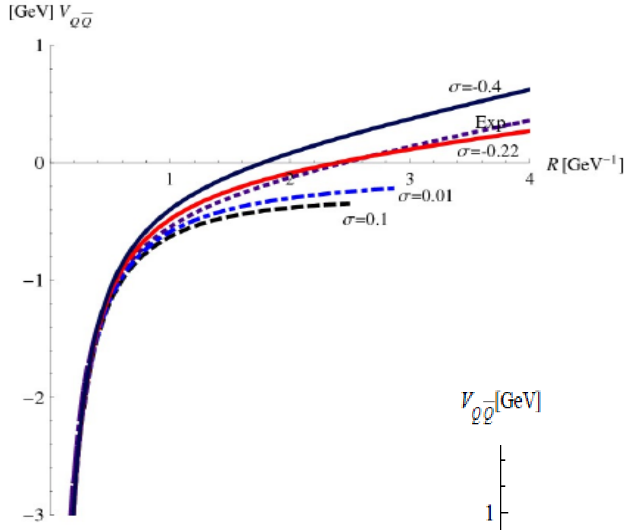
$$
\begin{aligned}
ds^2 = G^s_{\mu\nu} dX^\mu dX^\nu &= \frac{h(z)L^2}{z^2}\left(dt^2 + d\vec{x}^2 + dz^2\right) \\
&= e^{2\mathcal{A}_s(z)}\left(dt^2 + d\vec{x}^2 + dz^2\right).
\end{aligned}
$$

$$
h(z) = \exp\left(-\frac{\sigma z^2}{2} - c_0 \ln\left(\frac{z_{IR} - z}{z_{IR}}\right)\right)
$$

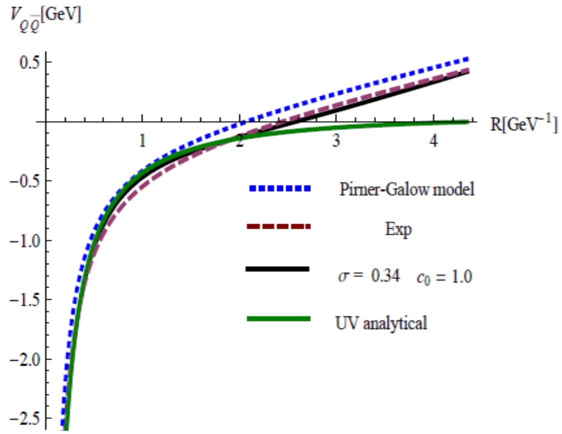
$$h(z) = \exp\left(-\frac{\sigma z^2}{2} - c_0 \ln\left(\frac{z_{IR} - z}{z_{IR}}\right)\right)$$

S. He, M. H., Q. S. Yan, arXiv:1004.1880, PRD2011

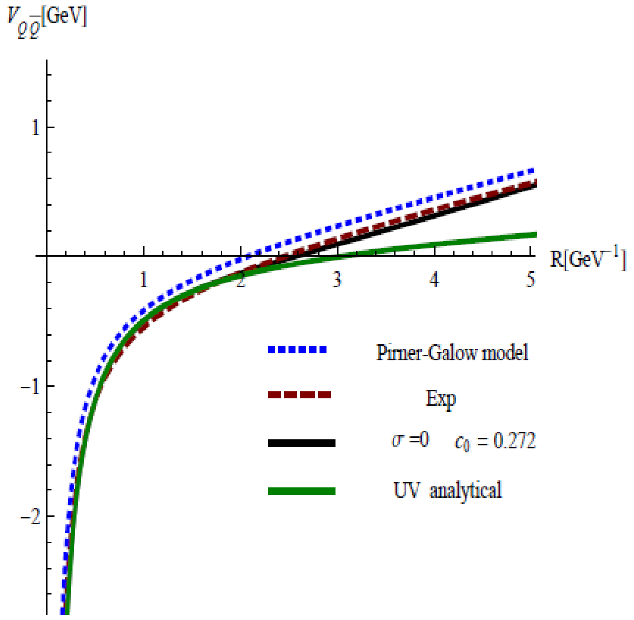
With only quadratic correction:
Andreev-Zakharov model



With both quadratic and logarithmic correction:



With only logarithmic correction:



A systematic framework: Graviton-dilaton system

$$S_{5D} = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g^E} \left(R - \frac{4}{3}\partial_\mu \phi \partial^\mu \phi - V_E(\phi) \right)$$

N=4 Super YM

conformal

AdS$_5$

$$ds^2 = \frac{L^2}{z^2}\left(dt^2 + d\vec{x}^2 + dz^2\right)$$

$$V_E(\phi) = -\frac{12}{L^2}$$

QCD

nonconformal

deformed AdS$_5$

$$ds^2 = \frac{h(z)L^2}{z^2}\left(dt^2 + d\vec{x}^2 + dz^2\right)$$

Dilaton field breaks conformal symmetry

Metric structure, Dilaton field and dilaton potential should be solved self-consistently from the Einstein equations.

Connection
between QCD dynamics and the geometry

Through the AdS/CFT dictionary, for any dilaton field Φ, we have

$$\lim_{\Phi\to 0} V(\Phi) = -\frac{12}{L^2} + \frac{1}{2L^2}\Delta(\Delta-4)\Phi^2 + O(\Phi^4).$$

$$\Delta(\Delta - 4) = M_\Phi^2 L^2$$

For positive quadratic correction in the metric, the solution of the dilaton field has dimension of 2 :

$$M_\Phi^2 = -\frac{4}{L^2}, \qquad \Delta = 2$$

Indication of dimension-2 gluon condensate

2, Deconfinement phase transition

D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

5D graviton-dilaton action:

$$S_{5D} = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g^E} \left(R - \frac{4}{3} \partial_\mu \phi \partial^\mu \phi - V_E(\phi) \right)$$

$$ds_S^2 = \frac{L^2 e^{2A_s}}{z^2} \left(-f(z)dt^2 + \frac{dz^2}{f(z)} + dx^i dx^i \right),$$

Metric structure, blackhole, Dilaton field and Dilaton potential should be solved self-consistently from the Einstein equations.

Experiences in constructing holographic QCD model tells us that: a quadratic correction in the deformed warp factor is responsible for the linear confinement.

$$A_s(z) = ck^2 z^2$$

$$\begin{aligned}
\phi(z) &= \phi_0 + \phi_1 \int_0^z \frac{e^{2A_s(x)}}{x^2}\, dx + \frac{3A_s(z)}{2} \\
&\quad + \frac{3}{2}\int_0^z \frac{e^{2A_s(x)} \int_0^x y^2 e^{-2A_s(y)} A_s'(y)^2 dy}{x^2}\, dx, \\
f(z) &= f_0 + f_1 \left(\int_0^z x^3 e^{2\phi(x) - 3A_s(x)}\, dx \right), \\
V_E(\phi) &= \frac{e^{\frac{4\phi(z)}{3} - 2A_s(z)}}{L^2} \\
&\quad \left(z^2 f''(z) - 4f(z) \left(3z^2 A_s''(z) - 2z^2\phi''(z) + z^2\phi'(z)^2 + 3 \right) \right).
\end{aligned}$$

D.N, Li, S. He, M. H., Q. S. Yan, arXiv:1103.5389, JHEP2011

Deconfinement phase transition

$$L(\vec{x}) = \frac{1}{N_c} \mathrm{tr}\, \mathcal{P}(\vec{x}) \text{ with } \mathcal{P}(\vec{x}) = \mathrm{P}\, \mathrm{e}^{ig \int_0^\beta dt\, A_0(t,\vec{x})}$$

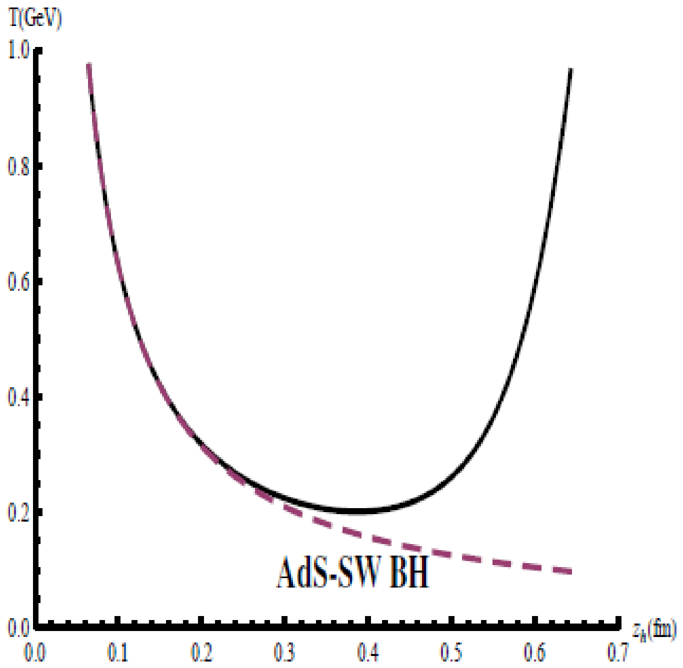
$$\langle P(\vec{x}) \rangle = \begin{cases} 0 & \text{unbroken } Z_N \text{ symmetry phase} \\ \text{non-zero} & \text{broken } Z_N \text{ symmetry phase} \end{cases}$$

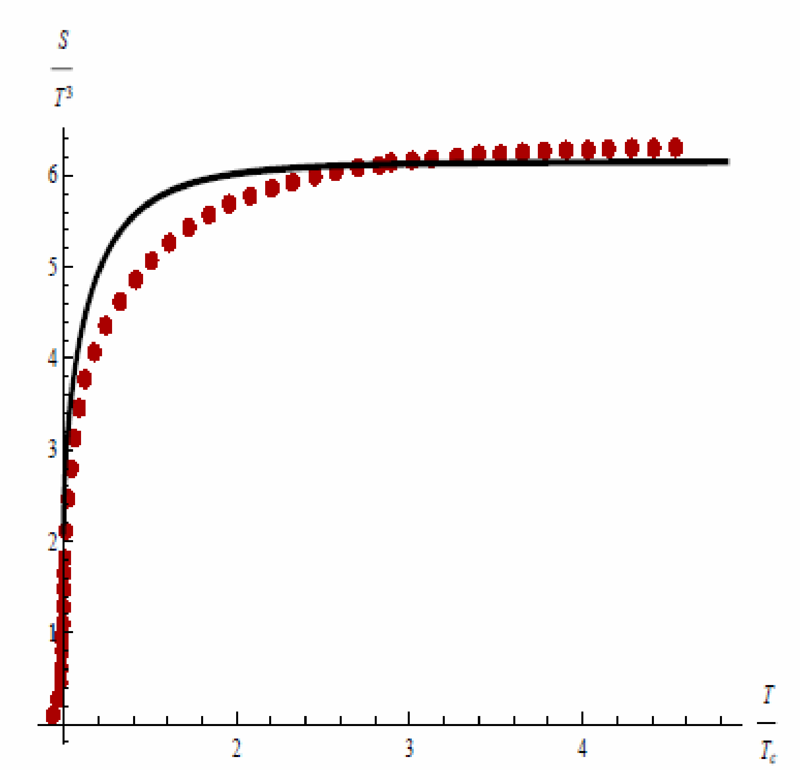
The positive quadratic correction deformed AdS5 model can fit all the finite temperature lattice QCD data for pure gauge SU(3) theory!

Lattice data for EOS: G.Boyd et.al., NPB469(1996),419
Lattice data for loop operators:
G.S.Bali et.al. PRL71(1993),3059
S.Gupta et.al. PRD77(2008),034503

$$T = \frac{|f'(z_h)|}{4\pi}.$$

$$s = \frac{A_{area}}{4G_5 V_3} = \frac{L^3}{4G_5}\left(\frac{e^{A_s - \frac{2}{3}\phi}}{z}\right)^3.$$



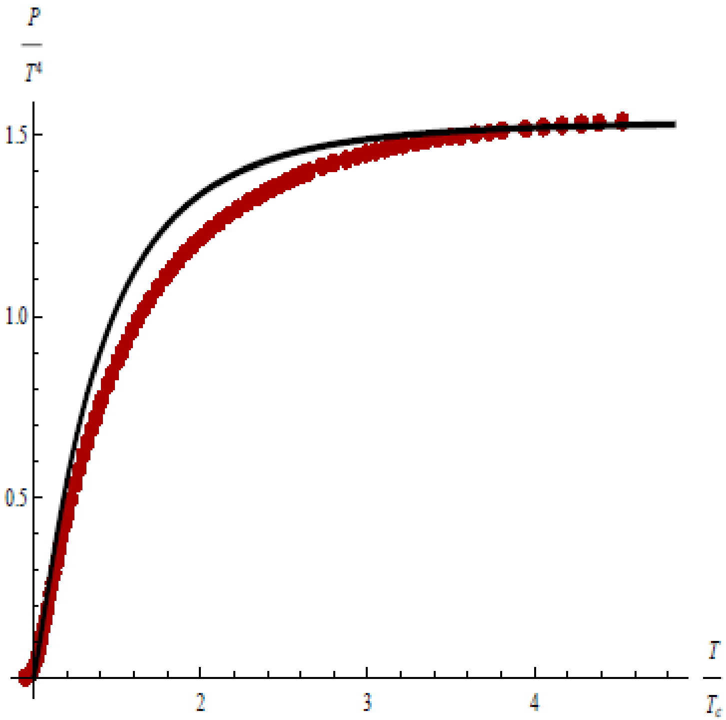


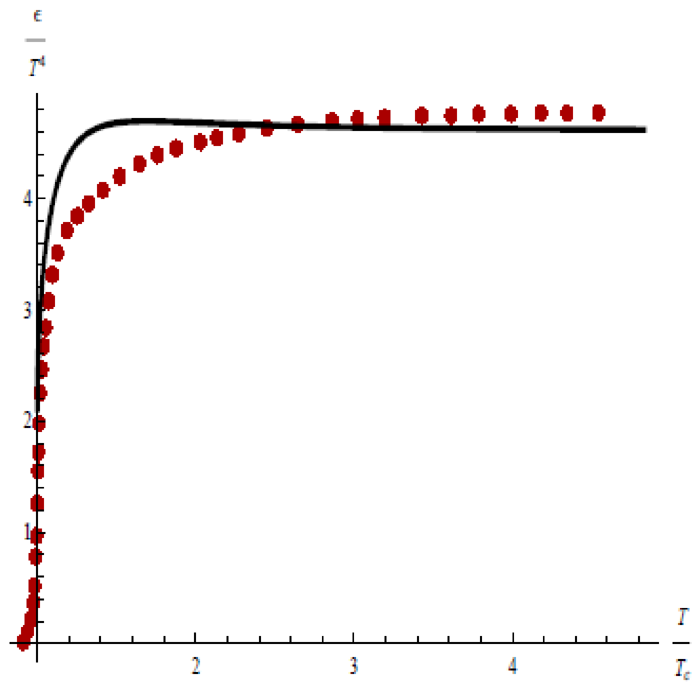
$T_c = 201\text{MeV}$

D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

$$\frac{dp(T)}{dT} = s(T).$$

$$\epsilon = -p + sT.$$

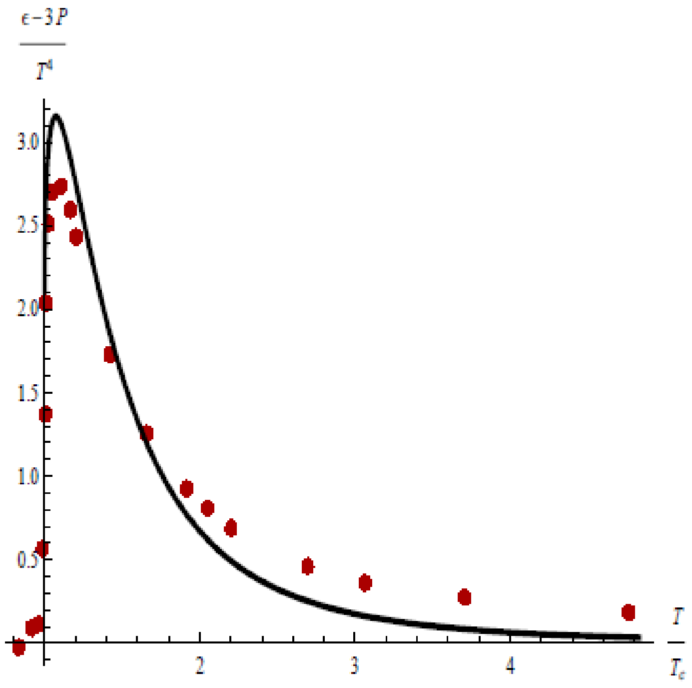


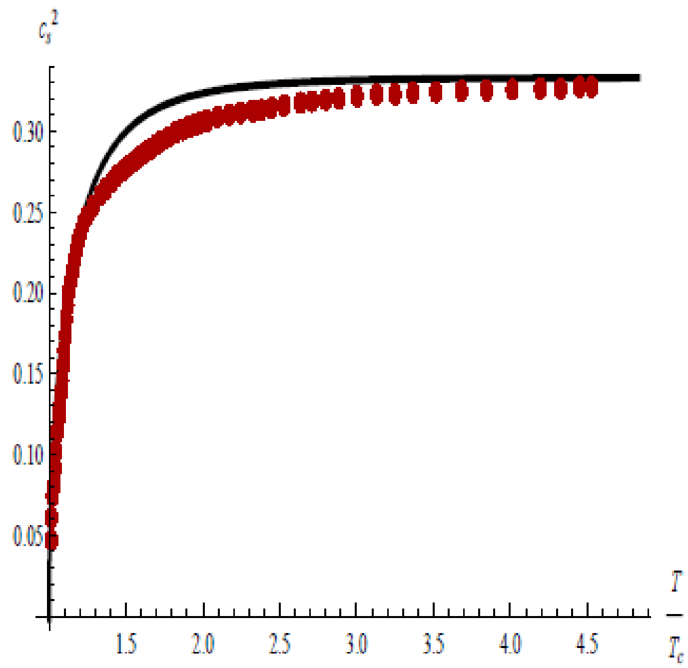


D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

Trace anomaly

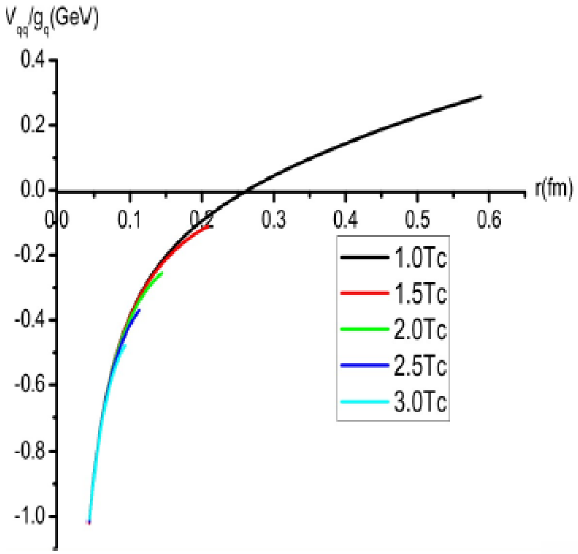
$$c_s^2 = \frac{d\log T}{d\log s} = \frac{s}{T ds/dT},$$



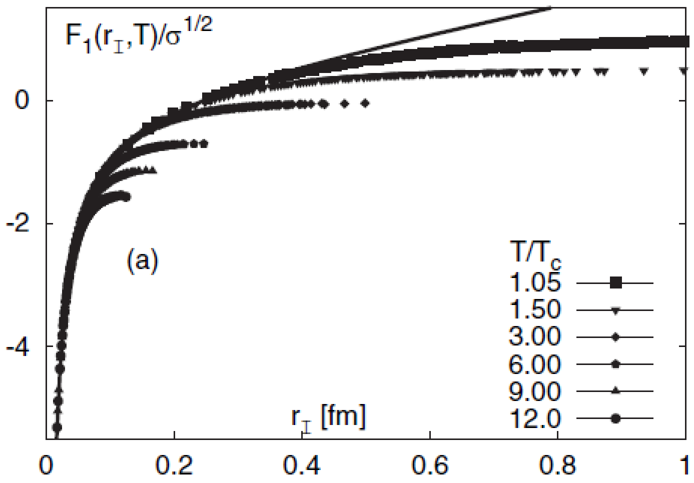


D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

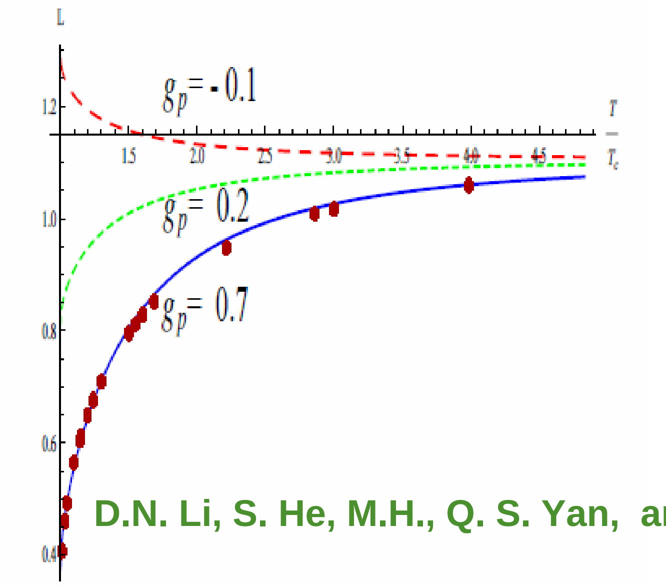
Electric screening



Heavy quark potential

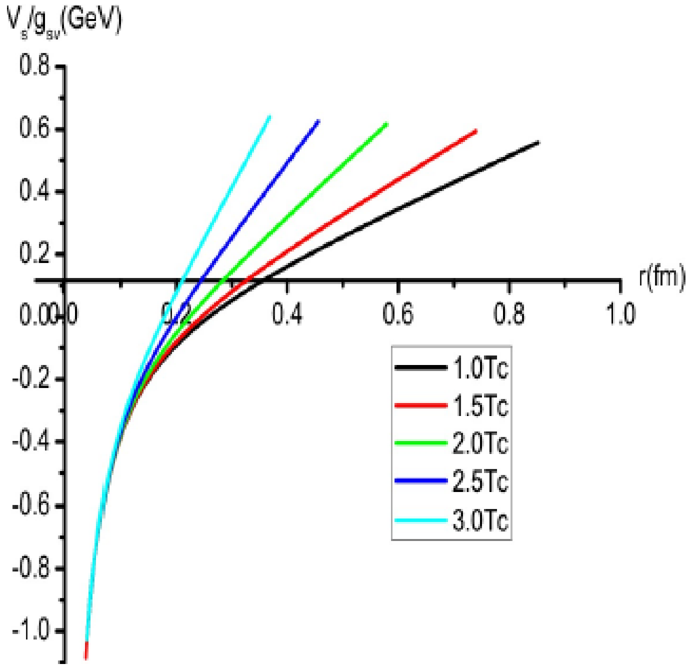


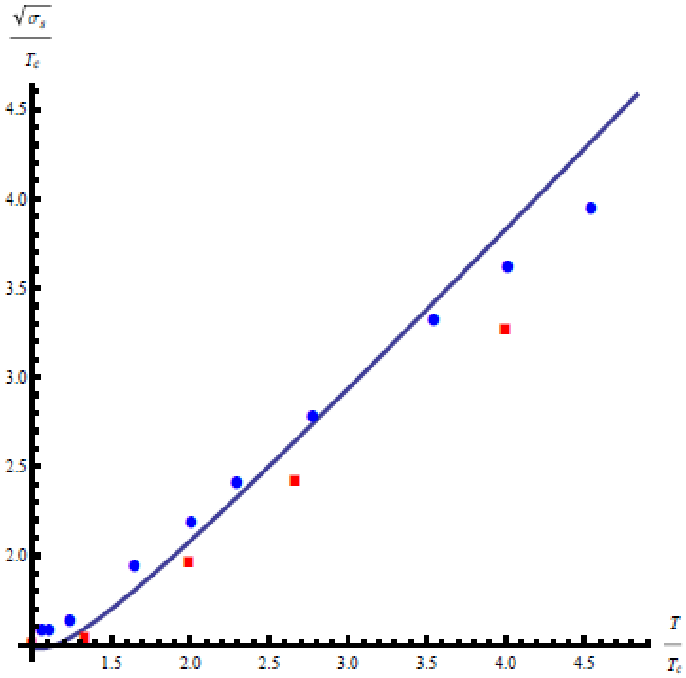
Polyakov loop: color electric deconfinement



D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

Magnetic screening and magnetic confinement





spatial Wilson loop　　spatial string tension

D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

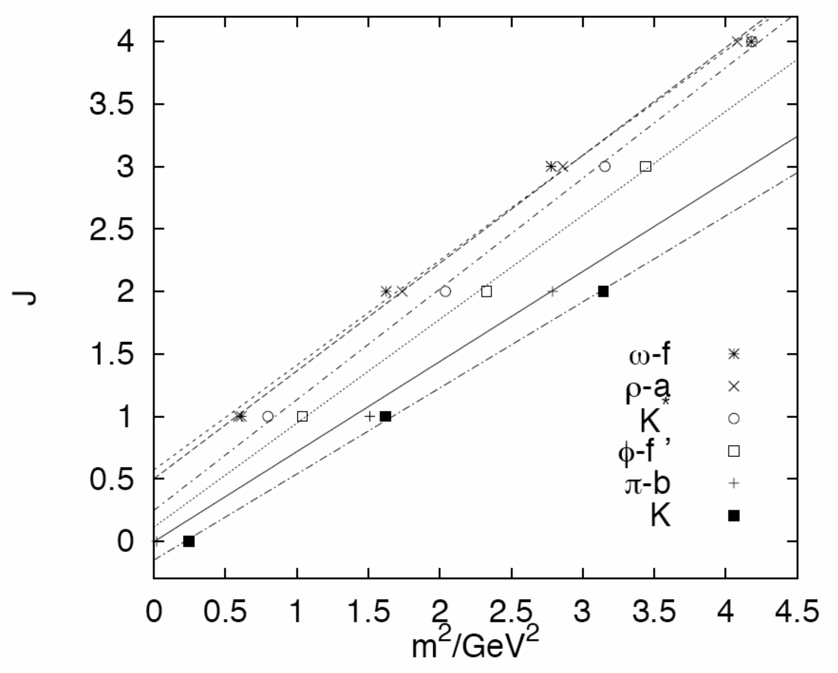
III. Linear Regge behavior and linear quark potential in graviton-dilaton-scalar system

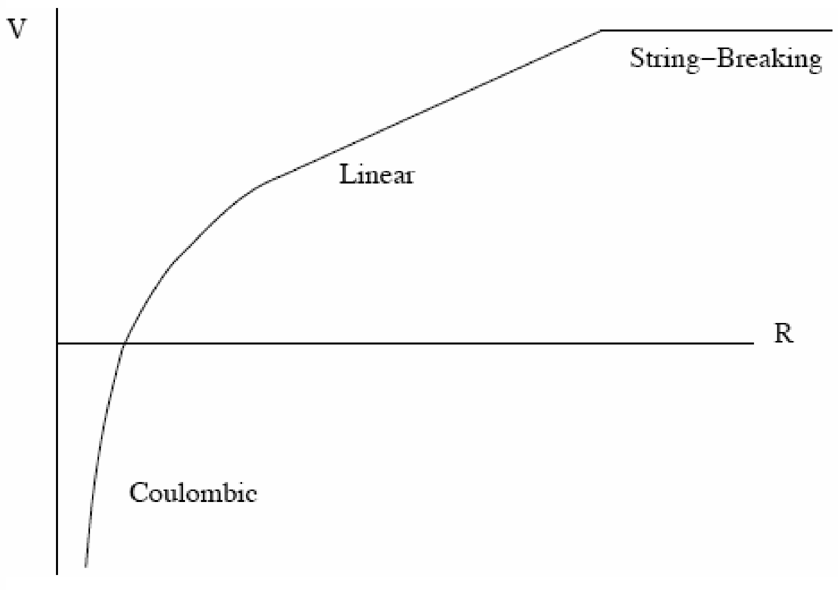
Add flavor dynamics on gluodynamic background

D.N. Li, M.H., Q. S. Yan, arXiv:1206.2824

Confinement:

Regge behavior and linear quark potential





Deformed AdS$_5$ models for hadron spectra:

hard-wall AdS$_5$ model
soft-wall AdS$_5$ model: quadratic dilaton model

1, Hard-wall AdS$_5$ model:

L. Da Rold and A. Pomarol, Nucl. Phys. B **721**, 79 (2005)

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005)

AdS$_5$ metric

$$ds^2 = \frac{1}{z^2}(-dz^2 + dx^\mu dx_\mu), \qquad 0 < z \le z_m.$$

5D hadron action

$$S = \int d^5x \sqrt{g}\, \mathrm{Tr}\left\{|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}$$

Lowest excitations: 80-90% agreement

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005)

Observable	Measured (MeV)	Model A (MeV)	Model B (MeV)
m_π	139.6 ± 0.0004 [8]	139.6^*	141
m_ρ	775.8 ± 0.5 [8]	775.8^*	832
m_{a_1}	1230 ± 40 [8]	1363	1220
f_π	92.4 ± 0.35 [8]	92.4^*	84.0
$F_\rho^{1/2}$	345 ± 8 [15]	329	353
$F_{a_1}^{1/2}$	433 ± 13 [6]	486	440
$g_{\rho\pi\pi}$	6.03 ± 0.07 [8]	4.48	5.29

$z_m = 1/(323\ \text{MeV})$ $z_m = 1/(346\ \text{MeV})$

However, no Regge behavior in the hard-wall AdS_5 model !

$\boxed{m_n^2 \text{ grow as } n^2.}$

2, Soft-wall AdS_5 model or KKSS model

A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006)

AdS_5 metric

$$g_{MN}\, dx^M dx^N = e^{2A(z)}(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu)$$

$$A(z) = -\ln z,\ \Phi(z) = z^2$$

A dilaton field to restore Regge behavior

$$I = \int d^5x\, e^{-\Phi(z)}\sqrt{g}\left\{-|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}$$

$$\boxed{M_{n,S}^2 = 4n + 4S}$$

However: only Coulomb potential, no linear quark potential

How to produce linear Regge behavior

& linear quark potential in a unified model?

Graviton-dilaton-scalar coupling system

D.N. Li, M.H., Q. S. Yan, arXiv:1206.2824
Also see poster by D.N. Li

Action for pure gluon system: Graviton-dilaton coupling

$$S_{GD} = \frac{1}{16\pi G_5} \int d^5x \sqrt{g_s} e^{-2\phi} \left(R + 4\partial_m\phi\partial^m\phi - V_\phi\right)$$

Action for light hadrons: KKSS model

$$S_M = -\frac{N_f}{N_c} \int d^5x \sqrt{g_s} e^{-\phi} Tr\left(|DX|^2 + V_X + \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right)$$

Total action: $S = S_{GD} + S_M$

Quadratic dilaton field $\longleftrightarrow$ Dimension-2 gluon condensate

$$S_{GD} = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\phi}(R+4\partial_m\phi\partial^m\phi-V_\phi)$$

Quadratic dilaton field: $\phi = \mu^2 z^2$

$$V_\Phi^E = -12\,\frac{{}_0F_1(1/4;\frac{\Phi^2}{24})^2}{L^2} + 2\,\frac{{}_0F_1(5/4;\frac{\Phi^2}{24})^2\Phi^2}{L^2} \qquad \phi = \sqrt{\tfrac{3}{8}}\Phi$$

Dimension-2 gluon condensate:

$$V_\Phi^E \xrightarrow{\Phi\to 0} -\frac{12}{L^2} + \frac{1}{2}M_\Phi^2\Phi^2 \qquad M_\Phi^2 L^2 = -4$$

$$\Delta(\Delta-4) = M_\Phi^2 L^2 \qquad \Delta = 2$$

$$\boxed{< A_\mu^2 >= \mu^2}$$

Background with dimension-2 gluon condensate $\phi = \mu^2 z^2$

and quark-antiquark condensate $<X> = \frac{\chi(z)}{2}$

$$S_{vac} = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}\{e^{-2\phi}(R_s + 4\partial_m\phi\partial^m\phi - V_\phi) - \lambda e^{-\phi}(\frac{1}{2}\partial_m\chi\partial^m\chi + V_\chi)\}.$$

$$dS_s^2 = B_s^2(-dt^2 + d\vec{x}^2 + dz^2) \qquad B_s^2 \equiv e^{2A_s} \equiv L^2 b_s^2.$$

$$\boxed{\begin{aligned} -A_s'' + A_s'^2 + \frac{2}{3}\phi'' - \frac{4}{3}A_s'\phi' - \frac{\lambda}{6}e^{\phi}\chi'^2 &= 0, \\ \phi'' + (3A_s' - 2\phi')\phi' - \frac{3\lambda}{16}e^{\phi}\chi'^2 & \\ -\frac{3}{8}e^{2A_s - \frac{4}{3}\phi}\partial_\phi\left(e^{4/3\phi}V_\phi + \lambda e^{7/3\phi}V_\chi\right) &= 0, \\ \chi'' + (3A_s' - \phi')\chi' - e^{2A_s}\partial_\chi V_\chi &= 0. \end{aligned}}$$

UV asymptotic form:

$$\boxed{\chi(z) \xrightarrow{z\to 0} m_q \zeta z + \frac{\sigma}{\zeta} z^3}$$

IR asymptotic form constrained by linear potential:

$$V_{q\bar{q}}(z_0) = \frac{L^2}{\pi \alpha_s} \int_0^{z_0} dz \frac{b_s^2(z)}{\sqrt{1 - \frac{b_s^4(z_0)}{b_s^4(z)}}}$$

$$R_{q\bar{q}}(z_0) = 2 \int_0^{z_0} dz \frac{1}{\sqrt{1 - \frac{b_s^4(z_0)}{b_s^4(z)}}} \frac{b_s^2(z_0)}{b_s^2(z)}$$

$$\boxed{\frac{V_{q\bar{q}}}{R_{q\bar{q}}} \longrightarrow b_s^2(z_c)}$$

$$\boxed{A_s'(z) \xrightarrow{z\to\infty} 0, A_s(z) \xrightarrow{z\to\infty} \mathrm{Const.}}$$

$$-A_s'' + A_s'^2 + \frac{2}{3}\phi'' - \frac{4}{3}A_s'\phi' - \frac{\lambda}{6}e^{\phi}\chi'^2 = 0,$$

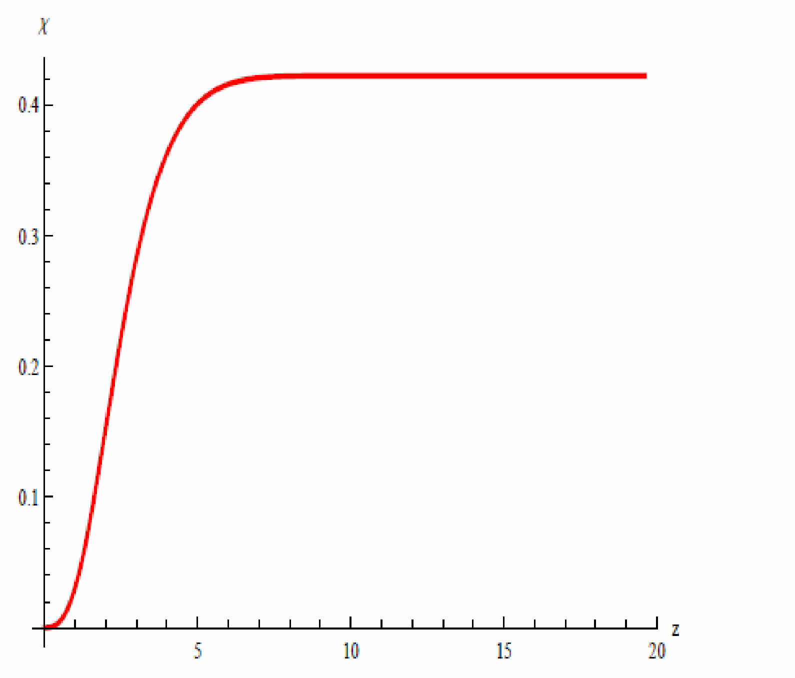
$$\Rightarrow \quad \frac{2}{3}\phi'' - \frac{\lambda}{6}e^{\phi}\chi'^2 = 0$$

$$\boxed{\chi(z) \xrightarrow{z\to\infty} \sqrt{8/\lambda}\,\mu e^{-\phi/2}}$$

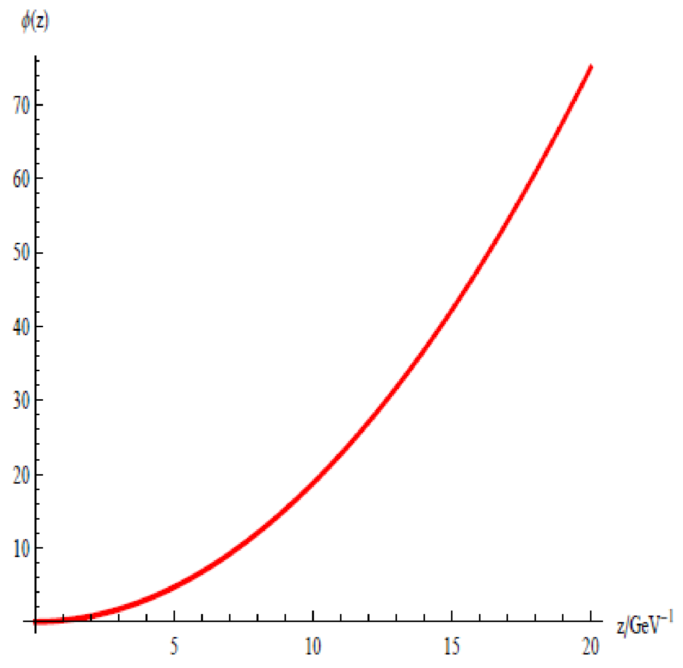
$$\phi = \mu^2 z^2 \qquad \chi'(z) = \sqrt{8/\lambda}\,\mu e^{-\phi/2}(1 + c_1 e^{-\phi} + c_2 e^{-2\phi})$$

$$\boxed{-A_s'' + A_s'^2 + \frac{2}{3}\phi'' - \frac{4}{3}A_s'\phi' - \frac{\lambda}{6}e^{\phi}\chi'^2 = 0}$$

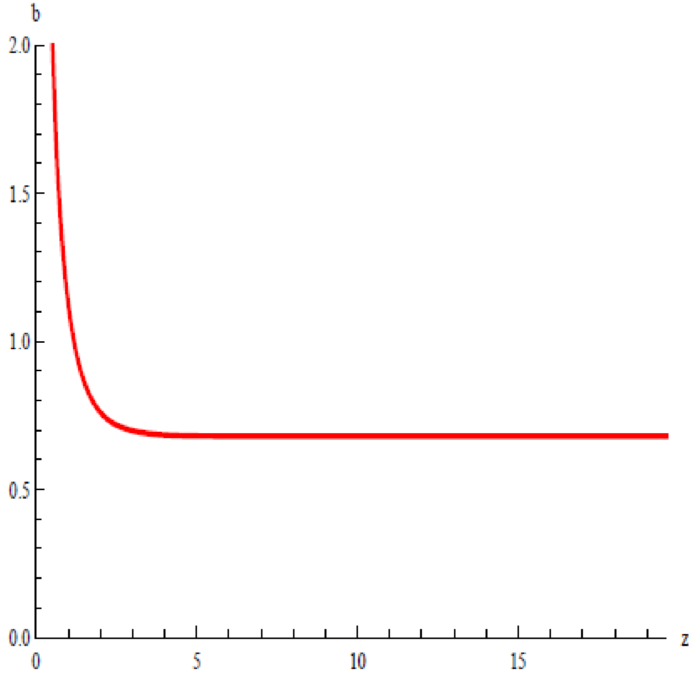
Chiral field



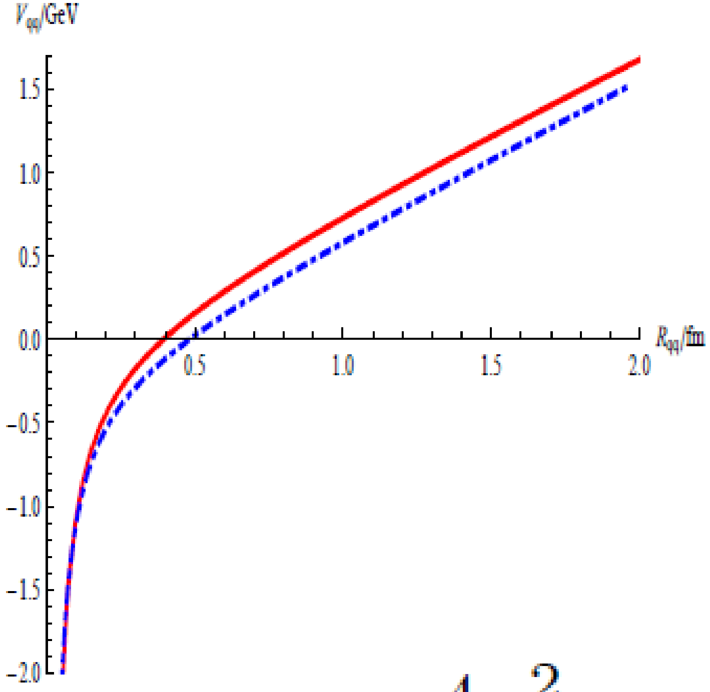
Dilaton field



Solved Metric

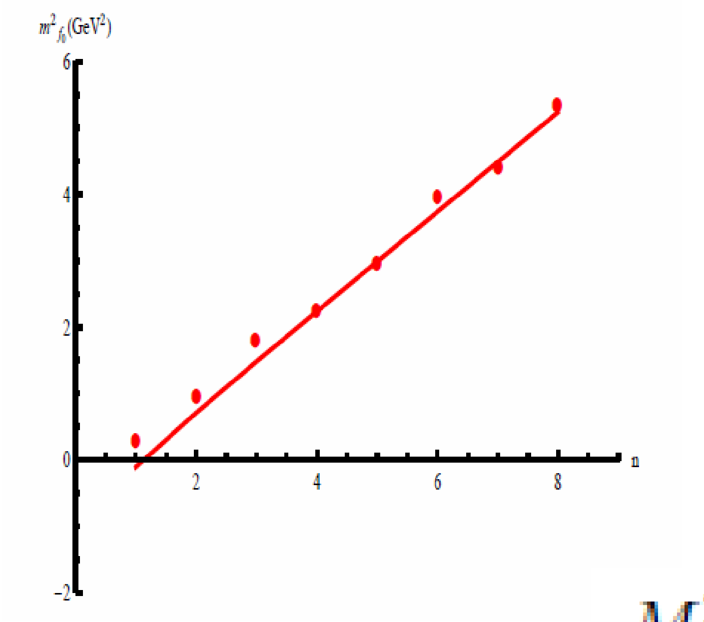


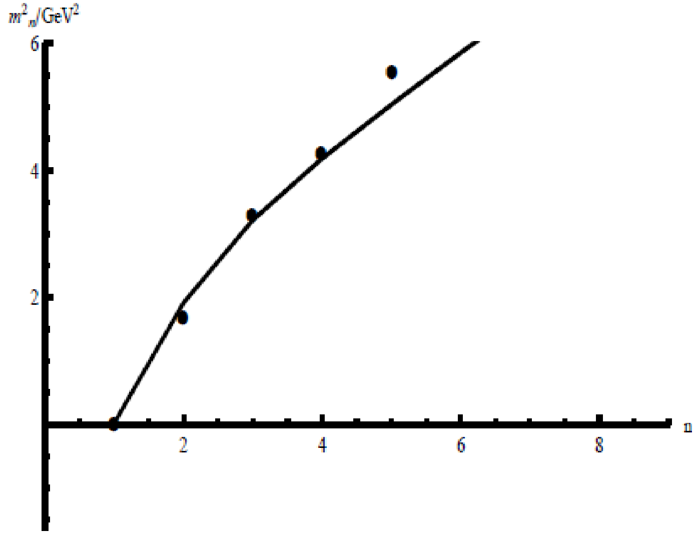
Produced quark potential compared with Cornell potential



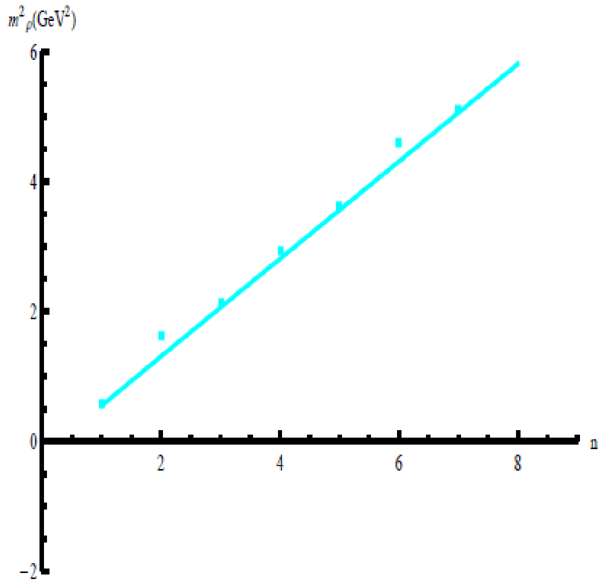
$$\sigma_s \sim 4\mu^2$$

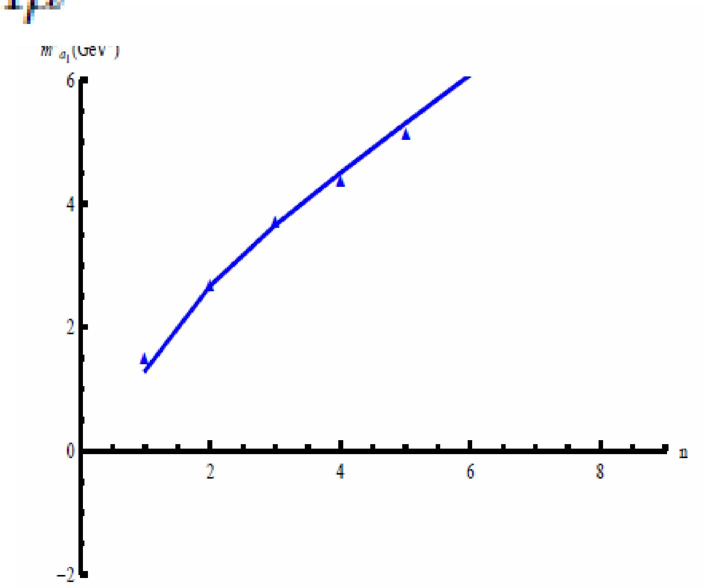
Produced hadron spectra compared with data





$$M_n^2/n \sim 4\mu^2$$





IV. Conclusion and discussion

1, A systematic framework connecting QCD dynamics and geometry:

graviton-dilaton for pure gluon system

graviton-dilaton-scalar for hadron spectra

2, The linear Regge behavior as well as linear quark potential can be produced in a dynamical holographic model!

The dimension-2 gluon condensate induces the linear confinement.

$$\sigma_s \sim 4\mu^2 \qquad \boxed{< A_\mu^2 >= \mu^2}$$

Dimension-2 gluon condensate & linear confinement

$\langle g^2 A^2 \rangle$

F.V. Gubarev, L. Stodolsky and V.I. Zakharov
Phys. Rev. Lett. 86, 2220-2222 (2001)

R. Akhoury and V.I. Zakharov
Phys. Lett. B 438, 165-172 (1998)

K. I. Kondo, Phys. Lett. B **514**, 335 (2001)

$$\alpha_s(Q^2) = \alpha_s(Q^2)_{pert}\left[1 + \frac{g_R^2\langle \mathscr{A}_\mu^2\rangle_R}{4(N_c^2-1)}\frac{9}{Q^2} + O(\alpha)\right]$$

$$V(r) = -C_F\frac{\alpha_s(r)}{r} + \sigma_s r \qquad \boxed{\sigma_s \cong g_R^2\langle \mathscr{A}_\mu^2\rangle_R}$$

4D pure gluodynamical model: Fukun Xu, M.H. arXiv:1111.5152

$$\mathscr{L}_G = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a \qquad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$A^a_\mu(x) := \mathbb{A}^a_\mu + \mathscr{A}^a_\mu(x) \qquad [\,\mathbb{A}^\mu_a\,]_{\rm cl} = \varphi_0 \eta^\mu_a \quad \langle \mathrm{vac} \,|\, \mathbb{A}^a_\mu \,|\, \mathrm{vac} \rangle = 0$$

$$\left\langle \mathrm{vac} \left| \frac{\alpha_s}{\pi} \mathbb{G}^a_{\mu\nu}(0) \mathbb{G}^{\mu\nu}_a(0) \right| \mathrm{vac} \right\rangle = \frac{3}{32\pi^2} [\langle \mathrm{vac} \,|\, g^2 \mathbb{A}^\mu_b(0) \mathbb{A}^b_\mu(0) \,|\, \mathrm{vac} \rangle]^2$$

$$\mathscr{L}_G(\mathscr{A}^a_\mu, \varphi_0) = -\tfrac{1}{4}[\mathscr{G}^a_{\mu\nu}(x) \mathscr{G}^{\mu\nu}_a(x)] + \frac{m_G{}^2}{2} \mathscr{A}^b_\mu(x) \mathscr{A}^\mu_b(x) - b\varphi_0{}^4$$

$$m_G{}^2 = [\tfrac{2}{3}] \tfrac{3}{8} g^2 \varphi_0{}^2 ,$$

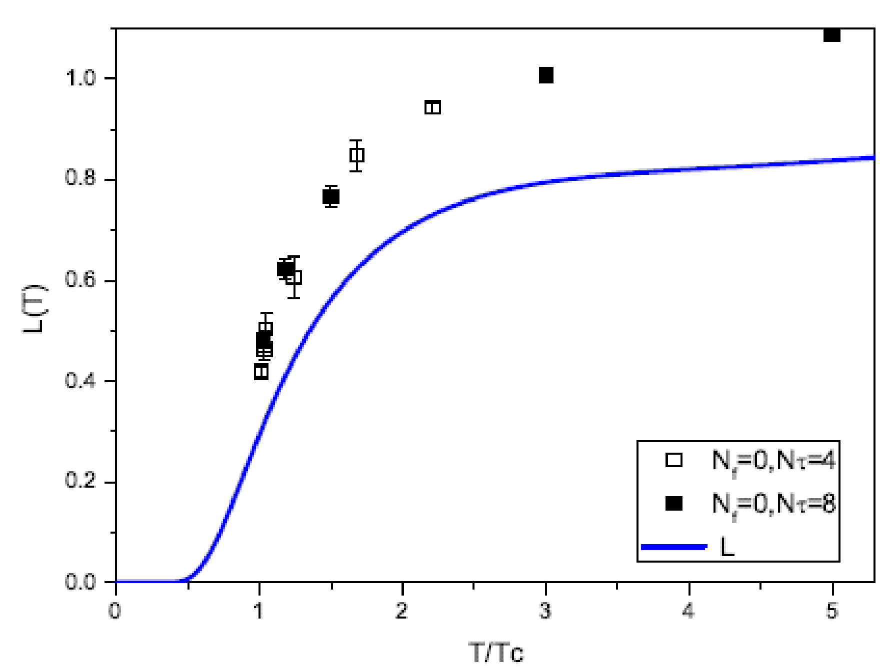
$$b = [\tfrac{8}{13}] \tfrac{3}{32} g^2 .$$

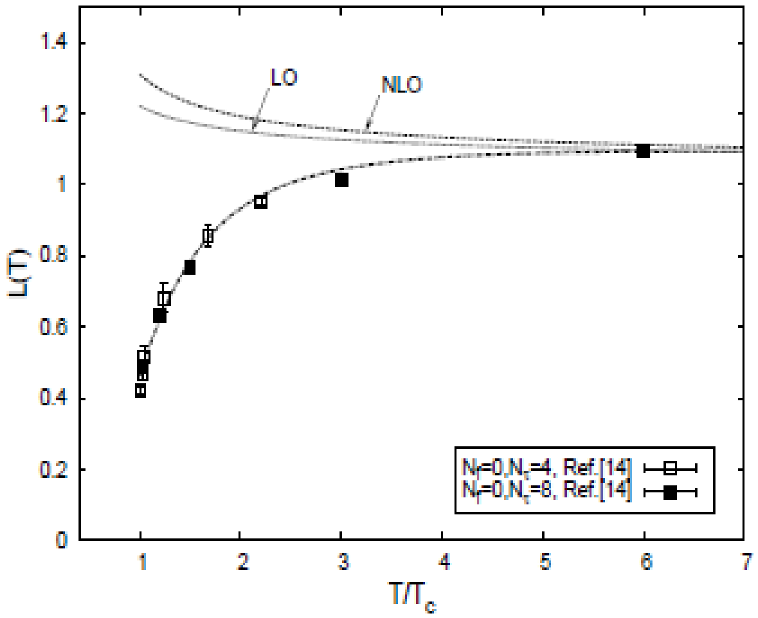
"pairing" of gluons
-> gluon condensate at zero momentum
-> effective gluon mass

Celenza, Shakin, PRD34(1986) 1591

Polyakov loop comparing with lattice result

Kaczmarek-Karsch-Petreczky-Zantow 2002



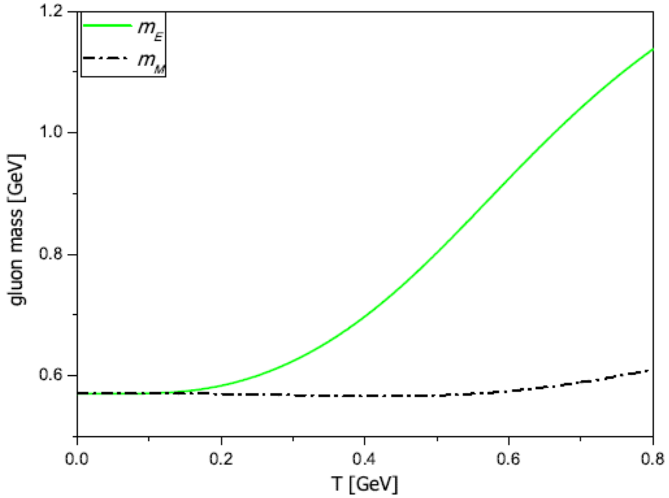


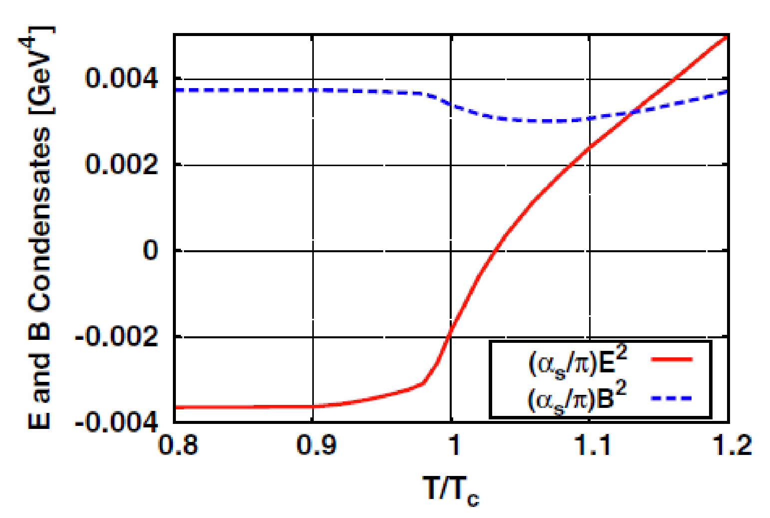
Fukun Xu, M.H. , arXiv:1111.5152

$$L(x) = \mathscr{P}\exp[ig\int_0^\beta d\tau A_4(\mathbf{x},\tau)] \qquad < L >= \exp[-\frac{g^2 < A_4^2 >}{4N_c T^2}]$$

E. Megias, E. Ruiz Arriola and L. L. Salcedo, JHEP **0601**, 073 (2006)

Color Electric deconfinement & color magnetic confienment





Fukun Xu, M.H. , arXiv:1111.5152

S.H.Lee et.al., PRD79(2009),011501
Extracting lattice data from
Boyd et. al. NPB469(1996),419