

High amplitude oscillations in dense matter

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M. Alford, S. Reddy, K. Schwenzer, [arXiv:1110.6213](https://arxiv.org/abs/1110.6213)
(Phys. Rev. Lett. 108, 111102 (2012))

M. Alford, S. Mahmoodifar, K. Schwenzer,
Phys. Rev. D85 024007 and 044051 (2012)

Overview

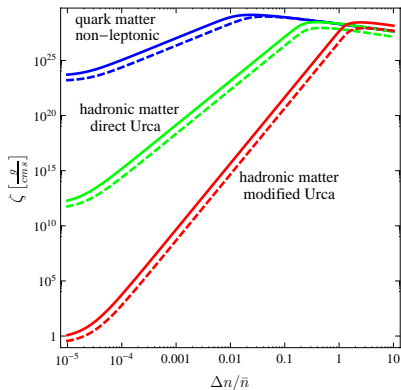
- ▶ Some oscillations of neutron stars can reach **high amplitude**.
E.g. violent local accretion impacts, or unstable “r-modes”.
- ▶ Response of different phases of dense matter to compression may provide **signatures** of their presence in neutron stars.
- ▶ At **high enough amplitude**, processes that depend on flavor equilibration have amplitude-dependent (“**suprathermal**”) enhancements.
- ▶ **Suprathermal** enhancement may be strong enough to overcome suppression due to
 - ▶ slowness of flavor equilibration
 - ▶ Cooper pairing of relevant fermions

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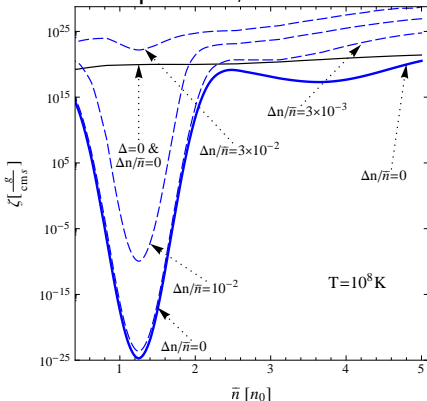
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- ▶ Consequences for **high amplitude** oscillations:
 - ▶ Enhancement of **heating and neutrino emission**
 - ▶ **Unsuppressed** bulk viscosity and neutrino emission in superfluid phases
 - ▶ Enhanced bulk viscosity is **capable of stopping r-mode growth**

Suprathermal enhancement of bulk viscosity

Unpaired matter, $T = 10^6$ K

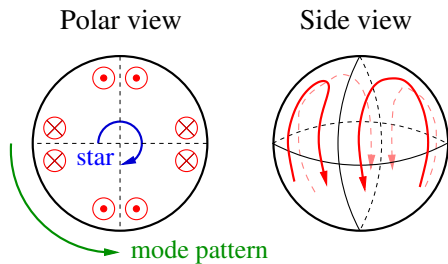


Nuclear superfluid, $T = 10^8$ K



Signature: *r*-mode-induced spindown

An *r*-mode is a mainly quadrupole flow that emits gravitational radiation. It becomes unstable (i.e. arises spontaneously) when a star **spins fast enough**, and if the **shear and bulk viscosity are low enough**.



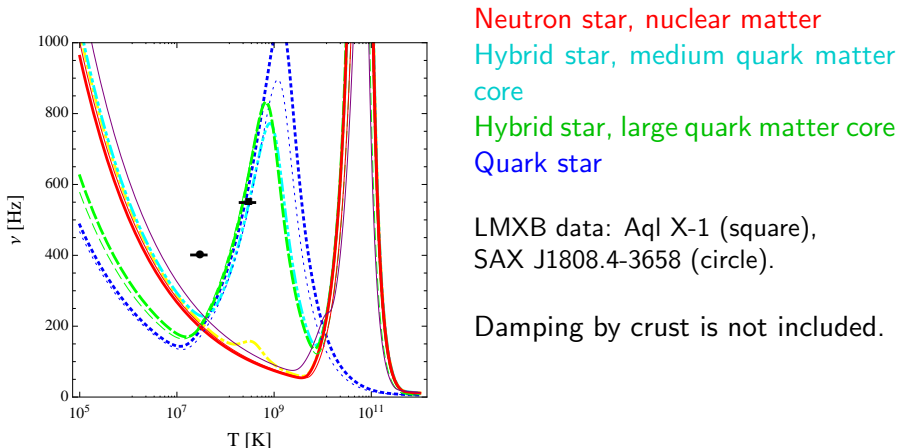
The unstable *r*-mode can spin the star down very quickly, exactly how fast depends on the amplitude at which it saturates.

(Andersson [gr-qc/9706075](#); Friedman and Morsink [gr-qc/9706073](#); Lindblom [astro-ph/0101136](#)).

So if we see a star spinning quickly, we can infer that the interior viscosity must be high enough to damp the *r*-modes.

Constraints from r-modes: current results

Regions above curves are forbidden \Leftarrow viscosity is too low to damp r -modes.



Alford, Mahmoodifar, Schwenzer arXiv:1012.4883

What is bulk viscosity?

Energy consumed in a
compression cycle:

$$\begin{aligned}V(t) &= \bar{V} + \delta V \sin(\omega t) \\ p(t) &= \bar{p} + \delta p \sin(\omega t + \phi)\end{aligned}$$

$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{\zeta}{\tau} \int_0^\tau (\operatorname{div} \vec{v})^2 dt = \frac{\zeta}{2} \omega^2 \frac{\delta V^2}{\bar{V}^2} = -\frac{1}{\tau \bar{V}} \int_0^\tau p(t) \frac{dV}{dt} dt$$

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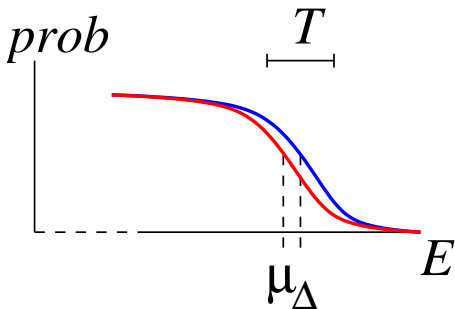
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- ▶ Bulk viscosity arises from re-equilibration processes.
- ▶ If some quantity goes out of equilibrium on compression, and re-equilibrates on a timescale comparable to τ , then pressure gets out of phase with volume.
- ▶ The driving force then does net work in each cycle.
- ▶ There is an exact analogy with V and Q in an R - C circuit.

Subthermal vs Suprathreshold

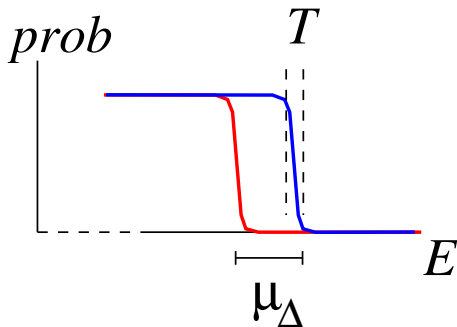
Subthermal

$$\mu_{\Delta} \ll T \ll \mu$$



Suprathreshold

$$T \ll \mu_{\Delta} \ll \mu$$



Calculating bulk viscosity

- ▶ Compression at frequency ω . Density of conserved charge oscillates as $n(t) = \bar{n} + \delta n \sin(\omega t)$
- ▶ One quantity “ Δ ” goes out of equilibrium (eg $S - D$ in quark matter). In equilibrium, $\mu_\Delta = 0$.
- ▶ EoS is characterized by susceptibilities B, C .

$$\zeta = -\frac{1}{\pi} \frac{\bar{n}}{\delta n} \frac{C}{B} \int_0^\tau \mu_\Delta(t) \cos(\omega t) dt$$

Bulk visc arises from component of μ_Δ that lags behind the forcing oscillation by a phase of 90° ; $\mu_\Delta(t)$ is given by

$$\frac{d\mu_\Delta}{dt} = \underbrace{C\omega \frac{\delta n}{\bar{n}} \cos(\omega t)}_{\text{forcing osc.}} - \underbrace{\Gamma(\mu_\Delta, T)}_{\text{equilibration}}$$

Suprathermal and subthermal bulk viscosity

Subthermal: assume $\mu_{\Delta} \ll T$,

$$\Gamma(\mu_{\Delta}, T) = \tilde{\Gamma} T^{2N} \frac{\mu_{\Delta}}{T}$$

Differential eqn is linear, so $\mu_{\Delta} \propto \delta n$, yielding

$$\zeta_{\text{sub}} = \frac{C^2}{B} \frac{\gamma_{\text{eff}}}{\omega^2 + \gamma_{\text{eff}}^2} \quad (\gamma_{\text{eff}} \equiv B \tilde{\Gamma} T^{2N})$$

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Suprathermal: allow $\mu_\Delta \gtrsim T$ (always assuming $\delta n \ll \bar{n}$).

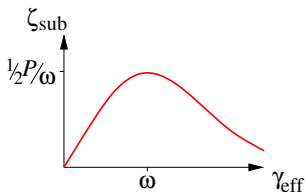
Differential eqn is nonlinear, $\mu_\Delta(t)$ may not be harmonic.

$$\frac{d\mu_\Delta}{dt} = C \frac{Dn}{\bar{n}} \cos(\omega t) - T \frac{\gamma_{\text{eff}}}{\omega} \mu_\Delta \left(1 + \chi_1 \frac{\mu_\Delta^2}{T^2} + \dots + \chi_N \frac{\mu_\Delta^{2N}}{T^{2N}} \right)$$

Has to be solved numerically.

The subthermal bulk viscosity

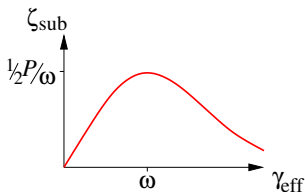
$$\zeta_{\text{sub}}(\omega, T) = P \frac{\gamma_{\text{eff}}(T)}{\gamma_{\text{eff}}(T)^2 + \omega^2}$$



- ▶ ζ_{sub} is independent of driving amplitude.
- ▶ Prefactor $P = C^2/B$ is a combination of susceptibilities.
- ▶ γ_{eff} is the effective re-equilibration rate per particle.

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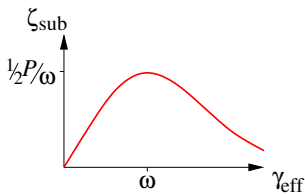
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- ▶ In phases where Fermi surface modes dominate equilibration (nuclear, unpaired quark matter, 2SC) P is constant for $T \ll \mu_q$, and subthermal bulk viscosity peaks when $\gamma_{\text{eff}}(T) = \omega$.
- ▶ In phases where bosons dominate equilibration (CFL, CFL-K0), $P(T)$ washes out the peak.

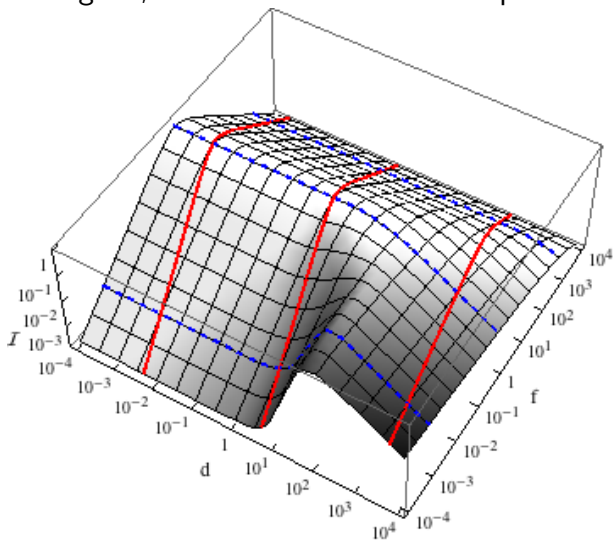
The general bulk viscosity

To include the suprathreshold regime, we have to solve the diffeq for $\mu_{\Delta}(\omega t)$ numerically.

$$\zeta = \frac{C^2}{2\omega B} \mathcal{I}(d, f)$$

$$f = \gamma_{\text{eff}}/\omega$$

$$d = (C/T) \delta n/\bar{n}$$

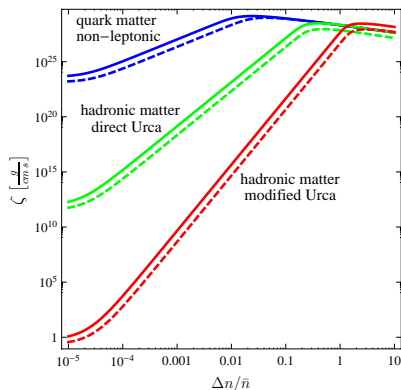


Nuclear matter,
modified Urca

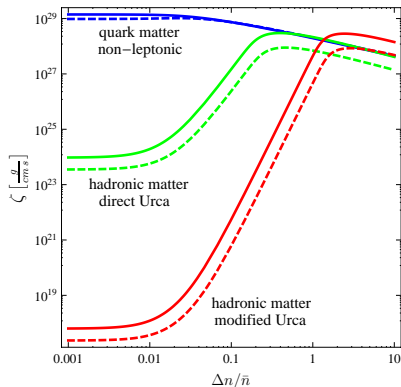
Suprathreshold enhancement of bulk viscosity

(ms pulsar)

$T = 10^6$ K



$T = 10^9$ K



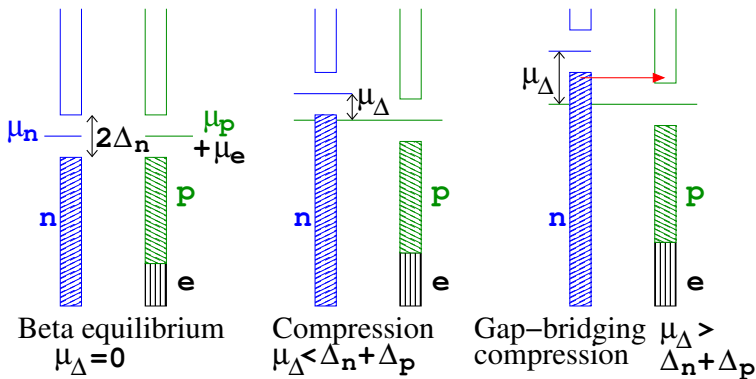
- ▶ Bulk visc rises very steeply in suprathreshold regime
- ▶ Max reached at $\delta n/n \sim 0.1$; max value indep of temperature
- ▶ Suprathreshold enhancement is greater at low T and for matter where ζ goes as higher power of $\mu\Delta$.

Consequences of suprathermal enhancement

- ▶ Superthermal bulk viscosity can stop r-mode growth, but only at very high amplitude $\alpha \sim 1$ ($\delta n/\bar{n} \sim 0.03$).
Other mechanisms may saturate r-mode first, e.g. mode-coupling at $\alpha \sim 10^{-4}$ (Bondarescu, Teukolsky, Wasserman, arXiv:0809.3448)
- ▶ Superthermal enhancement of bulk viscosity and neutrino emission may affect heating and cooling of stars undergoing r-mode spindown
- ▶ Response of stars to other high-amplitude compressions will also be affected.

Suprathermal enhancement in a superfluid ("gap-bridging")

If density amplitude is high enough, μ_Δ can be large enough to open up phase space above the gap, overcoming $\exp(-\Delta/T)$ suppression.

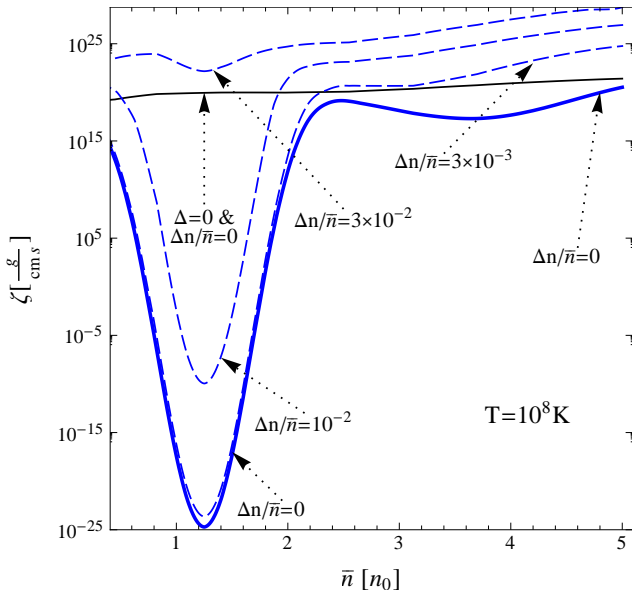


$$\mu_\Delta = C \frac{\delta n}{\bar{n}}$$

For APR, $C \sim 20$ to 150 MeV

Oscillation with $\delta n/\bar{n} \sim 0.01$ can overcome $\Delta \sim 1$ MeV.

Gap-bridging enhancement of bulk viscosity



Illustrative example:

direct Urca allowed
s-wave pairing for p
and n

Δ_p peak of 1 MeV at
 $n = 1.3n_0$

Δ_n peak of 0.12 MeV
at $n = 3.7n_0$ [Cas A]

There is similar
enhancement for
neutrino emissivity.

Future directions

Transport:

- ▶ Study suprathermal enhancement in other phases, e.g. Hyperonic nuclear matter, neutron star crust
- ▶ Gap-bridging: apply to realistic case, modified Urca, 3P_2 neutron pairing, etc.
- ▶ Investigate effect of multiple equilibrating quantities

Astrophysics:

- ▶ What is the saturation amplitude of an r-mode?
- ▶ Evolution of r-mode spindown, trajectory in (T, Ω) space (requires assumed r-mode saturation amplitude and a cooling model)
- ▶ Semi-analytic expressions for freq of star spun down by r-modes, insensitive to microscopic details and sat. ampl. (Alford & Schwenzer, forthcoming)
- ▶ Complications with r-modes: layered stars, role of crust, etc
- ▶ Apply to other modes, e.g. pulsations, f-modes (which emit grav waves), violent accretion events