Non-leptonic decays in an extended chiral quark model

J.O.E. , Phys. Dept., Univ. Oslo
Talk at: QCD@work, Lecce, june 2012

June 14, 2012

Model for Color suppressed mode $\overline{B_d^0} \to \pi^0\pi^0$

Outline

- Introduction
- Color suppression in non-leptonic decays
- Effective Theories at quark level (HQEFT and LEET $\longrightarrow$ SCET)
- Mesonic picture - (HL)χPT
- Chiral Quark Models (χQM, HLχQM, LEχQM)
- Color suppression for $\overline{B_d^0} \longrightarrow \pi^0\pi^0$
- Conclusions, further comments

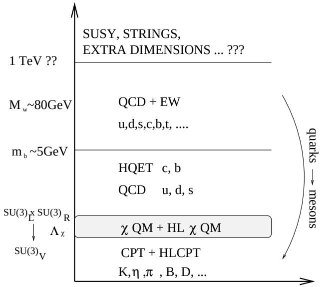
Introduction

* Since 1970's : Non-leptonic decays always difficult-
Perturbative QCD worked well;
BUT: "Hadronic uncertainties" (use Lattice, Quark Models)

From 1999, BBNS: *QCD factorization* For $B \to \pi\pi, \pi K, ...$
Corrections to factorization:
$\frac{\alpha_s}{\pi}$ (calculable), $\frac{\Lambda_{QCD}}{m_b}$ (not calculable).
For $\overline{B_d^0} \to \pi^0\pi^0$, tremendous effort (QCD fact, SCET, QCD sum rules) , but amplitude factor ~ 2 off.

Try new LEχQM !

Energy/Mass Scales of the SM



High mass particles may go in loops and affect low-energy decays!

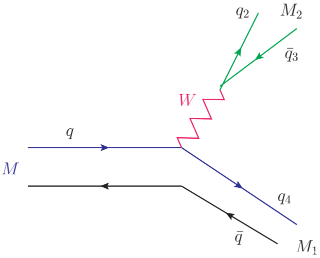
Chiral quark models bridge between the *quark* and *meson* picture

Bridge:Chiral quark models (χQM's)

Since 1990's used (HL)χQM (combined with χPT).
$K \to \gamma\gamma$, $K \to \pi\pi$, especially ϵ'/ϵ !! - $K - \overline{K}$ and $B - \overline{B}$ mixing, $D \to K\overline{K}$, $B \to D\overline{D}$, $D\eta'$, $D^*\gamma$, $K\eta'$, $D\pi$.
Collaborators: Ivica Picek, Marco Fabbrichesi, Stefano Bertolini, Svjetlana Fajfer, Aksel Hiorth, A.Polosa, J. Zupan, A. Prapotnik, J.A. Macdonald Sørensen, Krešimir Kumerički, L.E. Leganger

(HL)χQM also used by Bijnens et al, Pich and de Rafael, Ebert et al, Nardulli et al,.....

Quark Diagrams for Non-Leptonic Decays



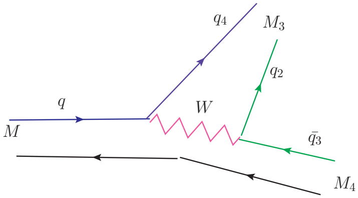


Figure: Factorizable and Color-suppressed diagrams

Color suppression in Non-Leptonic Decays

Effective non-leptonic Lagrangian at quark level:

$$\mathcal{L}_W = \sum_i C_i(\mu)\ \hat{Q}_i(\mu)\ ,$$

$\mu =$ renormalization scale. C_i = Wilson coeff. Typically $\hat{Q}_i$'s are products of two quark currents
For "flavor mismatch", use Fierz transf:

$$\hat{Q}_A \rightarrow \hat{Q}_A^{Fierz} = \frac{1}{N_c}\,\hat{Q}_B + \ \hat{Q}_B^{color}$$

where $\hat{Q}_B^{color}$ is a product of two colored currents.

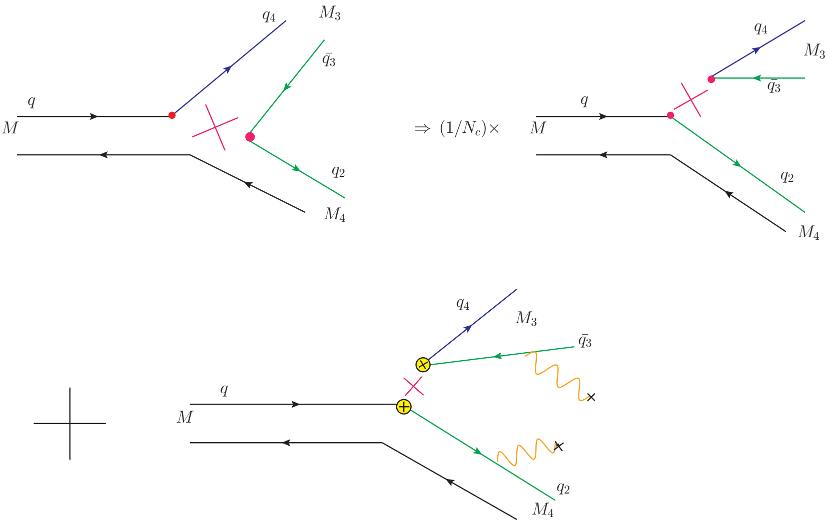


Figure: Diagrammatic illustration of the Fierz transf.

Colored operators might dominate!

Generic, for non-leptonic processes with two numerically relevant operators $\hat{Q}_{X,Y}$:

$$\langle M_1\, M_2 | \mathcal{L}_W | M \rangle \;=\; \left(C_X + \frac{C_Y}{N_c} \right) \langle M_1 |\, j_L^{\alpha}(1) | 0 \rangle \, \langle M_2 | j^L_{\alpha}(2) | M \rangle \;+\; C_Y \, \langle M_1\, M_2 | \, \hat{Q}^{color} \, | M \rangle$$

where $\hat{Q}^{color}$ is the product of the colored currents. Some cases: $\left(C_X + \frac{C_Y}{N_c} \right)$ close to zero. How to calculate $\langle M_1\, M_2 | \, \hat{Q}^{color} \, | M \rangle$?

Heavy Quark Effective Theory (*HQET*)

Project out movement of heavy quark : $p_Q = m_Q \, v + k \;\; , \;\; v^2 = 1.$
Effective Lagrangian:

$$\mathcal{L}_{HQET} = \overline{Q_v} i v \cdot D Q_v + \mathcal{O}(m_Q^{-1})$$

Heavy quark propagator:

$$S(p_Q) \;\rightarrow\; \frac{(1 + \gamma \cdot v)}{2 \, k \cdot v}$$

Replacements in quark operators

$$b \longrightarrow Q_{v_b} \quad , \quad c \longrightarrow Q_{v_c}$$

Large Energy Eff. Th. (*LEET* $\to$ *SCET*)

Project out movement of light energetic quark: $p_q^\mu = E\, n^\mu + k^\mu$,

$$n(\text{ or } \tilde{n}) \;=\; (1, 0, 0, \pm\eta)$$

$$\eta = \sqrt{1-\delta^2}\,, \quad n^2 = \tilde{n}^2 = \delta^2\,, \; v\cdot n = v\cdot\tilde{n} = 1.$$

$$\delta \sim \frac{\Lambda_{QCD}}{m_Q}\;(\to \frac{m}{E})\;; \quad S(p_q) \to \frac{\gamma\cdot n}{2n\cdot k}$$

Effective Lagrangian for reduced light energetic quark field q_n:

$$\mathcal{L}_{LEET\delta} \;=\; \bar{q}_n \left(\frac{1}{2}(\gamma\cdot\tilde{n}+\delta)\right)(in\cdot D)\, q_n + \mathcal{O}(E^{-1})\;,$$

In the formal limits $M_H \to \infty$ and $E \to \infty$, $\langle P \mid V^\mu \mid H \rangle$ of the form (Orsay group ; Charles et al 1999):

$$\langle P|V^\mu|H\rangle = 2E\left[\zeta^{(v)}(M_H,E)\,n^\mu + \zeta_1^{(v)}(M_H,E)\,v^\mu\right] \;,$$

where

$$\zeta^{(v)} = C\frac{\sqrt{M_H}}{E^2} \quad , \quad C \sim (\Lambda_{QCD})^{3/2} \qquad , \quad \frac{\zeta_1^{(v)}}{\zeta^{(v)}} \sim \delta \sim \frac{1}{E}$$

Behavior constistent with the energetic quark having x close to one, where x = quark momentum fraction of the outgoing pion.

Mesonic picture : (HL)χPT

$\Rightarrow$ Effective Theor. contains meson fields

Heavy meson field: $H^{(\pm)} = P_{\pm}(v)\,(P_{\mu}^{(\pm)}\gamma^{\mu} - iP_{5}^{(\pm)}\gamma_5)$

Light, soft meson fields $(\pi, K, \eta_8\ )$. $\Lambda_{\chi}\ \sim\ 4\pi f\ \sim\ 1$ GeV

$$\mathcal{A}_{\mu}\ =\ \frac{1}{2i}(\xi^{\dagger}\partial_{\mu}\xi - \xi\partial_{\mu}\xi^{\dagger})\quad ;\ \ \xi \equiv \exp\left(\frac{i}{2f}\sum_{a}\lambda^{a}\pi^{a}(x)\right)$$

For HLχPT, should have (??) $(m_b - m_c) < \Lambda_{\chi}$, or $(m_b - 2m_c) < \Lambda_{\chi}$
Few ideal cases, but $B - \overline{B}$ mixing should work well...
Bosonization of currents (to lead. order for $H = B, D$).

$$\overline{q_L}\,\gamma^{\mu}\,Q_v\ \longrightarrow\ \frac{1}{2}\,f_H\,\sqrt{M_H}\,Tr\left[\xi^{\dagger}\gamma^{\mu}L\,H_v\right] + \mathcal{O}(1/m_Q) + \mathcal{O}(m_q) + \mathcal{O}(1/N_c)$$

The Chiral Quark Model (χQM)

To be used for colored operators! Light $q = u, d, s$ sector:

$$\mathcal{L}_{\chi QM} = \mathcal{L}_{QCD} - m\left(\overline{q}_R\,\Sigma\,q_L + \overline{q}_L\,\Sigma^\dagger q_R\right)$$

$\Rightarrow$ Meson-quark couplings. Modelling confinement!!
m = *constituent* light quark mass, due to chiral symmetry breaking
"Rotated version" ; flavour rotated "constituent quark fields"

$$\chi_L = \xi q_L \quad , \quad \chi_R = \xi^\dagger q_R \quad ; \qquad \xi \cdot \xi = \Sigma$$

$$\mathcal{L}_{\chi QM} = \overline{\chi}\left[\gamma^\mu(iD_\mu + \mathcal{V}_\mu + \gamma_5 \mathcal{A}_\mu) - m\right]\chi + \mathcal{O}(m_q)$$

Color suppressed $M \to M_1\, M_2$ in χ QM

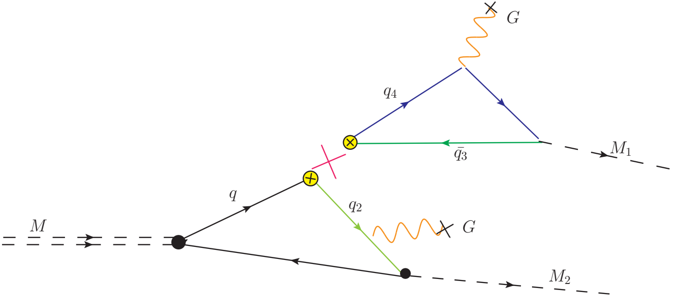


Figure: Bosoniztion of colored operator $\hat{Q}^{color} \;\; \to \;\; 1/N_c \; Tr(\ldots) \times Tr(\ldots)$

Soft gluon emision

Loop momenta in χQM's should be $< \Lambda_\chi$

Gluon condensates (model dep.) using (Novikov et al.):

$$g_s^2 G^a_{\mu\nu} G^a_{\alpha\beta} \;\rightarrow 4\pi^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle \frac{1}{12} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}) \, ,$$

Identification of logarithmic divergent loop integral I_2 and quadratic div. int. I_1 (Esprui and Taron, Bijnens et al, Pich and de Rafael,...):

$$f_\pi^2 = -i4m^2 N_c I_2 + \frac{1}{24m^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle$$

$$\langle \, \overline{q} q \, \rangle = -4imN_c I_1 - \frac{1}{12m} \langle \frac{\alpha_s}{\pi} G^2 \rangle$$

Prescription :

- Integrate out W and heavy quarks:

$$E > \mu \quad : \; \mathcal{L}_W = \sum_i C_i(\mu)\, \hat{Q}_i(\mu) \;,$$

- Bosonize : Integrate out light quark fields and Q_i:

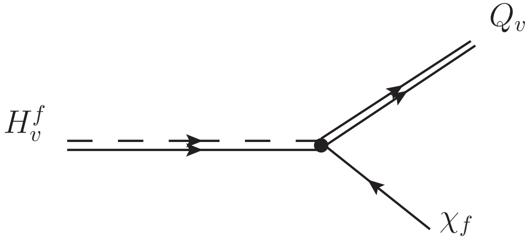
$$\hat{Q}_i(\mu) \;\rightarrow\; \sum_j G_{ij} \hat{\mathcal{L}}_j \;,$$

- Meson loops (chiral from $\hat{\mathcal{L}}_j$?) at meson level (FSI -some loops suppressed)

The $HL\chi QM$. Including heavy quarks

$$\mathcal{L}_{HL\chi QM} = \mathcal{L}_{HQET} + \mathcal{L}_{\chi QM} + \mathcal{L}_{Int}$$

$HL\chi$QM ansatz (also: Ebert et al, Bardeen and Hill, Nardulli et al):



$$\mathcal{L}_{Int} = -G_H\, \overline{Q_v}\, H_v^f\, \chi_f \; + \; h.c$$

Integrating out quarks (by loop diagrams) should give the known HLχPT terms! $\Rightarrow$ Physical and model dep. param. $f_\pi, \langle \overline{q}q \rangle, f_H, g_A$ linked to (divergent) loop integrals in HLχQM!

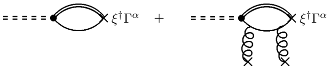


Figure: Bosonization of left handed current

Fit in strong sector: $m \sim 220$ MeV, $\langle \frac{\alpha_s}{\pi} G^2 \rangle^{1/4} \sim 315$ MeV,
${G_H}^2 = \frac{2m}{f^2}\,\rho$ where $\rho \sim 1$ and ρ depend on $f_\pi, \langle \frac{\alpha_s}{\pi} G^2 \rangle, m, g_A$.

Ideal case $B - \overline{B}$-mixing

$$\hat{B}_{B_q} = \frac{3}{4}\,\widetilde{b}\left[1 + \frac{1}{N_c}\left(1 - \delta_G^B\right) + \frac{\tau_b}{m_b} + \frac{\tau_\chi}{32\pi^2 f^2}\right]$$

The $LE\chi QM$

Ansatz(with L.E. Leganger, PRD 82 (2010)):

$$\mathcal{L}_{intq} = G_A \bar{q}\, \gamma_\mu \gamma_5 (\partial^\mu M)\, q_n \; + \; h.c \qquad , \; M = \text{meson fields}$$

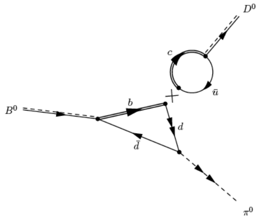


Figure: Factorizable contribution to the $B^0 \to D^0 \pi^0$ decay, including the current matrix element for $B \to \pi$. Here: Small Wilson coeff. combination for factorized term. Colored operator dominates.

Coupling G_A determined by loop diagram for $\zeta^{(v)}$.

Recall from LEET

$$\langle P|V^{\mu}|H\rangle = 2E\left[\zeta^{(v)}(M_H,E)\,n^{\mu} + \zeta_1^{(v)}(M_H,E)\,v^{\mu}\right] \ ,$$

where

$$p_H^{\mu} = m_H\,v^{\mu} \ ; \quad p^{\mu} = En^{\mu}$$

Recall:

$$\zeta_1^{(v)}/\zeta^{(v)} \ \sim \ \delta \sim 1/E$$

Property satisfied in LEχQM !!

$B \to \pi_n$ current (M_n = 3 $\times$ 3 matrix of energetic mesons):

$$J^{\mu}_{tot}(H_v \to M_n) \; = \; \left(-i \frac{G_H \, G_A}{2} \, m^2 \, F \right) \mathrm{Tr} \left\{ \gamma^{\mu} L H_v \left[\gamma \cdot n \right] \xi^{\dagger} M_n \right\} \; ,$$

where $F = N_c/(16\pi) + ... \sim 10^{-1}$. Use $\delta = m/E$:

$$G_A \;\sim\; \frac{1}{N_c} \, \frac{1}{E^{\frac{3}{2}}}$$

Coupling G_A fixed from knowledge of $\zeta^{(v)}$
(From light cone sum rules : $\zeta^{(v)} \simeq 0.3$).

Color suppression in $\overline{B^0} \to D^0\pi^0$

Factorized amplitude dominates for $\overline{B^0} \to D^+\pi^-$. BUT: $\overline{B^0} \to D^0\pi^0$ has very small factorized amplitude (-small Wilson coeff.)

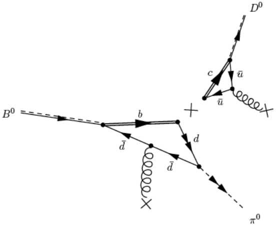


Figure: Non-factorizable contributions to $\overline{B^0} \to D^0\pi^0$ from the colored operators within LEχQM. Ampl. account for 2/3 of the experimental amplitude. Additional meson loop ampl.(?)

Bosonization of colored current for outgoing D-meson ($c \to Q_{v_c}$):

$$(\overline{q_L}\, t^a \,\gamma^\alpha \,Q_{v_c})_{1G} \;\longrightarrow\; -\frac{G_H\, g_s}{64\pi}\, G^a_{\mu\nu} Tr\left[\xi \gamma^\alpha L\, \sigma_{\mu\nu}\, \overline{H_{v_c}}\right] \;+ \ldots$$

$\overline{B^0} \to \pi^0\pi^0$ in LEχQM

with T. Palmer, PRD 83 (2011)

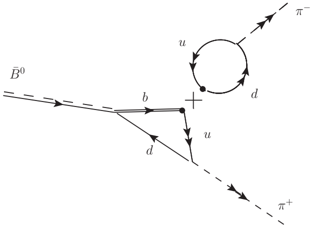


Figure: Factorized contribution to the $\overline{B^0_d} \to \pi^+\pi^-$ decay, as described in combined χQM, HLχQM and LEχQM. Factorized contrib to $\overline{B^0_d} \to \pi^0\pi^0$ small due to small Wilson coeff.

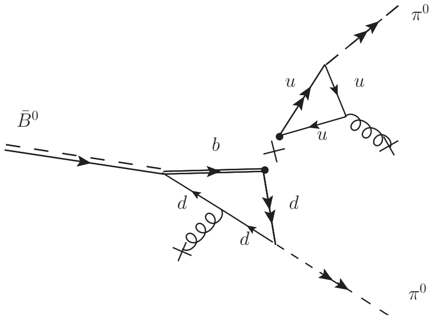


Figure: Non-factorizable contribution to $B \to \pi^0\pi^0$ containing large energy light fermions and mesons. Also corresponding diagram where the outgoing anti-quark $\overline{u}$ is hard.

The colored $B \to \pi_n$ current

$$J^{\mu}_{1G}(H_b \to M)^a \;=\; g_s\, G^a_{\alpha\beta}\, \frac{G_H\, G_A}{128\pi} \epsilon^{\sigma\alpha\beta\lambda}\, n_\sigma \mathrm{Tr}\left(\gamma^\mu L H_v \gamma_\lambda\, \xi^\dagger M_n\right) \;,$$

The colored current for outgoing hard $\pi_{\tilde{n}}$:

$$J^{\mu}_{1G}(M_{\tilde{n}})^a \;=\; g_s G^a_{\alpha\beta}\; 2\left(-\frac{G_A E}{4}\right)\, Y\; \tilde{n}_\sigma \epsilon^{\sigma\alpha\beta\mu}\, \mathrm{Tr}\left[\lambda^X\, M_{\tilde{n}}\right] \;,$$

where λ^X = appropriate SU(3) flavor matrix. Loop factor:

$$Y \;=\; \frac{f_\pi^2}{4m^2 N_c}\left(1 \;-\; \frac{1}{24 m^2 f_\pi^2}\langle\frac{\alpha_s}{\pi} G^2\rangle\right) \;.$$

$$r \equiv \frac{\mathcal{M}(\overline{B_d^0} \to \pi^0\pi^0)_{\text{Non-Fact}}}{\mathcal{M}(\overline{B_d^0} \to \pi^+\pi^-)_{\text{Fact}}} = \left(\frac{c_A}{c_f}\,\kappa\,\hat{c}\right)\frac{1}{N_c}\,\frac{m}{E}\,.$$

κ = model-dependent hadronic factor, dimension-less and $\sim (N_c)^0$:

$$\kappa = \left(\frac{\pi\, N_c\, \langle \frac{\alpha_s}{\pi} G^2 \rangle}{2\,F^2\, m^4 \sqrt{2\rho}}\right)\; Y.$$

Our calculations show that the ratio $r \sim 1/N_c$ and $r \sim m/2E \simeq \Lambda_{QCD}/m_b$ as it should acc. to BBNS

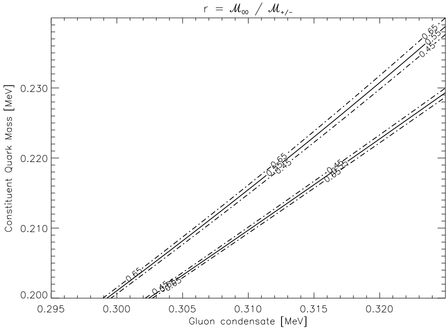


Figure: Plot for the ratio r in terms of m and $\langle \frac{\alpha_s}{\pi} G^2 \rangle^{1/4}$. For reasonable values of these parameters the ratio r can take a wide range of values such that fine-tuning is required to reproduce the experimental value.

Experimental value of $\overline{B_d^0} \to \pi^0\pi^0$ ampl. can be accomodated for $m \sim$ 220 Mev and $\langle \frac{\alpha_s}{\pi} G^2 \rangle^{1/4} \sim$ 315 MeV. - as in previous work. BUT: Result very sensitive to variations of m and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$, as seen by loop factor Y.

In addition,- meson loops:

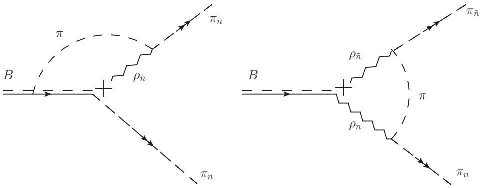


Figure: Suppressed and Non-suppr. meson loops for $\overline{B_d^0} \to \pi\pi$.

Extensions- Under construction

Meson loops

Include also light vectors (ρ, ...)

Need coupling for $V_n \rightarrow M_n + (\text{soft} \;\; \pi)$

Conclusions

- Have constructed LEχQM in accordance with $\langle \pi_n | j_V^\mu | B \rangle$
- 2/3 of $B \to \pi^0 \overline{D^0}$ described. Rest meson interactions?
- Good news: Color suppressed ($\sim 1/N_c$) ampl. $\overline{B_d^0} \to \pi^0 \pi^0$ can be accomodated numerically! And: amp. $\sim m/2E \simeq \Lambda_{QCD}/m_b$
- Bad news: Obtained ampl. very sensitive to m and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$
- extension (light vectors, meson loops) coming
- Systematic expansion in $1/N_c$ and m/E. Bosonized expressions obtained, BUT: Models not suited for precision tests