Quark confinement due to non-Abelian magnetic monopoles in SU(3) Yang-Mills theory

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In SU(3) Yang-Mills theory, confinement of fundamental quarks is realized based on the dual superconductivity caused by non-Abelian magnetic monopoles.

§ Introduction

In this talk we reconsider the dual superconductor picture in SU(3) Yang-Mills theory as a mechanism for quark confinement. We focus on how the SU(3) case is different from SU(2) one.

The basic ingredient for dual superconductivity is the existence of magnetic monopoles to be condensed and the realization of the dual Meissner effect.

What kind of magnetic monopoles can be defined in Yang-Mills theory without adjoint scalar fields?

We follow the well-known Wilson criterion for quark confinement, i.e., area law of the Wilson loop average.

We obtain the exact relationship between the Wilson loop and the magnetic monopole.

Consequently,

(1) the magnetic monopole can be defined in a gauge-invariant way,

(2) the magnetic monopole (inherent in the Wilson loop) can be detected by the Wilson loop.

Three steps of our strategy:

1) We give a definition of a gauge-invariant magnetic monopole which is inherent in the Wilson loop operator even in SU(N) Yang-Mills theory without adjoint scalar fields.

$\leftarrow$ a non-Abelian Stokes theorem for the Wilson loop operator (a path-integral representation of the Wilson loop operator using the coherent state for the Lie group) [Diakonov & Petrov, 1989,...],[Kondo & Taira, 2000],[Kondo, 2008]

2) We develop an optimal description of the magnetic monopole derived in 1).

$\leftarrow$ a reformulation of Yang-Mills theory using new field variables [Cho, 1980],[Duan & Ge, 1979],[Faddeev & Niemi, 1999],[Shabanov, 1999],[Kondo, Shinohara and Murakami, 2005,2008]

3) For SU(3), we confirm the infrared dominance of the restricted variables and the non-Abelian U(2) magnetic monopole dominance for quark confinement (in the string tension). We demonstrate the dual Meissner effect due to non-Abelian magnetic monopoles. cf. [infrared Abelian dominance and magnetic monopole dominance in MA gauge]
$\leftarrow$ a lattice version of the reformulation of the Yang-Mills theory and numerical simulations on a lattice
[Kato, Kondo, Shibata, Shinohara, Murakami and Ito, 2006, 2007, ...]

§ Wilson loop operator and magnetic monopole

The $SU(N)$ Wilson loop operator in a given representation specified by $|\Lambda\rangle$:

$$W_C[\mathscr{A}] := \mathrm{tr}\left[\mathscr{P}\exp\left\{ig_{\mathrm{YM}}\oint_C dx^\mu \mathscr{A}_\mu(x)\right\}\right]/\mathrm{tr}(\mathbf{1}), \quad \mathscr{A}_\mu(x) = \mathscr{A}_\mu^A(x)\lambda^A/2 \in su(N)$$

can be cast into an equivalent form without the path-ordering $\mathscr{P}$

$$W_C[\mathscr{A}] = \int d\mu_C(g) \exp\left[ig_{\mathrm{YM}}\oint_C A\right], \quad A := A_\mu(x)dx^\mu, \tag{1}$$

where g_{YM} is the Yang-Mills coupling constant,

$$d\mu_C(g) := \prod_{x\in C} d\mu(g_x), \quad d\mu(g) : \text{an invariant measure on } G = SU(N) \tag{2}$$

$$A := A_\mu(x)dx^\mu,\ A_\mu(x) = \mathrm{tr}\{\rho[g_x^\dagger \mathscr{A}_\mu(x) g_x + i g_{\mathrm{YM}}^{-1} g_x^\dagger \partial_\mu g_x]\},\ g_x \in G = SU(N). \tag{3}$$

$$\rho := |\Lambda\rangle\langle\Lambda|. \tag{4}$$

$|\Lambda\rangle$: a reference state (highest–weight state of the rep.) making a rep. of the Wilson loop we consider. $\mathrm{tr}(\rho) = \langle\Lambda|\Lambda\rangle = 1$ follows from the normalization of $|\Lambda\rangle$.

Then it is rewritten into the surface-integral form using a usual Stokes theorem:

$$W_C[\mathscr{A}] = \int d\mu_\Sigma(g) \exp\left[ig_{\rm YM} \int_{\Sigma:\partial\Sigma=C} F\right], \tag{5}$$

where $d\mu_\Sigma(g) := \prod_{x\in\Sigma} d\mu(g_x)$, the resulting two-form $F := dA = \frac{1}{2}F_{\mu\nu}(x)dx^\mu \wedge dx^\nu$ is given by

$$F_{\mu\nu}(x) = \sqrt{2(N-1)/N}[\mathscr{G}_{\mu\nu}(x) + ig_{\rm YM}^{-1}{\rm tr}\{\rho g_x^\dagger[\partial_\mu, \partial_\nu]g_x\}], \tag{6}$$

with the field strength $\mathscr{G}_{\mu\nu}$ defined by

$$\begin{aligned}\mathscr{G}_{\mu\nu}(x) :=& \partial_\mu {\rm tr}\{\boldsymbol{n}(x)\mathscr{A}_\nu(x)\} - \partial_\nu {\rm tr}\{\boldsymbol{n}(x)\mathscr{A}_\mu(x)\} \\ &+ \frac{2(N-1)}{N} ig_{\rm YM}^{-1}{\rm tr}\{\boldsymbol{n}(x)[\partial_\mu \boldsymbol{n}(x), \partial_\nu \boldsymbol{n}(x)]\},\end{aligned} \tag{7}$$

by introducing the so-called color field

$$\boldsymbol{n}(x) := \sqrt{N/[2(N-1)]}g_x\left[\rho - \mathbf{1}/{\rm tr}(\mathbf{1})\right]g_x^\dagger, \tag{8}$$

which is normalized ($\boldsymbol{n}(x)\cdot\boldsymbol{n}(x) = 1$) and traceless (${\rm tr}\{\boldsymbol{n}(x)\} = 0$).

Finally, the Wilson loop operator in the fundamental rep. of $SU(N)$ reads [Kondo, arXiv:0801.1274, Phys.Rev.D77, 085029 (2008)] [Kondo, hep-th/0009152]

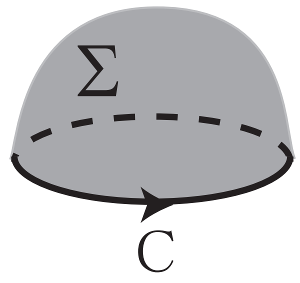
$$W_C[\mathscr{A}] = \int d\mu_\Sigma(g) \exp\left\{ig(\Xi_\Sigma, k) + ig(N_\Sigma, j)\right\}, \quad C = \partial\Sigma$$

$$k := \delta^* F = {}^* dF, \quad \Xi_\Sigma := \delta^* \Theta_\Sigma \triangle^{-1} \leftarrow \quad \text{(D-3)-forms}$$

$$j := \delta F, \quad N_\Sigma := \delta \Theta_\Sigma \triangle^{-1} \leftarrow \quad \text{1-forms (D-indep.)}$$

$$\Theta_\Sigma^{\mu\nu}(x) = \int_\Sigma d^2 S^{\mu\nu}(x(\sigma))\delta^D(x - x(\sigma))$$

$$(k, \Xi_\Sigma) := \frac{1}{(D-3)!}\int d^D x k^{\mu_1\cdots\mu_{D-3}}(x)\Xi_\Sigma^{\mu_1\cdots\mu_{D-3}}(x), \tag{9}$$



k,j: "magnetic current" k and "electric current" j which are conserved, $\delta k = 0 = \delta j$.
$\Delta := d\delta + \delta d$: the D-dimensional Laplacian,
Θ: the vorticity tensor, which has the support on the surface Σ (with the surface element $dS^{\mu\nu}(x(\sigma))$) whose boundary is the loop C.

The last part $ig_{\rm YM}^{-1}{\rm tr}\{\rho g_x^\dagger[\partial_\mu, \partial_\nu]g_x\}$ in F corresponds to the Dirac string, which is not gauge invariant and does not contribute to the Wilson loop (unless the gauge is fixed). **The gauge-invariant magnetic monopole k is inherent in the Wilson loop operator.**

⊙ **SU(2) case**: For the fundamental rep. of $SU(2)$, the highest-weight state $|\Lambda\rangle$ yields

$$|\Lambda\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \rho := |\Lambda\rangle\langle\Lambda| = \begin{pmatrix} 1 \\ 0 \end{pmatrix}(1,0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \rho - \frac{1}{2}\mathbf{1} = \frac{\sigma_3}{2}, \tag{10}$$

$$\Longrightarrow \boldsymbol{n}(x) = g_x \frac{\sigma_3}{2} g_x^\dagger \in SU(2)/U(1) \simeq S^2 \simeq CP^1. \tag{11}$$

Then the gauge–invariant magnetic–monopole current (D-3)-form $k = \frac{1}{2}\delta^* f$ is given by

$$\begin{aligned} f_{\mu\nu} =& \partial_\mu[\boldsymbol{n}^A \mathscr{A}_\nu^A] - \partial_\nu[\boldsymbol{n}^A \mathscr{A}_\mu^A] - g_{\rm YM}^{-1}\epsilon^{ABC}\boldsymbol{n}^A\partial_\mu\boldsymbol{n}^B\partial_\nu\boldsymbol{n}^C \\ =& 2{\rm tr}\left\{\boldsymbol{n}\mathscr{F}_{\mu\nu} + ig_{\rm YM}^{-1}\boldsymbol{n}[D_\mu\boldsymbol{n}, D_\nu\boldsymbol{n}]\right\}. \end{aligned} \tag{12}$$

The existence of magnetic monopole is suggested from a nontrivial Homotopy class of the map $\boldsymbol{n} : S^2 \to SU(2)/U(1)$

$$\pi_2(SU(2)/U(1)) = \pi_1(U(1)) = \mathbb{Z}. \tag{13}$$

Magnetic charge obeys the quantization condition a la Dirac:

$$Q_m := \int d^3x k^0 = 4\pi g_{\rm YM}^{-1}\ell, \ \ \ell \in \mathbb{Z}. \tag{14}$$

cf. the Abelian magnetic monopole of 't Hooft–Polyakov type:

$$\boldsymbol{n}^A \leftrightarrow \hat{\phi}^A(x)/|\hat{\phi}(x)|. \tag{15}$$

For SU(2), if we choose a special gauge in which the color field is uniform:

$$\boldsymbol{n}(x) = (n_1(x), n_2(x), n_3(x)) = (0, 0, 1), \tag{16}$$

then

$$f_{\mu\nu} = \partial_\mu \mathscr{A}_\nu^3 - \partial_\nu \mathscr{A}_\mu^3. \tag{17}$$

The Wilson loop operator reduces to the "Abelian-projected" form:

$$W_C[\mathscr{A}] = \exp\left[i g_{\rm YM} \int_{\Sigma:\partial\Sigma=C} F \right], \tag{18}$$

where the two-form $F := dA = \frac{1}{2} F_{\mu\nu}(x) dx^\mu \wedge dx^\nu$ is defined by

$$F_{\mu\nu}(x) = \mathscr{G}_{\mu\nu}(x) = \frac{1}{2} f_{\mu\nu}(x). \tag{19}$$

cf. Abelian projection, See reviews by Chernodub & Polikarpov, Greensite, ...

⊙ **SU(3) case**: For the fundamental rep. of $SU(3)$, the highest-weight sate $|\Lambda\rangle$ yields

$$|\Lambda\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \rho := |\Lambda\rangle\langle\Lambda| = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \rho - \frac{1}{3}\mathbf{1} = \frac{-1}{3}\begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{20}$$

$$\Longrightarrow \boldsymbol{n}(x) = g_x \frac{-1}{2\sqrt{3}}\begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} g_x^\dagger \in SU(3)/U(2) \simeq CP^2 \ (g_x \in SU(3)). \tag{21}$$

Then the gauge–invariant magnetic–monopole current (D-3)-form $k = \delta^* f$ is given by

$$\mathscr{G}_{\mu\nu} := \partial_\mu \mathrm{tr}\{\boldsymbol{n}\mathscr{A}_\nu\} - \partial_\nu \mathrm{tr}\{\boldsymbol{n}\mathscr{A}_\mu\} + \frac{4}{3} i g_{\mathrm{YM}}^{-1} \mathrm{tr}\{\boldsymbol{n}[\partial_\mu \boldsymbol{n}, \partial_\nu \boldsymbol{n}]\}. \tag{22}$$

Homotopy class of the map $\boldsymbol{n} : S^2 \to SU(3)/U(2)$

$$\pi_2(SU(3)/[SU(2)\times U(1)]) = \pi_1(SU(2)\times U(1)) = \pi_1(U(1)) = \mathbb{Z}. \tag{23}$$

Magnetic charge obeys the quantization condition:

$$Q_m := \int d^3x k^0 = 2\pi\sqrt{3} g_{\mathrm{YM}}^{-1} \ell, \ \ell \in \mathbb{Z}. \tag{24}$$

The maximal stability subgroup $\tilde{H}$ is defined by

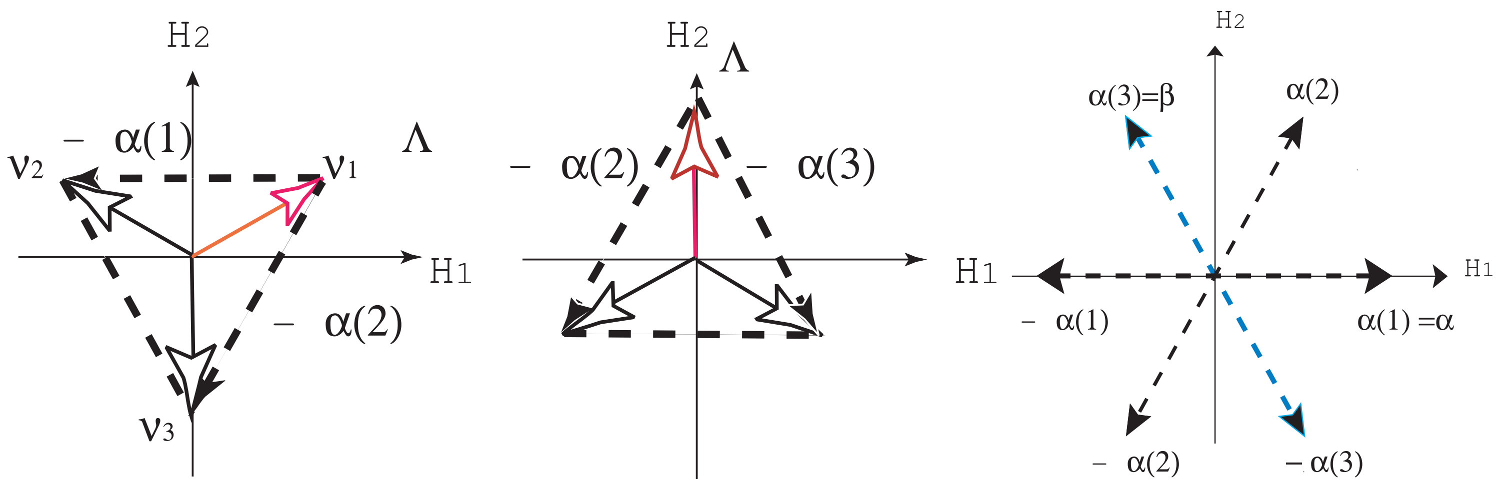
$$h \in \tilde{H} \Longleftrightarrow h|\Lambda\rangle = |\Lambda\rangle e^{i\phi(h)}, \tag{25}$$

for a reference state $|\Lambda\rangle$ of a given representation of a Lie group G.

Every representation R of $SU(3)$ with the Dynkin index [m,n] belongs to (I) or (II):

(I) $m \neq 0$ and $n \neq 0 \Longrightarrow \tilde{H} = H = U(1) \times U(1)$. maximal torus
e.g., adjoint rep.[1,1], $\{H_1, H_2\} \in u(1) + u(1)$,

(II) $m = 0$ or $n = 0 \Longrightarrow \tilde{H} = U(2)$.
when the weight vector Λ is orthogonal to some of the root vectors,
e.g., fundamental rep. [1,0], $\{H_1, H_2, E_\beta, E_{-\beta}\} \in u(2)$, where $\Lambda \perp \beta, -\beta$.



The target space of the color field is specified by the maximal stability group $\tilde{H}$:

$$\boldsymbol{n}(x) = g_x \mathrm{diag.}(\lambda_1, \lambda_2, \lambda_3) g_x^\dagger \in G/\tilde{H}, \tag{26}$$

The gauge-invariant magnetic monopoles inherent in the SU(3) Wilson loop operator for the fundamental rep. are non-Abelian $U(2)$ magnetic monopole in the sense of Goddard–Nuyts–Olive–Weinberg.

c.f. Abelian projection method=the partial gauge fixing from an original gauge group G to the maximal torus subgroup H:

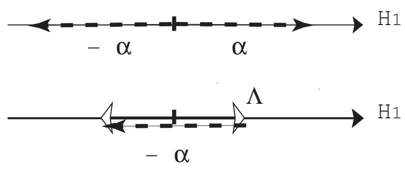
$$G = SU(3) \to H = U(1) \times U(1) \tag{27}$$

$$\pi_2(SU(3)/[U(1) \times U(1)]) = \pi_1(U(1) \times U(1)) = \mathbb{Z}^2. \tag{28}$$

$\Longrightarrow$ two kinds of Abelian U(1) magnetic monopoles for any rep.

No such difference for SU(2). For any rep. of $SU(2)$, magnetic monopole is U(1), since

$$\tilde{H} = H = U(1). \tag{29}$$



§ Reformulating Yang-Mills theory using new variables

We can construct the decomposition:

$$\mathscr{A}_\mu(x) = \mathscr{V}_\mu(x) + \mathscr{X}_\mu(x), \tag{1}$$

such that (a) $\mathscr{V}_\mu$ alone reproduces the Wilson loop operator:

$$W_C[\mathscr{A}] = W_C[\mathscr{V}], \tag{2}$$

and that (b) $\mathscr{F}_{\mu\nu}[\mathscr{V}] := \partial_\mu \mathscr{V}_\nu - \partial_\nu \mathscr{V}_\mu - ig_{\rm YM}[\mathscr{V}_\mu, \mathscr{V}_\nu]$ in $\boldsymbol{n}$ direction agrees with $\mathscr{G}_{\mu\nu}$:

$$\mathscr{G}_{\mu\nu}(x) = \mathrm{tr}\{\boldsymbol{n}(x)\mathscr{F}_{\mu\nu}[\mathscr{V}](x)\}. \tag{3}$$

For this purpose, we impose the defining equations:
(I) $\boldsymbol{n}(x)$ is a covariant constant in the background $\mathscr{V}_\mu(x)$:

$$0 = D_\mu[\mathscr{V}]\boldsymbol{n}(x) := \partial_\mu \boldsymbol{n}(x) - ig_{\rm YM}[\mathscr{V}_\mu(x), \boldsymbol{n}(x)], \Longrightarrow (b) \tag{4}$$

(II) $\mathscr{X}^\mu(x)$ does not have the $\tilde{H}$-commutative part:

$$\mathscr{X}^\mu(x)_{\tilde{H}} := \left(\mathbf{1} - 2\frac{N-1}{N}[\boldsymbol{n}, [\boldsymbol{n}, \cdot]]\right) \mathscr{X}^\mu(x) = 0. \Longrightarrow (a) \tag{5}$$

By solving the defining equations, such fields $\mathscr{V}_\mu(x)$ and $\mathscr{X}_\mu(x)$ are determined uniquely:

$$\begin{aligned}
\mathscr{X}_\mu =& - ig_{\rm YM}^{-1}\frac{2(N-1)}{N}[\boldsymbol{n}, \mathscr{D}_\mu[\mathscr{A}]\boldsymbol{n}] \in \mathscr{L}ie(G/\tilde{H}), \\
\mathscr{V}_\mu =& \mathscr{C}_\mu + \mathscr{B}_\mu \in \mathscr{L}ie(G), \\
& \mathscr{C}_\mu = \mathscr{A}_\mu - \frac{2(N-1)}{N}[\boldsymbol{n}, [\boldsymbol{n}, \mathscr{A}_\mu]] \in \mathscr{L}ie(\tilde{H}), \\
& \mathscr{B}_\mu = ig_{\rm YM}^{-1}\frac{2(N-1)}{N}[\boldsymbol{n}, \partial_\mu \boldsymbol{n}] \in \mathscr{L}ie(G/\tilde{H}).
\end{aligned} \tag{6}$$

At the same time, the color field

$$\boldsymbol{n}(x) \in \mathscr{L}ie(G/\tilde{H})$$

must be obtained by solving the reduction condition $\boldsymbol{\chi} = 0$ for a given $\mathscr{A}$, e.g.,

$$\boldsymbol{\chi}[\mathscr{A}, \boldsymbol{n}] := [\boldsymbol{n}, D^\mu[\mathscr{A}]D_\mu[\mathscr{A}]\boldsymbol{n}] \in \mathscr{L}ie(G/\tilde{H}). \tag{7}$$

For the gauge transformations of the original field $\mathscr{A}_\mu(x)$

$$\mathscr{A}_\mu(x) \to \mathscr{A}'_\mu(x) := U(x)\mathscr{A}_\mu(x)U(x)^{-1} + ig_{\rm YM}^{-1}U(x)\partial_\mu U(x)^{-1}. \tag{8}$$

new fields obey the gauge transformation:

$$\begin{aligned}\mathscr{X}_\mu(x) &\to \mathscr{X}'_\mu(x) := U(x)\mathscr{X}_\mu(x)U(x)^{-1}, \\ \mathscr{V}_\mu(x) &\to \mathscr{V}'_\mu(x) := U(x)\mathscr{V}_\mu(x)U(x)^{-1} + ig_{\rm YM}^{-1}U(x)\partial_\mu U(x)^{-1},\end{aligned} \tag{9}$$

if the color field obeys

$$\boldsymbol{n}(x) \to \boldsymbol{n}'(x) := U(x)\boldsymbol{n}(x)U(x)^{-1}. \tag{10}$$

The gauge invariance of $\mathscr{G}_{\mu\nu}(x)$ and hence the magnetic current k follow from these transformation rules.

We consider the Wilson loop average

$$W(C) := \langle W_C[\mathscr{A}]\rangle_{\rm YM} = Z_{\rm YM}^{-1} \int \mathcal{D}\mathscr{A}\, e^{-S_{\rm YM}[\mathscr{A}]} W_C[\mathscr{A}]. \tag{11}$$

	original YM		reformulated YM
field variables	$\mathscr{A}_\mu^A \in \mathscr{L}(G)$	$\Longrightarrow$	$\boldsymbol{n}^\beta, \mathscr{C}_\nu^k, \mathscr{X}_\nu^b$
action	$S_{\rm YM}[\mathscr{A}]$	$\Longrightarrow$	$\tilde{S}_{\rm YM}[\boldsymbol{n}, \mathscr{C}, \mathscr{X}]$
integration measure	$\mathcal{D}\mathscr{A}_\mu^A$	$\Longrightarrow$	$\mathcal{D}n^\beta \mathcal{D}\mathscr{C}_\nu^k \mathcal{D}\mathscr{X}_\nu^b \tilde{J}\delta(\tilde{\boldsymbol{\chi}})\Delta_{\rm FP}^{\rm red}[\boldsymbol{n}, c, \mathscr{X}]$
Wilson loop operator	$W_C[\mathscr{A}]$	$\Longrightarrow$	$\int d\mu_\Sigma(g) \exp\{ig_{\rm YM}(k, \Xi_\Sigma) + ig_{\rm YM}(j, N_\Sigma)\}$

Thus, we have arrived at the Wilson loop average in the reformulated YM theory:

$$\langle W_C[\mathscr{A}]\rangle_{\rm YM'} = Z_{\rm YM'}^{-1} \int d\mu_\Sigma(g) \int \mathcal{D}n^\beta \mathcal{D}\mathscr{C}_\nu^k \mathcal{D}\mathscr{X}_\nu^b \tilde{J}\delta(\tilde{\boldsymbol{\chi}})\Delta_{\rm FP}^{\rm red} e^{-\tilde{S}_{\rm YM}[\boldsymbol{n},\mathscr{C},\mathscr{X}]}$$

$$\times \exp\{ig_{\rm YM}(k, \Xi_\Sigma) + ig_{\rm YM}(j, N_\Sigma)\}, \tag{12}$$

where $\tilde{\boldsymbol{\chi}} = 0$ is the reduction condition written in terms of the new variables:

$$\tilde{\boldsymbol{\chi}} := \tilde{\boldsymbol{\chi}}[\boldsymbol{n}, \mathscr{C}, \mathscr{X}] := D^\mu[\mathscr{V}]\mathscr{X}_\mu, \tag{13}$$

and $\Delta_{\rm FP}^{\rm red}$ is the Faddeev-Popov determinant associated with the reduction condition:

$$\Delta_{\rm FP}^{\rm red} := \det\left(\frac{\delta \boldsymbol{\chi}}{\delta \boldsymbol{\theta}}\right)_{\boldsymbol{\chi}=0} = \det\left(\frac{\delta \boldsymbol{\chi}}{\delta \boldsymbol{n}^{\theta}}\right)_{\boldsymbol{\chi}=0}. \tag{14}$$

which is obtained by the BRST method as $\Delta_{\rm FP}^{\rm red}[\boldsymbol{n}, c, \mathscr{X}] = \det\{-D_\mu[\mathscr{V}+\mathscr{X}]D_\mu[\mathscr{V}-\mathscr{X}]\}$.

The Jacobian $\tilde{J}$ is very simple, irrespective of the choice of the reduction condition:

$$\tilde{J} = 1. \tag{15}$$

[Kondo, Shinohara & Murakami, arXiv:0803.0176, Prog.Theor.Phys. **120**, 1–50 (2008)]

§ Lattice reformulation and numerical simulations

Lattice reformulation [Kondo, Shibata, Shinohara, Murakami, Kato and Ito, Phys. Lett. B**669**, 107(2008)] [Shibata, Kondo and Shinohara, Phys. Lett. B**691**, 91(2010)]

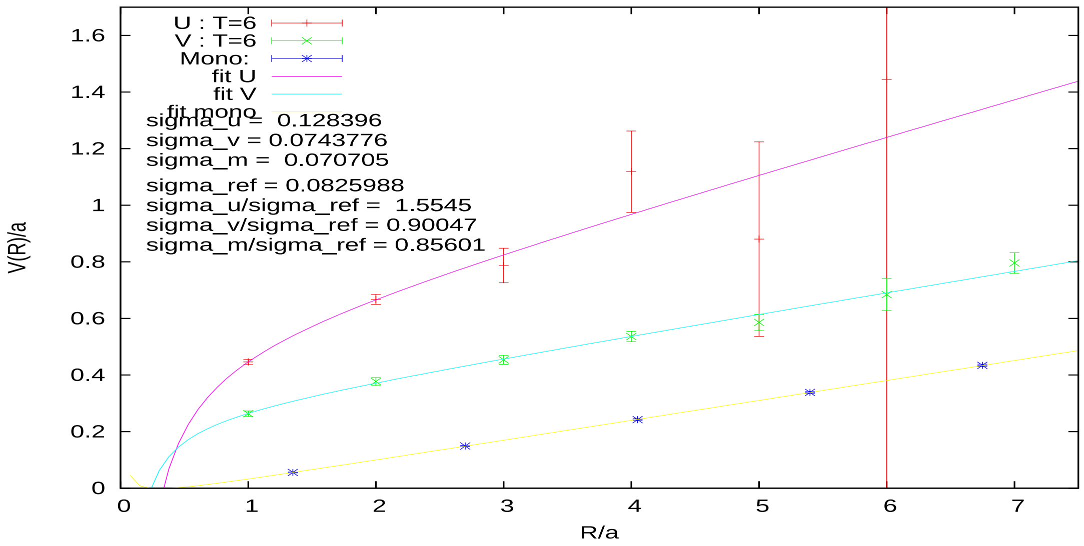


Figure 1: quark-antiquark potential: (from above to below) full SU(3) potential $V_f(r)$, restricted part $V_a(r)$ and magnetic–monopole part $V_m(r)$ at $\beta = 5.7$ on 16^4 lattice

$$\langle W_C[\mathscr{A}]\rangle_{\rm YM} \to V_f(r) \text{ full } SU(3) \text{ quark-antiquark potential},$$

$$\langle W_C[\mathscr{V}]\rangle_{\rm YM'} \to V_a(r) \text{ restricted field part}$$

$$\to \text{infrared } \mathscr{V} \text{ dominance in the string tension (85–90\%)},$$

$$\langle e^{ig_{\rm YM}(k,\Xi_\Sigma)}\rangle\rangle_{\rm YM'} \to V_m(r) \text{ magnetic–monopole part}$$

$$\to U(2) \text{ magnetic monopole dominance in the string tension (75\%)},$$

We study the color flux produced by a quark-antiquark pair, see the left-panel of Fig.2.

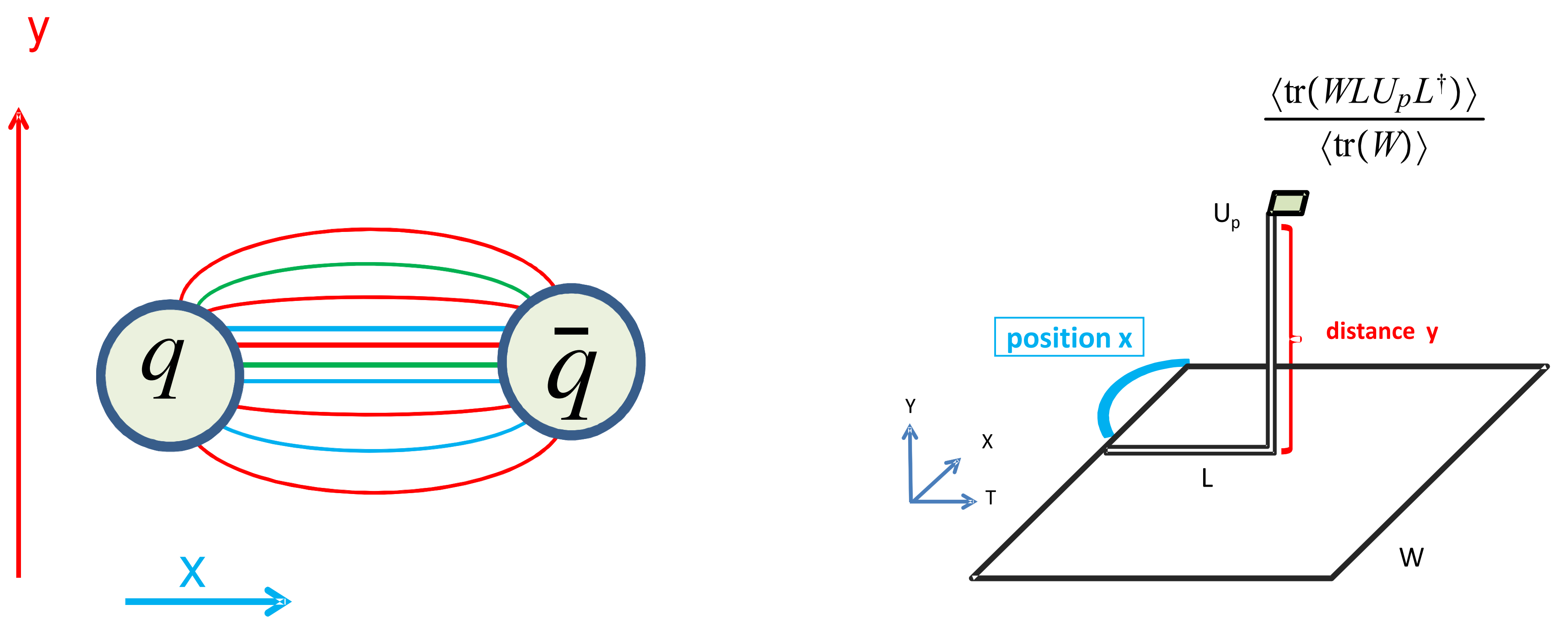


Figure 2: (Left) The setup for measuring the color flux produced by a quark–antiquark pair. (Right) The connected correlator between a plaquette P and the Wilson loop W.

In order to explore the color flux in the gauge invariant way, we use the connected correlator ρ_W of the Wilson line (see the right panel of Fig.2),

$$\rho_W := \frac{\left\langle \mathrm{tr}\left(U_P L^\dagger W L\right)\right\rangle}{\langle \mathrm{tr}\,(W)\rangle} - \frac{1}{N}\frac{\langle \mathrm{tr}\,(U_P)\,\mathrm{tr}\,(W)\rangle}{\langle \mathrm{tr}\,(W)\rangle}, \tag{1}$$

In the naive continuum limit, ρ_W reduces to the field strength:

$$\rho_W \overset{\varepsilon\to 0}{\simeq} g\epsilon^2 \left\langle \mathcal{F}_{\mu\nu} \right\rangle_{q\bar{q}} := \frac{\left\langle \mathrm{tr}\left(g\epsilon^2 \mathcal{F}_{\mu\nu} L^\dagger W L \right) \right\rangle}{\left\langle \mathrm{tr}\left(W \right) \right\rangle} + O(\epsilon^4), \tag{2}$$

Thus, the color filed strength produced by a $q\bar{q}$ pair is given by $F_{\mu\nu} = \sqrt{\frac{\beta}{2N}\rho_W}$.

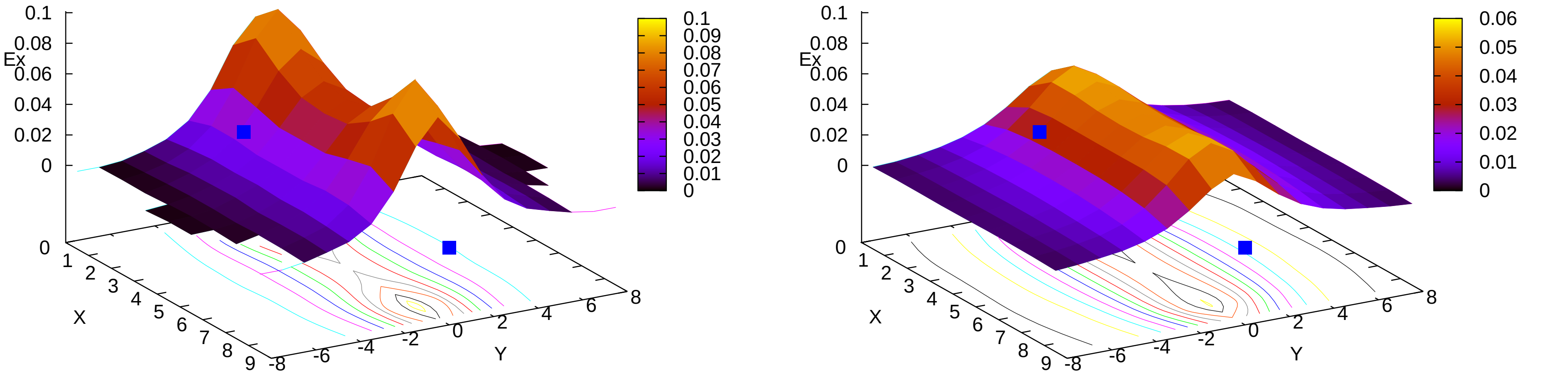


Figure 3: Measurement of chromo-electric flux in the SU(3) Yang-Mills theory. (Left) E_x from the original field $\mathscr{A}$. (Right) E_x from the restricted field $\mathscr{V}$.

These are numerical evidences supporting "non-Abelian" dual superconductivity due to non-Abelian magnetic monopoles as a mechanism for quark confinement in SU(3) Yang-Mills theory.

§ Conclusion and discussion

1) We have defined a gauge-invariant magnetic monopole k inherent in the Wilson loop operator by using a non-Abelian Stokes theorem for the Wilson loop operator, even in SU(N) Yang-Mills theory without adjoint scalar fields.

For quarks in the fundamental representation, $\tilde{H} = U(N)$ for $G = SU(N)$
G=SU(2) Abelian magnetic monopole SU(2)/U(1)
G=SU(3) non-Abelian magnetic monopole SU(3)/U(2)

2) We have constructed a new reformulation of Yang-Mills theory using new field variables, which gives an optimal description of the magnetic monopole derived in 1).

The reformulation allows options discriminated by the maximal stability group $\tilde{H}$.

For $G = SU(3)$, two options are possible:

• The minimal option $\tilde{H} = U(2)$ gives an optimized description of quark confinement through the Wilson loop in the fundamental representation.

• The maximal option, $\tilde{H} = H = U(1) \times U(1)$, the new theory reduces to a manifestly gauge-independent reformulation of the conventional Abelian projection in the maximal Abelian gauge.

The idea of using new variables is originally due to Cho, and Faddeev & Niemi, where $N-1$ color fields $\boldsymbol{n}_{(j)}$ $(j=1,...,N-1)$ are introduced.
However, our reformulation in the minimal option is new for $SU(N), N \geq 3$: we introduce only a single color field $\boldsymbol{n}$ for any N, which is enough for reformulating the quantum Yang-Mills theory to describe confinement of the fundamental quark.

By constructing a lattice version of the reformulation of the SU(N) Yang-Mills theory and performing numerical simulations on a lattice,

3) For SU(3), we have confirmed the infrared dominance of the restricted variables $\mathscr{V}$ and the non-Abelian magnetic monopole dominance for quark confinement (in the string tension),
cf. [infrared Abelian dominance and magnetic monopole dominance in MA gauge]

4) We have shown the evidence of the dual Meissner effect caused by non-Abelian magnetic monopoles in SU(3) Yang-Mills theory: the tube-shaped flux of the chromo-electric field originating from the restricted field including the non-Abelian magnetic monopoles.

To confirm the non-Abelian dual superconductivity picture in SU(3) Yang-Mills theory, we plan to do further checkes, e.g., determination of the type of dual superconductor, measurement of the penetraing depth, induced magnetc current around color flux due to magnetic monopole condensations, and so on.

Thank you very much for your attention!

Questions:

• breaking of the dual gauge symmetry, generation of dual gluon mass

• relationship between magnetic monopoles and vortex.

We can define a gauge-invariant vortex which ends on the non-Abelian magnetic monopole.

• relationship between magnetic monopoles and instantons or merons.
SU(2) case
Kondo, Fukui, Shibata and Shinohara, arXiv:0806.3913[hep-th], Phys.Rev.D78, 065033 (2008)
Fukui, Kondo, Shibata and Shinohara, arXiv:1005.3157[hep-th], Phys.Rev.D82, 045015 (2010)

• Extension to finite temperature case: magnetic–monopole liquid $(T_c < T < 2T_c)$, magnetic–monopole gas $(T > 2T_c)$, proposed by [Chernodub & Zakharov,2007]

• Skyrme-Fadeev-Niemi model as an low-energy effective theory, a non-linear sigma model written in terms of $\boldsymbol{n} \in CP^{N-1} = SU(N)/U(N-1)$

• Large N analysis

To obtain correlation functions of field variables, we need to fix the gauge and we have adopted the Landau gauge.
Fig.4 shows two-point correlation functions of color field, indicating the global $SU(3)$ color symmetry preservation, no specific direction in color space:

$$\langle n^A(0) n^B(r) \rangle = \delta^{AB} D(r)$$

.

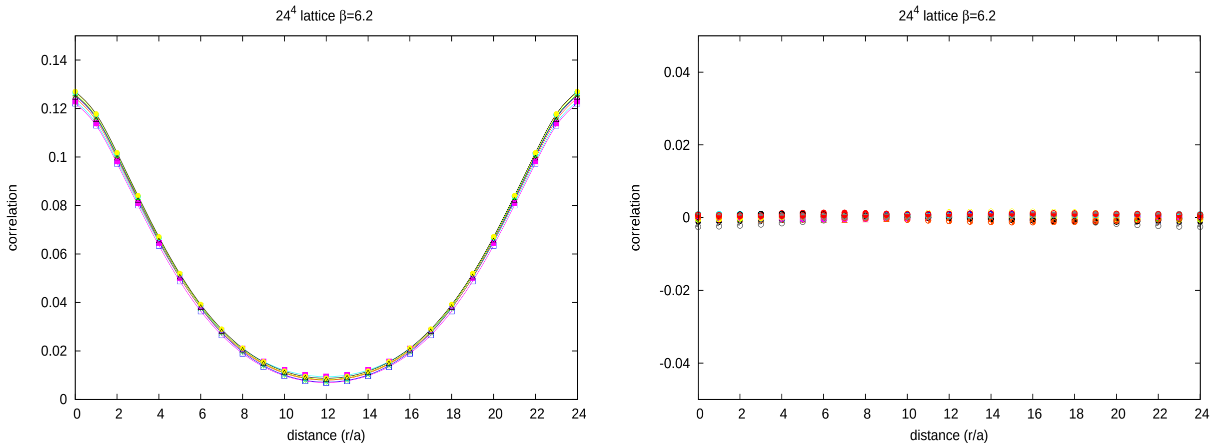


Figure 4: Color field correlators $\langle n^A(0) n^B(r) \rangle$ (Left) $A = B$, (Right) $A \neq B$ $(A, B = 1, \cdots, 8)$ measured at $\beta = 6.2$ on 24^4 lattice, using 500 configurations under the Landau gauge.

We have also checked that one-point functions vanish,

$$\langle n^A(x) \rangle = \pm 0.002 \simeq 0$$

.

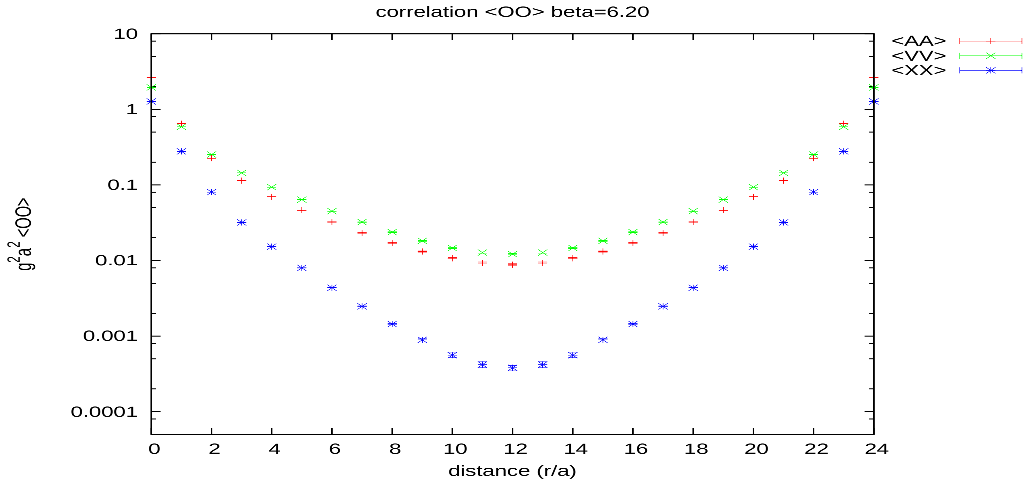


Figure 5: Field correlators as functions of r (from above to below) $\langle \mathscr{V}_\mu^A(0)\mathscr{V}_\mu^A(r)\rangle$, $\langle \mathscr{A}_\mu^A(0)\mathscr{A}_\mu^A(r)\rangle$, and $\langle \mathscr{X}_\mu^A(0)\mathscr{X}_\mu^A(r)\rangle$.

Fig. 5 shows correlators of new fields $\mathscr{V}$, $\mathscr{X}$, and original fields $\mathscr{A}$, indicating the infrared dominance of restricted correlation functions $\langle \mathscr{V}_\mu^A(0)\mathscr{V}_\mu^A(r)\rangle$ in the sense that the variable $\mathscr{V}$ is dominant in the long distance, while the correlator $\langle \mathscr{X}_\mu^A(0)\mathscr{X}_\mu^A(r)\rangle$ of $SU(3)/U(2)$ variable $\mathscr{X}$ decreases quickly.

For $\mathscr{X}$, at least, we can introduce a gauge-invariant "mass" term $\frac{1}{2}M_X^2\mathscr{X}_\mu^A\mathscr{X}_\mu^A$, since $\mathscr{X}$ transforms like an adjoint matter field under the gauge transformation. The naively estimated "mass" of $\mathscr{X}$ is $M_X = 2.409\sqrt{\sigma_{\text{phys}}} = 1.1$ GeV. This value should be compared with the result in MA gauge.

The above form is obtained from the coherent state for $SU(N)$ group:

$$|g_x, \Lambda\rangle := g_x|\Lambda\rangle, \quad \langle\Lambda, g_x| := \langle\Lambda|g_x^\dagger,\ g_x \in G = SU(N). \tag{3}$$

by inserting the complete set at each partition point,

$$\mathbf{1} = \int d\mu(g_x)|g_x, \Lambda\rangle\langle\Lambda, g_x| \left(= \int d\mu(g_x) g_x|\Lambda\rangle\langle\Lambda|g_x^\dagger \right), \tag{4}$$

and replacing the trace by the integral,

$$\mathrm{tr}(\mathscr{O})/\mathrm{tr}(\mathbf{1}) = \int d\mu(g_0)\langle\Lambda, g_0|\mathscr{O}|g_0, \Lambda\rangle \left(= \int d\mu(g_0)\langle\Lambda|g_0^\dagger \mathscr{O} g_0|\Lambda\rangle \right). \tag{5}$$

In fact,

$$\begin{aligned} W_C[\mathscr{A}] &= \int \prod_{x\in C} d\mu(g_x) \prod_{x\in C} \langle\Lambda|g_x^\dagger \exp[i g_{\mathrm{YM}} \Delta x^\mu \mathscr{A}_\mu(x)] g_{x+\Delta x}|\Lambda\rangle \\ &= \int \prod_{x\in C} d\mu(g_x) \prod_{x\in C} \exp\left[i g_{\mathrm{YM}} \Delta x^\mu \mathrm{tr}\{\rho[g_x^\dagger \mathscr{A}_\mu(x) g_x + i g_{\mathrm{YM}}^{-1} g_x^\dagger \partial_\mu g_x]\}\right]. \end{aligned} \tag{6}$$