The decay $B \to \gamma \ell \nu_\ell$ and B-meson distribution amplitude

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based on

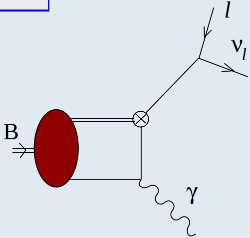
V.M. Braun, A. Khodjamirian, work in progress

QCD@Work, 20.06.2012

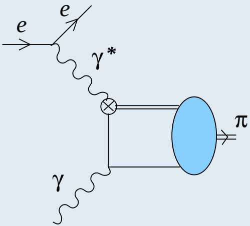
Gold-plated hard exclusive reactions

- Collinear factorization to leading power in $1/m_B$, $1/Q^2$
- Simular structure of power-suppressed corrections (subject of this talk)

$B \to \gamma \ell \nu_\ell$



$\gamma^* \gamma \to \pi$



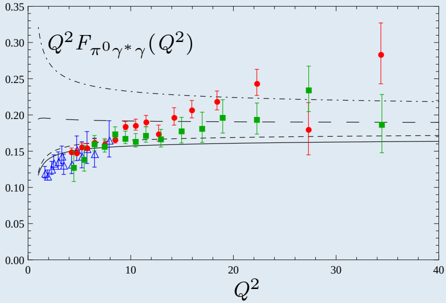
specially for flavor physics:

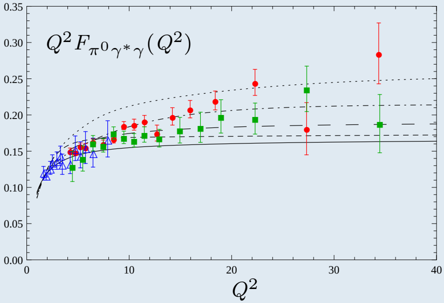
- Access to B-meson Distribution Amplitude; Crucial input for $B \to \pi\pi$ etc.
- Background to $B \to \ell \bar{\nu}_\ell$

Lessons from $\gamma^*\gamma \to \pi$

Left panel: **NLO collinear factorization**

Right panel: **NLO collinear factorization + Higher Twists + Soft Corrections**





- **Nonperturbative corrections at $Q^2 \sim 5$GeV^2 are large**

- **High-precision data expected from BES-III and later super-KEK**
- **Pion DA will be constrained from lattice calculations**

$\Rightarrow$ **expect a much better understanding of theoretical accuracy**

$B \to \gamma\ell\nu_\ell$ in the Heavy Quark Expansion

a good summary: M. Beneke and J. Rohrwild, Eur. Phys. J. C **71**, 1818 (2011)

The decay amplitude

$$A(B^- \to \gamma\ell\bar{\nu}_\ell) \quad = \quad \frac{G_F V_{ub}}{\sqrt{2}} \langle \ell\bar{\nu}_l\gamma|\bar{\ell}\gamma^\nu(1-\gamma_5)\nu_\ell \bar{u}\gamma_\nu(1-\gamma_5)b|B^-\rangle$$

can be written in terms of two form factors:

$$\begin{aligned} T_{\mu\nu}(p,q) &= -i\int d^4x\, e^{ipx}\langle 0|T\{j^{em}_\mu(x)\,\bar{u}(0)\gamma_\nu(1-\gamma_5)b(0)\}|B^-(p+q)\} \\ &= \epsilon_{\mu\nu\tau\rho}p^\tau v^\rho F_V + i\Big[-g_{\mu\nu}(pv) + v_\mu p_\nu\Big] F_A + \ldots \end{aligned}$$

The differential decay width is

$$\frac{d\Gamma}{dE_\gamma} = \frac{\alpha_{\rm em} G_F^2|V_{ub}|^2}{6\pi^2} m_B E_\gamma^3 \left(1 - \frac{2E_\gamma}{m_B}\right)\left(|F_V|^2 + |\widetilde{F}_A|^2\right), \qquad \widetilde{F}_A = F_A + \frac{e_\ell f_B}{E_\gamma}$$

$B \to \gamma \ell \nu_\ell$ in the Heavy Quark Expansion (2)

At large photon energies E_γ

$$F_V(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma,\mu) + \left[\xi(E_\gamma) + \frac{e_b f_B m_B}{2E_\gamma m_b} + \frac{e_u f_B m_B}{(2E_\gamma)^2}\right]$$

$$F_A(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma,\mu) + \left[\xi(E_\gamma) - \frac{e_b f_B m_B}{2E_\gamma m_b} - \frac{e_u f_B m_B}{(2E_\gamma)^2}\right]$$

where

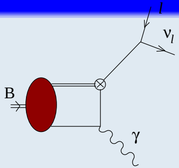
$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\phi_+(\omega,\mu)$$

is the most important nonperturbative parameter in QCD factorization for B-decays

- **$R(E_\gamma,\mu)$ stands for calculable radiative corrections (known to NLO)**
- **Terms in brackets are power corrections in $1/m_B$ and/or $1/E_\gamma$**
  - *— Symmetry-violating power corrections are calculable in HQE*
  - *— Symmetry-preserving power corrections are parametrized by a function $\xi(E_\gamma)$*
- **Beneke & Rohrwild estimate**

$$\xi(E_\gamma) = c \cdot \frac{f_B}{2E_\gamma}, \qquad c \in [-1,1]$$

$B \to \gamma \ell \nu_\ell$ in the Heavy Quark Expansion (3): soft vs. collinear regions



$$F(E_\gamma) \sim \int_0^\infty d\omega \int d^2k_\perp \, \frac{\Phi_+(\omega, k_\perp)}{2E_\gamma \omega + k_\perp^2}$$

$$= \int_{\mu/m_B}^\infty d\omega \int d^2k_\perp \, \frac{\Phi_+(\omega, k_\perp)}{2E_\gamma \omega + k_\perp^2} + \int_0^{\mu/m_B} d\omega \int d^2k_\perp \, \frac{\Phi_+(\omega, k_\perp)}{2E_\gamma \omega + k_\perp^2}$$

$\boxed{\omega \sim \Lambda_{\rm QCD}}$ $\boxed{\omega \sim \Lambda^2_{\rm QCD}/m_B}$

$$= \frac{1}{2E_\gamma} \int_{\mu/m_B}^\infty \frac{d\omega}{\omega} \int d^2k_\perp \, \Phi_+(\omega, k_\perp)$$ leading term

$$- \frac{1}{4E_\gamma^2} \int_{\mu/m_B}^\infty \frac{d\omega}{\omega^2} \int d^2k_\perp \, k_\perp^2 \Phi_+(\omega, k_\perp) + \ldots$$ factorisable correction (higher twist)

$$+ \int_0^{\mu/m_B} d\omega \int d^2k_\perp \, \frac{\Phi_+(\omega, k_\perp)}{2E_\gamma \omega + k_\perp^2}$$ nonfactorisable correction (soft, or end-point)

- **How can one estimate the soft end-point correction?**
  - — **Models**
  - — **Dispersion relations and duality (this talk)**

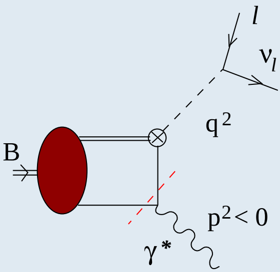
$B \to \gamma\ell\nu_\ell$ from dispersion relations and duality

Main idea:

Consider a generalized (unphysical) process $B \to \gamma^*\ell\nu_\ell$ with spacelike photon

- For sufficiently large photon virtualities calculable in HQE (analogue of $\gamma^*\gamma^*\pi$)
- Real photon limit using dispersion relations and the information on hadron spectrum

$F(q^2, p^2) =$



$B \to \gamma \ell \nu_\ell$ from dispersion relations and duality (2)

- **The $B \to \gamma^*$ form factors satisfy unsubtracted dispersion relation in p^2 for fixed q^2**
  Separating the contribution of the lowest-lying states ρ, ω

$$F_{B\to\gamma^*}(q^2,p^2) = \frac{\sqrt{2} f_\rho F_{B\to\rho}(q^2)}{m_\rho^2 - p^2} + \frac{1}{\pi}\int_{s_0}^\infty ds\, \frac{\mathrm{Im} F_{B\to\gamma^*}(q^2,s)}{s-p^2}$$

  ...and the real photon limit is recovered by the substitution $p^2 \to 0$

- **The same form factors can be calculated in HQE to arbitrary powers in $1/m_B$, $1/E_\gamma$**
  The result satisfies a similar dispersion relation

$$F^{\mathrm{HQE}}_{B\to\gamma^*}(q^2,p^2) = \frac{1}{\pi}\int_0^\infty ds\, \frac{\mathrm{Im} F^{\mathrm{HQE}}_{B\to\gamma^*}(q^2,s)}{s-p^2}$$

  ...but the real photon limit cannot be taken directly as $p^2 \to 0$ limit is singular

- **Duality assumption:**
  physical spectral density above the threshold s_0 coincides with QCD spectral density as given by the HQE

$$\mathrm{Im} F_{B\to\gamma^*}(q^2,s) = \mathrm{Im} F^{HQE}_{B\to\gamma^*}(q^2,s) \qquad \text{for} \quad s > s_0$$

  ...at least after averaging with a smooth function

$B \to \gamma \ell \nu_\ell$ from dispersion relations and duality (3)

- **Asymptotic freedom: Expect QCD (HQE) reproduces physical result at $p^2 \to -\infty$. Hence**

$$\sqrt{2} f_\rho F_{B\to\rho}(q^2) = \frac{1}{\pi} \int_0^{s_0} ds\, \mathrm{Im} F^{\mathrm{QCD}}_{B\to\gamma^*}(q^2, s)$$

. . . a refinement borrowed from QCD sum rules: Borel parameter

$$\sqrt{2} f_\rho F_{B\to\rho}(q^2) = \frac{1}{\pi} \int_0^{s_0} ds\, e^{-(s-m_\rho^2)/M^2}\, \mathrm{Im} F^{\mathrm{QCD}}_{B\to\gamma^*}(q^2, s)\,, \qquad M^2 \sim 1 - 2\,\mathrm{GeV}^2$$

- **Using this substitution obtain:**

$$\boxed{F_{B\to\gamma}(q^2) = \frac{1}{\pi} \int_0^{s_0} \frac{ds}{m_\rho^2} \mathrm{Im} F^{\mathrm{HQE}}_{B\to\gamma^*}(q^2, s) e^{-(s-m_\rho^2)/M^2} + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{ds}{s} \mathrm{Im} F^{\mathrm{HQE}}_{B\to\gamma^*}(q^2, s)}$$

Leading-Order Example

- **Handbag diagram neglecting terms $\sim \omega/E_\gamma$:**

$$F^{(0)}_{B\to\gamma^*}(E_\gamma,p^2) = e_u f_B m_B \int_0^\infty d\omega \frac{\phi_+(\omega,\mu)}{2E_\gamma\omega - p^2}\,,$$

In this case ω-integral easily converted to a form of a disp. relaton by $s = 2E_\gamma\omega$. Obtain

$$F^{(0)}_{B\to\gamma}(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma}\left[(2E_\gamma)\int\limits_0^{s_0/(2E_\gamma)} \frac{d\omega}{m_\rho^2}\,\phi_+(\omega,\mu)e^{-(2E_\gamma\omega - m_\rho^2)/M^2} + \int\limits_{s_0/(2E_\gamma)}^{\infty} \frac{d\omega}{\omega}\,\phi_+(\omega,\mu)\right].$$

or, completing the second integral to infinity and subtracting the correction from the first term

$$F^{(0)}_{B\to\gamma}(E_\gamma) \quad = \quad \underbrace{\left(\frac{e_u f_B m_B}{2E_\gamma\lambda_B(\mu)}\right)}_{\text{LO HQE}} + \underbrace{\frac{e_u f_B m_B}{2E_\gamma}\int\limits_0^{s_0/(2E_\gamma)} d\omega\left[\frac{2E_\gamma}{m_\rho^2}e^{-(2E_\gamma\omega - m_\rho^2)/M^2} - \frac{1}{\omega}\right]\phi_+(\omega,\mu)}_{\text{soft correction}}$$

Leading-Order Example (2)

- We define a rescaled soft correction

$$
\begin{aligned}
F_{B\to\gamma}(E_\gamma) &= \left(\frac{e_u f_B m_B}{2E_\gamma\lambda_B(\mu)}\right)\left(1+\frac{\widehat{\xi}_{B\to\gamma\ell\nu}}{2E_\gamma}\right)+\ldots \\
\widehat{\xi}^{(0)}_{B\to\gamma\ell\nu}(E_\gamma) &= 2E_\gamma\lambda_B \int\limits_0^{s_0/(2E_\gamma)} d\omega\left[\frac{2E_\gamma}{m_\rho^2}\,e^{-(2E_\gamma\omega-m_\rho^2)/M^2}-\frac{1}{\omega}\right]\phi_+(\omega,\mu)
\end{aligned}
$$

- and compare to the similar correction to the $\pi\gamma^*\gamma$ form factor

$$
\begin{aligned}
Q^2F_{\gamma^*\gamma\to\pi}(Q^2) &= \frac{\sqrt{2}f_\pi}{3}\left\{\int_0^1\frac{dx}{x}\phi_\pi(x)+\int_0^{x_0}dx\left[\frac{Q^2}{\bar{x}m_\rho^2}e^{(\bar{x}m_\rho^2-xQ^2)/(\bar{x}M^2)}-\frac{1}{x}\right]\phi_\pi(x)\right\} \\
&\equiv \frac{\sqrt{2}f_\pi}{3}\left(\int_0^1\frac{dx}{x}\phi_\pi(x)\right)\left[1+\frac{\widehat{\xi}_{\gamma^*\gamma\to\pi}(Q^2)}{Q^2}\right], \qquad x_0=s_0/(s_0+Q^2)
\end{aligned}
$$

Leading-Order Soft Corrections in $B \to \gamma\ell\nu_\ell$ vs. $\gamma^*\gamma \to \pi$

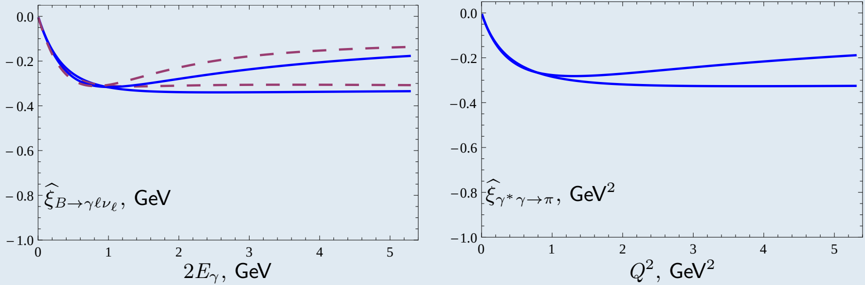


Figure: The leading-order soft correction $\widehat{\xi}(E_\gamma)$ to the $B \to \gamma\ell\nu_\ell$ form factor (left panel) for two models of the B-meson DA, compared to the soft correction for the $\gamma^*\gamma \to \pi$ transition form factor (right panel) in the same approximation. In both cases the lower and the higher curves correspond to $M^2 = 1$ GeV^2 and $M^2 = 1.5$ GeV^2, respectively.

- **For this plot assumed $\lambda_B = 500$ MeV and asymptotic pion DA**

Leading-Order Soft Corrections: Numerical Estimates

- **Rescaled soft correction $\widehat{\xi}_{B\to\gamma\ell\nu_\ell}$, GeV:**

	$\lambda_B = 0.3$	$\lambda_B = 0.4$	$\lambda_B = 0.5$	$\lambda_B = 0.6$
Model I	$-0.49^{+0.05}_{-0.13}$	$-0.35^{+0.06}_{-0.12}$	$-0.27^{+0.06}_{-0.11}$	$-0.22^{+0.05}_{-0.10}$
Model II			$-0.23^{+0.07}_{-0.11}$	

- **Beneke-Rohrwild soft parameter:**

$$c \quad = \quad \frac{e_u}{\lambda_B}\widehat{\xi}(E_\gamma)\frac{m_B}{2E_\gamma} \simeq -(0.36 \pm 0.12)\left(\frac{500\text{ MeV}}{\lambda_B}\right)^{2.2\pm 0.5}\frac{m_B}{2E_\gamma}$$

Phenomenology

- **Decay width with a cut on low energy photons:**

B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **80** (2009) 111105

$$\int_{E_0}^{m_B/2} dE_\gamma \frac{d\Gamma}{dE_\gamma} = \frac{\alpha_{\rm em} G_F^2 |V_{ub}|^2}{6\pi^2} m_B \int_{E_0}^{m_B/2} dE_\gamma E_\gamma^3 \left(1 - \frac{2E_\gamma}{m_B}\right) \left(|F_V|^2 + |\widetilde{F}_A|^2\right)$$

$\hookleftarrow$ present (published) experimental limit too weak

- **The ratio of "photoleptonic" and pure leptonic widths:**

$$\frac{dBR(B \to \gamma \ell \nu_\ell)/dE_\gamma}{BR(B \to \tau \nu_\tau)} = \frac{4\alpha_{em}}{3\pi} \frac{(1 - 2E_\gamma/m_B) E_\gamma^3}{m_\tau^2 (1 - m_\tau^2/m_B^2)} \left[\frac{|F_V|^2 + |\widetilde{F}_A|^2}{f_B^2}\right]$$

$\hookleftarrow$ is independent of V_{ub} and f_B

Preliminary results

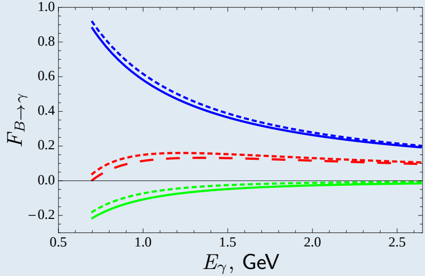


Figure: The form factors $F_V(E_\gamma)$ (blue) and $\widetilde{F}_A(E_\gamma)$ (red) for $\lambda_B = 500$ MeV. The green curve shows the soft-overlap part $\xi(E_\gamma)$

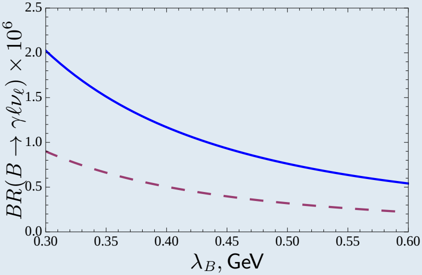


Figure: The partial integrated branching fraction for $E_{min} = 1.0$ GeV (upper) and $E_{min} = 1.7$ GeV (lower) at central input

Outlook

- **Need complete effective theory calculations with p^2 serving as IR cutoff**

$$F_{B\to\gamma}(q^2) = \frac{1}{\pi}\int_0^{s_0}\frac{ds}{m_\rho^2}\mathrm{Im}F^{\mathrm{HQE}}_{B\to\gamma^*}(q^2,s)e^{-(s-m_\rho^2)/M^2} + \frac{1}{\pi}\int_{s_0}^{\infty}\frac{ds}{s}\mathrm{Im}F^{\mathrm{HQE}}_{B\to\gamma^*}(q^2,s)$$

  - NLO radiative corrections with or without resummation
  - Higher-twist DAs, including three-particle
    — automatically protected from end-point divergences
    — normalization related to parameters of leading DA in some schemes

- **Need high precision data on both $B\to\gamma\ell\nu_\ell$ and $\gamma^*\gamma\to\pi$ to control theoretical accuracy**