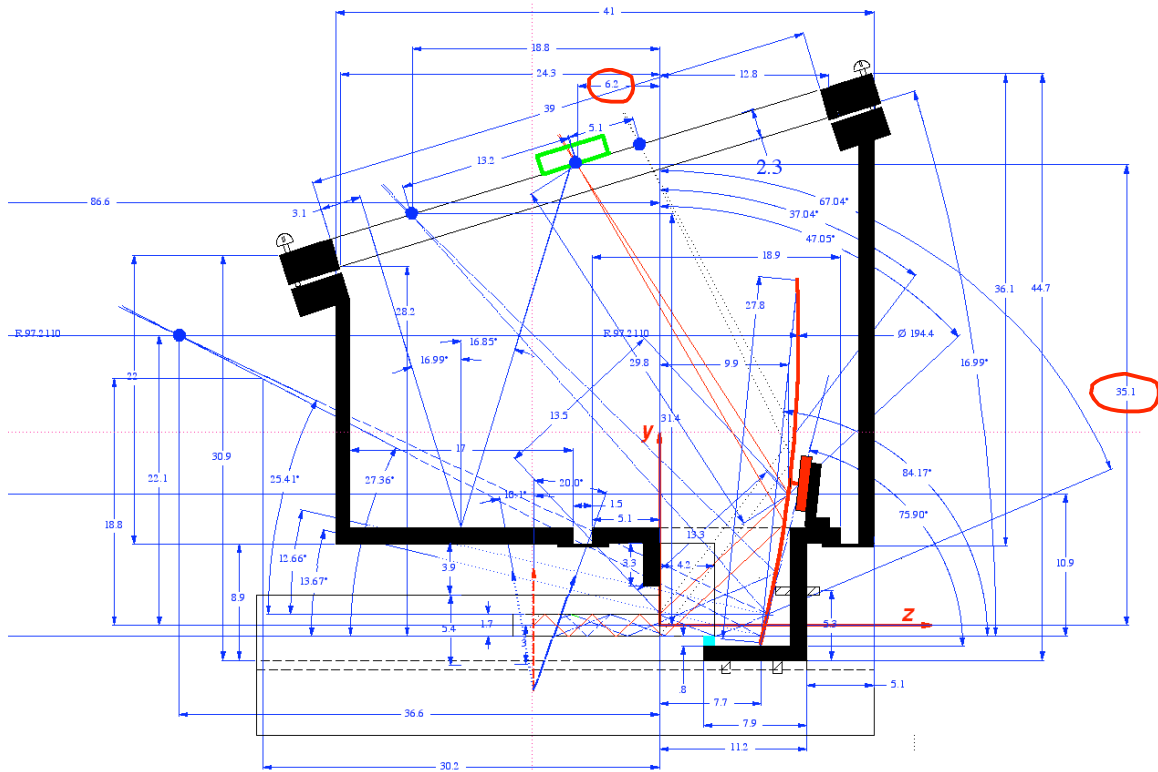


The initial look at the FDIRC Optical Design

J. Va'vra



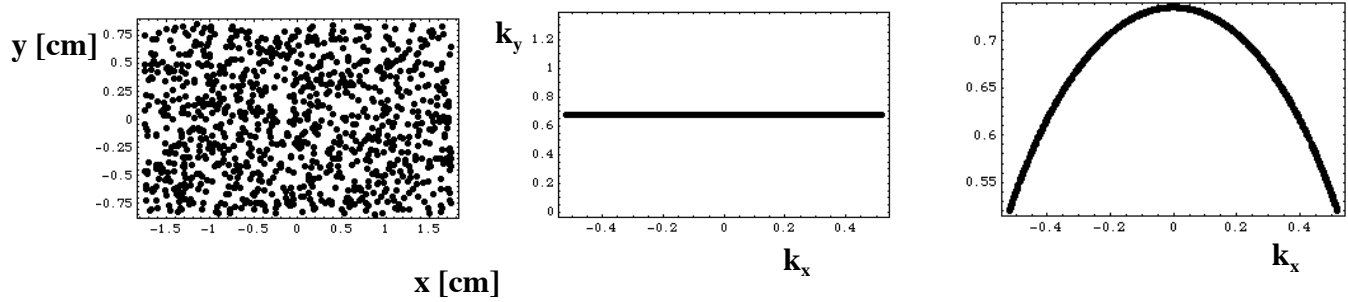
- ## The point of this study:

- 2

Photon x-y distribution at the end of the bar (z = 0):

Photon starting point in the middle of a bar:

```
nrefr=1.47;
barw = 3.5; barh = 1.7; barl = 144*2.54;
x0 = 0; y0 = (0+Random[]*barh-barh/2); z0 = -barl/2;
zbarstart = -barl; zbarend = 0;
Theta = 90/(180/Pi); Phi = 90/(180/Pi);
Thetac = 47.3/(180/Pi); Phic = (180+Random[]*2*45-45)/(180/Pi);
```



Assume a general quadratic shape for the mirror:

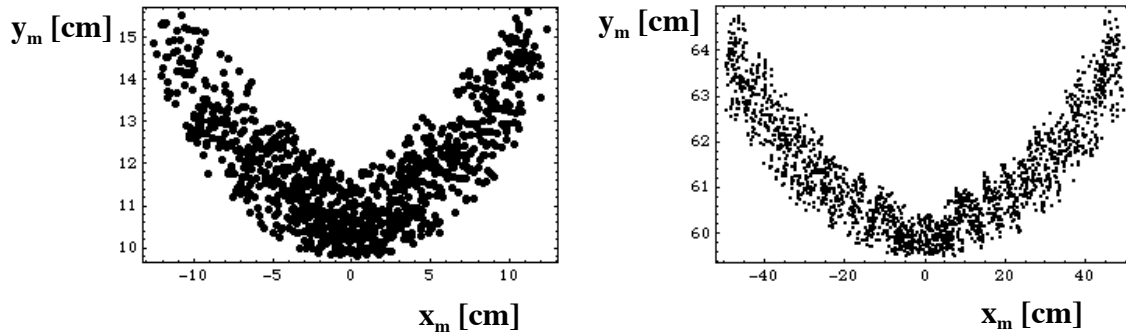
A general form of 3D **quadratic** shape is:

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz + 2a_{14}x + 2a_{24}y + 2a_{34}z + a_{44} = 0$$

Intersection of photons and mirror:

xm0 = 0; ym0 = 23.0; zm0 = -86.6; r = 2*(49.5);

zm0 = -86.6 + 60; r = 2*(49.5); - mirror moved along z by 60cm



Note: Based on the Vellum program of the FDIRC prototype, expect for x_m ~ -8 to 8, y_m ~ 8-11 in the bar coordinate system. The periodicity in this image can be observed if one moves the mirror a bit further along the z axis.

Intersection of photons and FDIRC detector plane:

a) Spherical mirror & flat detector plane:

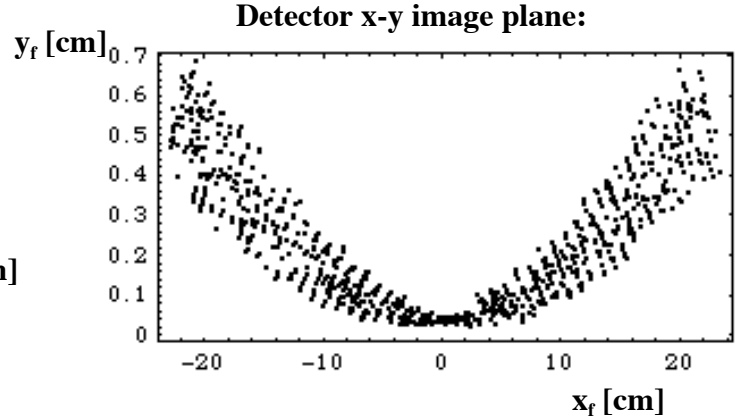
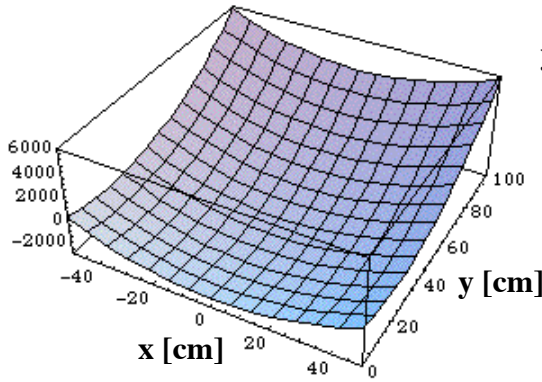
$$(x - x_{m0})^2 + (y - y_{m0})^2 + (z - z_{m0})^2 - r^2 = 0$$

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz + 2a_{14}x + 2a_{24}y + 2a_{34}z + a_{44} = 0$$

$x_{m0} = 0$; $y_{m0} = 23.0$; $z_{m0} = -86.6$; $r = 2*(49.5)$; ← CRID mirror parameters

$a_{11} = 1$; $a_{22} = 1$; $a_{33} = 1$; $a_{12} = 0$; $a_{13} = 0$; $a_{23} = 0.0$; $a_{14} = -x_{m0}$; $a_{24} = -y_{m0}$; $a_{34} = -z_{m0}$; $a_{44} = x_{m0}^2 + y_{m0}^2 + z_{m0}^2 - r^2$;

For the FDIRC prototype geometry with a **spherical mirror** one gets a familiar image (in the bar coordinate system, and after a subtraction of a constant of 35.55 from y_f):



Note: Based on the Vellum program of the FDIRC prototype, expect for $x_f \sim -20$ to 20 , $y_f \sim 35$, $z_f \sim -(3-6)$ in the bar coordinate system. The calculation is not that far from the expected numbers.

b) Circular cylindrical mirror & flat detector plane:

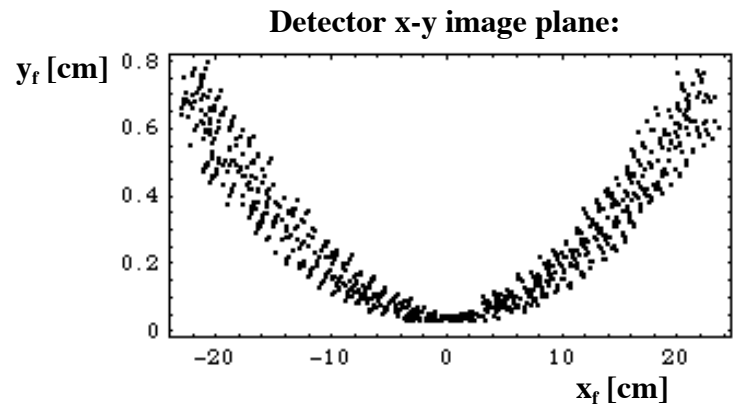
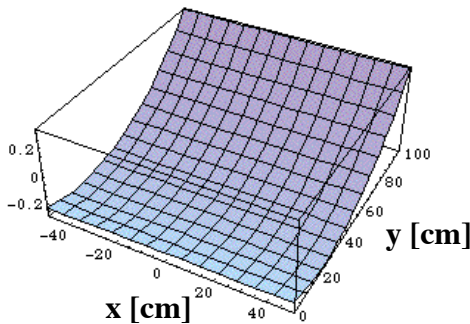
$$(y - y_{m0}/a)^2 + (z - z_{m0}/b)^2 - 1 = 0 \text{ (axis along the x-axis)}$$

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz + 2a_{14}x + 2a_{24}y + 2a_{34}z + a_{44} = 0$$

$x_{m0} = 0$; $y_{m0} = 23.0$; $z_{m0} = -86.6$; $r = 2*(49.5)$; $a = r$; $b = r$;

$a_{11} = 0$; $a_{22} = 1/(a^2)$; $a_{33} = 1/(b^2)$; $a_{12} = 0$; $a_{13} = 0$; $a_{23} = 0.0$; $a_{14} = 0$; $a_{24} = -y_{m0}/(a^2)$; $a_{34} = -z_{m0}/(b^2)$; $a_{44} = y_{m0}^2/a^2 + z_{m0}^2/b^2 - 1$;

If the FDIRC prototype would have a **circular cylindrical mirror** and similar dimensions, one would get this image (in a bar coordinate system):



The resolution loss seems to be smaller compared to the spherical mirror, perhaps by a factor of two.

c) Parabolic cylindrical mirror & flat detector plane:

$$(y - y_{m0})^2 + b(z - z_{m0}) = 0 \text{ (axis along the x-axis)}$$

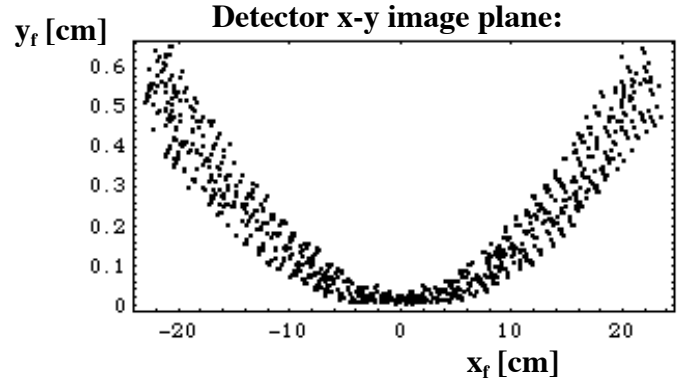
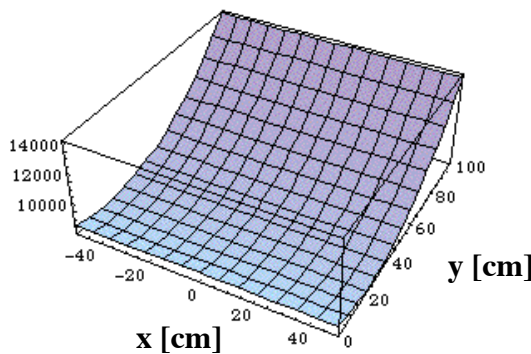
$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz + 2a_{14}x + 2a_{24}y + 2a_{34}z + a_{44} = 0$$

$$x_{m0} = 0; y_{m0} = 23.0; z_{m0} = -86.6; r = 2*(49.5);$$

$$b = r;$$

$$a_{11} = 0; a_{22} = 1; a_{33} = 0; a_{12} = 0; a_{13} = 0; a_{23} = 0.0; a_{14} = 0; a_{24} = -y_{m0}; a_{34} = b/2; a_{44} = y_{m0}^2 - b*z_{m0};$$

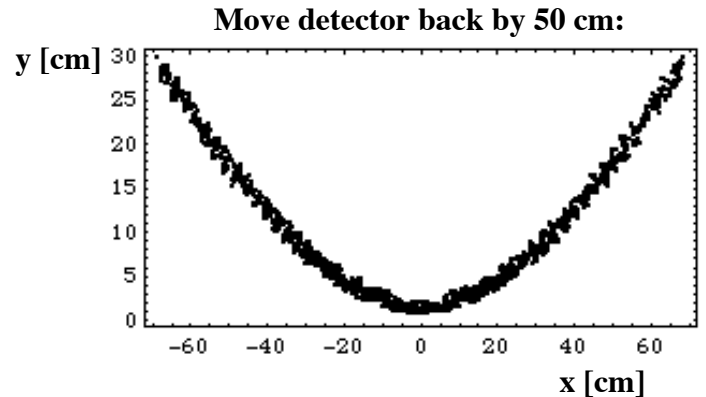
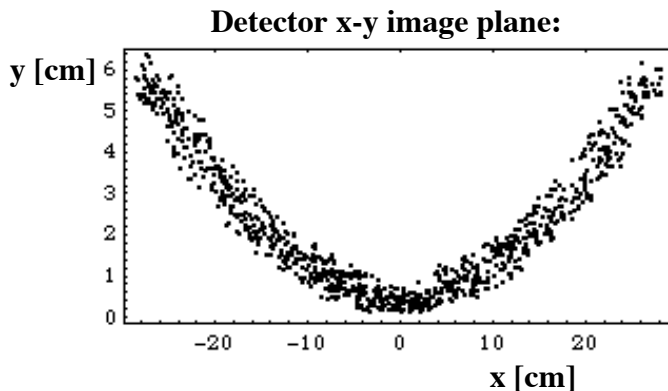
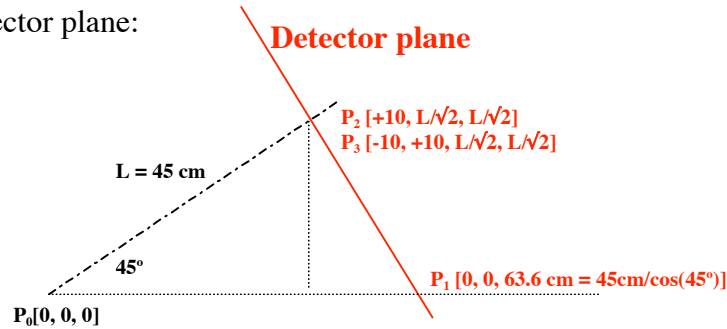
For the FDIRC prototype geometry with a **parabolic cylindrical mirror** one gets this image (in a bar coordinate system):



Similar result to that of the circular cylindrical mirror.

d) Flat detector plane, no mirror:

3 points defining the detector plane:



Indeed, there is a periodic kaleidoscopic pattern even without a mirror. It is created by the square bar. To see it better, move the detector plane back by 50 cm. Since the wiggles originate from the bar, i.e., the mirror has nothing to do with it, it cannot be easily fixed.

Find a detector focal surface in 3D

a) Spherical mirror:

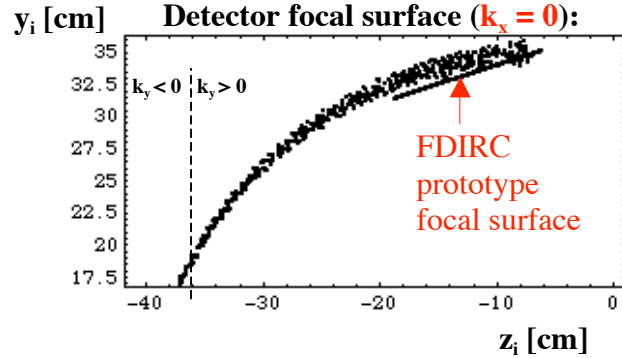
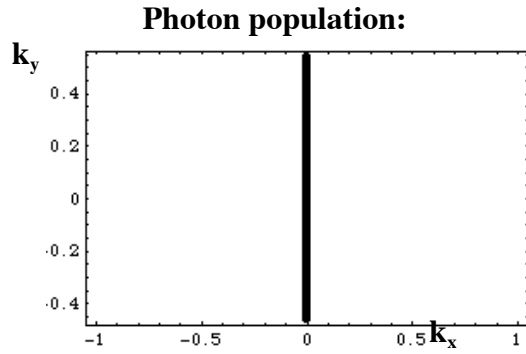
zbarstart = - barl; zbarend = 0;

Theta = (60 + Random[]*2*34 - 40)/(180/Pi); Phi = 90/(180/Pi);

Thetac = 47.3/(180/Pi); Phic = 180/(180/Pi); Vary Phic

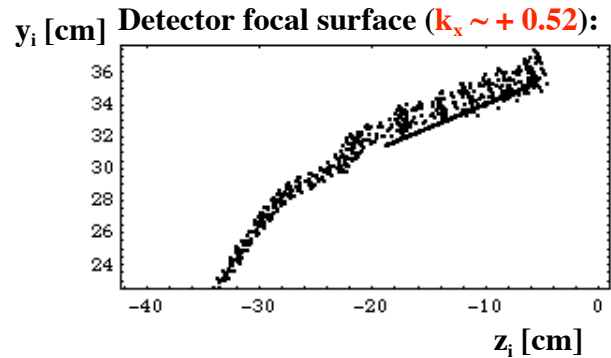
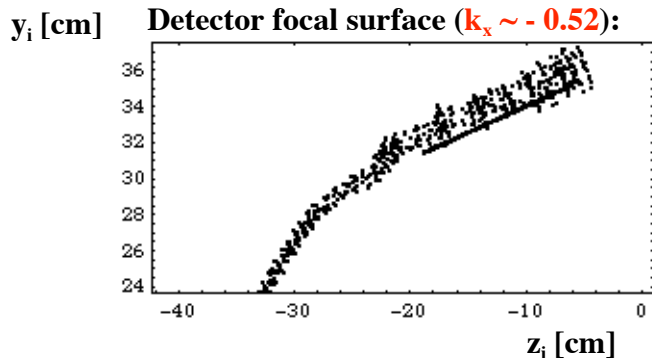
xm0 = 0; ym0 = 23.0; zm0 = -86.6; r = 2*(48.6); **Choose r as one has in the FDIRC prototype Vellum study**

a11 = 1; a22 = 1; a33 = 1; a12 = 0; a13 = 0; a23 = 0.0; a14 = - xm0; a24 = -ym0; a34 = - zm0; a44 = xm0*xm0 + ym0*ym0 + zm0*zm0 - r*r;

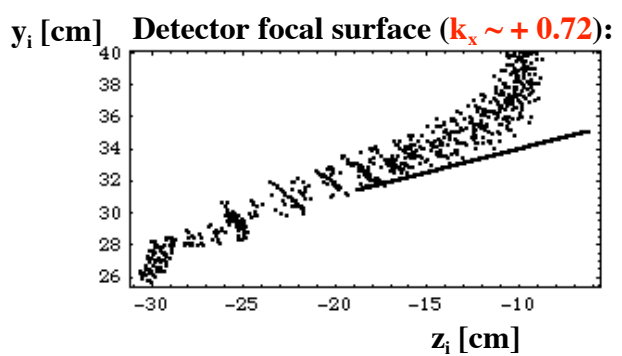
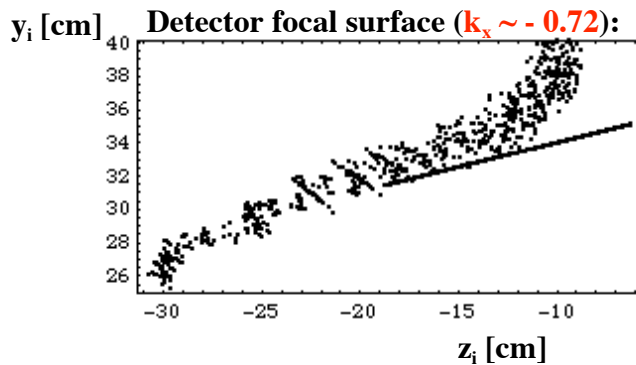


In the region, where the FDIRC prototype works ($z \sim -6$ cm), the calculated focal plane is close to a straight line. So our solution with a flat window was OK. Our results would be perfect for $k_x = 0$ of the detector plane would have zig-zag shape.

Phic = 135/(180/Pi); or Phic = 225/(180/Pi);



Phic = 110/(180/Pi); or Phic = 250/(180/Pi);



The detector surface shape is changing as a function of k_x , mainly as k_y is approaching 0, which corresponds to photons going parallel to z axis.

b) Circular cylindrical mirror:

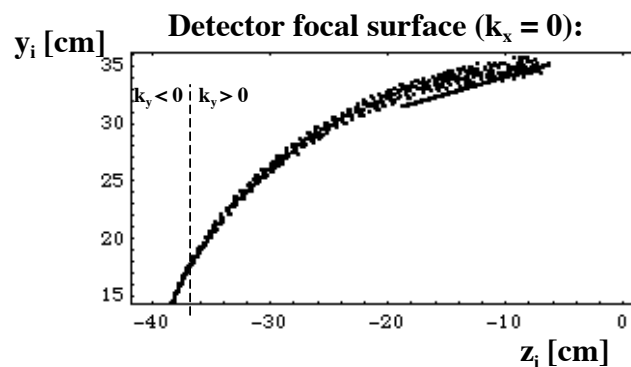
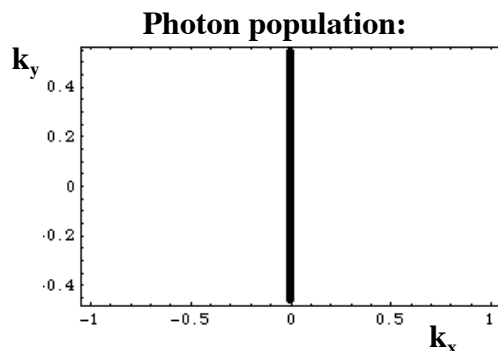
zbarstart = - barl; zbarend = 0;

Theta = (60 + Random[]*2*30 - 40)/(180/Pi); Phi = 90/(180/Pi);

Thetac = 47.3/(180/Pi); Phic = 180/(180/Pi); Vary Phic

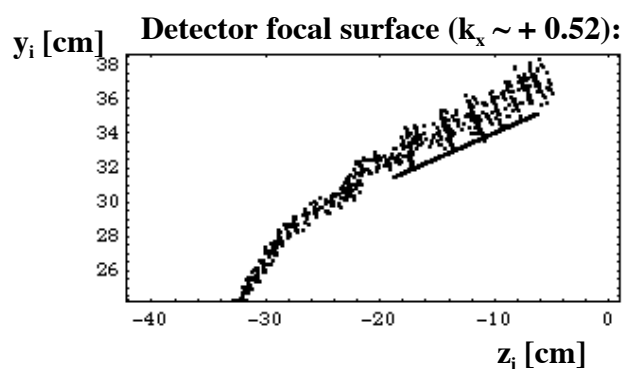
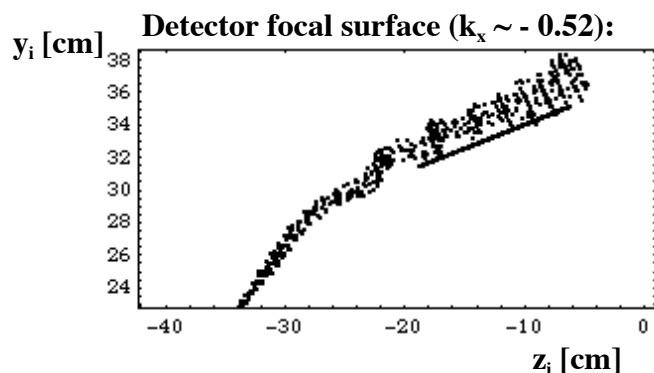
xm0 = 0; ym0 = 23.0; zm0 = -86.6; r = 2*(48.6); **Choose r as one has in the FDIRC prototype Vellum study**

a11 = 1; a22 = 1; a33 = 1; a12 = 0; a13 = 0; a23 = 0.0; a14 = - xm0; a24 = -ym0; a34 = - zm0; a44 = xm0*xm0 + ym0*ym0 + zm0*zm0 - r*r;

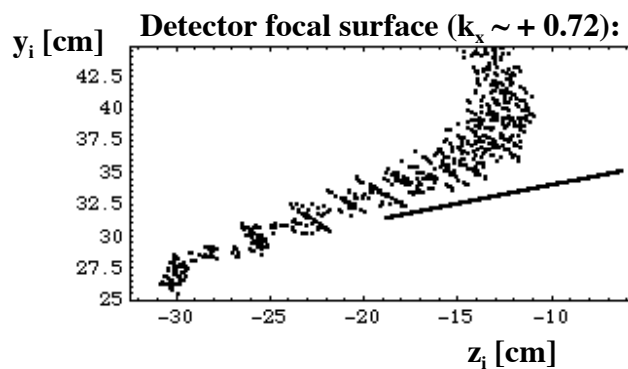
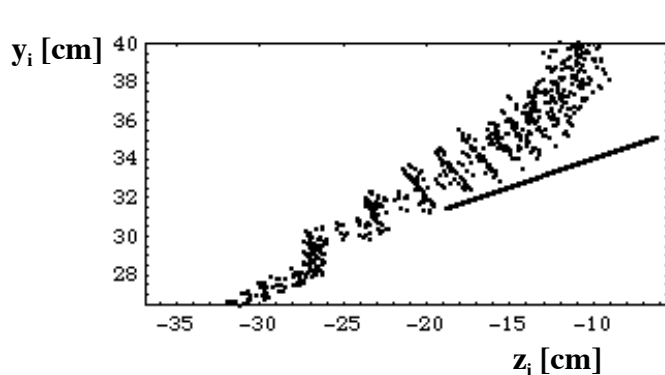


In the region, where the FDIRC prototype works ($z \sim -6$ cm), the calculated focal plane is close to a straight line. So our solution with a flat window was OK.

Phic = 135/(180/Pi); or Phic = 225/(180/Pi);



Phic = 110/(180/Pi); or Phic = 250/(180/Pi);



The cylindrical mirror is not much of a help !!