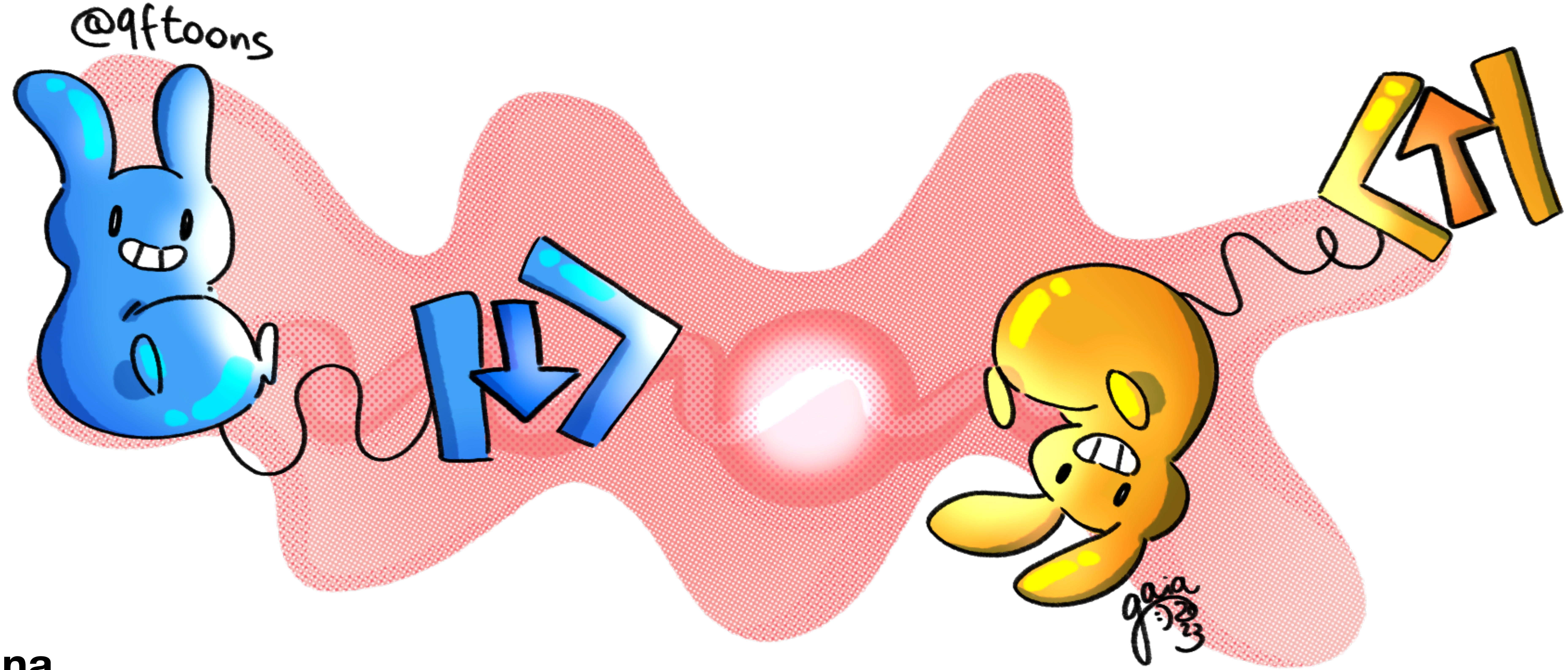
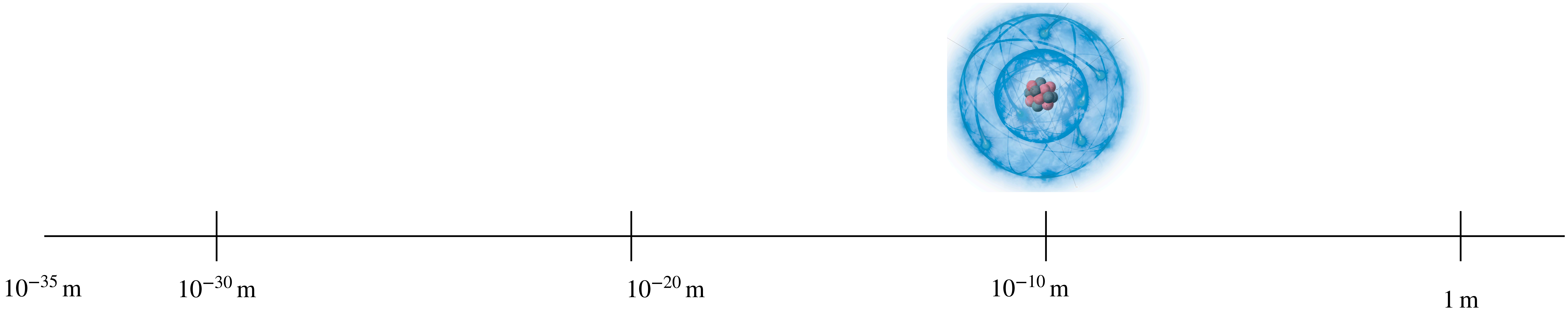
Quantum Observables in HEP



Fabio Maltoni
Università di Bologna
Université catholique de Louvain

Introduction

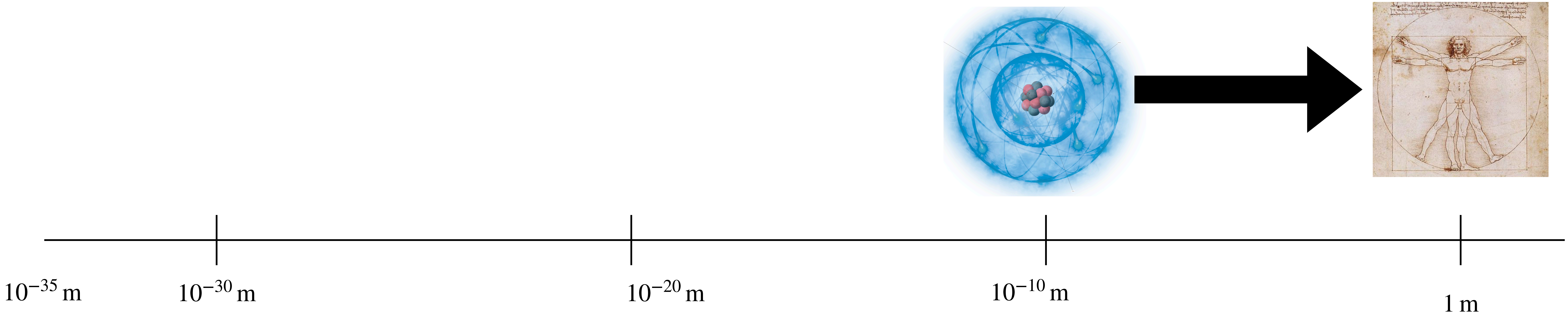
Quantum information/computing/technology: push quantum into the macroscopic world



Quantum computers, quantum communications, quantum devices,…

Introduction

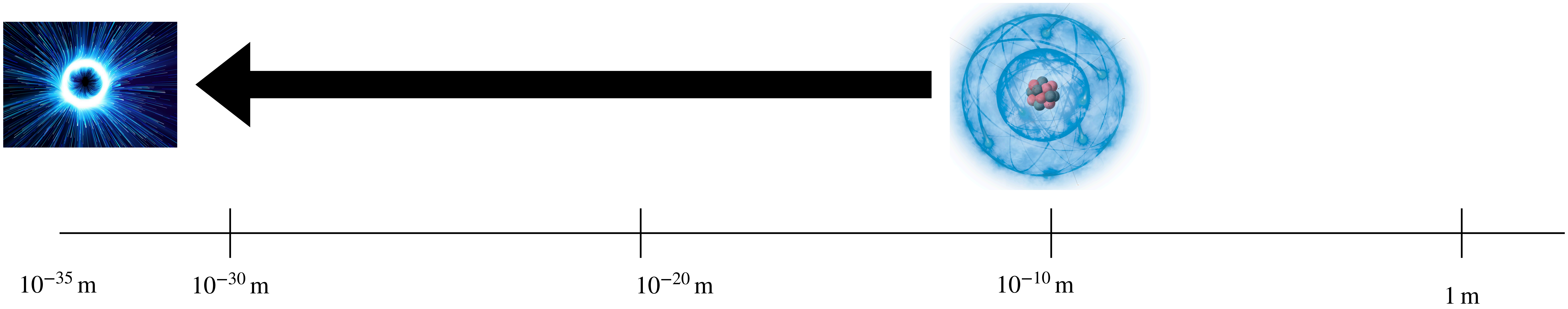
Quantum information/computing/technology: push quantum into the macroscopic world



Quantum computers, quantum communications, quantum devices,…

Introduction

Quantum information paradox (gravity+quantum mechanics):

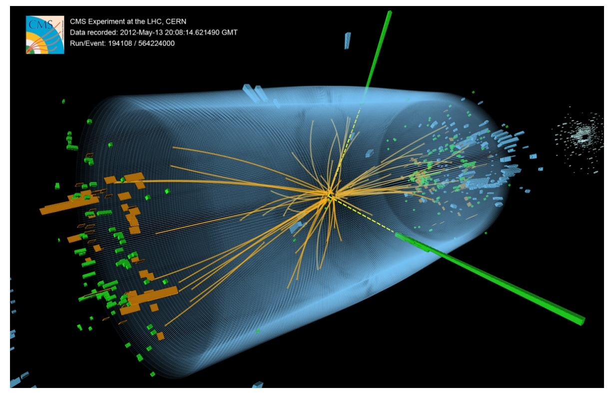


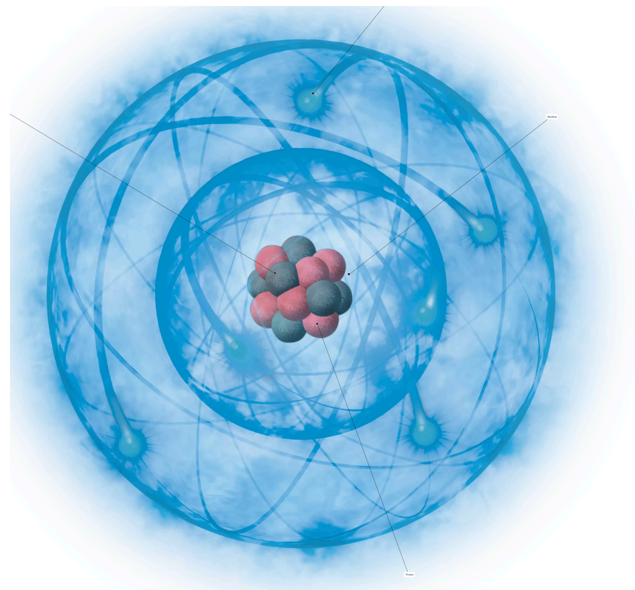
It from Qubit:

- Does spacetime emerge from entanglement?
- Do black holes have interiors?
- Does the universe exist outside our horizon?
- **What is the information-theoretic structure of quantum field theories?**
- Can quantum computers simulate all physical phenomena?
- How does quantum information flow in time?

Introduction

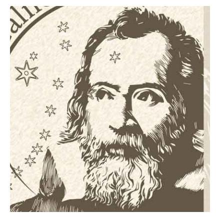
What can we learn on Fundamental Interactions from quantum information ideas/methods/techniques/results?





10^{-35} m $\quad$ 10^{-30} m $\quad$ 10^{-20} m $\quad$ 10^{-10} m $\quad$ 1 m

***Io stimo più il trovar un vero, benché di cosa leggiera, che 'l disputar lungamente delle massime questioni senza conseguir verità nissuna.**



Introduction

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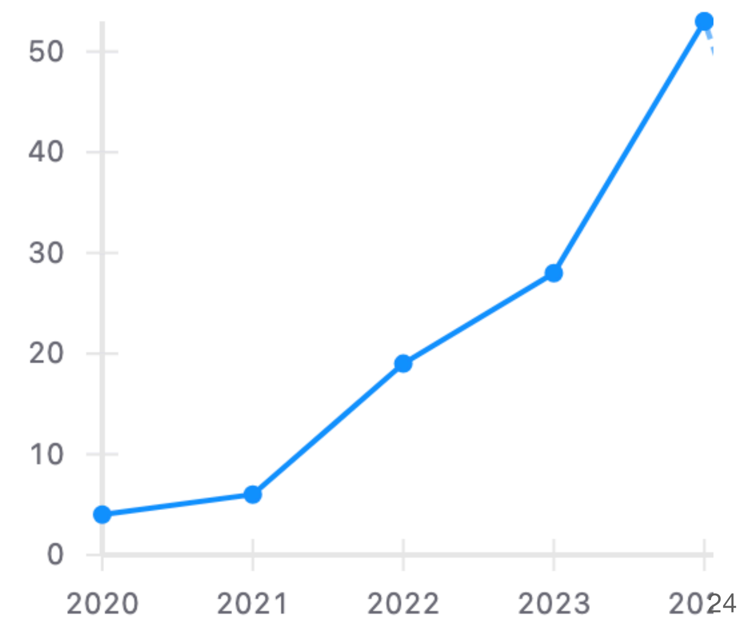
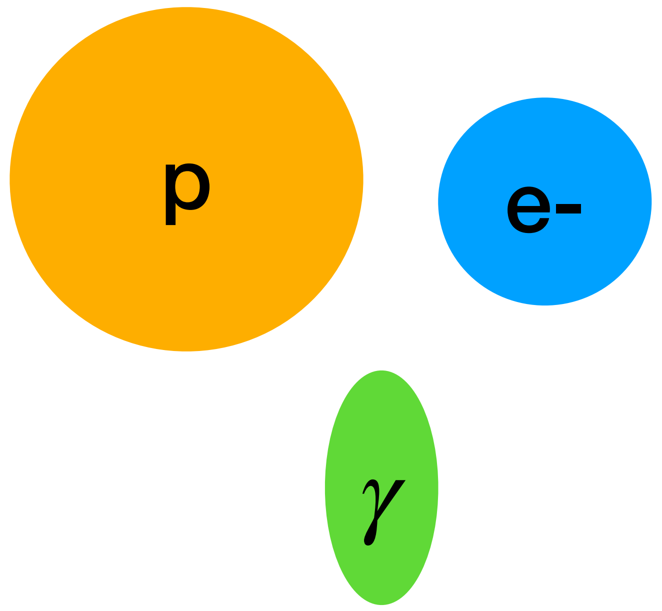


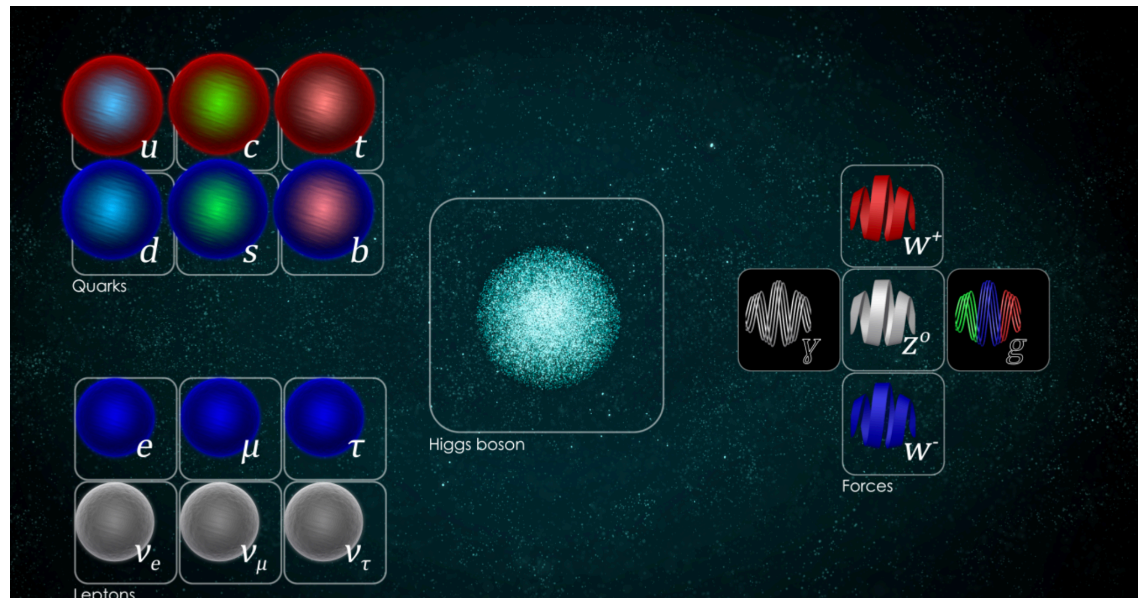
Citations to the Afik & de Nova paper

- Recent and growing interest in the high-Q^2 **collider community** with ~100 papers in the last few years.
- Broadening of interests in more formal and pheno aspects.
- First experimental results appeared in Nov 2023!

Introduction



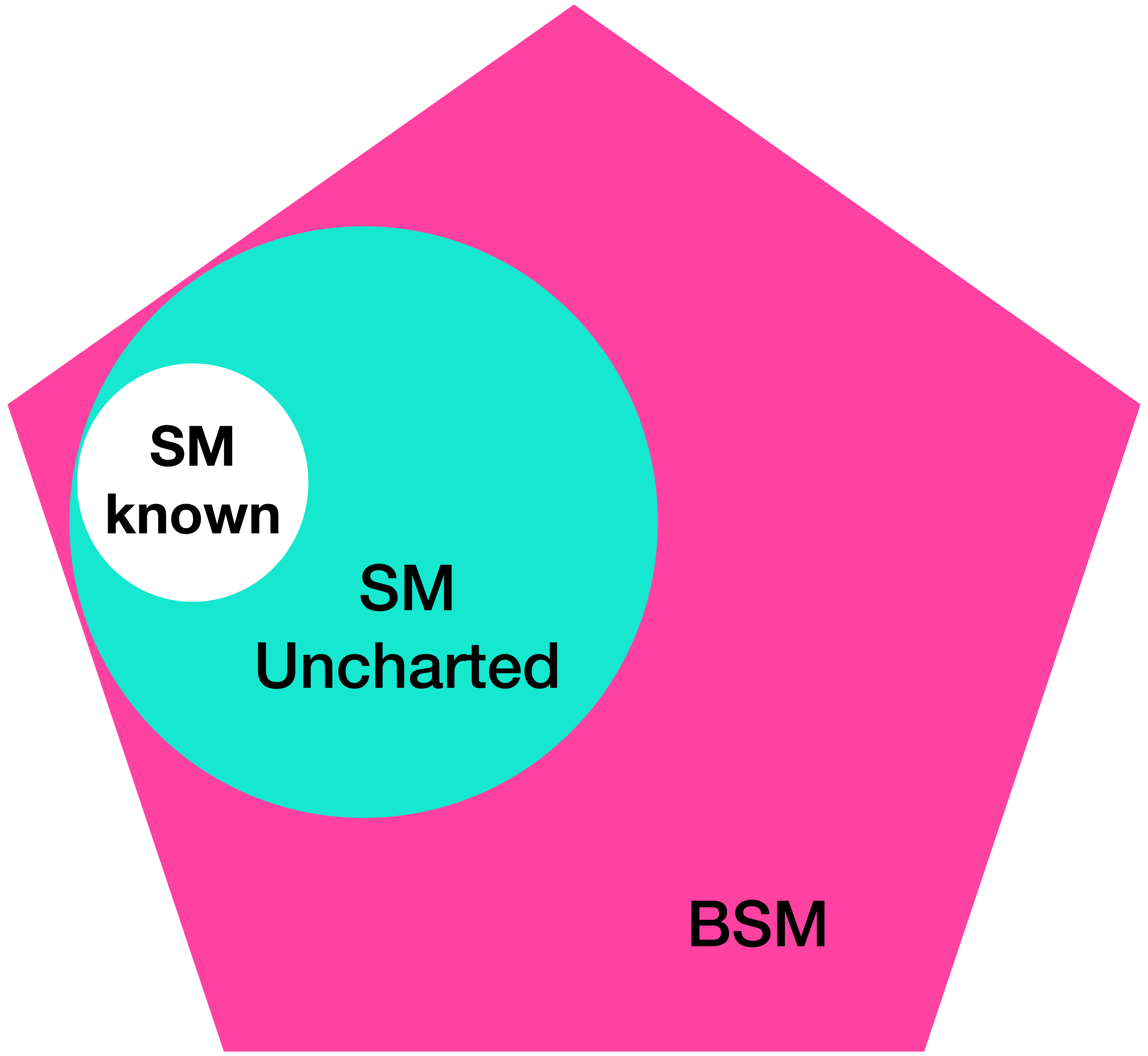
Vs



My personal motivations:

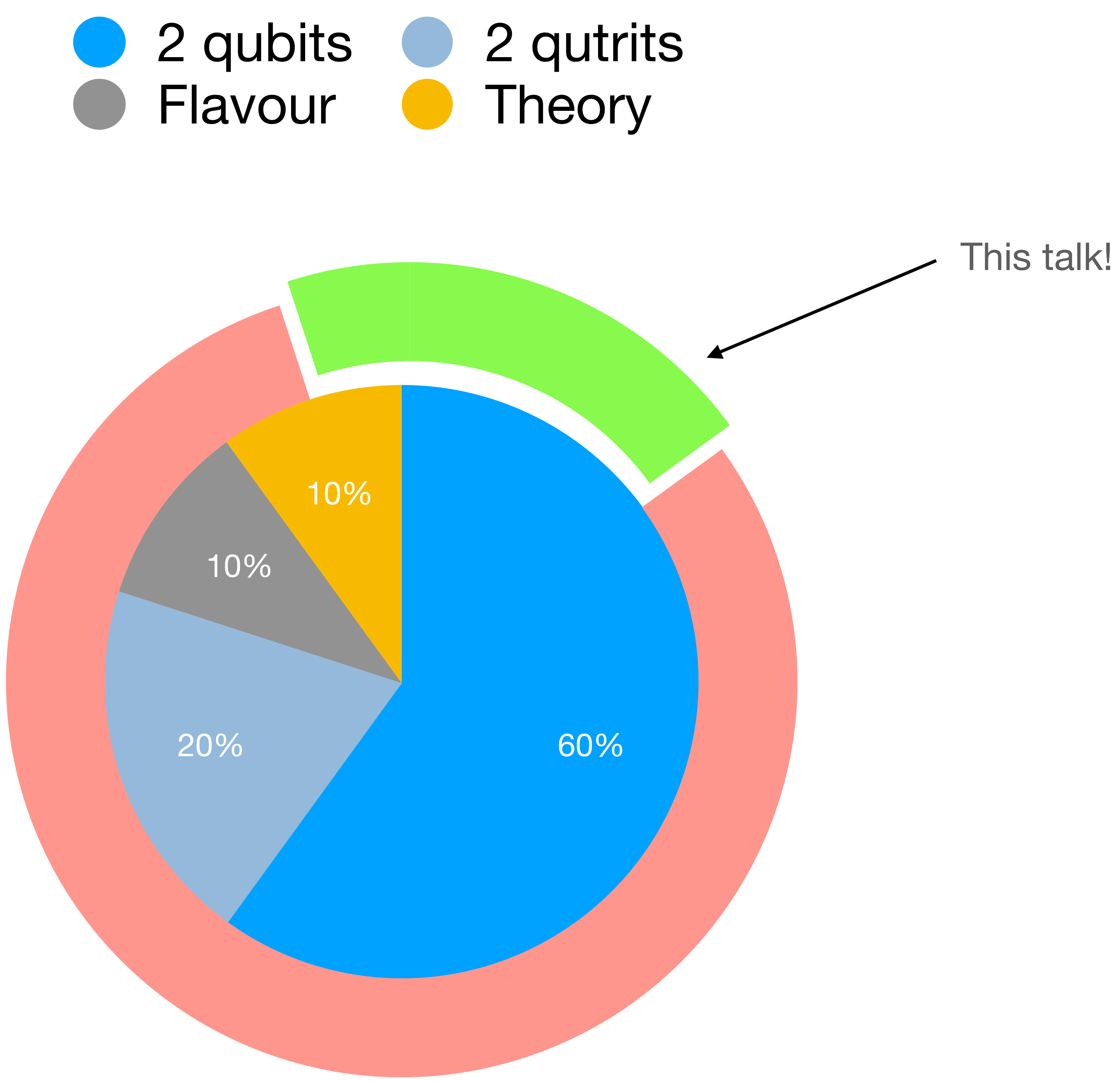
- **Community pride:** $\mathbb{QM} \subset \mathbb{QFT}$
- **Meaning:** A plethora of fundamental results (theorems!) for QI : what do they mean for HEP?
- **Impact:** And viceversa, what can be learnt on QI from particle physics?
- **Value:** Opportunity to elaborate (and communicate) what is important/interesting in our field:

New Physics ≡ Uncharted SM physics + BSM



Plan

- Crash course on QI tools
- $t\bar{t}$ production at the LHC
- Searching for New Physics
- Conclusions



For a nice review:

Quantum entanglement and Bell inequality violation at colliders

Alan J. Barr[a,b], Marco Fabbrichesi[c,*], Roberto Floreanini[c], Emidio Gabrielli[d,c,e], Luca Marzola[e]

https://arxiv.org/pdf/2402.07972.pdf

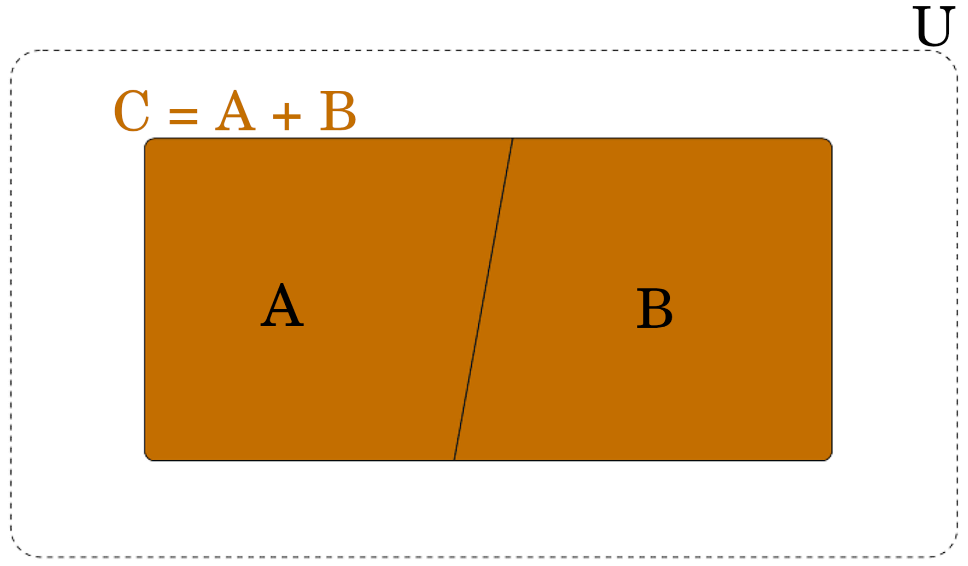
Basics

Density matrix : pure versus mixed

Schrödinger wave function (pure)
$\|\psi\rangle = \sum_n \alpha_n \|\phi_n\rangle$
$i\hbar \frac{d}{dt} \|\psi(t)\rangle = H \|\psi(t)\rangle$
$\langle A \rangle = \langle \psi \| A \| \psi \rangle$
$\langle \psi \| \psi \rangle = 1$
$\|\langle \phi \| \psi \rangle\|^2 \geq 0$

Basics
Density matrix : pure versus mixed

Schrödinger wave function (pure)	Pure	Generic (mixed)
$\lvert\psi\rangle = \sum_n \alpha_n \lvert\phi_n\rangle$	$\rho = \lvert\psi\rangle\langle\psi\rvert$	$\rho = \sum_j p_j \lvert\psi_j\rangle\langle\psi_j\rvert \quad (\sum_j p_j = 1, p_j \geq 0)$
$i\hbar \frac{d}{dt}\lvert\psi(t)\rangle = H\lvert\psi(t)\rangle$	$i\hbar \frac{d\rho}{dt} = [H, \rho]$	
$\langle A\rangle = \langle\psi\lvert A\rvert\psi\rangle$	$\langle A\rangle = \mathrm{Tr}[A\rho]$	
$\langle\psi\vert\psi\rangle = 1$	$\mathrm{Tr}[\rho] = 1$	
$\lvert\langle\phi\vert\psi\rangle\rvert^2 \geq 0$	$\mathrm{Tr}[\rho^2] = 1 \quad \rho = \rho^2$	$\mathrm{Tr}[\rho^2] < 1 \quad \rho \neq \rho^2$



Basics
Composite systems

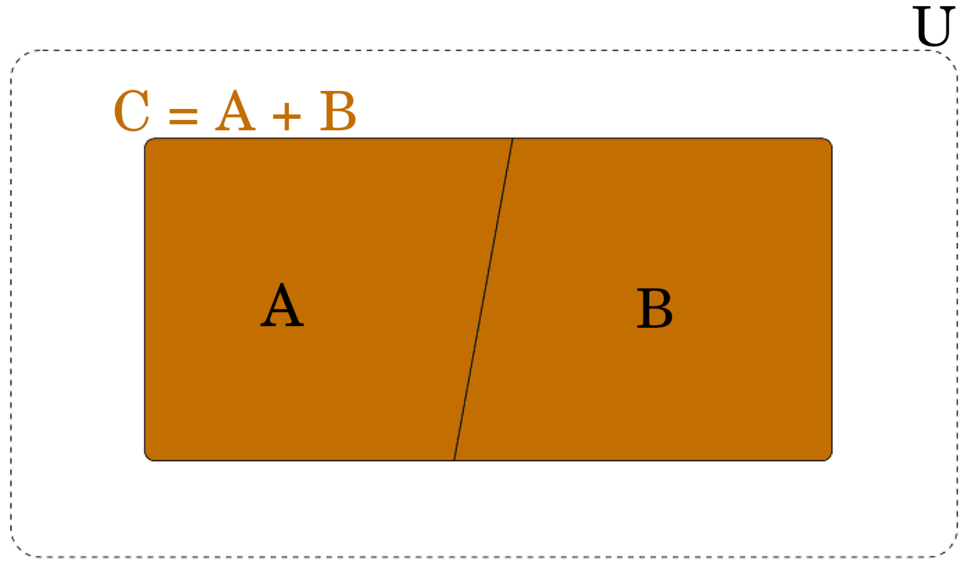
	Product pure state	Generic state
Pure	$|\psi\rangle = |a\rangle \otimes |b\rangle$	$|\psi\rangle = \sum_{ij} p_{ij} |a_i\rangle \otimes |b_j\rangle \quad p_{ij} \in \mathbb{C}, \sum_{ij} p_{ij} p_{ij}^* = 1$ $|a_i\rangle, |b_j\rangle$ orthonormal bases

	Separable	Non-separable
Mixed	$\rho = \sum_i p_i \, \rho_A^i \otimes \rho_B^i$! $p_i \geq 0, \sum_i p_i = 1$	$\rho = \sum_{ijkl} p_{ij} p_{kl}^* |a_i\rangle \otimes |b_j\rangle\langle a_k| \otimes \langle b_l| = \sum_{ijkl} p_{ij} p_{kl}^* |a_i\rangle\langle a_k| \otimes |b_j\rangle\langle b_l|$ $\rho_A = \mathrm{Tr}_B\left[\rho\right] = \sum_{ijl} p_{ij} p_{kj}^* |a_i\rangle\langle a_k|$ $\rho_B = \mathrm{Tr}_A\left[\rho\right] = \sum_{ijl} p_{ij} p_{il}^* |b_j\rangle\langle b_l|$

! Here one can use pure states.

The properties are different for pure and mixed states

Basics
Entanglement



Theorem:

If $|\psi\rangle$ is a **pure state** of the AB system, then two (orthonormal) bases (in A and B) exist such that

$$|\psi\rangle = \sum_i \sqrt{\lambda_i}\, |w_i\rangle_A \otimes |z_i\rangle_B \qquad \text{with} \qquad \sum_i \lambda_i = 1, \quad \lambda_i \geq 0$$

$$\rho_A = \sum_i \lambda_i |w_i\rangle_A\, {}_A\langle w_i|$$

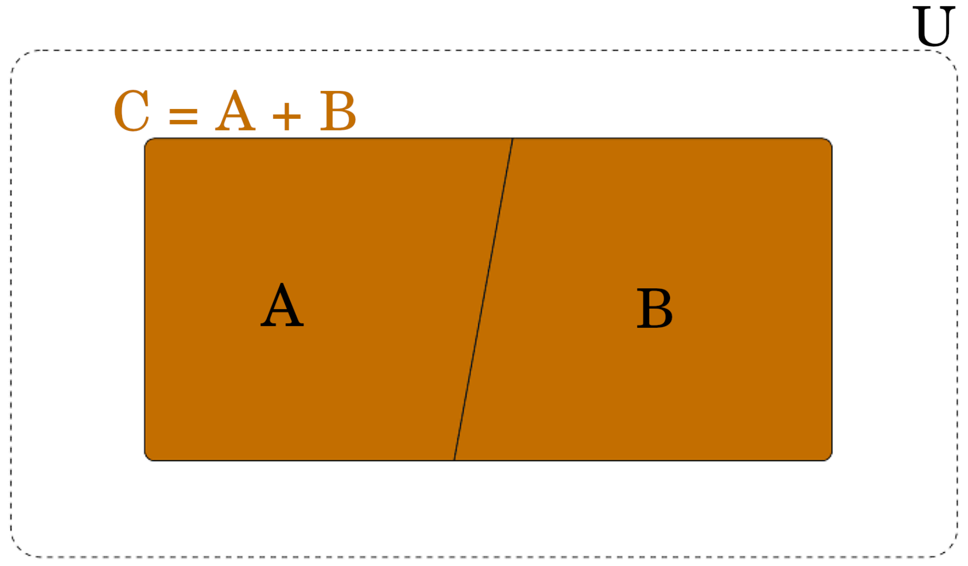
$$\rho_B = \sum_i \lambda_i |z_i\rangle_B\, {}_B\langle z_i|$$

$\Rightarrow$

- The states of the subsystems are mixed-states!
- They have the same eigenvalues => they are equally impure

Consequences:

1. One can always think of a mixed state as the trace out a subsystem in a large system (purification).
2. Two subsystems that partition a pure state are entangled IFF their reduced states are mixed.



Basics
Entanglement

Exercise 22: Consider the system C composed by two qubits A and B, to be in a maximally entangled pure Bell state:

$$|\Psi^-\rangle_C = \frac{|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B}{\sqrt{2}} .$$

Show that the two qubits are in maximally impure states.

Perhaps the most remarkable feature of quantum mechanics, a feature that clearly distinguishes it from classical physics, is this: for any composite system, there exist pure states of the system in which the parts of the system do not have pure states of their own. Such states are called entangled.

B. Wootters

Solution: We have:

$$\begin{aligned}\rho_C &= |\Psi^-\rangle\langle\Psi^-| = \frac{1}{2}\left(|01\rangle - |10\rangle\right)\left(\langle 01| - \langle 10|\right) \\ &= \frac{1}{2}\left(|01\rangle\langle 01| - |10\rangle\langle 01| - |01\rangle\langle 10| + |10\rangle\langle 10|\right) .\end{aligned}$$

We can now trace out one of the two qubits, for instance B:

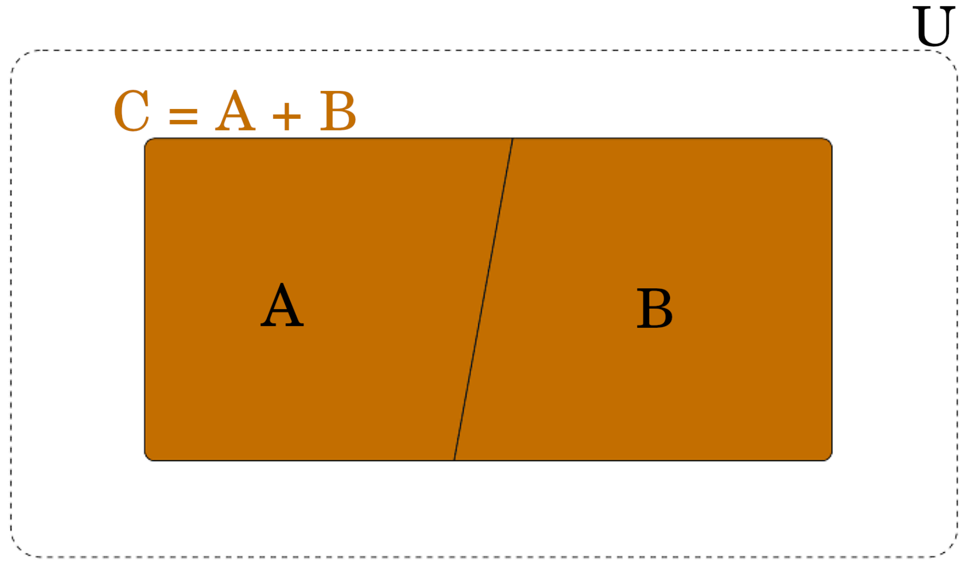
$$\begin{aligned}\rho_A = \mathrm{Tr}_B[\rho_C] &= \frac{1}{2}\mathrm{Tr}_B\left[|01\rangle\langle 01| - |10\rangle\langle 01| - |01\rangle\langle 10| + |10\rangle\langle 10|\right] \\ &= \frac{1}{2}\left(|0\rangle\langle 0|\langle 1|1\rangle - |1\rangle\langle 0|\langle 0|1\rangle - |0\rangle\langle 1|\langle 1|0\rangle + |1\rangle\langle 1|\langle 0|0\rangle\right) \\ &= \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right) \\ &= \frac{\mathbb{I}}{2} ,\end{aligned}$$

and the same is obtained if tracing over A

$$\rho_B = \mathrm{Tr}_A[\rho_C] = \frac{\mathbb{I}}{2} .$$

Since $\mathrm{Tr}_{A,B}[\rho^2_{A,B}] = 1/2$ the systems A and B are mixed, more precisely maximally mixed.

Basics
Concurrence



Take an entangled **pure** state between the two subsystems A and B. $\mathscr{H} = \mathscr{H}_A \otimes \mathscr{H}_B$

As a result, the states in A and B must be mixed and

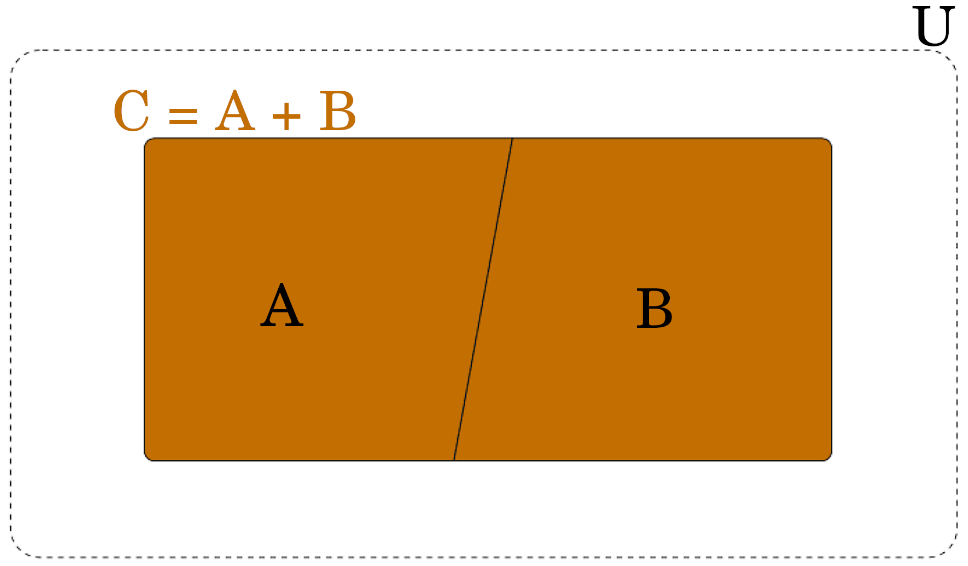
$\mathrm{Tr}\left[\rho_A^2\right] \leq 1$ and $\mathrm{Tr}\left[\rho_B^2\right] \leq 1$

The concurrence $C_{A|B}$ is defined as

$$0 \leq C_{A|B}^2 = 2(1 - \mathrm{Tr}[\rho_A^2]) = C_{B|A}^2 \leq 1$$

$$C_{A|B}^2 = 2S_2(\rho_A)$$ Tsallis-2 linear entropy

For mixed states, things are in general more complicated.



Basics
Peres-Horodecki criterium

This is a necessary (and for two qubits sufficient) criterium for separability of a mixed state of two subsystems A and B. Consider a generic state:

$$\rho = \sum_{ijkl} p_{ij} p^*_{kl} \, |a_i\rangle \otimes |b_j\rangle \langle a_k| \otimes \langle b_l| \qquad \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

And the partial transpose on B

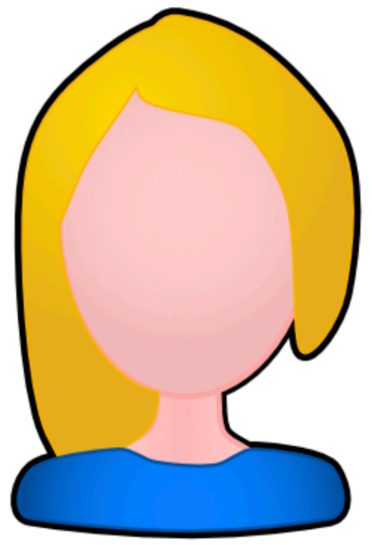
$$\rho^{T_B} = (I \otimes T)[\rho] = \sum_{ijkl} p_{ij} p^*_{kl} \, |a_i\rangle\langle a_k| \otimes (|b_j\rangle\langle b_l|)^T = \sum_{ijkl} p_{ij} p^*_{kl} \, |a_i\rangle\langle a_k| \otimes |b_l\rangle\langle b_j| = \sum_{ijkl} p_{il} p^*_{kj} \, |a_i\rangle\langle a_k| \otimes |b_j\rangle\langle b_l|$$

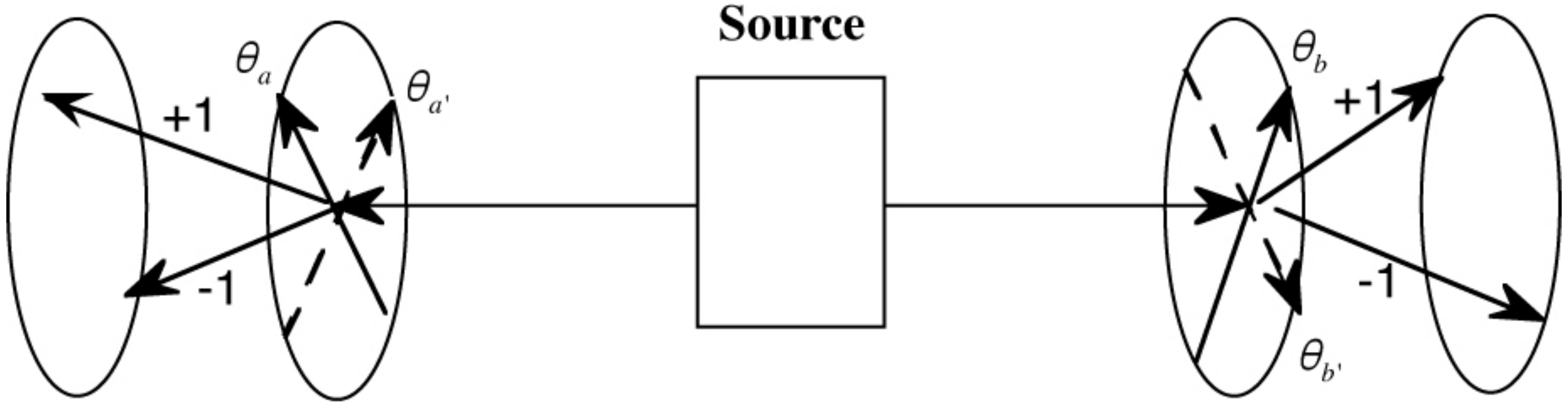
The criterion states that if ρ is separable then all the eigenvalues of ρ^{T_B} are non-negative.
In other words, if ρ^{T_B} has a negative eigenvalue, then the system is guaranteed to be entangled.

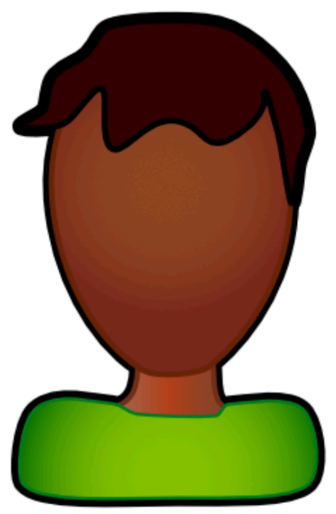
For 2 qubits or 1 qubit x 1 quitrit is a IFF

Basics

Bell (Clauser, Horne, Shimony, and Holt) inequalities







$$A = \pm 1$$
$$A' = \pm 1$$

Assuming:
1] Measurements reveal element of reality, physical properties present beforehand.
2] Alice and Bob are separated by a space-like distance

Then:

$$B = \pm 1$$
$$B' = \pm 1$$

$$E(AB) + E(AB') + E(A'B) - E(A'B') \leq 2$$

Basics
Bell (Clauser, Horne, Shimony, and Holt) inequalities

Proof:

$$E(AB) + E(AB') + E(A'B) - E(A'B')$$

$$= E(AB + A'B + A'B - A'B')$$

$$= E(A(B + B') + A'(B - B'))$$

Now $B + B' = 0 \Rightarrow B - B' = \pm 2$ and viceversa. So

$$AB + A'B + A'B - A'B' = \pm 2$$

So for the expectation value

$$E(AB) + E(AB') + E(A'B) - E(A'B') = \sum_{a,a',b,b'} p(a, a', b, b')(ab + ab' + a'b - a'b') \leq 2$$

Comments:

i) Bell inequalities have nothing to do with quantum mechanics.

ii) In the last 40 years experiments have proven them over larger and larger distances.

iii) Discussions on possible loopholes have heated and continue to keep the community of experts busy.

Basics

Bell (CHSH) inequalities : QM

$$|\Phi^{\pm}\rangle = \frac{|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle}{\sqrt{2}}$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle)$$

$$A(a) = \hat{S}_x \otimes I$$

$$A(a') = \hat{S}_z \otimes I$$

$$B(b) = -\frac{1}{\sqrt{2}} I \otimes (\hat{S}_z + \hat{S}_x)$$

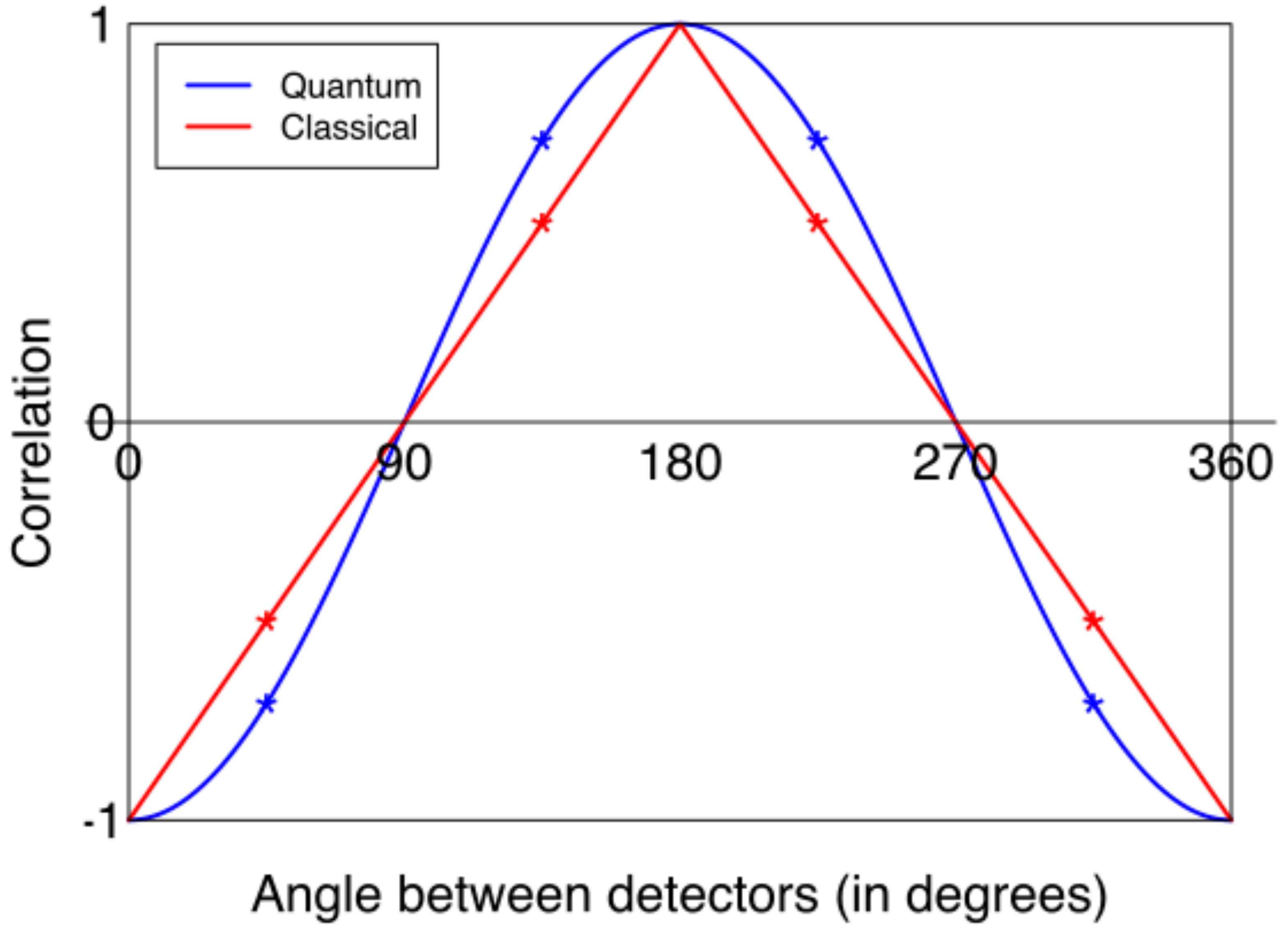
$$B(b') = \frac{1}{\sqrt{2}} I \otimes (\hat{S}_z - \hat{S}_x)$$

$$E(AB) = \langle A(a)B(b) > = \frac{1}{\sqrt{2}}$$

$$E(AB') = \langle A(a)B(b') > = \frac{1}{\sqrt{2}}$$

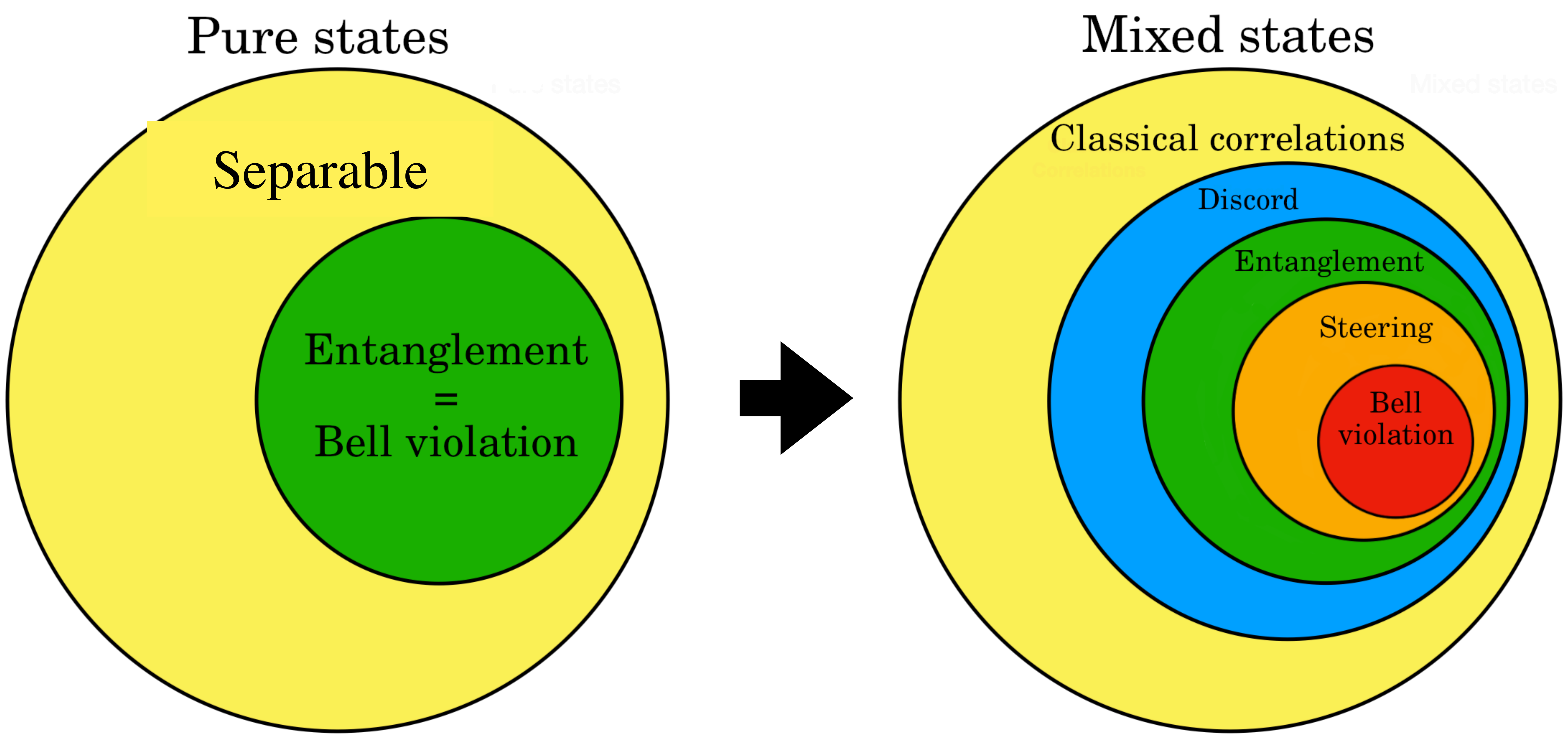
$$E(A'B) = \langle A(a')B(b) > = \frac{1}{\sqrt{2}}$$

$$E(A'B') = \langle A(a')B(b') > = -\frac{1}{\sqrt{2}}$$



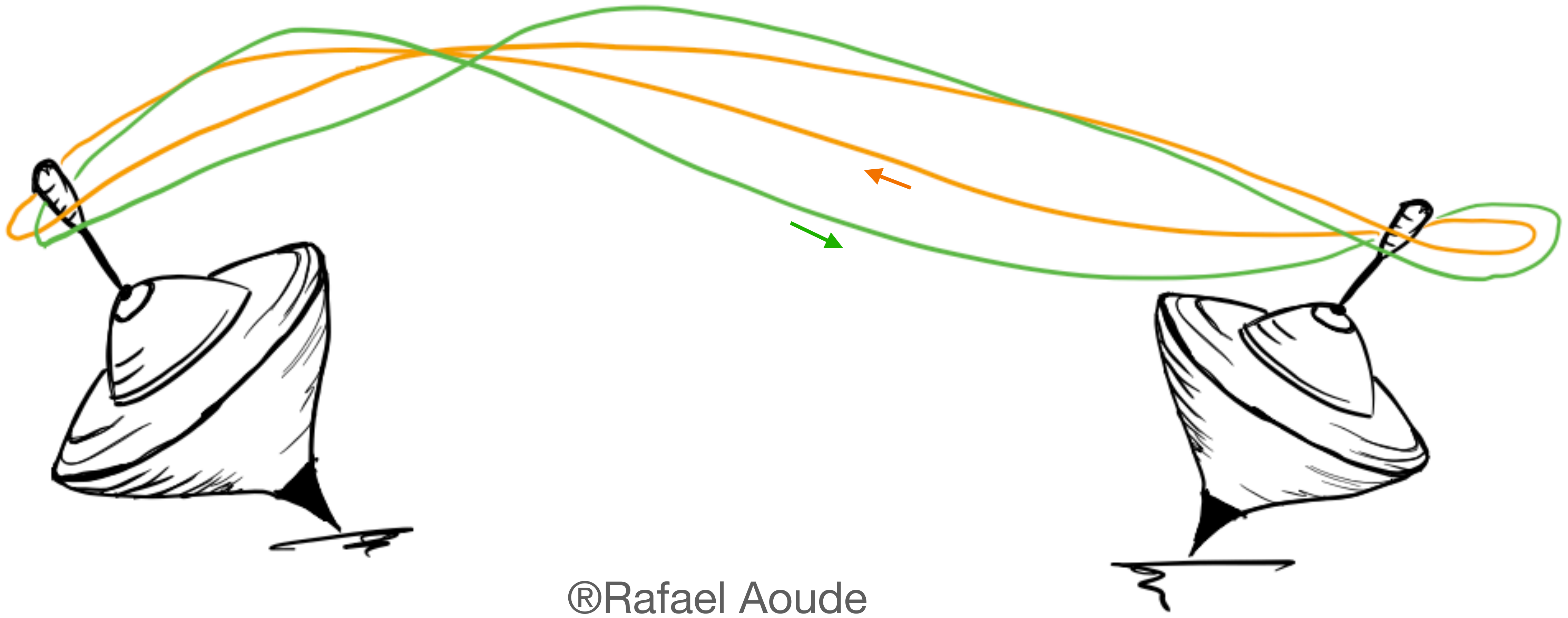
$$E(AB) + E(AB') + E(A'B) - E(A'B') = 2\sqrt{2}$$

Basics
Grading quantum correlations



Remember: Entanglement is a feature of QM. It does not make any sense of talking about entanglement OUTSIDE QM. On the other hand, Bell inequalities are general, independent on QM (which in fact violate them!). The plots above refer to QM.

Quantum tops @ LHC



®Rafael Aoude

Y. Afik and JRM de Nova: 2003.02280 [quant-ph]

M. Fabbrichesi, R. Floreanini. G. Panizzo: 2102.11883 [hep-ph]

C. Severi, C. Boschi, **FM**, M. Sioli : 2110.10112 [hep-ph]

Y. Afik and JRM de Nova: 2203.05582 [quant-ph]

R. Aoude, E. Madge, **FM**, L. Mantani: 2203.05619 [hep-ph]

J.A. Aguilar-Saavedra, J.A. Casas: 2205.00542 [hep-ph]

Y. Afik and JRM de Nova: 2209.03969 [quant-ph]

C. Severi, E. Vryonidou: 2210.09330 [hep-ph]

Z. Dong, D. Gonçalves, K. Kong, A. Navarro: 2305.07075 [hep-ph]

J.A. Aguilar-Saavedra : 2307.06991 [hep-ph]

T. Han, M. Low, TA Wu: 2310.17696 [hep-ph]

J.A. Aguilar-Saavedra, J.A. Casas: 2401.06854 [hep-ph]

C. Severi, **FM**, S. Tentori, E. Vryonidou: 2401.08751[hep-ph]

C. Severi, **FM**, S. Tentori, E. Vryonidou: 2404.08049[hep-ph]

K. Cheng,T. Han, M. Low: 2407.01672[hep-ph]

Many other papers on (H→)WW,ZZ,ZW, $\tau^+\tau^-$, tW,…

Why looking at tops?

- **LHC:** a top factory.
- **Top decay:** The decay occurs in two steps, t→Wb is the first one:

$\tau_{had} \approx h/\Lambda_{QCD} \approx 2 \cdot 10^{-24}$ s

$\tau_{top} \approx h/\,\Gamma_{top} = 1/(G_F\, m_t^3\, |V_{tb}|2/8\pi\sqrt{2}) \approx 5 \cdot 10^{-25}$ s (with h=6.6 10^{-25} GeV s)

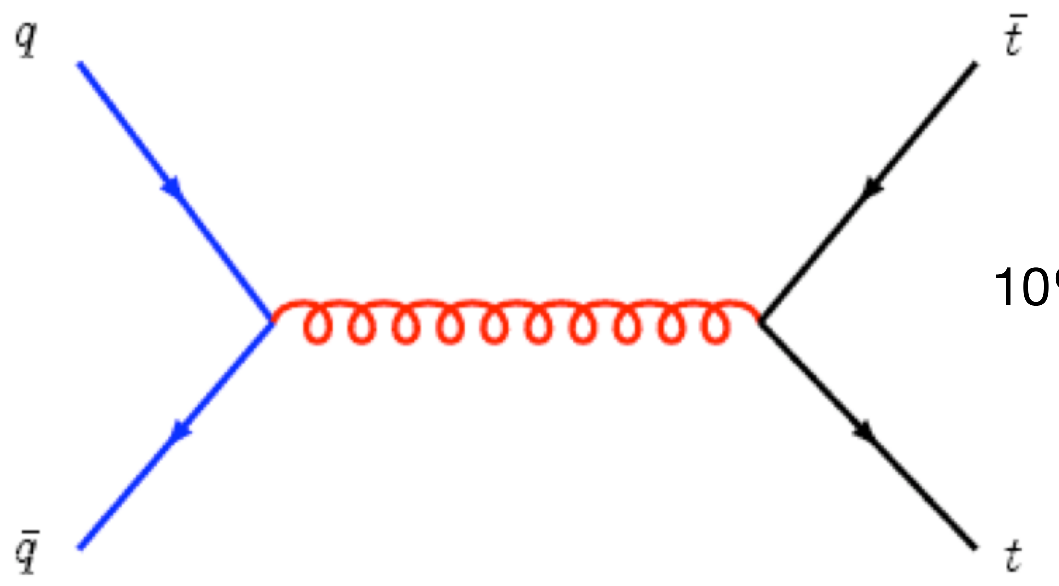
$$\tau_{spin\text{-}flip} \approx \left(\frac{\Lambda^2_{QCD}}{m_t}\right)^{-1} \gg \tau_{had}$$

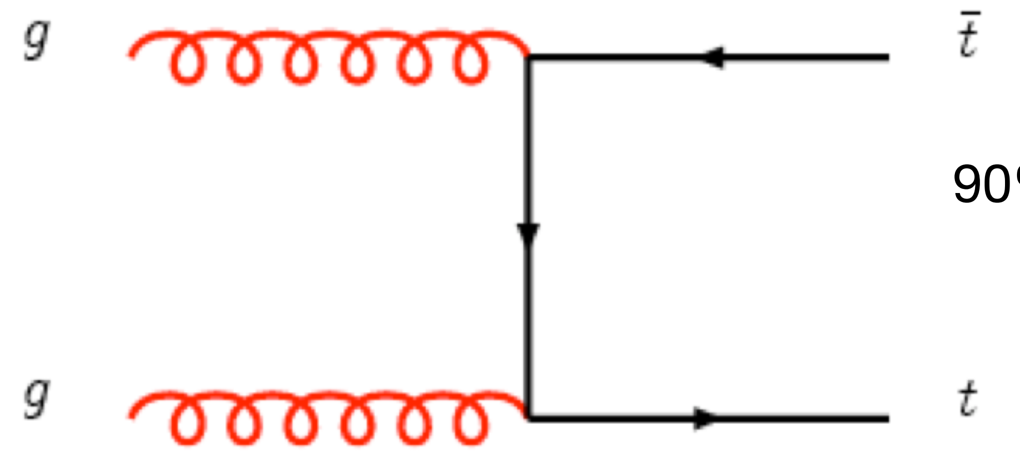
- Due to the structure of weak interactions, it “magically” turns out that the direction of the lepton is 100% correlated with that of the spin of the top.

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\varphi} = \frac{1+\alpha\cos\varphi}{2}$$

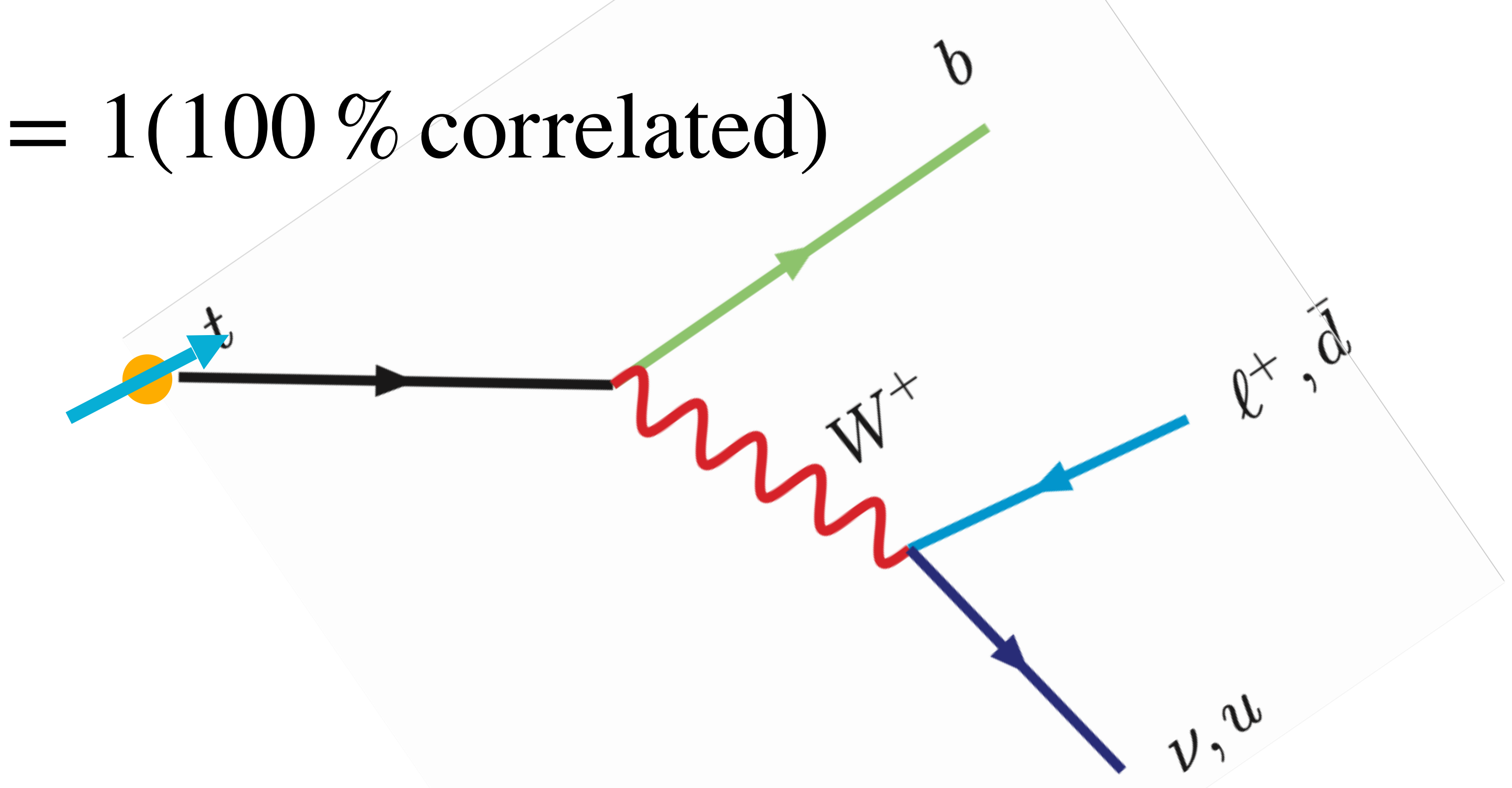
$$\alpha_d = 1, \alpha_u = -0.3, \alpha_b = -0.4, \alpha_W = 0.4$$

⇒ **The charged lepton is the best proxy for the spin**

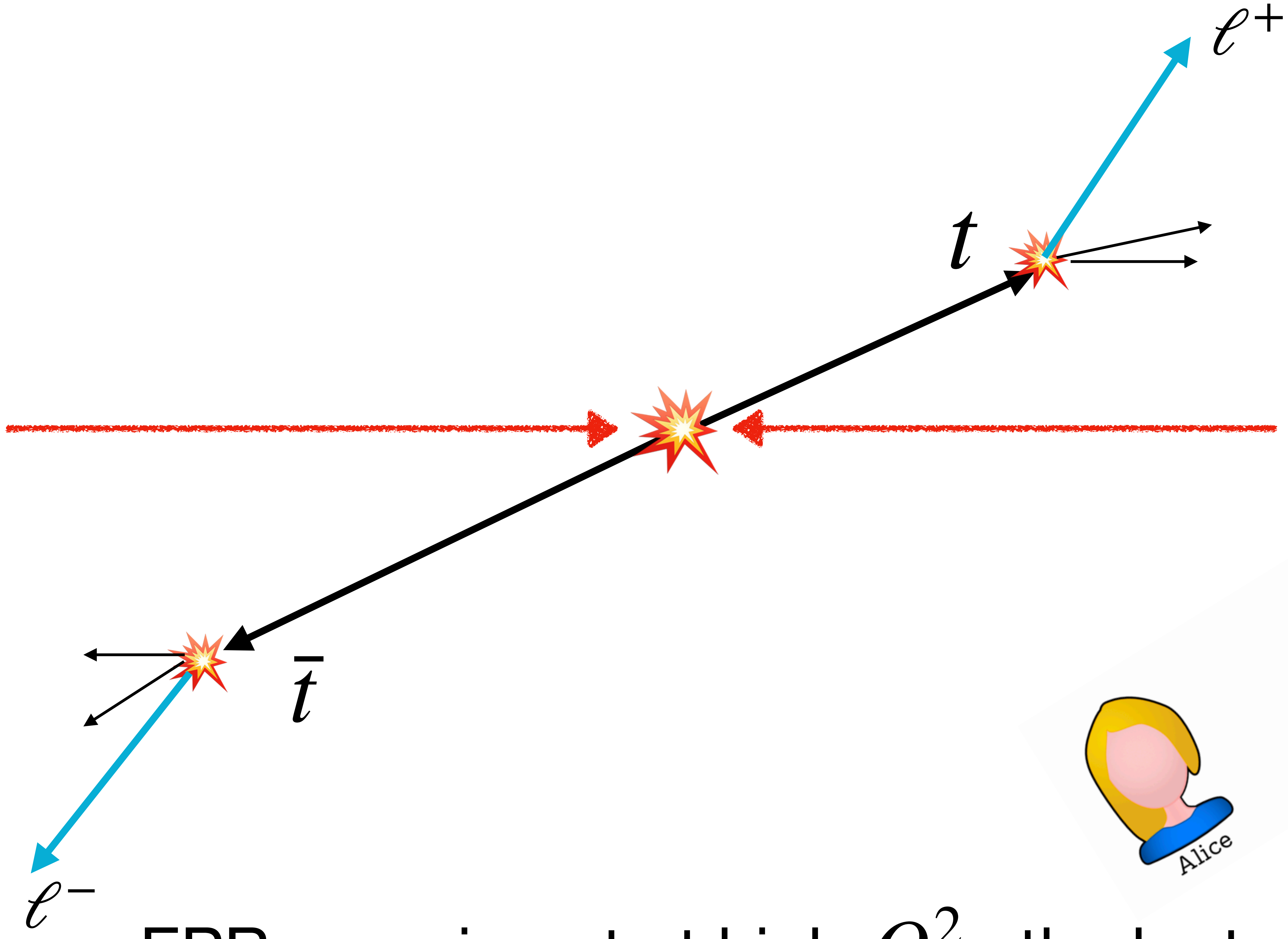


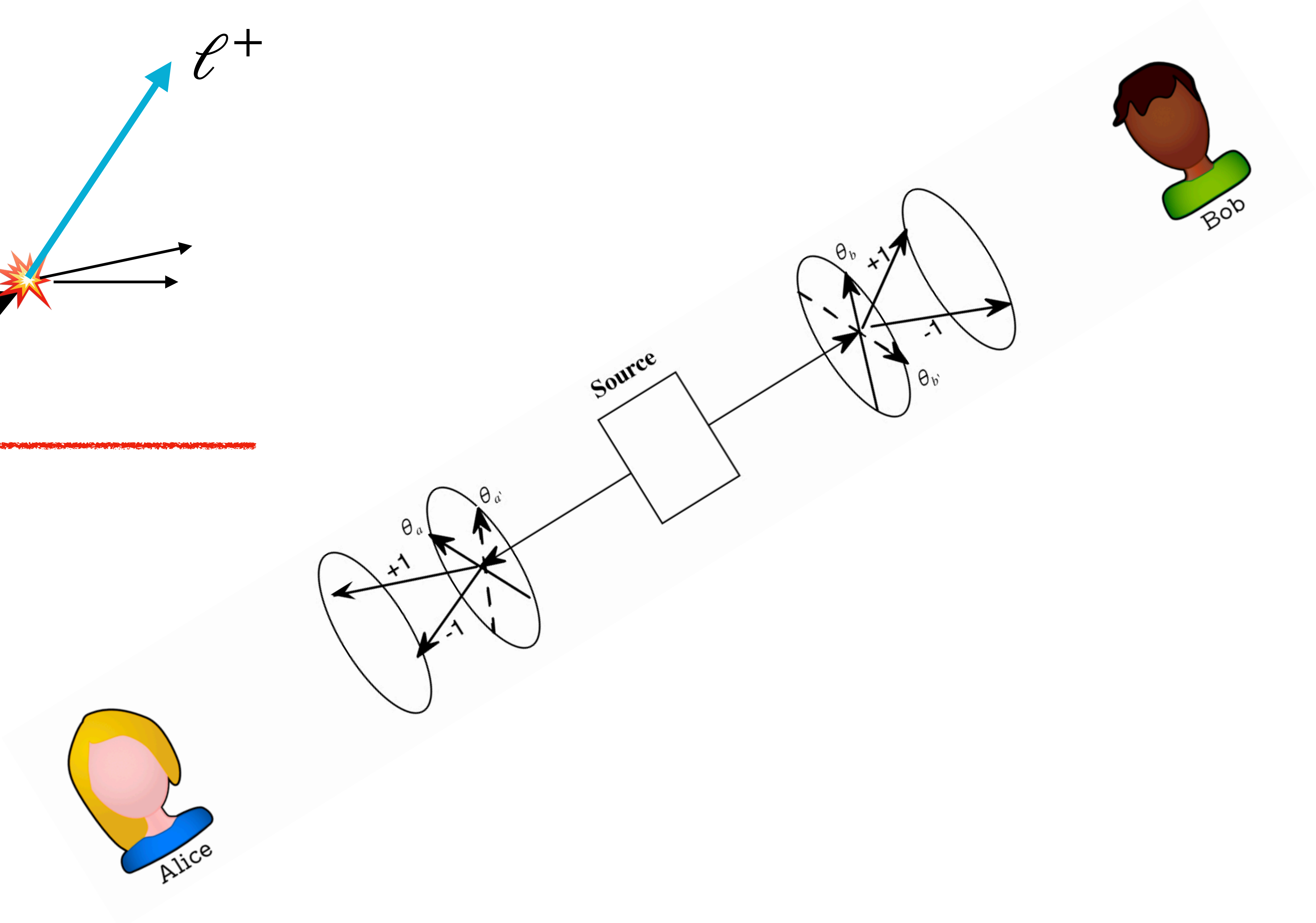


$\alpha_\ell = 1(100\,\%\,\text{correlated})$



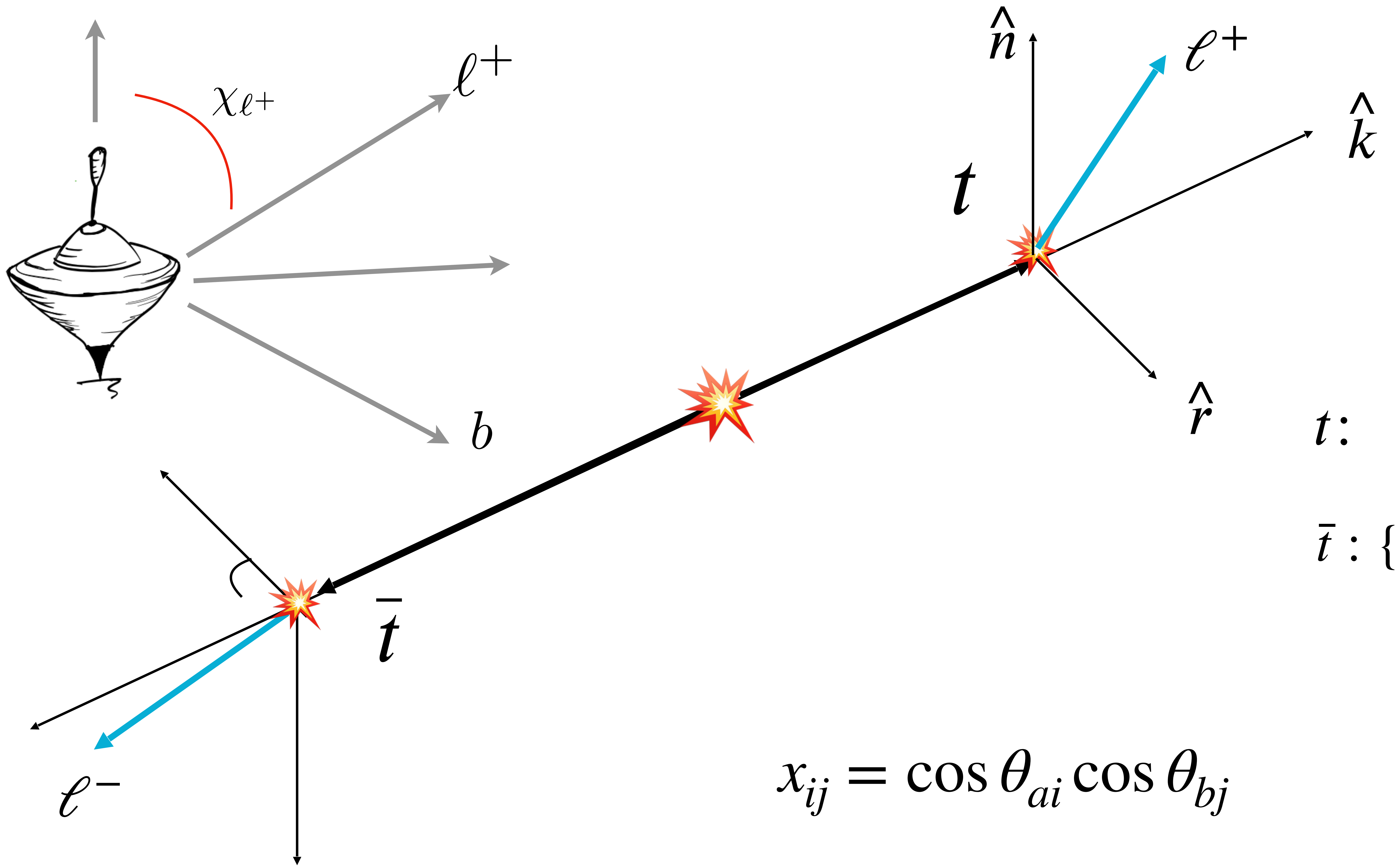
Quantum tops





EPR experiment at high-Q^2 : the lepton “measures” the spin.

$t\bar{t}$ tomography



$t:\quad \hat{k} = \text{top direction}, \quad \hat{r} = \frac{\hat{p} - \hat{k}\cos\theta}{\sin\theta}, \quad \hat{n} = \frac{\hat{p}\times\hat{k}}{\sin\theta}$

$\bar{t} : \{-\hat{k}, -\hat{r}, -\hat{n}\}$

$$x_{ij} = \cos\theta_{ai}\cos\theta_{bj}$$

$$\frac{1}{\sigma}\frac{d\sigma}{dx_{ij}} = \frac{C_{ij}x_{ij} - 1}{2}\log|x_{ij}|$$

Quantum tops

"The devil is in the details" : $t\bar{t}$ pair is not in a pure state.

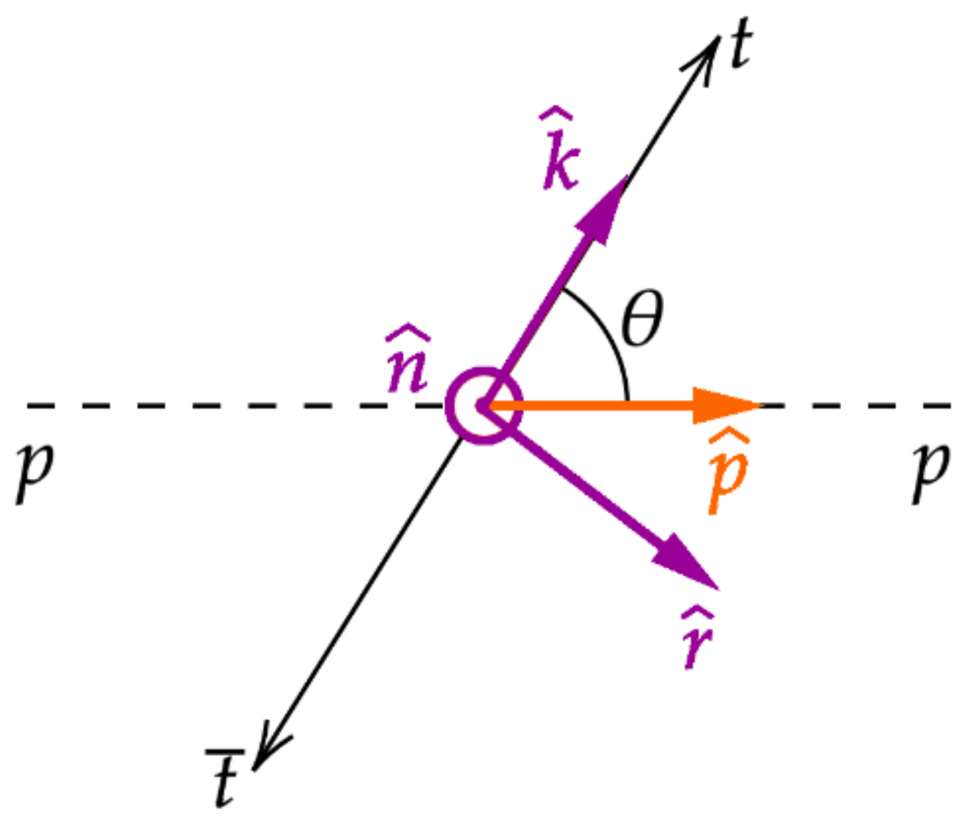
- The qubit-qubit system is described by the following density matrix

$$\rho = \frac{1}{4}\left(\mathbf{1}\otimes\mathbf{1} + \mathcal{B}\_ \cdot \boldsymbol{\sigma}\otimes\mathbf{1} + \bar{\mathcal{B}}\_ \cdot \mathbf{1}\otimes\boldsymbol{\sigma} + \mathcal{C}\cdot\boldsymbol{\sigma}\otimes\boldsymbol{\sigma}\right)$$

which, for $t\bar{t}$ can be approximated by $B_1 = B_2 = 0$, and C is symmetric (CP conservation) and almost diagonal in the helicity basis.

$$\langle S_i\rangle = B_i, \quad \langle \bar{S}_i\rangle = \bar{B}_j, \quad \langle S_i\bar{S}_j\rangle = C_{ij}$$

Quantum tops



$$\hat{k} = \text{top direction}, \quad \hat{r} = \frac{\hat{p} - \hat{k}\cos\theta}{\sin\theta}, \quad \hat{n} = \frac{\hat{p}\times\hat{k}}{\sin\theta}$$

We can write four sufficient conditions for entanglement:

$$-C_{kk} - C_{rr} - C_{nn} \equiv -3D^{(1)} > 1$$
$$-C_{kk} + C_{rr} + C_{nn} \equiv -3\,D^{(k)} > 1$$
$$+C_{kk} + C_{rr} - C_{nn} \equiv -3\,D^{(n)} > 1$$
$$+C_{kk} - C_{rr} + C_{nn} \equiv -3\,D^{(r)} > 1$$

$$D < -1/3 \qquad \Rightarrow \text{Entanglement}$$

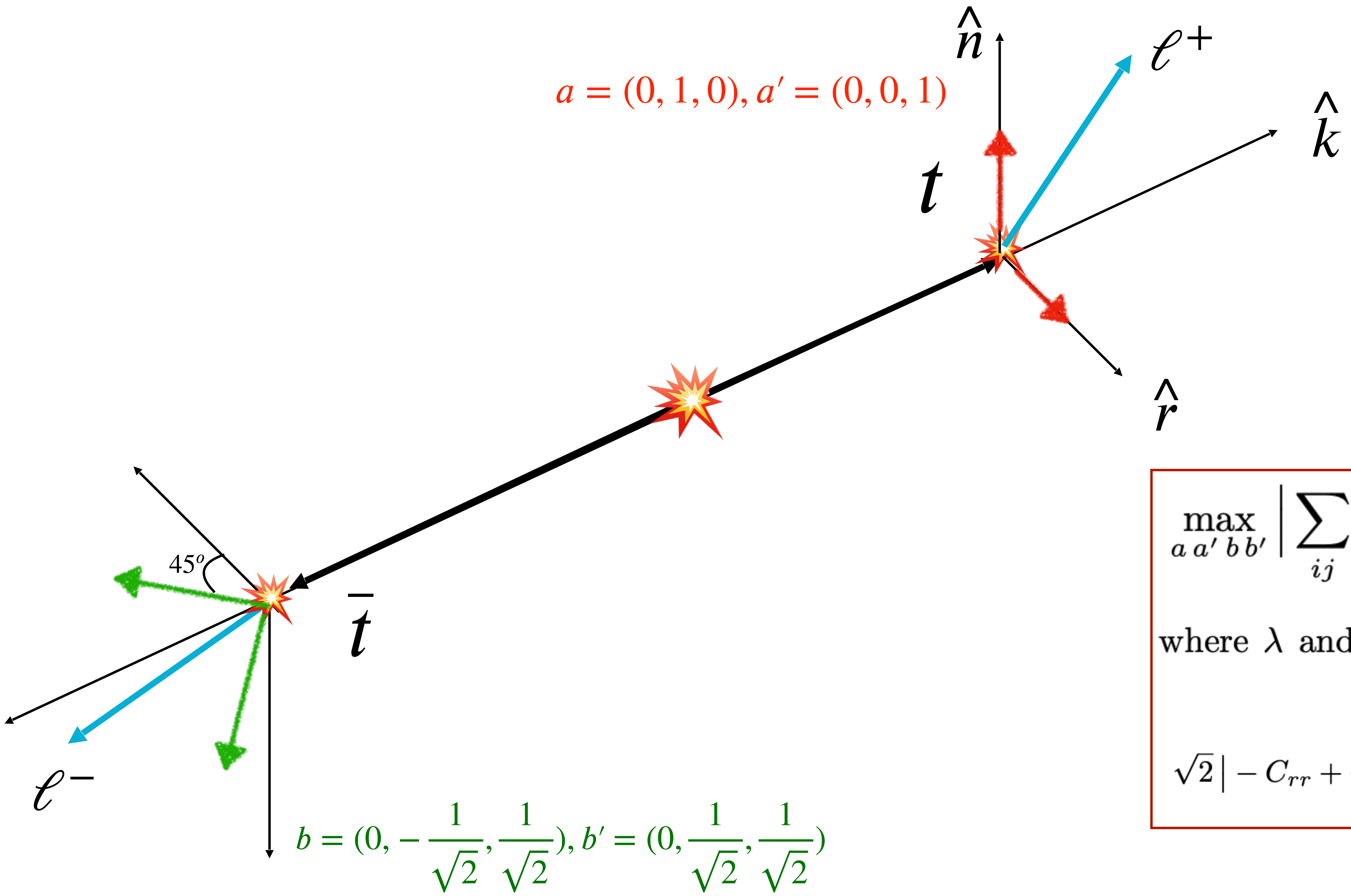
$$\mathcal{C}^{(\text{singlet})} = \begin{pmatrix} -\eta & 0 & 0 \\ 0 & -\eta & 0 \\ 0 & 0 & -\eta \end{pmatrix}, \; 0 < \eta \leq 1$$

$$\mathcal{C}^{(\text{triplet})} = \begin{pmatrix} \eta & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & -\eta \end{pmatrix}, \qquad 0 < \eta \leq 1$$

Where D=-η and in the limiting case of $\eta = 1$ we have the four Bell states:

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle\right),$$
$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle\right).$$

Kinematics



$$\max_{a\,a'\,b\,b'} \left| \sum_{ij} C_{ij}\,(a_i b_j - a_i b'_j + a'_i b_j + a'_i b'_j) \right| = 2\sqrt{\lambda + \lambda'}$$

where λ and λ' are the two largest eigenvalues of $C^T C$.

$\sqrt{2}\,|-C_{rr} + C_{nn}| \leq 2$ $\Rightarrow$ Bell condition (with axis choice)

Proof

6.4 Bell violations in mixed states

To complete our discussion relating the formal results we reached in the previous Sections to phenomenology, we briefly illustrate how to detect Bell violations in mixed states. We focus on the case of two qubits, which is the simplest algebraically, and also very relevant for HEP applications. Consider again the two-qubit density matrix:

$$\rho = \frac{1}{4}\mathbb{I}_2 \otimes \mathbb{I}_2 + \frac{1}{4}\sum_{i,j=1}^{3} C_{ij}\,(\sigma_i \otimes \sigma_j), \tag{6.33}$$

corresponding to a pair of spin $1/2$ particles with unpolarized spins, $\langle S \rangle = B = 0$, but non-zero spin correlations. The four observables entering the Bell operator (4.9) correspond to four directions upon which spin can be measured:

$$Q \equiv \hat{a},\;\; R \equiv \hat{a}',\;\; S \equiv \hat{b},\;\; T \equiv \hat{b}'. \tag{6.34}$$

To be consistent with the discussion in Section 4, we normalize the outcome of such spin measurements so they take values between -1 and 1. With this notation, the CHSH inequality reads:

$$\langle \mathcal{B} \rangle = \langle \hat{a}\hat{b} + \hat{a}'\hat{b} + \hat{a}\hat{b}' - \hat{a}'\hat{b}' \rangle \leq 2. \tag{6.35}$$

The expectation value of $\mathcal{B}$ on the state (6.33) is simply:

$$\langle \mathcal{B} \rangle = \mathrm{Tr}[\rho\,\mathcal{B}] = \sum_{ij} C_{ij}\,(\hat{a}_i\hat{b}_j + \hat{a}'_i\hat{b}_j + \hat{a}_i\hat{b}'_j - \hat{a}'_i\hat{b}'_j). \tag{6.36}$$

We now look for the **optimal directions**, that is, the choice of $\hat{a}, \hat{a}', \hat{b}, \hat{b}'$ such that $\langle \mathcal{B} \rangle$ is maximized. It is useful to introduce the auxiliary unit vectors:

$$\hat{d} = \frac{\hat{b} - \hat{b}'}{2\cos\varphi}, \qquad \hat{d}' = \frac{\hat{b} + \hat{b}'}{2\sin\varphi}, \tag{6.37}$$

where $\cos^2\varphi = 1/2(1 - \hat{b}\cdot\hat{b}')$, so that we can rewrite:

$$\langle \mathcal{B} \rangle = 2(\hat{a}\cdot C \cdot \hat{d}\,\cos\varphi + \hat{a}'\cdot C\cdot \hat{d}'\,\sin\varphi). \tag{6.38}$$

We now maximize this expression with respect to all the variables. First for φ:

$$\max_{\varphi}\langle \mathcal{B} \rangle = 2\sqrt{(\hat{a}\cdot C\cdot \hat{d})^2 + (\hat{a}'\cdot C\cdot \hat{d}')^2}. \tag{6.39}$$

To maximize with respect to $\hat{a}$ and $\hat{a}'$, notice $\hat{a}\cdot C\cdot \hat{d} = |C\cdot \hat{d}|\,\cos\theta$, where θ is the angle between $\hat{a}$ and $C\cdot\hat{d}$. To maximize (6.39), one then simply takes $\hat{a}$ aligned with $C\cdot\hat{d}$, and similarly $\hat{a}'$ aligned with $C\cdot\hat{d}'$:

$$\max_{\varphi,\hat{a},\hat{a}'}\langle \mathcal{B} \rangle = 2\sqrt{|C\cdot\hat{d}|^2 + |C\cdot\hat{d}'|^2}. \tag{6.40}$$

To finally maximize over $\hat{b}$ and $\hat{b}'$ (or, in the new variables, over $\hat{d}$ and $\hat{d}'$), consider the identity:

$$|C\cdot\hat{d}|^2 = \hat{d}^{\mathsf{T}}C^{\mathsf{T}}C\hat{d}. \tag{6.41}$$

Since $C^{\mathsf{T}}C$ is symmetric, it can be diagonalized preserving dot products:

$$C^{\mathsf{T}}C = \begin{pmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0\\ 0 & 0 & \lambda_3\end{pmatrix}, \qquad \lambda_1 > \lambda_2 > \lambda_3 \geq 0. \tag{6.42}$$

The maximum of (6.40), then, is obviously when $\hat{d}$ and $\hat{d}'$ are aligned with the two directions corresponding to the largest eigenvalues λ_1 and λ_2:

$$\max_{\varphi,\hat{a},\hat{a}',\hat{b},\hat{b}'}\langle \mathcal{B} \rangle = 2\sqrt{\lambda_1 + \lambda_2}. \tag{6.43}$$

This simple criterion relates a Bell violation to the **eigenvalues of** $C^{\mathsf{T}}C$. In particular, the Bell inequality is simply:

$$\lambda_1 + \lambda_2 < 1. \tag{6.44}$$

Tracing back our argument, we have also obtained that the optimal directions are:

$$\hat{a} = \hat{d}, \tag{6.45}$$

$$\hat{a}' = \hat{d}', \tag{6.46}$$

$$\hat{b} = \hat{d}\cos\varphi + \hat{d}'\sin\varphi \tag{6.47}$$

$$\hat{b}' = -\hat{d}\cos\varphi + \hat{d}'\sin\varphi, \tag{6.48}$$

where $\tan\varphi = \sqrt{\mu_1/\mu_2}$, and $\hat{d}$ and $\hat{d}'$ are the eigenvectors of $C^{\mathsf{T}}C$ corresponding to λ_1 and λ_2.

Summary

$$\hat{k} = \text{top direction}, \quad \hat{r} = \frac{\hat{p} - \hat{k}\cos\theta}{\sin\theta}, \quad \hat{n} = \frac{\hat{p}\times\hat{k}}{\sin\theta}$$

We can write four sufficient conditions for entanglement:

$$-C_{kk} - C_{rr} - C_{nn} \equiv -3D^{(1)} > 1$$

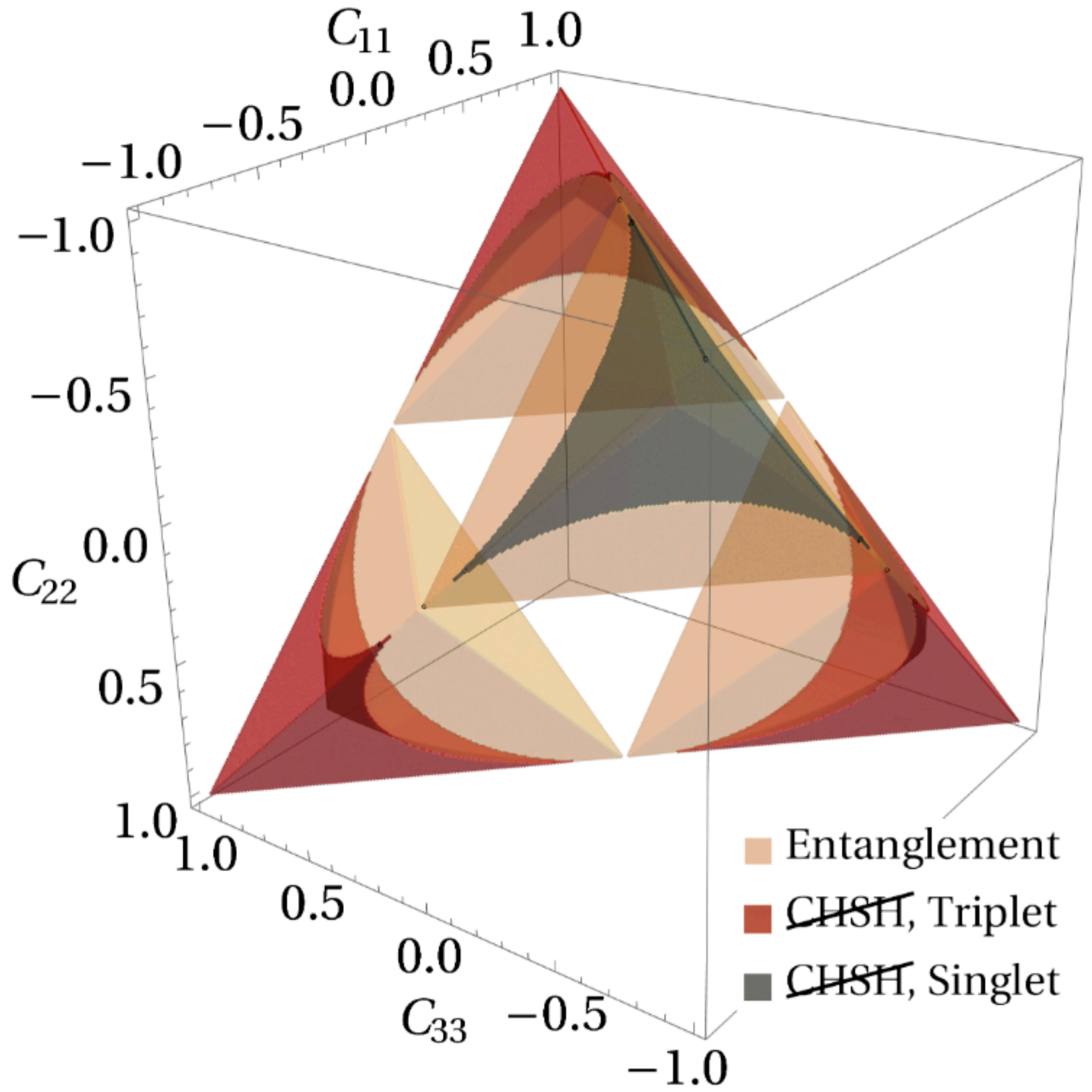
$$-C_{kk} + C_{rr} + C_{nn} \equiv -3\,D^{(k)} > 1$$

$$+C_{kk} + C_{rr} - C_{nn} \equiv -3\,D^{(n)} > 1$$

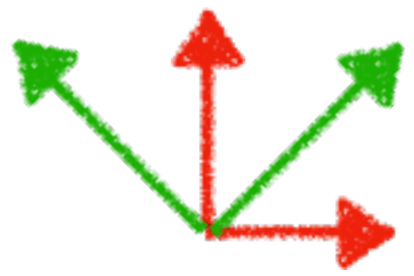
$$+C_{kk} - C_{rr} + C_{nn} \equiv -3\,D^{(r)} > 1$$

$D < -1/3 \quad \Rightarrow$ Entanglement

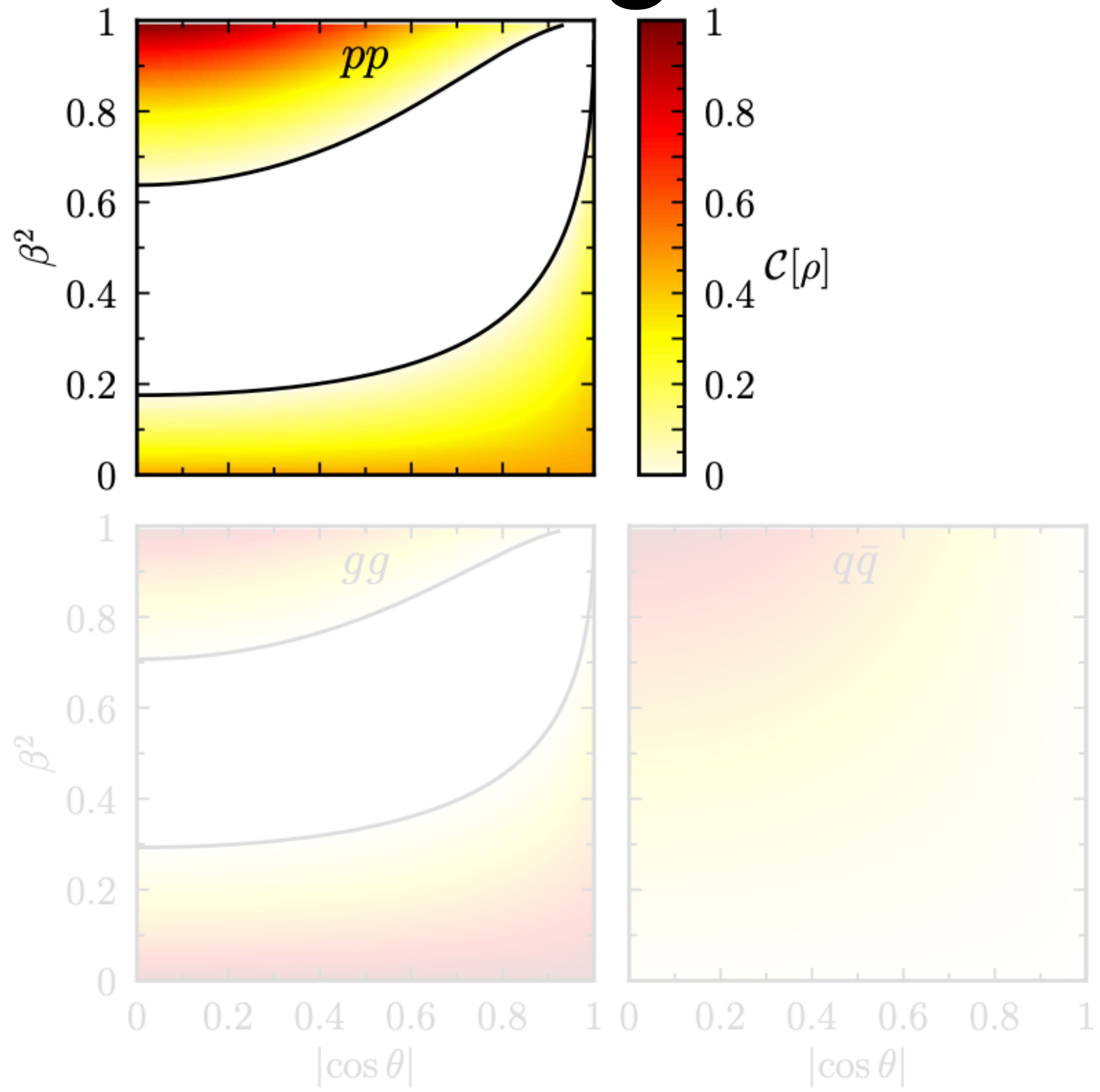
$\sqrt{2}\,|-C_{rr} + C_{nn}| \leq 2, \quad \Rightarrow$ Bell violation (with axis choice)



The specific case where $C_{xx} = C_{yy} = C_{zz} = -\eta$ with $0 < \eta < 1$ corresponds to the singlet Werner state, while $C_{xx} = C_{yy} = -C_{zz} = \eta$ and cyclic permutations correspond to a triplet of Werner states (with fidelity $F = \frac{3\eta+1}{4}$). It is known that for Werner states, $\eta > 1/3$ implies entanglement, while the CHSH inequality is violated when $\eta > \sqrt{2}/2$.[1]



SM Entanglement



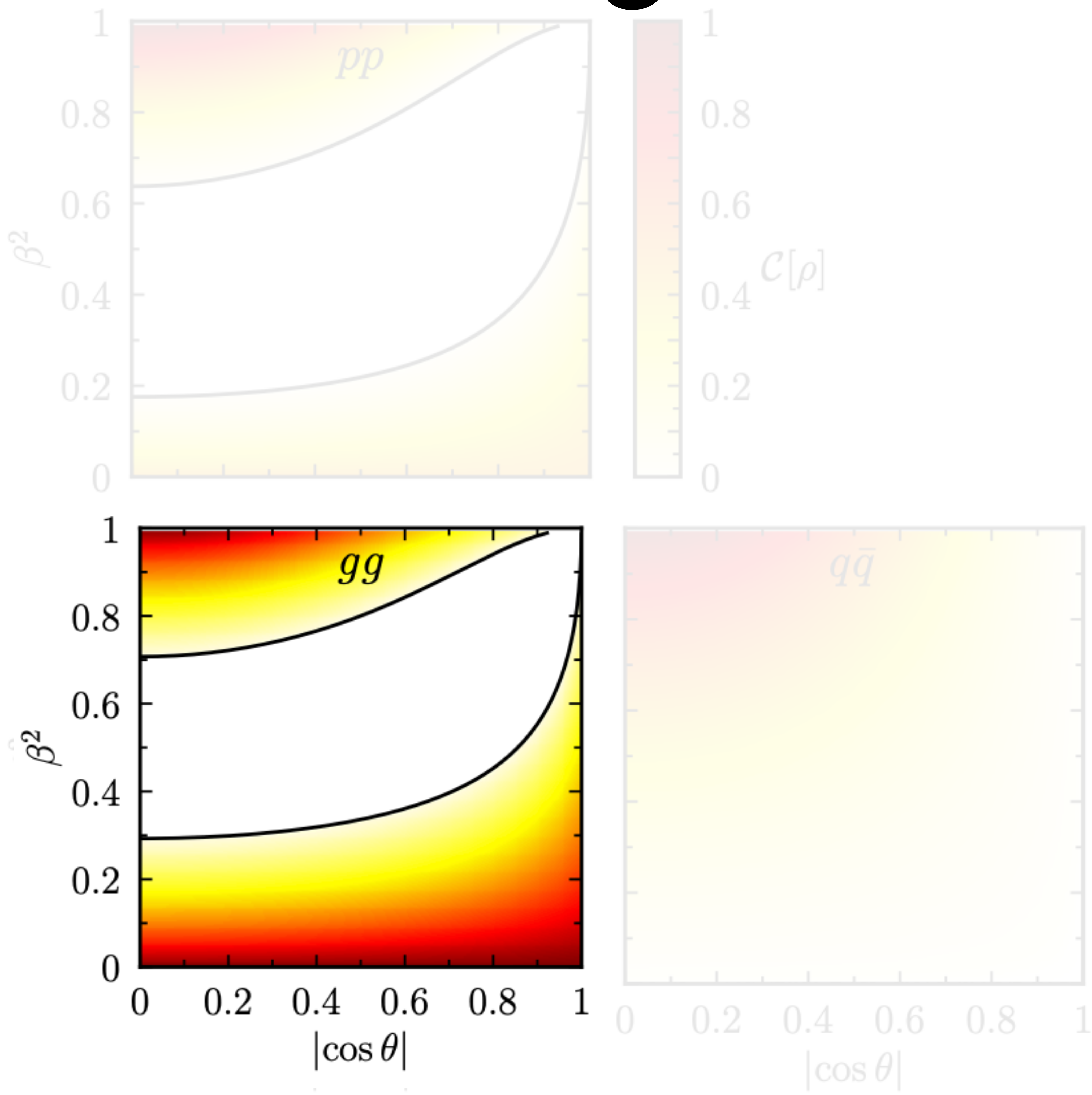
pp

White regions: zero-entanglement

Maximal entanglement points/regions:

- At threshold: $\beta^2 = 0, \forall\theta$
- high-E: $\beta^2 \to 1, \cos\theta = 0$

SM Entanglement



$$gg$$

Maximal entanglement points/regions:

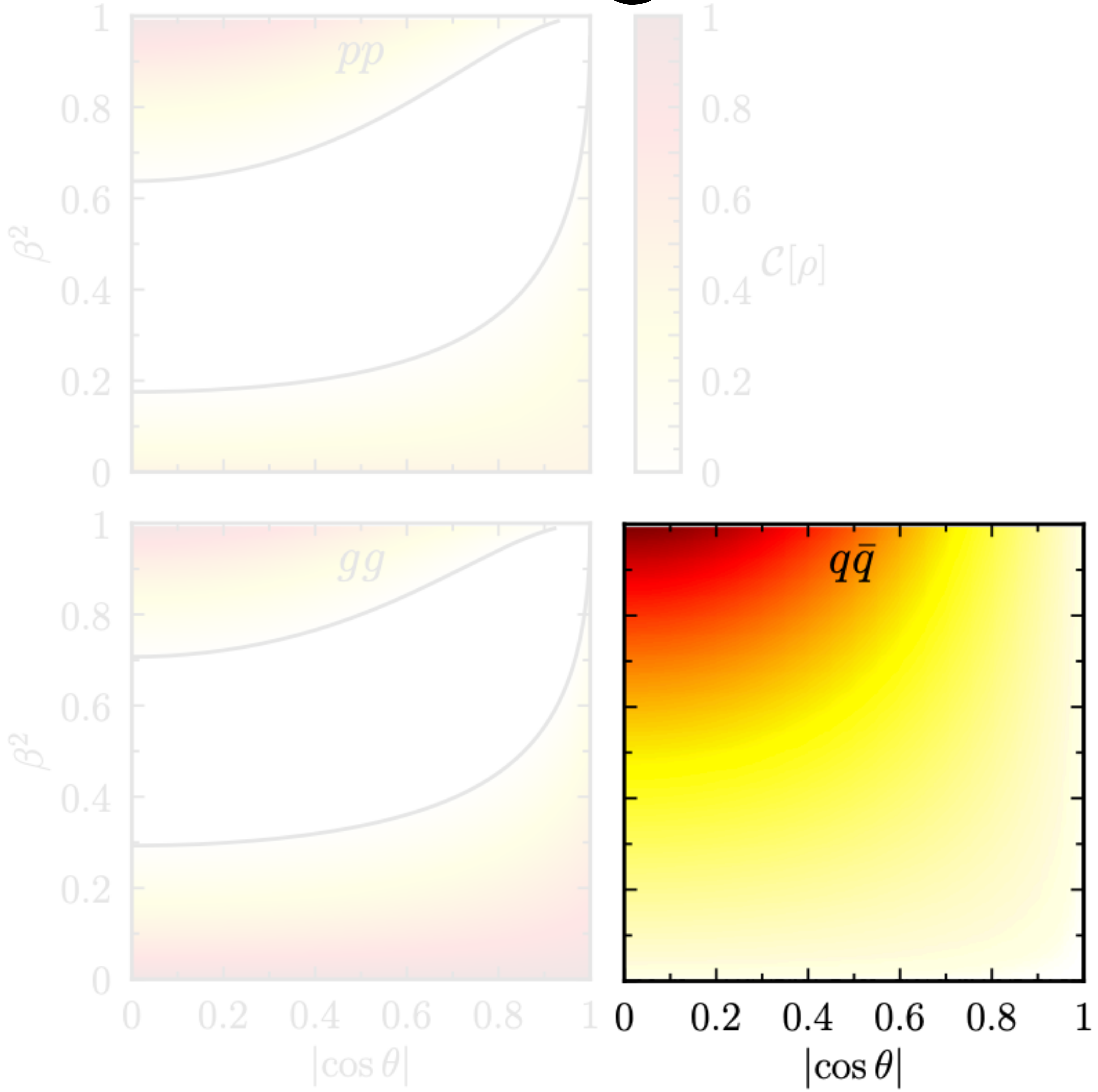
- At threshold: $\beta^2 = 0, \forall\theta$

$$\rho_{gg}^{\rm SM}(0, z) = |\Psi^-\rangle_{\boldsymbol{n}}\langle\Psi^-|_{\boldsymbol{n}}$$ (singlet)

- High-E: $\beta^2 \to 1, \cos\theta = 0$

$$\rho_{gg}^{\rm SM}(1, 0) = |\Psi^+\rangle_{\boldsymbol{n}}\langle\Psi^+|_{\boldsymbol{n}}$$ (triplet)

SM Entanglement



$q\bar{q}$

Maximal entanglement points/regions:

- At threshold: $\beta^2 = 0, \forall\theta$

  mixed but separable

- High-E: $\beta^2 \to 1, \cos\theta = 0$

$$\rho^{\text{SM}}_{q\bar{q}}(1,0) = |\Psi^+\rangle_{\boldsymbol{n}}\langle\Psi^+|_{\boldsymbol{n}}.$$ (triplet)

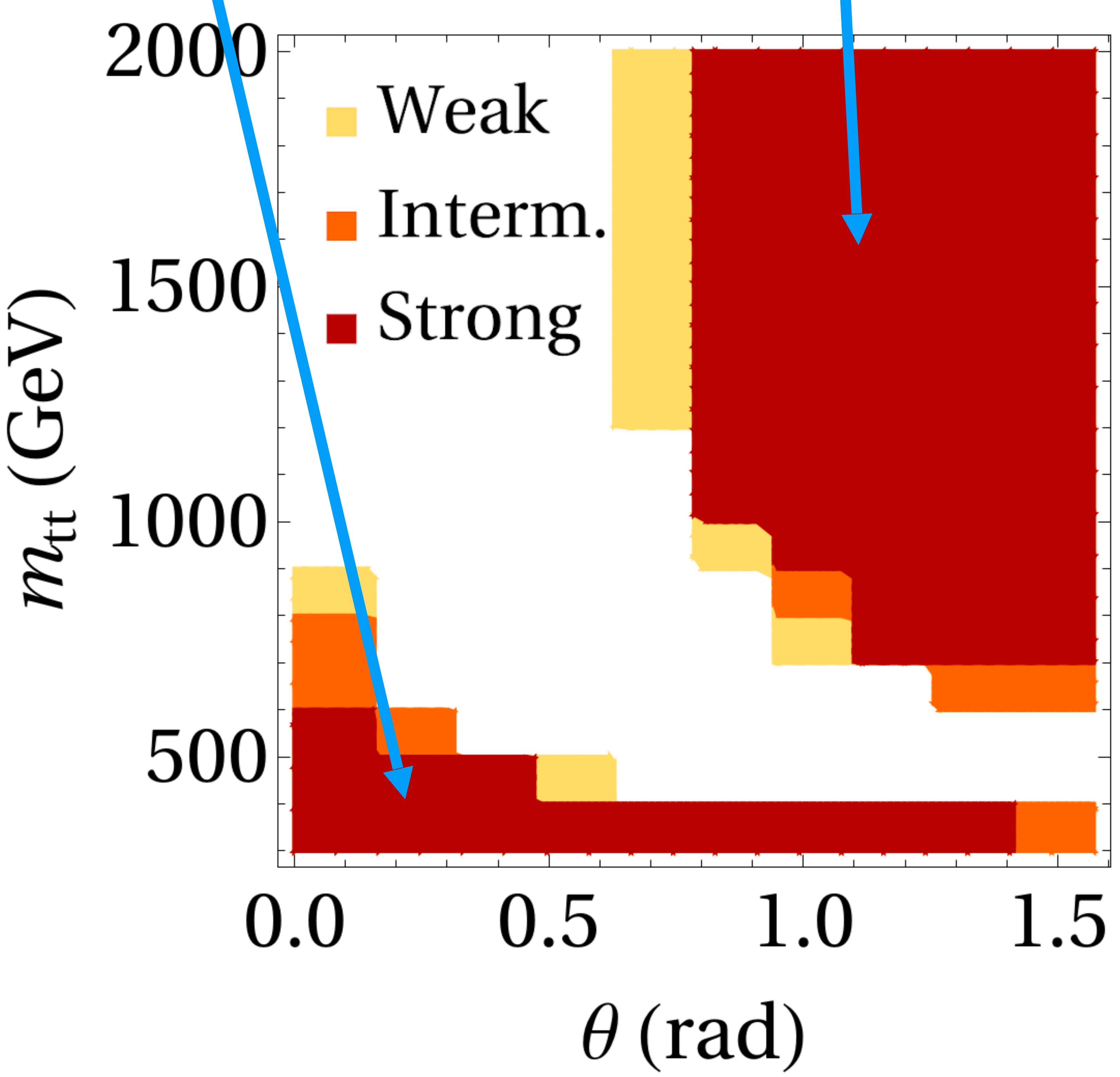
SM Entanglement

Region	Selection	Cross section	$\lvert C_{kk}+C_{rr}\rvert - C_{nn}$ Reconstructed	Significance for >1
Threshold	Weak	14 pb	1.31 ± 0.02	$\gg 5\sigma$
	Intermediate	12 pb	1.34 ± 0.02	$\gg 5\sigma$
	Strong	10 pb	1.38 ± 0.02	$\gg 5\sigma$
High-p_T	Weak	1.9 pb	1.32 ± 0.07	5σ
	Intermediate	1.5 pb	1.36 ± 0.08	4σ
	Strong	1.0 pb	1.42 ± 0.13	3σ

C. Severi, C. Boschi, **FM**, M. Sioli : 2110.10112 [hep-ph]

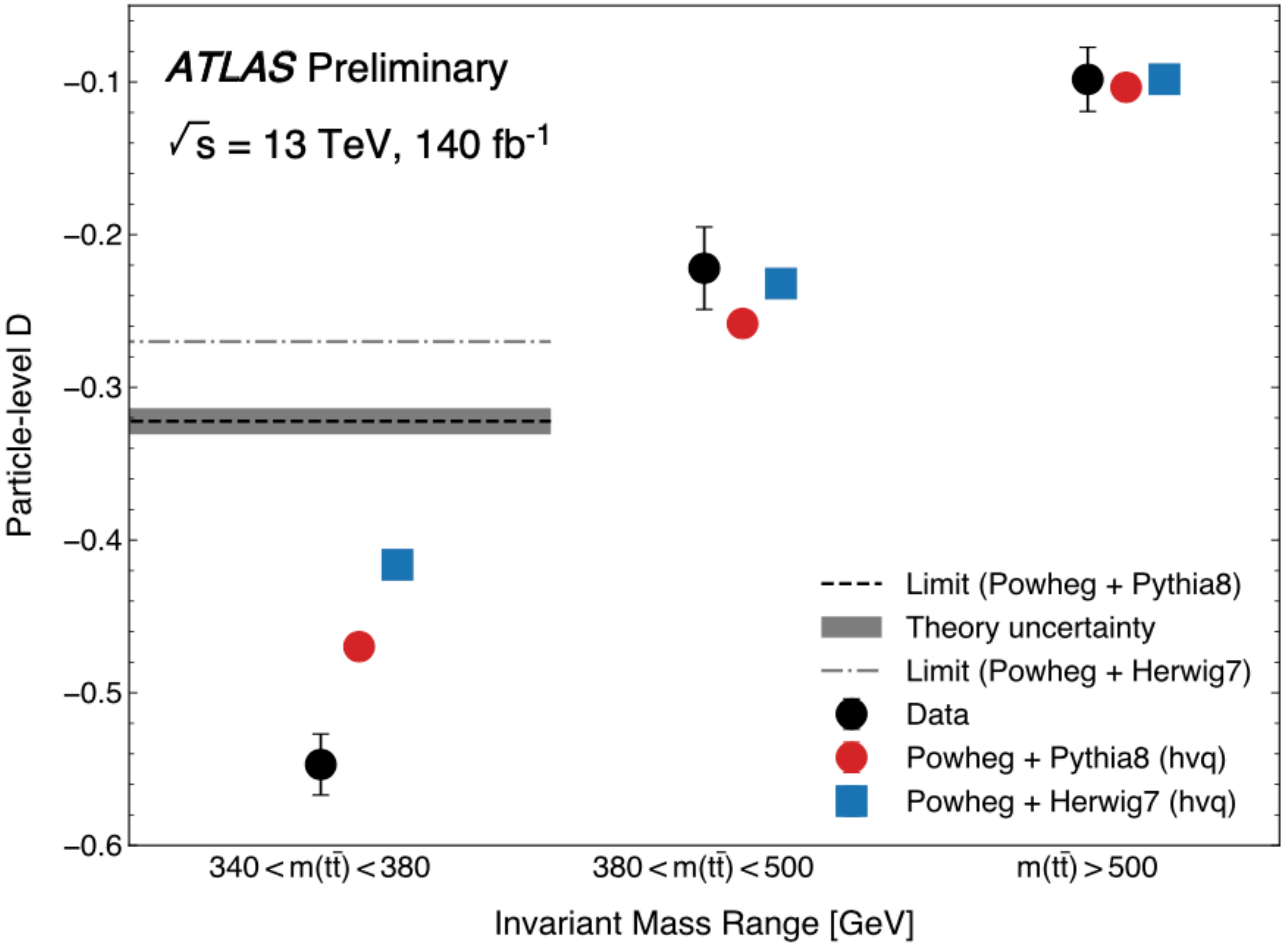
$C_{kk} + C_{rr} - C_{nn} > 1$

$-C_{kk} - C_{rr} - C_{nn} > 1$



Entanglement "easily" observable at the LHC!

First measurements
Dilepton channel at threshold



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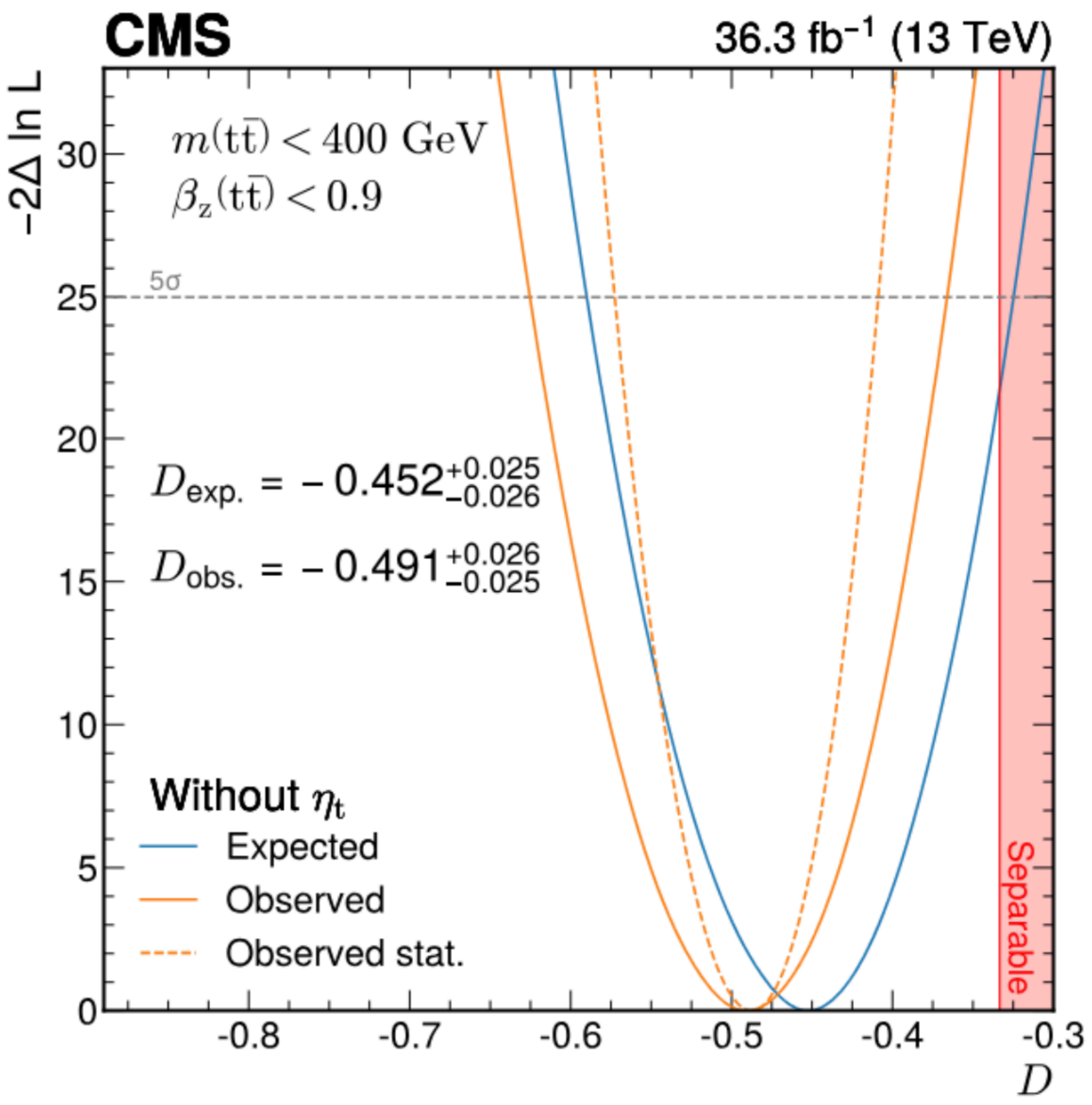

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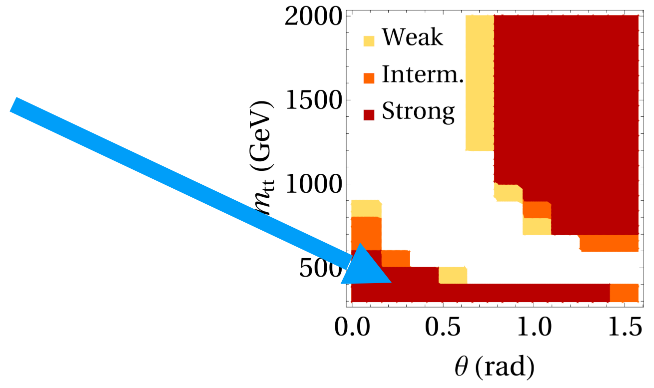
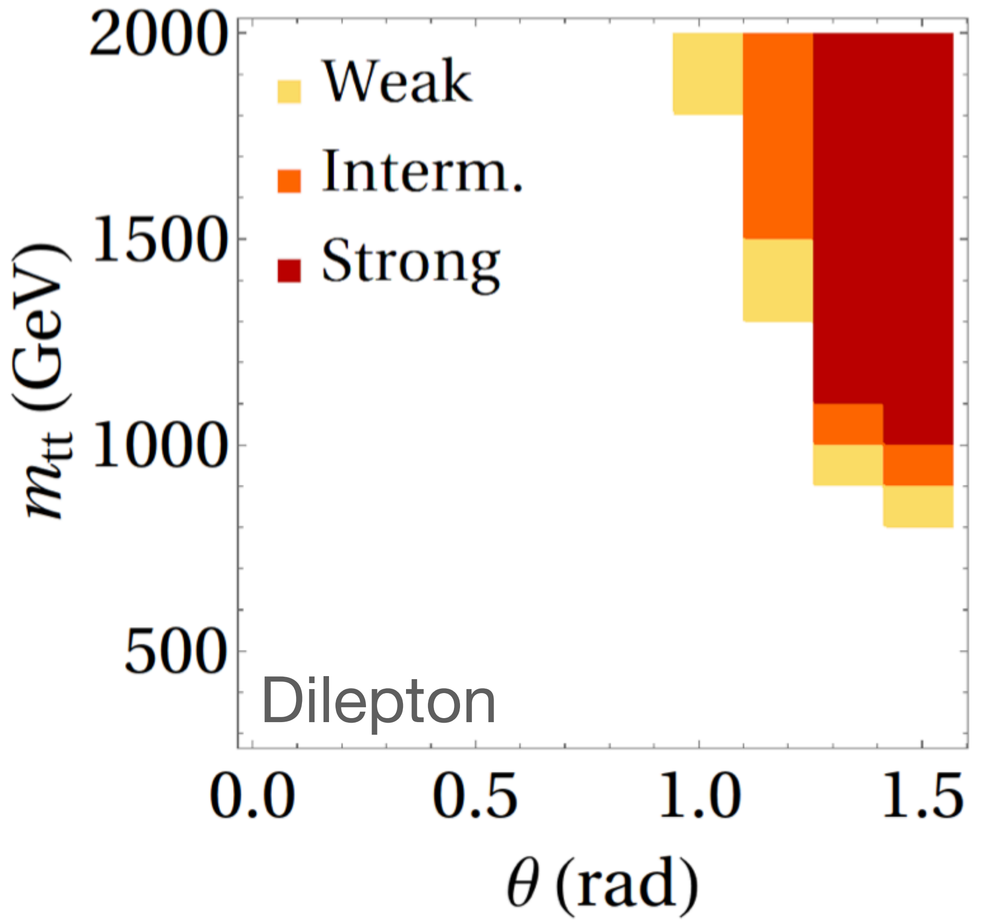


Entanglement observation by ATLAS

Entanglement observation by CMS

Bell inequalities violation

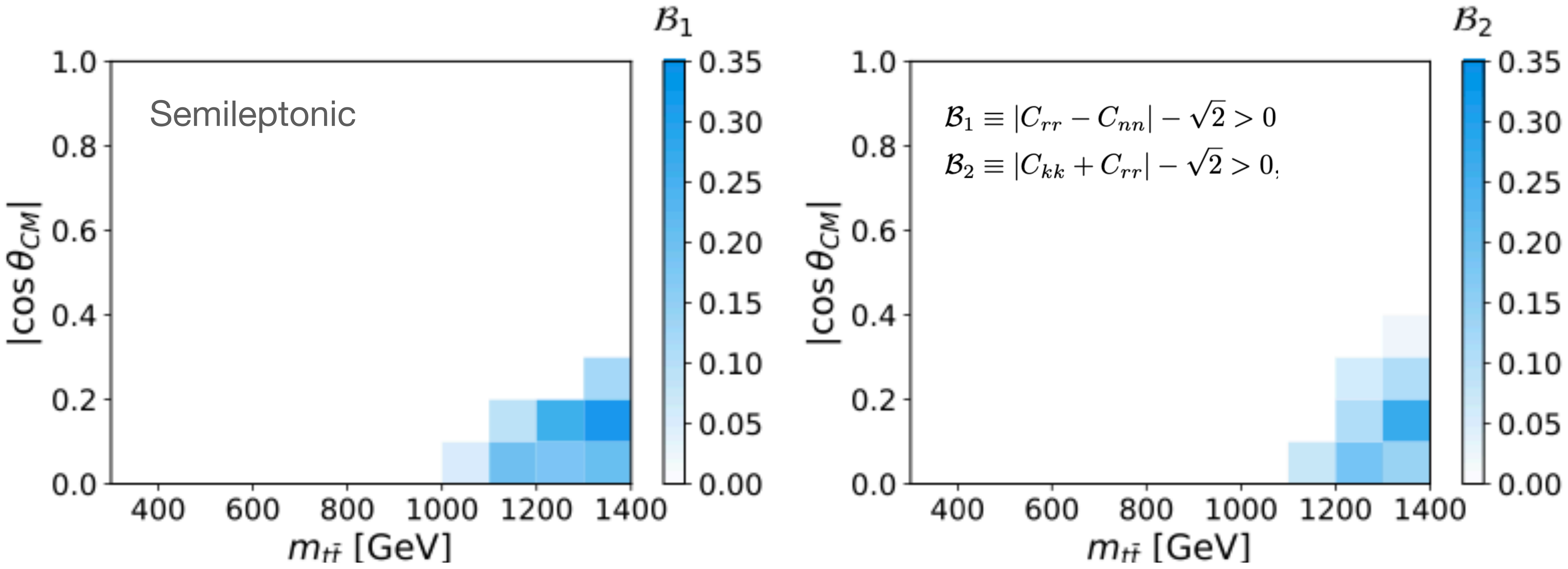
Severi et al. 2110.10112



High-p_T Selection	Cross section	Significance for > 2 w/ 3 ab^{-1}
Weak	0.58 pb	83% CL
Intermediate	0.31 pb	81% CL
Strong	0.17 pb	66% CL

Low statistics…

Z. Dong et al. 2305.07075



Indicator	Parton-level	Unfolded	Significance ($\mathcal{L} = 3$ ab^{-1})
$\mathcal{B}_1$	0.267 ± 0.023	0.274 ± 0.057	4.8
$\mathcal{B}_2$	0.204 ± 0.023	0.272 ± 0.058	4.7

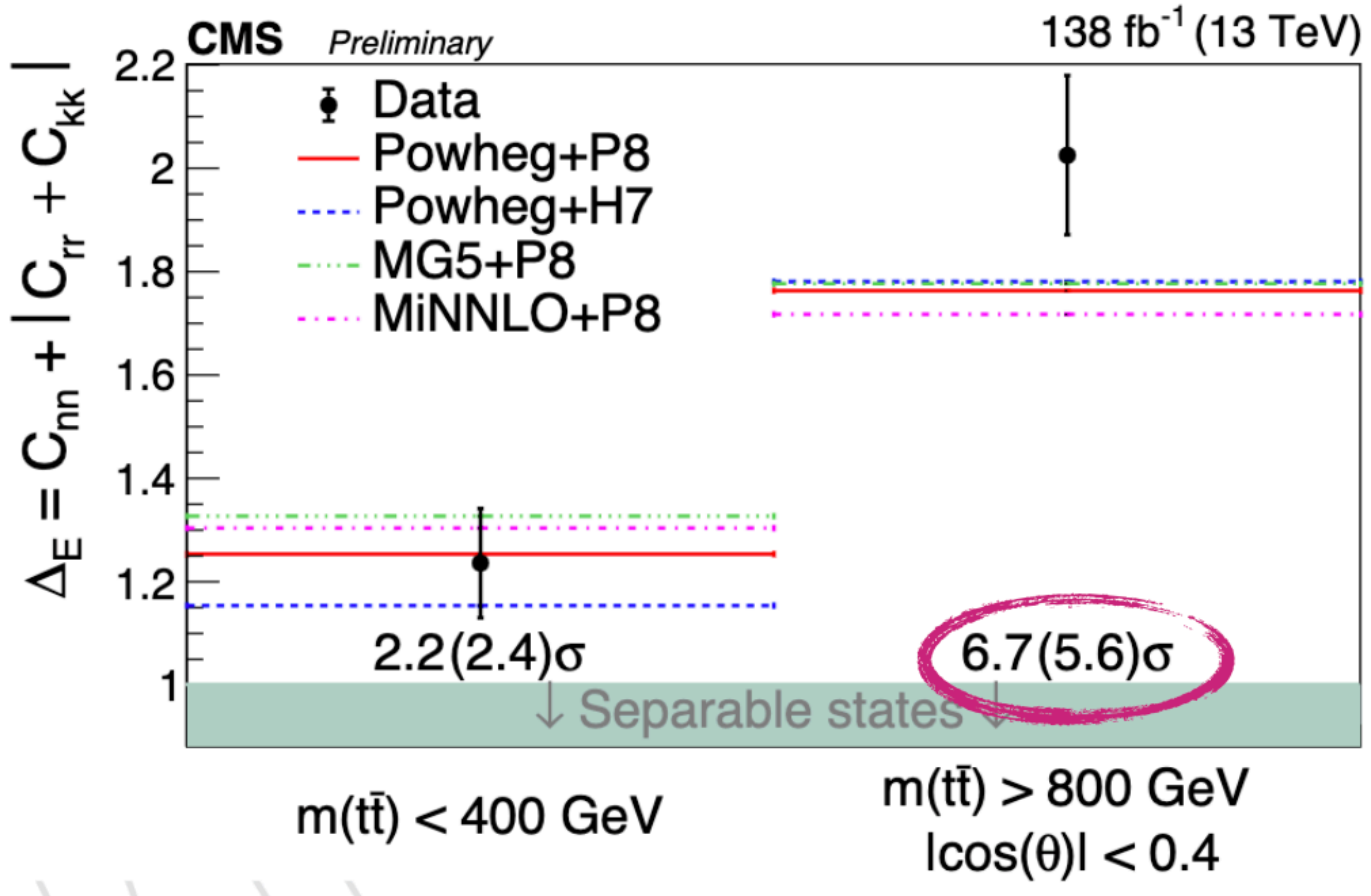
Better statistics, use of boosted top tagging

Challenging!!

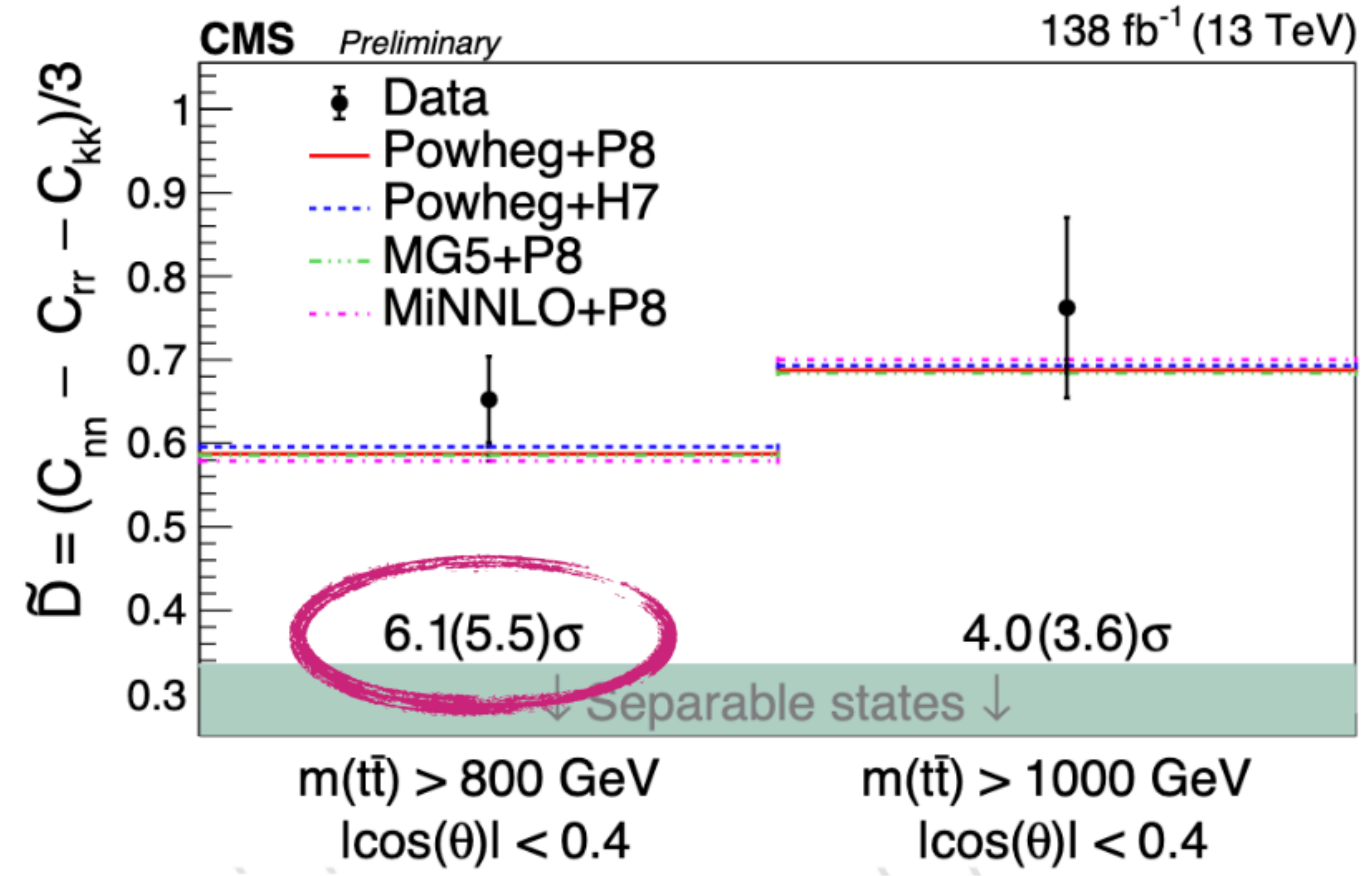
First measurements

Semileptonic channel

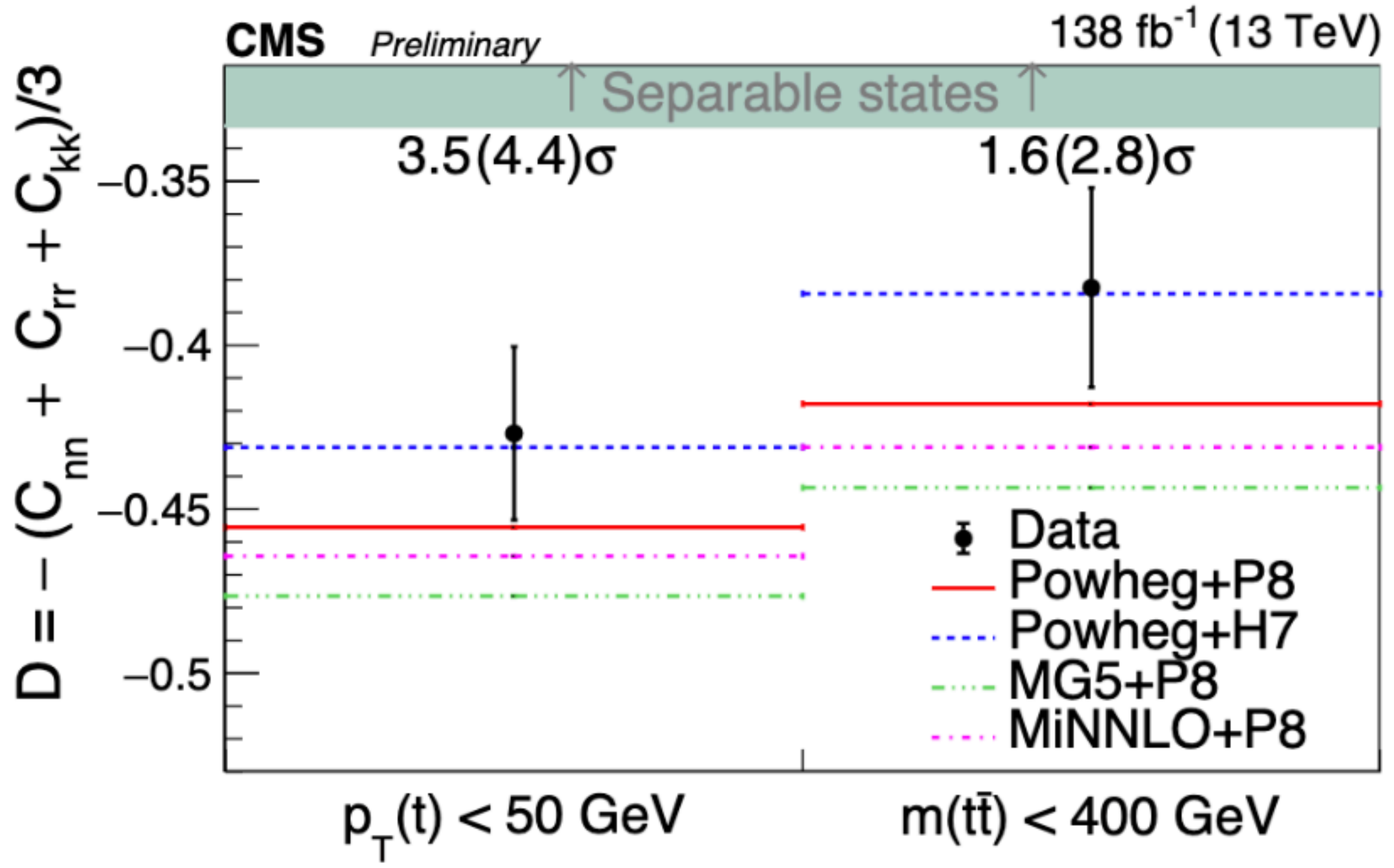
Full matrix $\Delta_E = C_{nn} + |C_{rr} + C_{kk}| > 1$

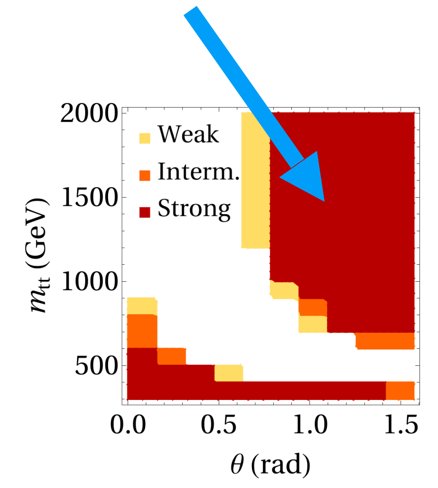


High $m_{t\bar{t}}$ $\tilde{D} > 1/3$



Low $m_{t\bar{t}}$ $D < -1/3$

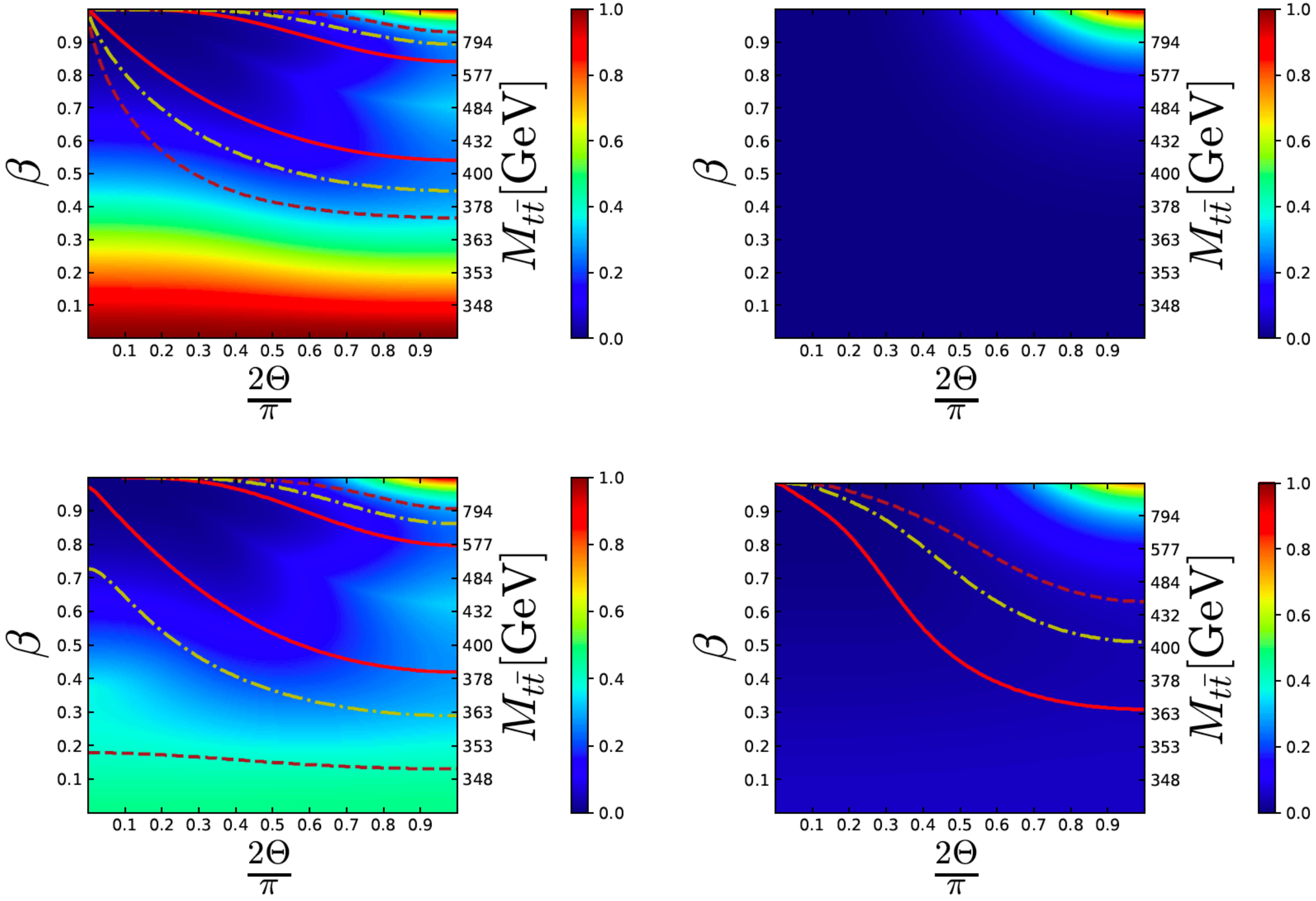




See Giulia Negro's seminar at CERN

Measurements going towards testing BIV (with many loopholes…)

Quantum correlations



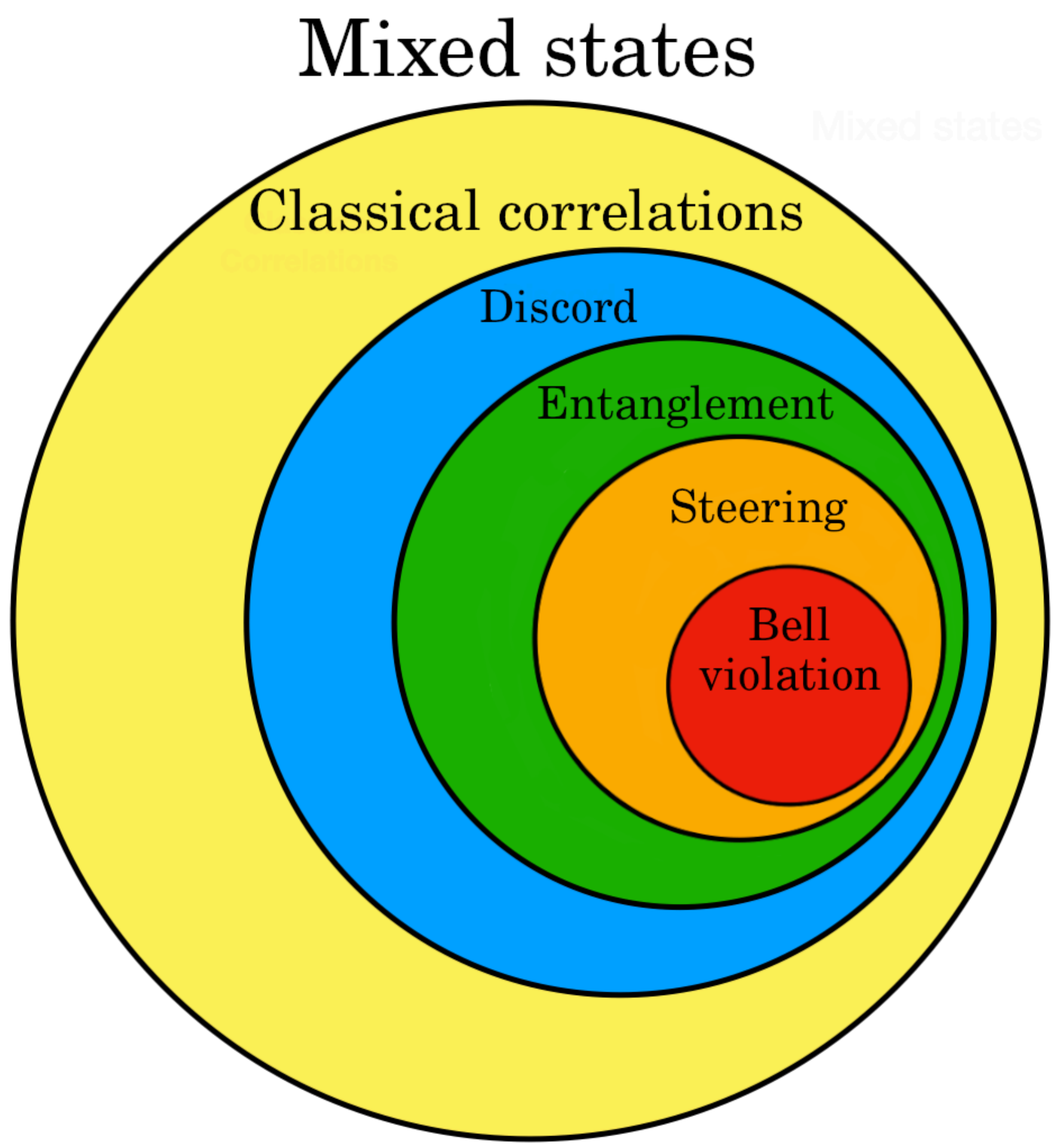


FIG. 2. Quantum discord of the spin density matrix $\rho(M_{t\bar{t}}, \hat{k})$ as a function of the top velocity β and the production angle Θ in the $t\bar{t}$ c.m. frame. All plots are symmetric under $\Theta \to \pi - \Theta$. Upper left: $gg \to t\bar{t}$. Upper right: $q\bar{q} \to t\bar{t}$. Lower left: $t\bar{t}$ production at the LHC for Run 2 c.m. energy, $\sqrt{s} = 13$ TeV [19]. Lower right: $t\bar{t}$ production at the Tevatron for $\sqrt{s} = 2$ TeV, close to its last-run c.m. energy $\sqrt{s} = 1.96$ TeV [15]. Solid red, dashed-dotted yellow, and dashed brown lines are the critical boundaries of separability, steerability, and Bell locality, respectively.

Is there a *Quantum Advantage* in the search for New Physics?

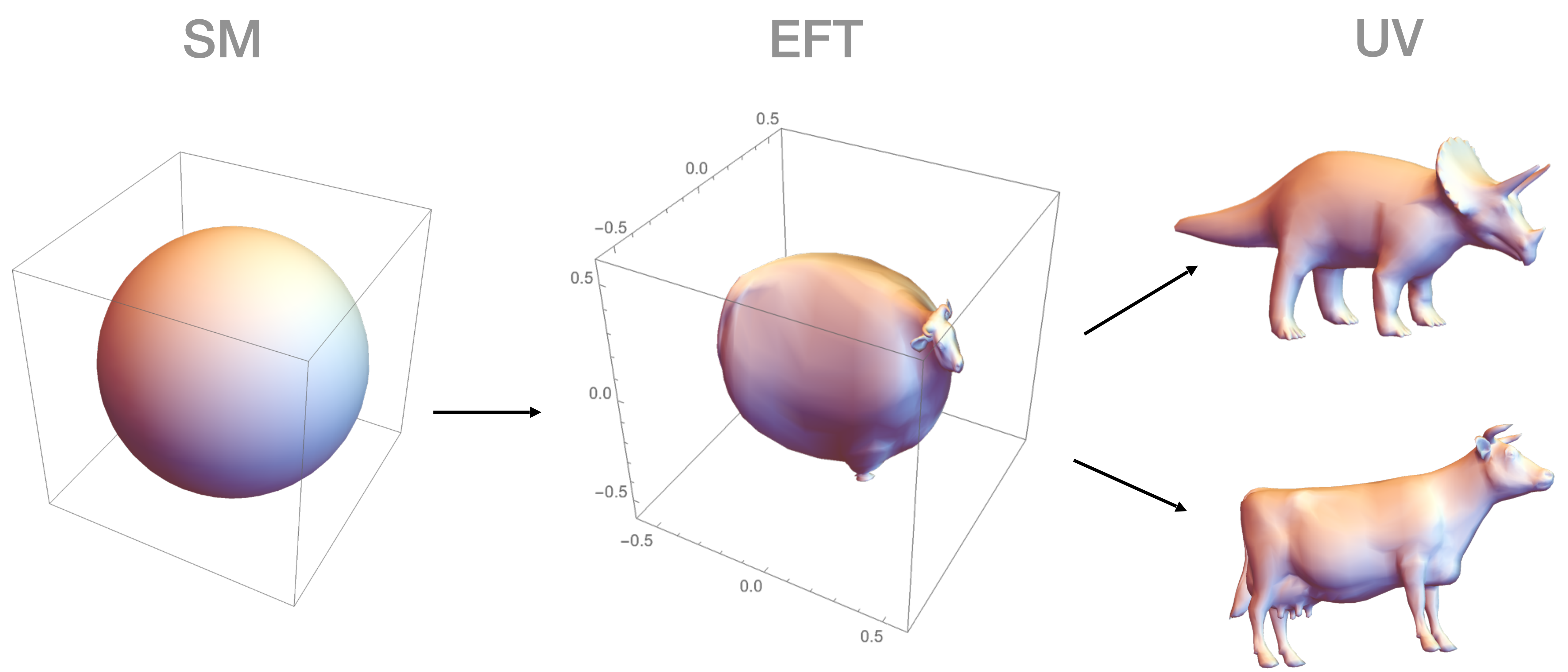
Two model-independent approaches:

1] BSM resides beyond the scales directly explored by the experiment
⇒ Effective Field Theory

2] BSM resonances can be directly produced
⇒ Simplified models

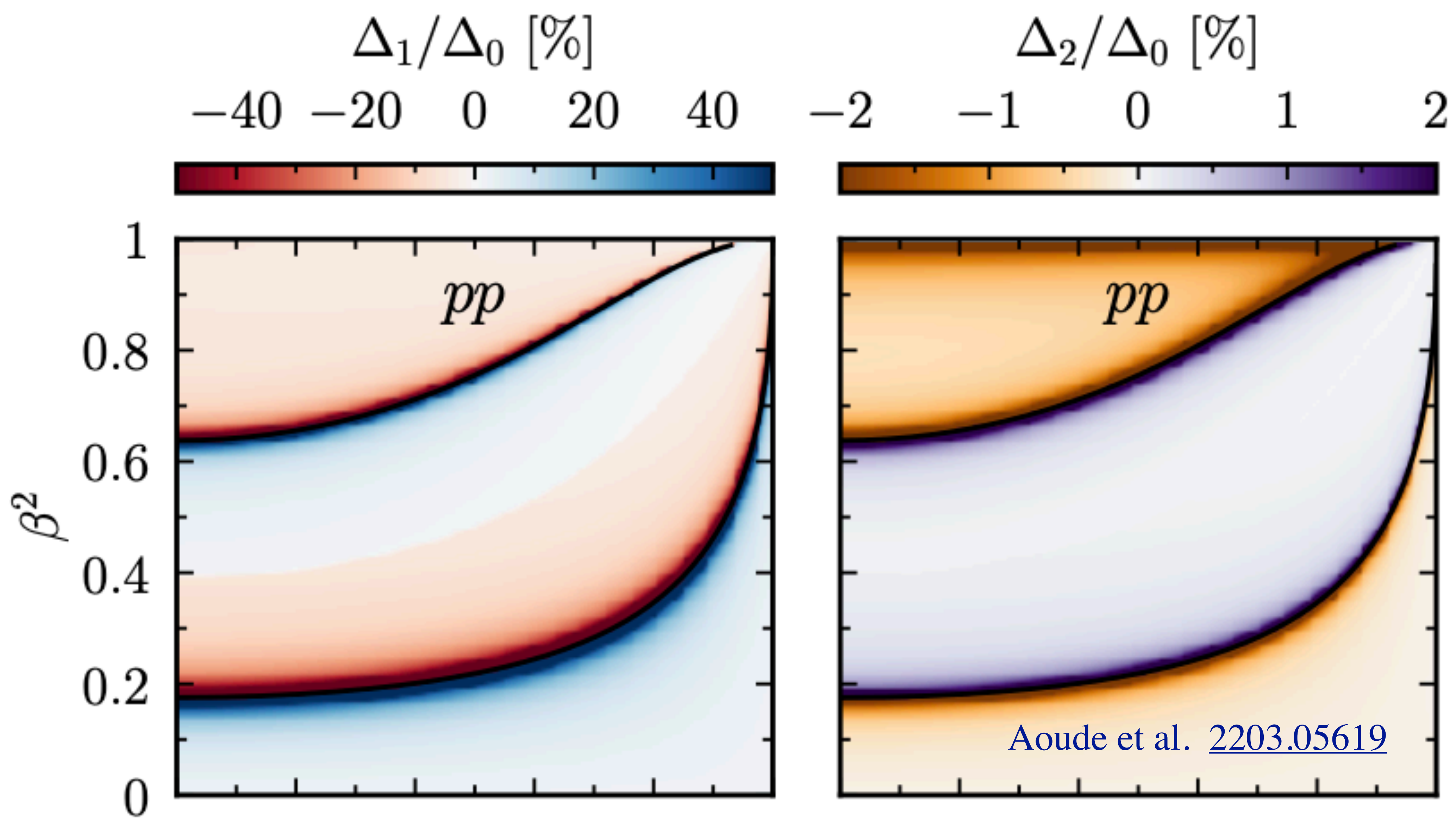
SMEFT

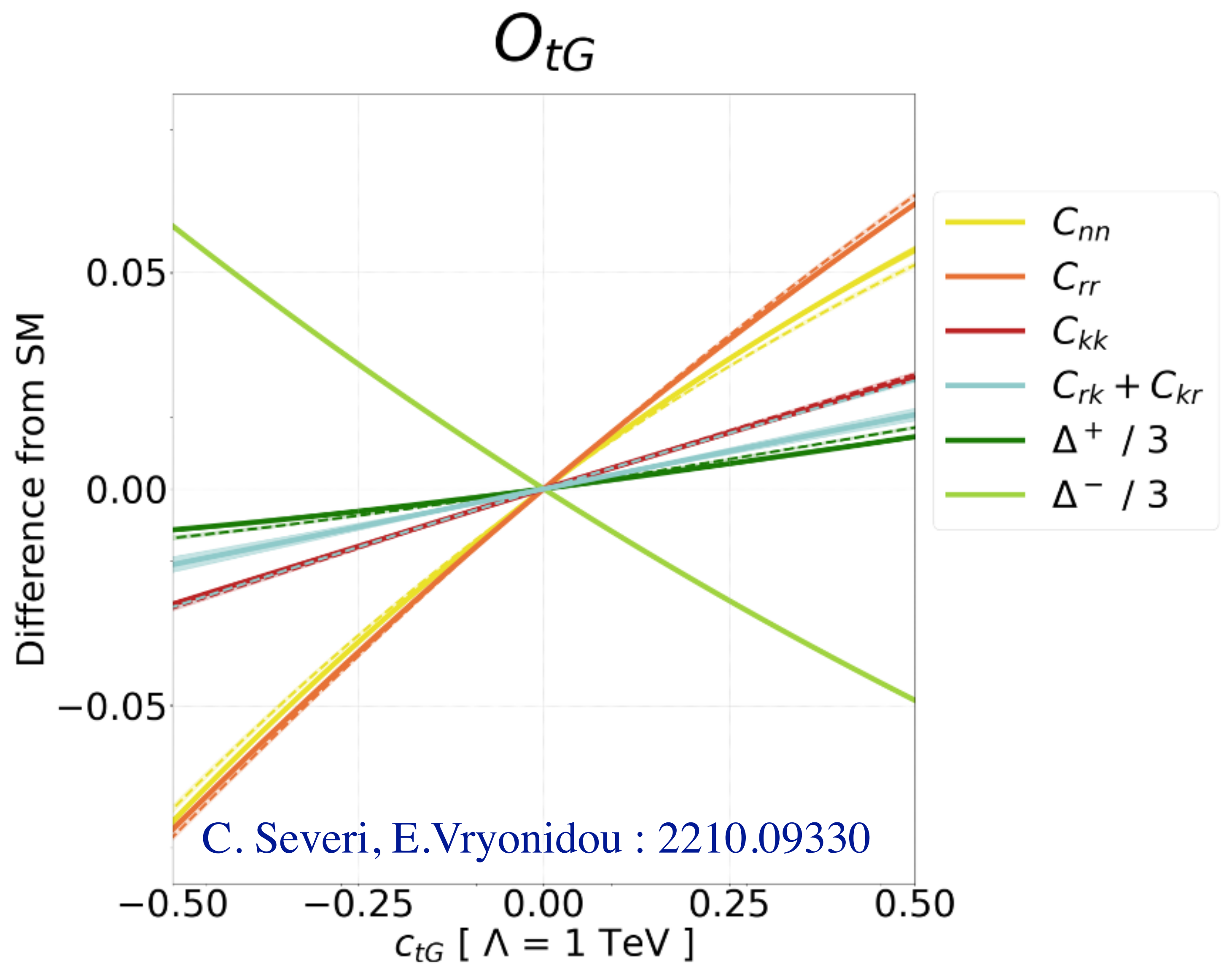
$$\mathscr{L}_{SM}^{(4)} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}i\not{D}\psi + (y_{ij}\bar{\psi}_L^i\phi\psi_R^j + \mathrm{h.c.}) + |D_\mu\phi|^2 - V(\phi)$$



$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}}^{(4)} + \frac{1}{\Lambda^2}\sum_i^{N_6} c_i\mathcal{O}_i^{(6)} + \frac{1}{\Lambda^4}\sum_j^{N_8} c_j\mathcal{O}_i^{(8)} + \dots$$

Quantum Advantage for SMEFT



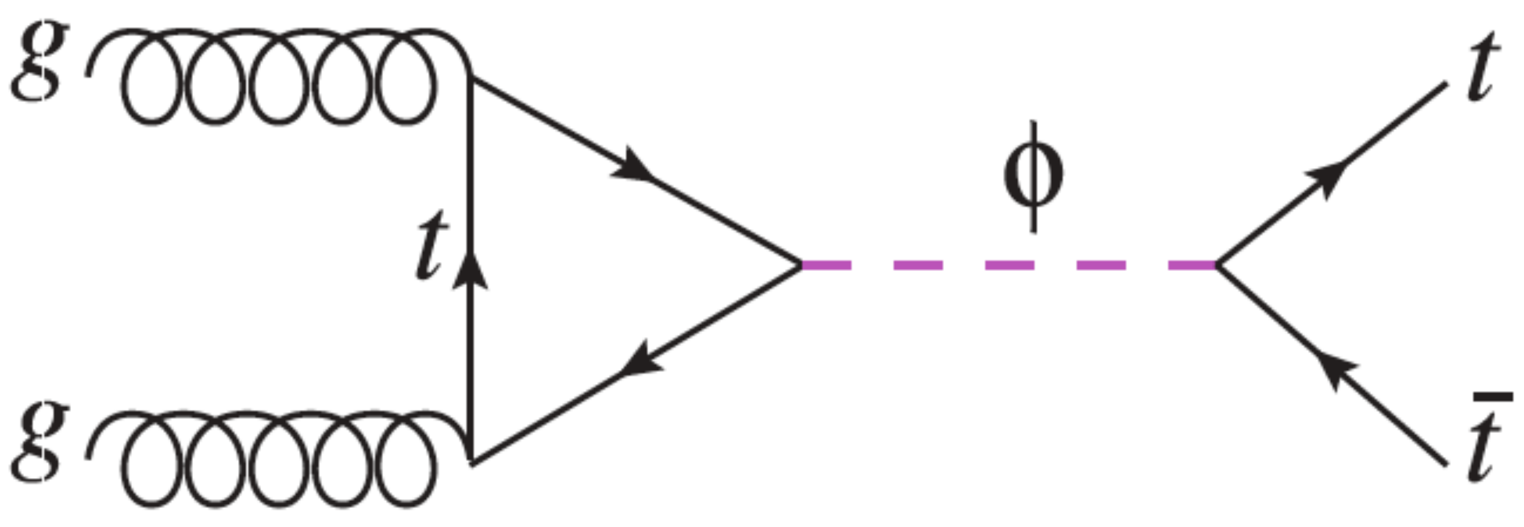


- New interactions modify both conventional and quantum observables
- Dimension-6 operators can modify the degree of entanglement between top quarks
- SMEFT introduce new structures, thus probing new linear combinations between coefficients
- QI observables can break degeneracies between operators when combined with standard observables

New sensitivity!

BSM resonances

Scalar

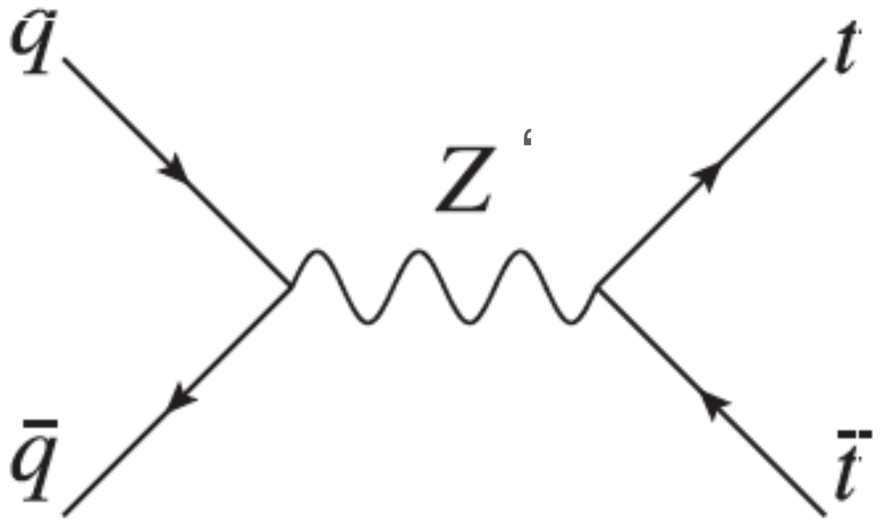


$$c_y \frac{y_t}{\sqrt{2}} \phi \, \bar{t} \, (\cos\alpha + i\gamma^5 \sin\alpha) \, t$$

$$\mathcal{C}^{[gg,\,\phi]}\big|_{\alpha=\pi/2} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\mathcal{C}^{[gg,\,\phi]}\big|_{\alpha=0} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

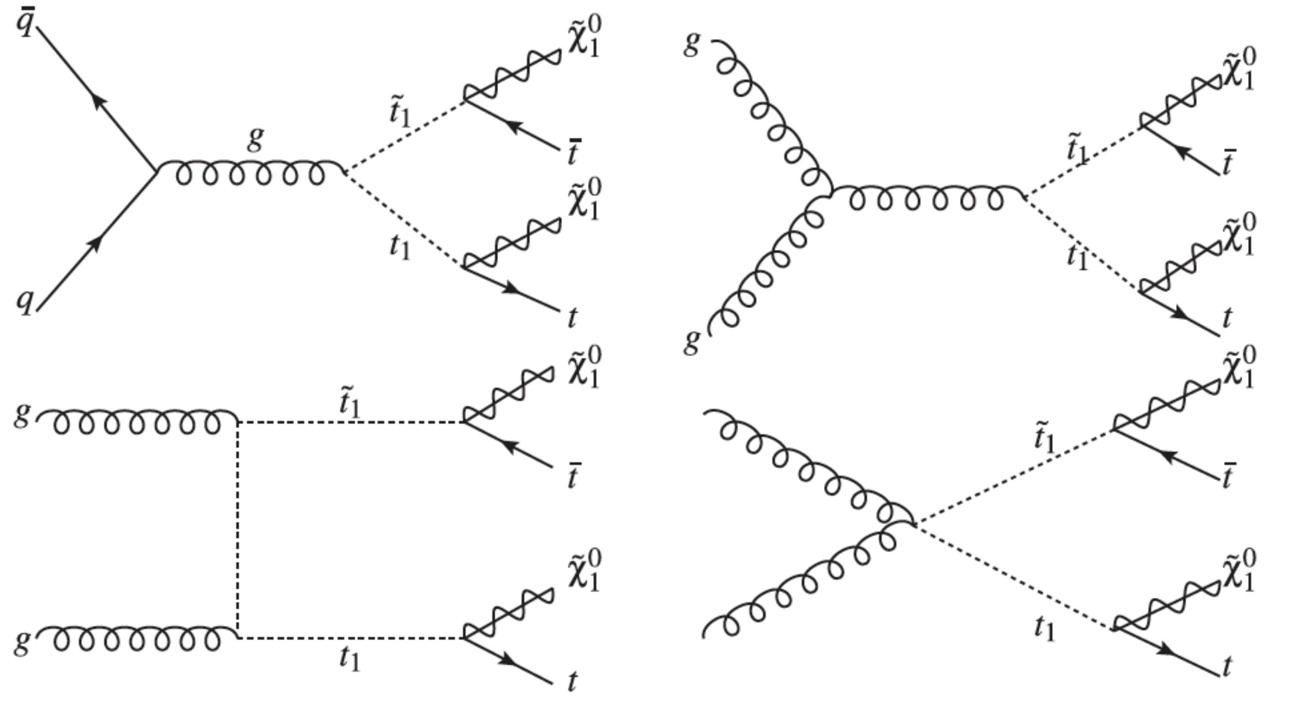
Z’



Sequential Z’

It interferes with SM EW top production.

Stops



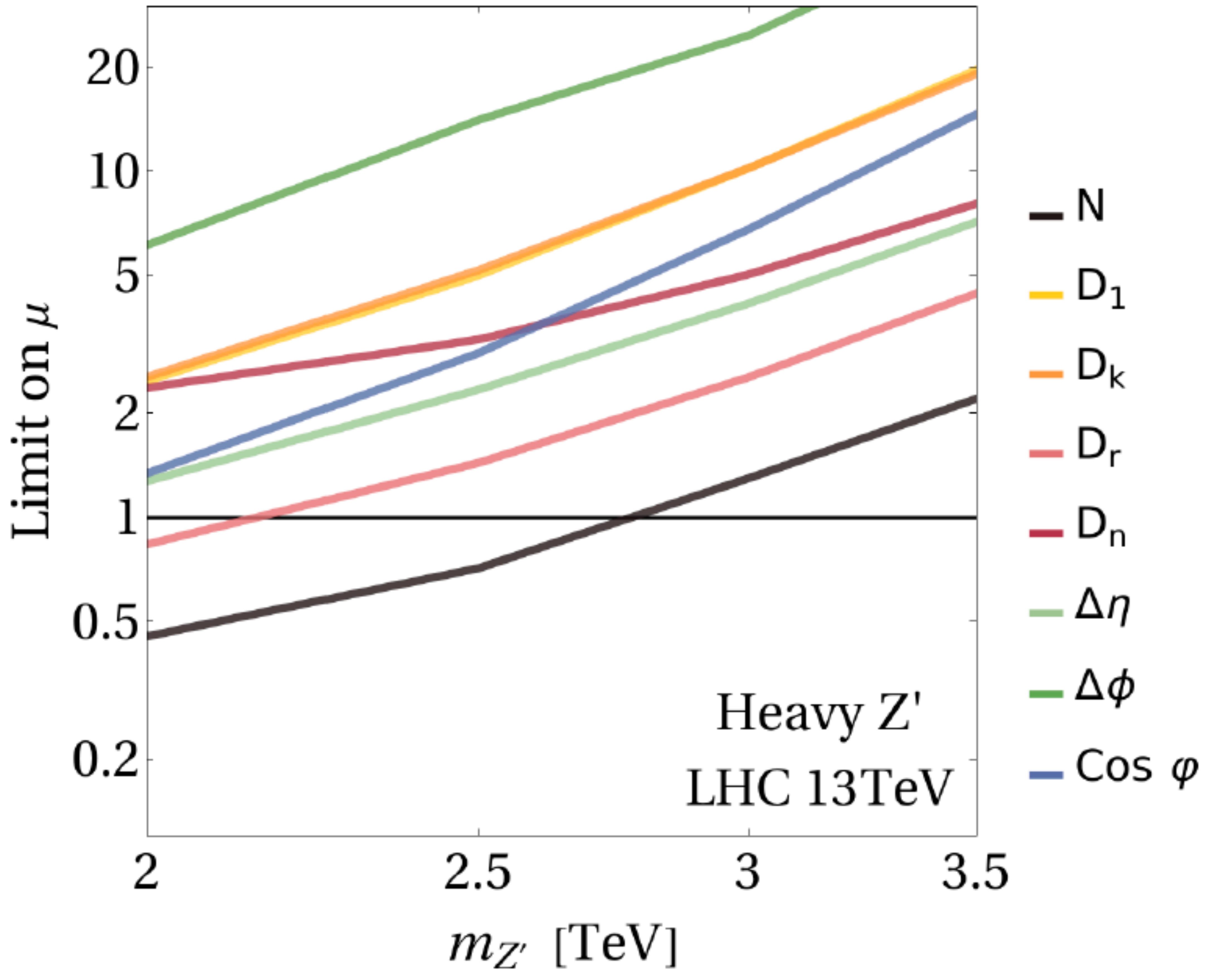
$$|m_{\tilde{t}_1} - m_{\tilde{\chi}_1^0} - m_t| \approx \Gamma_t,$$

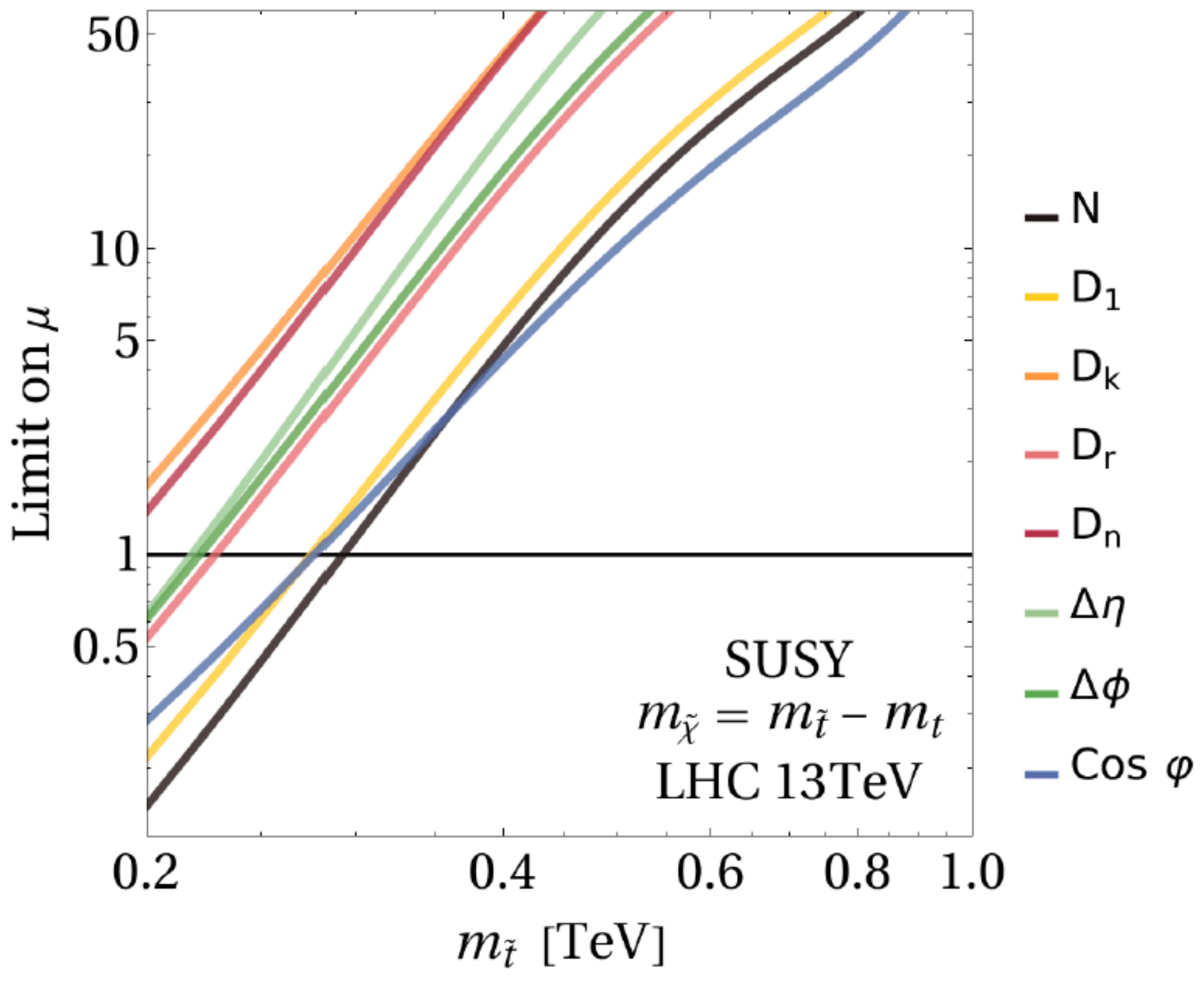
$$\mathcal{C}^{[\mathrm{SUSY}]} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

$$B_i = \frac{A^{[\mathrm{SM}]}}{A^{[\mathrm{TOT}]}} B_i^{[\mathrm{SM}]} + \frac{A^{[\mathrm{SUSY}]}}{A^{[\mathrm{TOT}]}} B_i^{[\mathrm{SUSY}]} \simeq \frac{A^{[\mathrm{SUSY}]}}{A^{[\mathrm{TOT}]}} B_i^{[\mathrm{SUSY}]},$$

$$\mathcal{C} = \frac{A^{[\mathrm{SM}]}}{A^{[\mathrm{TOT}]}} \mathcal{C}^{[\mathrm{SM}]} + \frac{A^{[\mathrm{SUSY}]}}{A^{[\mathrm{TOT}]}} \mathcal{C}^{[\mathrm{SUSY}]} = \frac{A^{[\mathrm{SM}]}}{A^{[\mathrm{TOT}]}} \mathcal{C}^{[\mathrm{SM}]},$$

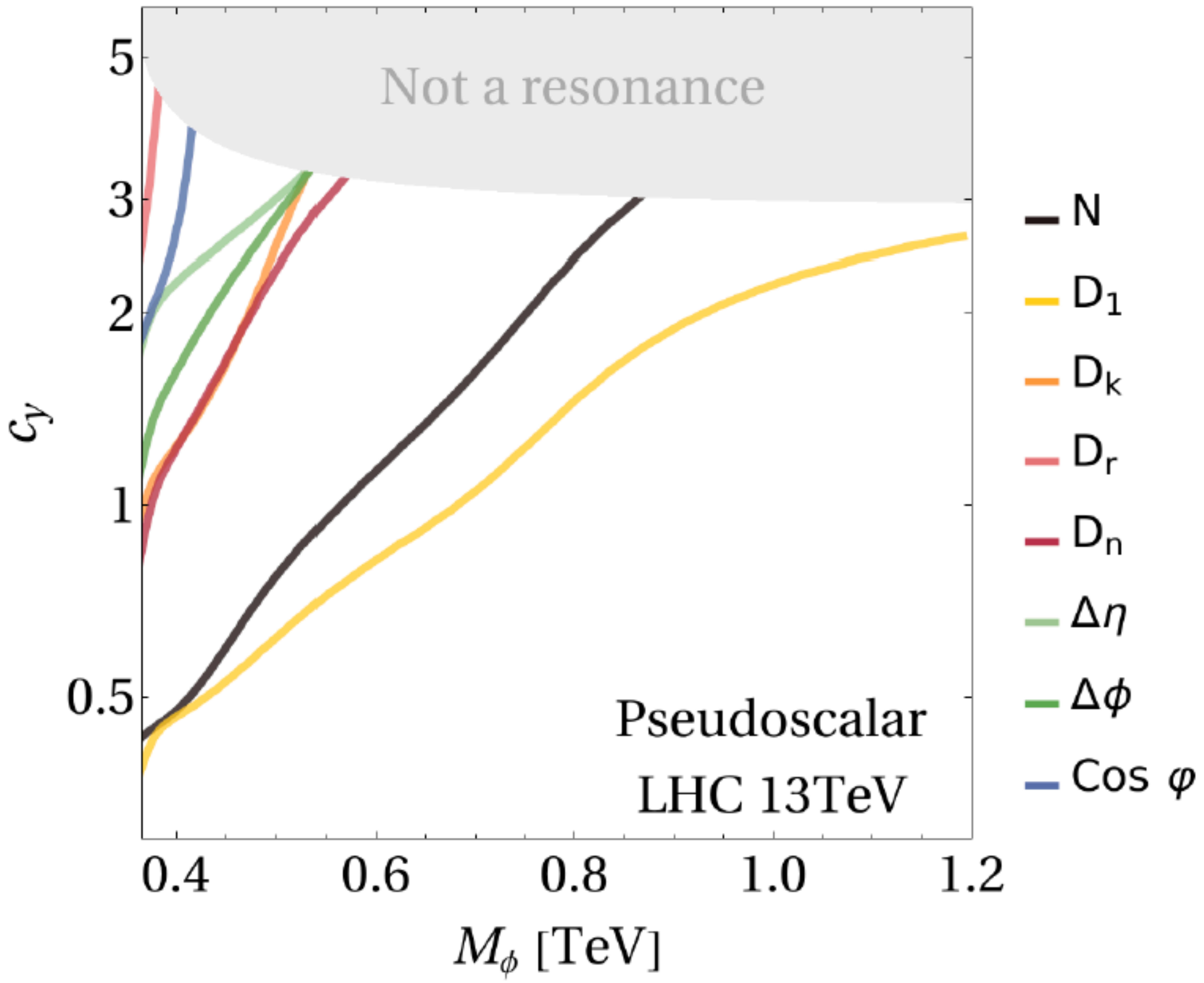
Quantum Advantage for BSM resonances

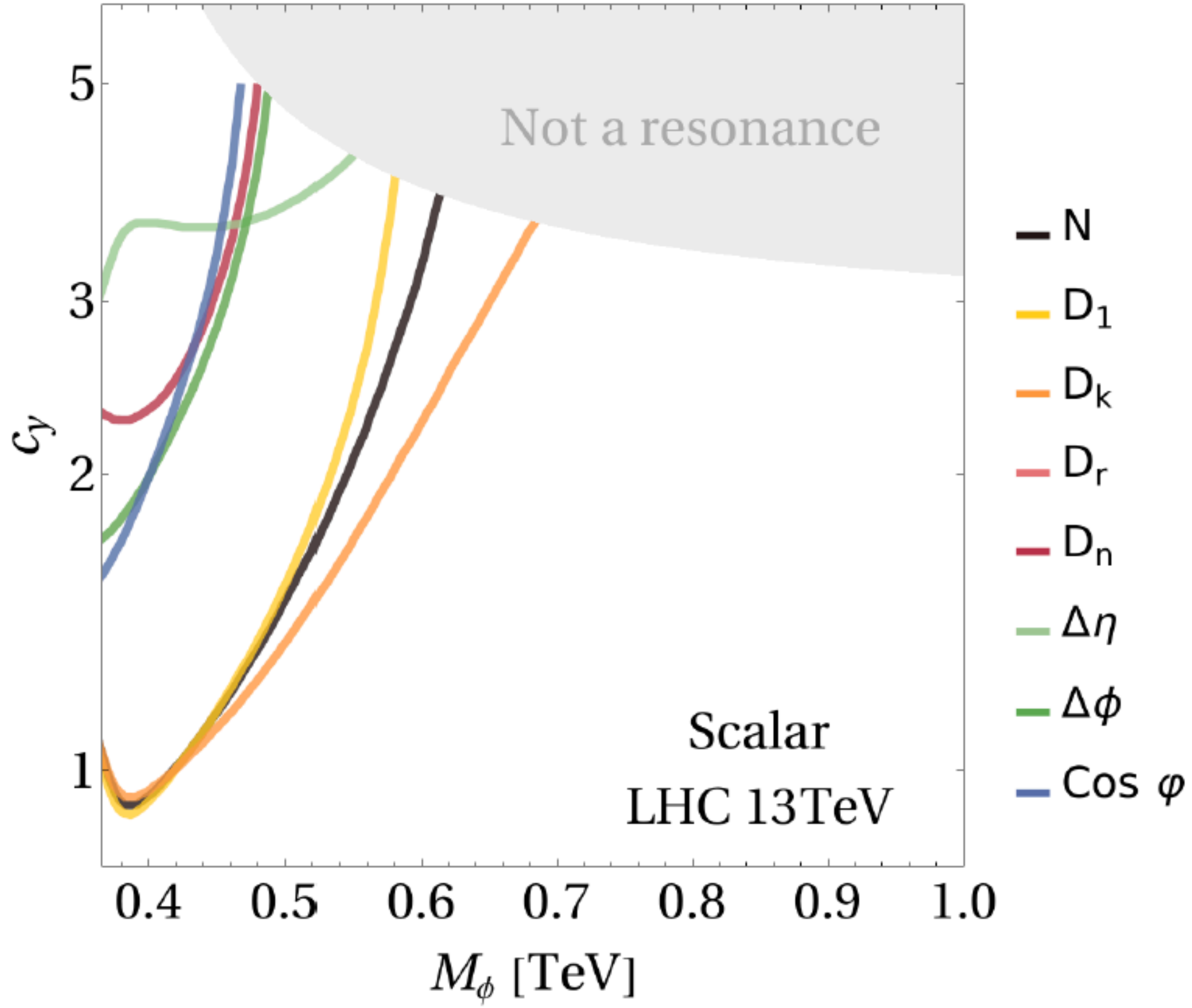




Not adding much…

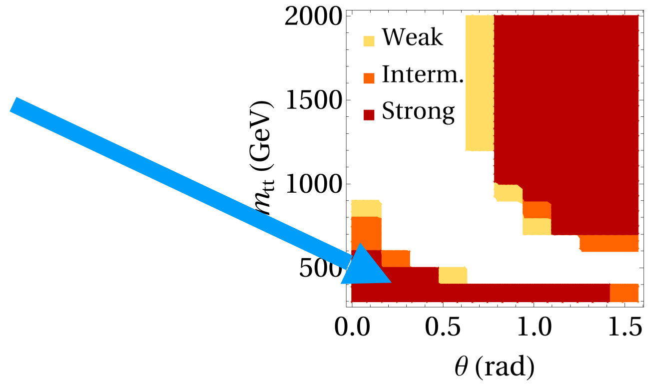
Quantum Advantage for BSM resonances

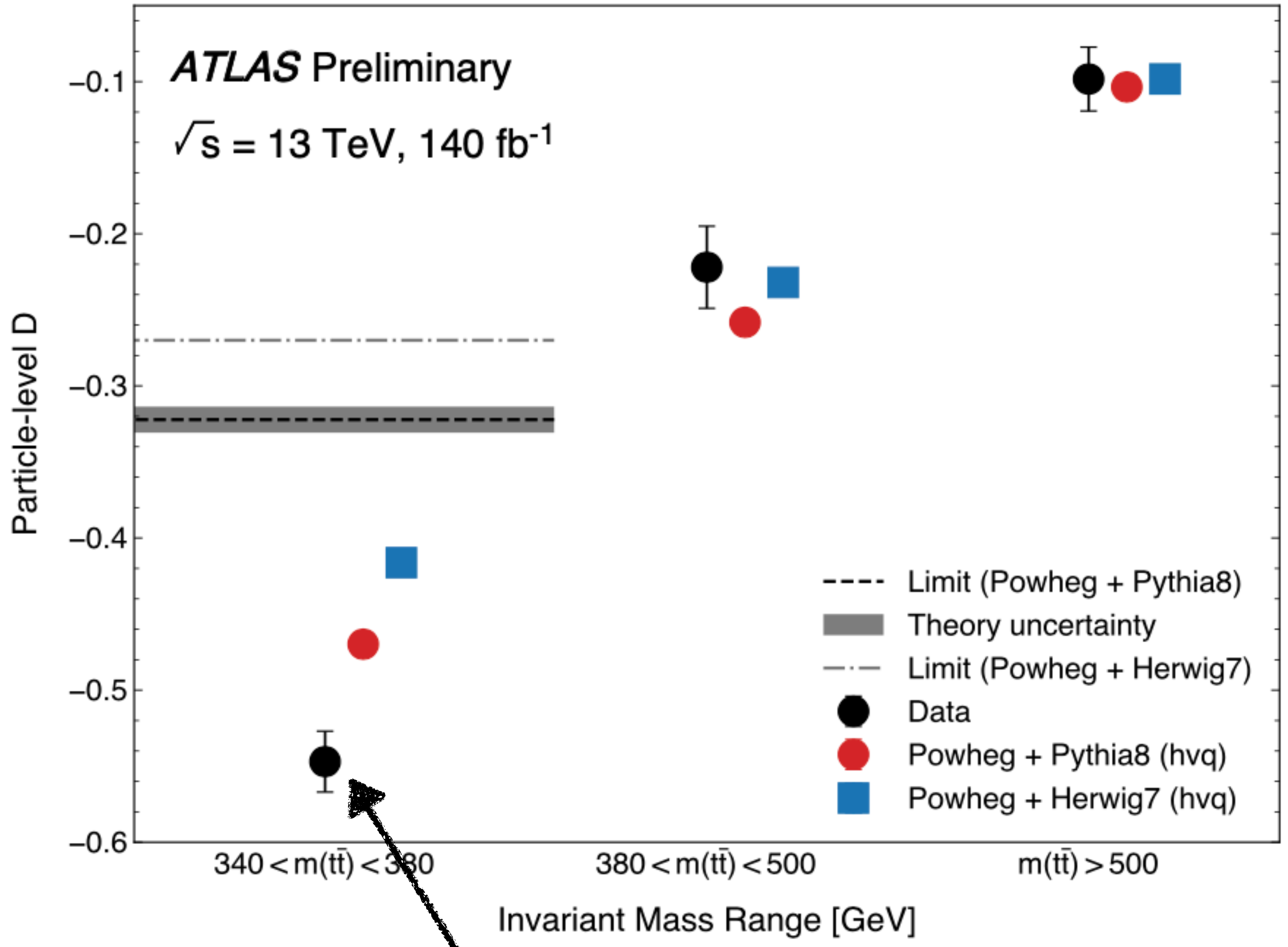




Promising!

First measurements
Dilepton channel at threshold





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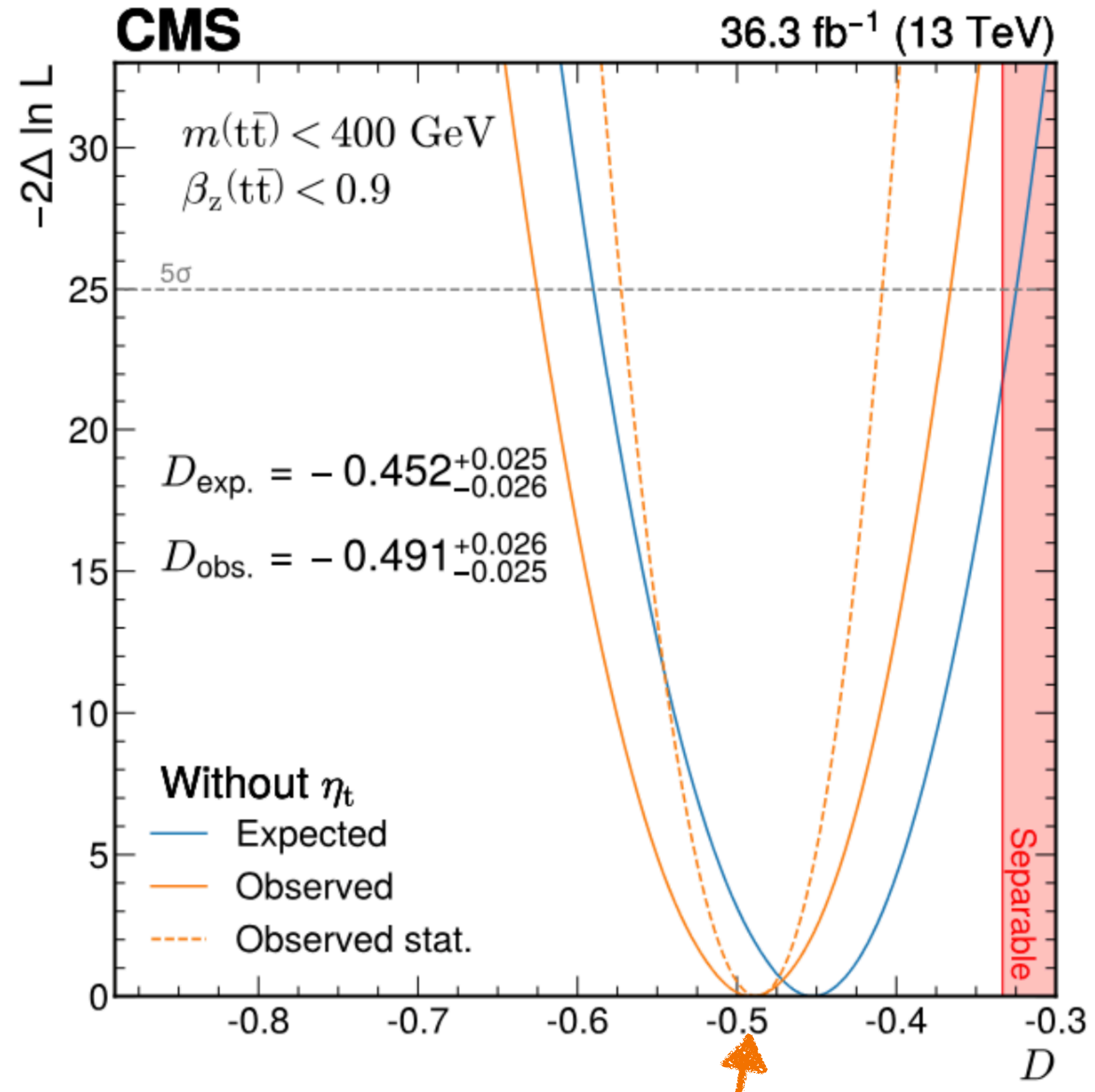
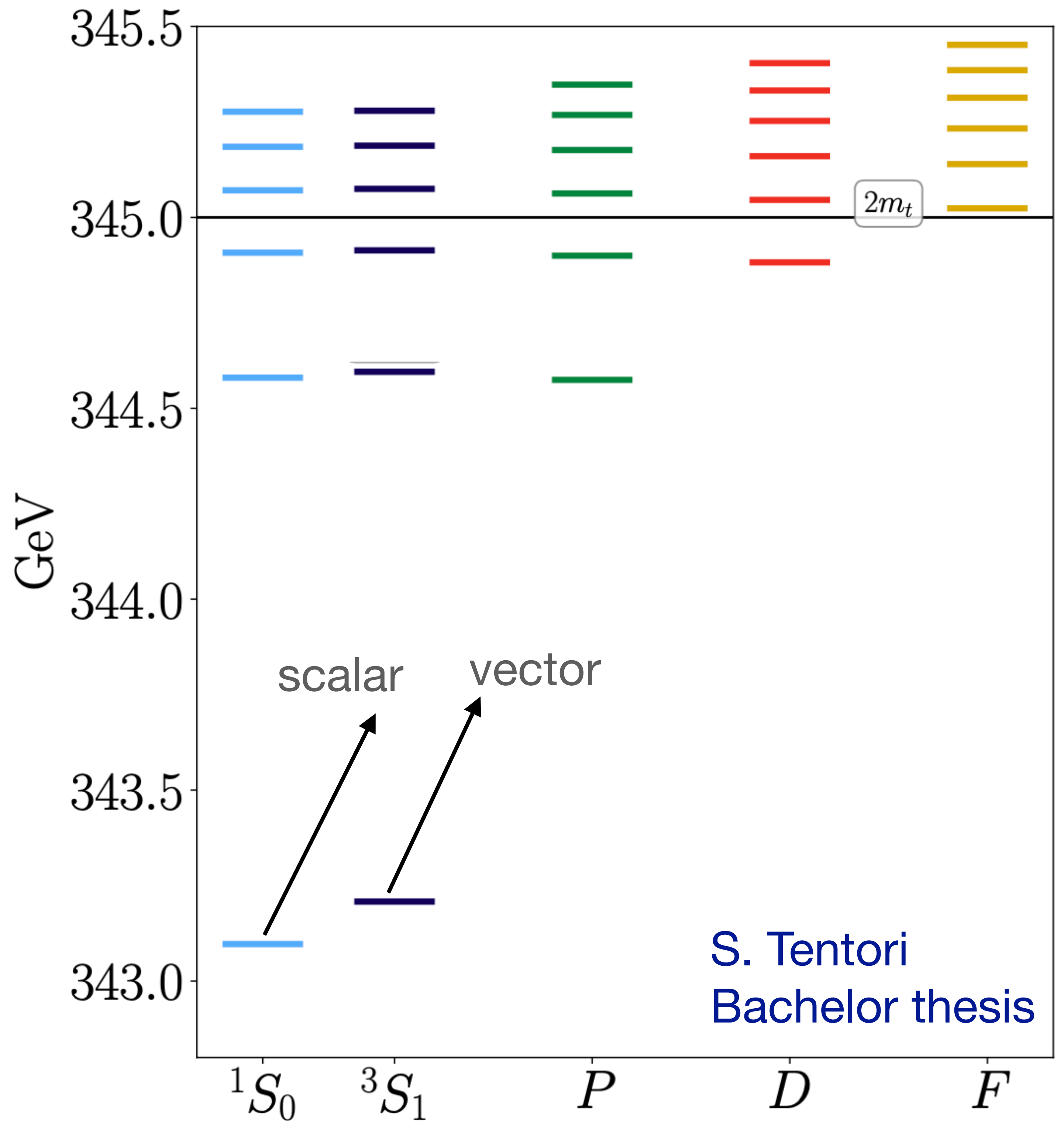


TOP-23-001-pas

More entanglement than predicted by NLO+PS QCD!

New SM Physics: Toponium

- Quasi-Bound State of top and antitop
- Energy states obtained by solving Schrödinger equation with QCD potential
- Described by NRQCD
- Ground state n=1 S-wave
- Spin-singlet vs spin-triplet depending on production mode
    - spin singlet for pp and spin triplet for e^+e^-

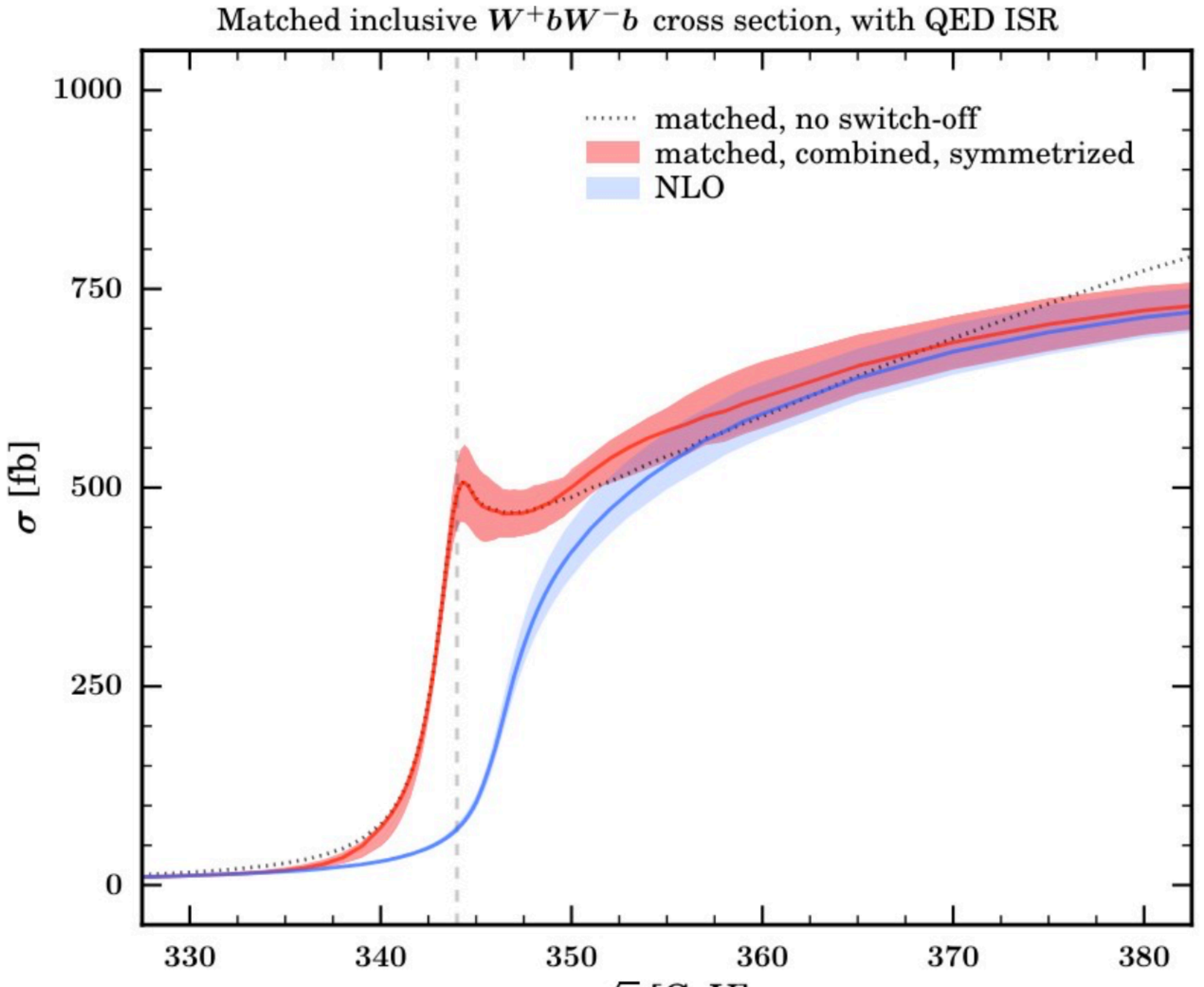


$$\left[(E+i\Gamma_t)-\left(\frac{\boldsymbol{\nabla}^2}{m_t}+V(\boldsymbol{r})\right)\right]G(\boldsymbol{r},E+i\Gamma_t)=\delta^{(3)}(\boldsymbol{r})$$

$$V_{\rm QCD}(r,\mu_B)=C^{[\rm col]}\,\frac{\alpha_s(\mu_B)}{r}\left[1+\frac{\alpha_s}{4\pi}\left(2\,\beta_0\log(e^{\gamma}\mu_B\,r)+\frac{31}{9}C_A-\frac{10}{9}n_f\right)+\mathcal{O}(\alpha_s^2)\right]$$

New SM Physics: Toponium

e^+e^- predictions

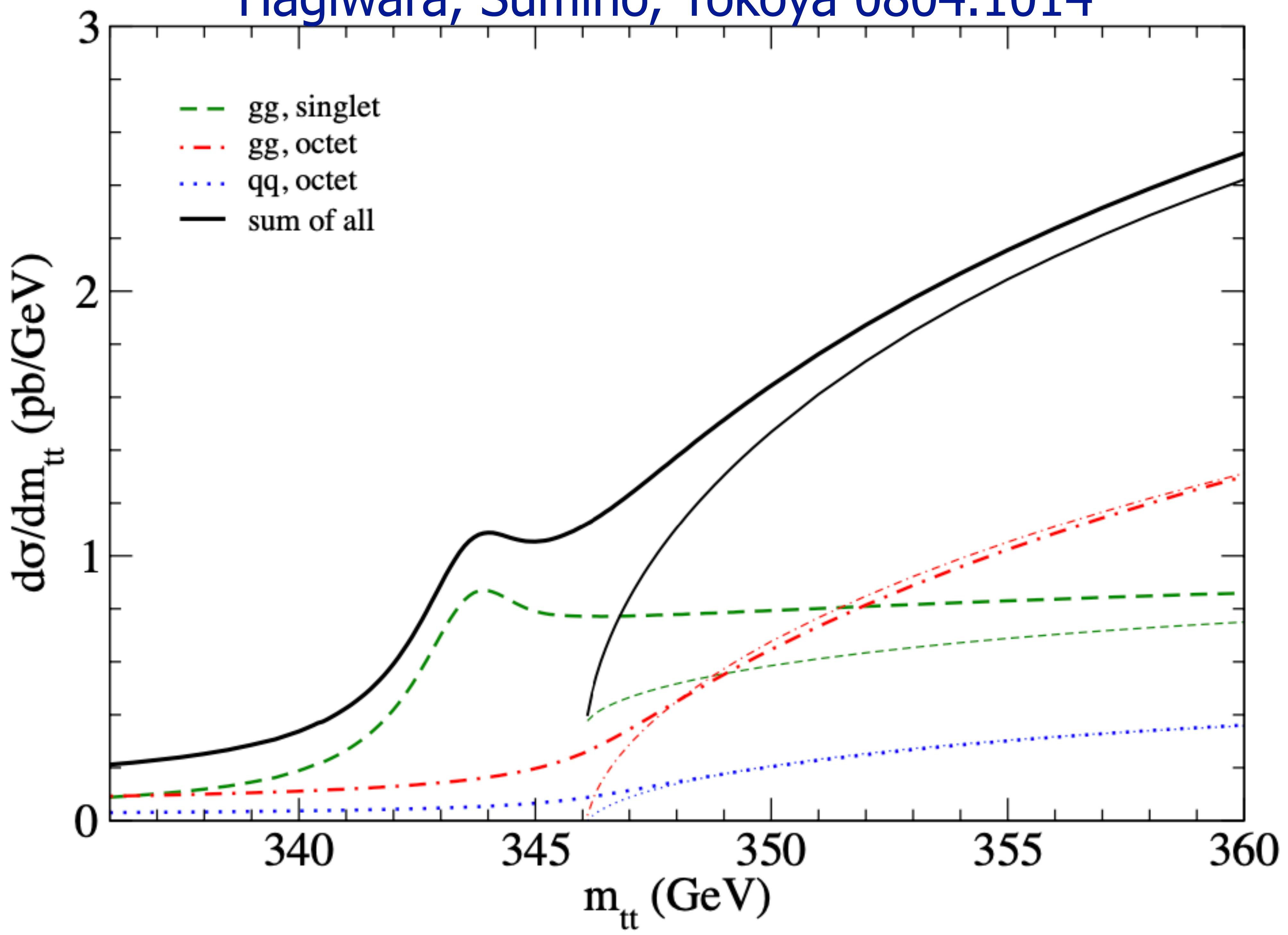


Bach et al. 1712.02220

Fully differential NLO+LL, Coulomb Resummation

LHC predictions

Hagiwara, Sumino, Yokoya 0804.1014



Coulomb Resummation

Needs matching between below threshold, toponium region, continuum

Toponium modelling

FM et al. 2404.08049

We can approximate the impact in the Monte Carlo by introducing a toy model with a resonance

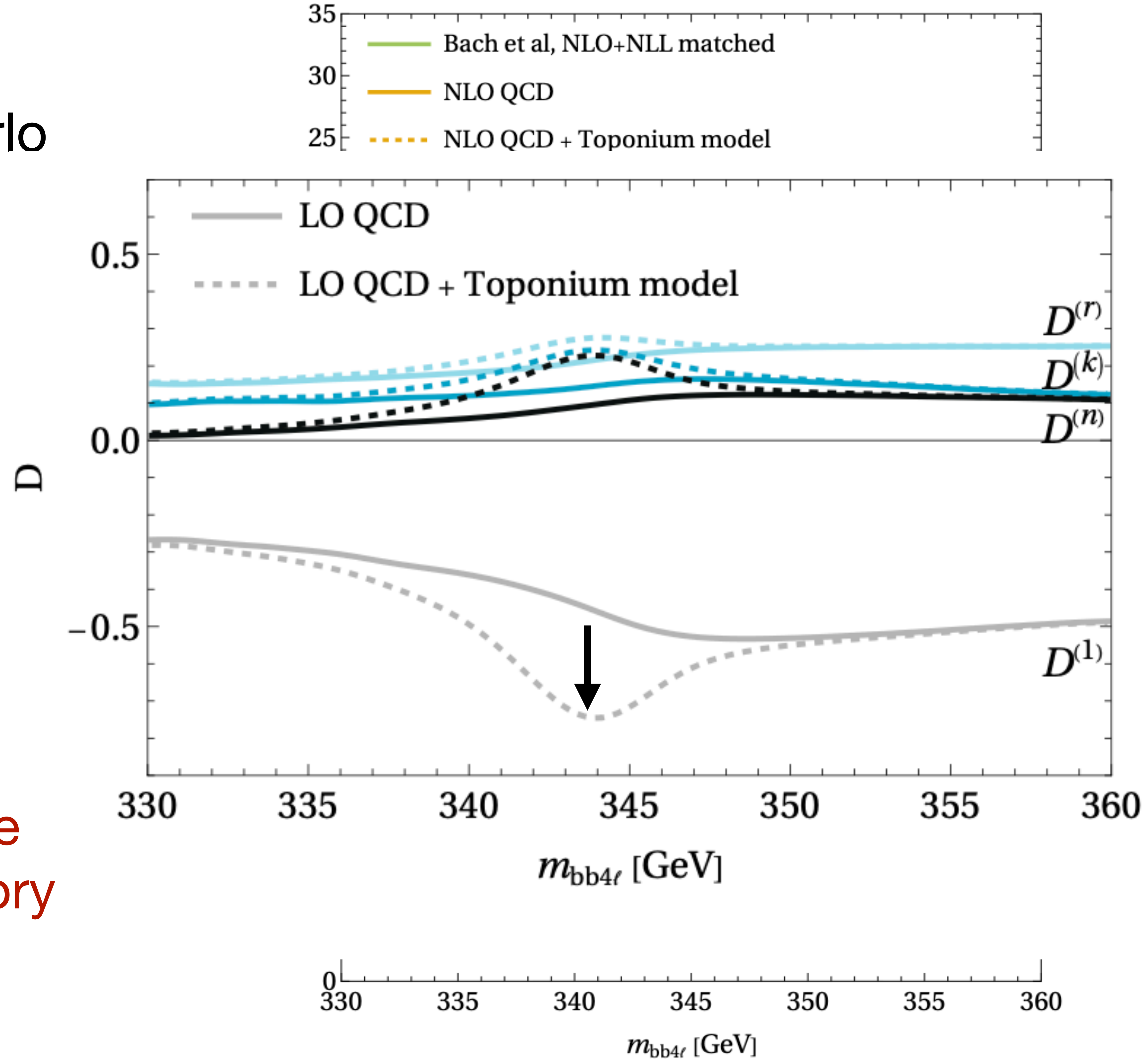
- vector resonance for lepton collisions
- psedoscalar resonance for proton collisions

$$m_\psi = m_\eta \simeq 2m_\mathrm{t} - 2\,\mathrm{GeV}, \quad \text{and} \quad \Gamma_\psi = \Gamma_\eta \simeq 2\,\Gamma_\mathrm{t}$$

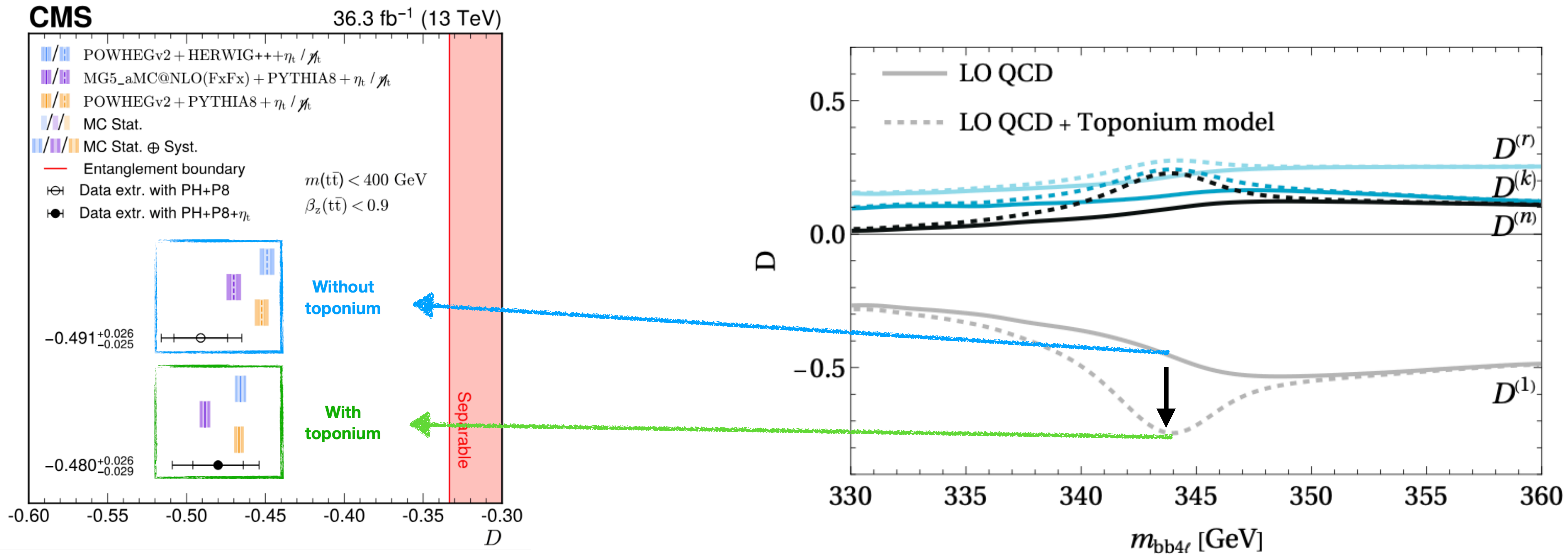
Peak of resonance fitted to match the results obtained by the resummed computation

Significant impact on entanglement markers, hence improvement of measurement agreement with theory

Pseudoscalar resonance leads to different spin correlations compared to QCD

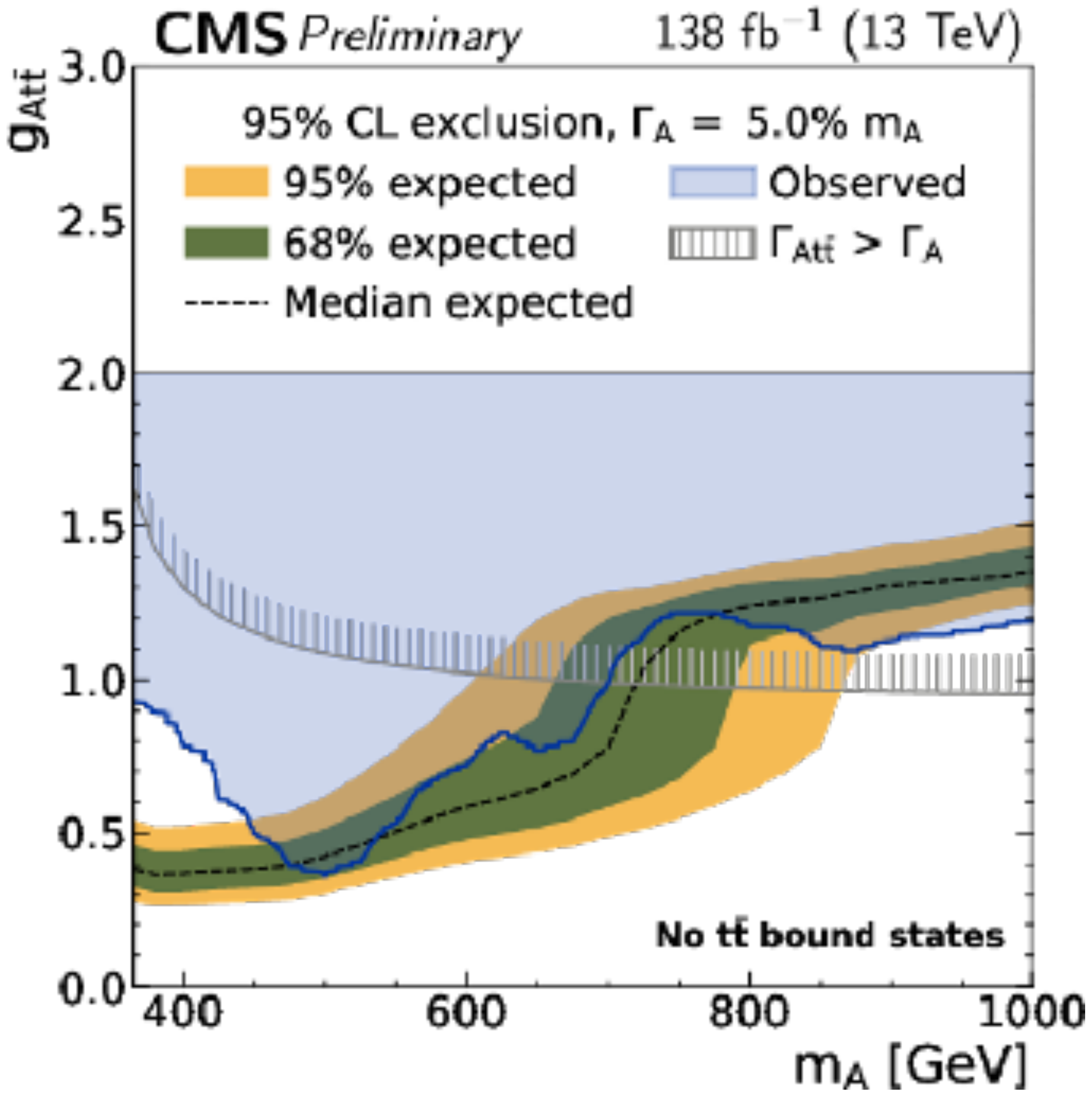


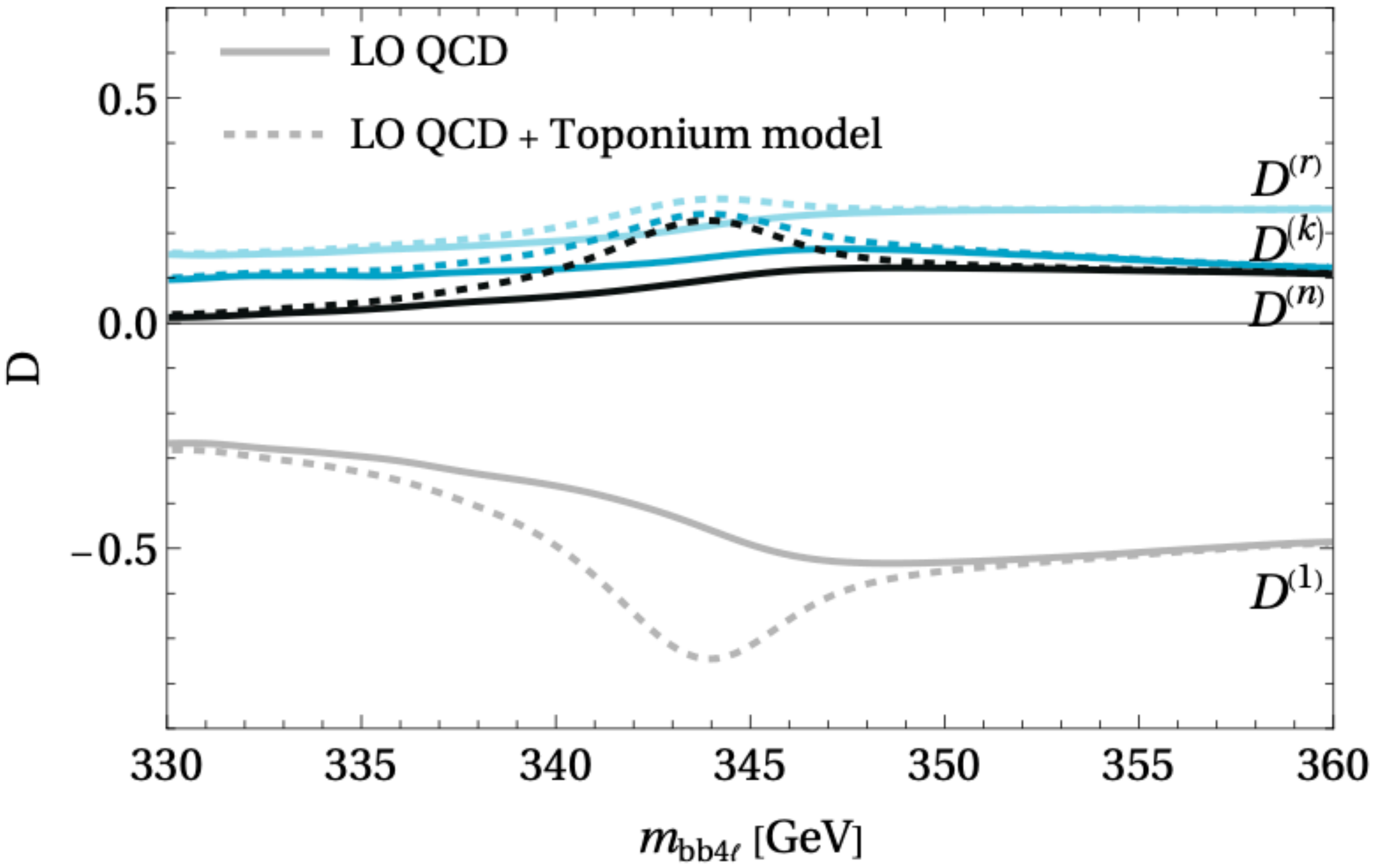
First measurements



Tantalising indication of New (SM) Physics!

First measurements



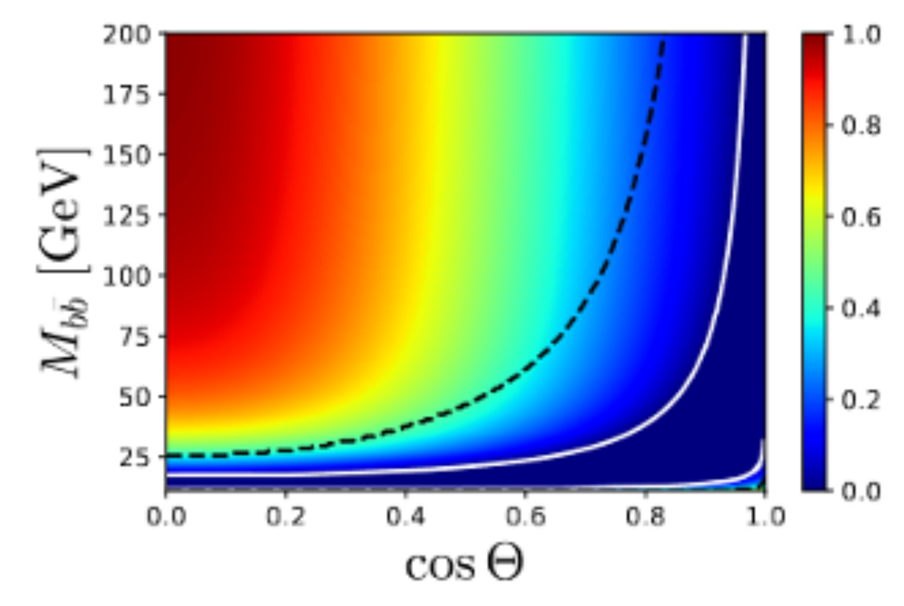


Tantalising indication of New (SM) Physics!

New directions…

Several proposal exist for measurement at HL-LHC:

- Measurement of entanglement between two quark b from Z decay
  - Exploit the propagation of polarization to baryon final state
  - Reconstruct the b spin from the charged lepton in the Λ_b decay chain

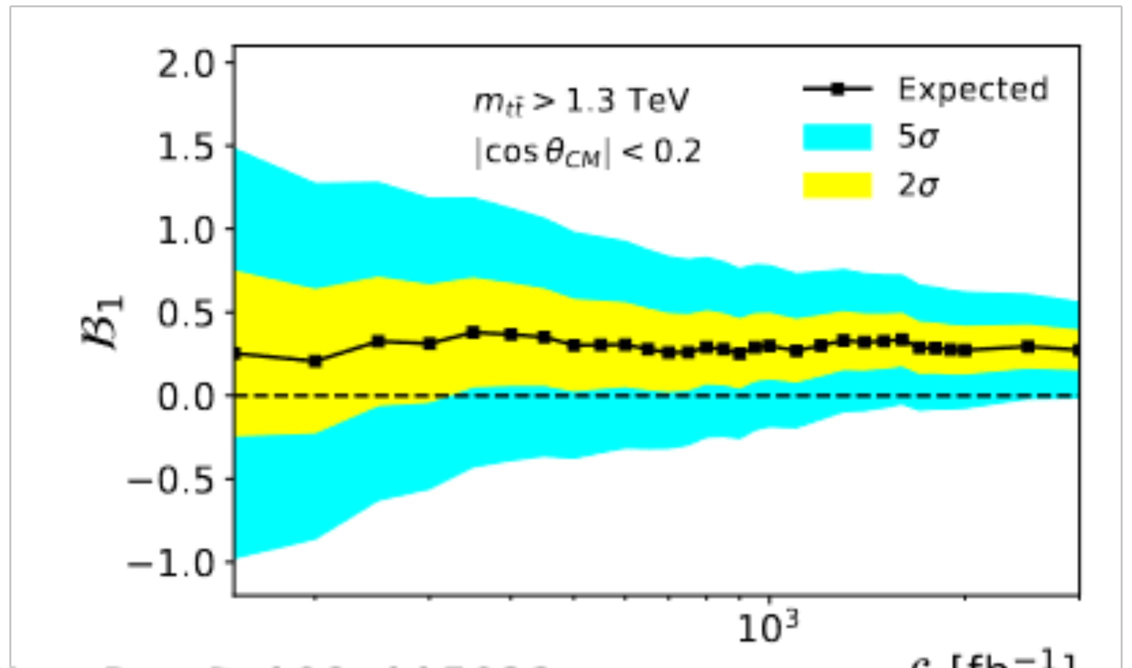


- Observation of entanglement and Bell's inequality violation in H→VV
  - Orthogonal final state compared to top-pair
  - Qutrit highly entangled across the whole phase space
  - Current main limitation is stat in ZZ final state and neutrino reconstruction in WW final state

- Measurement of entanglement in non-resonant di-boson final states
  - High potential for new physics constraints

- Measurement of the post-decay entanglement Phys. Rev. D 109, 096027
  - Study the evolution of entanglement after the decay of one of the particles
  - Proposed in top pair production, between the top and the W
  - **In lepton colliders with polarised beams could also be possible to observe an increase in the entanglement compared to top-pair**

- Observation of BIV violation in top-pair production



Phys. Rev. D 109, 115023

- Multibody entanglement:
  - Across multiple qubit in the final state
  - Between particles and the momentum of the system

- Several proposal based on mesons entanglement
  - Both in terms of flavour and spin
  - Flavour studies suggested to test different decoherence models

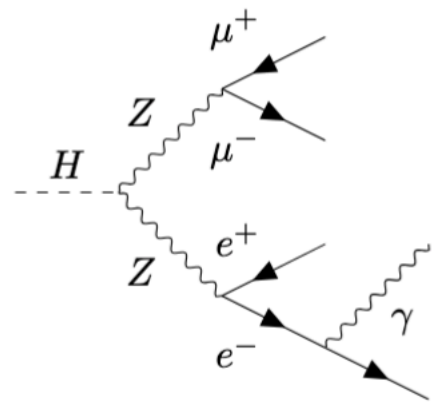
… and new ideas and questions every day!

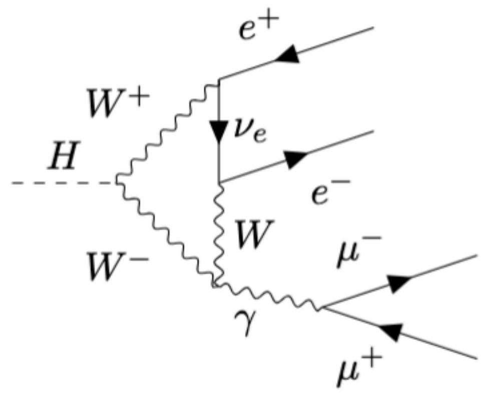
- Is there a relation between symmetries and entanglement? 1812.03138, 2210.12085
- What is the best frame for making quantum measurements? 2311.09166
- How is decoherence happening for collider final states? What about NLO?
- When is the actual measurement of spin really happening? 2401.06854
- Is there an optimal way to do quantum tomography? 2311.09166
- Is there a general approach to quantum measurements at colliders? 2201.03159
- Are there quantities in colliders that can be entangled beyond spin and flavour? Color?
- Is the information entropy a useful quantity in collider physics?
- Are SM interactions minimal in with respect to alternative theories? 2307.08112
- Can multi-partite systems be studied at colliders? 2310.01477
- Is entanglement conserved/augmented/lost in SM interactions? 1703.02989,2209.01405
- Is there a relation between scattering in QFT and computing in IS? 2312.02242, 2310.10838
- Can entanglement be used to do model building? 2307.08112
- What is the analogue of purification at collider processes?
- Can we test Bell-inequalities on an event by event basis?
- What is the analogue of distillation? 2401.06854
- What is the most general constraint on non-locality from scattering processes? 2401.01162
- Entanglement in neutrino oscillations? Many papers, see 2305.06095
- Entanglement and Bell in B0/B0-mixing? Several papers, see 2106.07399
- How should we think about virtual particles? 2211.05782
- Maximal or minimal entanglement as a guiding principle? 1703.02989 vs 2307.08112 and 2410.23343

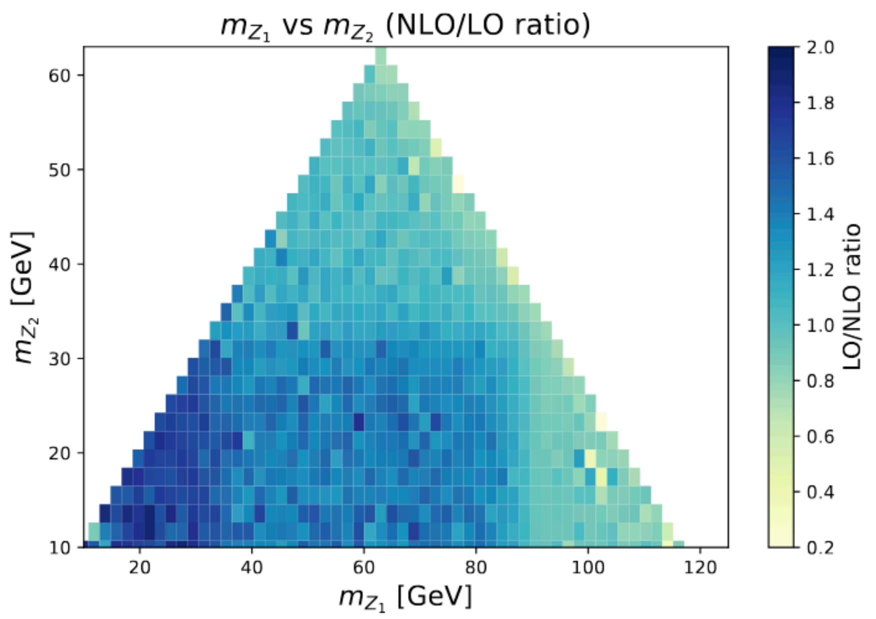
….. new ideas and questions every day!

Higher orders

Grossi, Pelliccioli, Vicini 2409.16731
Del Gratta, Fabbri, Lamba, FM, Pagani, 2504.03841





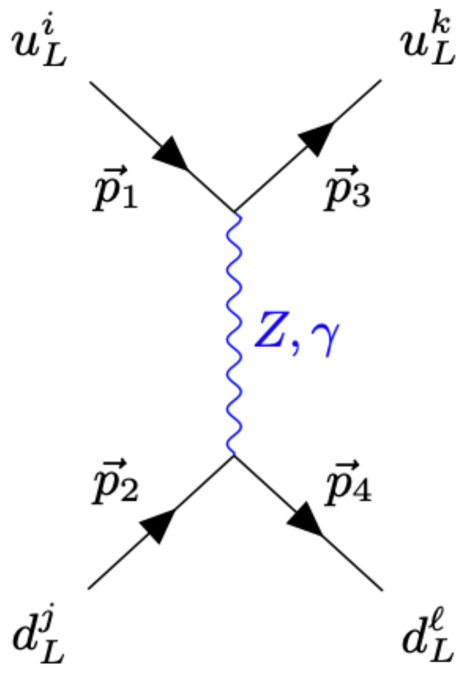


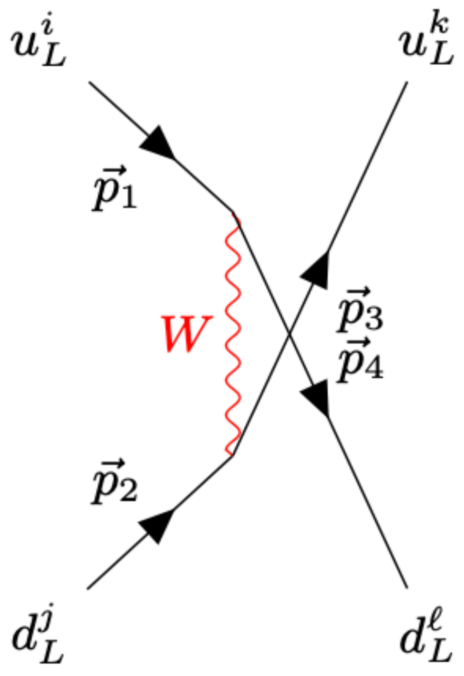
$$\frac{\rho_{\rm NLO}}{\rho_{\rm LO}} = \begin{pmatrix} 0.85 \pm 0.01 & 0.84 \pm 0.01 & 0.988 \pm 0.005 \\ 0.84 \pm 0.01 & 0.978 \pm 0.002 & 0.84 \pm 0.01 \\ 0.988 \pm 0.005 & 0.84 \pm 0.01 & 0.85 \pm 0.02 \end{pmatrix}$$

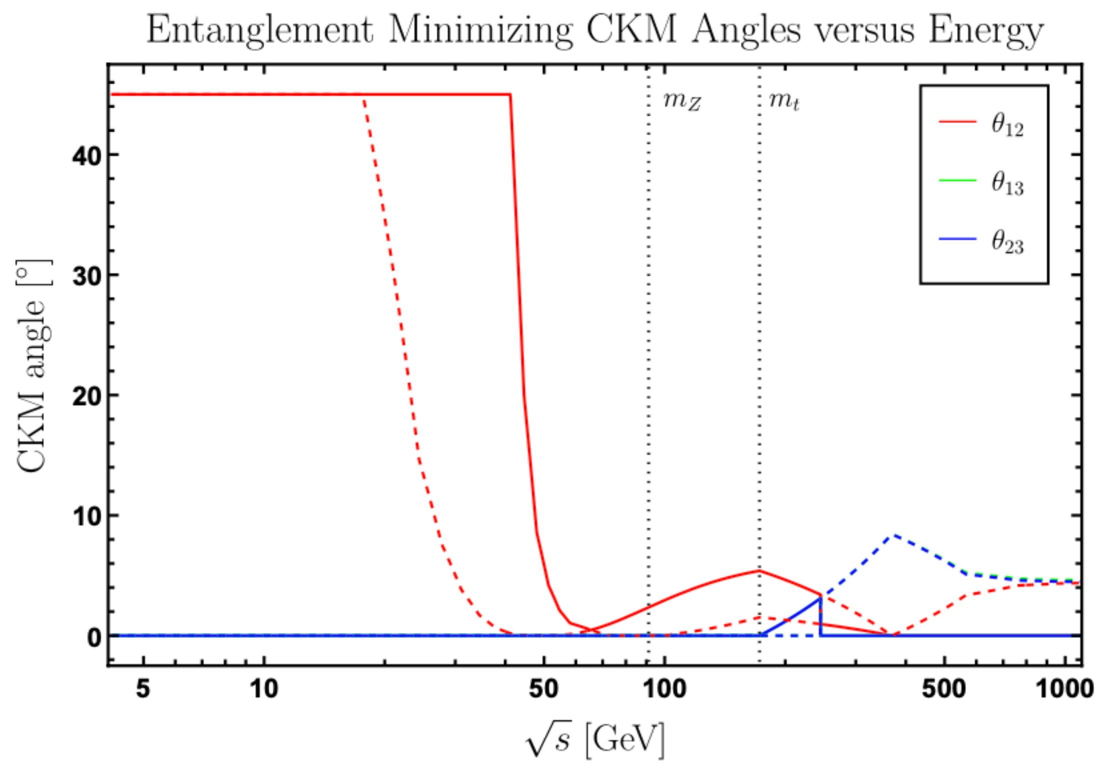
Sizeable effects from NLO EW corrections.
Connected to decoherence.

Max/Min Entanglement as a principle

Cervera-Lierta, Latorre, Rojo, Rottoli,1703.02989 [hep-th]
Thaler et al. 2410.23343 [hep-ph]







Ent. generated by scattering is minimized when the CKM matrix is almost (but not exactly) diagonal and when the PMNS matrix features two large angles and a smaller one,

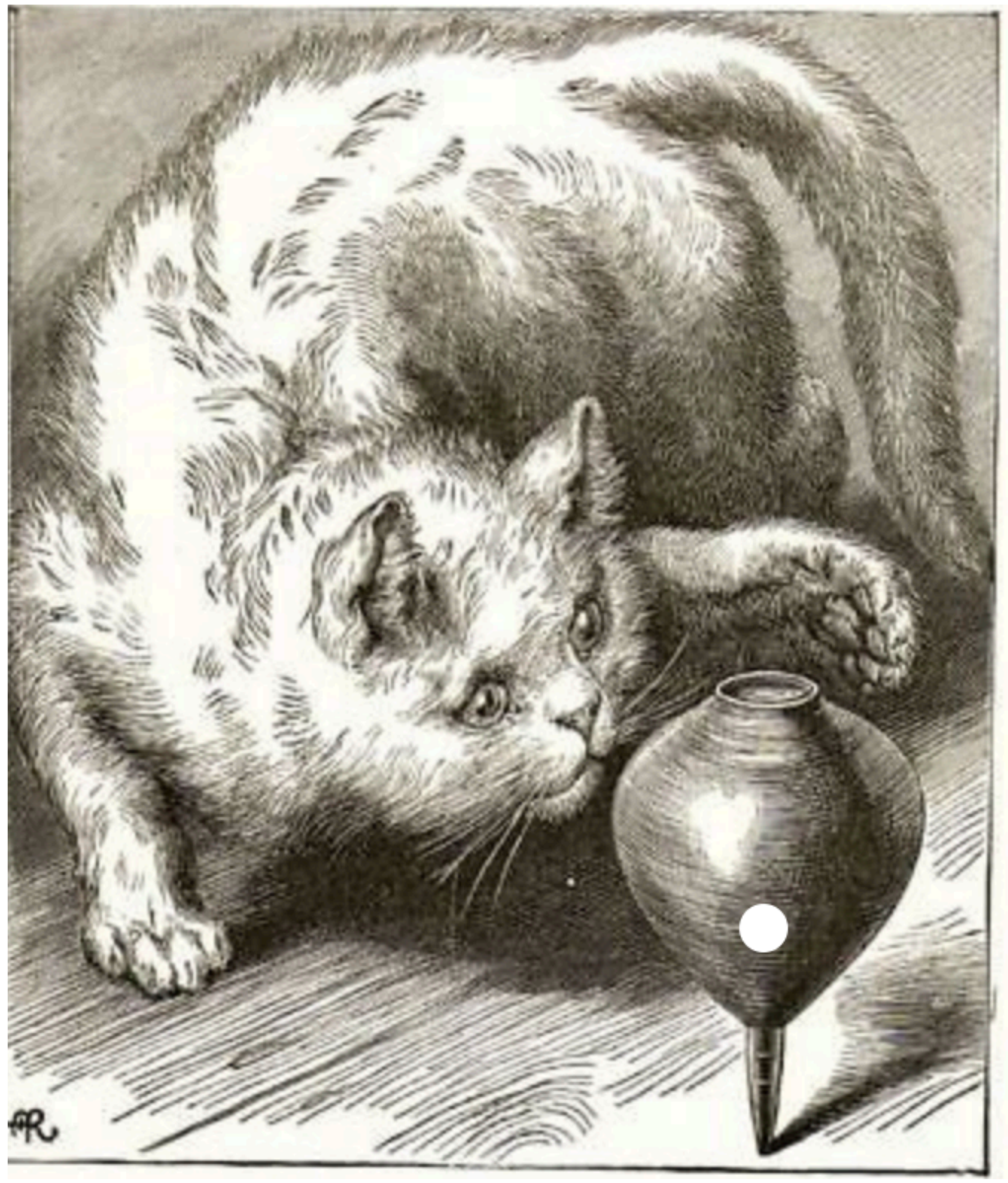
Conclusions

- ❖ Quantum information and computing is hyped up. It promises a quantum advantage that, while not yet proven, could bring to transformative applications.

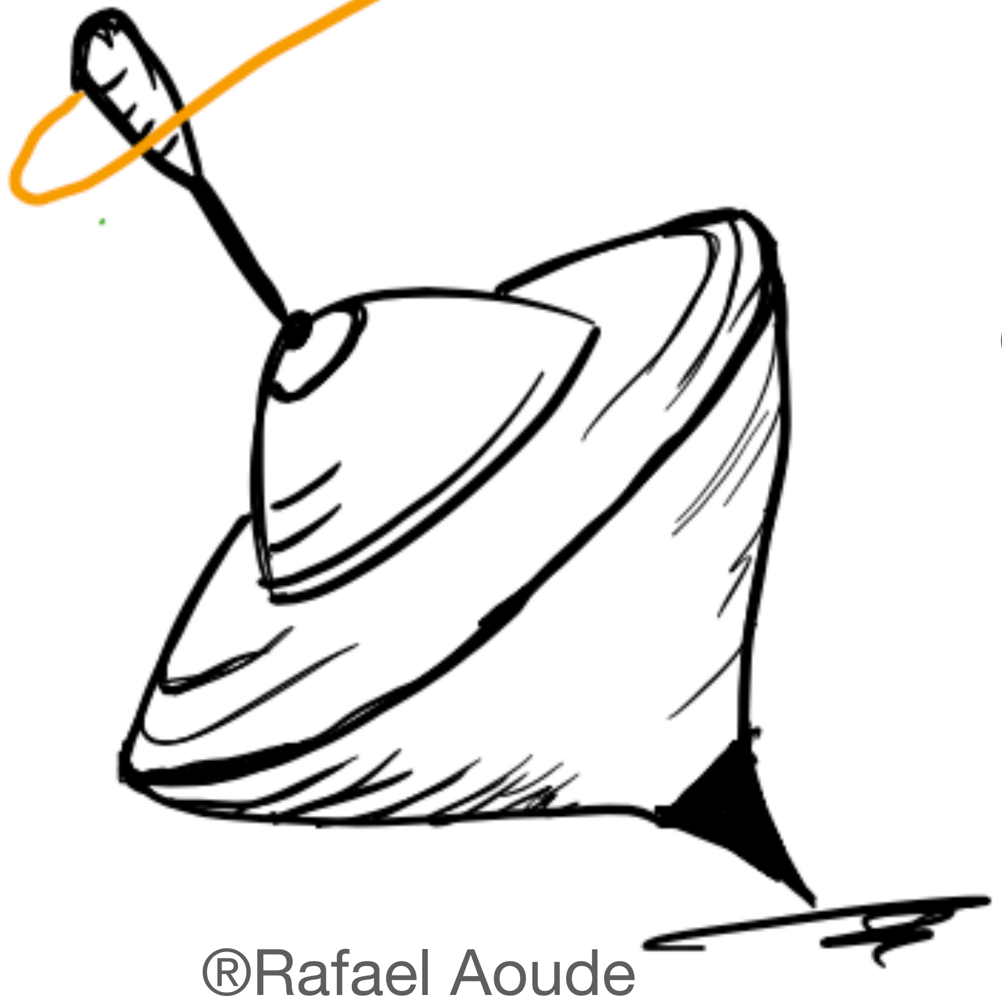
- ❖ The current status builds upon a number of theoretical and experimental advances in the last 30 years that have changed the way we think about quantum mechanics.

- ❖ Our current description of fundamental interactions, based on QFT, has QM at its core. Theoretically, it is embedded in our formalism so deeply that (sometimes) we do not even notice. Experimentally, however, most of our measurements are not correlations, but just counting experiments.

- ❖ A novel interest in looking at fundamental interactions at TeV scale with QI glasses has started since two/three years ago and has quickly lead to a variety of studies and interesting results, …

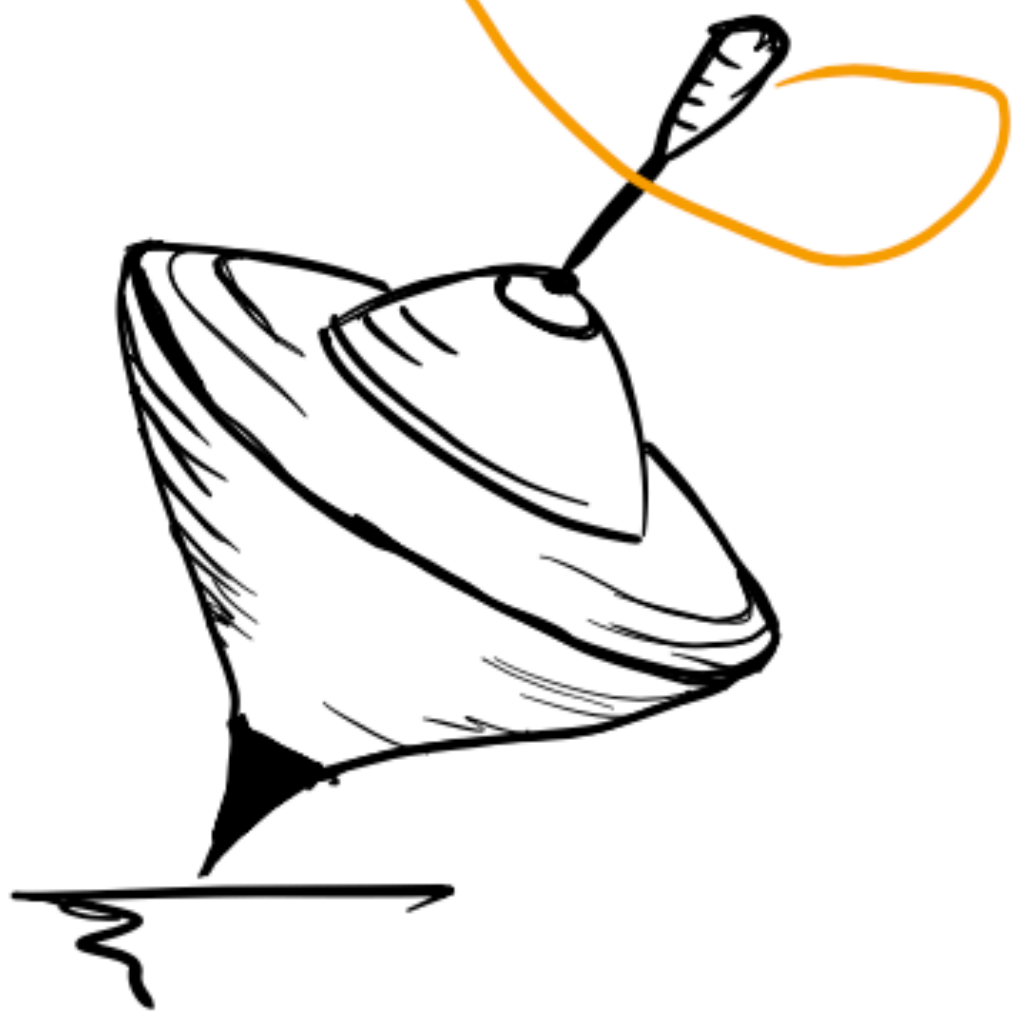


Thank you



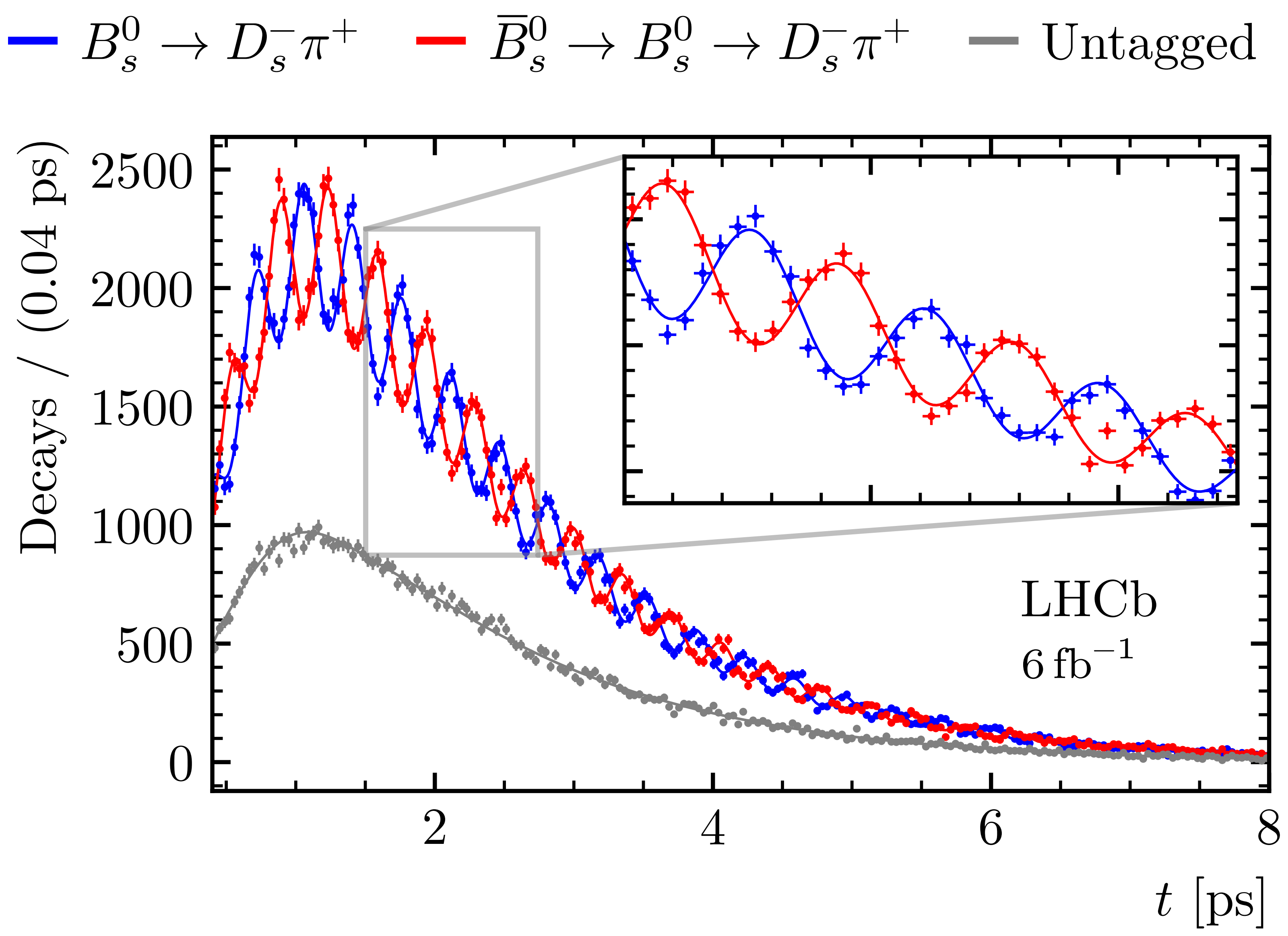
And all collaborators on QI:

Claudio Severi, Christian Degli Esposti Boschi, Max Sioli, Federica Fabbri, Rafael Aoude, Eric Madge, Luca Mantani, Eleni Vryodinou, Simone Tentori, Kentarou Mawatari, Kazuki Sakurai, Priyanka Lamba, Mohammad Altakach.

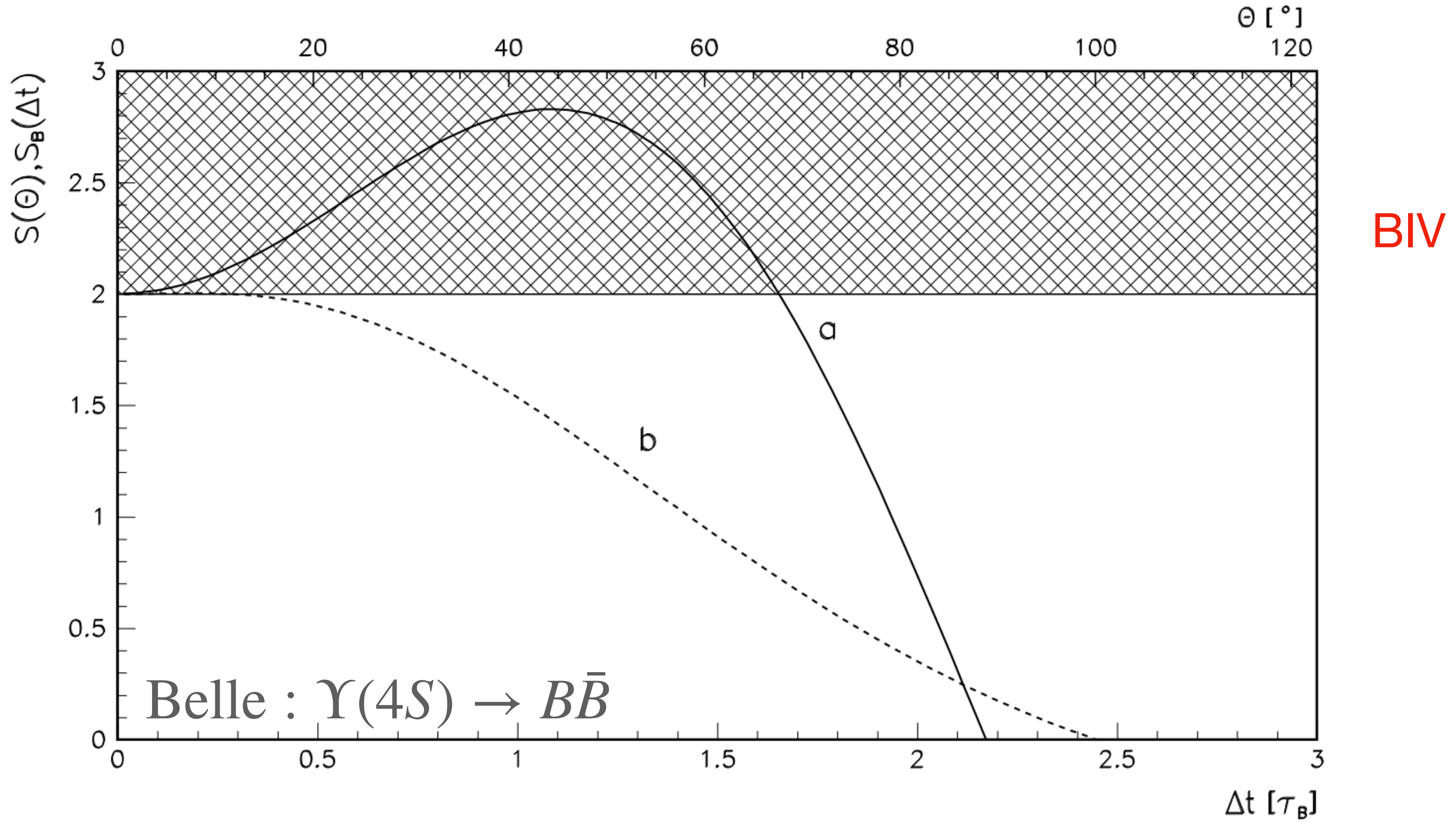


®Rafael Aoude

B-flavour oscillations

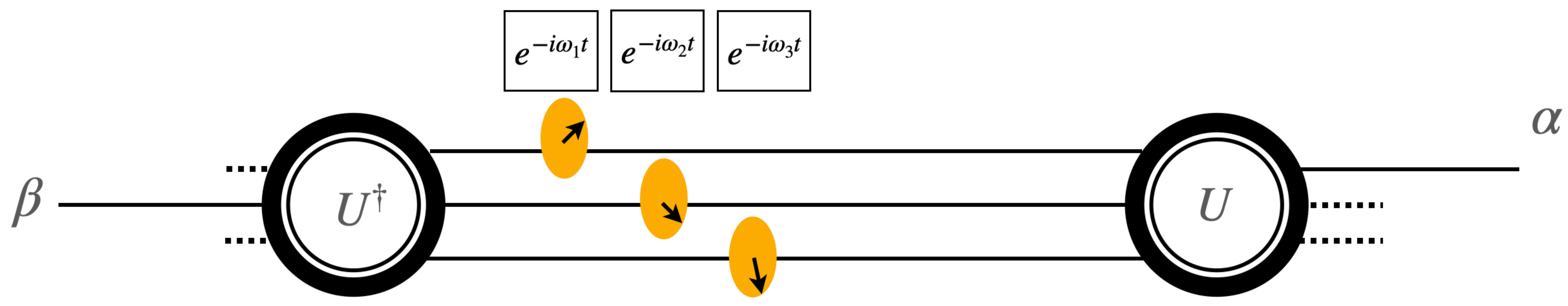


1-leg: Leggett-Garg K3



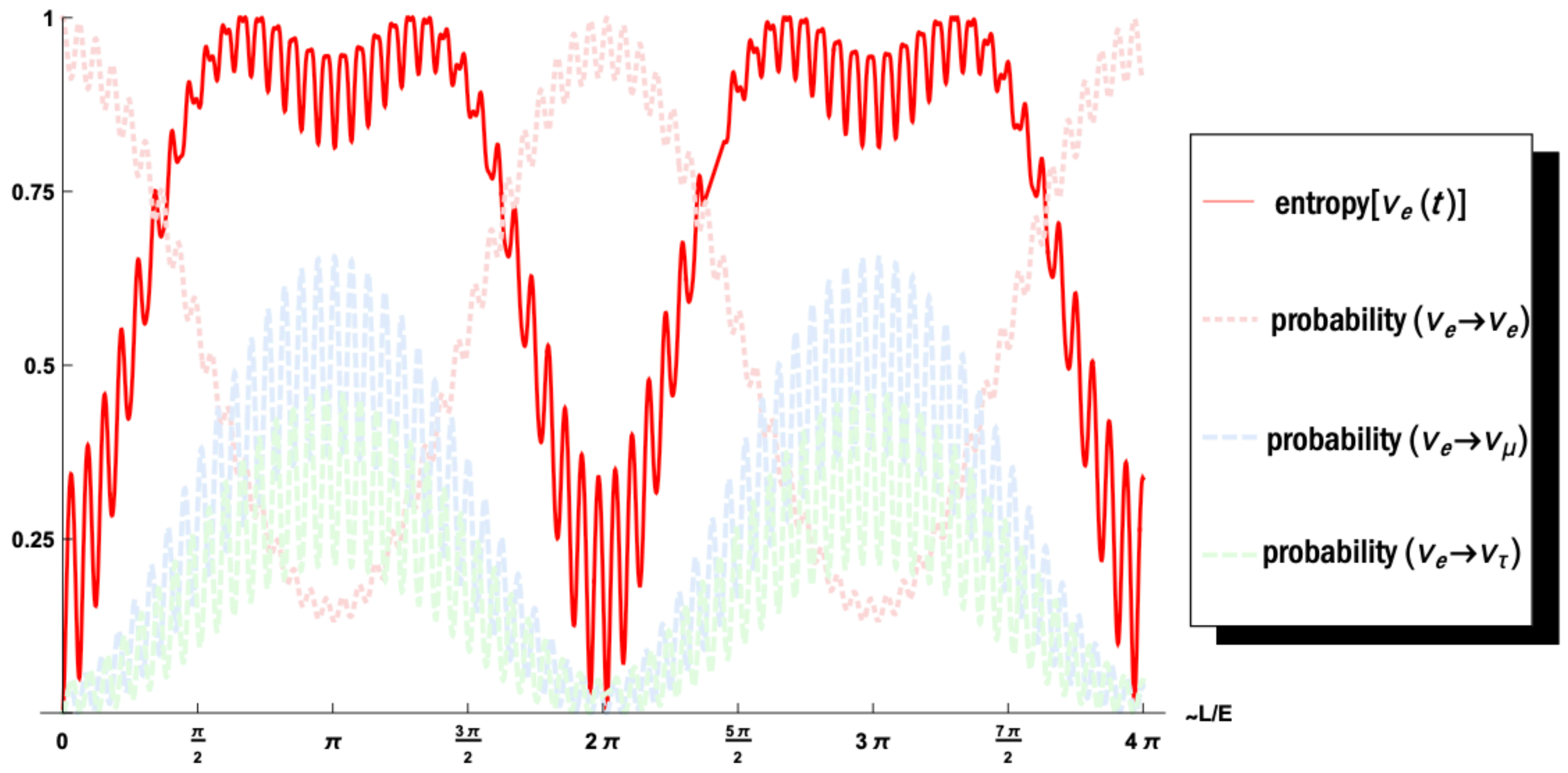
2-leg : Leggett-Garg K4 (like Bell)

Neutrino-flavour oscillations



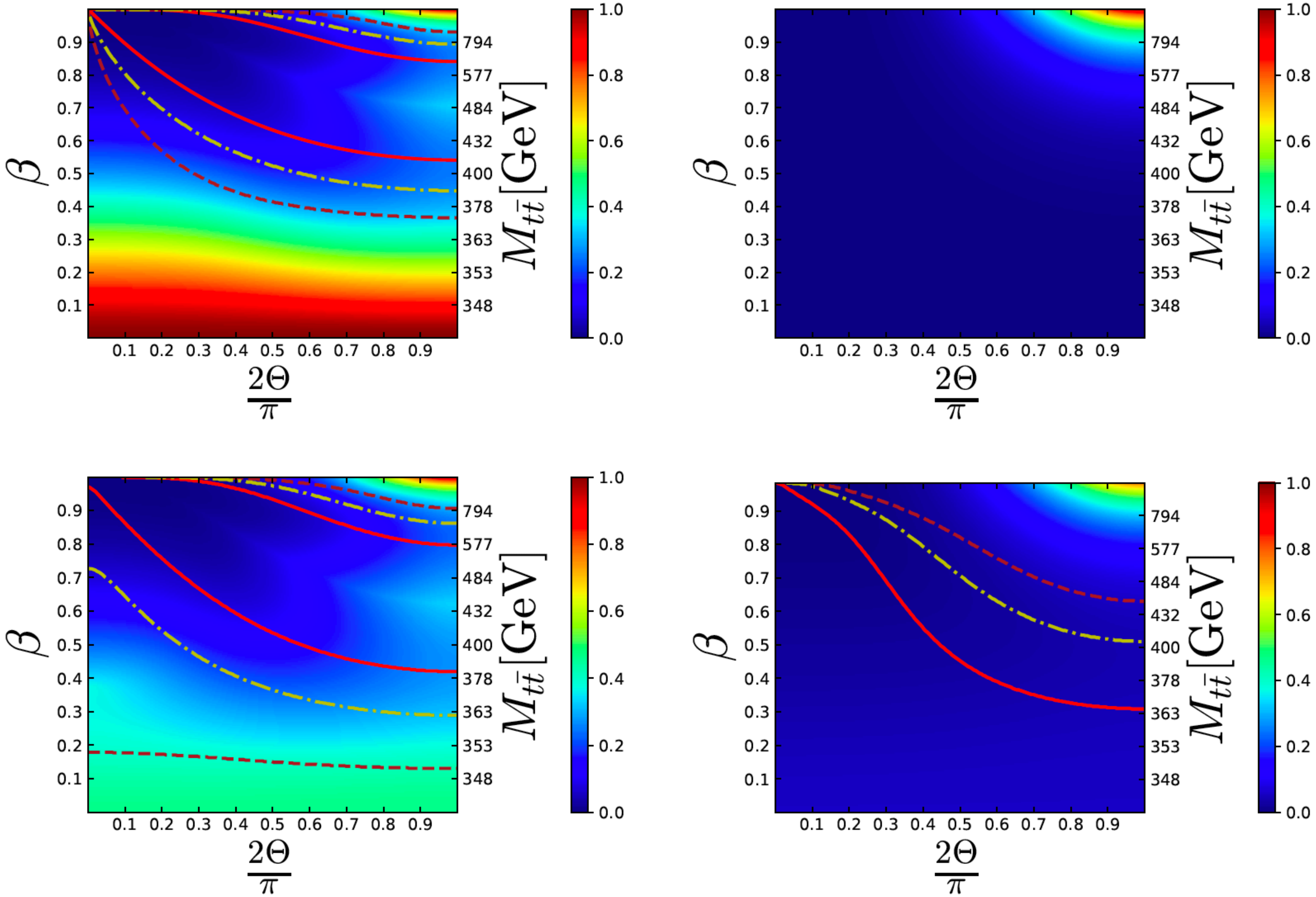
$$|\Psi(t)\rangle_\alpha = a_{\alpha e}(t)\ |100\rangle + a_{\alpha\mu}(t)\ |010\rangle + a_{\alpha\tau}(t)\ |001\rangle$$

Neutrino oscillations as bi- and tri-partite systems [Blasone et al.] to more recent analyses of neutrino oscillations [Banerjee et al.]. See also [Kumar et al.]. Possibility of using quantum observables to access the mass hierarchy [Dixit et al.], distinguishing between Majorana vs Dirac [Richter et al.]. For a very interesting proposal to use Leggett-Garg violations at different energies was made [Formaggio et al.].



Y. Afik and JRM de Nova: [2209.03969](https://arxiv.org/abs/2209.03969) [quant-ph]

Quantum correlations



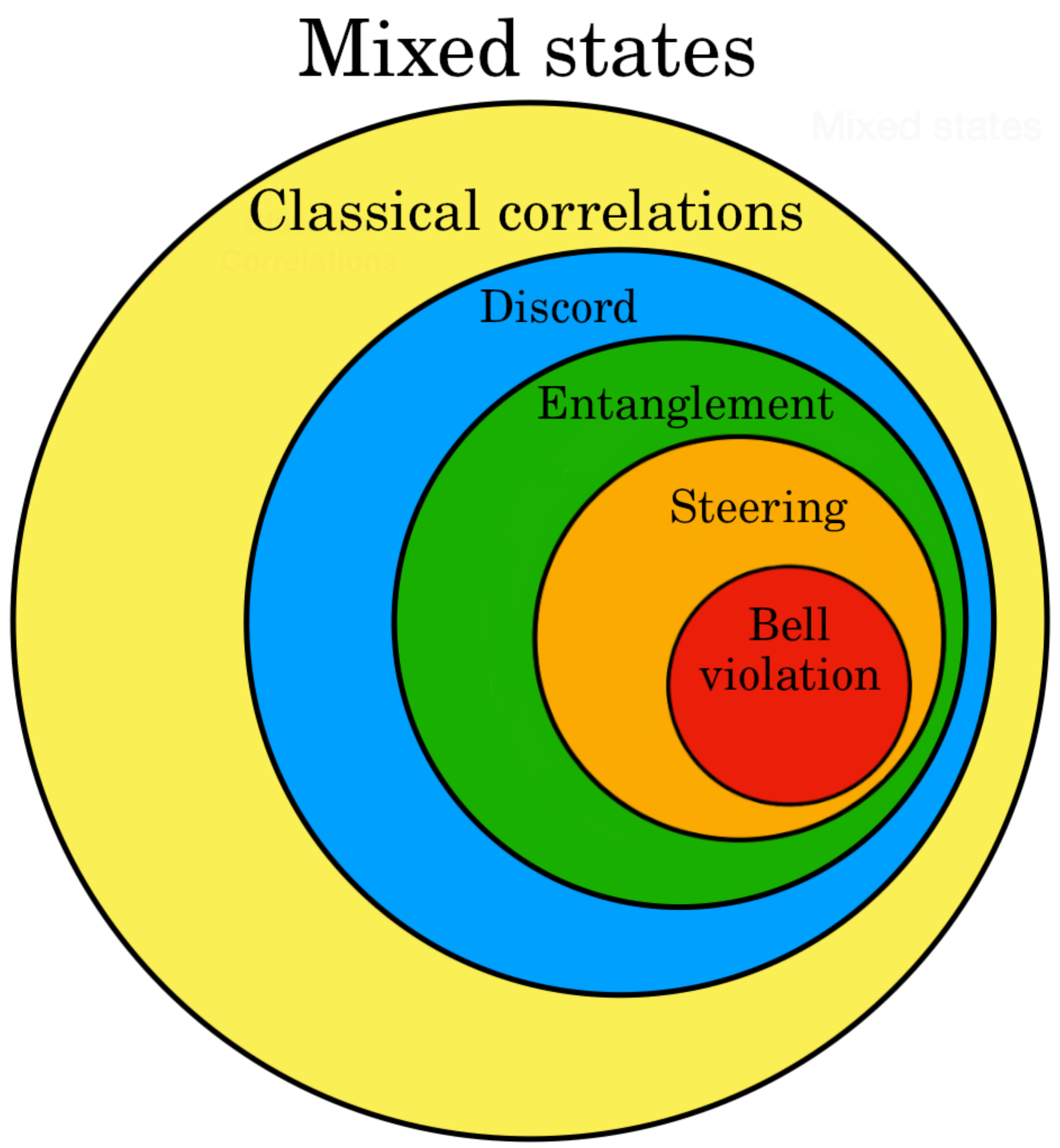


FIG. 2. Quantum discord of the spin density matrix $\rho(M_{t\bar{t}}, \hat{k})$ as a function of the top velocity β and the production angle Θ in the $t\bar{t}$ c.m. frame. All plots are symmetric under $\Theta \to \pi - \Theta$. Upper left: $gg \to t\bar{t}$. Upper right: $q\bar{q} \to t\bar{t}$. Lower left: $t\bar{t}$ production at the LHC for Run 2 c.m. energy, $\sqrt{s} = 13$ TeV [19]. Lower right: $t\bar{t}$ production at the Tevatron for $\sqrt{s} = 2$ TeV, close to its last-run c.m. energy $\sqrt{s} = 1.96$ TeV [15]. Solid red, dashed-dotted yellow, and dashed brown lines are the critical boundaries of separability, steerability, and Bell locality, respectively.

Question

- To say something about quantum, we need non-commuting observables. At the end we are not measuring spin, and amplitudes 2 → 6 depend only on momenta. We only measure momenta, and momenta commute. How can we gain any quantum insight?

- Answer: Our amplitudes have poles! And poles give space/time information.

$$G(t) = \int_{-\infty}^{\infty} \frac{e^{-iEt}\, dE}{E^2 - m_0^2 + i m_0 \Gamma_0} \quad \propto e^{-im_0 t} e^{-\frac{\Gamma_0}{2} t} \theta(t)$$