

# Vacua of Maximal Supergravity

&  $\Lambda_{\text{cosm}}$

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Based on Dall'Agata, G.I. arXiv:1112.3345 [hep-th]



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Nuclear Weak } described by Quantum Field Theory  
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$\hookrightarrow$  Candidate: **String Theory**

- \*  $D = 9 + 1$  spacetime dimensions (even  $10 + 1$ : M-theory)
- \* Supersymmetry!!
- \* ...

Strings  $D = 10 \longrightarrow$  Physics in  $D = 4$

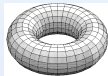
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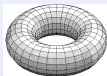
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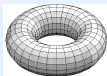
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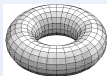
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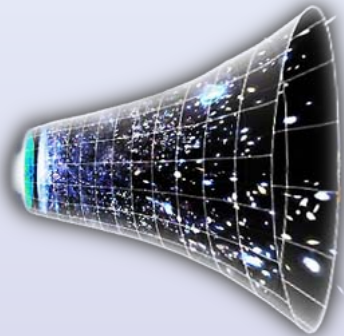
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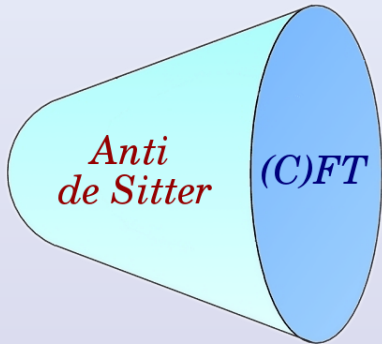
$$E_{vacuum} = \Lambda_{cosmological} > 0$$



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## Supergravity

$$E_{\text{vacuum}} = \Lambda_{\text{cosmological}} < 0$$



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$$V(\phi_{\text{scalar}})_{\text{@ stationary point}} = \Lambda_{\text{cosm.}}$$

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**AIM:** study gauge theories and their vacua  
in Maximal Supergravity



# Overview

- ✓ Supergravity &  $\Lambda_{cosm}$ 
  - Maximal Supergravity in 4D
  - Gauge Interactions, Vacua &  $\Lambda_{cosm}$

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- ✓ renewed interest for *finiteness??*



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## New Approach:

- choice parametrized by one matrix  $\Theta$  {  $G_{gauge}$  "charges"
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[de Wit et al. 2007]

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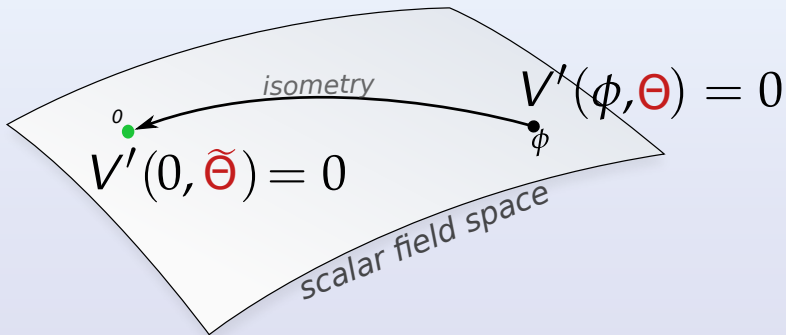
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- $E_{vacuum} = V(\phi_{scalars}, \Theta)$  @ stat. point  $\phi_0$
- ★ redundancy:  $V(\phi, \Theta) = V(0, \tilde{\Theta})$

[also Dibitetto et al 2011]

★ redundancy:  $V(\phi, \Theta) = V(0, \tilde{\Theta})$



Study  $\Theta$  and  $V(0, \Theta) \rightsquigarrow \left\{ \begin{array}{l} \text{several new vacua} \\ \text{different gauge theories!} \end{array} \right.$

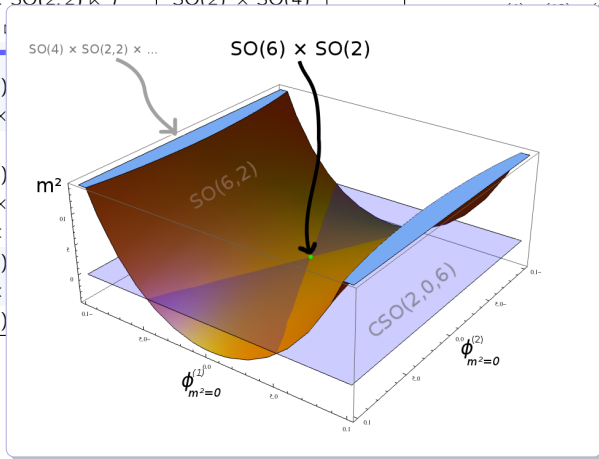
<i>new</i>	$G_{gauge}$	$G_{residual}$	$\Lambda_{cosm}$	$m_{scalars}^2$ (multi.)
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Thank You!





M-theory compactified on  $T^7 \rightarrow D = 4$  Maximal SUGRA

Classical theory enjoys a large duality group:  $E_{7(7)}(\mathbb{R})$

$$\mathfrak{e}_{7(7)} \ni t_{\alpha M}{}^N \equiv \begin{pmatrix} \Lambda_{AB}{}^{CD} & \Sigma_{ABCD} \\ \star \Sigma^{ABCD} & \Lambda'^{AB}{}_{CD} \end{pmatrix}$$

$\Lambda \in \mathfrak{sl}(8, \mathbb{R})$  generate global symmetries of  $S_{\text{ungauged}}$   
(this is a choice of symplectic frame)

**56** = **28** + **28**<sub>(dual)</sub> vector fields:

$$\delta A_{\mu}^M = A_{\mu}^N (\epsilon^{\alpha} t_{\alpha})_N{}^M$$

Scalar fields  $\phi$  parametrize  $E_{7(7)}/SU(8)$

$E_{7(7)}/SU(8)$  can be represented by coset generators:

$$t_{\text{Re}\phi} = \begin{pmatrix} \Lambda^{\text{sym}} & \\ & -\Lambda^{\text{sym}} \end{pmatrix}, \quad t_{\text{Im}\phi} = \begin{pmatrix} & \Sigma^+ \\ \star\Sigma^+ & \end{pmatrix}$$

$\hookrightarrow$  representatives  $L(\phi)_M{}^N$

This is a **real** basis. Complex  $SU(8)$  covariant basis?  
Cayley matrix + "trality":

$$S_M{}^N \equiv \frac{1}{4\sqrt{2}} \begin{pmatrix} \Gamma_{ij}{}^{AB} & \Gamma_{ijAB} \\ -i\Gamma^{ijAB} & i\Gamma^{ij}{}_{AB} \end{pmatrix} \longrightarrow t_\phi = \begin{pmatrix} & \phi \\ \bar{\phi} & \end{pmatrix}$$

Vacua of  $\theta$   $\xi$  gaugings

# Gauging in (Q)FT

In every Field Theory:

- $t_\alpha \equiv$  global symm. generators  $G$
- Vector bosons  $A_\mu^M$ ,  $M = 1, \dots, n_v$

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$$A_\mu^M \quad m_M^r \quad n_r^\alpha \quad t_\alpha$$

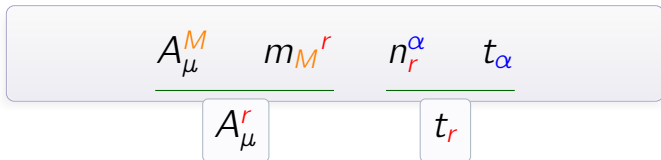
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$$A_\mu^M \quad \underline{m_M^r} \quad n_r^\alpha \quad t_\alpha$$

$$\Theta_M^\alpha$$

**Embedding Tensor**



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$$D_\mu = \partial_\mu - g A_\mu^M \Theta_M^\alpha t_\alpha$$

# $\Theta_M^\alpha$ in Maximal $D = 4$ SUGRA

$$\Theta_M^\alpha \in \mathbf{912} \text{ of } E_{7(7)}$$

$$\Theta_M = \begin{pmatrix} \Theta_\Lambda \\ \Theta^\Lambda \end{pmatrix} \longrightarrow \Theta^\Lambda \text{ couples to } e.m. \text{ dual vectors } A_{\mu\Lambda} !$$

$$\Rightarrow \text{Locality: } \Theta_\Lambda^{[\alpha} \Theta^{\Lambda\beta]} = 0$$

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$$CSO(p, q, r) : \Theta = \begin{pmatrix} \theta_{8 \times 8} \\ 0 \end{pmatrix} \text{ (very schematic...)}$$

$$2\theta^2 - \theta \text{Tr} \theta = 8\Lambda_{\text{cosm}}$$

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$$\text{here: } SO(p, q), SO(p, q) \times SO(p', q') \times Tr : \Theta = \begin{pmatrix} \theta_{8 \times 8} \\ \xi_{8 \times 8} \end{pmatrix}$$

$$\Lambda_{cosm} \sim 2 \text{Tr}(\theta^2 + \xi^2) - \theta \text{Tr} \theta - \xi \text{Tr} \xi \quad \theta \xi \propto \mathbb{1}$$

# Duality vs Symplectic frame

There are two embeddings to specify  
in a  $\mathcal{N} = 8$  SUGRA model:

embedding tensor – symplectic frame

$$\begin{array}{ccccc} & \Theta_M^\alpha & & \mathcal{E}_M^N & \\ & \text{yellow box} & & \text{blue box} & \\ G_{gauge} & \subset & E_{7(7)} & \subset & Sp(56, \mathbb{R}) \\ \text{symmetry} & & \text{duality} & & \text{????} \end{array}$$

**NB:** different action on fields!!

embedding tensor:

$$X_M = \Theta_M^\alpha t_\alpha \qquad \delta L(\phi) = X_M L(\phi)$$

$$\begin{array}{ccc} \cap & & \cap \\ \mathfrak{g}_{\text{gauge}} & \subset & \mathfrak{e}_{7(7)} \end{array}$$

symplectic frame:

$$\mathcal{E} \in GL(28) \backslash Sp(56, \mathbb{R}) / E_{7(7)}$$

$$t_\alpha \quad \rightarrow \quad t'_\alpha = \mathcal{E} t_\alpha \mathcal{E}^{-1}$$

$$\begin{array}{ccc} \cap & & \cap \\ \mathfrak{e}_{7(7)} & \neq & \mathfrak{e}'_{7(7)} \subset \mathfrak{sp}(56, \mathbb{R}) \end{array}$$

$$\Theta_M^\alpha \quad \rightarrow \quad \mathcal{E}_M^N \Theta_N^\alpha$$