Sviluppi del calcolo degli elementi di matrice nucleare per il doppio decadimento beta senza emissione di neutrini

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Meeting della Commissione Scientifica Nazionale II
7-9 Aprile 2025 - Palazzo Loredan, Venezia

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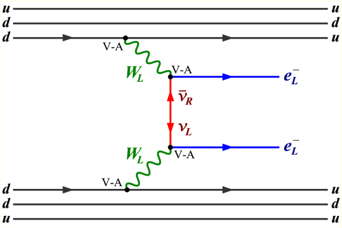
The neutrinoless double β-decay

The detection of the $0\nu\beta\beta$ decay is nowadays one of the main targets in many laboratories all around the world, since its detection would correspond to a violation of the conservation of the leptonic number, and may provide more informations on the nature of the neutrinos and its effective mass



- The inverse of the $0\nu\beta\beta$-decay half-life is proportional to the squared nuclear matrix element $M^{0\nu}$, which relates the parent and grand-daughter wave functions via the decay operator.

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\left|M^{0\nu}\right|^2\left|g_A^2\frac{\langle m_\nu\rangle}{m_e}\right|^2$$

- The calculation of $M^{0\nu}$ links $\left[T_{1/2}^{0\nu}\right]^{-1}$ to the neutrino effective mass $\langle m_\nu\rangle = \left|\sum_k m_k U_{ek}^2\right|$ (light-neutrino exchange)

The structure of $M^{0\nu}$

Within the light-neutrino exchange, the matrix elements $M^{0\nu}_{\alpha}$ are defined in terms of:

- the 1-body transition density matrix elements between the parent (i), daughter (k), grand-daughter (f) nuclei;
- the matrix elements of the Gamow-Teller (GT), Fermi (F), and tensor (T) decay-operators:

$$M^{0\nu}_{\alpha} = \sum_{k} \sum_{j_p j_{p'} j_n j_{n'}} \langle f | a^{\dagger}_{p} a_{n} | k \rangle \langle k | a^{\dagger}_{p'} a_{n'} | i \rangle \left\langle j_p j_{p'} \mid \tau^{-}_{1} \tau^{-}_{2} \Theta^{k}_{\alpha} \mid j_n j_{n'} \right\rangle$$

with $\alpha = (GT,\ F,\ T)$

$$\begin{aligned}
\Theta^{GT}_{12} &= \vec{\sigma}_1 \cdot \vec{\sigma}_2 H_{GT}(r) \\
\Theta^{F}_{12} &= H_F(r) \\
\Theta^{T}_{12} &= [3\,(\vec{\sigma}_1 \cdot \hat{r})\,(\vec{\sigma}_1 \cdot \hat{r}) \\
&\quad - \vec{\sigma}_1 \cdot \vec{\sigma}_2]\, H_T(r)
\end{aligned}$$

The neutrino potentials H_{α} depend on the energy of the initial, final, and intermediate states:

$$H_{\alpha}(r) = \frac{2R}{\pi} \int_0^{\infty} \frac{j_{\alpha}(qr) h_{\alpha}(q^2) q dq}{q + E_k - (E_i + E_f)/2}$$

The structure of $M^{0\nu}$: the closure approximation

- Only few nuclear models allow the inclusion of a number of intermediate k states, large enough to provide converging results.
- Then, for the most part, nuclear structure calculations resort to the closure approximation by replacing the intermediate-states energies with an average value.
- This approximation leads to shift from the product of 1-body transition-density to 2-body transition-density matrix elements, and simplifies the expression of the neutrino potentials $H_\alpha(r)$

$$E_k - (E_i + E_f)/2 \to \langle E \rangle$$

$$\sum_k \langle f | a_p^\dagger a_n | k \rangle \langle k | a_{p'}^\dagger a_{n'} | i \rangle = \langle f | a_p^\dagger a_n a_{p'}^\dagger a_{n'} | i \rangle$$

$$H_\alpha(r) = \frac{2R}{\pi} \int_0^\infty \frac{j_\alpha(qr) h_\alpha(q^2) q dq}{q + < E >}$$

This approximation works since $q \approx$ 100-200 MeV and model-space excitation energies $E_{exc} \approx$ 10 MeV

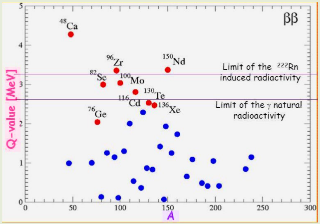
In *Phys. Rev. C* **88**, *064312 (2013)* the impact of the approximation has been evaluated as being within 10% of the exact value

The structure of $M^{0\nu}$: the closure approximation

Bottom line: the closure-approximation allows to write the $0\nu\beta\beta$ nuclear matrix element $M^{0\nu}$ in straightforward form:

$$M^{0\nu}_{\alpha} = \sum_{j_n j_{n'} j_p j_{p'}} \langle f | a^{\dagger}_{p} a_n a^{\dagger}_{p'} a_{n'} | i \rangle \langle j_p j_{p'} \mid \tau^{-}_{1} \tau^{-}_{2} \Theta_{\alpha} \mid j_n j_{n'} \rangle$$

The above expression underlines that to calculate $M^{0\nu}$ one needs to compute the matrix elements of the $0\nu\beta\beta$ decay operator, as well as the wave functions of the parent and grand-daughter nuclei.



However, all candidates of experimental interest lie in a mass region where ”exact” solutions of the nuclear eigenvalue problem cannot be obtained

Theoretical nuclear structure calculations

The study of the many-body Schrödinger equation, for system with $A > 4$, needs the introduction of truncations and approximations, and follows two main approaches:

Mean-field and collective models

- Energy Density Functional (EDF)
- Quasiparticle Random-Phase Approximation (QRPA)
- Interacting Boson Model IBM

Microscopic approaches

- *ab initio* methods
  - No-Core Shell Model (NCSM)
  - Coupled-Cluster Method (CCM)
  - In-Medium Similarity Renormalization Group (IMSRG)
  - Self-Consistent Green's Function approach (SCGF)
- Nuclear Shell Model

The many-body problem

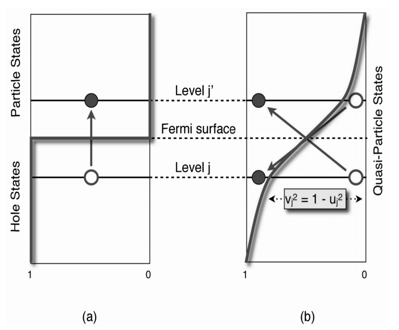
- Mean-field and collective models operate a drastic cut of the nuclear degrees of freedom, the computational problem is alleviated
- Their effective Hamiltonian H_{eff} cannot be derived from realistic nuclear forces and depend from parameters fitted to reproduce a selection of observables
- This reduces the predictive power, that is crucial to search “new physics”

- The degrees of freedom of *ab initio* methods and SM Hamiltonians are the microscopic ones of the single nucleons (very demanding calculations)
- Consequently, they may operate starting from realistic nuclear forces
- These features enhance the predictiveness and the calculated wave functions are more reliable

Collective models: the QRPA

The quasiparticle random-phase approximation is based on the concept of “pairing” among the nucleons.

Particles are substituted with “quasiparticles”.



- Advantage $\rightarrow$ The dimension of the hamiltonian does not scale rapidly with the mass number A as with the shell model.
- Shortcoming $\rightarrow$ Results are strongly dependent on the choice of the free renormalization-parameter g_{pp} (g_{ph} is determined from experiment), that is fixed to reproduce both spectroscopy and GT transitions

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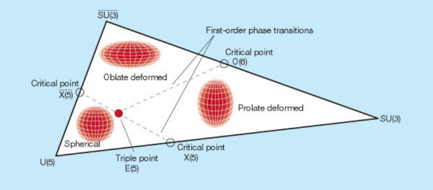
In the interacting boson model identical nucleons are paired so to generate bosons:

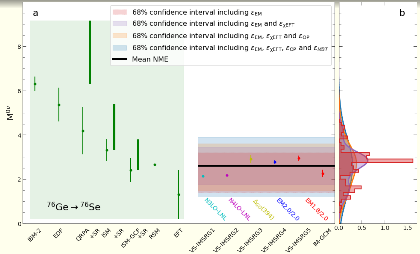
- $L = 0 \rightarrow s$-boson
- $L = 2 \rightarrow d$-boson



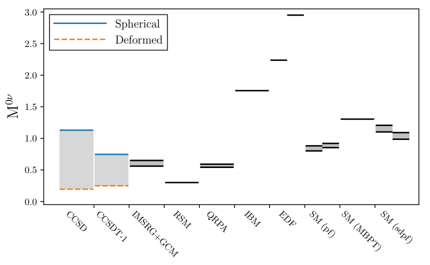
- Advantage $\rightarrow$ The computational complexity is drastically simplified
- Shortcoming $\rightarrow$ The configuration space is strongly reduced

Microscopic models: *Ab initio* methods

Coupled-cluster method CCM and in-medium SRG (IMRSG) calculations have recently performed to calculate $M^{0\nu}$ for the $0\nu\beta\beta$ decay of ^{48}Ca, ^{76}Ge, and ^{82}Se



In-medium SRG



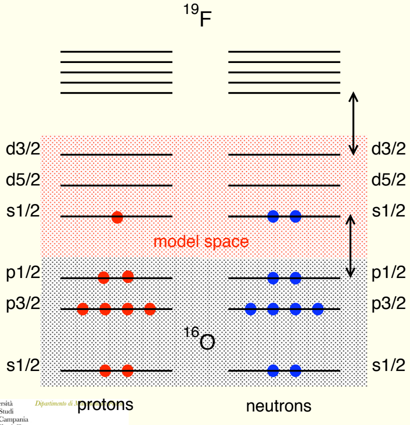
Coupled-Cluster Method

- Advantage → The degrees of freedom of *all* constituent nucleons are included, the number of correlations among nucleons is enormous
- Shortcoming → Highest-degree of computational complexity, the comparison with spectroscopic data is not yet satisfactory

Microscopic models: the realistic shell model

The nucleons are subject to the action of a mean field, that takes into account most of the interaction of the nuclear constituents.

Only valence nucleons interact by way of a residual two-body potential, within a reduced model space.



- Advantage $\rightarrow$ It is a microscopic and flexible model, the degrees of freedom of the valence nucleons are explicitly taken into account.
- Shortcoming $\rightarrow$ High-degree computational complexity.

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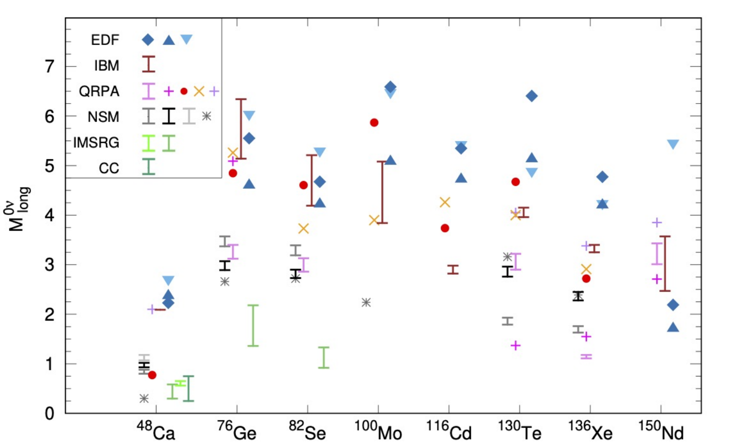
Microscopic models: the realistic shell-model

The nuclear shell model is a microscopic one, then it is possible to construct, within the many-body theory, effective Hamiltonians and decay operators starting from realistic nuclear potentials

Realistic shell model (RSM)

1. Choose a realistic *NN* potential (*NNN*)
2. Renormalize its short range correlations
3. Identify the model space better tailored to study the physics problem
4. Derive the effective shell-model Hamiltonian and consistently effective shell-model operators for decay amplitudes, by way of the many-body perturbation theory
5. Calculate the observables (energies, e.m. transition probabilities, β-decay amplitudes...), using only theoretical SP energies, two-body matrix elements, and effective SM operators.

Nuclear structure calculations



- The spread of nuclear structure calculations evidences inconsistencies among results obtained with different models

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M. Agostini, G. Benato, J. A. Detwiler, J. Menéndez, and F. Vissani, Rev. Mod. Phys. 95, 025002 (2023)

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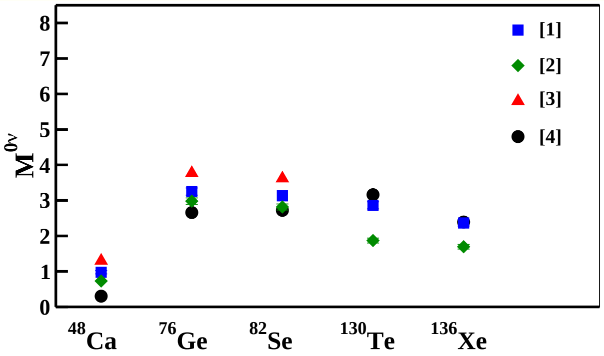
Shell-model calculations of $M^{0\nu}$



1 *J. Menéndez, J. Phys. G* **45***, 014003 (2018).*

2 *A. Neacsu and M. Horoi, Phys. Rev. C* **91***, 024309 (2015), R. A. Sen'kov and M. Horoi, Phys. Rev. C* **88***, 064312 (2013), R. A. Sen'kov, M. Horoi, and B. A. Brown, Phys. Rev. C* **89***, 054304 (2014).*

3 *J. D. Holt and J. Engel, Phys. Rev. C* **87***, 064315 (2013), A. A. Kwiatowski et al., Phys. Rev C* **89***, 045502 (2014).*

4 *LC, A. Gargano, N. Itaco, R. Mancino, and F. Nowacki, Phys. Rev. C* **101** *044315 (2020).*

Ab initio calculations of $M^{0\nu}$

- *Ab initio* methods that can be considered the most complex approach to the calculation of $M^{0\nu}$, in terms of the single-nucleons degrees of freedom of the single nucleons
- All those calculations start from a nuclear Hamiltonian that has been derived through chiral perturbative expansion of an EFT Lagrangian (ChEFT)
- This aspect, in conjuction with the property of *ab initio* methods of being ”size extensive”, allows, in principle, to estimate the theoretical error

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Ab initio calculations of $M^{0\nu}$: results

- The computational complexity of *ab initio* methods makes the calculation of a large number of intermediate states very demanding and complicated, so the validation through the computing of $2\nu\beta\beta$ nuclear matrix elements
- The most important outcome of *ab initio* methods is that the calculated $M^{0\nu}$s are sistematically much smaller than all other nuclear structure calculations

Decay	CCM $M^{0\nu}$	VS-IMSRG $M^{0\nu}$	$T^{0\nu}_{1/2}$ (in yr)
$^{48}\mathrm{Ca}_1 \rightarrow {}^{48}\mathrm{Ti}_1$	$0.25 \leq M^{0\nu} \leq 0.75$[1]	0.58(1)[2]	$> 2 \times 10^{29}$
$^{76}\mathrm{Ge}_1 \rightarrow {}^{76}\mathrm{Se}_1$		2.60 ± 1.4[3]	$> 2 \times 10^{28}$
$^{82}\mathrm{Se}_1 \rightarrow {}^{82}\mathrm{Kr}_1$		1.24(5)[2]	$> 2 \times 10^{28}$

1 *S. Novario et al., Phys. Rev. Lett.* **121** *182502 (2021)*

2 *A. Belley et al., Phys. Rev. Lett.* **126** *042502 (2021)*

3 *A. Belley et al., Phys. Rev. Lett.* **132** *182502 (2024)*

RSM calculations of $M^{0\nu}$: results

Decay	bare operator	$\Theta_{\rm eff}$	
$^{48}Ca \rightarrow {}^{48}Ti$	0.53	0.30	-40%
$^{76}Ge \rightarrow {}^{76}Se$	3.35	2.66	-20%
$^{82}Se \rightarrow {}^{82}Kr$	3.30	2.72	-20%
$^{100}Mo \rightarrow {}^{100}Ru$	3.96	2.24	-40%
$^{130}Te \rightarrow {}^{130}Xe$	3.27	3.16	-3%
$^{136}Xe \rightarrow {}^{136}Ba$	2.47	2.39	-3%

- Results obtained with the effective shell-model operator are relatively reduced with respect those with bare operator: quenching effect is much smaller than the two-neutrino double-β decay

Decay	q	bare operator	quenched operator	
$^{48}Ca \rightarrow {}^{48}Ti$	0.83	0.53	0.40	-20%
$^{76}Ge \rightarrow {}^{76}Se$	0.58	3.35	1.41	-60%
$^{82}Se \rightarrow {}^{82}Kr$	0.56	3.30	1.32	-60%
$^{100}Mo \rightarrow {}^{100}Ru$	0.48	3.96	1.33	-70%
$^{130}Te \rightarrow {}^{130}Xe$	0.68	3.27	1.78	-50%
$^{136}Xe \rightarrow {}^{136}Ba$	0.61	2.47	1.15	-50%

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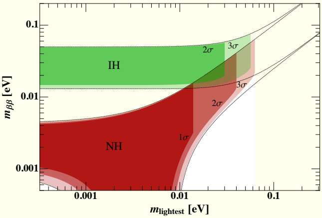
The calculation of $M^{0\nu}$: results



To rule out the Inverted Hierarchy of neutrino mass spectra, the upper bound of neutrino effective mass should be $\langle m_{\beta\beta} \rangle < 18.4 \pm 1.3$ meV.

We could then evaluate the lower bound of the half lives of the decay processes, accordingly to our calculated $M^{0\nu}$

$T^{0\nu}_{1/2}$ (in yr)	$^{76}\mathrm{Ge} \rightarrow ^{76}\mathrm{Se}$	$^{82}\mathrm{Se} \rightarrow ^{82}\mathrm{Kr}$	$^{100}\mathrm{Mo} \rightarrow ^{100}\mathrm{Ru}$	$^{130}\mathrm{Te} \rightarrow ^{130}\mathrm{Xe}$	$^{136}\mathrm{Xe} \rightarrow ^{136}\mathrm{Ba}$
	$> 2 \times 10^{28}$	$> 4 \times 10^{27}$	$> 4 \times 10^{27}$	$> 2 \times 10^{27}$	$> 4 \times 10^{27}$
	(0.3-2.8) $\times 10^{28}$	(1-6) $\times 10^{27}$	(0.4-4) $\times 10^{27}$	(0.5-11) $\times 10^{27}$	(0.9-17) $\times 10^{27}$

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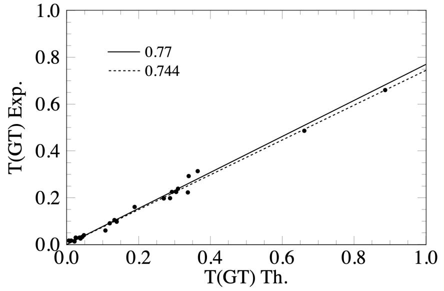
The quenching of g_A

A major issue in the calculation of quantities related to spin-isospin-dependent transitions is the need to quench the axial coupling constant g_A by a factor q in order to reproduce the data.



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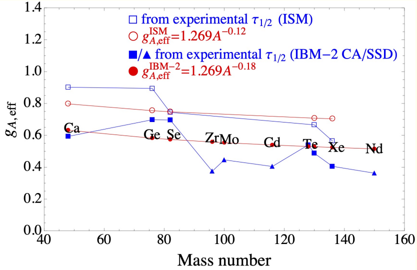

J. Barea, J. Kotila, and F. Iachello, Phys. Rev. C **91**, *034304 (2015)*

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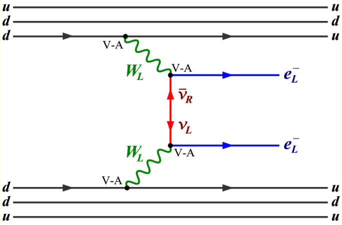
The quenching of g_A

This is an important question when studying $0\nu\beta\beta$ decay, in fact the need of a quenching factor largely affects the value of the half-life $T^{0\nu}_{1/2}$, since the latter would be enlarged by a factor q^{-4}



- The inverse of the $0\nu\beta\beta$-decay half-life is proportional to the squared nuclear matrix element $M^{0\nu}$

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G^{0\nu}\left|M^{0\nu}\right|^2\left|g_A^2\frac{\langle m_\nu\rangle}{m_e}\right|^2$$

- $M^{0\nu}$ links $\left[T^{0\nu}_{1/2}\right]^{-1}$ to the neutrino effective mass $\langle m_\nu\rangle = \left|\sum_k m_k U^2_{ek}\right|$ (light-neutrino exchange)

That is why experimentalists are deeply concerned about q, its value has a strong impact on the sensitivity of the experimental apparatus.

The quenching of g_A

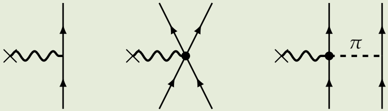
The two main sources of the need of a quenching factor q may be identified as:

Truncation of the nuclear configurations

Nuclear models operate a cut of the nuclear degrees of freedom in order to diagonalize the nuclear Hamiltonian $\Rightarrow$ effective Hamiltonians and decay operators must be considered to account for the neglected configurations in the nuclear wave function

Nucleon internal degrees of freedom

Nucleons are not point-like particles $\Rightarrow$ contributions to the free value of g_A come from two-body meson exchange currents:



- *K. Shimizu, M. Ichimura, and A. Arima, Nucl. Phys. A* **226**, *282 (1974)*
- *I. S. Towner, Phys. Rep.* **155**, *263 (1987)*

The effective operators for decay amplitudes

- Ψ_α eigenstates of the full Hamiltonian H with eigenvalues E_α
- Φ_α eigenvectors obtained diagonalizing $H_{\rm eff}$ in the model space P and corresponding to the same eigenvalues E_α

$$\Rightarrow |\Phi_\alpha\rangle = P\,|\Psi_\alpha\rangle$$

Obviously, for any decay-operator Θ:

$$\langle\Phi_\alpha|\Theta|\Phi_\beta\rangle \neq \langle\Psi_\alpha|\Theta|\Psi_\beta\rangle$$

We then require an effective operator $\Theta_{\rm eff}$ defined as follows

$$\Theta_{\rm eff} = \sum_{\alpha\beta} |\Phi_\alpha\rangle\,\langle\Psi_\alpha|\Theta|\Psi_\beta\rangle\,\langle\Phi_\beta|$$

Consequently, the matrix elements of $\Theta_{\rm eff}$ are

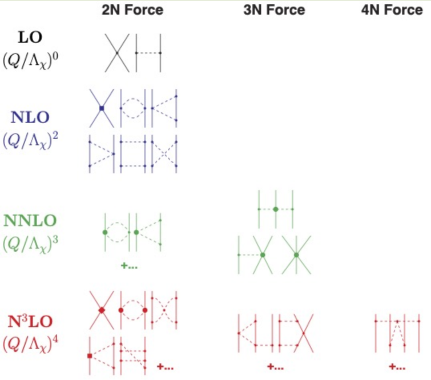
$$\langle\Phi_\alpha|\Theta_{\rm eff}|\Phi_\beta\rangle = \langle\Psi_\alpha|\Theta|\Psi_\beta\rangle$$

This means that the parameters characterizing $\Theta_{\rm eff}$ are renormalized with respect to $\Theta \Rightarrow g_A^{\rm eff} = q \cdot g_A \neq g_A$

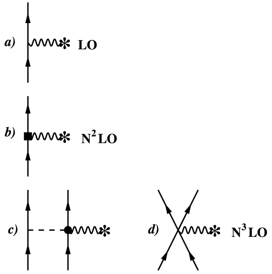
Two-body meson exchange currents

A powerful approach to the derivation of two-body currents (2BC) is to resort to effective field theories (EFT) of quantum chromodynamics.
In such a way, both nuclear potentials and 2BC may be consistently constructed, since in the EFT approach they appear as subleading corrections to the one-body Gamow-Teller (GT) operator $\sigma\tau^{\pm}$.

Nuclear Hamiltonian



Two-body currents



The impact of 2BC on the calculated β-decay properties has been investigated in terms of *ab initio* methods

Nuclear models and predictive power

Nuclear model ⇒ Accurate reproduction of experimental data

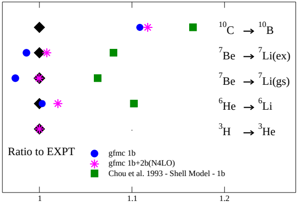
⇓

Predictive power

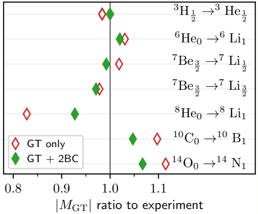
- The predictivity of the calculated $M^{0\nu}$s should be validated through the comparison of experimental β-decay observables and theory
- This validation should also evidence the ability to solve the "quenching puzzle"
- There is a large variety of observables that nuclear models should be able to reproduce, such as GT matrix elements of single- and double-β decay, GT running sums of GT strengths, forbidden β-decay energy spectra, etc.

Ab initio methods: β-decay in light nuclei

GT nuclear matrix elements of the β-decay of *p*-shell nuclei have been calculated with Green's function Monte Carlo (GFMC) and no-core shell model (NCSM) methods, including contributions from 2BC

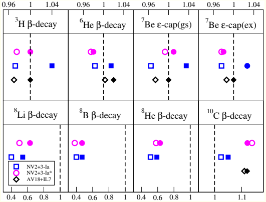


S. Pastore et al., Phys. Rev. C **97** *022501(R) (2018)*



P. Gysbers et al., Nat. Phys. **15** *428 (2019)*

The contribution of 2BC improves systematically the agreement between theory and experiment



G. B. King et al., Phys. Rev. C **102** *025501 (2020)*

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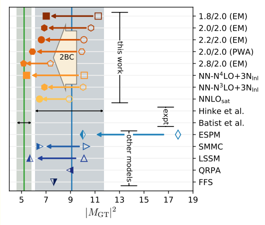
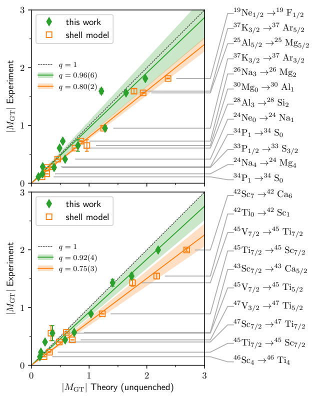
Ab initio methods: β-decay in medium-mass nuclei

Coupled-cluster method CCM and in-medium SRG (IMRSG) calculations have recently performed to overcome the quenching problem g_A to reproduce β-decay observables in heavier systems

P. Gysbers et al., Nat. Phys. **15** *428 (2019)*



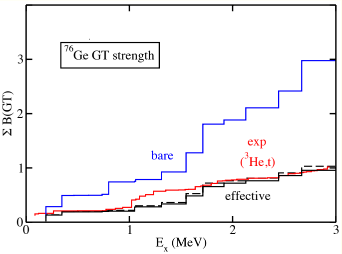
Coupled-Cluster Method

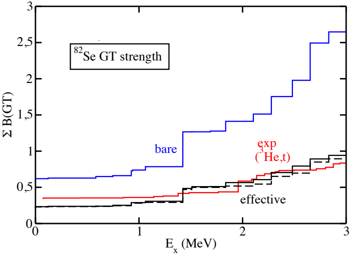


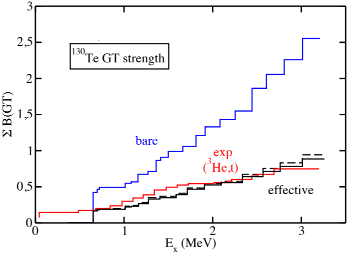
In-Medium SRG

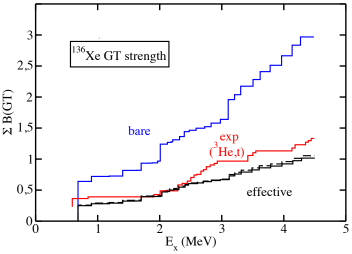
A proper treatment of nuclear correlations and consistency between GT two-body currents and 3N forces, derived in terms of ChPT, explains the "quenching puzzle"

RSM: GT$^-$ running sums









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$$B(p,n) = \frac{\left|\langle \Phi_f || \sum_j \vec{\sigma}_j \tau_j^- || \Phi_i \rangle\right|^2}{2J_i + 1}, \quad (1)$$

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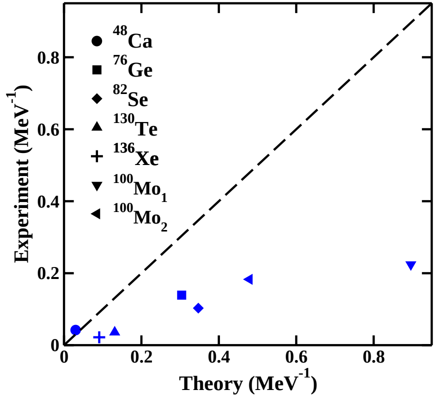
RSM: $2\nu\beta\beta$ nuclear matrix elements



Blue symbols: bare GT operator

Decay	Expt.	Bare
$^{48}\mathrm{Ca}_1 \rightarrow {}^{48}\mathrm{Ti}_1$	0.042 ± 0.004	0.030
$^{76}\mathrm{Ge}_1 \rightarrow {}^{76}\mathrm{Se}_1$	0.129 ± 0.005	0.304
$^{82}\mathrm{Se}_1 \rightarrow {}^{82}\mathrm{Kr}_1$	0.103 ± 0.001	0.347
$^{130}\mathrm{Te}_1 \rightarrow {}^{130}\mathrm{Xe}_1$	0.036 ± 0.001	0.131
$^{136}\mathrm{Xe}_1 \rightarrow {}^{136}\mathrm{Ba}_1$	0.0219 ± 0.0007	0.0910
$^{100}\mathrm{Mo}_1 \rightarrow {}^{100}\mathrm{Ru}_1$	0.224 ± 0.002	0.896
$^{100}\mathrm{Mo}_1 \rightarrow {}^{100}\mathrm{Ru}_2$	0.183 ± 0.006	0.479

Experimental data from *Thies et al, Phys. Rev. C* **86**, *044309 (2012); A. S. Barabash, Universe* **6**, *(2020)*

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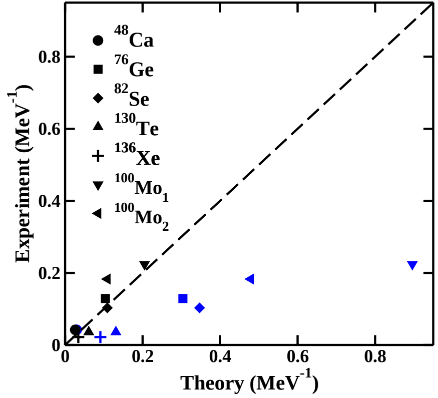
RSM: $2\nu\beta\beta$ nuclear matrix elements



Blue symbols: bare GT operator
Black symbols: effective GT operator

Decay	Expt.	Eff.
$^{48}\mathrm{Ca}_1 \rightarrow {}^{48}\mathrm{Ti}_1$	0.042 ± 0.004	0.026
$^{76}\mathrm{Ge}_1 \rightarrow {}^{76}\mathrm{Se}_1$	0.129 ± 0.005	0.104
$^{82}\mathrm{Se}_1 \rightarrow {}^{82}\mathrm{Kr}_1$	0.103 ± 0.001	0.109
$^{130}\mathrm{Te}_1 \rightarrow {}^{130}\mathrm{Xe}_1$	0.036 ± 0.001	0.061
$^{136}\mathrm{Xe}_1 \rightarrow {}^{136}\mathrm{Ba}_1$	0.0219 ± 0.0007	0.0341
$^{100}\mathrm{Mo}_1 \rightarrow {}^{100}\mathrm{Ru}_1$	0.224 ± 0.002	0.205
$^{100}\mathrm{Mo}_1 \rightarrow {}^{100}\mathrm{Ru}_2$	0.183 ± 0.006	0.109

Experimental data from *Thies et al, Phys. Rev. C* **86**, *044309 (2012); A. S. Barabash, Universe* **6**, *(2020)*

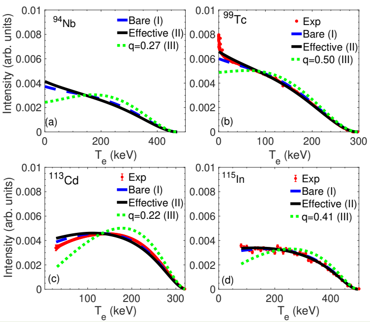
Decay	q
$^{48}\mathrm{Ca} \rightarrow {}^{48}\mathrm{Ti}$	0.83
$^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se}$	0.58
$^{82}\mathrm{Se} \rightarrow {}^{82}\mathrm{Kr}$	0.56
$^{100}\mathrm{Mo} \rightarrow {}^{100}\mathrm{Ru}$	0.48
$^{130}\mathrm{Te} \rightarrow {}^{130}\mathrm{Xe}$	0.68
$^{136}\mathrm{Xe} \rightarrow {}^{136}\mathrm{Ba}$	0.61

- *LC, L. De Angelis, T. Fukui, A. Gargano, N. Itaco, and F. Nowacki, Phys. Rev. C* **100**, *014316 (2019).*
- *LC, N. Itaco, G. De Gregorio, A. Gargano, R. Mancino, and F. Nowacki, Phys. Rev. C* **105** *034312 (2022).*

RSM: forbidden β-decay energy spectra

Forbidden β-decay logfts

Nucleus	Bare	Effective	Exp.
^{94}Nb	11.30	11.58	11.95 (7)
^{99}Tc	11.580	11.876	12.325 (12)
^{113}Cd	21.902	22.493	23.127 (14)
^{115}In	21.22	21.64	22.53 (3)



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G. De Gregorio, R. Mancino, LC, N. Itaco, Phys. Rev. C **110**, *014324 (2024)*

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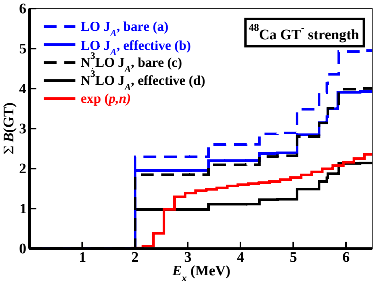
RSM and ChPT: Gamow-Teller observables



$$B(p,n) = \frac{|\langle \Phi_f || \sum \mathbf{J}_A || \Phi_i \rangle|^2}{2J_i + 1}$$

(a) bare $\mathbf{J}_A$ at LO in ChPT (namely the GT operator $g_A \boldsymbol{\sigma} \cdot \boldsymbol{\tau}$);

(b) effective $\mathbf{J}_A$ at LO in ChPT;

(c) bare $\mathbf{J}_A$ at N^3LO in ChPT (namely includy 2BC contributions too);

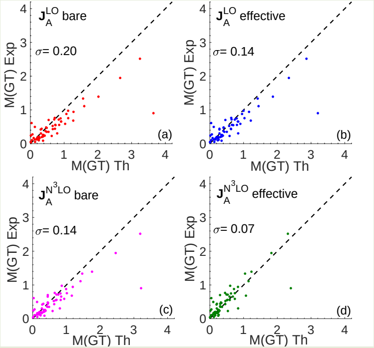
(d) effective $\mathbf{J}_A$ at N^3LO in ChPT.

Total GT$^-$ strength

	(a)	(b)	(c)	(d)	Expt
$\sum B(GT^-)$	24.0	17.5	20.9	11.2	15.3 ± 2.2

The impact of meson-exchange currents on the GT$^-$ matrix elements is $\approx 20\%$

RSM and ChPT: Gamow-Teller observables



GT matrix elements of 60 experimental decays of 43 $0f1p$-shell nuclei, only yrast states involved

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \hat{x}_i)^2}{n}}$$

(a) bare $\mathbf{J}_A$ at LO in ChPT (namely the GT operator $g_A \boldsymbol{\sigma} \cdot \boldsymbol{\tau}$);

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(d) effective $\mathbf{J}_A$ at N^{3}LO in ChPT.

$2\nu\beta\beta$ nuclear matrix element $M^{2\nu}$ ^{48}Ca$\rightarrow$^{48}Ti

$J_i^\pi \rightarrow J_f^\pi$	(a)	(b)	(c)	(d)	Expt
$0_1^+ \rightarrow 0_1^+$	0.057	0.048	0.033	0.019	0.042 ± 0.004
$0_1^+ \rightarrow 2_1^+$	0.131	0.102	0.097	0.057	≤ 0.023
$0_1^+ \rightarrow 0_2^+$	0.102	0.086	0.073	0.040	≤ 2.72

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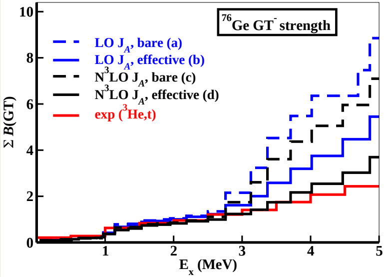
RSM and ChPT: Gamow-Teller observables



$$B(p,n) = \frac{|\langle \Phi_f || \sum \mathbf{J}_A || \Phi_i \rangle|^2}{2J_i + 1}$$

(a) bare $\mathbf{J}_A$ at LO in ChPT (namely the GT operator $g_A \boldsymbol{\sigma} \cdot \boldsymbol{\tau}$);

(b) effective $\mathbf{J}_A$ at LO in ChPT;

(c) bare $\mathbf{J}_A$ at N^{3}LO in ChPT (namely includy 2BC contributions too);

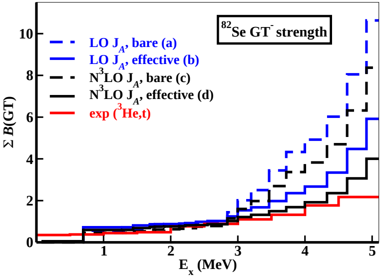
(d) effective $\mathbf{J}_A$ at N^{3}LO in ChPT.

Total GT$^-$ strength

	(a)	(b)	(c)	(d)	Expt
$\sum B(GT^-)$	15.8	10.8	12.8	7.4	$\sim$

The impact of meson-exchange currents on the GT$^-$ matrix elements is $\approx 18\%$

RSM and ChPT: Gamow-Teller observables



$$B(p,n) = \frac{|\langle \Phi_f || \sum \mathbf{J}_A || \Phi_i \rangle|^2}{2J_i + 1}$$

(a) bare $\mathbf{J}_A$ at LO in ChPT (namely the GT operator $g_A \boldsymbol{\sigma} \cdot \boldsymbol{\tau}$);

(b) effective $\mathbf{J}_A$ at LO in ChPT;

(c) bare $\mathbf{J}_A$ at N^{3}LO in ChPT (namely includy 2BC contributions too);

(d) effective $\mathbf{J}_A$ at N^{3}LO in ChPT.

Total GT^{-} strength

	(a)	(b)	(c)	(d)	Expt
$\sum B(GT^{-})$	19.0	11.4	14.9	7.5	$\sim$

The impact of meson-exchange currents on the GT^{-} matrix elements is $\approx 20\%$

RSM and ChPT: Gamow-Teller observables

$2\nu\beta\beta$ nuclear matrix element $M^{2\nu}$ ^{76}Ge$\rightarrow$^{76}Se

$J_i^\pi \rightarrow J_f^\pi$	(a)	(b)	(c)	(d)	Expt
$0_1^+ \rightarrow 0_1^+$	0.211	0.153	0.160	0.118	0.129 ± 0.004
$0_1^+ \rightarrow 2_1^+$	0.023	0.042	0.025	0.048	≤ 0.035
$0_1^+ \rightarrow 0_2^+$	0.009	0.086	0.016	0.063	≤ 0.089

$2\nu\beta\beta$ nuclear matrix element $M^{2\nu}$ ^{82}Se$\rightarrow$^{82}Kr

$J_i^\pi \rightarrow J_f^\pi$	(a)	(b)	(c)	(d)	Expt
$0_1^+ \rightarrow 0_1^+$	0.173	0.123	0.136	0.095	0.103 ± 0.001
$0_1^+ \rightarrow 2_1^+$	0.003	0.006	0.008	0.033	≤ 0.020
$0_1^+ \rightarrow 0_2^+$	0.018	0.007	0.013	0.007	≤ 0.052

L. C., N. Itaco, G. De Gregorio, A. Gargano, Z. H. Cheng, Y. Z. Ma, F. R. Xu, and M. Viviani, Phys. Rev. C **109**, *014301 (2024)*

Conclusions and Outlook

- The new developments of microscopic approaches to the nuclear many-body problem are leading towards reliable calculations of the $0\nu\beta\beta$ nuclear matrix elements.
  This goal may be achieved by focusing theoretical efforts on two main tasks:
  a) improving our knowledge of nuclear forces;
  b) estimation of the theoretical error from the application of many-body methods.
- The efforts of the EFT community are also providing new aspects of our knowledge of the $0\nu\beta\beta$ decay operator
- Nuclear-structure microscopic calculations, when carried out in a fully consistent framework, have proved that the so-called "quenching puzzle" in the study of β decay processes is no longer an issue.
- More benchmark calculations with different theoretical approaches need to to be performed, in order to narrow the spread among different theoretical results

The calculation of $M^{0\nu}$: LO contact transition operator

Within the framework of ChEFT, there is the need to introduce a LO short-range operator, which is missing in standard calculations of $M^{0\nu}$s, to renormalize the operator and make it independent of the ultraviolet regulator

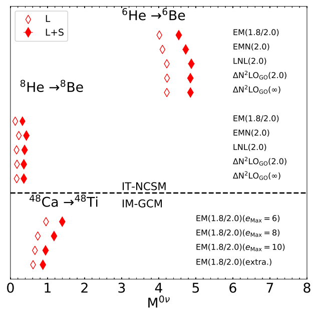
V. Cirigliano et al., Phys. Rev. Lett. **120** *202001 (2020)*

$$M^{0\nu}_{sr} = \frac{1.2A^{1/3}\,\mathrm{fm}}{g_A^2} \langle 0_f^+ | \sum_{n,m} \tau_m^- \tau_n^- \, \mathbf{1} \left[\frac{4 g_\nu^{NN}}{\pi} \int j_0(qr)\, f_S(p/\Lambda_S)\, q^2 dq \right] |0_i^+\rangle$$

The open question is the determination of the low-energy constant g_ν^{NN}

A recent attempt to fit g_ν^{NN} by computing the transition amplitude of the $nn \to ppe^- e^-$ process using nuclear NN and NNN interactions has shown that $M^{0\nu}_{sr}$ enlarges the $M^{0\nu}$ for ^{48}Ca $0\nu\beta\beta$ decay

R. Wirth et al., Phys. Rev. Lett. **127** *242502 (2021)*



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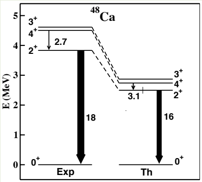
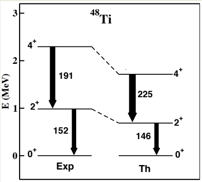
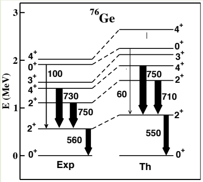
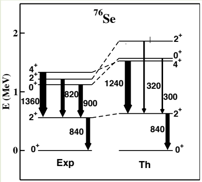
Backup slides

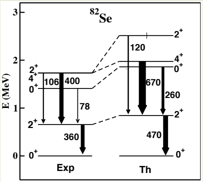
RSM: spectroscopic properties

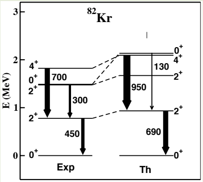






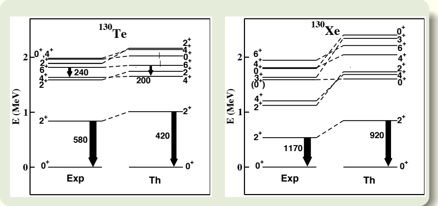


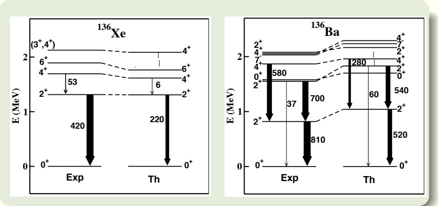


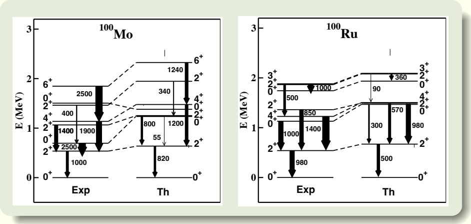


- *LC, L. De Angelis, T. Fukui, A. Gargano, N. Itaco, and F. Nowacki, Phys. Rev. C* **100***, 014316 (2019).*

RSM: spectroscopic properties







- *LC, L. De Angelis, T. Fukui, A. Gargano, and N. Itaco, Phys. Rev. C* **95***, 064324 (2017)*
- *LC, N. Itaco, G. De Gregorio, A. Gargano, R. Mancino, and F. Nowacki, Phys. Rev. C* **105** *034312 (2022)*

Ab initio vs NSM calculations