

Almost general analysis of CKM and MNS matrices
for hierarchical Yukawa structure

and

interpretation of Dirac CP phase
probed by DUNE and T2HK

[2410.08686](#)

Phys.Lett.B 860 (2025) 139169

Masaki Yang @ Saitama University

Current status of leptonic CPV

$$U_{\text{MNS}} = U_e^\dagger U_\nu \quad \text{MNS matrix (\textbf{PDG rep.})}$$

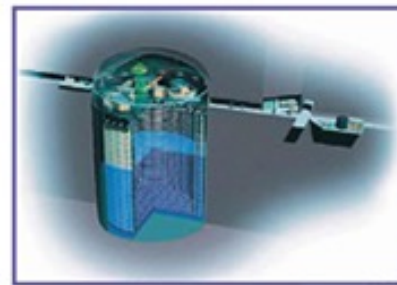
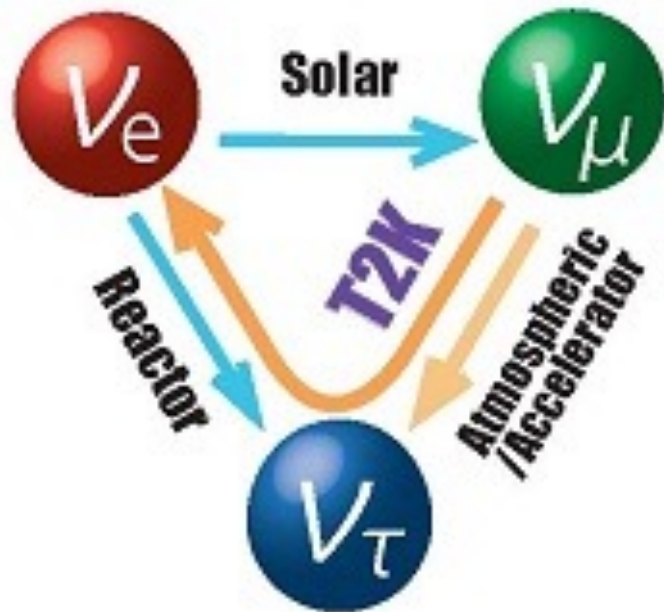
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Current status of leptonic CPV

$U_{\text{MNS}} = U_e^\dagger U_\nu$ MNS matrix (**PDG rep.**)

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

T2K exp.



Super-Kamiokande
(ICRR, Univ. Tokyo)

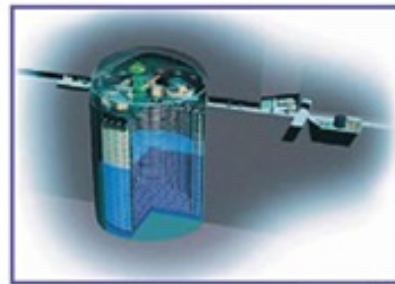
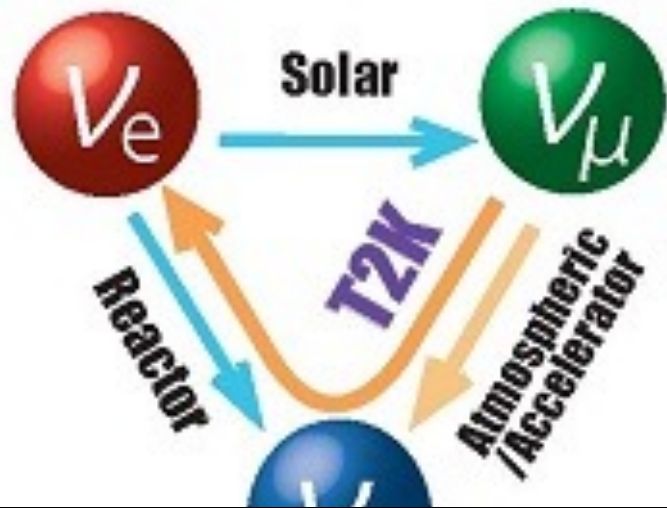


Current status of leptonic CPV

$$U_{\text{MNS}} = U_e^\dagger U_\nu \quad \text{MNS matrix (PDG rep.)}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

T2K exp.



Super-Kamiokande
(ICRR, Univ. Tokyo)



Measure the Dirac phase by comparing $P(\nu_\mu \rightarrow \nu_e)$ with $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$

Current status of leptonic CPV NuFit-6.0, 2410.05380

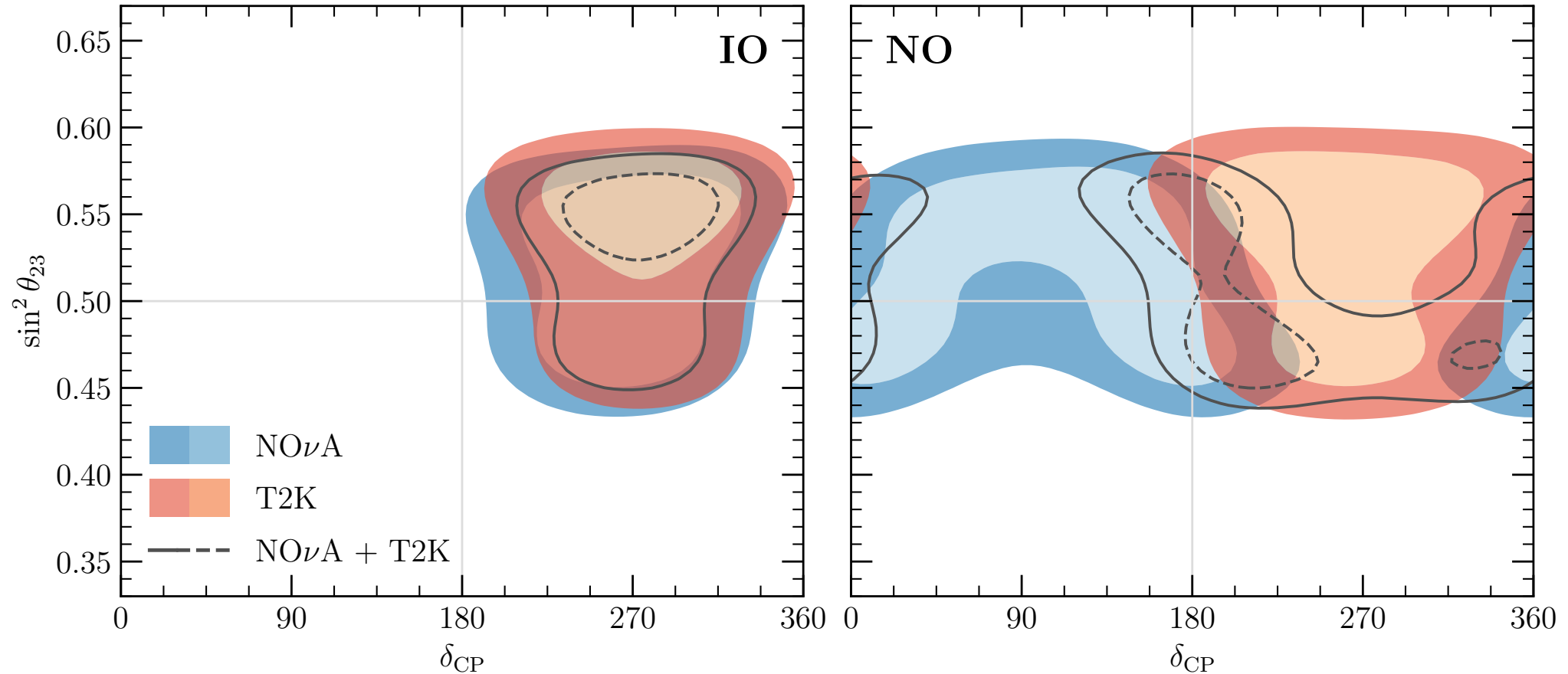


Figure 6. 1σ and 2σ allowed regions (2 dof) for T2K (red shading), NOvA (blue shading) and their combination (black curves). Contours are defined with respect to the local minimum for IO (left) or NO (right). We fix $\sin^2 \theta_{13} = 0.0222$, $\sin^2 \theta_{12} = 0.31$, $\Delta m_{21}^2 = 7.5 \times 10^{-5} \text{ eV}^2$ and minimize with respect to $|\Delta m_{3\ell}^2|$.

Current status of leptonic CPV

NuFit-6.0, 2410.05380

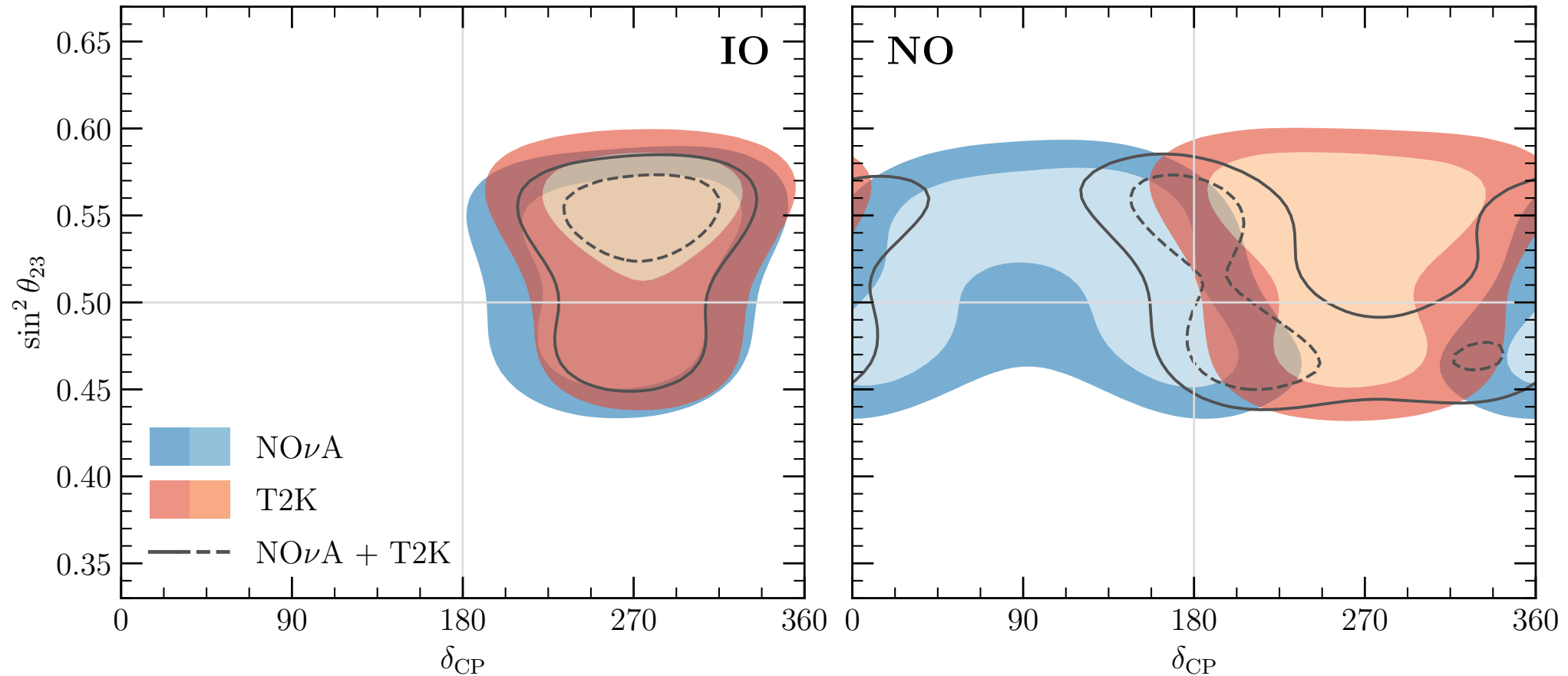
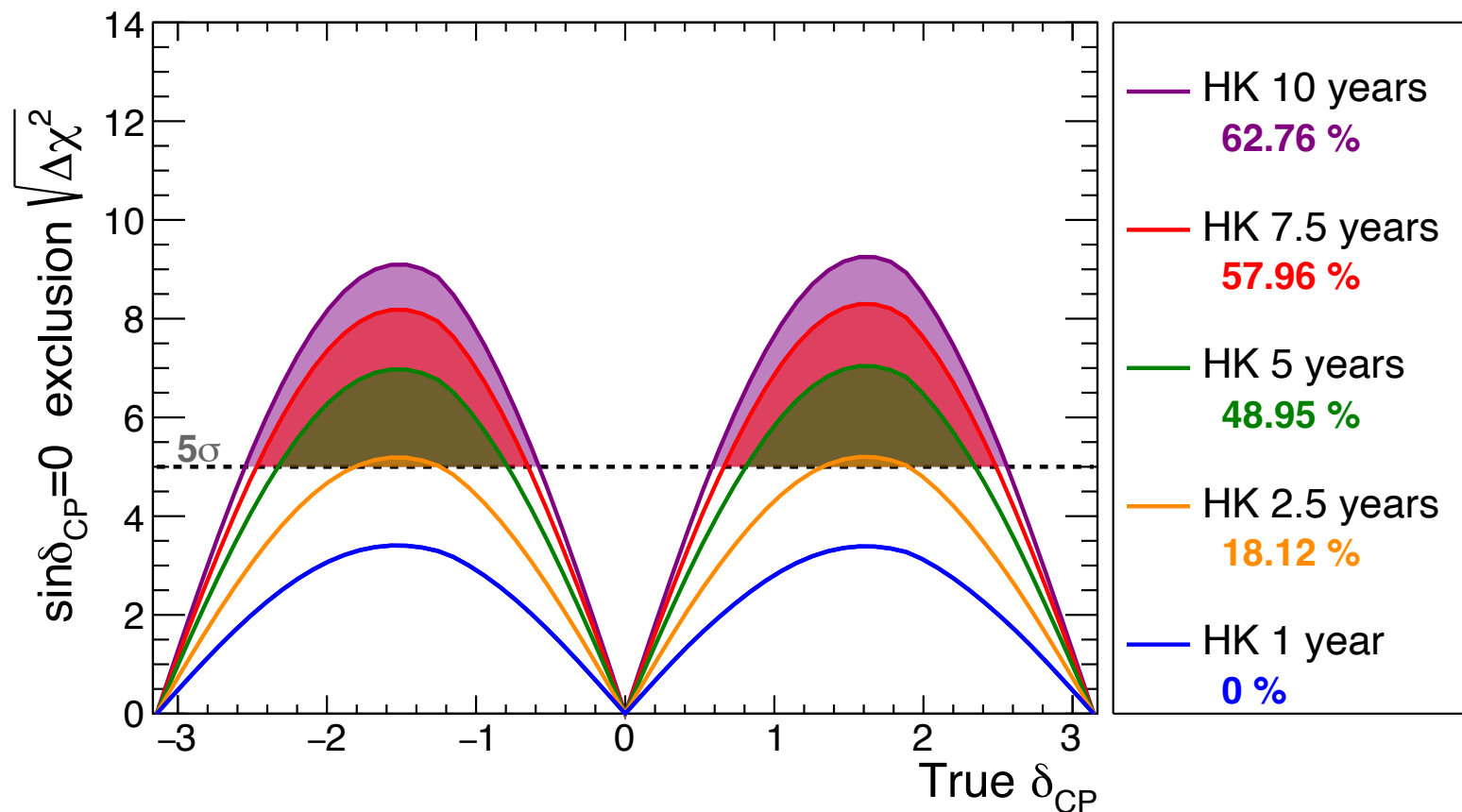


Figure 6: 1 σ and 2 σ allowed regions (2 dof) for T2K (red shading), NO ν A (blue shading) and their

combined region (dashed black line). The Dirac phase δ is still not well measured? respect to $|\Delta m_{3\ell}^2|$.

HK will be completed in 2027, HK 1 year $\sim$ SK 10 years

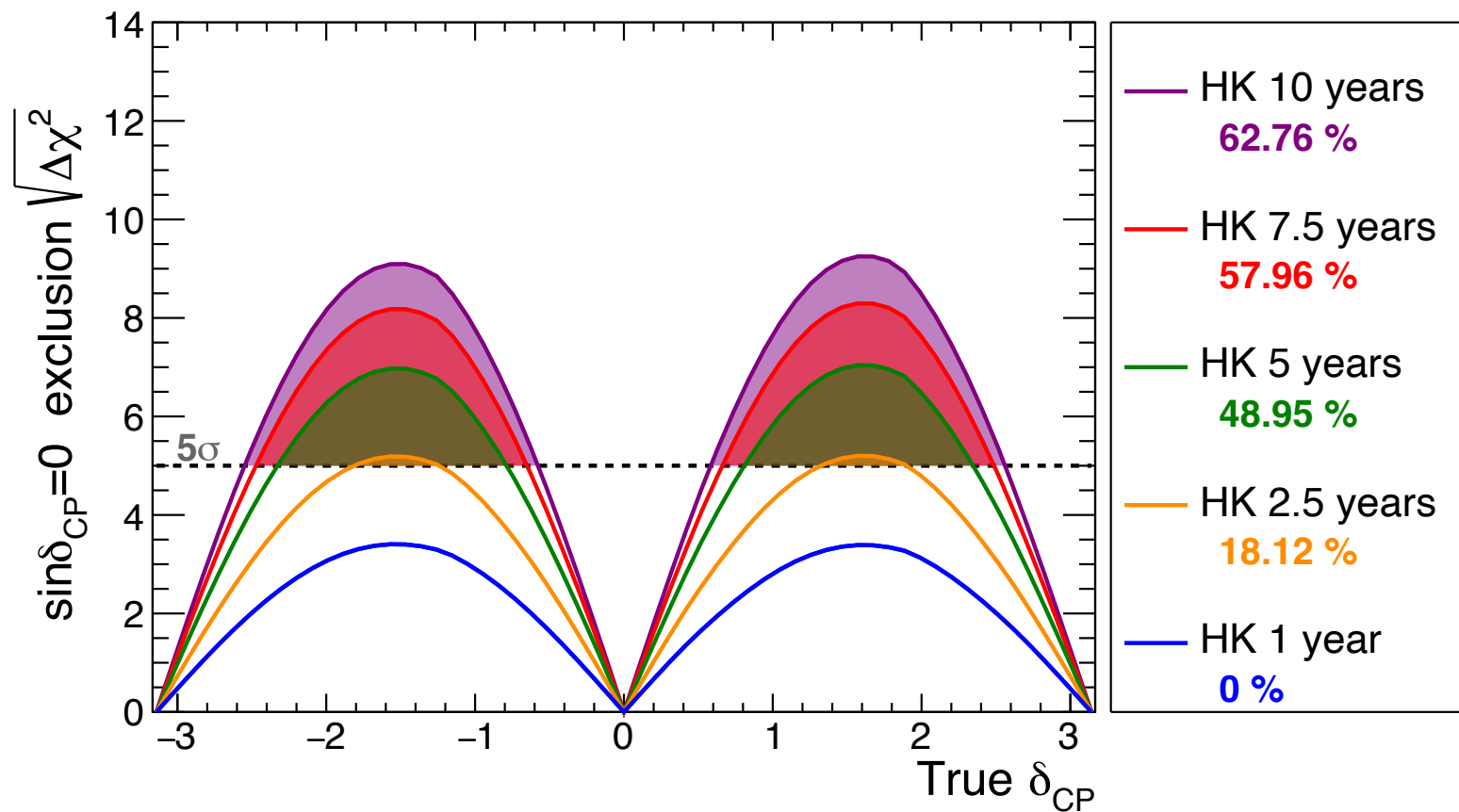


Hyper-K preliminary

True normal ordering (known), Improved systematics

$\sin^2 \theta_{13} = 0.0218 \pm 0.0007$, $\sin^2 \theta_{23} = 0.528$, $\Delta m_{32}^2 = 2.509 \times 10^{-3} \text{eV}^2/c^4$

HK will be completed in 2027, HK 1 year $\sim$ SK 10 years

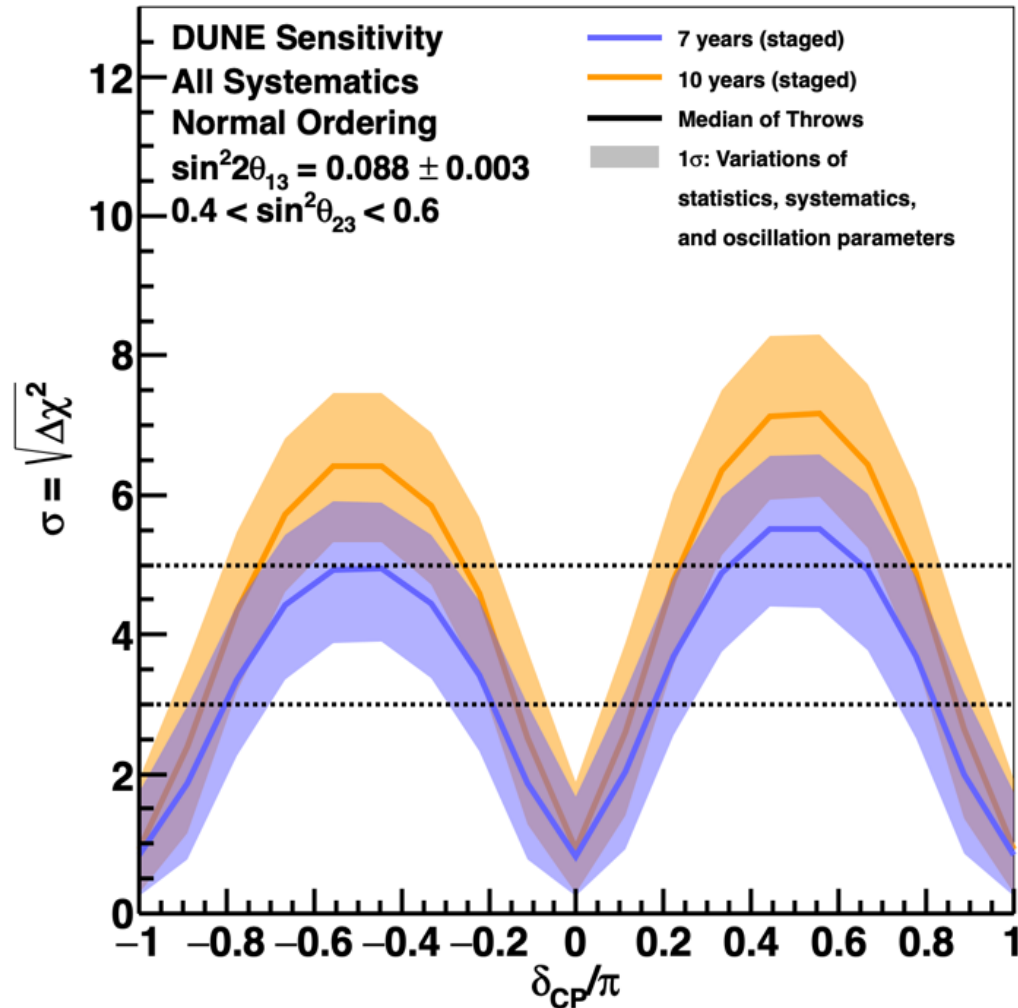


It can find about 30° in 10 years, $|\sin \delta| > 0.5$.

Future exp. (DUNE)

DUNE Collaboration, B. Abi et al,
EPJC 80 (2020) 10, 978, 2006.16043 [hep-ex]

DUNE will be completed in 202x?



Future exp. (DUNE)

DUNE Collaboration, B. Abi et al,
EPJC 80 (2020) 10, 978, 2006.16043 [hep-ex]

DUNE will be completed in 202x?

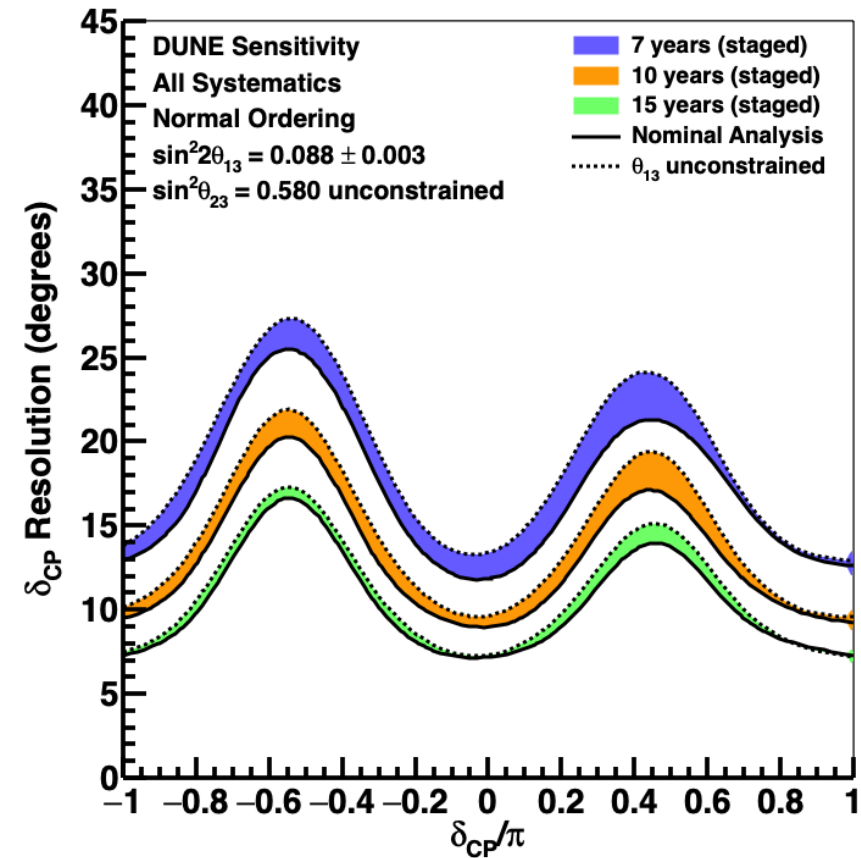
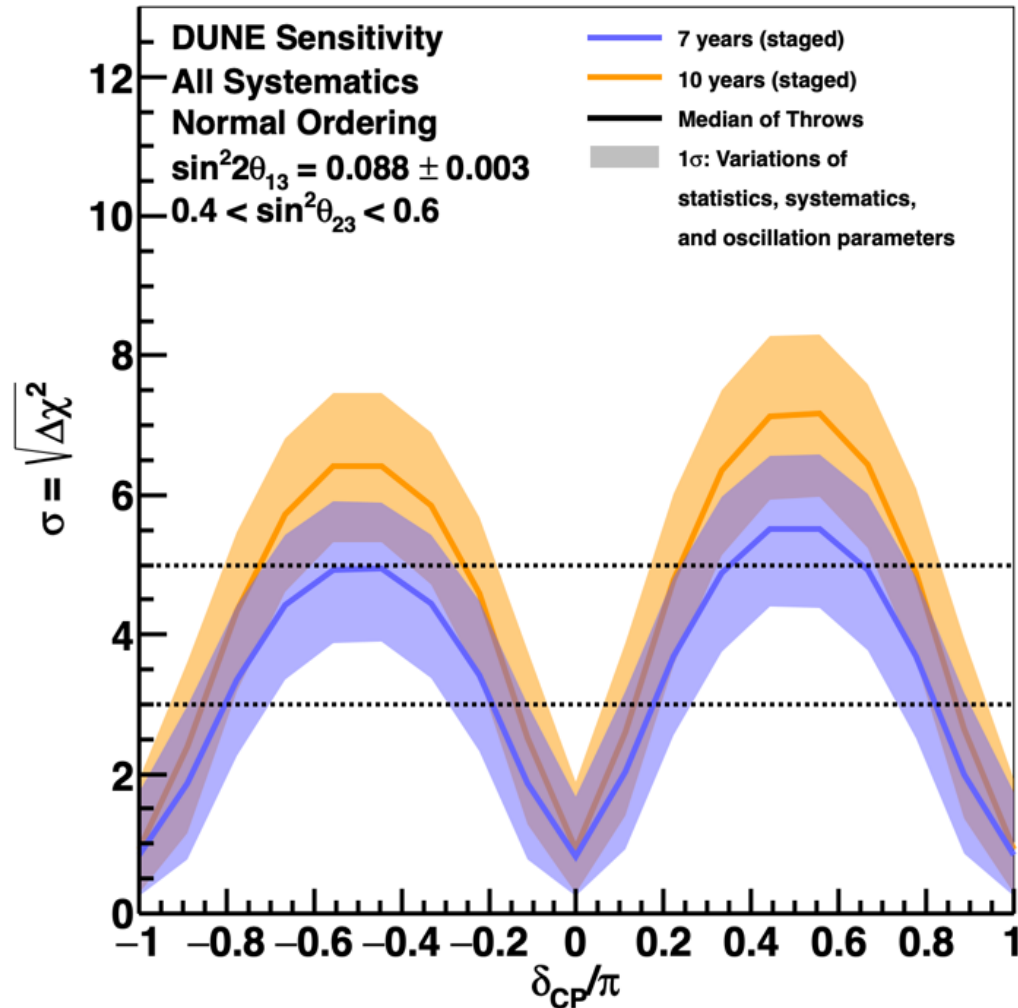


Fig. 21 Resolution in degrees for the DUNE measurement of δ_{CP} , as a function of the true value of δ_{CP} , for seven (blue), ten (orange), and fifteen (green) years of exposure. The width of the band shows the impact of applying an external constraint on θ_{13} .

Future exp. (DUNE)

DUNE Collaboration, B. Abi et al,
EPJC 80 (2020) 10, 978, 2006.16043 [hep-ex]

DUNE will be completed in 202x?

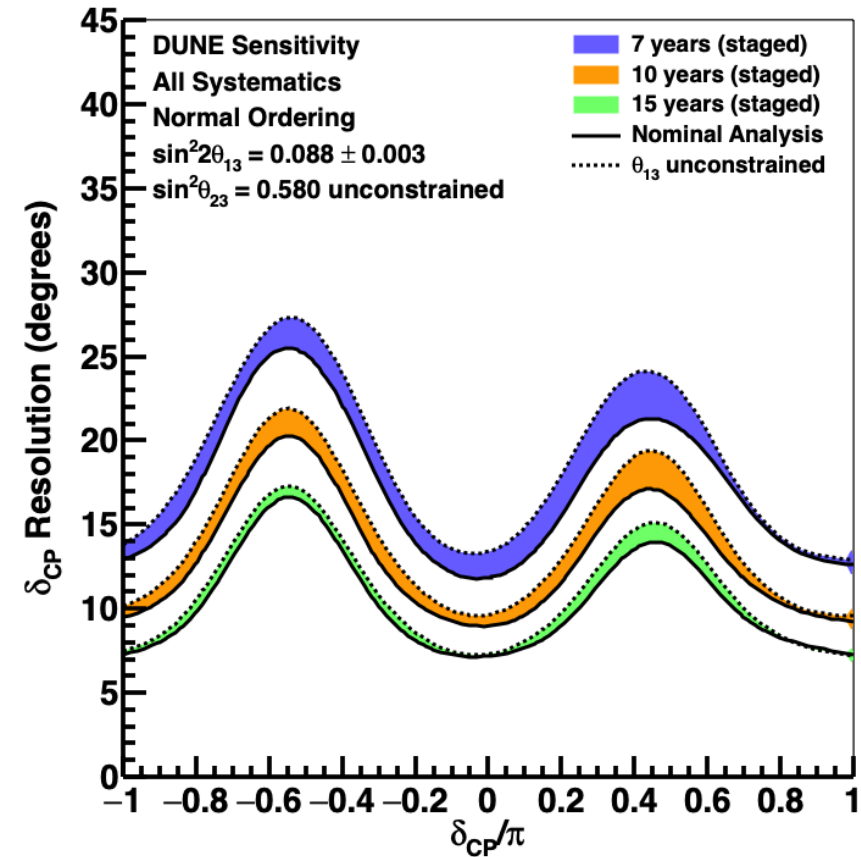
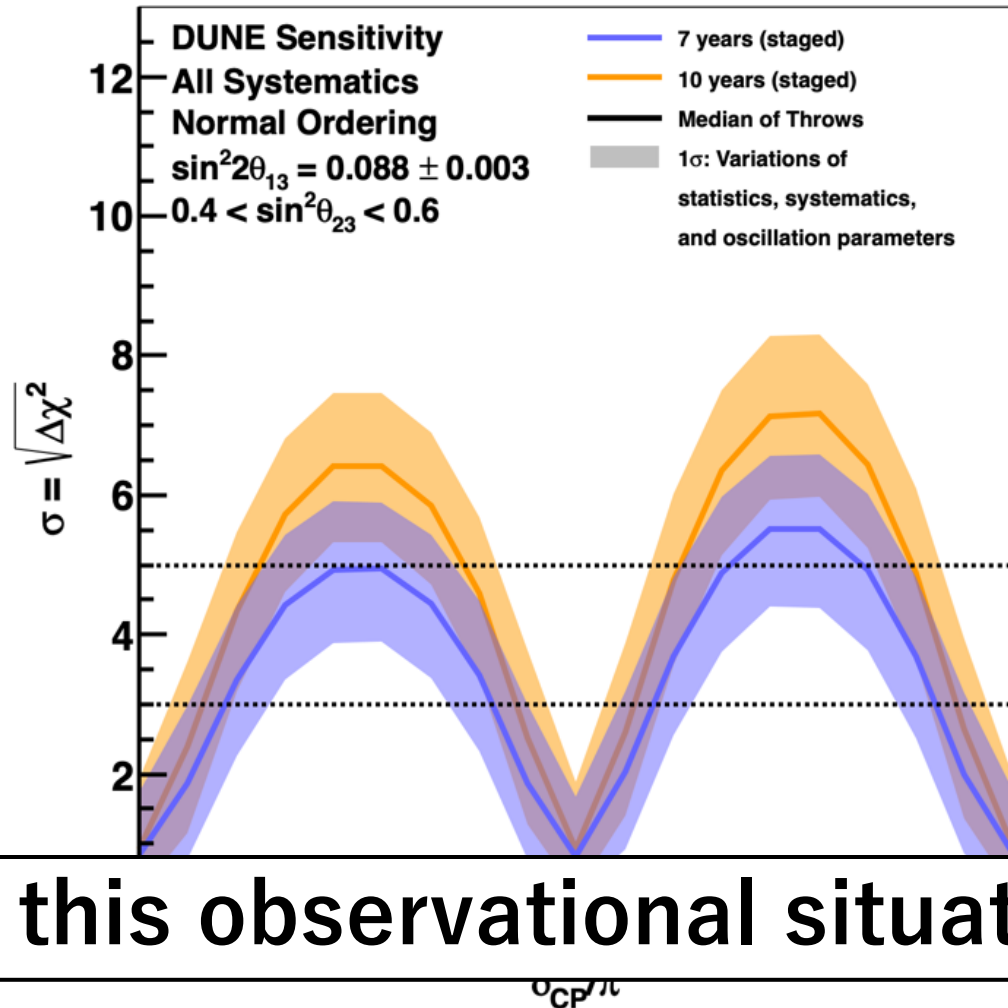


Fig. 21 Resolution in degrees for the DUNE measurement of

In this observational situation, what can be said about δ ?

straint on θ_{13} .

Contents

- Introduction
- Almost general analysis of CKM and MNS matrices and interpretation of Dirac phase
- What DUNE/HK can verify in the first 10 years:
perturbed $\mu - \tau$ reflection symmetry
- Summary (Blueprint for future)

PDG representation

General treatment of mixing matrices:

Any unitary matrix $\rightarrow$ PDG rep. and phases

$$U = \begin{pmatrix} e^{i\rho_{L1}} & 0 & 0 \\ 0 & e^{i\rho_{L2}} & 0 \\ 0 & 0 & e^{i\rho_{L3}} \end{pmatrix} U_{\text{PDG}} \begin{pmatrix} e^{i\rho_{R1}} & 0 & 0 \\ 0 & e^{i\rho_{R2}} & 0 \\ 0 & 0 & e^{i\rho_{R3}} \end{pmatrix}$$

Overall phase trf. can eliminate one of $\rho \Rightarrow 9$ parameters

PDG representation

General treatment of mixing matrices:

Any unitary matrix $\rightarrow$ PDG rep. and phases

For the mixing matrix, $U_{\text{MNS}} = U_e^\dagger U_\nu$

PDG rep. of each fermion will be,

$$U_{\nu,e} = \Phi_L^{\nu,e} U_{\nu,e}^0 \Phi_R^{\nu,e}$$

PDG representation

General treatment of mixing matrices:

Any unitary matrix $\rightarrow$ PDG rep. and phases

For the mixing matrix, $U_{\text{MNS}} = U_e^\dagger U_\nu$

PDG rep. of each fermion will be,

$$U_{\nu,e} = \Phi_L^{\nu,e} U_{\nu,e}^0 \Phi_R^{\nu,e}$$

$$U_\nu^0 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\nu & s_\nu \\ 0 & -s_\nu & c_\nu \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_\nu} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_\nu} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

PDG representation

General treatment of mixing matrices:

Any unitary matrix $\rightarrow$ PDG rep. and phases

For the mixing matrix, $U_{\text{MNS}} = U_e^\dagger U_\nu$

PDG rep. of each fermion will be,

$$U_{\nu,e} = \Phi_L^{\nu,e} U_{\nu,e}^0 \Phi_R^{\nu,e}$$

**claim: DUNE/HK
roughly observes
the intrinsic phase**

$$U_\nu^0 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\nu & s_\nu \\ 0 & -s_\nu & c_\nu \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_\nu} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_\nu} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

General treatment of CP phase (history)

- Fritzsch-Xing representation Fritzsch and Xing, 99, 00, 01
(neglect two 13 mixings and phases)

$$V_{\text{CKM}} = U_u^\dagger U_d = \begin{pmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c_q & s_q \\ 0 & -s_q & c_q \end{pmatrix} \begin{pmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

General treatment of CP phase (history)

- Fritzsch-Xing representation Fritzsch and Xing, 99, 00, 01
(neglect two 13 mixings and phases)

$$V_{\text{CKM}} = U_u^\dagger U_d = \begin{pmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c_q & s_q \\ 0 & -s_q & c_q \end{pmatrix} \begin{pmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Sum rule from discrete sym. S. T. Petcov, NPB 892, 400 (2015),
(neglect two 13 mixings) arXiv:1405.6006.

$$\cos \delta = \frac{\tan \theta_{23}}{\sin 2\theta_{12} \sin \theta_{13}} \left[\cos 2\theta_{12}^\nu + (\sin^2 \theta_{12} - \cos^2 \theta_{12}^\nu) (1 - \cot^2 \theta_{23} \sin^2 \theta_{13}) \right] .$$

PDG representation

Approximation: neglect 1-3 mixing for more hierarchical fermions (u,e)

$$U_e^0 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\tau & s_\tau \\ 0 & -s_\tau & c_\tau \end{pmatrix} \begin{pmatrix} c_{13}^e & 0 & \cancel{s_{13}^e e^{-i\delta_e}} \\ 0 & 1 & 0 \\ \cancel{-s_{13}^e e^{i\delta_e}} & 0 & c_{13}^e \end{pmatrix} \begin{pmatrix} c_e & s_e & 0 \\ -s_e & c_e & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

PDG representation

Approximation: neglect 1-3 mixing for more hierarchical fermions (u,e)

$$U_e^0 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\tau & s_\tau \\ 0 & -s_\tau & c_\tau \end{pmatrix} \begin{pmatrix} c_{13}^e & 0 & \cancel{s_{13}^e e^{-i\delta_e}} \\ 0 & 1 & 0 \\ \cancel{-s_{13}^e e^{i\delta_e}} & 0 & c_{13}^e \end{pmatrix} \begin{pmatrix} c_e & s_e & 0 \\ -s_e & c_e & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Justification: chiral symmetry of 1st gen. $\Rightarrow s_{13}^e \sim m_e/m_\tau \sim 10^{-3}$

Phenomenologically, $|V_{ub}| \sim 0.003 \ll |U_{e3}| \sim 0.15$

$$J_{\text{CKM}} \sim 3 \times 10^{-5} \ll J_{\text{MNS}} \sim 0.03$$

PDG representation

Approximation: neglect 1-3 mixing for more hierarchical fermions (u,e)

$$U_e^0 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\tau & s_\tau \\ 0 & -s_\tau & c_\tau \end{pmatrix} \begin{pmatrix} c_{13}^e & 0 & \cancel{s_{13}^e e^{-i\delta_e}} \\ 0 & 1 & 0 \\ \cancel{-s_{13}^e e^{i\delta_e}} & 0 & c_{13}^e \end{pmatrix} \begin{pmatrix} c_e & s_e & 0 \\ -s_e & c_e & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Justification: chiral symmetry of 1st gen. $\Rightarrow s_{13}^e \sim m_e/m_\tau \sim 10^{-3}$

Phenomenologically, $|V_{ub}| \sim 0.003 \ll |U_{e3}| \sim 0.15$

$$J_{\text{CKM}} \sim 3 \times 10^{-5} \ll J_{\text{MNS}} \sim 0.03$$

$U_e \sim U_{u,d} \Rightarrow s_{13}^e$ and δ_e can be **neglected**

PDG representation

Approximation: neglect 1-3 mixing for more hierarchical fermions (u,e)

$$U_e^0 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\tau & s_\tau \\ 0 & -s_\tau & c_\tau \end{pmatrix} \begin{pmatrix} c_{13}^e & 0 & \cancel{s_{13}^e e^{-i\delta_e}} \\ 0 & 1 & 0 \\ \cancel{-s_{13}^e e^{i\delta_e}} & 0 & c_{13}^e \end{pmatrix} \begin{pmatrix} c_e & s_e & 0 \\ -s_e & c_e & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

By merging relative phases and redefining the overall phase

$$U_{\text{MNS}} \equiv \Phi_R^{e\dagger} U_e^{0\dagger} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{-i\sigma} \end{pmatrix} U_\nu^0 \Phi_R^\nu,$$

The outer ϕ do not contribute to δ

PDG representation

Approximation: neglect 1-3 mixing for more hierarchical fermions (u,e)

$$U_e^0 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\tau & s_\tau \\ 0 & -s_\tau & c_\tau \end{pmatrix} \begin{pmatrix} c_{13}^e & 0 & \cancel{s_{13}^e e^{-i\delta_e}} \\ 0 & 1 & 0 \\ \cancel{-s_{13}^e e^{i\delta_e}} & 0 & c_{13}^e \end{pmatrix} \begin{pmatrix} c_e & s_e & 0 \\ -s_e & c_e & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

By merging relative phases and redefining the overall phase

$$U_{\text{MNS}} \equiv \Phi_R^{e\dagger} U_e^{0\dagger} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{-i\sigma} \end{pmatrix} U_\nu^0 \Phi_R^\nu,$$

The outer ϕ do not contribute to δ

$\Rightarrow$ CPV sources are **relative phases ρ , σ and δ_ν** in U_ν

Constraints from experiments

By merging the 2-3 mixing of ν and e and redefining phases,

$$U_{\text{MNS}} = \begin{pmatrix} c_e & -s_e & 0 \\ s_e & c_e & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\beta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\beta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

If s_{τ} (2-3 mixing of Ue) is small, $\alpha \sim \rho - \sigma$, $\beta \sim \delta_{\nu}$

4 mixing angles (s_e, s_{ij}) and 2 phases α, β = 6 parameters

← s_{ij} are solved by three observed mixing angles

Constraints from experiments

By merging the 2-3 mixing of ν and e and redefining phases,

$$U_{\text{MNS}} = \begin{pmatrix} c_e & -s_e & 0 \\ s_e & c_e & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\beta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\beta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

If s_{τ} (2-3 mixing of Ue) is small, $\alpha \sim \rho - \sigma$, $\beta \sim \delta_{\nu}$

4 mixing angles (s_e, s_{ij}) and 2 phases α, β = 6 parameters

← s_{ij} are solved by three observed mixing angles

U_{MNS} analytically written by s_e and α, β !

Jarlskog inv. $J = \text{Im} [U_{\mu 2} U_{\tau 3} U_{\mu 3}^* U_{\tau 2}^*]$

C. Jarlskog,
Phys. Rev. Lett. 55, 1039 (1985).

For small mixing s_e , (S_{ij} is the observed mixing angle)

$$\sin \delta \simeq \sin \beta + s_e \left(-\frac{S_{23} \cos \beta \sin(\alpha - \beta)}{S_{13}} + \right.$$

← δ is roughly $\beta \sim \delta$ ν , NLO is suppressed by s_e

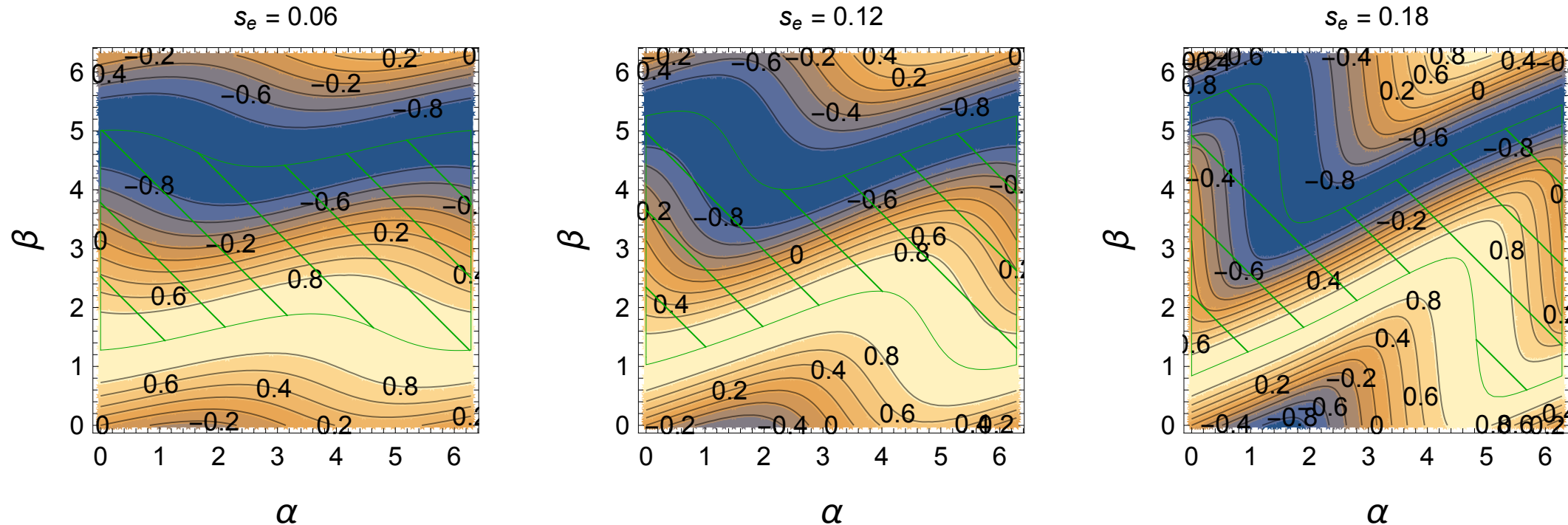
Jarlskog inv. $J = \text{Im} [U_{\mu 2} U_{\tau 3} U_{\mu 3}^* U_{\tau 2}^*]$

C. Jarlskog,
Phys. Rev. Lett. 55, 1039 (1985).

For small mixing s_e , (S_{ij} is the observed mixing angle)

$$\sin \delta \simeq \sin \beta + s_e \left(-\frac{S_{23} \cos \beta \sin(\alpha - \beta)}{S_{13}} + \right.$$

← δ is roughly $\beta \sim \delta \nu$, NLO is suppressed by s_e



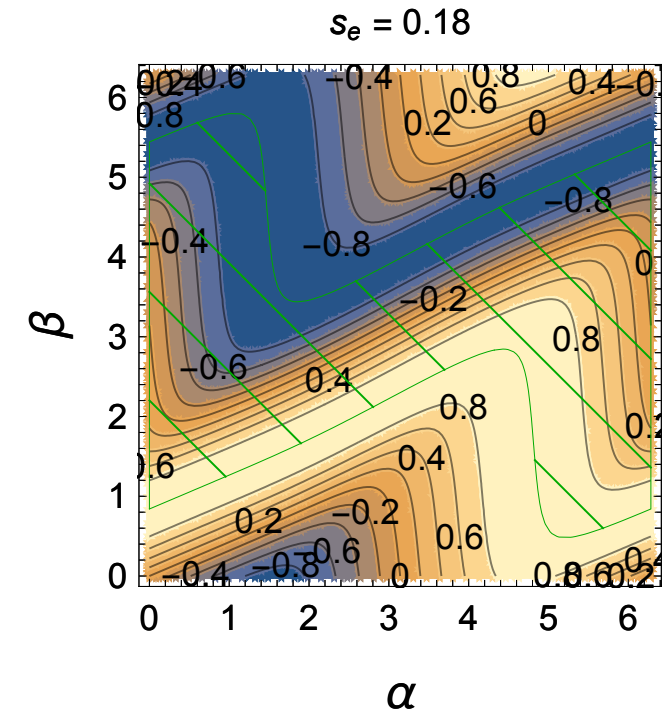
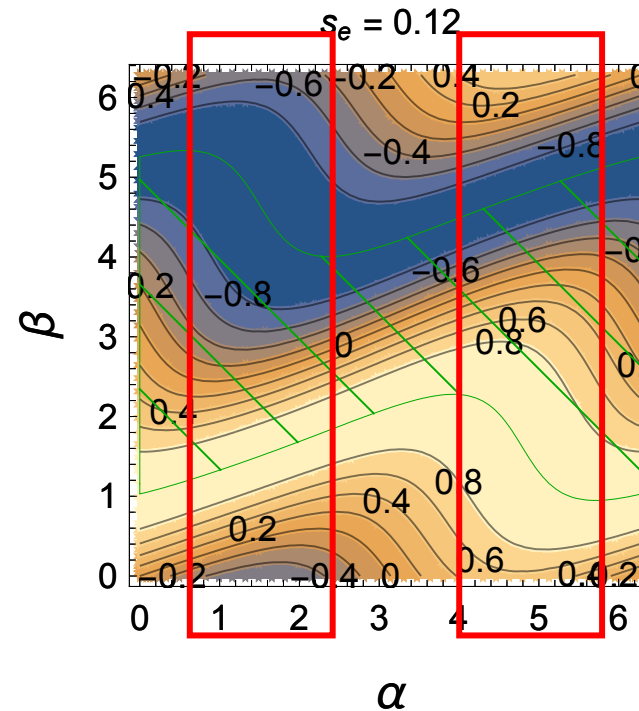
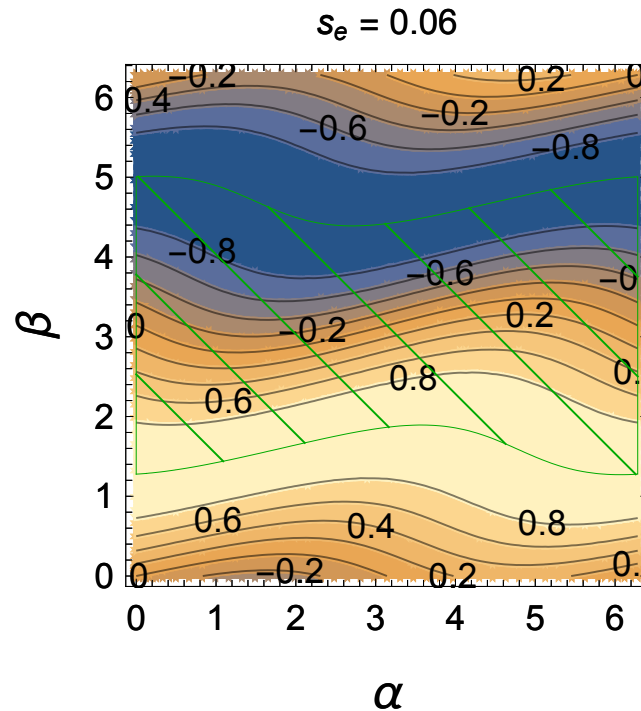
Jarlskog inv. $J = \text{Im} [U_{\mu 2} U_{\tau 3} U_{\mu 3}^* U_{\tau 2}^*]$

C. Jarlskog,
Phys. Rev. Lett. 55, 1039 (1985).

For small mixing s_e , (S_{ij} is the observed mixing angle)

$$\sin \delta \simeq \sin \beta + s_e \left(-\frac{S_{23} \cos \beta \sin(\alpha - \beta)}{S_{13}} + \right.$$

← δ is roughly $\beta \sim \delta$ ν , NLO is suppressed by s_e



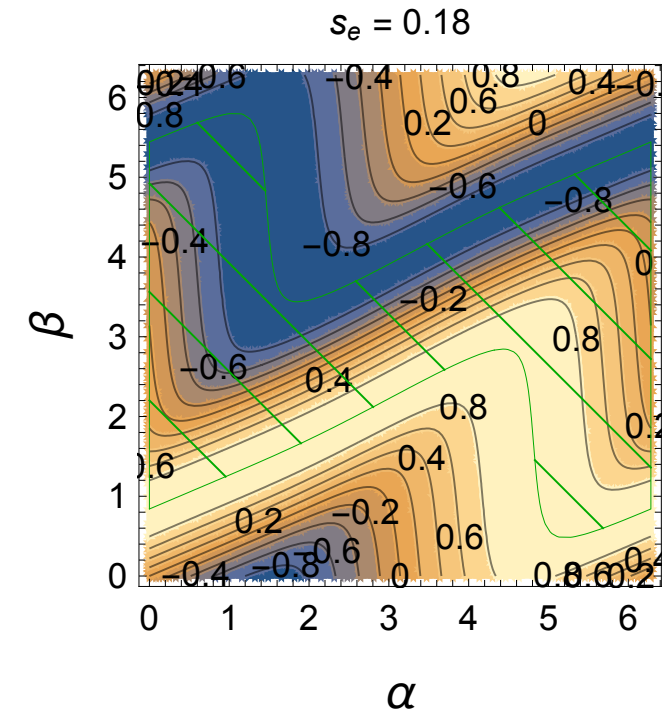
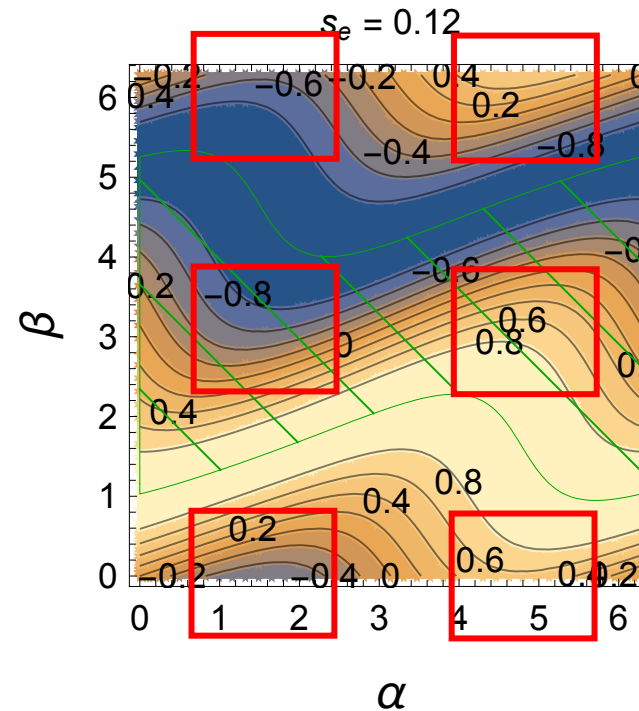
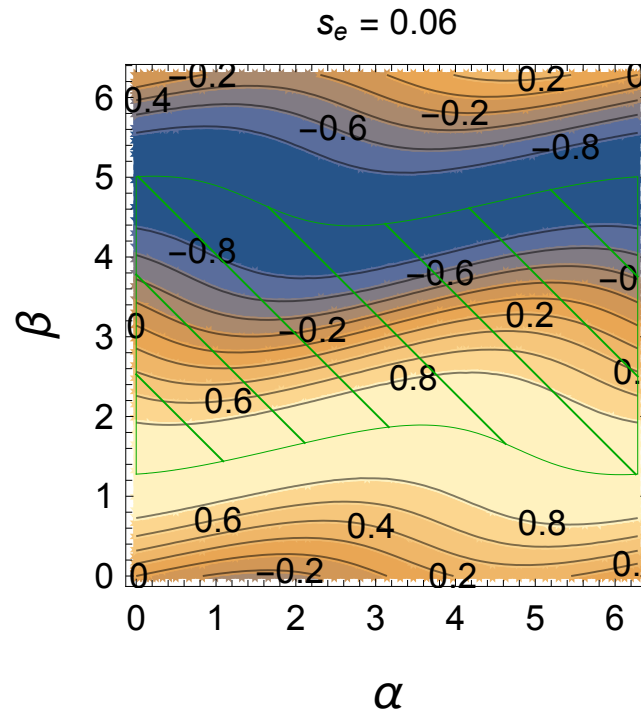
Jarlskog inv. $J = \text{Im} [U_{\mu 2} U_{\tau 3} U_{\mu 3}^* U_{\tau 2}^*]$

C. Jarlskog,
Phys. Rev. Lett. 55, 1039 (1985).

For small mixing s_e , (S_{ij} is the observed mixing angle)

$$\sin \delta \simeq \sin \beta + s_e \left(-\frac{S_{23} \cos \beta \sin(\alpha - \beta)}{S_{13}} + \right.$$

← δ is roughly $\beta \sim \delta \nu$, NLO is suppressed by s_e



Jarlskog inv. $J = \text{Im} [U_{\mu 2} U_{\tau 3} U_{\mu 3}^* U_{\tau 2}^*]$

C. Jarlskog,
Phys. Rev. Lett. 55, 1039 (1985).

For small mixing s_e , (S_{ij} is the observed mixing angle)

$$\sin \delta \simeq \sin \beta + s_e \left(-\frac{S_{23} \cos \beta \sin(\alpha - \beta)}{S_{13}} + \right.$$

← δ is roughly $\beta \sim \delta$ ν , NLO is suppressed by s_e

DUNE/T2HK can detect $|\sin \delta| > 0.5$ in about 10 years.

For $S_{23}/S_{13} \sim 5$ and $s_e \sim 0.1$,

Jarlskog inv. $J = \text{Im} [U_{\mu 2} U_{\tau 3} U_{\mu 3}^* U_{\tau 2}^*]$

C. Jarlskog,
Phys. Rev. Lett. 55, 1039 (1985).

For small mixing s_e , (S_{ij} is the observed mixing angle)

$$\sin \delta \simeq \sin \beta + s_e \left(-\frac{S_{23} \cos \beta \sin(\alpha - \beta)}{S_{13}} + \right.$$

← δ is roughly $\beta \sim \delta \nu$, NLO is suppressed by s_e

DUNE/T2HK can detect $|\sin \delta| > 0.5$ in about 10 years.

For $S_{23}/S_{13} \sim 5$ and $s_e \sim 0.1$,

Result (prospects for a probable future) :

Discovery of δ in 10 years leads to $\delta \nu \neq 0$

If not discovered, upper bounds imposed $\delta \nu$ and ρ, σ

Jarlskog inv. $J = \text{Im} [U_{\mu 2} U_{\tau 3} U_{\mu 3}^* U_{\tau 2}^*]$

C. Jarlskog,
Phys. Rev. Lett. 55, 1039 (1985).

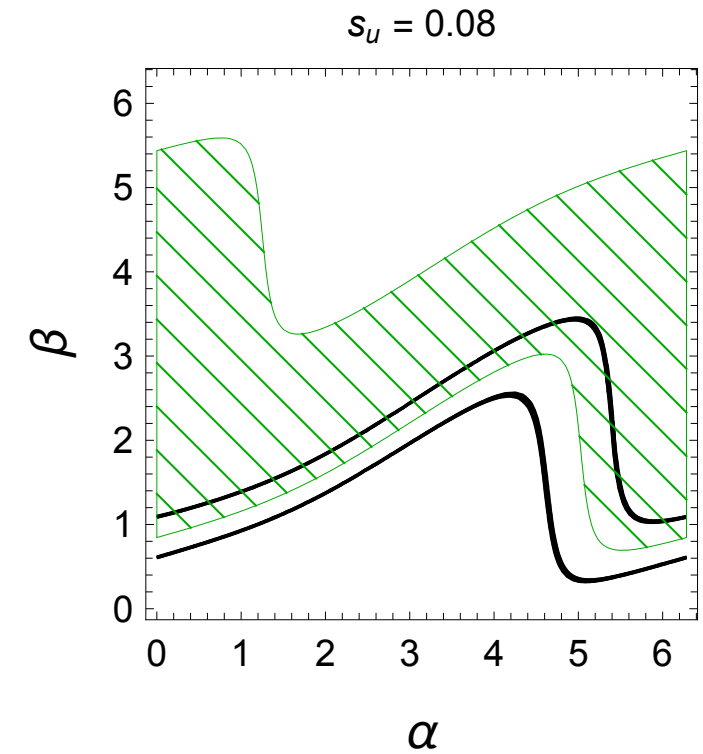
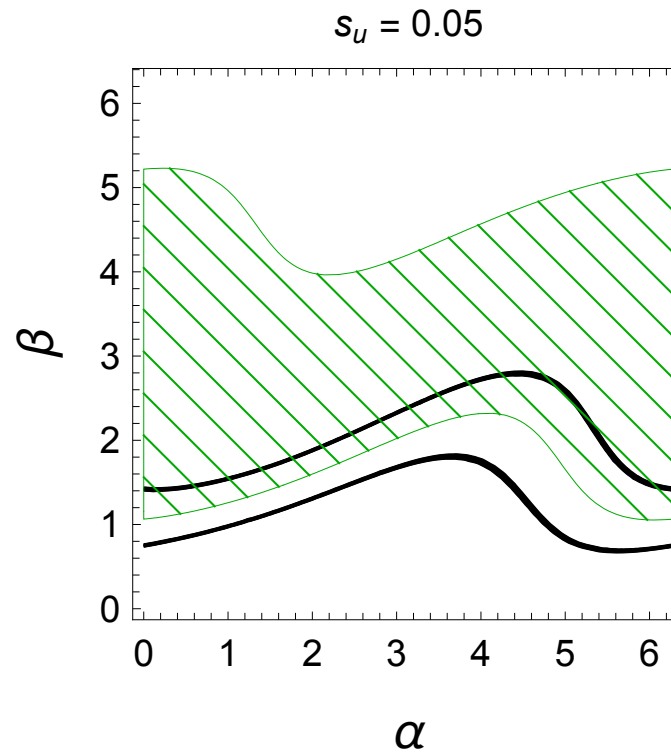
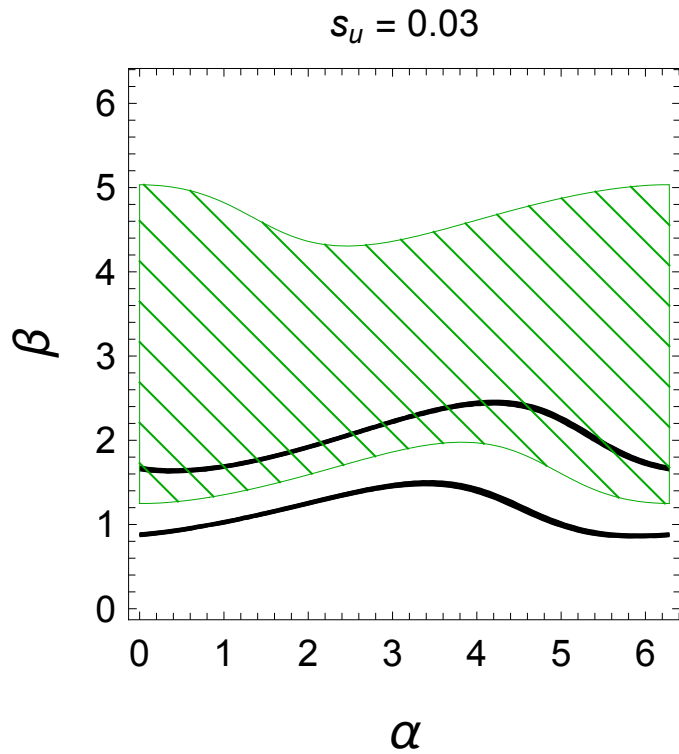
Physical understanding:

$$U_{\text{MNS}} \equiv U_e^{0\dagger} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{-i\sigma} \end{pmatrix} U_\nu^0,$$

1. CPV of U_e is ignored.
2. if $U_e=1$, relative phases are erased by redefinition, so they are suppressed by mixings of U_e
3. $\delta \sim \delta_\nu$, and the boundary is about 10 years for DUNE/HK.

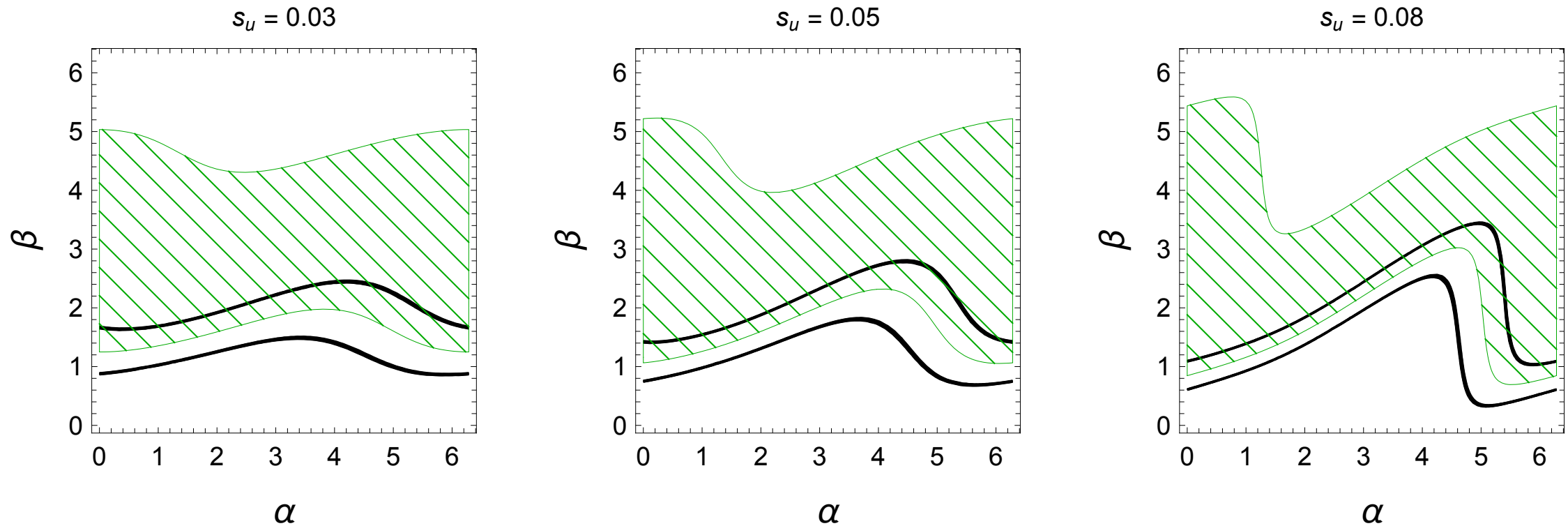
CKM matrix

$|V_{ub}| \sim m_{d1}/m_{d3} \gg m_{u1}/m_{u3}$, neglect the 1-3 mixing for up-quarks appears reasonable.



CKM matrix

$|V_{ub}| \sim m_{d1}/m_{d3} \gg m_{u1}/m_{u3}$, neglect the 1-3 mixing for up-quarks appears reasonable.



If $s_u \cos \beta \sin \alpha$ is not large, the observed $\delta_{\text{CKM}} \sim \beta \sim \delta d$

Middle Summary

1. Almost general analysis of CKM and MNS matrices

- **Approximation:** neglect 1-3 mixing for more hierarchical fermions (u,e)
- **justification:** chiral symmetry of 1st gen. $\Rightarrow s_{13}^e \sim m_e/m_\tau \sim 10^{-3}$

2. interpretation of Dirac CP phase probed by DUNE and T2HK

- **Result :** Discovery of δ in 10 years leads to $\delta \neq 0$.
If not discovered, upper bounds imposed δ and ρ, σ
- **Understanding :** if $U_{e3}=1$, relative phases are erased, and they are suppressed by U_{e3}

3. Analysis of general CP phases is important in GUT and leptogenesis.

**What DUNE/HK can verify in the first 10 years:
perturbed μ - τ reflection symmetry**

Arxiv:2504.00365

Generalized CP symmetry

Ecker, Grimus, Konetschny '80,
Holthausen et al, '12, Feruglio, et al, '12

- CP $\times$ unitary trf (different from generalised symmetry)

$$\psi'_i = U_{ij} \psi_j^c$$

Generalized CP symmetry

Ecker, Grimus, Konetschny '80,
Holthausen et al, '12, Feruglio, et al, '12

- CP $\times$ unitary trf (different from generalised symmetry)

$$\psi'_i = U_{ij} \psi_j^c$$

- $\mu - \tau$ reflection symmetry

Harrison & Scott, '02, Grimus & Lavoura '02

$$T m_\nu^* T = m_\nu, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \Rightarrow \theta_{23} = \pm 45^\circ, \\ \delta_{CP} = \pm 90^\circ,$$

$\mu - \tau$ reflection symmetry

Harrison & Scott, '02,
Grimus & Lavoura '02

$Tm_\nu^*T = m_\nu$, it restricts

$$m_\nu = \begin{pmatrix} a & b+ic & b-ic \\ b+ic & d+if & g \\ b-ic & g & d-if \end{pmatrix}$$

$$a \sim h \in \mathbb{R}$$

$\mu - \tau$ reflection symmetry

Harrison & Scott, '02,
Grimus & Lavoura '02

$Tm_\nu^*T = m_\nu$, it restricts

$$m_\nu = \begin{pmatrix} a & b+ic & b-ic \\ b+ic & d+if & g \\ b-ic & g & d-if \end{pmatrix} \quad a \sim h \in \mathbb{R}$$

Bi-maximal trf.

$$U_{BM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \quad U_{BM}m_fU_{BM}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -i \end{pmatrix} m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix},$$

$\mu - \tau$ reflection symmetry

Harrison & Scott, '02,
Grimus & Lavoura '02

$Tm_\nu^*T = m_\nu$, it restricts

$$m_\nu = \begin{pmatrix} a & b+ic & b-ic \\ b+ic & d+if & g \\ b-ic & g & d-if \end{pmatrix} \quad a \sim h \in \mathbb{R}$$

Bi-maximal trf.

$$U_{BM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \quad U_{BM}m_fU_{BM}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -i \end{pmatrix} m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix},$$

Inversely, $\theta_{23}=45^\circ$ & $\delta = \pm 90^\circ \Rightarrow$ we can find a basis that satisfies $\mu - \tau$ ref. sym.

$\mu - \tau$ reflection symmetry

Harrison & Scott, '02,
Grimus & Lavoura '02

$Tm_\nu^*T = m_\nu$, it restricts

$$m_\nu = \begin{pmatrix} a & b+ic & b-ic \\ b+ic & d+if & g \\ b-ic & g & d-if \end{pmatrix} \quad a \sim h \in \mathbb{R}$$

Bi-maximal trf.

$$U_{BM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \quad U_{BM}m_fU_{BM}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -i \end{pmatrix} m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix},$$

Inversely, $\theta_{23}=45^\circ$ & $\delta = \pm 90^\circ \Rightarrow$ we can find a basis that satisfies $\mu - \tau$ ref. sym.

How is it understood when δ deviates from $\pm \pi/2$?

Perturbation by the charged leptons

μ - τ ref. symmetry + hierarchical mass of charged leptons

$$T m_\nu^* T = m_\nu, \quad m_e \sim \begin{pmatrix} m_{e1} & * & * \\ * & m_{e2} & * \\ * & * & m_{e3} \end{pmatrix},$$

Perturbation by the charged leptons

$\mu - \tau$ ref. symmetry + hierarchical mass of charged leptons

$$T m_\nu^* T = m_\nu, \quad m_e \sim \begin{pmatrix} m_{e1} & * & * \\ * & m_{e2} & * \\ * & * & m_{e3} \end{pmatrix},$$

$$U_\nu^0 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\nu & s_\nu \\ 0 & -s_\nu & c_\nu \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_\nu} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_\nu} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$= \boldsymbol{\pi}/4 \qquad \qquad = \pm \boldsymbol{\pi}/2$

Perturbation by the charged leptons

$\mu - \tau$ ref. symmetry + hierarchical mass of charged leptons

$$T m_\nu^* T = m_\nu, \quad m_e \sim \begin{pmatrix} m_{e1} & * & * \\ * & m_{e2} & * \\ * & * & m_{e3} \end{pmatrix},$$

$$U_\nu^0 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\nu & s_\nu \\ 0 & -s_\nu & c_\nu \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_\nu} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_\nu} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$= \pi/4 \qquad \qquad = \pm \pi/2$

How these predictions are modified by charged leptons?

Merging of 2-3 mixings in $U_{\text{MNS}} = U_e^\dagger U_\nu$

Merging of 2-3 mixings defines new phases η and χ

$$\begin{pmatrix} c_\tau & -s_\tau \\ s_\tau & c_\tau \end{pmatrix} \begin{pmatrix} e^{i\sigma} & 0 \\ 0 & e^{-i\sigma} \end{pmatrix} \begin{pmatrix} c_\nu & s_\nu \\ -s_\nu & c_\nu \end{pmatrix} \equiv \begin{pmatrix} e^{i\chi} c_{23} & e^{i\eta} s_{23} \\ -e^{-i\eta} s_{23} & e^{-i\chi} c_{23} \end{pmatrix}$$

Merging of 2-3 mixings in $U_{\text{MNS}} = U_e^\dagger U_\nu$

Merging of 2-3 mixings defines new phases η and χ

$$\begin{pmatrix} c_\tau & -s_\tau \\ s_\tau & c_\tau \end{pmatrix} \begin{pmatrix} e^{i\sigma} & 0 \\ 0 & e^{-i\sigma} \end{pmatrix} \begin{pmatrix} c_\nu & s_\nu \\ -s_\nu & c_\nu \end{pmatrix} \equiv \begin{pmatrix} e^{i\chi} c_{23} & e^{i\eta} s_{23} \\ -e^{-i\eta} s_{23} & e^{-i\chi} c_{23} \end{pmatrix}$$

Relation of before and after merging

$$\frac{1 - \cos 2\sigma \sin 2\tau}{2} = s_{23}^2$$

The mixing, phase and s_{23} are related

Merging of 2-3 mixings in $U_{\text{MNS}} = U_e^\dagger U_\nu$

Merging of 2-3 mixings defines new phases η and χ

$$\begin{pmatrix} c_\tau & -s_\tau \\ s_\tau & c_\tau \end{pmatrix} \begin{pmatrix} e^{i\sigma} & 0 \\ 0 & e^{-i\sigma} \end{pmatrix} \begin{pmatrix} c_\nu & s_\nu \\ -s_\nu & c_\nu \end{pmatrix} \equiv \begin{pmatrix} e^{i\chi} c_{23} & e^{i\eta} s_{23} \\ -e^{-i\eta} s_{23} & e^{-i\chi} c_{23} \end{pmatrix}$$

Relation of before and after merging

$$\frac{1 - \boxed{\cos 2\sigma \sin 2\tau}}{2} = s_{23}^2 \quad \text{The mixing, phase and } s_{23} \text{ are related}$$

The shift of s_{23} from 45° depends on the lepton sector.

Merging of 2-3 mixings (entire)

By further redefinition of fields,

$$U_{\text{MNS}} = \begin{pmatrix} c_e & -s_e & 0 \\ s_e & c_e & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i(\rho-\chi)} & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i(\delta_\nu+\eta-\chi)} \\ 0 & 1 & 0 \\ -s_{13}e^{i(\delta_\nu+\eta-\chi)} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\alpha \equiv \rho - \chi, \quad \beta = \delta_\nu + \eta - \chi. \quad \eta - \chi \simeq 2 \sin 2\sigma \tan \tau$$

Merging of 2-3 mixings (entire)

By further redefinition of fields,

$$U_{\text{MNS}} = \begin{pmatrix} c_e & -s_e & 0 \\ s_e & c_e & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i(\rho-\chi)} & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i(\delta_\nu+\eta-\chi)} \\ 0 & 1 & 0 \\ -s_{13}e^{i(\delta_\nu+\eta-\chi)} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

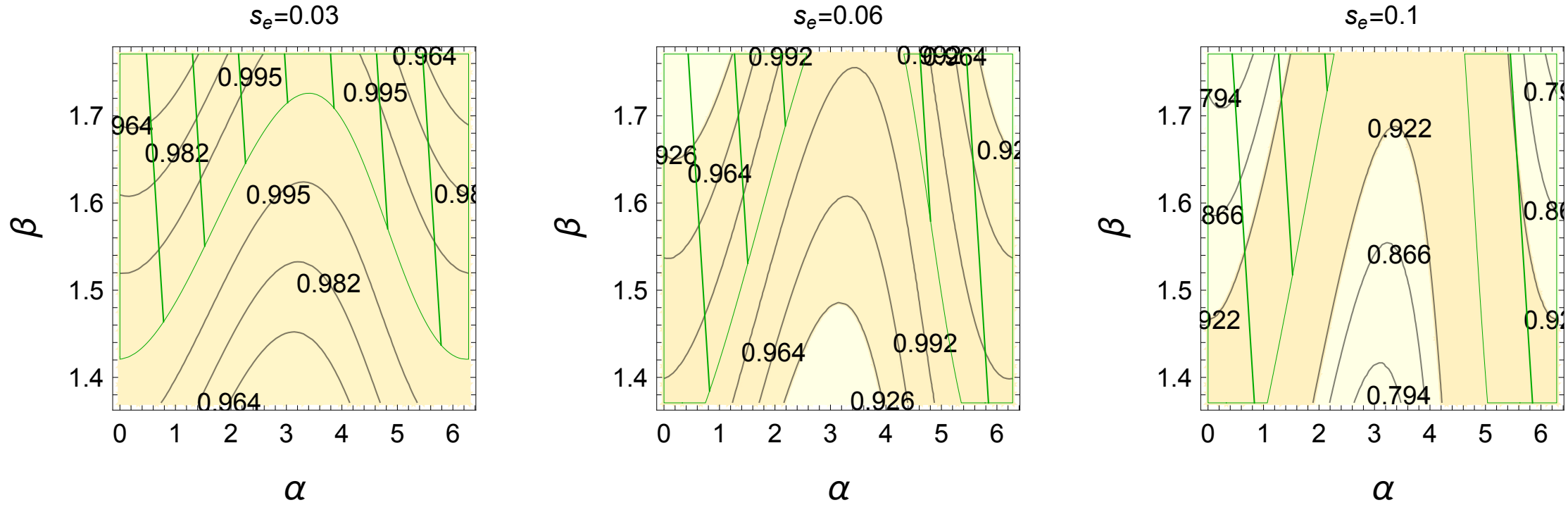
$$\alpha \equiv \rho - \chi, \quad \beta = \delta_\nu + \eta - \chi. \quad \eta - \chi \simeq 2 \sin 2\sigma \tan \tau$$

If U_e is about the CKM matrix, $s \tau < 0.1 \Rightarrow |\eta - \chi| < 0.2$

Region of β is restricted to $\beta = \pm \pi/2 \pm 0.2$.

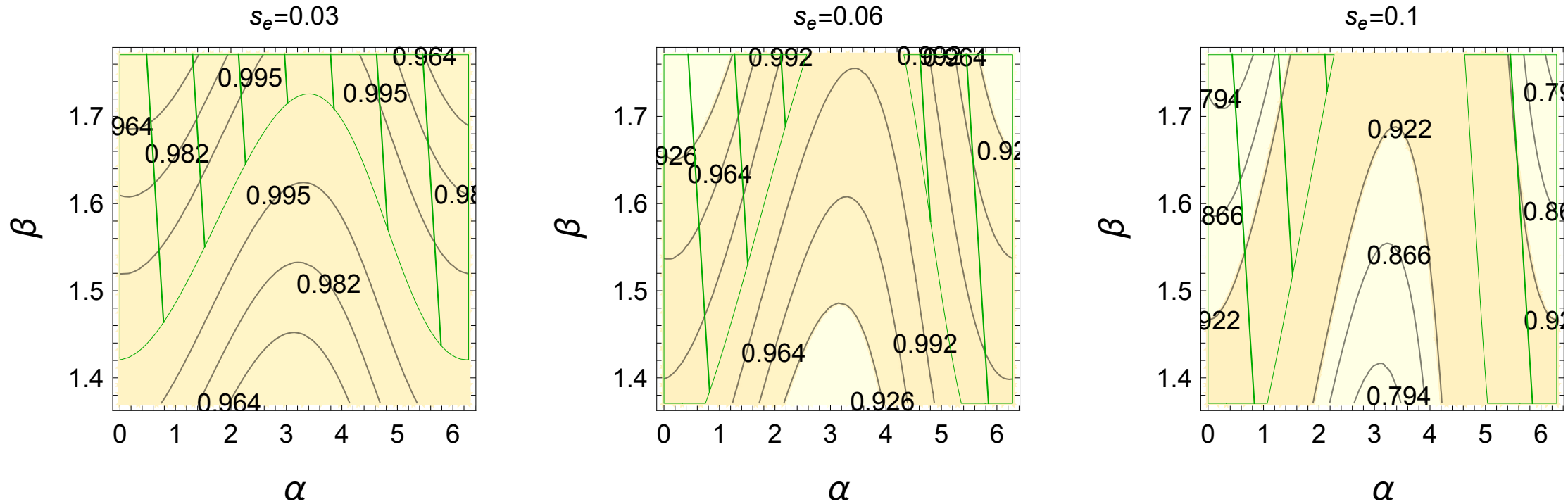
The Jarlskog inv. (again)

$$\Delta\delta \equiv \delta - \frac{\pm\pi}{2} = \pm 2s_\tau \sqrt{1 - \frac{(\Delta s_{23}^2)^2}{s_\tau^2}} - s_e \left(\frac{S_{23} \sin(\alpha - \beta)}{C_{13} S_{13}} \right)$$



The Jarlskog inv. (again)

$$\Delta\delta \equiv \delta - \frac{\pm\pi}{2} = \pm 2s_\tau \sqrt{1 - \frac{(\Delta s_{23}^2)^2}{s_\tau^2}} - s_e \left(\frac{S_{23} \sin(\alpha - \beta)}{C_{13} S_{13}} \right)$$



In most regions, this scenario predicts $|\sin \delta| > 0.95$
 $|\sin \delta| \sim 0.8$ only in a special region ($s_e \sim 0.1$, $\alpha - \beta \sim \pm \pi/2$)

The Jarlskog inv. (again)

$$\Delta\delta \equiv \delta - \frac{\pm\pi}{2} = \pm 2s_\tau \sqrt{1 - \frac{(\Delta s_{23}^2)^2}{s_\tau^2}} - s_e \left(\frac{S_{23} \sin(\alpha - \beta)}{C_{13} S_{13}} \right)$$

$\pm 0.2 \sim \pm 10^\circ$ $\pm 0.5 \sim \pm 30^\circ$

Interpretation for the very near future :

The Jarlskog inv. (again)

$$\Delta\delta \equiv \delta - \frac{\pm\pi}{2} = \pm 2s_\tau \sqrt{1 - \frac{(\Delta s_{23}^2)^2}{s_\tau^2}} - s_e \left(\frac{S_{23} \sin(\alpha - \beta)}{C_{13} S_{13}} \right)$$

$\pm 0.2 \sim \pm 10^\circ$ $\pm 0.5 \sim \pm 30^\circ$

Interpretation for the very near future :

1. $\delta = \pm 90^\circ \pm 10^\circ$

⇒ Explained by 1st term, 2nd term is strongly restricted

2. $0.8 < |\sin \delta| < 0.95$

⇒ The second term is necessary and s_e is large, $\alpha \sim 0$ or π

3. $|\sin \delta| < 0.8$ ($< 50^\circ$ or $> 130^\circ$ or ...)

⇒ This scenario is excluded, **about DUNE/HK 5 years**

The Jarlskog inv. (again)

Physical understanding:

$$U_{\text{MNS}} \equiv U_e^{0\dagger} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{-i\sigma} \end{pmatrix} U_\nu^0,$$

$$T m_\nu^* T = m_\nu ,$$

$\mathbf{s}_\nu = \pi/4$ $\boldsymbol{\delta}_\nu = \pm \pi/2$

1. Dirac phase is roughly $-\text{Arg}[(U_\nu)_{13}]$
2. 2nd and 3rd gen. (s τ , σ) do not have large effects
3. For 1st gen., large s_e & $\rho \sim 0$ makes smaller δ the most

Summary (a blueprint for the future)



t	$ \sin \delta $	
2027		HK (and DUNE?) completed
2030	1.0	$\mu - \tau$ reflection symmetry
	0.9	+perturb.
2032	0.8	ref.sym+perturb. is up to this point

Summary (a blueprint for the future)



t	$ \sin \delta $	
2027		HK (and DUNE?) completed
2030	1.0	$\mu - \tau$ reflection symmetry
	0.9	+perturb.
2032	0.8	ref.sym+perturb. is up to this point
		(difficult area to take limits?)
2037	0.5	relative phases are more important

That's all. Thank you!

Request:

Since I'm not good at **listening** English,
It would be nice if you ask me slowly!

Back ups

Redefining 2-3 mixing (back up)

The two 2-3 mixings are merged to define new arguments χ and η .

$$\begin{pmatrix} c_\tau & -s_\tau \\ s_\tau & c_\tau \end{pmatrix} \begin{pmatrix} e^{i\sigma} & 0 \\ 0 & e^{-i\sigma} \end{pmatrix} \begin{pmatrix} c_\nu & s_\nu \\ -s_\nu & c_\nu \end{pmatrix} = \begin{pmatrix} e^{i\sigma} c_\tau c_\nu + e^{-i\sigma} s_\tau s_\nu & e^{i\sigma} c_\tau s_\nu - e^{-i\sigma} s_\tau c_\nu \\ e^{i\sigma} s_\tau c_\nu - e^{-i\sigma} c_\tau s_\nu & e^{i\sigma} s_\tau s_\nu + e^{-i\sigma} c_\tau c_\nu \end{pmatrix} \quad (15)$$

$$= \begin{pmatrix} c_\sigma c_{\nu-\tau} + i s_\sigma c_{\nu+\tau} & c_\sigma s_{\nu-\tau} + i s_\sigma s_{\nu+\tau} \\ -c_\sigma s_{\nu-\tau} + i s_\sigma s_{\nu+\tau} & c_\sigma c_{\nu-\tau} - i s_\sigma c_{\nu+\tau} \end{pmatrix} \equiv \begin{pmatrix} e^{i\chi} c_{23} & e^{i\eta} s_{23} \\ -e^{-i\eta} s_{23} & e^{-i\chi} c_{23} \end{pmatrix}. \quad (16)$$

Relations between these arguments χ, η and parameters σ, ν, τ are

$$\tan \chi = \tan \sigma \frac{c_{\nu+\tau}}{c_{\nu-\tau}}, \quad \tan \eta = \tan \sigma \frac{s_{\nu+\tau}}{s_{\nu-\tau}}. \quad (17)$$

From this,

$$U_{\text{MNS}} = \begin{pmatrix} c_e & -s_e & 0 \\ s_e & c_e & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\chi} c_{23} & e^{i\eta} s_{23} \\ 0 & -e^{-i\eta} s_{23} & e^{-i\chi} c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_\nu} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_\nu} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (18)$$

Constraints by experiment

Fixes mixing angles by comparing the absolute values

$$\left| U_e^{0\dagger} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{-i\sigma} \end{pmatrix} U_\nu^0 \right| = \left| \begin{pmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{23}S_{13}e^{i\delta} & C_{12}c_{23} - S_{12}S_{23}S_{13}e^{i\delta} & S_{23}C_{13} \\ S_{12}S_{23} - C_{12}C_{23}S_{13}e^{i\delta} & -C_{12}S_{23} - S_{12}C_{23}S_{13}e^{i\delta} & C_{23}C_{13} \end{pmatrix} \right|$$

$$\Rightarrow \quad c_{13}c_{23} = C_{13}C_{23}, \quad s_{23} = \sqrt{1 - \frac{C_{13}^2 C_{23}^2}{c_{13}^2}}.$$

$$U_{13} = c_e s_{13} e^{i(\alpha-\beta)} - c_{13} s_{23} s_e,$$

We sequentially eliminate of variables

perturbative behavior

For $s_e \ll 1$,

- s_{23} : dependence is small because α and β is second order

$$s_{23} = S_{23} + O(s_{13}^2, s_e s_{13}, s_e^2) .$$

- s_{13} : depends only on the phase $\alpha - \beta$

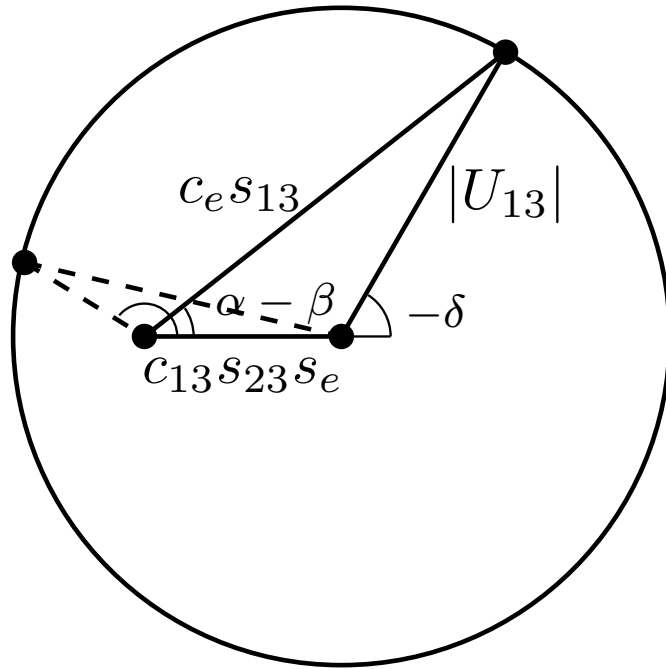
$$s_{13} = S_{13} + C_{13} S_{23} s_e \cos(\alpha - \beta) + O(s_e^2) .$$

- s_{12} : Dependence of β is second order and small

$$s_{12} \simeq S_{12} + C_{12} C_{23} s_e \cos \alpha .$$

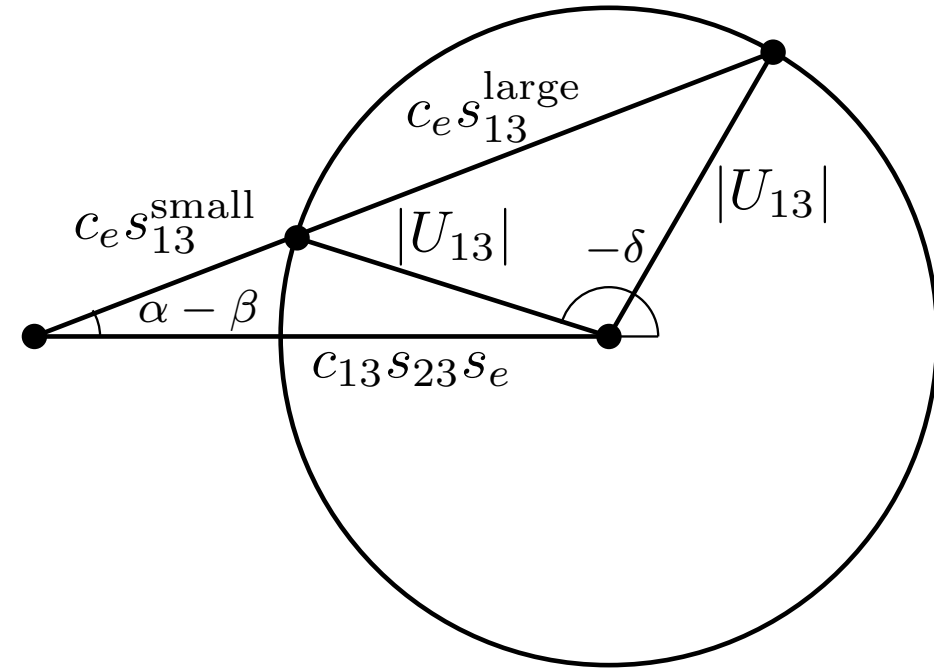
Critical angle of s13

$$U_{13} = c_e s_{13} e^{i(\alpha - \beta)} - c_{13} s_{23} s_e ,$$



s_e is less than the critical value

$$s_{13} = 0 \Rightarrow \frac{s_e^c}{c_e^c} = \frac{S_{13}}{C_{13} S_{23}} .$$



s_e is more than the critical value