Model testing with flavour results in *CKM*fitter

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Flavour in the quark sector

(Gauge couplings to fermions)

$$\mathcal{L}_{SM(NP)} \sim -\frac{1}{4}(F_{\mu\nu})^2 + i\bar{\psi}\not{D}\psi$$

(Higgs self-interaction)

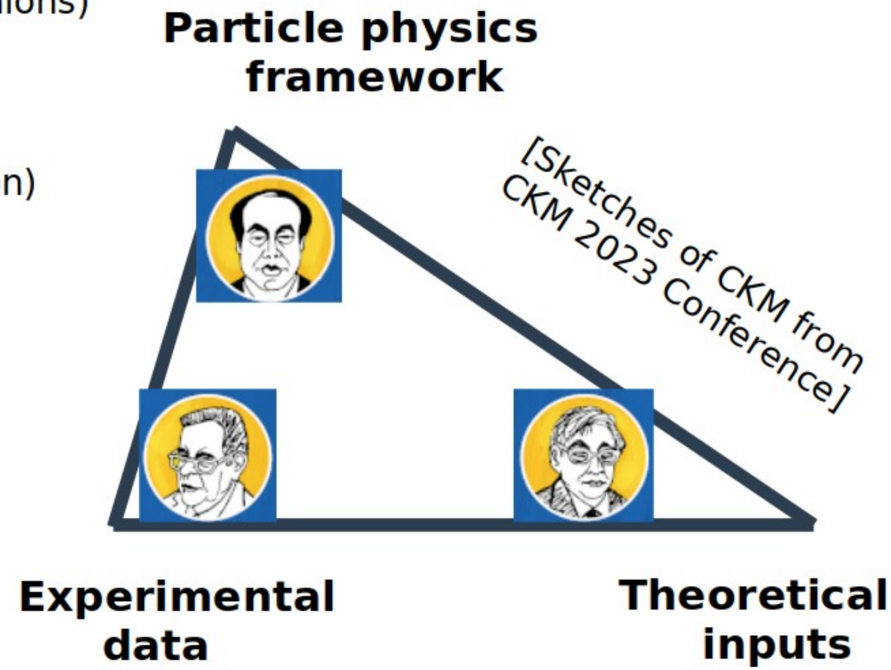
$$+ |D_\mu H|^2 - V(H)$$

(short-range weak interactions)

$$+ YH\bar{\psi}\psi + \text{h.c.}$$

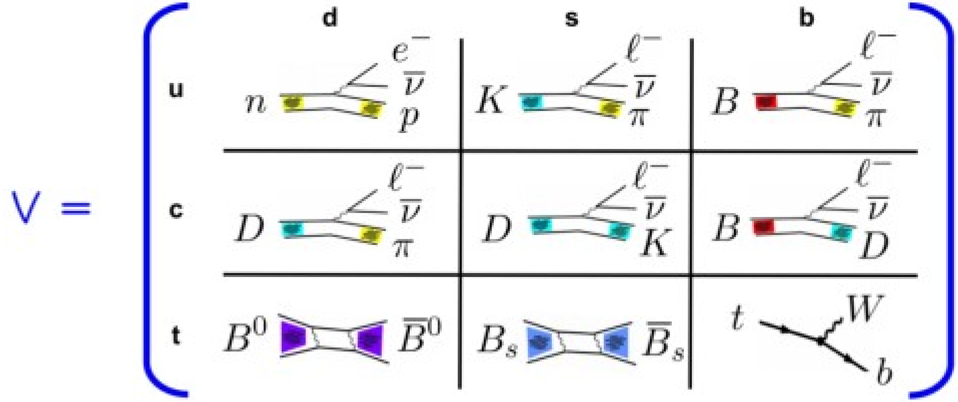
(spectrum of quark masses, CKM matrix)

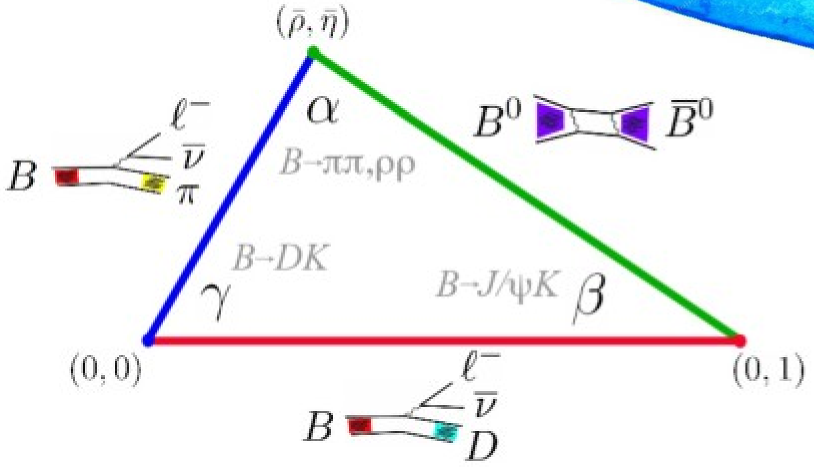
$$\left(+\sum_{d>4} \frac{1}{\Lambda_{heavy}^{d-4}} C_k O_k^d\right)$$



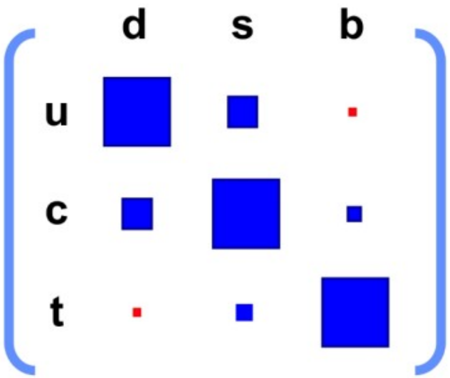
- Goal is **testing the SM/CKM mechanism**, and **point out possible tensions**
- Many flavour observables enjoy the status of precision physics
- Flavour transitions pattern is likely to change in the presence of NP

Measuring the CKM matrix





- *Tree*: e.g., $|V_{ub}|$
- *Loop*: e.g., Δm_d, Δm_s, ϵ_K, $\sin(2\beta)$
- $\mathcal{CP}$*-conserving*: e.g., $|V_{ub}|$, Δm_d, Δm_s
- $\mathcal{CP}$*-violating*: e.g., γ, ϵ_K, $\sin(2\beta)$
- *Exp. uncs.*: e.g., α, $\sin(2\beta)$, γ
- *Syst. uncs.*: e.g., $|V_{ub}|$, $|V_{cb}|$, ϵ_K, Δm_d, Δm_s



Theo. inputs: hadronic effects

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Meson-mixing	$B_{(s)}\overline{B}_{(s)}$, $K\overline{K}$: bag-parameters $\widehat{B}_{B_s}$, $\widehat{B}_{B_s}/\widehat{B}_{B_d}$, $\widehat{B}_K$ $\frac{2}{3}m_K^2 f_K^2 B_K = \langle\overline{K}\vert(\bar{s}\gamma^\mu P_L d)(\bar{s}\gamma_\mu P_L d)\vert K\rangle$
(semi-)leptonic decays	$\pi \to \ell\nu$, $K \to \pi\ell\nu$, etc.: decay constants, form factors Ex.: f_π, $f_+^{K\to\pi}(0)$ $-p_\mu f_\pi = \langle 0\vert(\bar{d}\gamma_\mu\gamma_5 u)\vert\pi(p)\rangle$, $f_+^{K\to\pi}(q^2)(p+p')_\mu + f_-^{K\to\pi}(q^2)(p-p')_\mu = \langle\pi(p')\vert(\bar{s}\gamma_\mu P_L u)\vert K(p)\rangle$

$\rightarrow$ Determine $\mathcal{L}^{eff}_{SM\,(NP)} \sim \Sigma_i C_i(\mu) \times O_i(\mu)$, where $\mu \sim \mathcal{O}$(few) GeV:
C_i collects *short*-distance physics; O_i collects *long*-distance physics

$\rightarrow$ Lattice QCD: extractions of non-perturbative parameters;
averages dominated by **systematic uncertainties**

Stat: essentially size of gauge configurations
Syst: fermion action, a → 0, L → ∞, mass extrapolations...

FLAG reports:
guide to sort results

Theo. uncertainties

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- **Statistical uncertainties** result from the intrinsic variability of data, typically distributed normally
- **Theoretical uncertainties** are different in nature: they are modeling parameters (ξ), fixed and unknown, that incorporate our incomplete knowledge about the properties of a distribution (<u>Ex.: truncation of a perturbative series</u>) [Punzi '01]
- Though *a priori* theoretical uncertainties are a universal issue, in the context of **quark flavor physics** they are particularly important, due to the **strong dynamics**

Statistical framework

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- ***CKM*fitter**: frequentist statistics based on a χ^2 analysis
- χ^2_{min}: **goodness-of-fit** under SM (or NP), **estimators** for q=V_{CKM}
- $\Delta\chi^2$ (χ^2-distributed): **Confidence Level** (CL) intervals

$$\mathcal{L}(q) = \prod_{\mathcal{O}} \mathcal{L}_{\mathcal{O}}(q)\,, \quad \chi^2(q) = -2\ell n\mathcal{L}(q) = \sum_{\mathcal{O}} \left(\frac{\mathcal{O}_{\rm th}(q) - \mathcal{O}_{\rm meas}}{\sigma_{\mathcal{O}_{\rm meas}}} \right)^2$$

max likelihood

ratio of likelihoods

$$\chi^2(\hat{q}) = \min_q \chi^2(q)\,, \quad \Delta\chi^2(q) = \chi^2(q) - \chi^2(\hat{q})$$

goodness-of-fit, estimators

confidence levels

*R*fit

***Range* fit** scheme incorporates theoretical uncertainties

- stat: agreement of data & prediction; theo: accuracy of QCD parameters
- Theoretical uncertainties strictly contained in a range. Ex.: $\xi \in [-\Delta, \Delta]$
- Different sources of syst. uncertainty are combined linearly

product of likelihoods

$$\mathcal{L} \overset{Rfit}{=} \mathcal{L}_{stat} \times \mathcal{L}_{theo},$$

$$\chi^2 = -2 \,\ell n \,\mathcal{L}$$

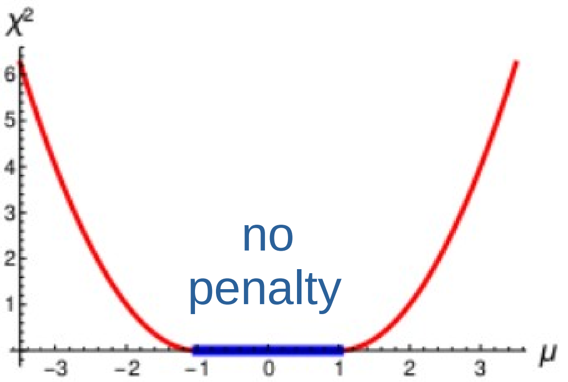
$\mathcal{L}_{stat}$: exp. data

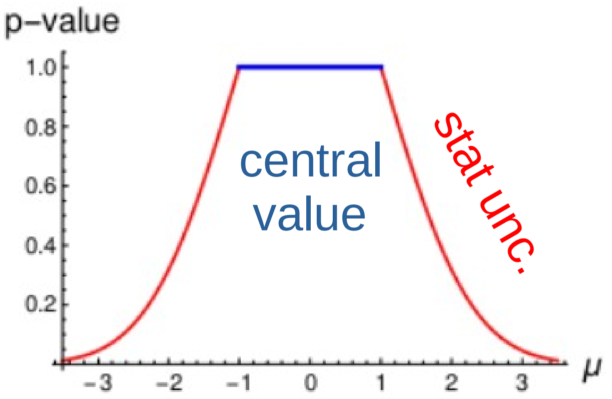
$\mathcal{L}_{theo}$: had. inputs

[cf. Charles, Descotes-G., Niess, LVS '17]

[Hoecker et al. '01, Charles et al. '04]

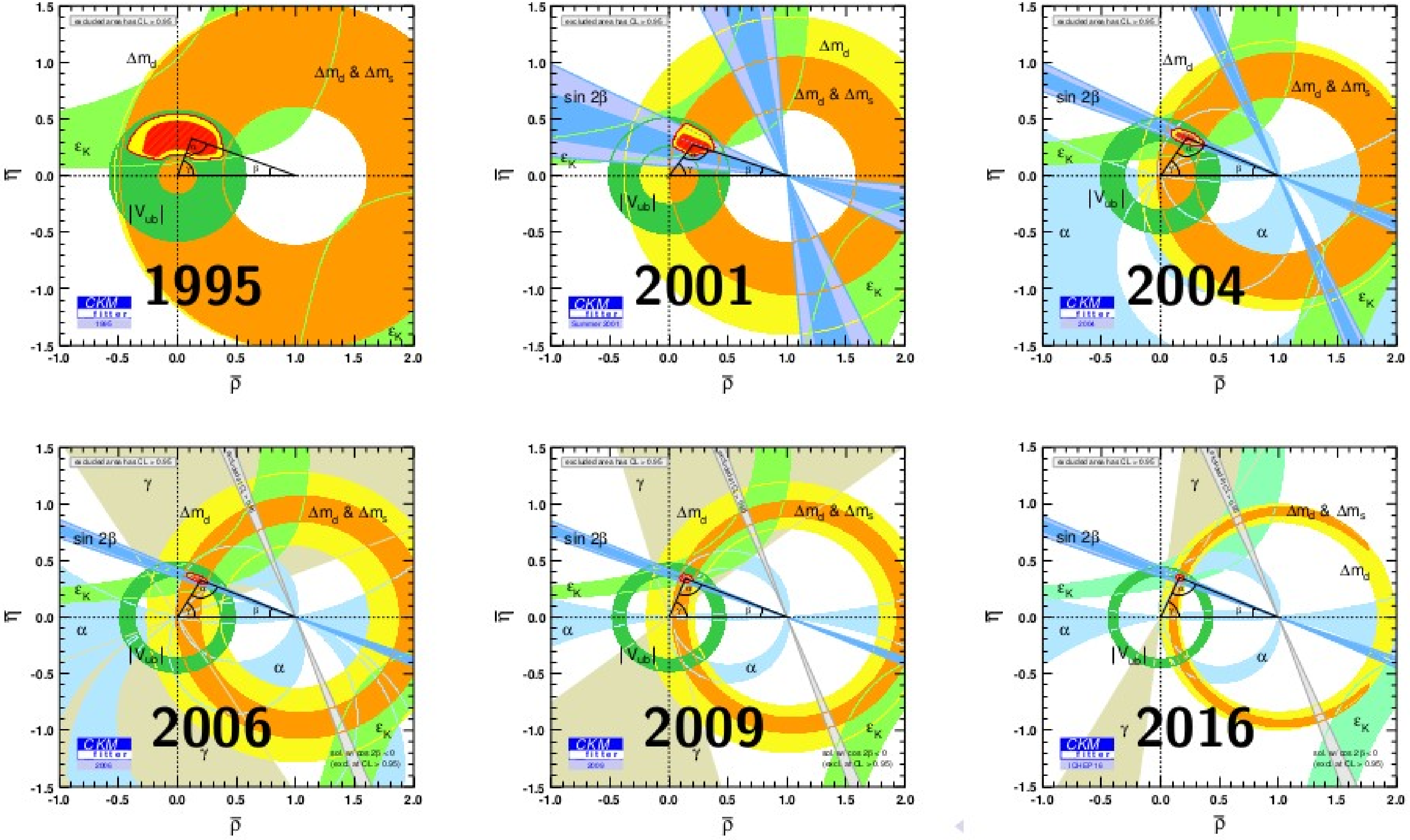
Example in 1D, $0 \pm 1_{stat} \pm 1_{theo}$ $(N_{dof} = 1)$





χ^2: flat bottom, quadratic walls

Progress over the years

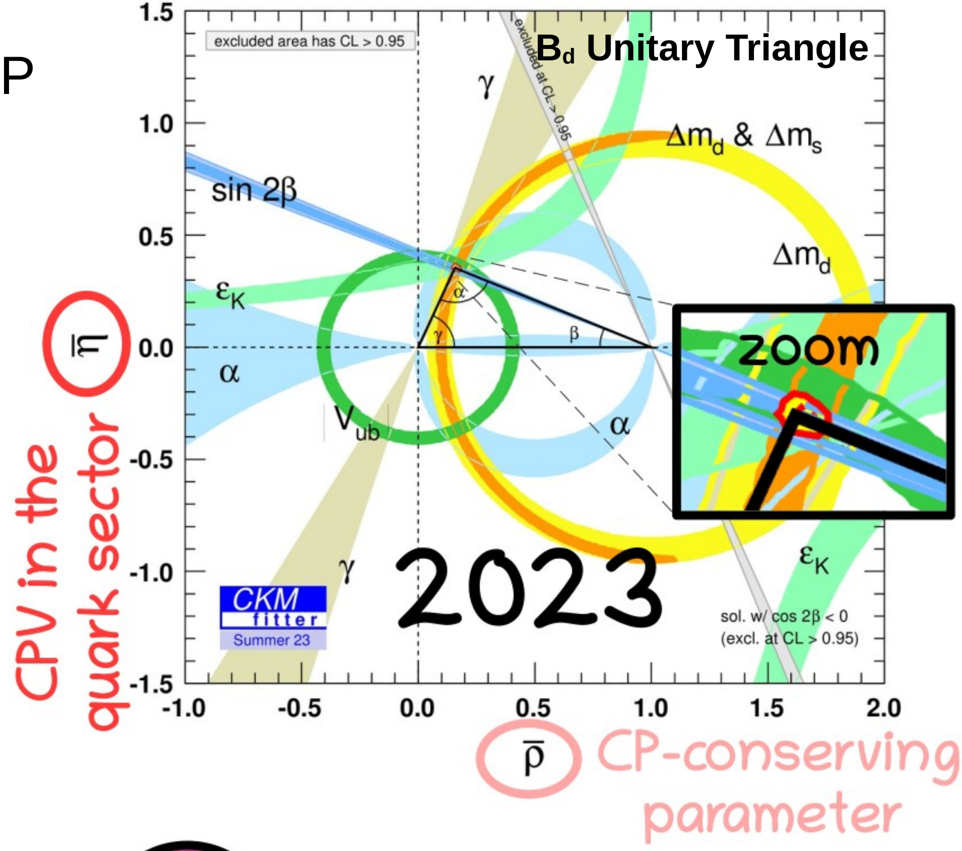


Current status of flavour

- A **single** phase must be responsible for CP violation across distinct flavour sectors
- Observables of very different natures

$$A = 0.8215\,^{+0.0047}_{-0.0082}\ (0.8\%\ \text{unc.})$$
$$\lambda = 0.22498\,^{+0.00023}_{-0.00021}\ (0.1\%\ \text{unc.})$$
$$\bar\rho = 0.1562\,^{+0.0112}_{-0.0040}\ (4.9\%\ \text{unc.})$$
$$\bar\eta = 0.3551\,^{+0.0051}_{-0.0057}\ (1.5\%\ \text{unc.})$$

68% C.L. intervals



Rephasing invariant:

$$\frac{|V_{us}|}{(|V_{ud}|^2+|V_{us}|^2)^{1/2}} = \lambda\,, \quad \frac{|V_{cb}|}{(|V_{ud}|^2+|V_{us}|^2)^{1/2}} = A\lambda^2\,, \quad -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} = \bar\rho + i\bar\eta$$

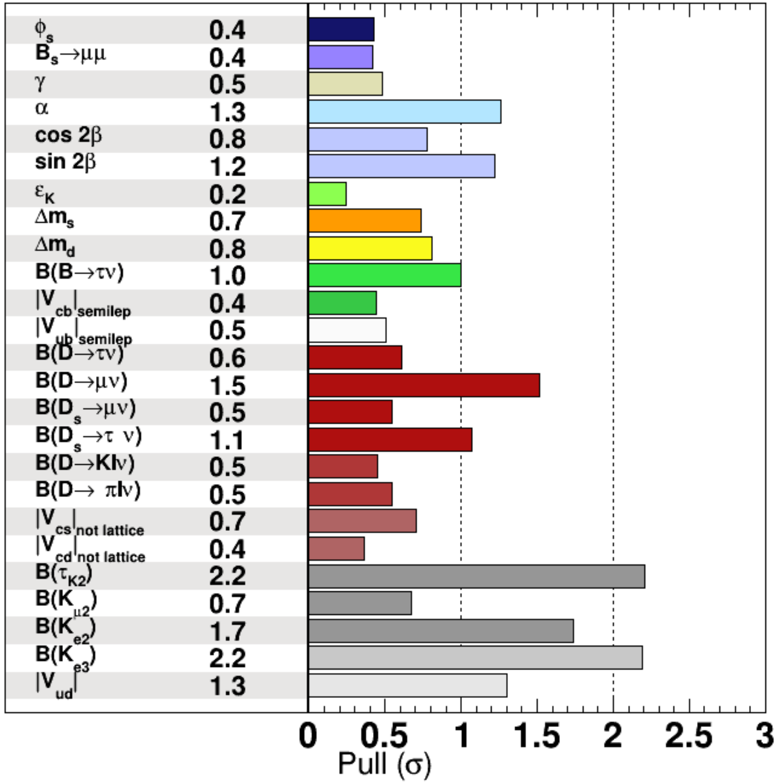
b : $|V_{xb}|$, α, **β**, γ, Δm_d, Δm_s

s : ε_K

Consistency among observables

$$pull_{\mathcal{O}_{exp}} = \sqrt{\chi^2_{min} - \chi^2_{min,!\mathcal{O}_{exp}}}$$

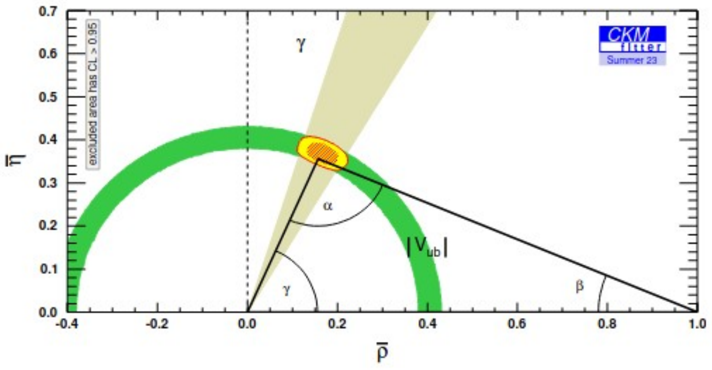
$$!\,\mathcal{O}_{exp}:\ \chi^2_{min}\ \text{w/o}\ \mathcal{O}_{exp}$$



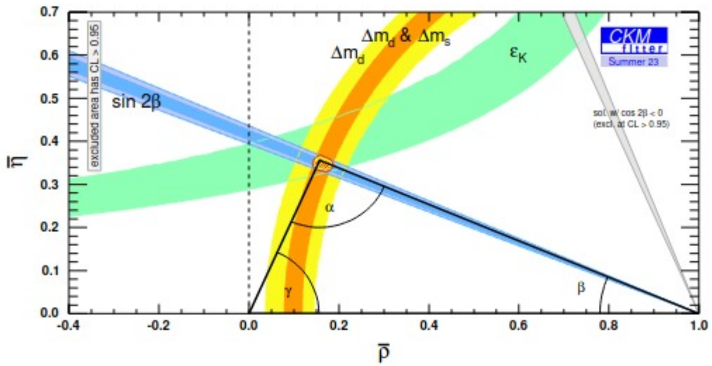
- If Gaussian uncs., uncorrelated random vars.: mean 0 and variance 1
- Here, correlations are expected; pulls can be 0 due to the *R*fit model for systematics
- Overall agreement w/ the SM, no clear indication of significant deviations from CKM picture

Consistency among observables

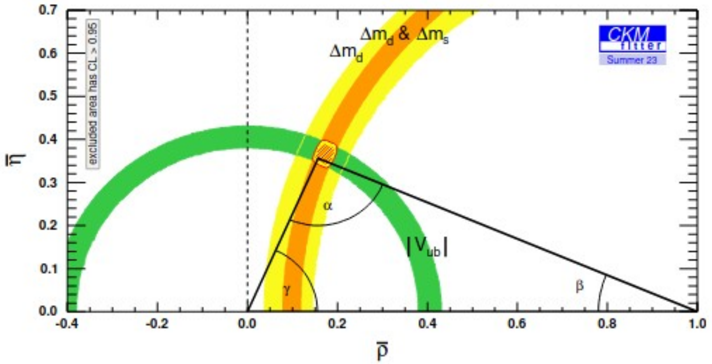
tree level



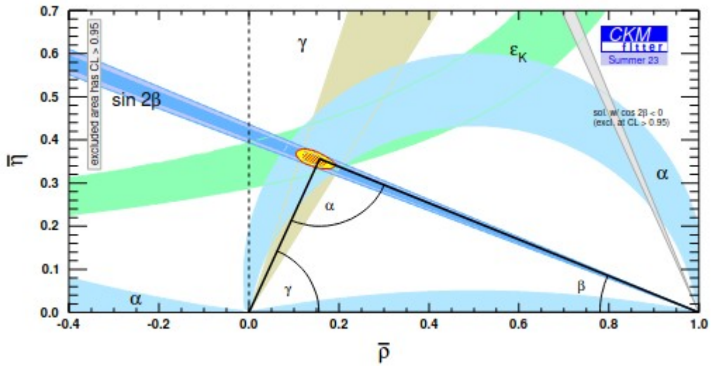
loop-induced



$\mathcal{CP}$−conserving



$\mathcal{CP}$−violating



Alternative frequentist schemes

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Properties of other statistical approaches to incorporate theoretical uncertainties; <u>impact on the extraction of the true values of parameters</u>

Given the one-dimensional (1D) case: $X \sim X_0 \pm \underbrace{\sigma}_{statistical} \pm \underbrace{\Delta}_{theoretical}$

$\rightarrow$ The true value of the theo. uncertainty ξ is fixed and unknown

$\rightarrow$ Being unknown, one quotes a range $\xi \in \Omega$ and **vary** ξ

$\rightarrow$ Usually, one has in mind that $\Omega = [-\Delta, \Delta]$, but this may miss an unexpectedly large value of ξ

$\rightarrow$ Were ξ known, we would quote instead $(X_0 + \xi) \pm \sigma$

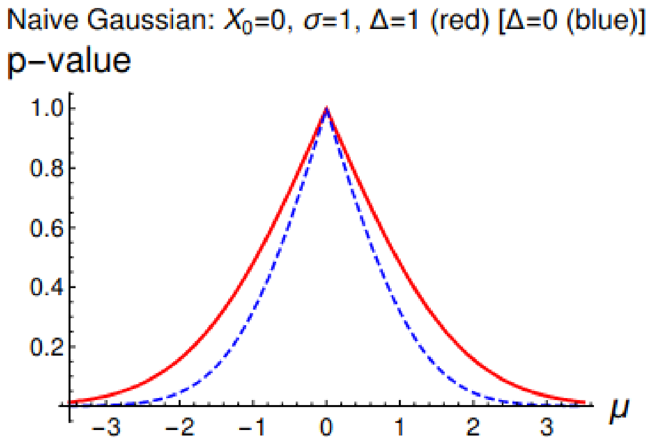
Modeling theo uncs: random

Random approach

$\rightarrow$ Different techniques of calculation lead to different predictions around the exact one (pseudo-randomly distributed)

$\rightarrow$ Naive Gaussian (nG): $\xi \sim \mathcal{N}_{(0,\Delta)}$

$\rightarrow$ MLR ($\mathcal{H}_\mu : x_t = \mu$): $T(X;\mu) = \frac{(X-\mu)^2}{\sigma^2+\Delta^2}$



(MLR: maximum likelihood ratio)

Modeling theo uncs: external

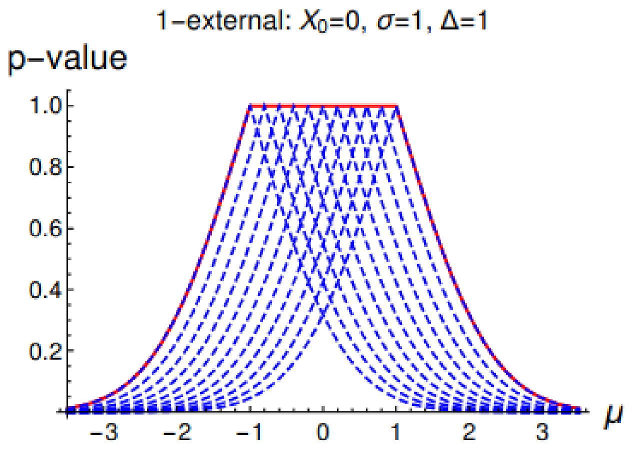
***n*-external approach** [Scan: Dubois-Felsmann et al.]

→ In a first step, assume that ξ is known

→ Family of hypotheses, $\mathcal{H}_\mu^{(\xi)} : x_t = \mu + \xi$

→ MLR $(\mathcal{H}_\mu^{(\xi)})$: $T(X;\mu) = \frac{(X-\mu-\xi)^2}{\sigma^2}$

→ Combine the p_ξ, for $\xi \in n \times [-\Delta, \Delta]$



Close to what some experiments interpret as theo. uncertainties

Simple 1D case: **Rfit** and **1-external** are equivalent

Modeling theo uncs: nuisance

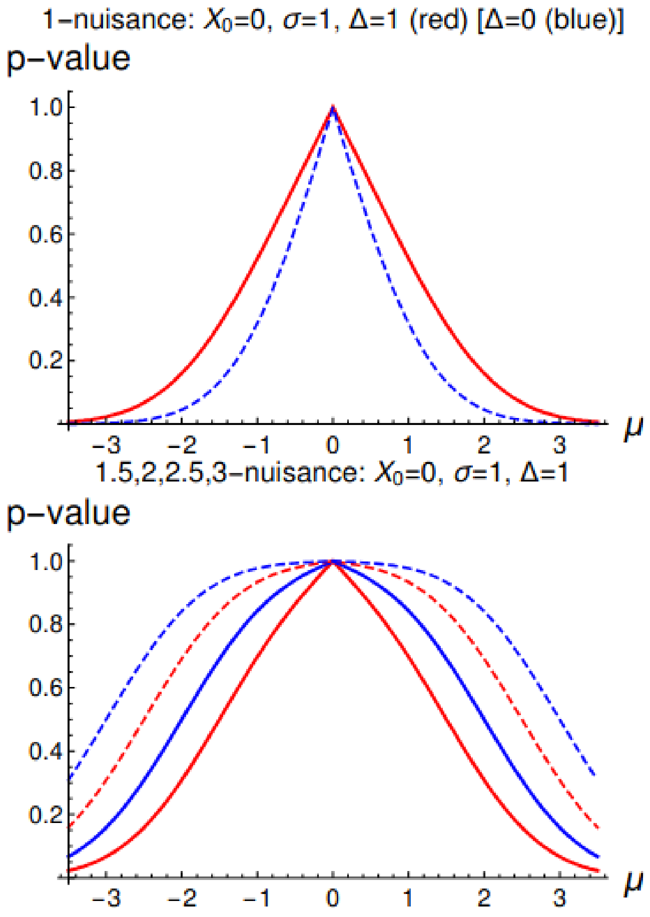
Fixed-n nuisance

$\rightarrow$ ~MLR ($\mathcal{H}_\mu : x_t = \mu$): $T(X;\mu) = \frac{(X-\mu)^2}{\sigma^2+\Delta^2}$

$\rightarrow$ ξ strictly found in $n \times [-\Delta, \Delta]$

$\rightarrow$ Small n may lead to reasonable CLs, but possibly uncovering

$\rightarrow$ Large n avoid uncovering, but lead to large CLs



Modeling theo uncs: nuisance

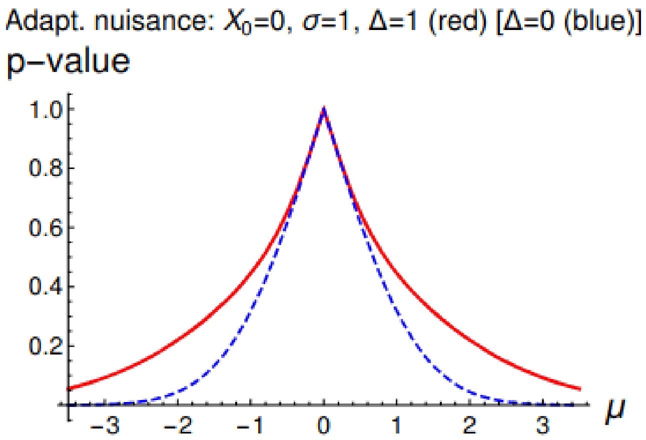
Adaptive nuisance

[Charles, Descotes-G., Niess, LVS]

→ ~MLR ($\mathcal{H}_\mu : x_t = \mu$): $T(X;\mu) = \frac{(X-\mu)^2}{\sigma^2+\Delta^2}$

→ The interval where we look for ξ grows w/ the CL interval we want to quote

→ n CL intervals: $\xi \in n \times [-\Delta, \Delta]$



Designed to deal with:

- Metrology/extraction of parameters ($1-2\,\sigma$ intervals)
- Minimizing Type-II (false positive) errors (above $\sim 5\,\sigma$)

Coverage in special cases

Limit case: the simulated ξ is at the edge of $[-\Delta, \Delta]$

$\Delta/\sigma = 3,\ \xi/\Delta = 1$	68.27% CL	95.45% CL	99.73% CL
nG	56.3%	100.0%	100.0%
1-nuisance	68.1%	95.5%	99.7%
adaptive nuisance	68.2%	100.0%	100.0%
1-external/Rfit	84.1%	97.7%	99.9%

Unfortunate case: the simulated ξ is outside $[-\Delta, \Delta]$

$\Delta/\sigma = 3,\ \xi/\Delta = 3$	68.27% CL	95.45% CL	99.73% CL
nG	0.00%	0.35%	68.7%
1-nuisance	0.00%	0.00%	0.07%
adaptive nuisance	0.00%	9.60%	99.8%
1-external/Rfit	0.00%	0.00%	0.13%

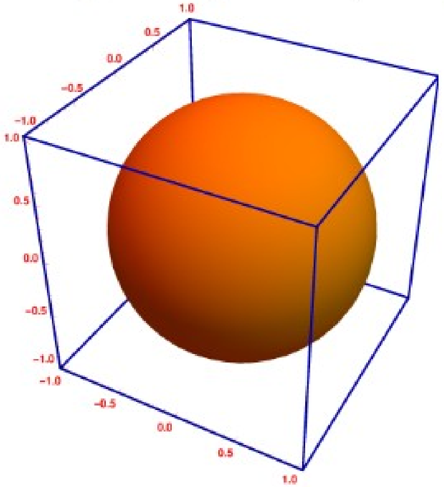
red: undercoverage
blue: exact or overcoverage

Multi-dimensional case

$$X_0^{(i)} \pm \sigma_i \pm \Delta_i,\ \xi_i \in [-\Delta_i, \Delta_i] \quad (\text{or } X_0 \pm \sigma \pm \Delta_1 \pm \ldots \pm \Delta_N)$$

Average: $\xi = \sum_{i=1}^{N} w_i\, \xi_i$, w/ weights $\sum_{i=1}^{N} w_i = 1\,,\, w_i \geq 0$

Edges: $[-\Delta_i, \Delta_i]$



Interval where the bias ξ_i is varied

Hyper-cube: assuming extreme values simultaneously $\widehat{\Delta} = \sum_{i=1}^{N} w_i \Delta_i$

Hyper-ball: $\widehat{\Delta} = \sqrt{\sum_{i=1}^{N} (w_i \Delta_i)^2}$

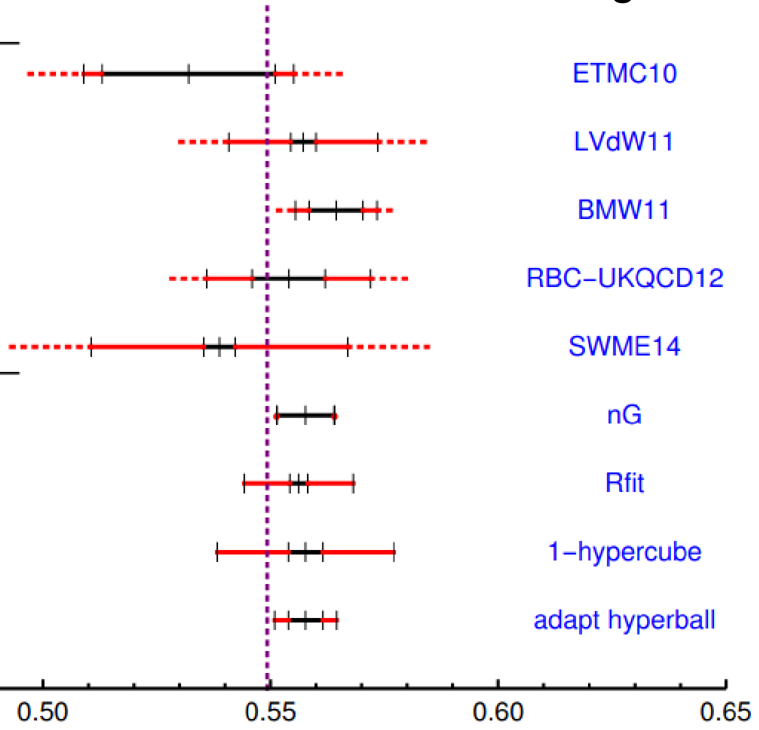
$$\Rightarrow \widehat{X}_0 \pm \widehat{\sigma} \pm \widehat{\Delta}$$

Combining data

Example: combination of different extractions of $B_K^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})$

(red edges: 1σ; purple: "naive" average of the CVs)

	CV	stat	(theo) quad	(theo) lin
ETMC10	0.532	(19)	(12)	(26)
LVdW11	0.557	(03)	(15)	(26)
BMW11	0.564	(06)	(06)	(10)
RBC-UKQCD12	0.554	(08)	(14)	(22)
SWME14	0.539	(03)	(27)	(44)
nG	0.5577	(63)	-	
*R*fit	0.556	(02)	(10)	
1-hypercube	0.558	(04)	(18)	
adapt hyperball	0.5577	(38)	(50)	



[further examples: Charles, Descotes-G., Niess, LVS '17]

Conclusions

- Global fit of a rich variety of processes sensitive to CP Violation and SM predictions in agreement
- We are then able to extract accurate values for the fundamental parameters describing the CKM matrix
- **Theoretical uncertainties** are omnipresent in flavor analyses and deserve a careful look; they carry an ill-defined nature
- The choice of the scheme has an impact on: confidence level intervals, metrology, significance of a tension, etc.
- Future: study properties of different treatments of theoretical uncertainties in the full CKM fit (coverage, separation of uncs, computing time, etc.)

CKMfitter Collaboration

MORE DETAILS @ ▸ CKMfitter

Jérôme Charles, Theory
Olivier Deschamps, LHCb
Sébastien Descotes-Genon, Theory
Stéphane Monteil, LHCb
Jean Orloff, Theory
Wenbin Qian, LHCb/BESIII
Vincent Tisserand, LHCb/BABAR
Karim Trabelsi, Belle/Belle II
Philip Urquijo, Belle/Belle II
Luiz Vale Silva, Theory

THANKS!

Status of NP in B meson mixing

app

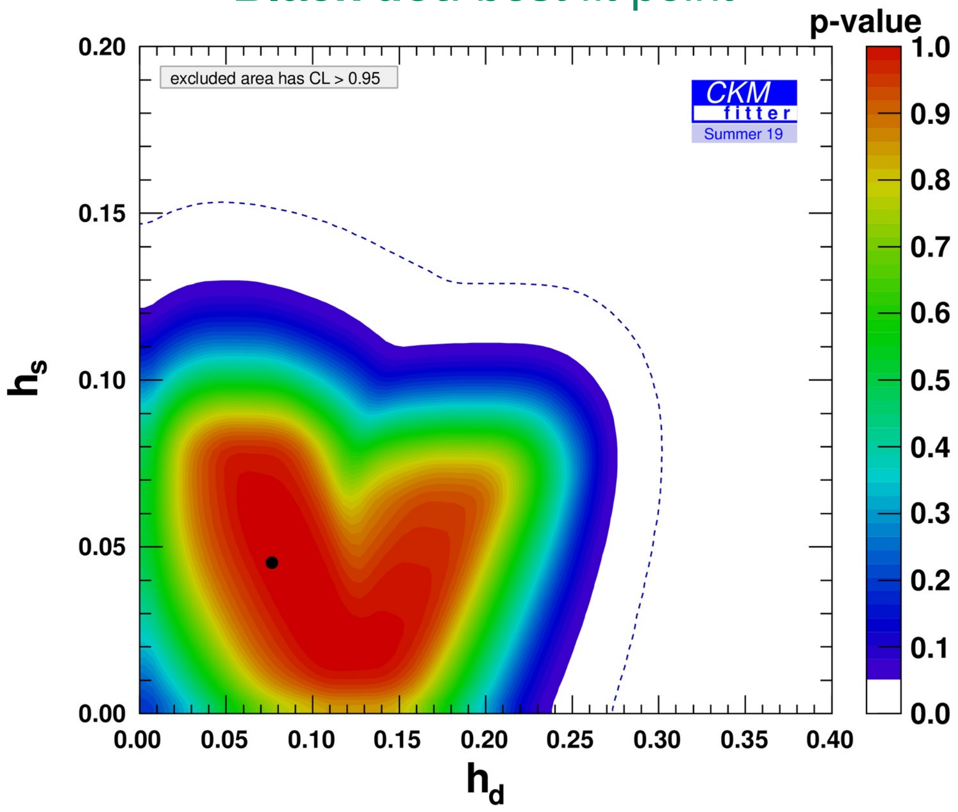
$$M_{12} = M_{12}^{\rm SM} \times (1 + h\, e^{2i\sigma})$$

NP parameters

- Agreement with the SM (h_d=h_s=0) at ~1σ
- Allowed size for NP at the level of O(20%)
- Extractions of ρ and η (Wolfenstein parameters) **degrade by factor ~3**

[Charles, Descotes-G., Ligeti, Monteil, Papucci, Trabelsi, LVS ‘20]

Black dot: best fit point



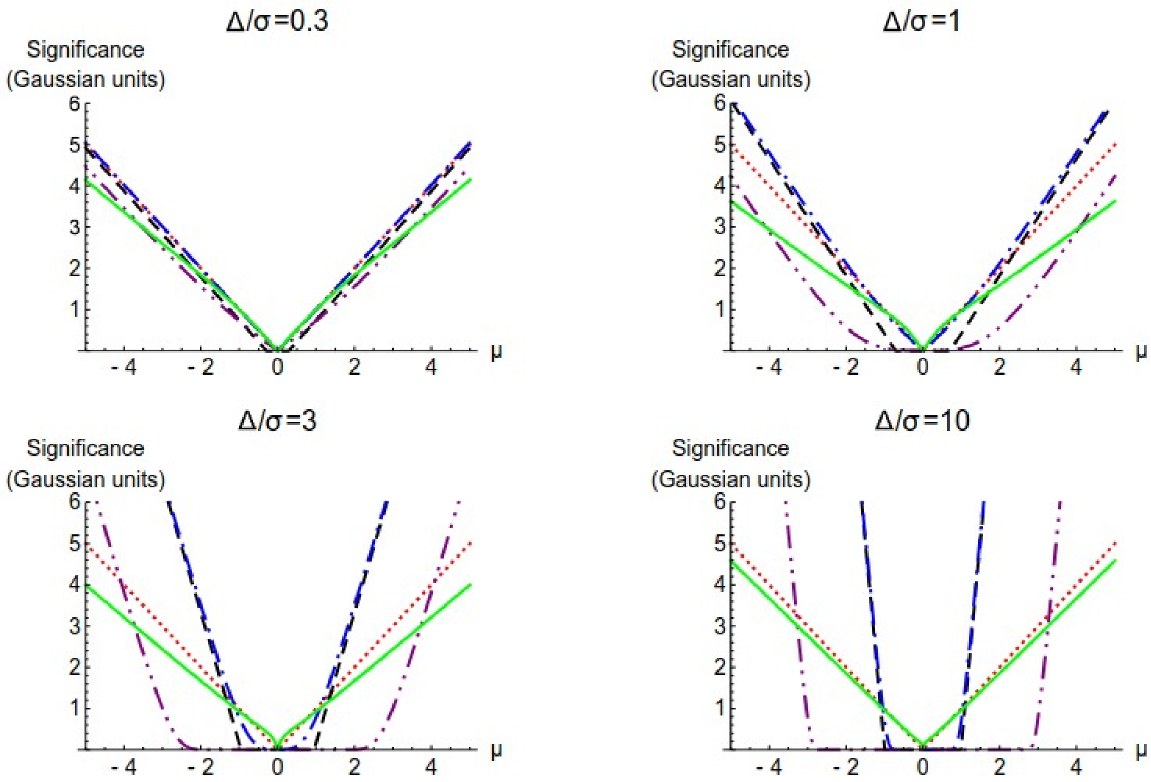
[See also: UTfit; De Bruyn, Fleischer, Malami, van Vliet '23]

Illustration of CL intervals, 1D

app

→ Consider $0 \pm \sigma \pm \Delta$, w/ fixed $\sigma^2 + \Delta^2 = 1$

→ Gaussian units: $\sqrt{2}\,\mathrm{Erf}^{-1}(1 - p(X_0;\mu))$



(**red**) naive Gaussian (nG);

(**black**) fixed-1 external/Rfit;

(**blue**) fixed-1 nuisance;

(**purple**) fixed-3 nuisance;

(**green**) adaptive nuisance

Combining data

app

Example: combination of different extractions of $B_K^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})$

(orange edges: 3σ; purple: "naive" average of the CVs)

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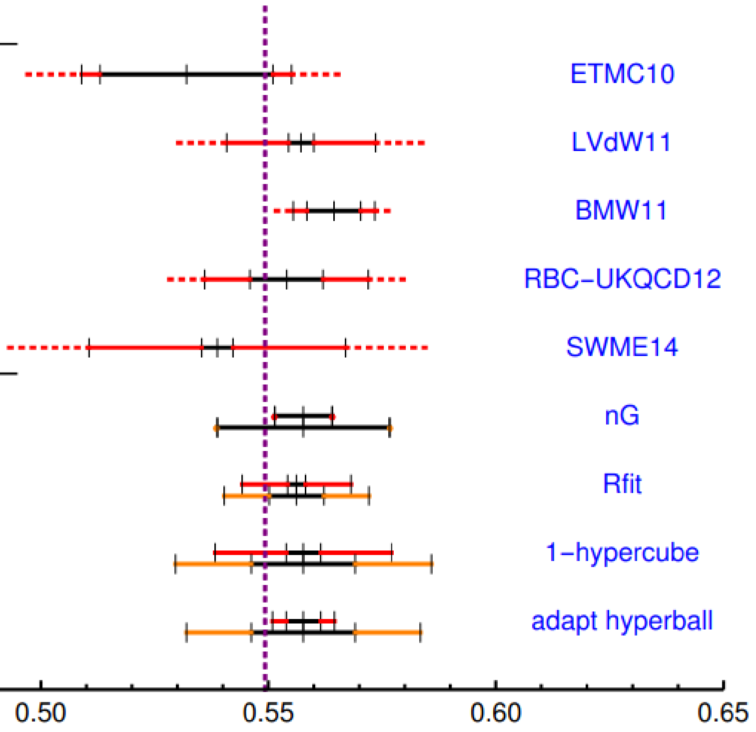

LVS

Different significances

app

$$\left(a_{muon}^{exp} - a_{muon}^{SM}\right) \times 10^{11} = 288 \pm 63_{exp} \pm 49_{SM}$$

significance of the tension	
nG	$3.6\,\sigma$
1-external/Rfit	$3.8\,\sigma$
1-nuisance	$3.9\,\sigma$
adapt. nuisance	$2.7\,\sigma$