HFLAV and Result Combination in Flavour Physics

Abi Soffer
Tel Aviv University

JENNIFER2 meeting March 2025

Outline

- The Heavy Flavor Averaging Group (HFLAV)
- Differences wrt. the PDG
- Statistical techniques
- Example cases
- Recommendations and requests

Heavy Flavor Averaging Group

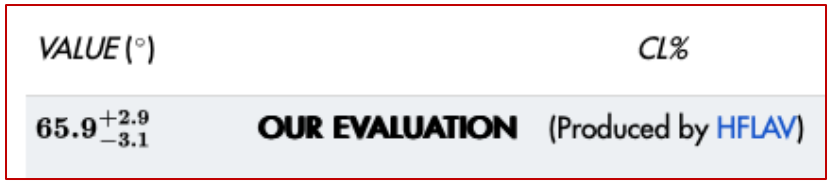
- https://hflav.web.cern.ch:
  - "The Heavy Flavor Averaging Group (HFLAV) was established at the May 2002 Flavor Physics and CP Violation Conference (Philadelphia) and continues the LEP Heavy Flavor Steering Group's tradition of providing regular updates to the world averages of heavy flavor quantities."
- Current leaders:
  - LHCb: Ulrik Egede
  - Belle II: Mirco Dorigo, recently replaced me (2018-2025)
- 45 additional members in 8 groups:
  - B oscillations, unitarity-triangle angles, semileptonic b & UT sides,
    $b \to c$ decays, rare b decays,
    charm CPV and oscillations, charm decays, τ properties
- The leaders and members are appointed by their collaborations
  - Appointment is for 4 years (decided at January 2025 meeting), renewable for w/o limit for 4 years at a time

HFLAV publications

- "Averages of *b*-hadron properties":
  - hep-ex/0412073
  - hep-ex/0505100
  - hep-ex/0603003
  - 0704.3575
- "Averages of *b*-hadron and *c*-hadron properties":
  - 0808.1297
- "Averages of *b*-hadron, *c*-hadron, and τ-lepton properties"
  - 1010.1589
  - 1207.1158
  - 1412.7515
    - EPJC 77 (2017) 12, 895, 1612.07233
    - EPJC 81 (2021) 3, 226, 1909.12524
    - PRD 107 (2023) 5, 052008, 2206.07501
    - (To appear in PRD) 2411.18639
- Updates at https://hflav.web.cern.ch, but starting to appear at https://zenodo.org
  - Provides a citable DOI for updated average between publications

HFLAV and PDG

- Unlike PDG:
  - HFLAV doesn't scale up systematic uncertainties if $\chi^2/\mathrm{N_{dof}} > 1$
    Rather, we discuss with the analysts and examine the systematic uncertainties to try and understand potential sources of discrepancy
  - HFLAV includes new, yet-unpublished results, as long as they are documented.
    Results get dropped if they remain unpublished for too long
- PDG adopts HFLAV averages in several areas (calculated with PDG procedures):
  - Unitarity triangle angles
  - B-meson oscillation frequencies
  - B-meson semileptonic branching fractions and V_{cb}, V_{ub}
- Right now these appear as a link to the general HFLAV page:
  - This makes it difficult to find the actual result
- Starting to improve the procedure:
  - Related groups of up-to-date HFLAV results appear as a https://zenodo.org entry
  - Facilitates precise citation by PDG and also by experimental and theoretical papers



VALUE (°)		*CL%*
$65.9^{+2.9}_{-3.1}$	**OUR EVALUATION**	(Produced by HFLAV)

Statistical method

- For N independent measurements of $x_1 \ldots x_n$, minimize

$$\chi^2(\boldsymbol{x}) = \sum_i^N (\boldsymbol{x}_i - \boldsymbol{x})^{\mathrm{T}} \boldsymbol{V}_i^{-1} (\boldsymbol{x}_i - \boldsymbol{x})$$

  to obtain the estimates $\hat{\boldsymbol{x}}$ and their covariance matrix $\hat{V}^{-1} = \sum_n^N V_i^{-1}$

- Experiments often measure functions $f_i(\boldsymbol{p})$ of parameters $\boldsymbol{p}$, e.g., ratios of branching fractions. This results in correlations among many measurements involving "normalization branching fractions". Then:

$$\chi^2(\boldsymbol{p}) = \sum_i^N (\boldsymbol{f}_i(\boldsymbol{p}) - \boldsymbol{x}_i)^{\mathrm{T}} \boldsymbol{V}_i^{-1} (\boldsymbol{f}_i(\boldsymbol{p}) - \boldsymbol{x}_i)$$

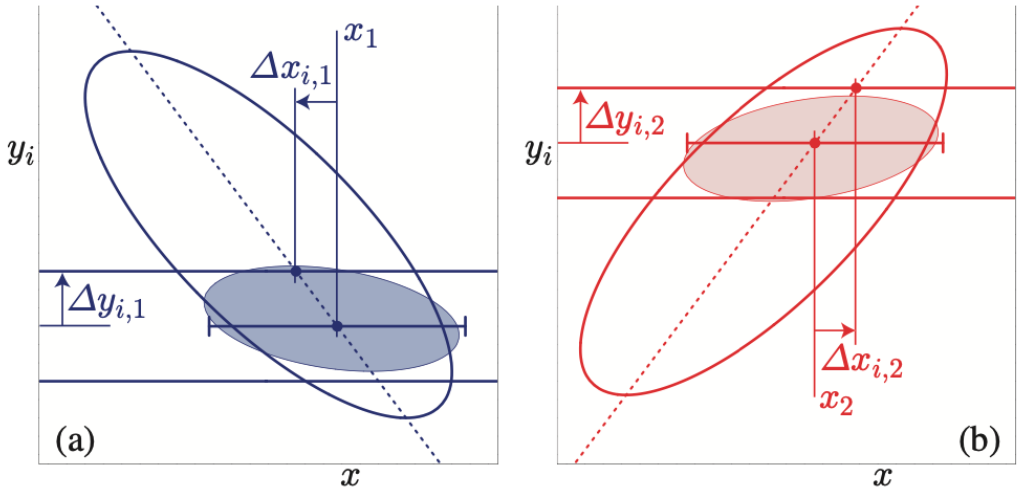
Correlated & updated systematics

Consider 2 measurements

$$x_1 \pm \delta x_1 \pm \Delta x_{1,1} \pm \Delta x_{1,2} \ldots$$
$$x_2 \pm \delta x_2 \pm \Delta x_{2,1} \pm \Delta x_{2,2} \ldots$$

Statistical Systematic

- $\Delta x_{k,i}$ may depend on measurements of quantities y_i, introducing correlations b/w x_1 & x_2. E.g., $x = B^0 - \bar{B}^0$ oscillation frequency, $y = B^0$ lifetime.
- Different values of y_i may have been used in x_1,x_2



- By now there might be yet a newer value $y_i' \pm \Delta y_i'$

Correlated & updated systematics

Recent measurement

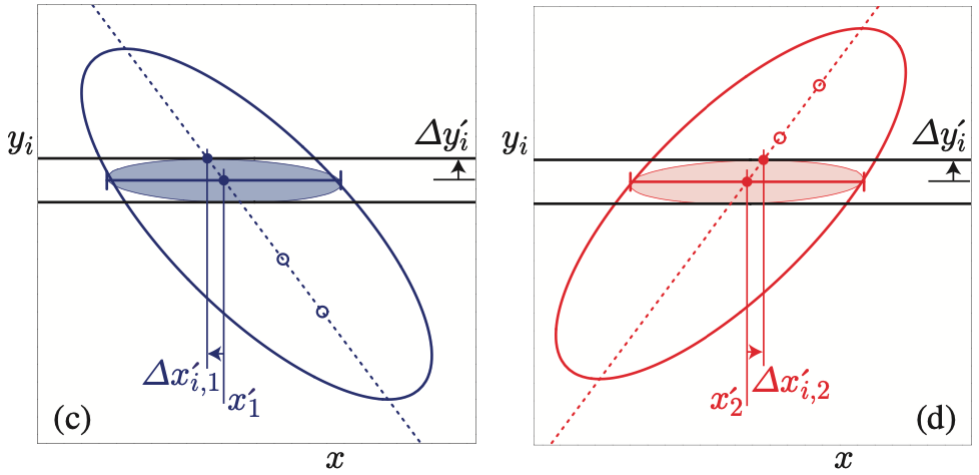
- Ideally, combine the likelihoods to obtain x, y_i simultaneously:

$$\mathcal{L}_{\text{comb}}(x, y_1, y_2, \ldots) \equiv \prod_k \mathcal{L}_k(x, y_1, y_2, \ldots) \exp\left[-\frac{1}{2}\left(\frac{y_i - y_i'}{\Delta y_i'}\right)^2\right]$$

- But likelihoods are usually not given, forcing us to make a Gaussian approximation:
- Adjust the old measurements to the new y_i' values:

$$x_k' = x_k + \sum_i \frac{\Delta x_{k,i}}{\Delta y_{k,i}} (y_i' - y_{k,i})$$

$$\Delta x_{k,i}' = \Delta x_{k,i} \frac{\Delta y_i'}{\Delta y_{k,i}} .$$

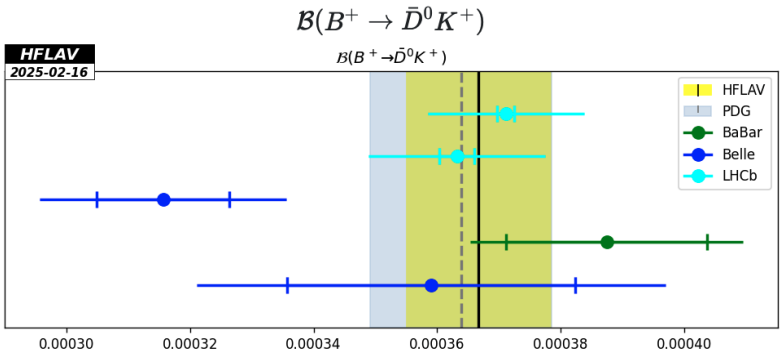


- Minimize the combined χ^2 wrt. x and y_i

$$\chi^2_{\text{comb}}(x, y_1, y_2, \ldots) \equiv \sum_k \frac{1}{\delta x_k^2}\left[x_k' - \left(x + \sum_i (y_i - y_i')\frac{\Delta x_{k,i}'}{\Delta y_i'}\right)\right]^2 + \sum_i \left(\frac{y_i - y_i'}{\Delta y_i'}\right)^2$$

- We don't quote the new values of y_i, since $\Delta x_{k,i}$ weren't estimated with this goal in mind

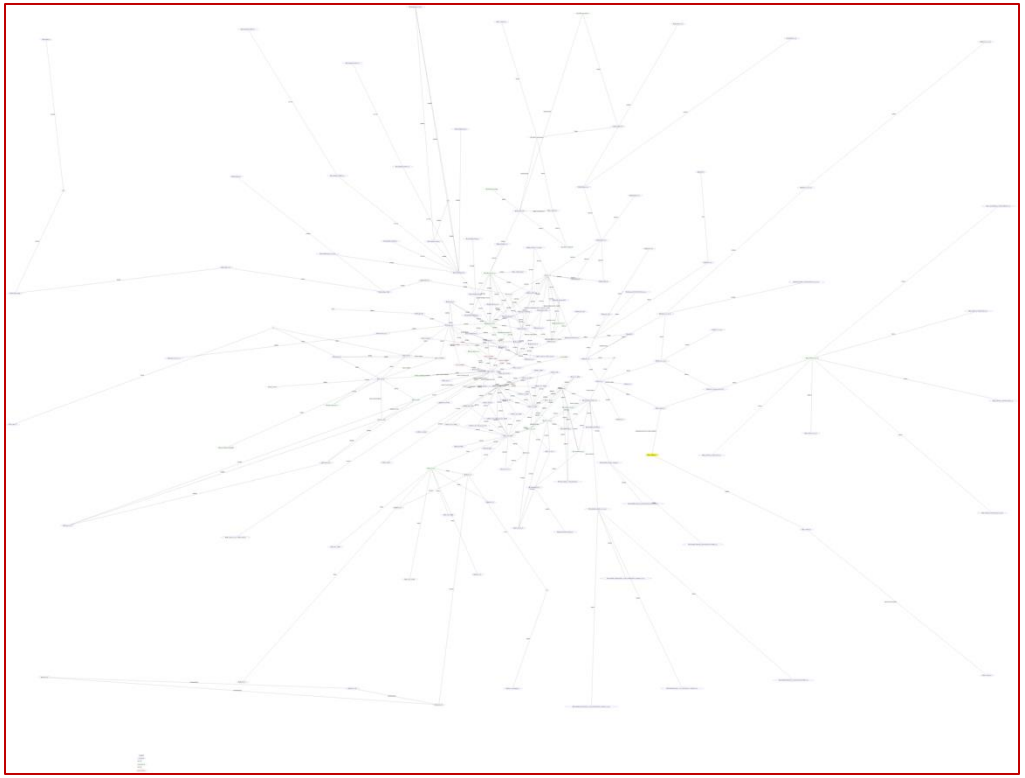
E.g., $Br(B^+ \to \bar{D}^0 K^+)$



Note difference wrt. PDG average

- No error scaling
- May include yet-unpublished results

Correlations with dozens of measurements:



The scale:

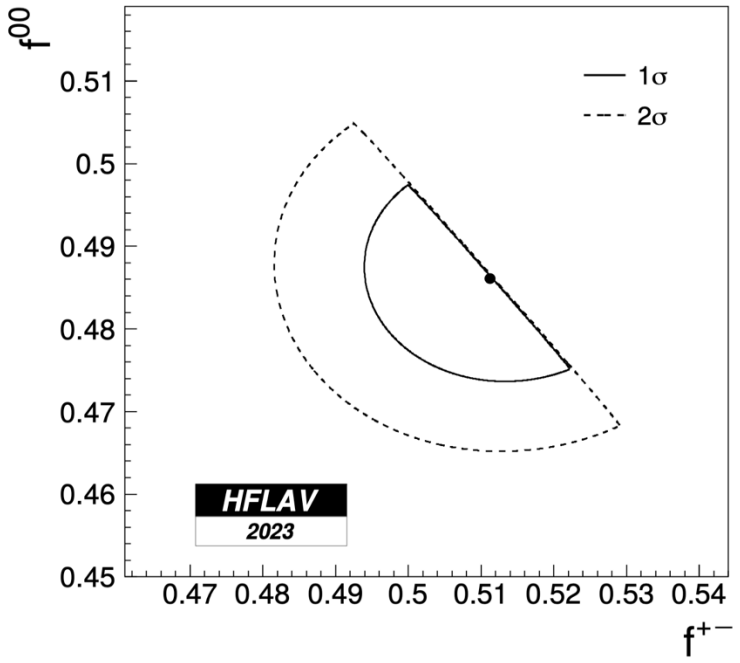
- Among the $b \to c$ decay modes, we combine 717 measurements from 305 papers into 503 averages
- Also O(100s) measurements of rare b -hadron decays, charm decays
- Other groups deal with fewer measurements

E.g., $Br(\Upsilon(4S) \to B\bar{B})$

- All reported B-meson branching fractions used to have the (usually unstated) assumption $Br(\Upsilon(4S) \to B^0\bar{B}^0) = Br(\Upsilon(4S) \to B^+B^-) = 0.5$
- HFLAV now recommends (and Belle II has adopted)

$$f^{00} = 0.4861^{+0.0074}_{-0.0080}, \quad f^{+-} = 0.5113^{+0.0073}_{-0.0108}, \quad f_{\not B} = 0.00264^{+0.0125}_{-0.0002}, \quad \frac{f^{+-}}{f^{00}} = 1.052 \pm 0.031$$

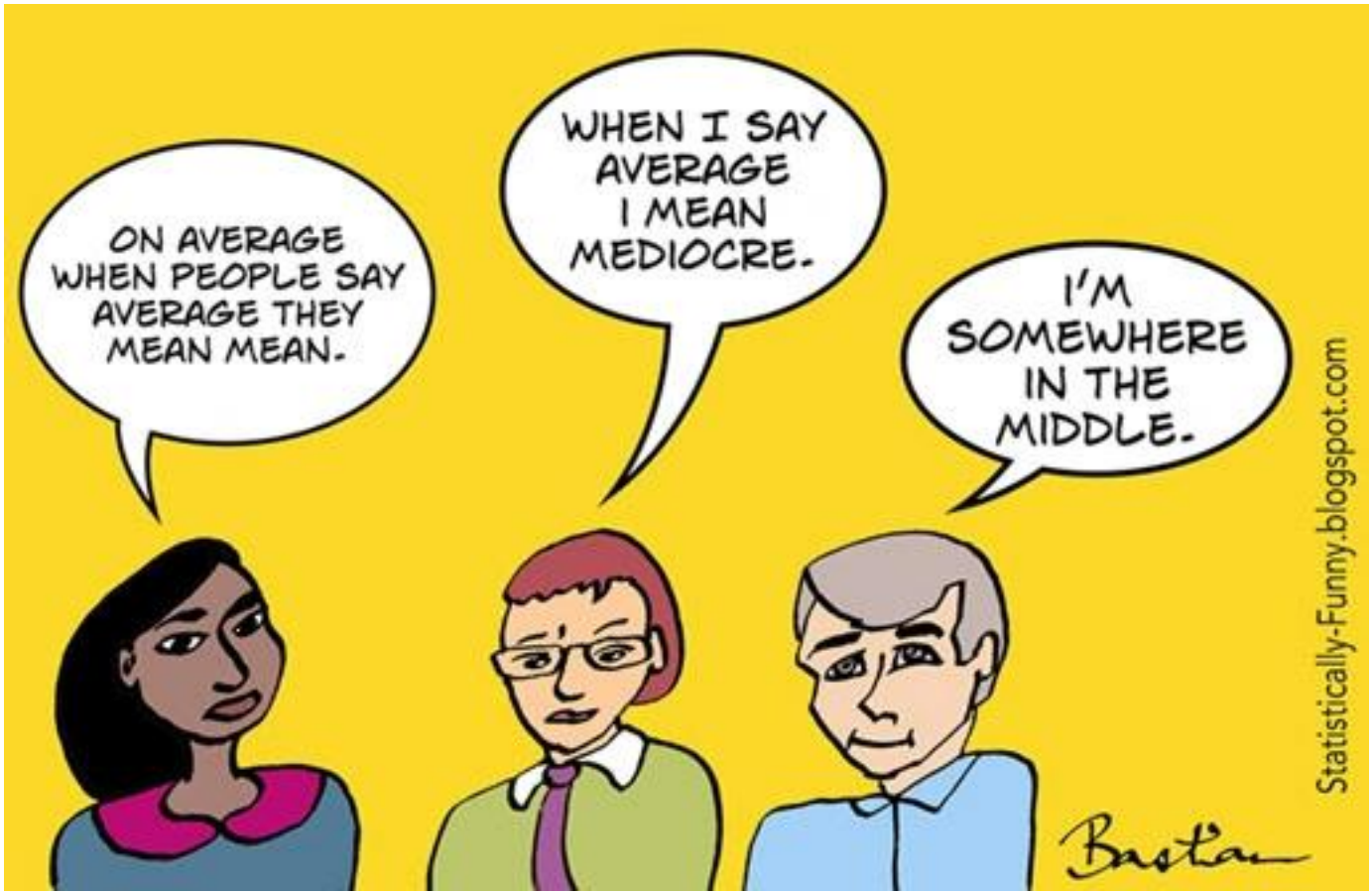
- The correlations are highly non-Gaussian:



Requests and recommendations

- When publishing limits, also publish the central value and uncertainty
  - Well-known rule, but you'd be surprised how often it's not followed…
- When possible, provide the likelihood function of your measurement with respect to all the parameters considered.
  - This will enable the most accurate calculation of averages while updating nuisance parameters
- In any case, provide the values and systematics associated with nuisance parameters to enable executing the y_i-update procedure
  - E.g., $Br(\Upsilon(4S) \to B^0\bar{B}^0)$
- Publish measurements of BR ratios (more precise than measurements of BRs)!
  - LHCb does this regularly since they can't measure the number of b hadrons produced.
  - e^+e^- experiments utilize their ability to measure N_B and $Br(\Upsilon(4S) \to B^0\bar{B}^0)$, so usually quote absolute BRs, which are less precise.
    Therefore, averaging requires accounting for correlations, which can be difficult (different samples taken at slightly different energies by same/different experiments)

Thank you


ON AVERAGE WHEN PEOPLE SAY AVERAGE THEY MEAN MEAN.
WHEN I SAY AVERAGE I MEAN MEDIOCRE.
I'M SOMEWHERE IN THE MIDDLE.
Statistically-Funny.blogspot.com
Bastian