Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA
INFN Istituto Nazionale di Fisica Nucleare Laboratori Nazionali di Frascati

The Yukawa sector in Grand Unified Theories

Vasja Susič

Laboratori Nazionali di Frascati (LNF), INFN

PRIN 2022K4B58X

Neutrinos and Flavour: a stairway to New Physics
Pisa, 2025-04-02

Outline

(1) An introduction to **Grand Unified Theories** (GUTs)

- → unification of SM forces
- → matter unification (at least partial)
- → proton decay

(2) Yukawa sector in GUTs:

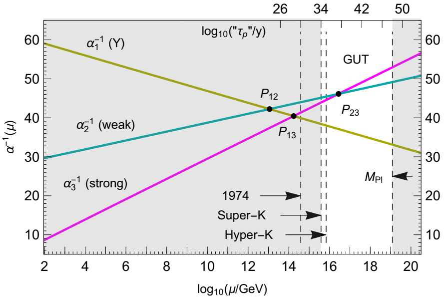
charged and neutrino sectors in minimal setups of

- → $SU(5)$ GUT
- → $SO(10)$ GUT
- → some alternatives

Grand Unified Theories (GUTs) — Motivation

- Do SM gauge couplings unify at a high scale?

Shown: 2-loop RGE in SM:



Grand Unified Theories (GUTs) — Motivation

- Do SM gauge couplings unify at a high scale?

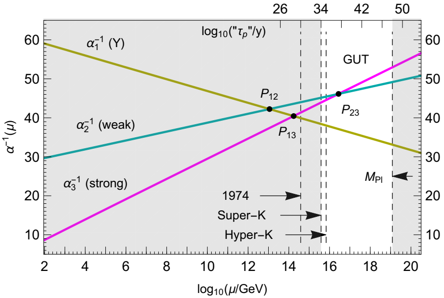
→ Unification of SM forces [1] in G:

(1) G is a **simple group** (one factor)

(2) $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset G$

(3) G has **complex irreps** (SM chiral)

Shown: 2-loop RGE in SM:



Grand Unified Theories (GUTs) — Motivation

- Do SM gauge couplings unify at a high scale?

→ Unification of SM forces [1] in G:

(1) G is a **simple group** (one factor)

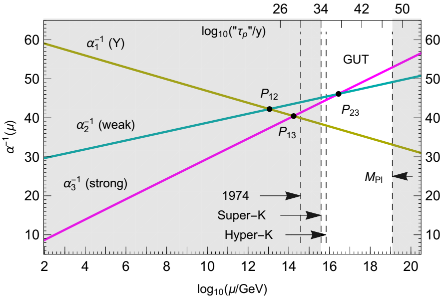
(2) $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset G$

(3) G has **complex irreps** (SM chiral)

→ Classification of simple groups:

- Infinite families:
  $\mathrm{SU}(n), \quad \mathrm{SO}(n), \quad \mathrm{Sp}(2n)$
- Exceptional groups:
  $\mathrm{G}_2, \quad \mathrm{F}_4, \quad \mathrm{E}_6, \quad \mathrm{E}_7, \quad \mathrm{E}_8$

Shown: 2-loop RGE in SM:



Grand Unified Theories (GUTs) — Motivation

- Do SM gauge couplings unify at a high scale?

→ Unification of SM forces [1] in G:

(1) G is a **simple group** (one factor)

(2) $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset G$

(3) G has **complex irreps** (SM chiral)

→ Classification of simple groups:

- Infinite families:

  $\mathrm{SU}(n), \quad \mathrm{SO}(n), \quad \mathrm{Sp}(2n)$

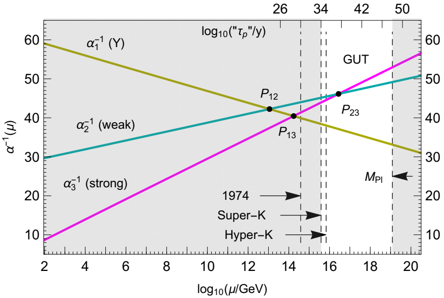
- Exceptional groups:

  $\mathrm{G}_2, \quad \mathrm{F}_4, \quad \mathrm{E}_6, \quad \mathrm{E}_7, \quad \mathrm{E}_8$

→ Groups with $\mathbb{C}$ irreps:

$$\boxed{\mathrm{SU}(n), \quad \mathrm{SO}(4n+2), \quad \mathrm{E}_6}$$

Shown: 2-loop RGE in SM:



Grand Unified Theories (GUTs) — Motivation

- Do SM gauge couplings unify at a high scale?

→ Unification of SM forces [1] in G:

(1) G is a **simple group** (one factor)

(2) $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset G$

(3) G has **complex irreps** (SM chiral)

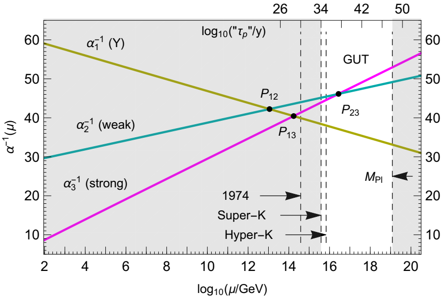
→ Classification of simple groups:

- Infinite families:
  $\mathrm{SU}(n), \quad \mathrm{SO}(n), \quad \mathrm{Sp}(2n)$
- Exceptional groups:
  $\mathrm{G}_2, \quad \mathrm{F}_4, \quad \mathrm{E}_6, \quad \mathrm{E}_7, \quad \mathrm{E}_8$

→ Groups with $\mathbb{C}$ irreps:

$$\boxed{\mathrm{SU}(n), \quad \mathrm{SO}(4n+2), \quad \mathrm{E}_6}$$

Shown: 2-loop RGE in SM:



Minimal choices for unified group:

$$\mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{E}_6$$

GUTs — fermions

- SM fermions: 3 families of

$$Q \sim (\mathbf{3},\mathbf{2},+\tfrac{1}{6}), \qquad u^c \sim (\bar{\mathbf{3}},\mathbf{1},-\tfrac{2}{3}), \qquad d^c \sim (\bar{\mathbf{3}},\mathbf{1},+\tfrac{1}{3}), \tag{1}$$

$$L \sim (\mathbf{1},\mathbf{2},-\tfrac{1}{2}), \qquad e^c \sim (\mathbf{1},\mathbf{1},+1). \tag{2}$$

Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA
INFN Istituto Nazionale di Fisica Nucleare Laboratori Nazionali di Frascati

GUTs — fermions

- SM fermions: 3 families of

$$Q \sim (\mathbf{3}, \mathbf{2}, +\tfrac{1}{6}), \qquad u^c \sim (\bar{\mathbf{3}}, \mathbf{1}, -\tfrac{2}{3}), \qquad d^c \sim (\bar{\mathbf{3}}, \mathbf{1}, +\tfrac{1}{3}), \tag{1}$$

$$L \sim (\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}), \qquad e^c \sim (\mathbf{1}, \mathbf{1}, +1). \tag{2}$$

- SM fermions of 1 family can be embedded into $\mathrm{SU}(5)$ irreps

$$\mathbf{10} = \begin{pmatrix} 0 & {u^c}_3 & -{u^c}_2 & u^1 & d^1 \\ -{u^c}_3 & 0 & {u^c}_1 & u^2 & d^2 \\ {u^c}_2 & -{u^c}_1 & 0 & u^3 & d^3 \\ -u^1 & -u^2 & -u^3 & 0 & e^c \\ -d^1 & -d^2 & -d^3 & -e^c & 0 \end{pmatrix}, \qquad \bar{\mathbf{5}} = \begin{pmatrix} {d^c}_1 \\ {d^c}_2 \\ {d^c}_3 \\ e \\ -\nu \end{pmatrix}. \tag{3}$$

GUTs — fermions

- SM fermions: 3 families of

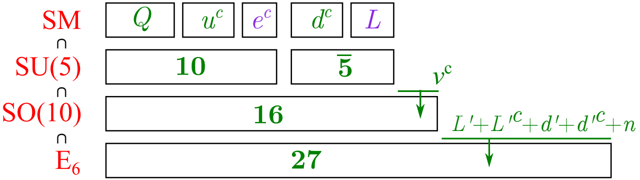
$$Q \sim (\mathbf{3},\mathbf{2},+\tfrac{1}{6}), \qquad u^c \sim (\bar{\mathbf{3}},\mathbf{1},-\tfrac{2}{3}), \qquad d^c \sim (\bar{\mathbf{3}},\mathbf{1},+\tfrac{1}{3}), \tag{1}$$

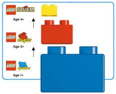
$$L \sim (\mathbf{1},\mathbf{2},-\tfrac{1}{2}), \qquad e^c \sim (\mathbf{1},\mathbf{1},+1). \tag{2}$$

- SM fermions of 1 family can be embedded into $\mathrm{SU}(5)$ irreps

$$\mathbf{10} = \begin{pmatrix} 0 & {u^c}_3 & -{u^c}_2 & u^1 & d^1 \\ -{u^c}_3 & 0 & {u^c}_1 & u^2 & d^2 \\ {u^c}_2 & -{u^c}_1 & 0 & u^3 & d^3 \\ -u^1 & -u^2 & -u^3 & 0 & e^c \\ -d^1 & -d^2 & -d^3 & -e^c & 0 \end{pmatrix}, \qquad \bar{\mathbf{5}} = \begin{pmatrix} {d^c}_1 \\ {d^c}_2 \\ {d^c}_3 \\ e \\ -\nu \end{pmatrix}. \tag{3}$$

- General embedding into GUT: SM chiral content, anomaly free





GUTs — proton/nucleon decay 1

- Universal GUT prediction: proton/nucleon decay . . .

GUTs — proton/nucleon decay 1

- Universal GUT prediction: proton/nucleon decay . . .
- Nucleon decay in SMEFT appears at $d = 6$: $qqql$

$$\mathcal{O}_{RL} := (du)(QL), \quad (4)$$
$$\mathcal{O}_{LR} := (QQ)(ue), \quad (5)$$
$$\mathcal{O}_{LL} := (QQ)(QL), \quad (6)$$
$$\mathcal{O}_{RR} := (du)(ue). \quad (7)$$

$\rightarrow$ 4 operators

$\rightarrow$ $\not{B}$, $\not{L}$; preserve $B - L$

$\rightarrow$ flavor dependence: $\mathcal{O}^{IJKL}$

GUTs — proton/nucleon decay 1

- Universal GUT prediction: proton/nucleon decay ...
- Nucleon decay in SMEFT appears at $d = 6$: qql

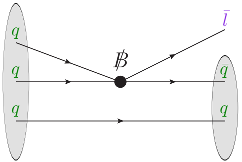
$$\mathcal{O}_{RL} := (du)(QL), \quad (4)$$
$$\mathcal{O}_{LR} := (QQ)(ue), \quad (5)$$
$$\mathcal{O}_{LL} := (QQ)(QL), \quad (6)$$
$$\mathcal{O}_{RR} := (du)(ue). \quad (7)$$

→ 4 operators

→ $\not{B}$, $\not{L}$; preserve $B - L$

→ flavor dependence: $\mathcal{O}^{IJKL}$

- Experiment: baryon → meson + anti-lepton



GUTs — proton/nucleon decay 1

- Universal GUT prediction: proton/nucleon decay ...
- Nucleon decay in SMEFT appears at $d = 6$: $qqql$

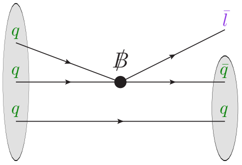
$$\mathcal{O}_{RL} := (du)(QL), \quad (4)$$
$$\mathcal{O}_{LR} := (QQ)(ue), \quad (5)$$
$$\mathcal{O}_{LL} := (QQ)(QL), \quad (6)$$
$$\mathcal{O}_{RR} := (du)(ue). \quad (7)$$

$\rightarrow$ 4 operators

$\rightarrow$ $\not{B}$, $\not{L}$; preserve $B - L$

$\rightarrow$ flavor dependence: $\mathcal{O}^{IJKL}$

- Experiment: baryon $\rightarrow$ meson + anti-lepton



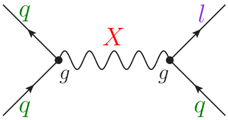
Lower bounds from Super-K (examples)

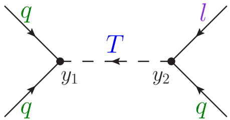
channel	$\tau/\mathcal{B}$ [y]		dominant in
$p \to \pi^0 e^+$	$2.4 \cdot 10^{34}$	[2]	non-SUSY GUT
$p \to \pi^0 \mu^+$	$1.6 \cdot 10^{34}$	[2]	
$p \to K^+ \bar{\nu}$	$5.9 \cdot 10^{33}$	[3]	SUSY GUT

GUTs — proton/nucleon decay 2

- $\mathcal{O}$ from tree-level mediation of "leptoquarks": gauge bosons or scalars

$$\Gamma \sim \frac{g^4\, m_p^5}{m_X^4}$$





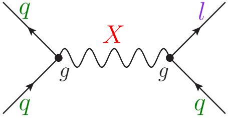
$$\Gamma \sim \frac{|y_1 y_2|^2\, m_p^5}{m_T^4}$$

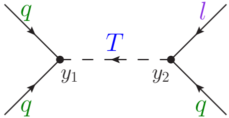
Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA
INFN Istituto Nazionale di Fisica Nucleare Laboratori Nazionali di Frascati

GUTs — proton/nucleon decay 2

- $\mathcal{O}$ from tree-level mediation of "leptoquarks": gauge bosons or scalars

$$\Gamma \sim \frac{g^4\, m_p^5}{m_X^4}$$





$$\Gamma \sim \frac{|y_1 y_2|^2\, m_p^5}{m_T^4}$$

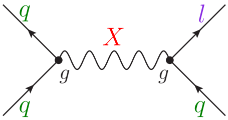
Irreps: $(\mathbf{3},\mathbf{2},-\tfrac{5}{6})$, $(\mathbf{3},\mathbf{2},+\tfrac{1}{6})$; $(\mathbf{3},\mathbf{1},-\tfrac{1}{3})$, $(\mathbf{3},\mathbf{1},-\tfrac{4}{3})$, $(\mathbf{3},\mathbf{3},-\tfrac{1}{3})$

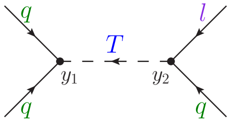
Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA
INFN Istituto Nazionale di Fisica Nucleare Laboratori Nazionali di Frascati

GUTs — proton/nucleon decay 2

- $\mathcal{O}$ from tree-level mediation of "leptoquarks": gauge bosons or scalars

$$\Gamma \sim \frac{g^4\, m_p^5}{m_X^4}$$





$$\Gamma \sim \frac{|y_1 y_2|^2\, m_p^5}{m_T^4}$$

Irreps: $(\mathbf{3},\mathbf{2},-\tfrac{5}{6})$, $(\mathbf{3},\mathbf{2},+\tfrac{1}{6})$; $(\mathbf{3},\mathbf{1},-\tfrac{1}{3})$, $(\mathbf{3},\mathbf{1},-\tfrac{4}{3})$, $(\mathbf{3},\mathbf{3},-\tfrac{1}{3})$

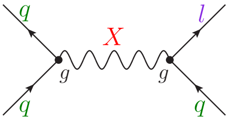
- Decomposition of $\mathrm{SU}(5)$ gauge bosons:

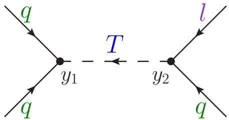
$$\mathbf{24} = (\mathbf{8},\mathbf{1},0) \oplus (\mathbf{1},\mathbf{3},0) \oplus (\mathbf{1},\mathbf{1},0) \oplus \overbrace{(\mathbf{3},\mathbf{2},-\tfrac{5}{6})_{\mathbb{C}}}^{X} \tag{8}$$

GUTs — proton/nucleon decay 2

- $\mathcal{O}$ from tree-level mediation of "leptoquarks": gauge bosons or scalars

$$\Gamma \sim \frac{g^4\, m_p^5}{m_X^4}$$





$$\Gamma \sim \frac{|y_1 y_2|^2\, m_p^5}{m_T^4}$$

Irreps: $(\mathbf{3},\mathbf{2},-\tfrac{5}{6})$, $(\mathbf{3},\mathbf{2},+\tfrac{1}{6})$; $(\mathbf{3},\mathbf{1},-\tfrac{1}{3})$, $(\mathbf{3},\mathbf{1},-\tfrac{4}{3})$, $(\mathbf{3},\mathbf{3},-\tfrac{1}{3})$

- Decomposition of $\mathrm{SU}(5)$ gauge bosons:

$$\mathbf{24} = (\mathbf{8},\mathbf{1},0) \oplus (\mathbf{1},\mathbf{3},0) \oplus (\mathbf{1},\mathbf{1},0) \oplus \overbrace{(\mathbf{3},\mathbf{2},-\tfrac{5}{6})_{\mathbb{C}}}^{X} \tag{8}$$

- Unified group $G \supset \mathrm{SU}(5) \Rightarrow$ proton decays in any GUT (via X)

  → In non-SUSY: **gauge mediation** usually dominates

$$g \approx 0.5 \qquad \Rightarrow \qquad m_X \gtrsim 10^{15.4}\,\mathrm{GeV} \tag{9}$$

GUTs — what is needed for a realistic model?

- Choose G (group theory tables: [4, 5])

  → Then choose G-irreps for fermions and scalars

GUTs — what is needed for a realistic model?

- Choose G (group theory tables: [4, 5])
  - $\rightarrow$ Then choose G-irreps for fermions and scalars
- Considerations in GUTs:

GUTs — what is needed for a realistic model?

- Choose G (group theory tables: [4, 5])

  $\rightarrow$ Then choose G-irreps for fermions and scalars

- Considerations in GUTs:

  (a) **Spontaneous symmetry breaking**: $G \rightarrow \ldots \rightarrow \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$

  $\rightarrow$ Perhaps through intermediate stages (possible only in $\mathrm{SO}(10)$ and E_6)

  $\rightarrow$ SUSY breaking in SUSY GUTs (independent; exception: radiative breaking [6])

GUTs — what is needed for a realistic model?

- Choose G (group theory tables: [4, 5])
  $\rightarrow$ Then choose G-irreps for fermions and scalars
- Considerations in GUTs:
  (a) **Spontaneous symmetry breaking**: $G \rightarrow \ldots \rightarrow \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
    $\rightarrow$ Perhaps through intermediate stages (possible only in $\mathrm{SO}(10)$ and E_6)
    $\rightarrow$ SUSY breaking in SUSY GUTs (independent; exception: radiative breaking [6])
  (b) Realistic **Yukawa sector**:
    $\rightarrow$ embed SM fermions for correct chiral content; embed SM Higgs H into scalars
    $\rightarrow$ fit to SM parameters has to work (masses, CKM, PMNS)

GUTs — what is needed for a realistic model?

- Choose G (group theory tables: [4, 5])
  $\to$ Then choose G-irreps for fermions and scalars
- Considerations in GUTs:
  (a) **Spontaneous symmetry breaking**: $G \to \ldots \to \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
    $\to$ Perhaps through intermediate stages (possible only in $\mathrm{SO}(10)$ and E_6)
    $\to$ SUSY breaking in SUSY GUTs (independent; exception: radiative breaking [6])
  (b) Realistic **Yukawa sector**:
    $\to$ embed SM fermions for correct chiral content; embed SM Higgs H into scalars
    $\to$ fit to SM parameters has to work (masses, CKM, PMNS)
  (c) **Unification** of gauge couplings (constraint on spectrum)
  (d) Consistent with **proton decay** bounds (M_{GUT} is high)
  (e) **Other** model-specific considerations/bounds

GUTs — what is needed for a realistic model?

- Choose G (group theory tables: [4, 5])
  $\rightarrow$ Then choose G-irreps for fermions and scalars
- Considerations in GUTs:
  (a) **Spontaneous symmetry breaking**: $G \rightarrow \ldots \rightarrow \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
  - $\rightarrow$ Perhaps through intermediate stages (possible only in $\mathrm{SO}(10)$ and E_6)
  - $\rightarrow$ SUSY breaking in SUSY GUTs (independent; exception: radiative breaking [6])

  (b) Realistic **Yukawa sector**:
  - $\rightarrow$ embed SM fermions for correct chiral content; embed SM Higgs H into scalars
  - $\rightarrow$ fit to SM parameters has to work (masses, CKM, PMNS)

  (c) **Unification** of gauge couplings (constraint on spectrum)
  (d) Consistent with **proton decay** bounds (M_{GUT} is high)
  (e) **Other** model-specific considerations/bounds
- This presentation: focus just on **Yukawa sector** in GUTs ...

Yukawa sector in $\mathrm{SU}(5)$ — part 1

- Fermions of one SM family: $\mathbf{10}_F \oplus \bar{\mathbf{5}}_F$

Yukawa sector in $\mathrm{SU}(5)$ — part 1

- Fermions of one SM family: $\mathbf{10}_F \oplus \bar{\mathbf{5}}_F$
- SM Higgs $H \sim (\mathbf{1}, \mathbf{2}, +\frac{1}{2})$: embedded in $\mathbf{5}$, $\mathbf{45}$ or $\mathbf{70}$

  List of $\mathrm{SU}(5)$-irreps: $\mathbf{1}$, $\mathbf{5}$, $\mathbf{10}$, $\mathbf{15}$, $\mathbf{24}$, $\mathbf{35}$, $\mathbf{40}$, $\mathbf{45}$, $\mathbf{50}$, $\mathbf{70}$, $\mathbf{70}'$, $\mathbf{75}$, ...

Yukawa sector in $\mathrm{SU}(5)$ — part 1

- Fermions of one SM family: $\mathbf{10}_F \oplus \bar{\mathbf{5}}_F$
- SM Higgs $H \sim (\mathbf{1}, \mathbf{2}, +\frac{1}{2})$: embedded in $\mathbf{5}$, $\mathbf{45}$ or $\mathbf{70}$

  List of $\mathrm{SU}(5)$-irreps: $\mathbf{1}$, $\mathbf{5}$, $\mathbf{10}$, $\mathbf{15}$, $\mathbf{24}$, $\mathbf{35}$, $\mathbf{40}$, $\mathbf{45}$, $\mathbf{50}$, $\mathbf{70}$, $\mathbf{70}'$, $\mathbf{75}$, ...
- Minimal model for Yukawa sector: $\mathbf{5}$

$$\mathcal{L}_Y = Y_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{5} + Y_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{5}^*. \tag{10}$$

Yukawa sector in $\mathrm{SU}(5)$ — part 1

- Fermions of one SM family: $\mathbf{10}_F \oplus \bar{\mathbf{5}}_F$
- SM Higgs $H \sim (\mathbf{1}, \mathbf{2}, +\frac{1}{2})$: embedded in $\mathbf{5}$, $\mathbf{45}$ or $\mathbf{70}$

  List of $\mathrm{SU}(5)$-irreps: $\mathbf{1}$, $\mathbf{5}$, $\mathbf{10}$, $\mathbf{15}$, $\mathbf{24}$, $\mathbf{35}$, $\mathbf{40}$, $\mathbf{45}$, $\mathbf{50}$, $\mathbf{70}$, $\mathbf{70'}$, $\mathbf{75}$, ...
- Minimal model for Yukawa sector: $\mathbf{5}$

$$\mathcal{L}_Y = Y_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{5} + Y_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{5}^*. \tag{10}$$

$\rightarrow$ matrices: $Y_{10}, Y_5 \in \mathbb{C}_{3\times 3}$, (Y_{10}) is symmetric, $\langle \mathbf{5} \rangle =: v$

Yukawa sector in $\mathrm{SU}(5)$ — part 1

- Fermions of one SM family: $\mathbf{10}_F \oplus \bar{\mathbf{5}}_F$
- SM Higgs $H \sim (\mathbf{1}, \mathbf{2}, +\frac{1}{2})$: embedded in $\mathbf{5}$, $\mathbf{45}$ or $\mathbf{70}$

  List of $\mathrm{SU}(5)$-irreps: $\mathbf{1}$, $\mathbf{5}$, $\mathbf{10}$, $\mathbf{15}$, $\mathbf{24}$, $\mathbf{35}$, $\mathbf{40}$, $\mathbf{45}$, $\mathbf{50}$, $\mathbf{70}$, $\mathbf{70}'$, $\mathbf{75}$, ...
- Minimal model for Yukawa sector: $\mathbf{5}$

$$\mathcal{L}_Y = Y_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{5} + Y_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{5}^*. \tag{10}$$

→ matrices: $Y_{10}, Y_5 \in \mathbb{C}_{3\times 3}$, (Y_{10}) is symmetric, $\langle \mathbf{5} \rangle =: v$

→ Prediction for mass matrices (sectors u, d, e):

$$M_u = Y_{10}\, v, \qquad M_d = M_e^{\mathsf{T}} = Y_5\, v^* \tag{11}$$

Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA
INFN Istituto Nazionale di Fisica Nucleare Laboratori Nazionali di Frascati

Yukawa sector in $\mathrm{SU}(5)$ — part 1

- Fermions of one SM family: $\mathbf{10}_F \oplus \bar{\mathbf{5}}_F$
- SM Higgs $H \sim (\mathbf{1}, \mathbf{2}, +\frac{1}{2})$: embedded in $\mathbf{5}$, $\mathbf{45}$ or $\mathbf{70}$

  List of $\mathrm{SU}(5)$-irreps: $\mathbf{1}$, $\mathbf{5}$, $\mathbf{10}$, $\mathbf{15}$, $\mathbf{24}$, $\mathbf{35}$, $\mathbf{40}$, $\mathbf{45}$, $\mathbf{50}$, $\mathbf{70}$, $\mathbf{70}'$, $\mathbf{75}$, ...
- Minimal model for Yukawa sector: $\mathbf{5}$

$$\mathcal{L}_Y = Y_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{5} + Y_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{5}^*. \tag{10}$$

→ matrices: $Y_{10}, Y_5 \in \mathbb{C}_{3\times 3}$, (Y_{10}) is symmetric, $\langle \mathbf{5} \rangle =: v$

→ Prediction for mass matrices (sectors u, d, e):

$$M_u = Y_{10}\, v, \qquad\qquad M_d = M_e^{\mathsf{T}} = Y_5\, v^* \tag{11}$$

→ Redundancies: (i) family rotation of $(\mathbf{10}_F)_I$ can make Y_{10} $\mathbb{R}$-diagonal
(ii) rotation in $(\bar{\mathbf{5}}_F)_I$ can make Y_5 triangular

Yukawa sector in $\mathrm{SU}(5)$ — part 1

- Fermions of one SM family: $\mathbf{10}_F \oplus \bar{\mathbf{5}}_F$
- SM Higgs $H \sim (\mathbf{1}, \mathbf{2}, +\frac{1}{2})$: embedded in $\mathbf{5}$, $\mathbf{45}$ or $\mathbf{70}$

  List of $\mathrm{SU}(5)$-irreps: $\mathbf{1}$, $\mathbf{5}$, $\mathbf{10}$, $\mathbf{15}$, $\mathbf{24}$, $\mathbf{35}$, $\mathbf{40}$, $\mathbf{45}$, $\mathbf{50}$, $\mathbf{70}$, $\mathbf{70}'$, $\mathbf{75}$, ...
- Minimal model for Yukawa sector: $\mathbf{5}$

$$\mathcal{L}_Y = Y_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{5} + Y_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{5}^*. \tag{10}$$

→ matrices: $Y_{10}, Y_5 \in \mathbb{C}_{3\times 3}$, (Y_{10}) is symmetric, $\langle \mathbf{5} \rangle =: v$

→ Prediction for mass matrices (sectors u, d, e):

$$M_u = Y_{10}\, v, \qquad M_d = M_e^{\mathsf{T}} = Y_5\, v^* \tag{11}$$

→ Redundancies: (i) family rotation of $(\mathbf{10}_F)_I$ can make Y_{10} $\mathbb{R}$-diagonal
(ii) rotation in $(\bar{\mathbf{5}}_F)_I$ can make Y_5 triangular

→ **Problem**: $M_d = M_e^{\mathsf{T}}$ at M_{GUT} not a good fit for today's precision!

Yukawa sector in $\mathrm{SU}(5)$ — part 2

- Extend scalars to $\mathbf{5}_1 \oplus \mathbf{5}_2$?
  → 2nd copy doesn't help ...

$$M_d = M_e^{\mathsf{T}} = Y_{5_1} {v_1}^* + Y_{5_2} {v_2}^* \quad (12)$$

Yukawa sector in $\mathrm{SU}(5)$ — part 2

- Extend scalars to $\mathbf{5}_1 \oplus \mathbf{5}_2$?
  $\rightarrow$ 2nd copy doesn't help ...

$$M_d = M_e^{\mathsf{T}} = Y_{5_1} {v_1}^* + Y_{5_2} {v_2}^* \quad (12)$$

- Realistic for ude sectors: $\mathbf{5} \oplus \mathbf{45}$

$$\begin{aligned} \mathcal{L}_Y &= Y_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{5} + Y_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{5}^* \\ &+ Y'_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{45} + Y'_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{45}^* \end{aligned} \quad (13)$$

Yukawa sector in $\mathrm{SU}(5)$ — part 2

- Extend scalars to $\mathbf{5}_1 \oplus \mathbf{5}_2$?

  $\rightarrow$ 2nd copy doesn't help ...

$$M_d = M_e^{\mathsf{T}} = Y_{5_1} {v_1}^* + Y_{5_2} {v_2}^* \quad (12)$$

- Realistic for ude sectors: $\mathbf{5} \oplus \mathbf{45}$

$$\begin{aligned}\mathcal{L}_Y = &\; Y_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{5} + Y_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{5}^* \\ &+ Y'_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{45} + Y'_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{45}^*\end{aligned} \quad (13)$$

$\rightarrow$ Predictions at M_{GUT}:

$$M_u = Y_{10}\, v_5 + Y_{10}\, v_{45} \quad (14)$$

$$M_d = Y_5\, {v_5}^* + Y'_5\, {v_{45}}^* \quad (15)$$

$$M_e^{\mathsf{T}} = Y_5\, {v_5}^* - 3Y'_5\, {v_{45}}^* \quad (16)$$

Yukawa sector in $\mathrm{SU}(5)$ — part 2

- Extend scalars to $\mathbf{5}_1 \oplus \mathbf{5}_2$?

  → 2nd copy doesn't help . . .

$$M_d = M_e^{\mathsf{T}} = Y_{5_1} v_1{}^* + Y_{5_2} v_2{}^* \quad (12)$$

- Realistic for ude sectors: $\mathbf{5} \oplus \mathbf{45}$

$$\mathcal{L}_Y = Y_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{5} + Y_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{5}^* + Y'_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{45} + Y'_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{45}^* \quad (13)$$

→ Predictions at M_{GUT}:

$$M_u = Y_{10}\, v_5 + Y_{10}\, v_{45} \quad (14)$$

$$M_d = Y_5\, v_5{}^* + \ Y'_5\, v_{45}{}^* \quad (15)$$

$$M_e^{\mathsf{T}} = Y_5\, v_5{}^* - 3Y'_5\, v_{45}{}^* \quad (16)$$

- General observations for GUT:
  - → Need **multiple Higgs irreps** with different Clebsches to break $M_d \propto M_e^{\mathsf{T}}$
  - → Higgs must be **admixed** in all doublets where EW VEVs needed

Yukawa sector in $\mathrm{SU}(5)$ — part 2

- Extend scalars to $\mathbf{5}_1 \oplus \mathbf{5}_2$?

  $\rightarrow$ 2nd copy doesn't help ...

$$M_d = M_e^{\mathsf{T}} = Y_{5_1} v_1{}^* + Y_{5_2} v_2{}^* \quad (12)$$

- Realistic for ude sectors: $\mathbf{5} \oplus \mathbf{45}$

$$\mathcal{L}_Y = Y_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{5} + Y_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{5}^* + Y'_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{45} + Y'_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{45}^* \quad (13)$$

$\rightarrow$ Predictions at M_{GUT}:

$$M_u = Y_{10}\, v_5 + Y_{10}\, v_{45} \quad (14)$$

$$M_d = Y_5\, v_5{}^* + Y'_5\, v_{45}{}^* \quad (15)$$

$$M_e^{\mathsf{T}} = Y_5\, v_5{}^* - 3Y'_5\, v_{45}{}^* \quad (16)$$

- General observations for GUT:

  $\rightarrow$ Need **multiple Higgs irreps** with different Clebsches to break $M_d \propto M_e^{\mathsf{T}}$

  $\rightarrow$ Higgs must be **admixed** in all doublets where EW VEVs needed

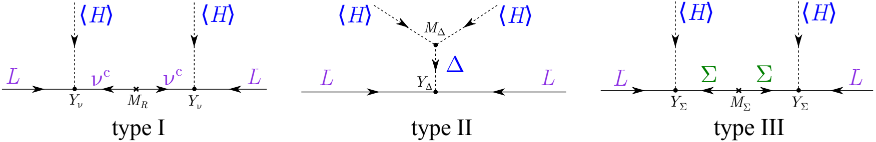
- What about **neutrinos**!? ... Masless in above setup

Neutrinos and see-saw

- ν masses in SMEFT: $\not{L}$ operator $LLHH$ (if ν Majorana)

Neutrinos and see-saw

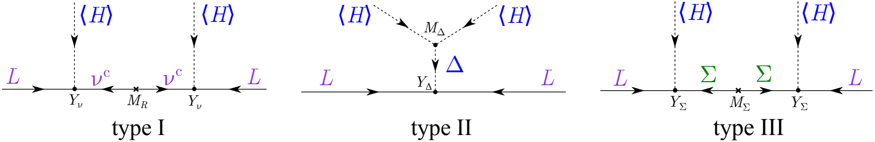
- ν masses in SMEFT: $\not{L}$ operator $LLHH$ (if ν Majorana)
- Realization via tree-level mediation: **see-saw**



$$\nu^c \sim (\mathbf{1},\mathbf{1},0) \qquad \Delta \sim (\mathbf{1},\mathbf{3},+1) \qquad \Sigma \sim (\mathbf{1},\mathbf{3},0) \tag{17}$$

Neutrinos and see-saw

- ν masses in SMEFT: $\not{L}$ operator $LLHH$ (if ν Majorana)
- Realization via tree-level mediation: **see-saw**

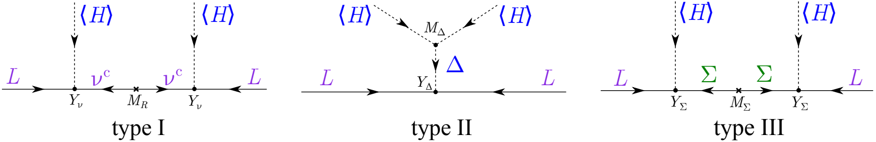


$$\nu^c \sim (\mathbf{1},\mathbf{1},0) \qquad \Delta \sim (\mathbf{1},\mathbf{3},+1) \qquad \Sigma \sim (\mathbf{1},\mathbf{3},0) \tag{17}$$

- $M_\nu \sim v^2/M_{\mathsf{ss}}$. Seesaw scale $M_{\mathsf{ss}} \sim 10^{14}\,\mathrm{GeV}$ to get $M_\nu \sim 0.1\,\mathrm{eV}$.

Neutrinos and see-saw

- ν masses in SMEFT: $\not{L}$ operator $LLHH$ (if ν Majorana)
- Realization via tree-level mediation: **see-saw**



$$\nu^c \sim (\mathbf{1}, \mathbf{1}, 0) \qquad \Delta \sim (\mathbf{1}, \mathbf{3}, +1) \qquad \Sigma \sim (\mathbf{1}, \mathbf{3}, 0) \tag{17}$$

- $M_\nu \sim v^2/M_{\text{ss}}$. Seesaw scale $M_{\text{ss}} \sim 10^{14}\,\text{GeV}$ to get $M_\nu \sim 0.1\,\text{eV}$.
- In GUT context:
  - (a) Typically need a new scale below M_{GUT}
  - (b) ν^c in **16** of $\mathrm{SO}(10)$ (and **27** of E_6) $\rightarrow$ automatically a theory of ν mass

Neutrinos in $\mathrm{SU}(5)$

- Option (1): add e.g. $\nu^c \sim \mathbf{1}$ by hand
  Option (2): add non-singlets, radiative mass generation

Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA
INFN Istituto Nazionale di Fisica Nucleare Laboratori Nazionali di Frascati

Neutrinos in $\mathrm{SU}(5)$

- Option (1): add e.g. $\nu^c \sim \mathbf{1}$ by hand
  Option (2): add non-singlets, radiative mass generation
- Minimal realistic $\mathrm{SU}(5)$ model: proposed in 2019 [7, 8, 9]
  $\rightarrow$ Fermions: $3 \times (\mathbf{10}_F \oplus \bar{\mathbf{5}}_F) \oplus (\mathbf{15}_F \oplus \overline{\mathbf{15}}_F)$
  $\rightarrow$ Scalars: $\mathbf{5} \oplus \mathbf{24} \oplus \mathbf{35}$

$$\mathbf{15} = (\mathbf{3}, \mathbf{2}, +\tfrac{1}{6}) \oplus (\mathbf{1}, \mathbf{3}, +1) \oplus (\mathbf{6}, \mathbf{1}, -\tfrac{2}{3})$$

Neutrinos in $\mathrm{SU}(5)$

- Option (1): add e.g. $\nu^c \sim \mathbf{1}$ by hand
  Option (2): add non-singlets, radiative mass generation
- Minimal realistic $\mathrm{SU}(5)$ model: proposed in 2019 [7, 8, 9]
  → Fermions: $3 \times (\mathbf{10}_F \oplus \bar{\mathbf{5}}_F) \oplus (\mathbf{15}_F \oplus \overline{\mathbf{15}}_F)$
  → Scalars: $\mathbf{5} \oplus \mathbf{24} \oplus \mathbf{35}$ $\qquad \mathbf{15} = (\mathbf{3}, \mathbf{2}, +\tfrac{1}{6}) \oplus (\mathbf{1}, \mathbf{3}, +1) \oplus (\mathbf{6}, \mathbf{1}, -\tfrac{2}{3})$
  →

$$\begin{aligned} \mathcal{L}_Y = {} & Y_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{5} + Y_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{5}^* + \mathbf{15}_F\, \overline{\mathbf{15}}_F\, (M_{15} + y\, \mathbf{24}) \\ & + y_a\, \bar{\mathbf{5}}_F\, \mathbf{15}_F\, \mathbf{5}^* + y_b\, \bar{\mathbf{5}}_F\, \overline{\mathbf{15}}_F\, \mathbf{35}^* + y_c\, \mathbf{10}_F\, \overline{\mathbf{15}}_F\, \mathbf{24} + h.c. \end{aligned} \tag{18}$$

Neutrinos in SU(5)

- Option (1): add e.g. $\nu^c \sim \mathbf{1}$ by hand
  Option (2): add non-singlets, radiative mass generation
- Minimal realistic $\mathrm{SU}(5)$ model: proposed in 2019 [7, 8, 9]
  → Fermions: $3 \times (\mathbf{10}_F \oplus \bar{\mathbf{5}}_F) \oplus (\mathbf{15}_F \oplus \overline{\mathbf{15}}_F)$
  → Scalars: $\mathbf{5} \oplus \mathbf{24} \oplus \mathbf{35}$ $\qquad \mathbf{15} = (\mathbf{3},\mathbf{2},+\tfrac{1}{6}) \oplus (\mathbf{1},\mathbf{3},+1) \oplus (\mathbf{6},\mathbf{1},-\tfrac{2}{3})$
  →

$$\mathcal{L}_Y = Y_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{5} + Y_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{5}^* + \mathbf{15}_F\, \overline{\mathbf{15}}_F\, (M_{15} + y\, \mathbf{24}) + y_a\, \bar{\mathbf{5}}_F\, \mathbf{15}_F\, \mathbf{5}^* + y_b\, \bar{\mathbf{5}}_F\, \overline{\mathbf{15}}_F\, \mathbf{35}^* + y_c\, \mathbf{10}_F\, \overline{\mathbf{15}}_F\, \mathbf{24} + h.c. \quad (18)$$

→ Charged sectors: $\langle \mathbf{5} \rangle \equiv v \ll V \equiv \langle \mathbf{24} \rangle$

$$M_u = \begin{pmatrix} Y_{10}\, v & -\sqrt{5/48}\, y_c\, V \\ 0 & M_{15} + \sqrt{3/20}\, y\, V \end{pmatrix}\Bigg|_{[3\times 3]} \quad (19)$$

$$M_d = \begin{pmatrix} Y_5\, v^* & -\sqrt{5/48}\, y_c\, V \\ y_a^{\mathsf{T}}\, v^* & M_{15} + \sqrt{3/20}\, y\, V \end{pmatrix}\Bigg|_{[3\times 3]} \quad (20)$$

$$M_e = \begin{pmatrix} Y_5^{\mathsf{T}}\, v^* & y_a\, v^* \\ 0 & M_{15} + y\, V/4/\sqrt{15} \end{pmatrix}\Bigg|_{[3\times 3]} \quad (21)$$

Neutrinos in $\mathrm{SU}(5)$

- Option (1): add e.g. $\nu^c \sim \mathbf{1}$ by hand
  Option (2): add non-singlets, radiative mass generation
- Minimal realistic $\mathrm{SU}(5)$ model: proposed in 2019 [7, 8, 9]
  - → Fermions: $3 \times (\mathbf{10}_F \oplus \bar{\mathbf{5}}_F) \oplus (\mathbf{15}_F \oplus \overline{\mathbf{15}}_F)$
  - → Scalars: $\mathbf{5} \oplus \mathbf{24} \oplus \mathbf{35}$ $\qquad \mathbf{15} = (\mathbf{3},\mathbf{2},+\tfrac{1}{6}) \oplus (\mathbf{1},\mathbf{3},+1) \oplus (\mathbf{6},\mathbf{1},-\tfrac{2}{3})$
  - →

$$\begin{aligned}\mathcal{L}_Y &= Y_{10}\, \mathbf{10}_F\, \mathbf{10}_F\, \mathbf{5} + Y_5\, \mathbf{10}_F\, \bar{\mathbf{5}}_F\, \mathbf{5}^* + \mathbf{15}_F\, \overline{\mathbf{15}}_F\, (M_{15} + y\, \mathbf{24}) \\ &\quad + y_a\, \bar{\mathbf{5}}_F\, \mathbf{15}_F\, \mathbf{5}^* + y_b\, \bar{\mathbf{5}}_F\, \overline{\mathbf{15}}_F\, \mathbf{35}^* + y_c\, \mathbf{10}_F\, \overline{\mathbf{15}}_F\, \mathbf{24} + h.c. \end{aligned} \tag{18}$$

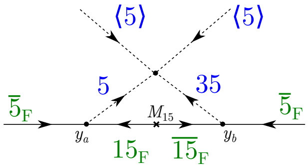
→ Charged sectors: $\langle \mathbf{5} \rangle \equiv v \ll V \equiv \langle \mathbf{24} \rangle$

$$M_u = \begin{pmatrix} Y_{10}\, v & -\sqrt{5/48}\, y_c\, V \\ 0 & M_{15} + \sqrt{3/20}\, y\, V \end{pmatrix}\Bigg|_{[3\times 3]} \tag{19}$$

$$M_d = \begin{pmatrix} Y_5\, v^* & -\sqrt{5/48}\, y_c\, V \\ y_a^{\mathsf{T}}\, v^* & M_{15} + \sqrt{3/20}\, y\, V \end{pmatrix}\Bigg|_{[3\times 3]} \tag{20}$$

$$M_e = \begin{pmatrix} Y_5^{\mathsf{T}}\, v^* & y_a\, v^* \\ 0 & M_{15} + y\, V/4/\sqrt{15} \end{pmatrix}\Bigg|_{[3\times 3]} \tag{21}$$

→ Neutrino masses:



Yukawa sector in $\mathrm{SO}(10)$ GUT — setup

- Fermions: $3 \times \mathbf{16}_F$ (matter unification)

Yukawa sector in $\mathrm{SO}(10)$ GUT — setup

- Fermions: $3 \times \mathbf{16}_F$ (matter unification)
- Scalar possibilities: all irreps in the product have Higgs doublets

$$\mathbf{16} \otimes \mathbf{16} = \mathbf{10} \oplus \mathbf{120} \oplus \mathbf{126} \tag{22}$$

Yukawa sector in $\mathrm{SO}(10)$ GUT — setup

- Fermions: $3 \times \mathbf{16}_F$ (matter unification)
- Scalar possibilities: all irreps in the product have Higgs doublets

$$\mathbf{16} \otimes \mathbf{16} = \mathbf{10} \oplus \mathbf{120} \oplus \mathbf{126} \tag{22}$$

- Most **general** renormalizable Yukawa sector: $\mathbf{10}_{\mathbb{C}} \oplus \mathbf{120}_{\mathbb{C}} \oplus \overline{\mathbf{126}}$

$$\begin{aligned} \mathcal{L}_Y = & \, Y_{10}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{10}_{\mathbb{C}} + Y_{120}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{120}_{\mathbb{C}} + Y_{126}\, \mathbf{16}_F \mathbf{16}_F\, \overline{\mathbf{126}} \\ & + Y'_{10}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{10}^*_{\mathbb{C}} + Y'_{120}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{120}^*_{\mathbb{C}} \end{aligned} \tag{23}$$

Yukawa sector in $\mathrm{SO}(10)$ GUT — setup

- Fermions: $3 \times \mathbf{16}_F$ (matter unification)
- Scalar possibilities: all irreps in the product have Higgs doublets

$$\mathbf{16} \otimes \mathbf{16} = \mathbf{10} \oplus \mathbf{120} \oplus \mathbf{126} \tag{22}$$

- Most **general** renormalizable Yukawa sector: $\mathbf{10}_{\mathbb{C}} \oplus \mathbf{120}_{\mathbb{C}} \oplus \overline{\mathbf{126}}$

$$\begin{aligned} \mathcal{L}_Y = &\, Y_{10}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{10}_{\mathbb{C}} + Y_{120}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{120}_{\mathbb{C}} + Y_{126}\, \mathbf{16}_F \mathbf{16}_F\, \overline{\mathbf{126}} \\ &+ Y'_{10}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{10}^*_{\mathbb{C}} + Y'_{120}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{120}^*_{\mathbb{C}} \end{aligned} \tag{23}$$

$\rightarrow$ [anti]-(symmetric) matrices: (Y_{10}), $[Y_{120}]$, (Y_{126}), (Y'_{10}), $[Y'_{120}]$

Yukawa sector in $\mathrm{SO}(10)$ GUT — setup

- Fermions: $3 \times \mathbf{16}_F$ (matter unification)
- Scalar possibilities: all irreps in the product have Higgs doublets

$$\mathbf{16} \otimes \mathbf{16} = \mathbf{10} \oplus \mathbf{120} \oplus \mathbf{126} \tag{22}$$

- Most **general** renormalizable Yukawa sector: $\mathbf{10}_{\mathbb{C}} \oplus \mathbf{120}_{\mathbb{C}} \oplus \overline{\mathbf{126}}$

$$\begin{aligned} \mathcal{L}_Y = &\, Y_{10}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{10}_{\mathbb{C}} + Y_{120}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{120}_{\mathbb{C}} + Y_{126}\, \mathbf{16}_F \mathbf{16}_F\, \overline{\mathbf{126}} \\ &+ Y'_{10}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{10}^*_{\mathbb{C}} + Y'_{120}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{120}^*_{\mathbb{C}} \end{aligned} \tag{23}$$

→ [anti]-(symmetric) matrices: (Y_{10}), $[Y_{120}]$, (Y_{126}), (Y'_{10}), $[Y'_{120}]$

→ EW VEVs of $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$: v^u_{10}, v^d_{10}, v^u_{120}, v^d_{120}, v^e_{120}, v^ν_{120}, v^u_{126}, v^d_{126}

Yukawa sector in $\mathrm{SO}(10)$ GUT — setup

- Fermions: $3 \times \mathbf{16}_F$ (matter unification)
- Scalar possibilities: all irreps in the product have Higgs doublets

$$\mathbf{16} \otimes \mathbf{16} = \mathbf{10} \oplus \mathbf{120} \oplus \mathbf{126} \tag{22}$$

- Most **general** renormalizable Yukawa sector: $\mathbf{10}_{\mathbb{C}} \oplus \mathbf{120}_{\mathbb{C}} \oplus \overline{\mathbf{126}}$

$$\begin{aligned}\mathcal{L}_Y = &\, Y_{10}\ \mathbf{16}_F\mathbf{16}_F\,\mathbf{10}_{\mathbb{C}} + Y_{120}\ \mathbf{16}_F\mathbf{16}_F\,\mathbf{120}_{\mathbb{C}} + Y_{126}\ \mathbf{16}_F\mathbf{16}_F\,\overline{\mathbf{126}} \\ &+ Y'_{10}\ \mathbf{16}_F\mathbf{16}_F\,\mathbf{10}^*_{\mathbb{C}} + Y'_{120}\ \mathbf{16}_F\mathbf{16}_F\,\mathbf{120}^*_{\mathbb{C}}\end{aligned} \tag{23}$$

→ [anti]-(symmetric) matrices: (Y_{10}), $[Y_{120}]$, (Y_{126}), (Y'_{10}), $[Y'_{120}]$

→ EW VEVs of $(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$: v^u_{10}, v^d_{10}, v^u_{120}, v^d_{120}, v^e_{120}, v^ν_{120}, v^u_{126}, v^d_{126}

→ If $\mathbf{10}_{\mathbb{R}}$ or $\mathbf{120}_{\mathbb{R}}$: $Y'_{10} = Y'_{120} = 0$, $v^d_{10} = {v^u_{10}}^*$, $v^d_{120} = {v^u_{120}}^*$, $v^e_{120} = {v^\nu_{120}}^*$

Yukawa sector in SO(10) GUT — setup

- Fermions: $3 \times \mathbf{16}_F$ (matter unification)
- Scalar possibilities: all irreps in the product have Higgs doublets

$$\mathbf{16} \otimes \mathbf{16} = \mathbf{10} \oplus \mathbf{120} \oplus \mathbf{126} \tag{22}$$

- Most **general** renormalizable Yukawa sector: $\mathbf{10}_{\mathbb{C}} \oplus \mathbf{120}_{\mathbb{C}} \oplus \overline{\mathbf{126}}$

$$\begin{aligned}\mathcal{L}_Y = {} & Y_{10}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{10}_{\mathbb{C}} + Y_{120}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{120}_{\mathbb{C}} + Y_{126}\, \mathbf{16}_F \mathbf{16}_F\, \overline{\mathbf{126}} \\ & + Y'_{10}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{10}^*_{\mathbb{C}} + Y'_{120}\, \mathbf{16}_F \mathbf{16}_F\, \mathbf{120}^*_{\mathbb{C}}\end{aligned} \tag{23}$$

→ [anti]-(symmetric) matrices: (Y_{10}), $[Y_{120}]$, (Y_{126}), (Y'_{10}), $[Y'_{120}]$

→ EW VEVs of $(\mathbf{1}, \mathbf{2}, \pm\tfrac{1}{2})$: $v^u_{10}, v^d_{10}, \quad v^u_{120}, v^d_{120}, v^e_{120}, v^\nu_{120}, \quad v^u_{126}, v^d_{126}$

→ If $\mathbf{10}_{\mathbb{R}}$ or $\mathbf{120}_{\mathbb{R}}$: $Y'_{10} = Y'_{120} = 0, \quad v^d_{10} = v^u_{10}{}^*, \quad v^d_{120} = v^u_{120}{}^*, v^e_{120} = v^\nu_{120}{}^*$

→ Usually **omit** Y'_{10}, Y'_{120}: forbidden by SUSY or PQ symmetry [10, 11] (to make the fit more predictive)

Yukawa sector in SO(10) GUT — predictions

- Predictions at GUT scale [12]:

$$M_u = Y_{10} v_{10}^u + Y_{120} v_{120}^u + Y_{126} v_{126}^u + Y'_{10} {v_{10}^d}^* + Y'_{120} {v_{120}^d}^* \quad (24)$$

$$M_d = Y_{10} v_{10}^d + Y_{120} v_{120}^d + Y_{126} v_{126}^d + Y'_{10} {v_{10}^u}^* + Y'_{120} {v_{120}^u}^* \quad (25)$$

$$M_e = Y_{10} v_{10}^d + Y_{120} v_{120}^e - 3Y_{126} v_{126}^d + Y'_{10} {v_{10}^u}^* + Y'_{120} {v_{120}^\nu}^* \quad (26)$$

$$M_\nu^D = Y_{10} v_{10}^u + Y_{120} v_{120}^\nu - 3Y_{126} v_{126}^u + Y'_{10} {v_{10}^d}^* + Y'_{120} {v_{120}^e}^* \quad (27)$$

$$M_R = Y_{126}\, V_R \quad (28)$$

$$M_L = Y_{126}\, v_L \quad (29)$$

Yukawa sector in $\mathrm{SO}(10)$ GUT — predictions

- Predictions at GUT scale [12]:

$$M_u = Y_{10} v_{10}^u + Y_{120} v_{120}^u + Y_{126} v_{126}^u + Y'_{10} v_{10}^{d\,*} + Y'_{120} v_{120}^{d\,*} \tag{24}$$

$$M_d = Y_{10} v_{10}^d + Y_{120} v_{120}^d + Y_{126} v_{126}^d + Y'_{10} v_{10}^{u\,*} + Y'_{120} v_{120}^{u\,*} \tag{25}$$

$$M_e = Y_{10} v_{10}^d + Y_{120} v_{120}^e - 3 Y_{126} v_{126}^d + Y'_{10} v_{10}^{u\,*} + Y'_{120} v_{120}^{\nu\,*} \tag{26}$$

$$M_\nu^D = Y_{10} v_{10}^u + Y_{120} v_{120}^\nu - 3 Y_{126} v_{126}^u + Y'_{10} v_{10}^{d\,*} + Y'_{120} v_{120}^{e\,*} \tag{27}$$

$$M_R = Y_{126}\, V_R \tag{28}$$

$$M_L = Y_{126}\, v_L \tag{29}$$

- In $\overline{\mathbf{126}}$: $V_R \equiv \langle(\mathbf{1},\mathbf{1},0)\rangle \sim M_{\mathrm{ss}}$
  $v_L \equiv \langle(\mathbf{1},\mathbf{3},+1)\rangle \sim M_W^2/M_{\mathrm{ss}}$ (induced VEV)

Yukawa sector in SO(10) GUT — predictions

- Predictions at GUT scale [12]:

$$M_u = Y_{10} v^u_{10} + Y_{120} v^u_{120} + Y_{126} v^u_{126} + Y'_{10} {v^d_{10}}^* + Y'_{120} {v^d_{120}}^* \quad (24)$$

$$M_d = Y_{10} v^d_{10} + Y_{120} v^d_{120} + Y_{126} v^d_{126} + Y'_{10} {v^u_{10}}^* + Y'_{120} {v^u_{120}}^* \quad (25)$$

$$M_e = Y_{10} v^d_{10} + Y_{120} v^e_{120} - 3Y_{126} v^d_{126} + Y'_{10} {v^u_{10}}^* + Y'_{120} {v^\nu_{120}}^* \quad (26)$$

$$M^D_\nu = Y_{10} v^u_{10} + Y_{120} v^\nu_{120} - 3Y_{126} v^u_{126} + Y'_{10} {v^d_{10}}^* + Y'_{120} {v^e_{120}}^* \quad (27)$$

$$M_R = Y_{126}\, V_R \quad (28)$$

$$M_L = Y_{126}\, v_L \quad (29)$$

- In $\overline{\mathbf{126}}$: $V_R \equiv \langle(\mathbf{1},\mathbf{1},0)\rangle \sim M_{ss}$
  $v_L \equiv \langle(\mathbf{1},\mathbf{3},+1)\rangle \sim M_W^2/M_{ss}$ (induced VEV)
- Mass matrix of active neutrinos ($\overline{\mathbf{126}}$ needed):

$$M_\nu = -\underbrace{M^D_\nu M_R^{-1} M^{D\mathsf{T}}_\nu}_{\text{type I}} + \underbrace{M_L}_{\text{type II}} \quad (30)$$

Yukawa sector in $\mathrm{SO}(10)$ GUT — fits

- Performing the fit:

  expressions at $M_{\mathsf{GUT}} \xrightarrow{\mathsf{RGE}} \chi^2$ at M_Z (masses, CKM, PMNS, λ, g_i)

Yukawa sector in $\mathrm{SO}(10)$ GUT — fits

- Performing the fit:

  expressions at $M_{\mathsf{GUT}} \xrightarrow{\mathsf{RGE}} \chi^2$ at M_Z (masses, CKM, PMNS, λ, g_i)

- Which fits work? [after θ_{13}^{ν} measurement in 2012]

Yukawa sector in $\mathrm{SO}(10)$ GUT — fits

- Performing the fit:

  expressions at $M_{\mathsf{GUT}} \xrightarrow{\mathsf{RGE}} \chi^2$ at M_Z (masses, CKM, PMNS, λ, g_i)

- Which fits work? [after θ_{13}^{ν} measurement in 2012]

  (1) $\mathbf{10}_{\mathbb{C}} \oplus \overline{\mathbf{126}}$ with PQ/SUSY: predictive [13, 14, 15, 16, 17]

Yukawa sector in $\mathrm{SO}(10)$ GUT — fits

- Performing the fit:

  expressions at $M_{\mathsf{GUT}} \xrightarrow{\mathsf{RGE}} \chi^2$ at M_Z (masses, CKM, PMNS, λ, g_i)

- Which fits work? [after θ^{ν}_{13} measurement in 2012]

  (1) $\mathbf{10}_{\mathbb{C}} \oplus \overline{\mathbf{126}}$ with PQ/SUSY: predictive [13, 14, 15, 16, 17]

  (2) $\mathbf{10}_{\mathbb{C}} \oplus \mathbf{120}_{\mathbb{C}} \oplus \overline{\mathbf{126}}$ with PQ/SUSY: exact fit [18, 14]

Yukawa sector in SO(10) GUT — fits

- Performing the fit:

  expressions at $M_{\mathsf{GUT}} \xrightarrow{\mathsf{RGE}} \chi^2$ at M_Z (masses, CKM, PMNS, λ, g_i)

- Which fits work? [after θ_{13}^{ν} measurement in 2012]

  (1) $\mathbf{10}_{\mathbb{C}} \oplus \overline{\mathbf{126}}$ with PQ/SUSY: predictive [13, 14, 15, 16, 17]

  (2) $\mathbf{10}_{\mathbb{C}} \oplus \mathbf{120}_{\mathbb{C}} \oplus \overline{\mathbf{126}}$ with PQ/SUSY: exact fit [18, 14]

  (3) $\mathbf{10}_{\mathbb{R}} \oplus \mathbf{120}_{\mathbb{R}} \oplus \overline{\mathbf{126}}$: minimal in number of parameters [19]

Yukawa sector in $\mathrm{SO}(10)$ GUT — fits

- Performing the fit:

  expressions at $M_{\mathrm{GUT}} \xrightarrow{\text{RGE}} \chi^2$ at M_Z (masses, CKM, PMNS, λ, g_i)

- Which fits work? [after θ^{ν}_{13} measurement in 2012]
  1. $\mathbf{10}_{\mathbb{C}} \oplus \overline{\mathbf{126}}$ with PQ/SUSY: predictive [13, 14, 15, 16, 17]
  2. $\mathbf{10}_{\mathbb{C}} \oplus \mathbf{120}_{\mathbb{C}} \oplus \overline{\mathbf{126}}$ with PQ/SUSY: exact fit [18, 14]
  3. $\mathbf{10}_{\mathbb{R}} \oplus \mathbf{120}_{\mathbb{R}} \oplus \overline{\mathbf{126}}$: minimal in number of parameters [19]

- General features and considerations:
  (a) (Y) and $[Y]$ with PQ: no good [18, 14] ($\mathbf{10}_{\mathbb{C}} \oplus \mathbf{120}_{\mathbb{C}}$ or $\mathbf{120}_{\mathbb{C}} \oplus \overline{\mathbf{126}}$)
  (b) **Normal hierarchy** of ν highly preferred over inverted hierarchy
  (c) See-saw: **type I** usually dominates over type II
  (d) **Predictions** for ν-observables: m_{ν_i}, CP phases (Dirac and Majorana)
  (e) **Precision**: consideration of full model, EFT tower needed [20, 21, 22]

Yukawa sector in GUTs — various alternatives

- E_6 GUT: fermions in $3 \times \mathbf{27}_F$ [23, 24, 25]
  - → Analogy with $\mathrm{SO}(10)$: $\mathbf{27}_F \supset \mathbf{16}_F$, $\mathbf{27} \supset \mathbf{10}_{\mathbb{C}}$, $\mathbf{351} \supset \mathbf{120}_{\mathbb{C}}$, $\mathbf{351}' \supset \mathbf{126}$
  - → M_d, M_e with vector-like states, M_ν has $3 \times (\nu_L \oplus 2 \times \nu_R \oplus (\nu \oplus \bar{\nu}))$
  - → $\mathrm{SO}(10)$-like fit possible (no mix with vector-like), general one not yet done

Yukawa sector in GUTs — various alternatives

- E_6 GUT: fermions in $3 \times \mathbf{27}_F$ [23, 24, 25]
  - → Analogy with $\mathrm{SO}(10)$: $\mathbf{27}_F \supset \mathbf{16}_F$, $\mathbf{27} \supset \mathbf{10}_{\mathbb{C}}$, $\mathbf{351} \supset \mathbf{120}_{\mathbb{C}}$, $\mathbf{351}' \supset \mathbf{126}$
  - → M_d, M_e with vector-like states, M_ν has $3 \times (\nu_L \oplus 2 \times \nu_R \oplus (\nu \oplus \bar{\nu}))$
  - → $\mathrm{SO}(10)$-like fit possible (no mix with vector-like), general one not yet done
- **Vector-like family** and one Higgs irrep: can break $M_d = M_e^\mathsf{T}$ relation
  - → $\mathrm{SU}(5)$: add $\mathbf{5}_F \oplus \overline{\mathbf{5}}_F$ with only $\mathbf{5}$, SUSY [26]
  - → $\mathrm{SO}(10)$: add $\mathbf{16}_F \oplus \overline{\mathbf{16}}_F$ with only $\overline{\mathbf{126}}$ [27]

Yukawa sector in GUTs — various alternatives

- E_6 GUT: fermions in $3 \times \mathbf{27}_F$ [23, 24, 25]
  - → Analogy with $\mathrm{SO}(10)$: $\mathbf{27}_F \supset \mathbf{16}_F$, $\mathbf{27} \supset \mathbf{10}_{\mathbb{C}}$, $\mathbf{351} \supset \mathbf{120}_{\mathbb{C}}$, $\mathbf{351}' \supset \mathbf{126}$
  - → M_d, M_e with vector-like states, M_ν has $3 \times (\nu_L \oplus 2 \times \nu_R \oplus (\nu \oplus \bar{\nu}))$
  - → $\mathrm{SO}(10)$-like fit possible (no mix with vector-like), general one not yet done
- **Vector-like family** and one Higgs irrep: can break $M_d = M_e^{\mathsf{T}}$ relation
  - → $\mathrm{SU}(5)$: add $\mathbf{5}_F \oplus \overline{\mathbf{5}}_F$ with only $\mathbf{5}$, SUSY [26]
  - → $\mathrm{SO}(10)$: add $\mathbf{16}_F \oplus \overline{\mathbf{16}}_F$ with only $\overline{\mathbf{126}}$ [27]
- Add **non-renormalizable** Yukawa operators:
  - → Can correct Georgi-Glashow $\mathrm{SU}(5)$ with $\mathbf{5} \oplus \mathbf{24}$ if $\Lambda \sim 10 M_{\mathrm{GUT}}$ [28]

Yukawa sector in GUTs — various alternatives

- E_6 GUT: fermions in $3 \times \mathbf{27}_F$ [23, 24, 25]
  - → Analogy with $\mathrm{SO}(10)$: $\mathbf{27}_F \supset \mathbf{16}_F$, $\mathbf{27} \supset \mathbf{10}_{\mathbb{C}}$, $\mathbf{351} \supset \mathbf{120}_{\mathbb{C}}$, $\mathbf{351}' \supset \mathbf{126}$
  - → M_d, M_e with vector-like states, M_ν has $3 \times (\nu_L \oplus 2 \times \nu_R \oplus (\nu \oplus \bar{\nu}))$
  - → $\mathrm{SO}(10)$-like fit possible (no mix with vector-like), general one not yet done
- **Vector-like family** and one Higgs irrep: can break $M_d = M_e^{\mathsf{T}}$ relation
  - → $\mathrm{SU}(5)$: add $\mathbf{5}_F \oplus \overline{\mathbf{5}}_F$ with only $\mathbf{5}$, SUSY [26]
  - → $\mathrm{SO}(10)$: add $\mathbf{16}_F \oplus \overline{\mathbf{16}}_F$ with only $\overline{\mathbf{126}}$ [27]
- Add **non-renormalizable** Yukawa operators:
  - → Can correct Georgi-Glashow $\mathrm{SU}(5)$ with $\mathbf{5} \oplus \mathbf{24}$ if $\Lambda \sim 10 M_{\mathsf{GUT}}$ [28]
- **SUSY GUT**: unification with one step $\not{G}$, but ambiguities from SU$\!\!\!/$SY
  - → threshold corrections to Yukawa couplings at M_{SUSY}
  - → if single operator dominance (in each family) and e.g. mSUGRA: SUSY spectrum predicted (a few TeV), see e.g. [29, 30, 31, 32]

Final thoughts and outlook

(1) GUTs are an attractive framework

- (1a) **unification** of SM forces
- (1b) "smoking gun" prediction: **proton decay**
- (1c) but also: structural constraints on **Yukawa sector**

Final thoughts and outlook

(1) GUTs are an attractive framework

- (1a) **unification** of SM forces
- (1b) "smoking gun" prediction: **proton decay**
- (1c) but also: structural constraints on **Yukawa sector**

(2) Yukawa sector predictions from GUTs

- (2a) GUT relations **obscured** by RG running
- (2b) **independent Yukawa fits**: important guide for model building . . .
- (2c) . . . but for precision: **full models** should be considered
- (2d) Future: confrontation of neutrino hierarchy/mass, CP phase predictions with **experiments** (JUNO, Hyper-K, DUNE, KATRIN, Project 8, etc.)

Final thoughts and outlook

(1) GUTs are an attractive framework

- (1a) **unification** of SM forces
- (1b) "smoking gun" prediction: **proton decay**
- (1c) but also: structural constraints on **Yukawa sector**

(2) Yukawa sector predictions from GUTs

- (2a) GUT relations **obscured** by RG running
- (2b) **independent Yukawa fits**: important guide for model building . . .
- (2c) . . . but for precision: **full models** should be considered
- (2d) Future: confrontation of neutrino hierarchy/mass, CP phase predictions with **experiments** (JUNO, Hyper-K, DUNE, KATRIN, Project 8, etc.)

Thank you for your attention!

References I

[1] H. Georgi and S. L. Glashow, "Unity of All Elementary Particle Forces," *Phys. Rev. Lett.* **32** (1974) 438–441.

[2] **Super-Kamiokande** Collaboration, A. Takenaka *et al.*, "Search for proton decay via $p \to e^+\pi^0$ and $p \to \mu^+\pi^0$ with an enlarged fiducial volume in Super-Kamiokande I-IV," *Phys. Rev. D* **102** no. 11, (2020) 112011, `arXiv:2010.16098 [hep-ex]`.

[3] **Super-Kamiokande** Collaboration, K. Abe *et al.*, "Search for proton decay via $p \to \nu K^+$ using 260 kiloton·year data of Super-Kamiokande," *Phys. Rev. D* **90** no. 7, (2014) 072005, `arXiv:1408.1195 [hep-ex]`.

[4] R. Slansky, "Group Theory for Unified Model Building," *Phys. Rept.* **79** (1981) 1–128.

[5] R. Feger, T. W. Kephart, and R. J. Saskowski, "LieART 2.0 – A Mathematica application for Lie Algebras and Representation Theory," *Comput. Phys. Commun.* **257** (2020) 107490, `arXiv:1912.10969 [hep-th]`.

[6] B. Bajc, S. Lavignac, and T. Mede, "Supersymmetry Breaking Induced by Radiative Corrections," *JHEP* **07** (2012) 185, `arXiv:1202.2845 [hep-ph]`.

[7] I. Doršner and S. Saad, "Towards Minimal $SU(5)$," *Phys. Rev. D* **101** no. 1, (2020) 015009, `arXiv:1910.09008 [hep-ph]`.

[8] I. Doršner, E. Džaferović-Mašić, and S. Saad, "Parameter space exploration of the minimal SU(5) unification," *Phys. Rev. D* **104** no. 1, (2021) 015023, `arXiv:2105.01678 [hep-ph]`.

[9] I. Doršner, E. Džaferović-Mašić, S. Fajfer, and S. Saad, "Gauge and scalar boson mediated proton decay in a predictive SU(5) GUT model," *Phys. Rev. D* **109** no. 7, (2024) 075023, `arXiv:2401.16907 [hep-ph]`.

[10] R. D. Peccei and H. R. Quinn, "CP Conservation in the Presence of Instantons," *Phys. Rev. Lett.* **38** (1977) 1440–1443.

[11] R. D. Peccei and H. R. Quinn, "Constraints Imposed by CP Conservation in the Presence of Instantons," *Phys. Rev. D* **16** (1977) 1791–1797.

[12] B. Bajc, A. Melfo, G. Senjanovic, and F. Vissani, "Yukawa sector in non-supersymmetric renormalizable SO(10)," *Phys. Rev. D* **73** (2006) 055001, `arXiv:hep-ph/0510139`.

References II

[13] G. Altarelli and D. Meloni, "A non supersymmetric SO(10) grand unified model for all the physics below M_{GUT}," *JHEP* **08** (2013) 021, arXiv:1305.1001 [hep-ph].

[14] A. Dueck and W. Rodejohann, "Fits to SO(10) Grand Unified Models," *JHEP* **09** (2013) 024, arXiv:1306.4468 [hep-ph].

[15] T. Fukuyama, K. Ichikawa, and Y. Mimura, "Revisiting fermion mass and mixing fits in the minimal SUSY $SO(10)$ GUT," *Phys. Rev. D* **94** no. 7, (2016) 075018, arXiv:1508.07078 [hep-ph].

[16] S. M. Boucenna, T. Ohlsson, and M. Pernow, "A minimal non-supersymmetric SO(10) model with Peccei–Quinn symmetry," *Phys. Lett. B* **792** (2019) 251–257, arXiv:1812.10548 [hep-ph]. [Erratum: Phys.Lett.B 797, 134902 (2019)].

[17] T. Ohlsson and M. Pernow, "Fits to Non-Supersymmetric SO(10) Models with Type I and II Seesaw Mechanisms Using Renormalization Group Evolution," *JHEP* **06** (2019) 085, arXiv:1903.08241 [hep-ph].

[18] A. S. Joshipura and K. M. Patel, "Fermion Masses in SO(10) Models," *Phys. Rev. D* **83** (2011) 095002, arXiv:1102.5148 [hep-ph].

[19] K. S. Babu, B. Bajc, and S. Saad, "Yukawa Sector of Minimal SO(10) Unification," *JHEP* **02** (2017) 136, arXiv:1612.04329 [hep-ph].

[20] D. Meloni, T. Ohlsson, and S. Riad, "Renormalization Group Running of Fermion Observables in an Extended Non-Supersymmetric SO(10) Model," *JHEP* **03** (2017) 045, arXiv:1612.07973 [hep-ph].

[21] T. Ohlsson and M. Pernow, "Running of Fermion Observables in Non-Supersymmetric SO(10) Models," *JHEP* **11** (2018) 028, arXiv:1804.04560 [hep-ph].

[22] K. S. Babu, B. Bajc, and S. Saad, "Resurrecting Minimal Yukawa Sector of SUSY SO(10)," *JHEP* **10** (2018) 135, arXiv:1805.10631 [hep-ph].

[23] B. Bajc and V. Susič, "Towards the minimal renormalizable supersymmetric E_6 model," *JHEP* **02** (2014) 058, arXiv:1311.0775 [hep-ph].

References III

[24] K. S. Babu, B. Bajc, and V. Susič, "A minimal supersymmetric E_6 unified theory," *JHEP* **05** (2015) 108, arXiv:1504.00904 [hep-ph].

[25] K. S. Babu, B. Bajc, and V. Susič, "A realistic theory of E_6 unification through novel intermediate symmetries," *JHEP* **06** (2024) 018, arXiv:2403.20278 [hep-ph].

[26] K. S. Babu, B. Bajc, and Z. Tavartkiladze, "Realistic Fermion Masses and Nucleon Decay Rates in SUSY SU(5) with Vector-Like Matter," *Phys. Rev. D* **86** (2012) 075005, arXiv:1207.6388 [hep-ph].

[27] K. S. Babu, B. Bajc, and S. Saad, "New Class of SO(10) Models for Flavor," *Phys. Rev. D* **94** no. 1, (2016) 015030, arXiv:1605.05116 [hep-ph].

[28] G. Senjanović and M. Zantedeschi, "Minimal SU(5) theory on the edge: The importance of being effective," *Phys. Rev. D* **109** no. 9, (2024) 095009, arXiv:2402.19224 [hep-ph].

[29] S. Antusch and M. Spinrath, "New GUT predictions for quark and lepton mass ratios confronted with phenomenology," *Phys. Rev. D* **79** (2009) 095004, arXiv:0902.4644 [hep-ph].

[30] S. Antusch and C. Sluka, "Predicting the Sparticle Spectrum from GUTs via SUSY Threshold Corrections with SusyTC," *JHEP* **07** (2016) 108, arXiv:1512.06727 [hep-ph].

[31] S. Antusch and C. Hohl, "Predictions from a flavour GUT model combined with a SUSY breaking sector," *JHEP* **10** (2017) 155, arXiv:1706.04274 [hep-ph].

[32] S. Antusch, C. Hohl, C. K. Khosa, and V. Susic, "Predicting δ^{PMNS}, $\theta_{23}^{\mathrm{PMNS}}$ and fermion mass ratios from flavour GUTs with CSD2," *JHEP* **12** (2018) 025, arXiv:1808.09364 [hep-ph].