April 2 – 3, 2025

Pisa

Neutrino and Flavour BSM models: a review

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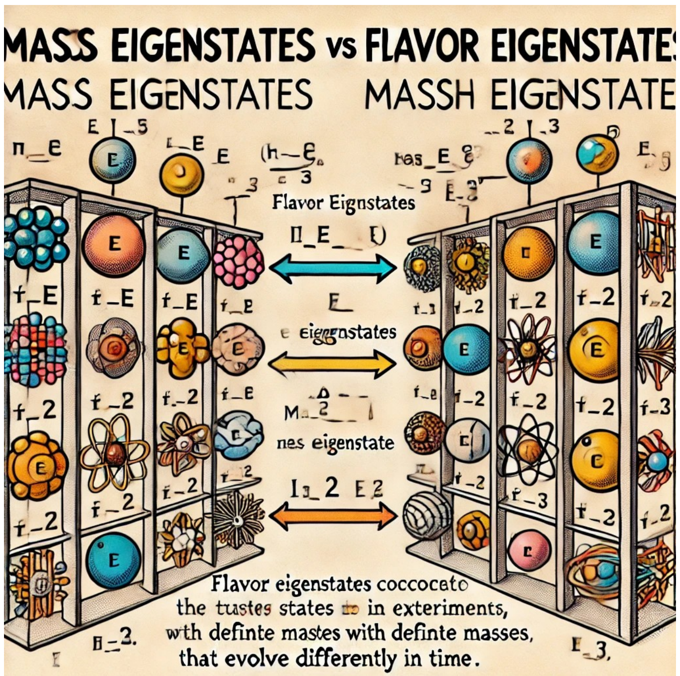
What we know: flavor mixing

Maybe the right time to ask AI agents to solve the ***flavor problem***



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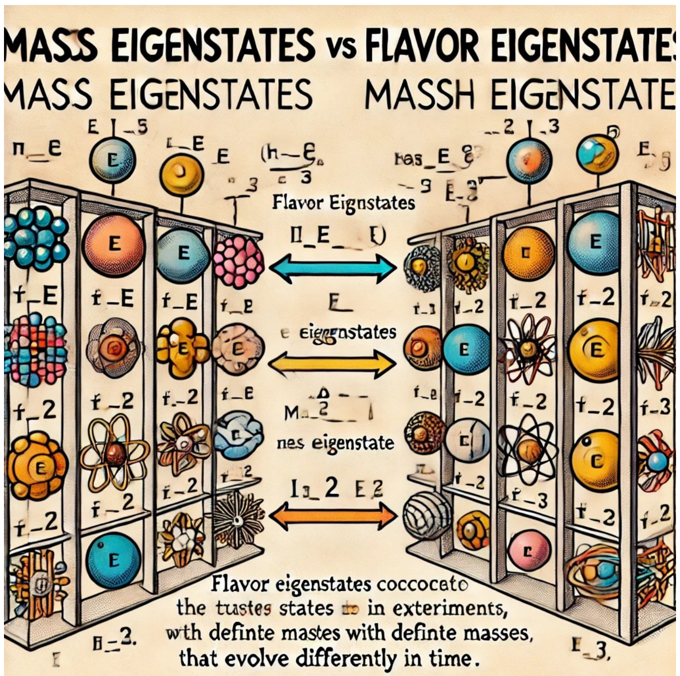
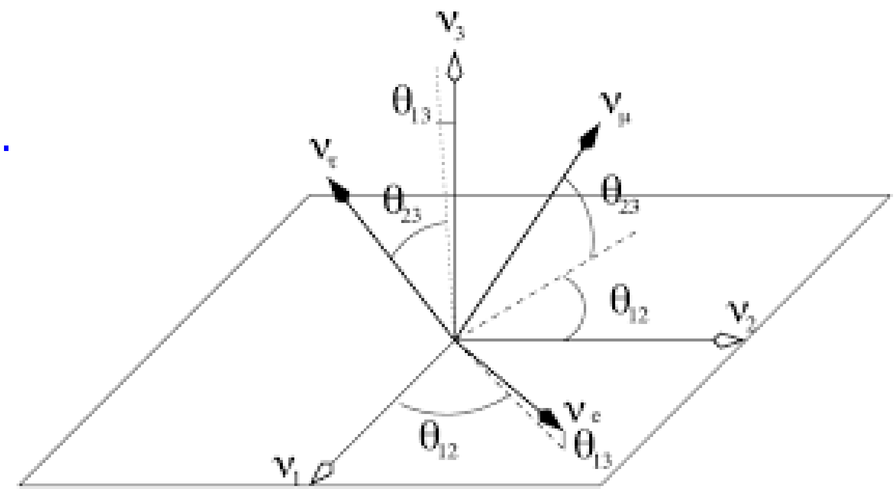
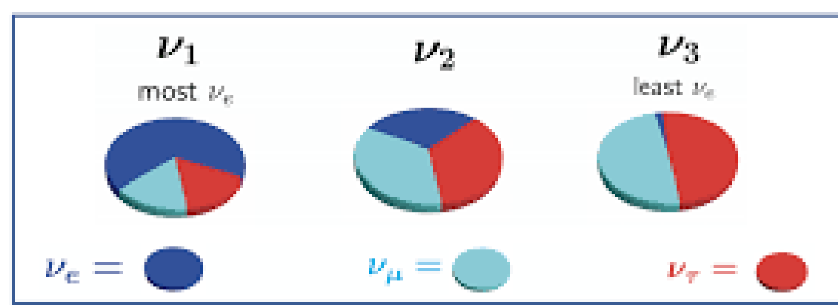


after several trials...

...I gave up and took from the web

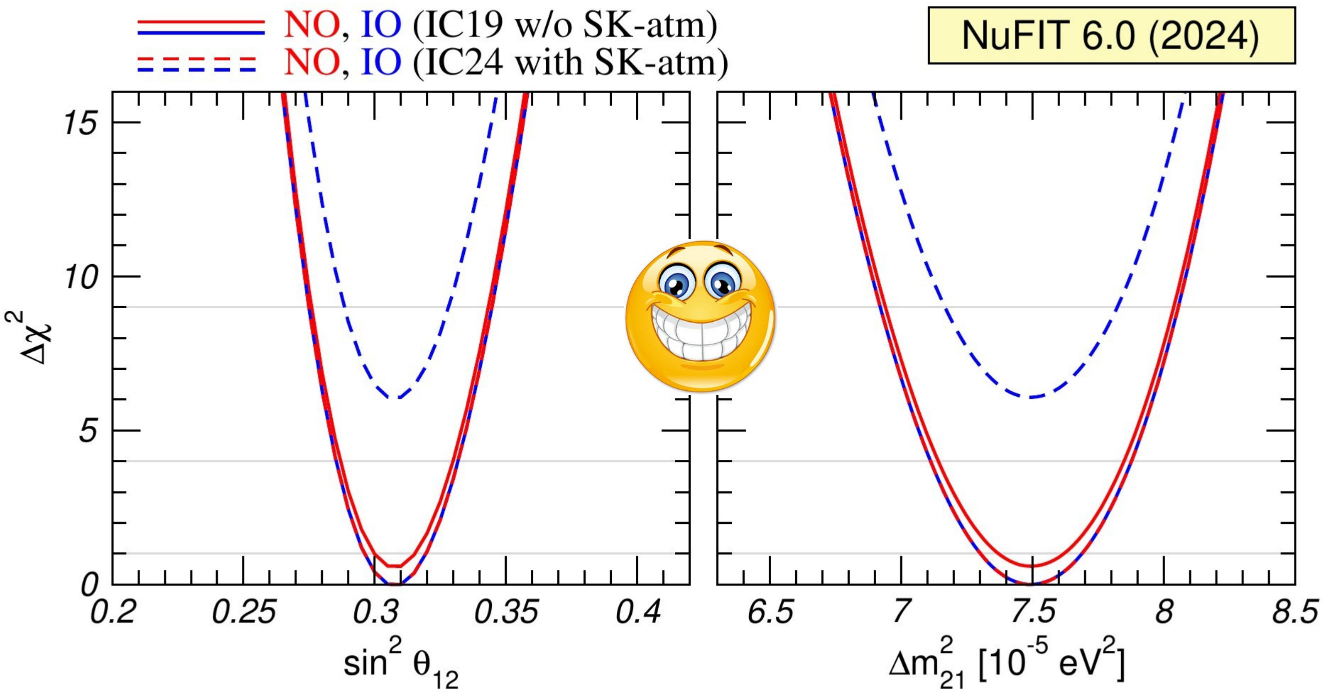


$$
\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} .
$$



Flavor mixing: current situation

three-neutrino fit based on data available in September 2024



	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.1$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$
$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$

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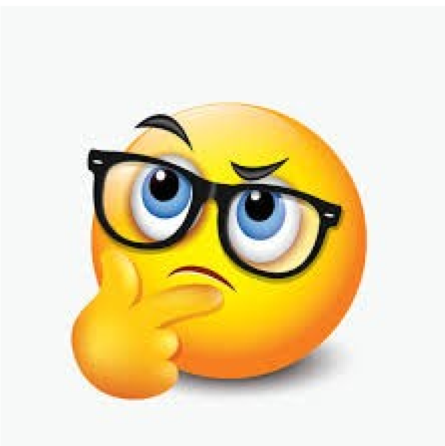
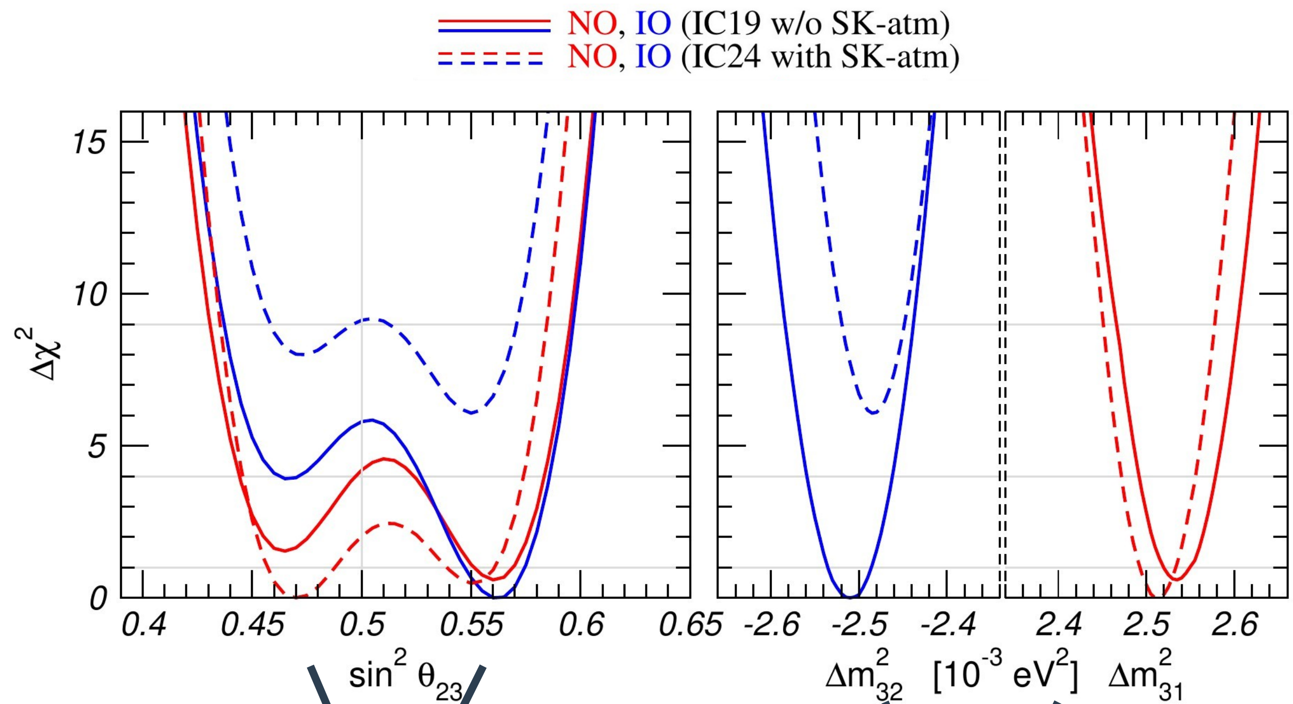




octant degeneracy

mass hierarchy

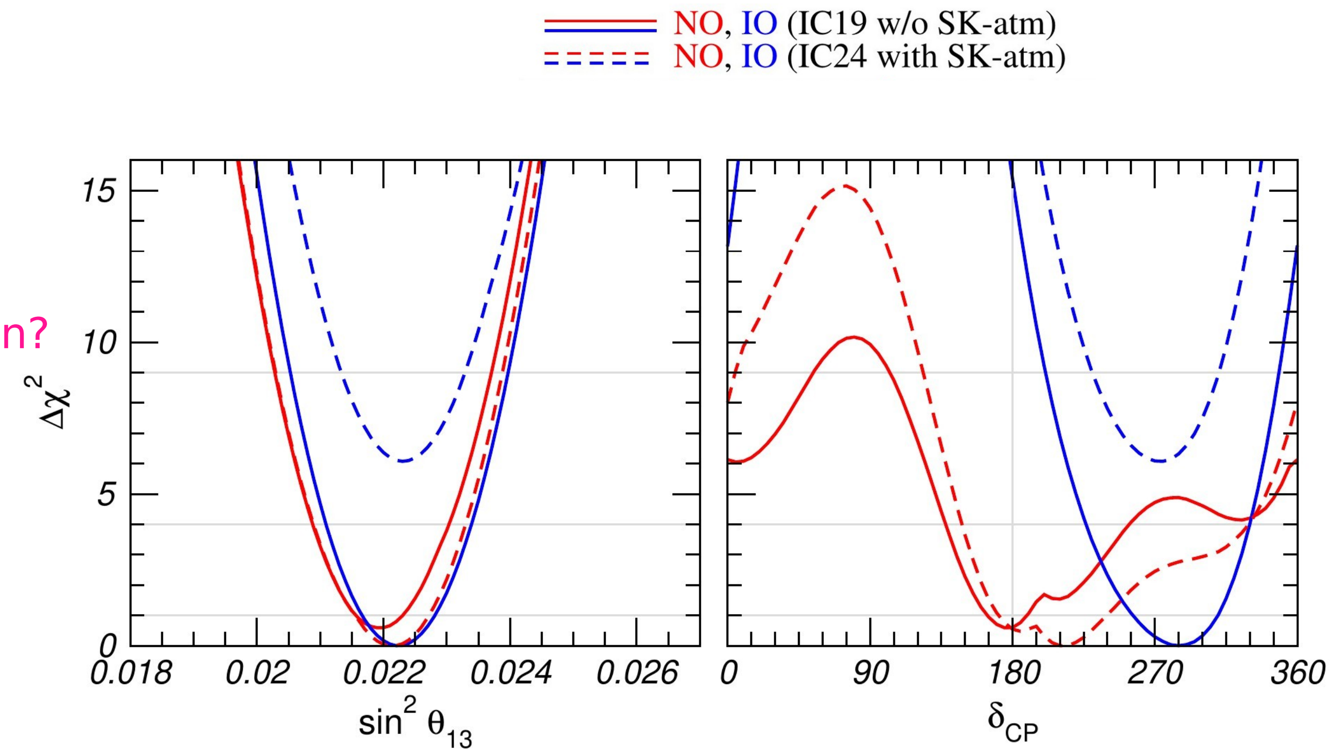
$\sin^2\theta_{23}$	$0.470^{+0.017}_{-0.013}$	$0.435 \to 0.585$	$0.550^{+0.012}_{-0.015}$	$0.440 \to 0.584$
$\theta_{23}/^\circ$	$43.3^{+1.0}_{-0.8}$	$41.3 \to 49.9$	$47.9^{+0.7}_{-0.9}$	$41.5 \to 49.8$
$\sin^2\theta_{13}$	$0.02215^{+0.00056}_{-0.00058}$	$0.02030 \to 0.02388$	$0.02231^{+0.00056}_{-0.00056}$	$0.02060 \to 0.02409$
$\theta_{13}/^\circ$	$8.56^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.59^{+0.11}_{-0.11}$	$8.25 \to 8.93$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.513^{+0.021}_{-0.019}$	$+2.451 \to +2.578$	$-2.484^{+0.020}_{-0.020}$	$-2.547 \to -2.421$

$$P_{\alpha\beta} \sim \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$

Flavor mixing: current situation

three-neutrino fit based on data available in September 2024

measuring CP violation?



$$P_{\alpha\beta}^{CPV} \sim \sin\left(\theta_{13}\right)\left(\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}\right)\sin\delta_{CP}$$

$\sin^2\theta_{13}$	$0.02215^{+0.00056}_{-0.00058}$	$0.02030 \to 0.02388$	$0.02231^{+0.00056}_{-0.00056}$	$0.02060 \to 0.02409$
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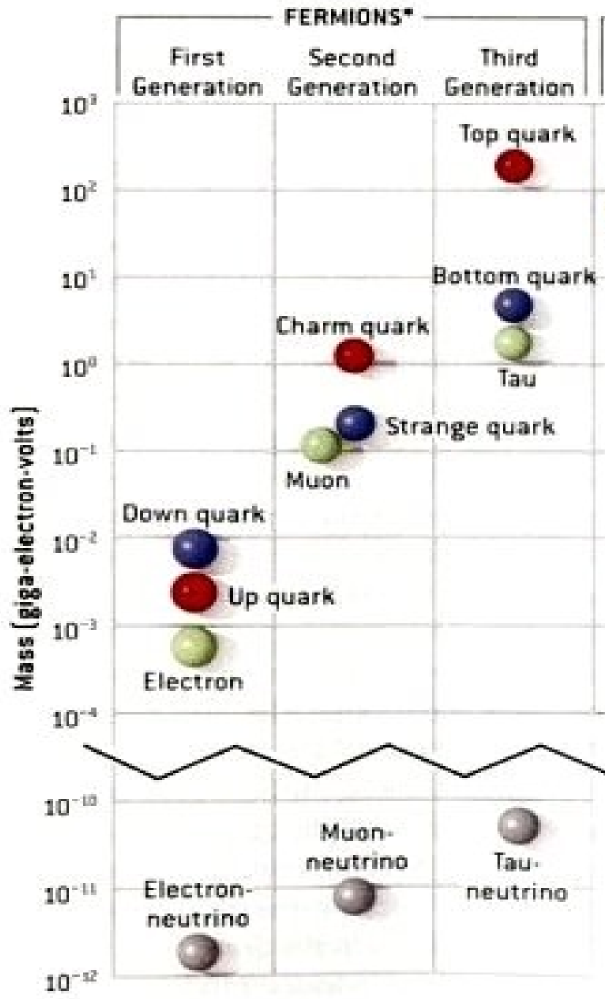
$\delta_{CP}/^\circ$	212^{+26}_{-41}	$124 \to 364$	274^{+22}_{-25}	$201 \to 335$

The Flavor Problem (I)



very small neutrino masses

Mass hierarchies

$$m_d \ll m_s \ll m_b\,, \quad \frac{m_d}{m_s} = 5.02 \times 10^{-2}\,,$$

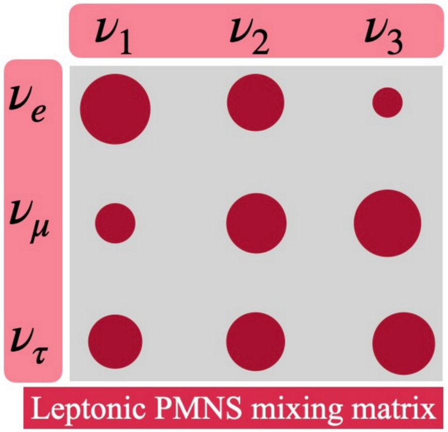
$$m_u \ll m_c \ll m_t\,, \quad \frac{m_u}{m_c} = 1.7 \times 10^{-3}\,,$$

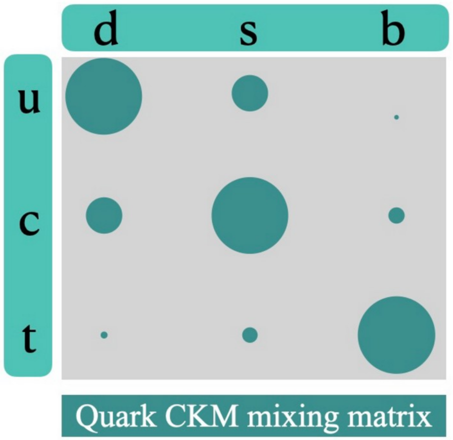
$$\frac{m_s}{m_b} = 2.22 \times 10^{-2}\,, \quad m_b = 4.18 \text{ GeV};$$

$$\frac{m_c}{m_t} = 7.3 \times 10^{-3}\,, \quad m_t = 172.9 \text{ GeV};$$

The Flavor Problem (II)

Fermion mixing





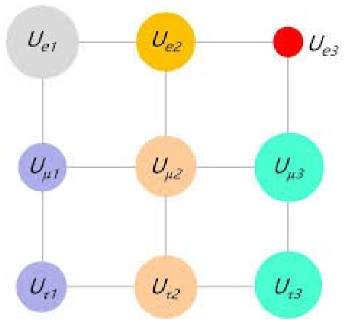
all mixing are large but the 13 element

almost a diagonal matrix

Why are they so different?

Let us focus on mixing: Some suggested solutions

Let us analyze the magnitude of
the PMNS matrix elements



NuFIT 6.0 (2024)

$$|U|_{3\sigma}^{\text{IC19 w/o SK-atm}} = \begin{pmatrix} 0.801 \to 0.842 & 0.519 \to 0.580 & 0.142 \to 0.155 \\ 0.248 \to 0.505 & 0.473 \to 0.682 & 0.649 \to 0.764 \\ 0.270 \to 0.521 & 0.483 \to 0.690 & 0.628 \to 0.746 \end{pmatrix}$$

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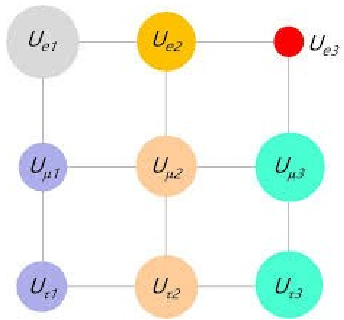

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some useful approximations

$\tan\theta_{12} = 1/\phi$, with $\phi = (1+\sqrt{5})/2$

$$T_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Bi-maximal mixing

$$T_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tri-Bi-maximal mixing

$$T_{GR} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & -\frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Golden Ratio

Some suggested solutions

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why do not we take them seriously?

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Golden Ratio

why do not we take them seriously?

nice

$\tan(\theta_{12}) = 1$ $\quad$ $\tan(\theta_{12}) = \frac{1}{\sqrt{2}}$ $\quad$ $\tan(\theta_{12}) = \frac{2\sqrt{5}}{\sqrt{5+\sqrt{5}}}$

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accceptable

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accceptable

$\tan(\theta_{23}) = 1$ | $\tan(\theta_{23}) = 1$ | $\tan(\theta_{23}) = 1$

bad

$\sin(\theta_{13}) = 0$ | $\sin(\theta_{13}) = 0$ | $\sin(\theta_{13}) = 0$

corrections are necessary

Non-abelian discrete symmetries

Non-abelian Discrete symmetries

S_4: permutation group of four elements

Altarelli et al., 0903.1940

easy to get the neutrino mass matrix:

$$\begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

diagonalized by

$$\longrightarrow$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

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Corrections are needed from charged lepton diagonalization

$$U^{PMNS} = U_{cl}^{+} \cdot U_{\nu} \qquad U_{cl} \sim \begin{pmatrix} 1 & \lambda_C & \lambda_C \\ \lambda_C & 1 & 0 \\ \lambda_C & 0 & 1 \end{pmatrix}$$

Altarelli et al., 0903.1940

introduced by *hand* *(me/mu ~ λ^2_c)*

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introduced by *hand* *(me/mu ~ λ^2_c)*

good results:

$$\begin{cases} \sin^2\theta_{12} = \frac{1}{2} - O(\lambda_C) \\ \sin^2\theta_{23} = \frac{1}{2} \\ \sin\theta_{13} = \frac{1}{\sqrt{2}} O(\lambda_C) \end{cases}$$

$$[\theta_{13}^{PMNS} = O(1) \cdot \theta_{12}^{CKM}]$$

Non-abelian discrete symmetries

Non-abelian Discrete symmetries

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\downarrow$$

$$\left\{ \begin{array}{l} \sin^2\theta_{12} = \frac{1}{2} - O(\lambda_C) \\ \sin^2\theta_{23} = \frac{1}{2} \\ \sin\theta_{13} = \frac{1}{\sqrt{2}} O(\lambda_C) \end{array} \right.$$

Non-abelian discrete symmetries

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+-----------------------------+
| 1. Choice of S_4 Reps
| - S_4: 1_1, 1_2, 2, 3_1, 3_2
| - Assign SM fermions
+-----------------------------+

↓

+-----------------------------+
| 2. Charge Assignments
| - Left-handed fields
| - e_R, μ_R, τ_R
| - Higgs/Flavons
+-----------------------------+

↓

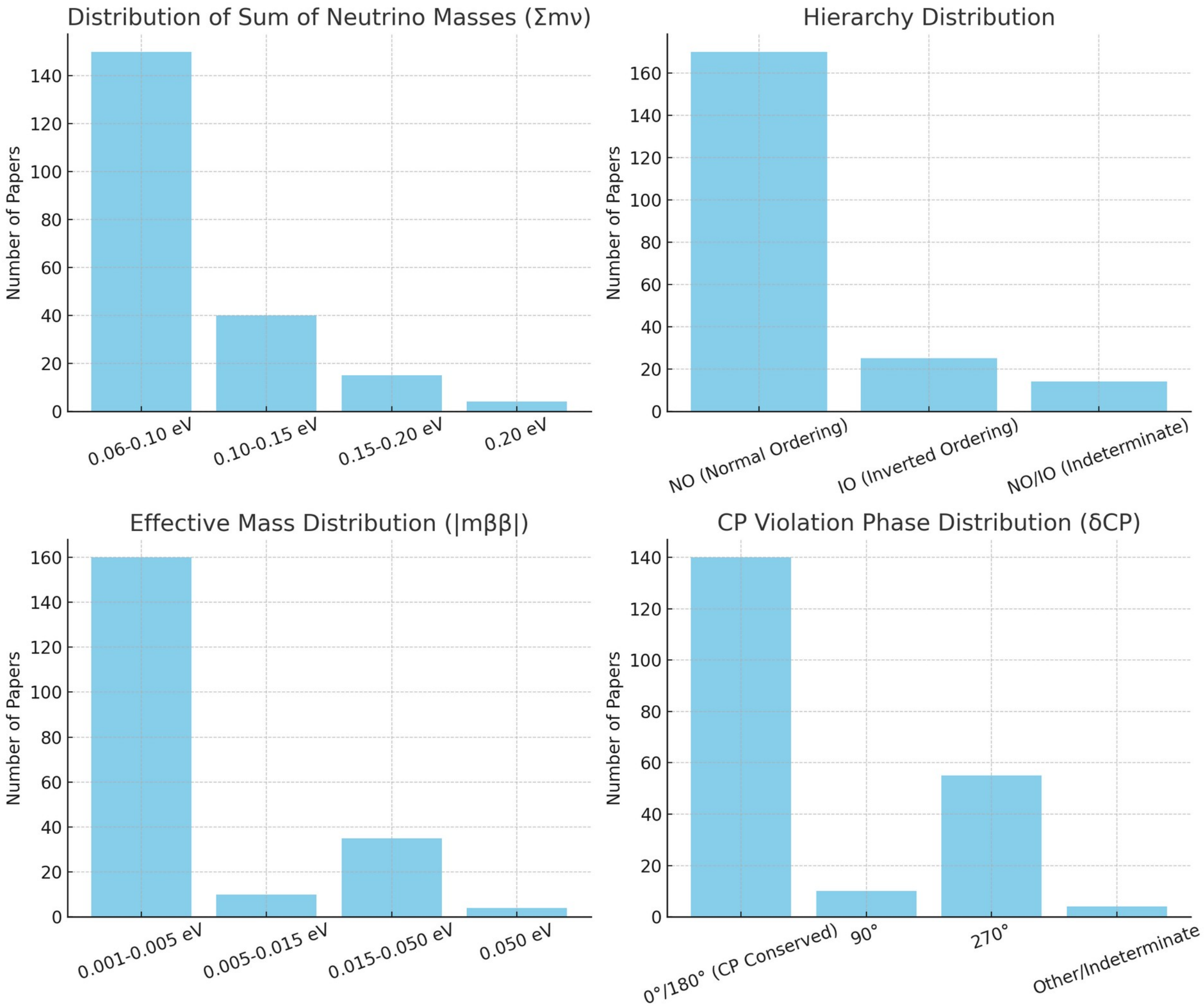
+-----------------------------+
| 3. Yukawa Interactions
| - renormalizable
| - higher-dim operators
+-----------------------------+

↓

+-----------------------------+
| 4. Flavon Fields & Breaking
| - scalar sector
+-----------------------------+

A (very very very) preliminar scan of A_4 models

Asked **Grok** to analyze the abstract of more than 200 papers related to A_4



Sum of Masses (Σmν): A heavily skewed distribution towards the 0.06-0.10 eV range, with a decreasing tail for higher masses

Hierarchy: A dominant peak for NO, a smaller bar for IO, and a minor bar for indeterminate models (both hierarchies are fine).

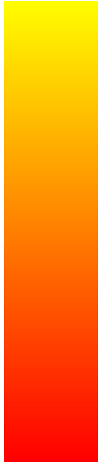
Effective Mass (|mββ|): A pronounced peak in the 0.001-0.005 eV range, a smaller peak in the 0.015-0.050 eV range, showing a bimodal distribution.

CP Violation Phase (δCP): A major peak for CP conservation (0°/180°), a smaller bar for 270°, and minimal presence for 90°

Non-abelian discrete symmetries

my personal perspective

Issue	Why Flavor Symmetries Don't Fully Solve It
Origin of Yukawa Couplings	Flavor symmetries impose textures but do not explain why Yukawa couplings take their specific values.
Connection to Quark Sector	Most models treat neutrinos and quarks separately, failing to explain the observed patterns in both sectors simultaneously.
Hierarchy of Masses	Symmetries predict mass structures but do not fully explain why neutrino masses are so small compared to quarks and charged leptons.
CP Violation	Many flavor symmetry models struggle to naturally generate the observed leptonic CP phase without additional assumptions.
Lack of Unique Prediction	Different symmetries (A_4, S_4, $\Delta(96)$, etc.) can fit the same data, making it unclear which one (if any) is the fundamental symmetry of nature.
Breaking Mechanisms	Many models require ad-hoc symmetry breaking sectors, introducing additional fields and parameters, reducing predictivity.



colors are ordered from least to most problematic

Most probably we need a new perspective

Quark-lepton complementarity

Let me rewrite the **intriguing** relations:

$$\begin{cases} \sin^2\theta_{12} = \dfrac{1}{2} - O(\lambda_C) \\ \sin^2\theta_{23} = \dfrac{1}{2} \\ \sin\theta_{13} = \dfrac{1}{\sqrt{2}}\, O(\lambda_C) \end{cases}$$

typical leptonic quantity

typical hadronic quantity

are they connected at a more **profund** level?

Quark-lepton complementarity

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typical hadronic quantity

typical leptonic quantity

are they connected at a more **profund** level?

$$\theta_{12}^{PMNS} + \theta_{12}^{CKM} \sim \pi/4$$

~33° ~13°

$$\theta_{23}^{PMNS} + \theta_{23}^{CKM} \sim \pi/4$$

~45° ~2°

Quark-lepton complementarity

complete failure: $\theta_{13}^{PMNS} + \theta_{13}^{CKM} \sim 10^\circ$

θ_{13}^{PMNS}: ~8.5°

θ_{13}^{CKM}: ~0.2°

we need to replace the bad relation with a promising one:

Quark-lepton complementarity

complete failure: $\theta_{13}^{PMNS} + \theta_{13}^{CKM} \sim 10^{\circ}$

~8.5° ~0.2°

we need to replace the bad relation with a promising one:

$$\theta_{13}^{PMNS} = O(1) \cdot \theta_{12}^{CKM}$$

same order of magnitude

Flavor symmetries do the job, Cabibbo angle needed to fit the charged lepton mass ratios

GUT? Promising but not viable in its simplest application

Experimental facts

GUT: simple example from SU(5)

Let us take the electron and down quark relation:

$$m_e = m_d^T$$

$$U^{PMNS} = U_{cl}^{+} \cdot U_{\nu} \qquad V^{CKM} = U_u^{+} \cdot U_d$$

Experimental facts

<u>GUT: simple example from SU(5)</u>

Let us take the electron and down quark relation:

$$m_e = m_d^T$$

$$U^{PMNS} = U_{cl}^{+} \cdot U_{\nu} \qquad V^{CKM} = U_u^{+} \cdot U_d$$

Let us diagonalize the matrices:

$$\underbrace{U_{cl}\, m_e\, E_R^{+} = m_e^D \qquad U_d\, m_d\, D_R^{+} = m_d^D}_{\text{this implies}}$$

$$U_{cl} = D_R^{*}$$

relations involve <u>unobservable</u> right-handed rotations

A new Perspective: Modular symmetry

Feruglio [1706.08749]

minimal yet powerful alternative to
conventional discrete flavor symmetries

typical Yukawa term: $$y\,\overline{\psi}\;h\;\;\psi$$

$$\Downarrow$$

A new Perspective: Modular symmetry

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minimal yet powerful alternative to
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typical Yukawa term: $y\,\overline{\psi}\;h\;\psi$

Modular approach: $y(\tau)\,\overline{\psi}\;h\;\psi$

modulus,
complex
variable

Step 1.

Yukawa couplings treated as functions of τ

fermion masses depend of τ

Modular symmetry

Step 2.

τ has well defined transformation properties under the **modular group**

the group of 2x2 matrices with integer entries
modulo N and determinant equals to one modulo N

$$\boxed{\gamma\,\tau = \frac{a\,\tau + b}{c\,\tau + d}}$$

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, Z), \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} (Mod\,N) \right\}$$

Modular symmetry

Step 3.

Y(τ) has well defined transformation properties under the group: they are called **Modular Forms**

$$Y(\gamma\tau) \to (c\tau + d)^k \rho(\gamma)_{ij} Y(\tau)$$

representative element of Γ_N

unitary representation of Γ_N

Modular symmetry

Step 3.

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$$Y(\gamma\tau) \rightarrow (c\tau+d)^k \rho(\gamma)_{ij} Y(\tau)$$

representative element of Γ_N

unitary representation of Γ_N

k = weigth, N = level

K < 0:
no modular forms

K = 0:
constant functions

K > 0 (even number):
linear space of finite dimension

N	$d_k(\Gamma(N))$	Γ_N
2	$k/2+1$	S_3
3	$k+1$	A_4
4	$2k+1$	S_4
5	$5k+1$	A_5

R. C. Gunning, Lectures on Modular Forms, Princeton, New Jersey USA, Princeton University Press 1962

Modular symmetry

Step 3.

$Y(\tau)$ has well defined transformation properties under the group: they are called **Modular Forms**

$$Y(\gamma\tau) \rightarrow (c\tau + d)^k \rho(\gamma)_{ij} Y(\tau)$$

representative element of Γ_N

unitary representation of Γ_N

K < 0:
no modular forms

K = 0:
constant functions

K > 0 (even number):
linear space of finite dimension

- few combinations available
- once k is fixed, it is so for combinations of modular forms

R. C. Gunning, Lectures on Modular Forms, Princeton, New Jersey USA, Princeton University Press 1962

Modular symmetry

Step 3.

Y(τ) has well defined transformation properties under the group: they are called **Modular Forms**

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for N ≤ 5, the finite modular groups Γ_N are isomorphic to non-Abelian discrete groups

$\Gamma_2 \simeq S_3$ $\Gamma_3 \simeq A_4$ $\Gamma_4 \simeq S_4$ $\Gamma_5 \simeq A_5$

Modular symmetry

Step 4.

Fields have well defined transformation properties under the **modular group**

$$\chi(x)_i \rightarrow (c\,\tau + d)^{-k_i} \rho(\gamma)_{ij}\, \chi(x)_j$$

not modular forms !
No restrictions on ki

Modular symmetry

Step 4.

Fields have well defined transformation properties under the **modular group**

$$\chi(x)_i \rightarrow (c\,\tau + d)^{-k_i} \rho(\gamma)_{ij}\, \chi(x)_j$$

not modular forms !
No restrictions on ki

putting all steps together

$$L_{eff} \in \; Y(\tau) \times \chi^{(1)} \ldots \chi^{(n)}$$

invariance requires:

$$Y_i(\gamma\,\tau) \rightarrow (c\,\tau + d)^{k} \rho(\gamma)_{ij}\, Y_j(\tau)$$

$$\chi(x)_i \rightarrow (c\,\tau + d)^{-k_i} \rho(\gamma)_{ij}\, \chi(x)_j$$

$$\rho_f \otimes \rho_{\chi_1} \otimes \ldots \otimes \rho_{\chi_n} \supset I$$

$$k = \Sigma_i k_i$$

Modular symmetry

Final results of this contruction:

small number of operators (few free parameters) → ***predictability***

no new matter fields → ***minimality***

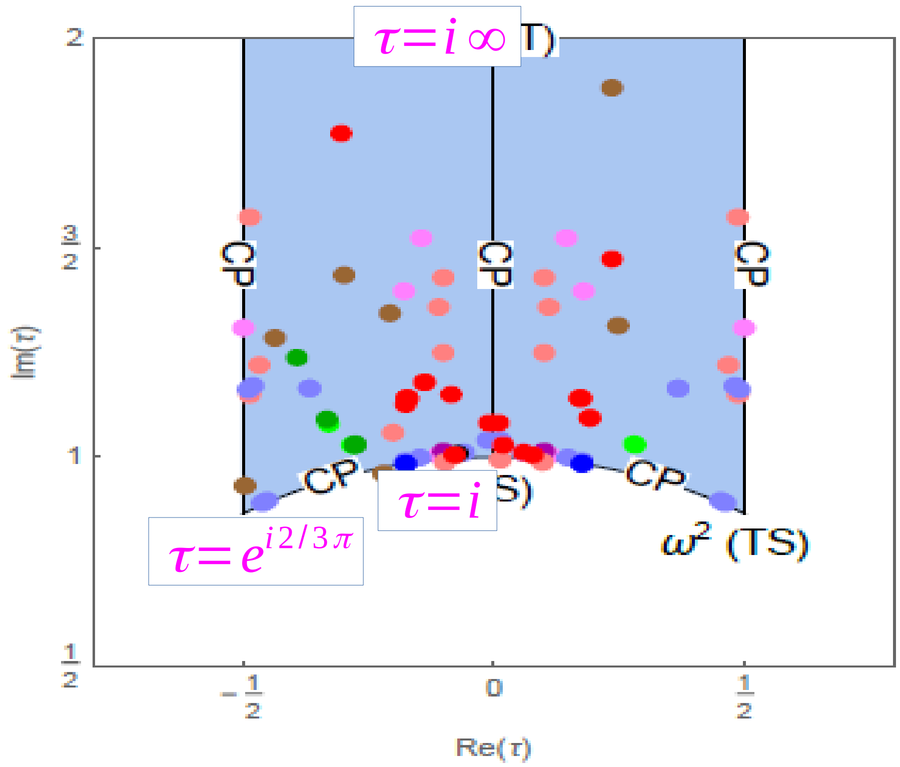
no new scalar fields beside Higgs(es) → ***symmetry breaking dictated by the vev of*** τ

charged lepton hierarchy by symmetry arguments → "***appealing***"

Modular symmetry

Feruglio:
2211.00659

Dots are the best fit values of τ
in selected models



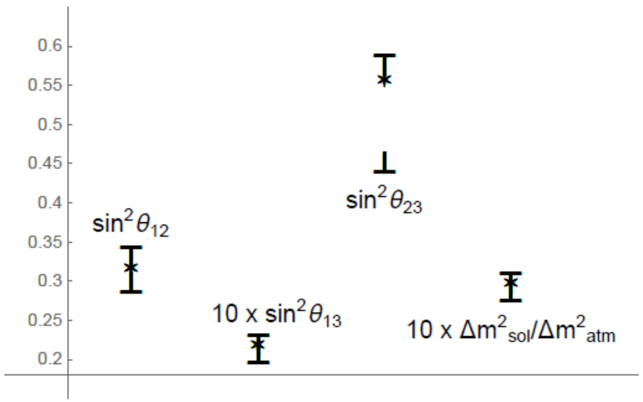
Clustering of points
fall close to the self-dual
point: $|\tau - \iota| <$ 0: 25

Typical “modular” predictions

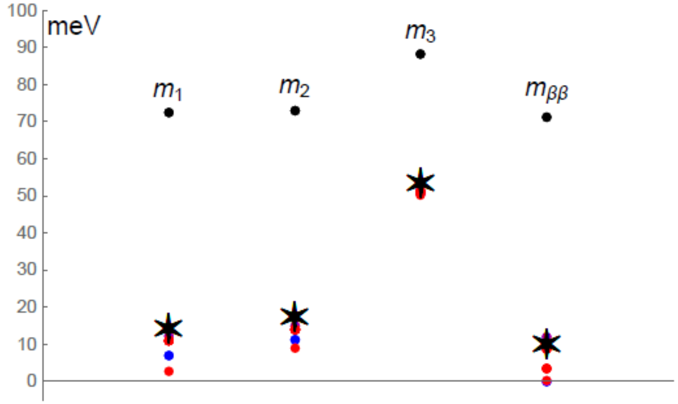
Analysis of 14 models, several N

Feruglio:
2211.00659

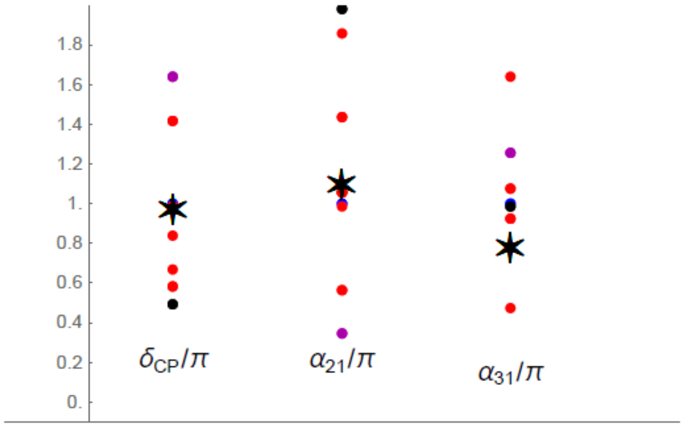
Mixing parameters



masses



phases



Typical “modular” predictions

Analysis of 14 models, several N

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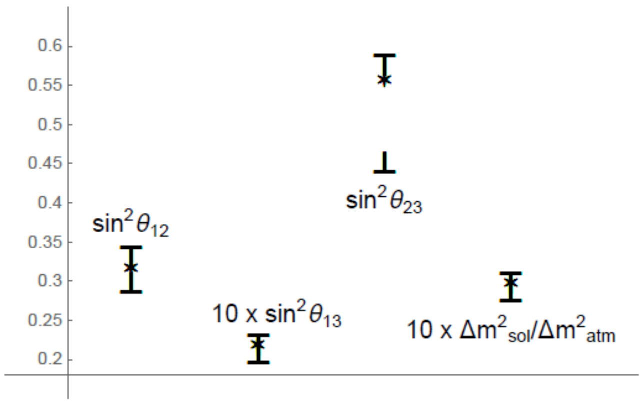
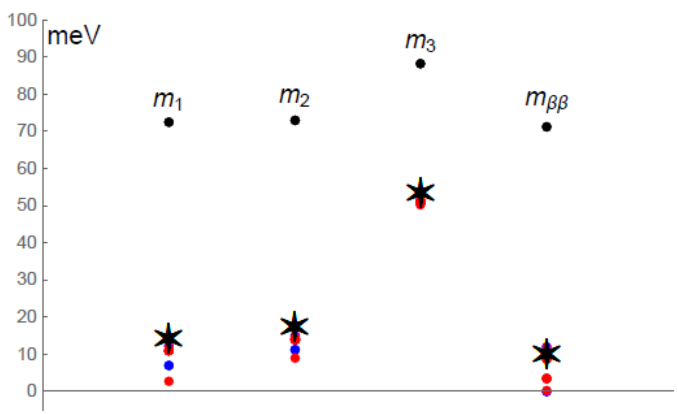
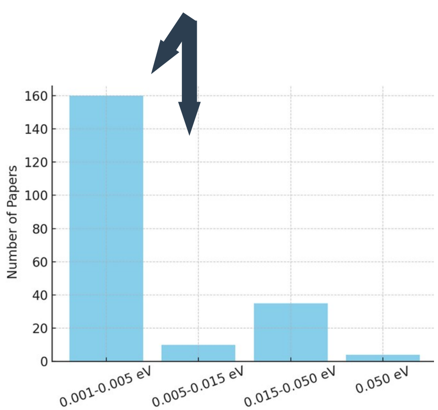
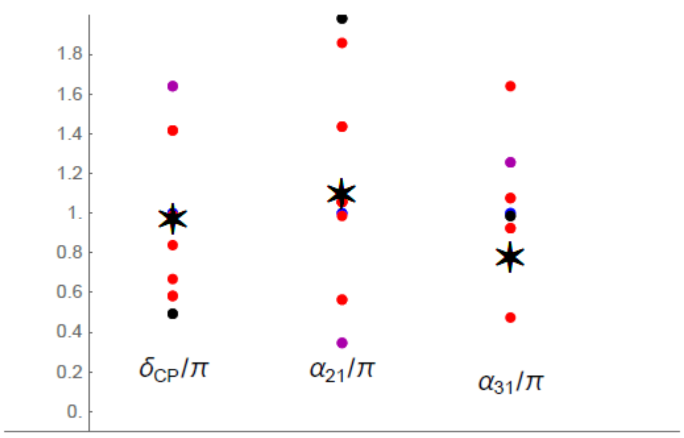


masses





phases

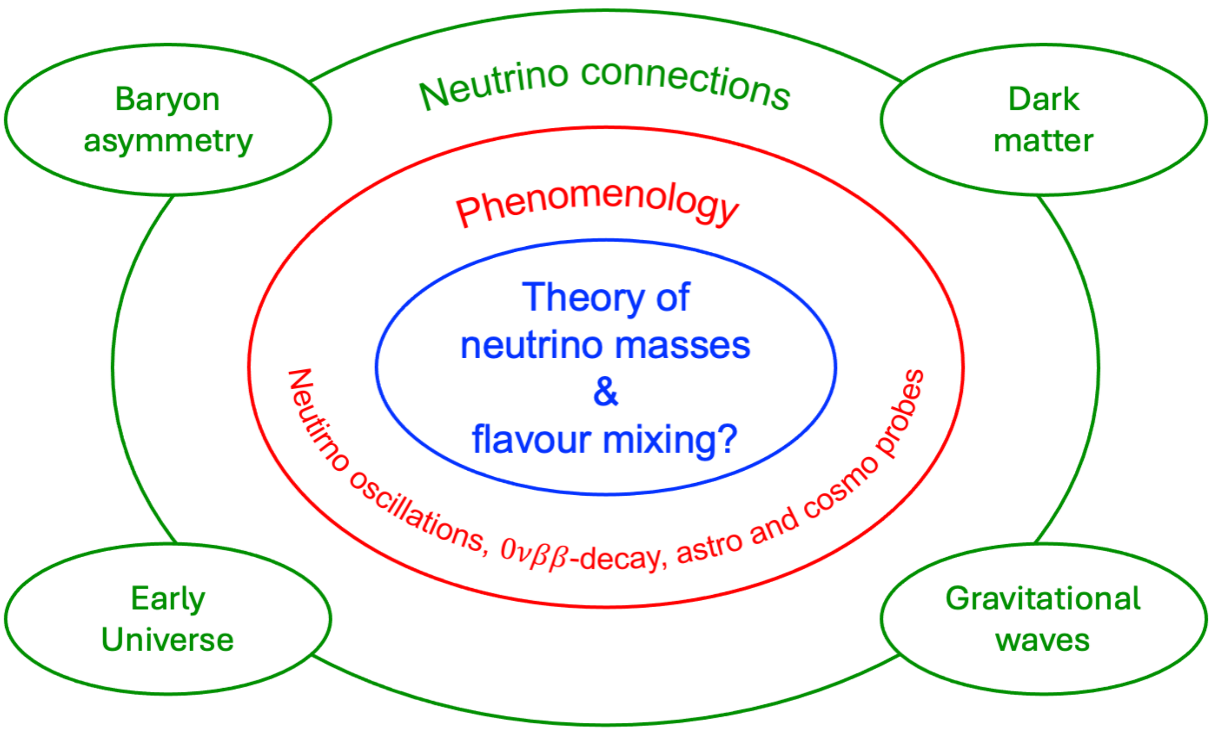


Conclusions

Approach	Advantages	Disadvantages
Discrete Non-Abelian Symmetries	Flexibility: Wide choice of groups (e.g., A_4, S_4) and representations to fit data. Established Precedents: Well-studied and widely applied Intuitive Structure: Directly imposes order on mass matrices via group invariants.	Complexity: Requires additional scalar fields and non-trivial vacuum alignment. Free Parameters: Many degrees of freedom (flavon VEVs, coupling constants) reduce predictivity. Ad Hoc: Phenomenologically motivated, less tied to fundamental theories.
Modular Forms	Elegance: Reduces the number of fields (only τ as a 'flavon-like' field), simplifying the model. Predictivity: Yukawa couplings constrained by modular forms, with fewer free parameters. Theoretical Foundation: Naturally arises from string theory or extra-dimensional geometries.	τ Stabilization: Determining the modulus τ's value is challenging. Limited Flexibility: The form of couplings is fixed by modular weight and level. Mathematical Complexity: Requires familiarity with modular forms and the $SL(2,\mathbb{Z})$ group.

Disclaimer

A highly debated topic: I present my point of view and what fascinates me about BSM connected to neutrinos.



Backup slides

Modular symmetry

Generators of Γ_N : elements S and T satisfying

$$S^2=1, \quad (ST)^3=1, \quad T^N=1$$

$$S=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T=\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$$

corresponding to:

$$\tau \xrightarrow{S} -\frac{1}{\tau} \qquad \tau \xrightarrow{T} \tau+1$$

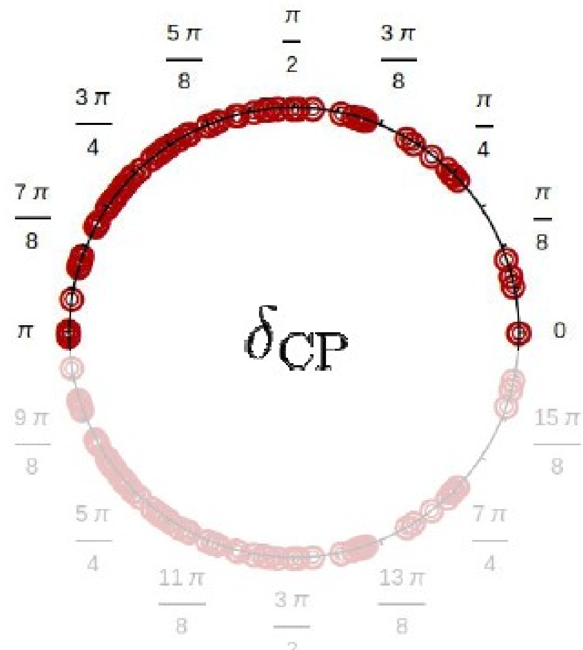
modular invariance completely broken everywhere but at ***three*** *fixed points*

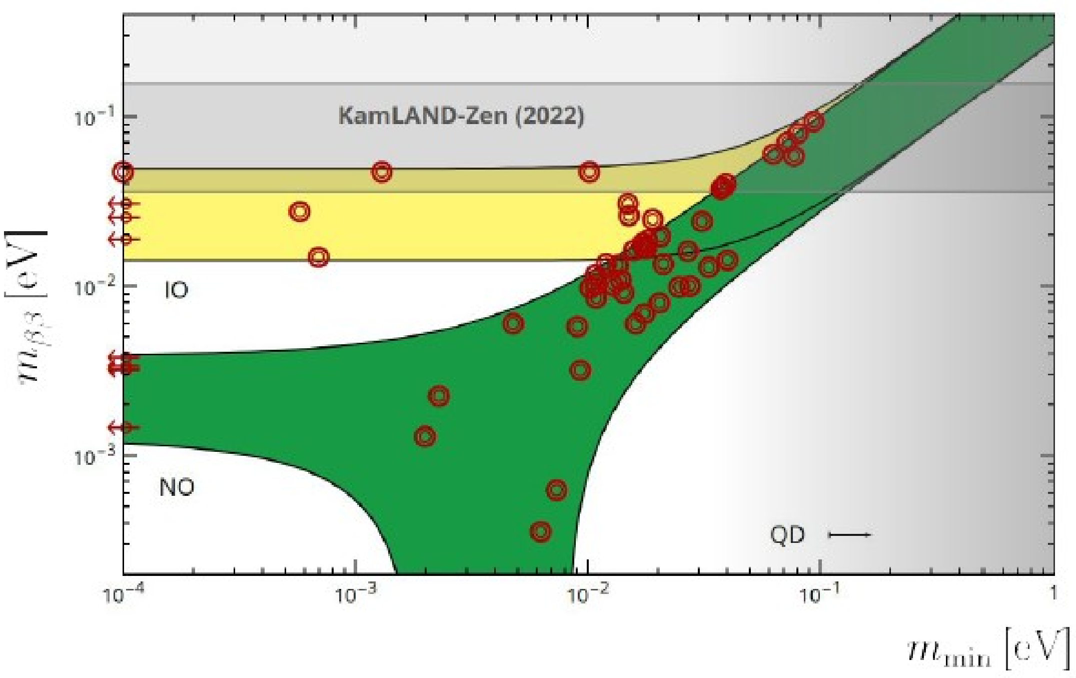
$$\boxed{\tau=i} \qquad \tau \xrightarrow{S} -\frac{1}{\tau}$$

$$\boxed{\tau=e^{i2/3\pi}} \qquad \tau \xrightarrow{ST} -\frac{1}{\tau+1}$$

$$\boxed{\tau=i\infty} \qquad \tau \xrightarrow{T} \tau+1$$

Modular symmetry





Courtesy by Joao Penedo

Modular Symmetry

We start from

Feruglio, 1706.08749

$$\Gamma(N)=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix}\in SL(2,Z),\begin{pmatrix} a & b \\ c & d \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}(Mod\,N)\right\}$$

the group of 2x2 matrices with integer entries
modulo N and determinant equals to one modulo N

Γ(1)=SL(2, Z) = *special linear group* = the group of 2x2 matrices with integer entries and determinant equals to one, called ***homogeneous modular group Γ***

Γ(N), N>=2 are infinite normal subgroups of Γ

the group Γ(N) acts on the complex variable τ (Im τ >0)

$$\gamma\,\tau=\frac{a\,\tau+b}{c\,\tau+d}$$

Modular Symmetry

Important observation for N=1:

a transformation characterized by parameters {a, b, c, d} is identical to the one defined by {-a, -b, -c, -d}

Γ**(1)** is isomorphic to **PSL(2, Z) = SL(2, Z)/{±1} =** $\overline{\Gamma}$

inhomogeneous modular group **(or simply Modular Group)**

In addition:

$$\overline{\Gamma}(2) = \Gamma(2)/\{1, -1\}$$

since 1 and -1 **cannot** be distinguished

$$\overline{\Gamma}(N) = \Gamma(N) \qquad N > 2$$

since 1 and -1 **can** be distinguished

Finite Modular Group:

$$\Gamma_N = \frac{\overline{\Gamma}}{\overline{\Gamma}(N)}$$

Modular Forms

Modular Forms:

holomorphic functions of the complex variable τ with well-defined transformation properties under the group **Γ(N)**

$$f(\gamma\tau)=(c\,\tau+d)^k f(\tau),\qquad \gamma=\begin{pmatrix} a & b \\ c & d \end{pmatrix}\in\Gamma(N)$$

k = weigth, N = level

K < 0:
no modular forms

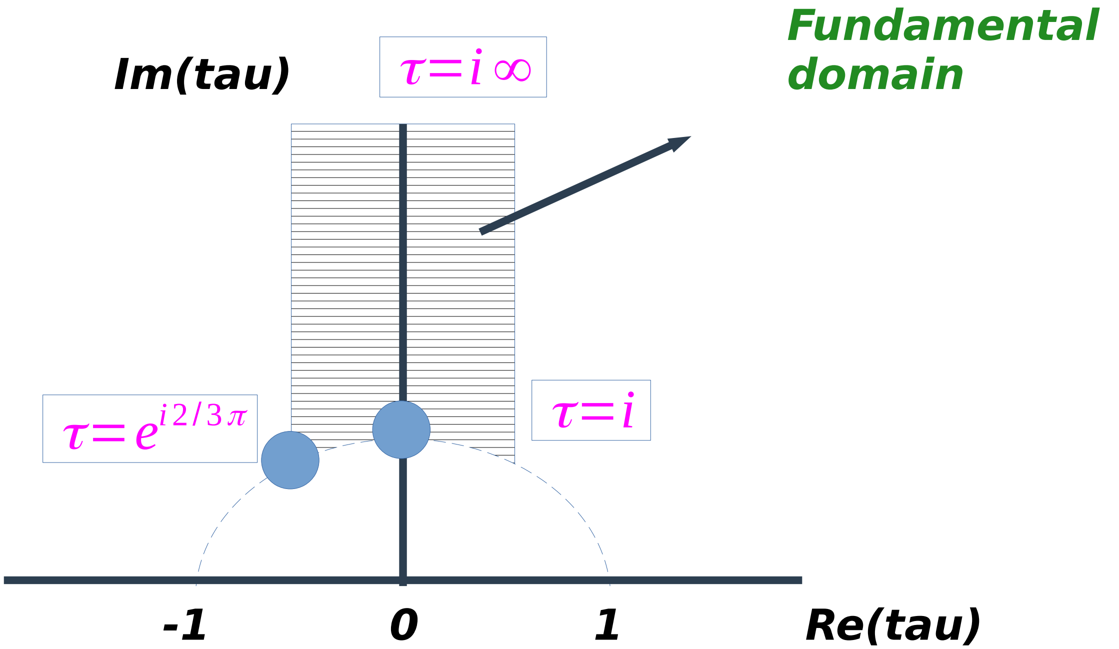
K = 0:
constant functions

K > 0:
linear space of finite dimension

N	$d_k(\Gamma(N))$	Γ_N
2	$k/2+1$	S_3
3	$k+1$	A_4
4	$2k+1$	S_4
5	$5k+1$	A_5

R. C. Gunning, Lectures on Modular Forms, Princeton, New Jersey USA, Princeton University Press 1962

Fundamental Domain



<u>relevant for model building:</u>

for N ≤ 5, the finite modular groups Γ_N are isomorphic to non-Abelian discrete groups

$$\boxed{\Gamma_2 \simeq S_3 \qquad \Gamma_3 \simeq A_4 \qquad \Gamma_4 \simeq S_4 \qquad \Gamma_5 \simeq A_5}$$

Then the question is: why Modular Symmetry ?

Model Building

Key points:

1. Modular forms of weight 2k and level N ≥ 2 are invariant, up to the factor $(c\tau + d)^k$ under **Γ(N)** but they transform under Γ_N !

$$f_i(\gamma\tau) = (c\tau + d)^k \rho(\gamma)_{ij} f_j(\tau)$$

representative element of Γ_N

unitary representation of Γ_N

2. in addition, one assumes that the fields of the theory χ_i transforms non-trivially under Γ_N

$$\chi(x)_i \rightarrow (c\tau + d)^{-k_i} \rho(\gamma)_{ij} \chi(x)_j$$

not modular forms !
No restrictions on ki

Model Building

Key points:

1. Modular forms of weight 2k and level N ≥ 2

2. fields of the theory χ_i transforms non-trivially under Γ_N

Invariance requires:

3. Combine modular forms and fields:

$$k = \Sigma_i k_i$$

$$\rho_f \otimes \rho_{\chi_1} \otimes \ldots \otimes \rho_{\chi_n} \supset I$$

$$L_{eff} \in Y(\tau) \times \chi^{(1)} \ldots \chi^{(n)}$$

modular
forms

Model Building in the Simplest Case: $\Gamma_2 \sim S_3$

Let us find the functions f(τ) !

N	$d_k(\Gamma(N))$	Γ_N
2	$k/2+1$	S_3
3	$k+1$	A_4
4	$2k+1$	S_4
5	$5k+1$	A_5

The group $\mathbf{S_3}$ contains 1 + 1' + 2

two independent modular forms can fit into a doublet of S_3

Dedekind eta functions $\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1-q^n)$ $\qquad q \equiv e^{i2\pi\tau}$

S: $\eta(-1/\tau) = \sqrt{-i\tau}\,\eta(\tau)$, T: $\eta(\tau+1) = e^{i\pi/12}\,\eta(\tau)$

η^{24} is a modular form of weight 12

Model Building

Constructing the Modular Forms

Crucial observation:

if $$g(\tau) \rightarrow e^{i\alpha}(c\tau+d)^k g(\tau)$$

then $$\frac{d}{d\tau}\log[g(\tau)] \rightarrow (c\tau+d)^2\frac{d}{d\tau}\log[g(\tau)]+k\,c\,(c\tau+d)$$

this term prevents of having a modular form of weight **2 k = 2**

The inhomogeneous term can be removed if we combine several $f_i(\tau)$ with weights k_i

$$\frac{d}{d\tau}\Sigma_i\log[g_i(\tau)] \rightarrow (c\tau+d)^2\frac{d}{d\tau}\Sigma_i\log[g_i(\tau)]+\left(\Sigma_i k_i\right)c\,(c\tau+d)$$

with $\Sigma_i k_i=0$

A case study: $\Gamma_2 \sim S_3$

Dedekind eta functions

$$\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n) \qquad q \equiv e^{i2\pi\tau}$$

Under **T**:

$$\begin{cases} \eta(2\tau) \to e^{i\pi/6}\,\eta(2\tau) \\ \eta(\tau/2) \to \eta((\tau+1)/2) \\ \eta((\tau+1)/2) \to e^{i\pi/12}\,\eta(\tau/2) \end{cases}$$

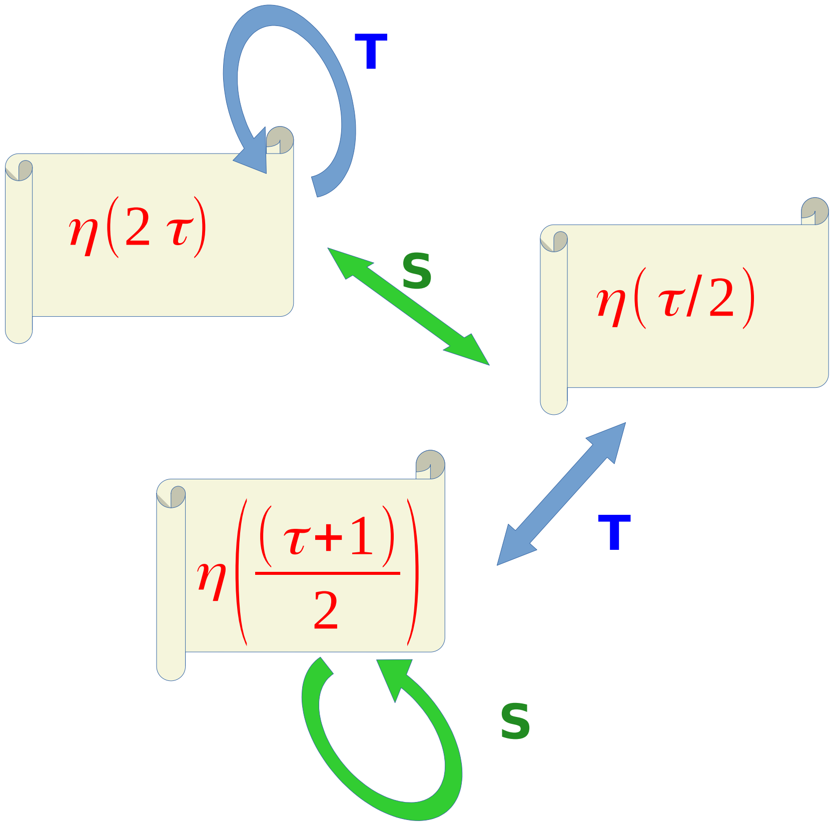
Under **S**:

$$\begin{cases} \eta(2\tau) \to \sqrt{-i\,\tau/2}\;\eta(\tau/2) \\ \eta(\tau/2) \to \sqrt{-2\,i\,\tau}\;\eta(2\tau) \\ \eta\left(\frac{(\tau+1)}{2}\right) \to e^{-i\pi/12}\sqrt{-i\,\tau(\sqrt{3}-i)}\;\eta\left(\frac{(\tau+1)}{2}\right) \end{cases}$$

Model Building in the Simplest Case: $\Gamma_2 \sim S_3$

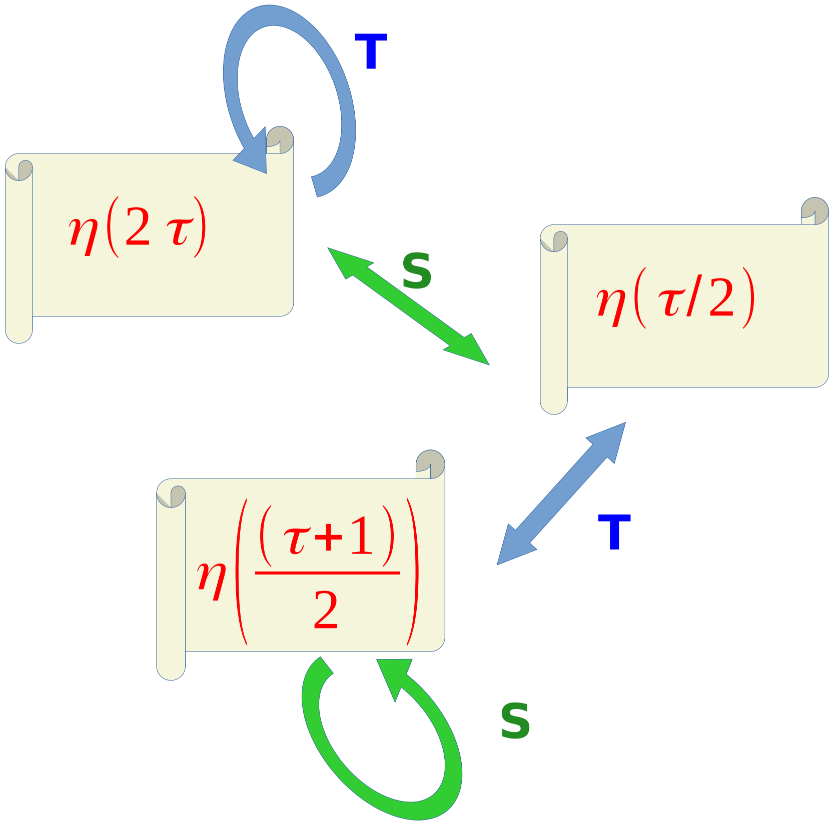
Constructing the Modular Forms

the system is closed under modular transformation



Model Building in the Simplest Case: $\Gamma_2 \sim S_3$

Constructing the Modular Forms



the system is closed under modular transformation

candidate modular form

$$Y(\alpha,\beta,\gamma)=\frac{d}{d\tau}\left[\alpha \log\eta(\tau/2)+\beta \log\eta((\tau+1)/2)+\gamma \log\eta(2\tau)\right]$$

$$\alpha+\beta+\gamma=0$$

A case study: $\Gamma_2 \sim S_3$

Constructing the Modular Forms

Under **T**: $Y(\alpha, \beta, \gamma) \rightarrow Y(\gamma, \beta, \alpha)$

Under **S**: $Y(\alpha, \beta, \gamma) \rightarrow \tau^2 \; Y(\gamma, \alpha, \beta)$

representation of generators

$$\rho(S) = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \qquad \rho(T) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$(\rho(S))^2 = \mathbb{I}, \qquad (\rho(S)\rho(T))^3 = \mathbb{I}, \qquad (\rho(T))^2 = \mathbb{I}$$

Model Building in the Simplest Case: $\Gamma_2 \sim S_3$

Constructing the Modular Forms

Equations to be satisfied:

$$\begin{pmatrix} Y_1(-1/\tau) \\ Y_2(-1/\tau) \end{pmatrix} = \tau^2 \rho(S) \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}, \qquad \begin{pmatrix} Y_1(\tau+1) \\ Y_2(\tau+1) \end{pmatrix} = \rho(T) \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}$$

representation of generators

$$\rho(S) = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \qquad \rho(T) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$(\rho(S))^2 = \mathbb{I}, \qquad (\rho(S)\rho(T))^3 = \mathbb{I}, \qquad (\rho(T))^2 = \mathbb{I}$$

$$Y_1(\alpha, \beta, \gamma) \sim Y(1, 1, -2) \qquad Y_2(\alpha, \beta, \gamma) \sim Y(1, -1, 0)$$

$$\left.\begin{aligned} Y_1(\tau) &= \frac{i}{4\pi}\left(\frac{\eta'(\tau/2)}{\eta(\tau/2)} + \frac{\eta'((\tau+1)/2)}{\eta((\tau+1)/2)} - \frac{8\eta'(2\tau)}{\eta(2\tau)}\right) \\ Y_2(\tau) &= \frac{\sqrt{3}i}{4\pi}\left(\frac{\eta'(\tau/2)}{\eta(\tau/2)} - \frac{\eta'((\tau+1)/2)}{\eta((\tau+1)/2)}\right), \end{aligned}\right\} \text{doublet of S3: } Y$$

Model Building in the Simplest Case: $\Gamma_2 \sim S_3$

3. Combine modular forms and fields:

Matteo Parriciatu & DM,
JHEP 09 (2023) 043, 2306.09028 [hep-ph]

$$\mathcal{W}_e^M = \alpha E_1^c H_d (D_\ell Y_{\mathbf{2}}^{(2)})_{\mathbf{1}} + \beta E_2^c H_d (D_\ell Y_{\mathbf{2}})_{\mathbf{1}'} + \gamma E_3^c H_d \ell_3$$

modular forms of weight 4

$$M_\ell = \begin{pmatrix} \alpha(Y_{\mathbf{2}}^{(2)})_1 & \alpha(Y_{\mathbf{2}}^{(2)})_2 & 0 \\ \beta Y_2 & -\beta Y_1 & 0 \\ 0 & 0 & \gamma \end{pmatrix} v_d$$

$$m_e = v_d \alpha \left(|Y_1^2| - \frac{7}{2}|Y_1^2||\epsilon|^2 + \mathcal{O}(\epsilon^3) \right)$$

$$m_\mu = v_d \alpha \left(\frac{\beta}{\alpha}|Y_1| + \frac{1}{2}\frac{\beta}{\alpha}|Y_1||\epsilon|^2 + \mathcal{O}(\epsilon^3) \right)$$

$$m_\tau = v_d \alpha \left(\frac{\gamma}{\alpha} \right).$$

for $|Y_1| \sim 7/100$

$$\boxed{(m_\tau, m_\mu, m_e) \sim m_\tau (1, |Y_1|, |Y_1|^2).}$$

Mass hierarchy scaling naturally reproduced ! (no fit so far...)

Model Building in the Simplest Case: $\Gamma_2 \sim S_3$

Ready for Neutrinos: key ingredient is to fix k_l to generate *Weinberg* operators

several possible choices. The best one gives (k_l=2):

$$m_\nu^{k_\ell=2} = \frac{2gv_u^2}{\Lambda}\left[\begin{pmatrix} -(Y_2^2-Y_1^2) & 2Y_1Y_2 & \frac{g'}{2g}2Y_1Y_2 \\ 2Y_1Y_2 & (Y_2^2-Y_1^2) & -\frac{g'}{2g}(Y_2^2-Y_1^2) \\ \frac{g'}{2g}2Y_1Y_2 & -\frac{g'}{2g}(Y_2^2-Y_1^2) & 0 \end{pmatrix} + \right.$$
$$\left. + \begin{pmatrix} \frac{g''}{g}(Y_1^2+Y_2^2) & 0 & 0 \\ 0 & \frac{g''}{g}(Y_1^2+Y_2^2) & 0 \\ 0 & 0 & \frac{g_p}{g}(Y_1^2+Y_2^2) \end{pmatrix}\right]$$

Independent parameters: Re(τ), Im(τ), β/α , γ/α , g'/g, g''/g, g_p/g

Model Building in the Simplest Case: $\Gamma_2 \sim S_3$

Numerical fit

Mass matrices against the experimental data

data

$r \equiv \Delta m^2_{\rm sol}/\lvert\Delta m^2_{\rm atm}\rvert$	0.0296 ± 0.0008
$\sin^2\theta_{12}$	$0.303^{+0.013}_{-0.013}$
$\sin^2\theta_{13}$	$0.0223^{+0.0007}_{-0.0006}$
$\sin^2\theta_{23}$	$0.455^{+0.018}_{-0.015}$
m_e/m_μ	0.0048 ± 0.0002
m_μ/m_τ	0.0565 ± 0.0045

$\chi^2 \sim$ O(0.1)

fit results

$\mathrm{Re}\,\tau$	$\pm 0.0895^{+0.0032}_{-0.0055}$
$\mathrm{Im}\,\tau$	$1.697^{+0.023}_{-0.016}$
β/α	$14.33^{+0.58}_{-0.38}$
γ/α	$17.39^{+1.38}_{-0.87}$
g'/g	$31.57^{+27.59}_{-10.29}$
g''/g	$7.17^{+6.36}_{-2.36}$
g_p/g	$8.51^{+7.99}_{-3.03}$

predictions

Ordering	**NO**
δ/π	$\pm 1.594^{+0.007}_{-0.010}$
m_1 [eV]	$0.0182^{+0.0018}_{-0.0014}$
m_2 [eV]	$0.0201^{+0.0017}_{-0.0013}$
m_3 [eV]	$0.0537^{+0.0006}_{-0.0005}$
$\sum_i m_i$ [eV]	$0.092^{+0.004}_{-0.003}$
$\lvert m_{\beta\beta}\rvert$ [meV]	$18.89^{+1.90}_{-1.47}$
$m_\beta^{\rm eff}$ [meV]	$20.26^{+1.69}_{-1.30}$
α_1/π	$\pm 1.124^{+0.014}_{-0.017}$
α_2/π	$\pm 0.949^{+0.005}_{-0.005}$

!!!

Origin of modular symmetry

Two periods in complex functions f : C → C

elliptic function: $$f(z+\omega_1) = f(z+\omega_2) = f(z)$$

periods ∈ C such that $\omega 2/\omega 1 \notin \Re$

a lattice Λ can be generated in the complex plane, spanned by the two directions ω_1, ω_2

$$\Lambda = \mathbb{Z}\omega_1 \oplus \mathbb{Z}\omega_2 = \{n_1\omega_1 + n_2\omega_2 \,|\, n_1, n_2 \in \mathbb{Z}\}$$

elliptic functions are translation-invariant in this lattice: f (z + λ) = f (z) for λ ∈ Λ

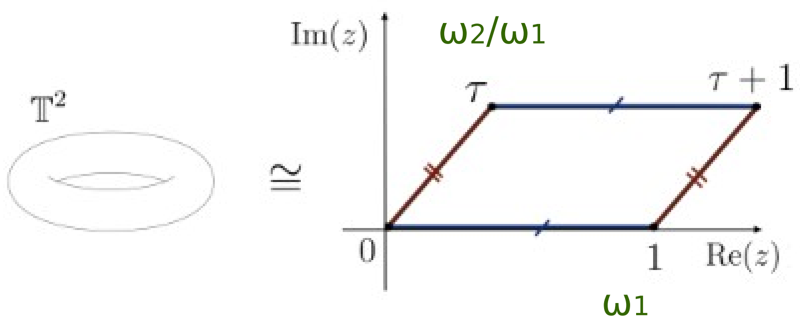
Thus, an elliptic function is single-valued on the quotient C/Λ, which is topologically known as a torus (T_2).

Origin of modular symmetry

Rescaling of the periods:

$\omega_1 = 1$ and $\omega_2/\omega_1 = \tau$, where τ is called the *modulus*

the torus is represented by a parallelogram with vertices $z = 0$, $z = 1$, $z = \tau$ and $z = \tau + 1$ where the opposite sides are pairwise identified



courtesy by Matteo Parriciatu, Master Thesis

The lattice Λ can be equivalently described by a different basis (ω'_1, ω'_2) related to the old one by a linear map with integer parameters:

$$\begin{pmatrix} \omega'_2 \\ \omega'_1 \end{pmatrix} = \gamma \begin{pmatrix} \omega_2 \\ \omega_1 \end{pmatrix} \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \omega_2 \\ \omega_1 \end{pmatrix} \quad , \qquad a, b, c, d \in \mathbb{Z}$$

EDM

Eur. Phys. J. C (2024) 84:1329

$\lvert d_e \rvert$	$< 4.1 \times 10^{-30}$ e cm [18]	$\mathrm{Im}\,[\mathcal{C}'_{\substack{e\gamma\\ ee}}] < 1.8 \times 10^{-13}$
$\lvert d_\mu \rvert$	$< 1.80 \times 10^{-19}$ e cm [22]	$\mathrm{Im}\,[\mathcal{C}'_{\substack{e\gamma\\ \mu\mu}}] < 7.9 \times 10^{-3}$
$\lvert d_\tau \rvert$	$< 1.85 \times 10^{-17}$ e cm [23]	$\mathrm{Im}\,[\mathcal{C}'_{\substack{e\gamma\\ \tau\tau}}] < 8.2 \times 10^{-1}$

$$\mathcal{O}_{\substack{e\gamma\\ RL}} = \frac{v}{\sqrt{2}} \overline{E}_R \sigma^{\mu\nu} E_L F_{\mu\nu}, \qquad \mathcal{C}'_{\substack{e\gamma\\ RL}} = \begin{pmatrix} \mathcal{C}'_{\substack{e\gamma\\ ee}} & \mathcal{C}'_{\substack{e\gamma\\ e\mu}} & \mathcal{C}'_{\substack{e\gamma\\ e\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\ \mu e}} & \mathcal{C}'_{\substack{e\gamma\\ \mu\mu}} & \mathcal{C}'_{\substack{e\gamma\\ \mu\tau}} \\ \mathcal{C}'_{\substack{e\gamma\\ \tau e}} & \mathcal{C}'_{\substack{e\gamma\\ \tau\mu}} & \mathcal{C}'_{\substack{e\gamma\\ \tau\tau}} \end{pmatrix},$$

$$\mathcal{O}_{\substack{e\gamma\\ LR}} = \frac{v}{\sqrt{2}} \overline{E}_L \sigma^{\mu\nu} E_R F_{\mu\nu}, \qquad \mathcal{C}'_{\substack{e\gamma\\ LR}} = \mathcal{C}'^{\dagger}_{\substack{e\gamma\\ RL}}, \tag{22}$$

same structure of CL in Meloni-Parriciatu model

$$\Rightarrow \begin{pmatrix} -\tilde{\alpha}'_1(1-144\epsilon p) & 16\sqrt{3}\tilde{\alpha}'_1\sqrt{\epsilon}\,p' & 0 \\ 8\sqrt{3}\tilde{\beta}'_1\sqrt{\epsilon}\,p' & -\tilde{\beta}'_1(1+24\epsilon p) & 0 \\ 0 & 0 & \gamma'_1 \end{pmatrix}_{RL}$$

bounds of free parameters