Light quark and hadron properties from lattice QCD at the physical point

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Motivation

- Verify that QCD is theory of strong interaction at low energies $\leftrightarrow$ verify the validity of the computational framework
  - $\rightarrow$ light hadron masses (BMWc, Science 322 (2008))
  - $\rightarrow$ hadron widths
  - $\rightarrow$ look for exotics
  - $\rightarrow$ ...
- Fix fundamental parameters and help search for new physics
  - $\rightarrow$ $m_u, m_d, m_s, \ldots$ (BMWc, PLB 701 (2011); JHEP 1108 (2011))
  - $\rightarrow$ $\langle N|m_q\bar{q}q|N\rangle$, $q = u, d, s$ for dark matter (BMWc, PRD 85 (2012))
  - $\rightarrow$ $F_K/F_\pi \leftrightarrow \frac{G_q}{G_\mu}\left[|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2\right] = 1$? (BMWc, PRD 81 (2010))
  - $\rightarrow$ $B_K \leftrightarrow$ consistency of CPV in K and B decays ? (BMWc, PLB 705 (2011))
  - $\rightarrow$ ...
- Make predictions in nuclear physics?
- Full description of low energy particle physics $\rightarrow$ include QED

Today's triple feature

A CLASSIC

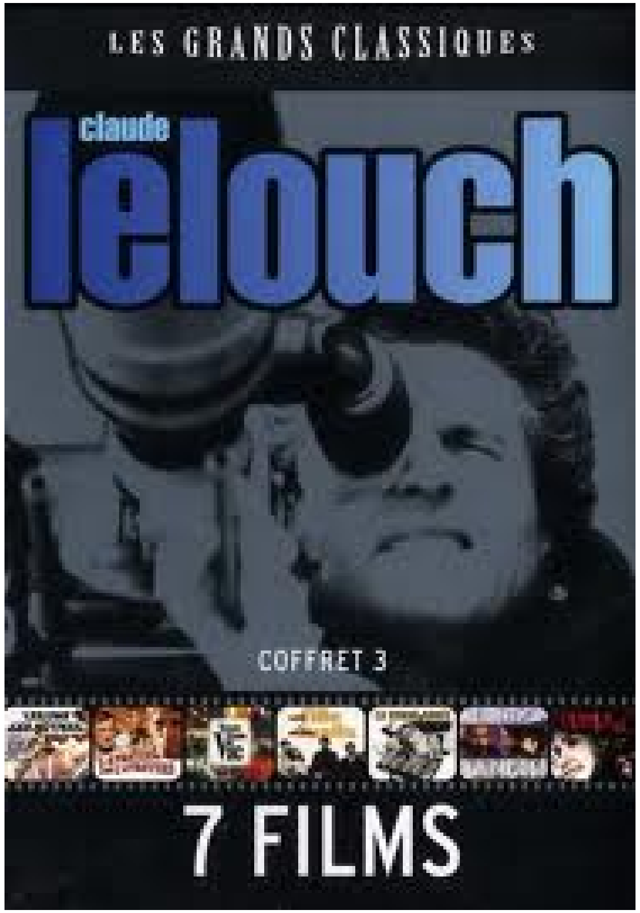
Ab initio determination of light hadron masses

Dürr et al (BMWc), Science 322 '08

LA NOUVELLE VAGUE

Lattice QCD at the physical point: light quark masses

Dürr et al (BMWc), PLB 701 '11 & JHEP 1108 '11



AN INDIE

Precision computation of the kaon bag parameter

Dürr et al (BMWc), PLB 705 '11

Why do we need lattice QCD (LQCD)?

- QCD fundamental d.o.f.: q and g
- QCD observed d.o.f.: p, n, π, K, ...
  - q and g are permanently confined w/in hadrons
  - hadrons hugely different from d.o.f. present in the Lagrangian

$\Rightarrow$ perturbation in α_s has no chance

$\Rightarrow$ Need a tool to solve low energy QCD:

- to compute hadronic and nuclear properties
- to subtract "parasitical" hadronic contributions to low energy observables important for uncovering new fundamental physics

$\longrightarrow$ numerical lattice QCD

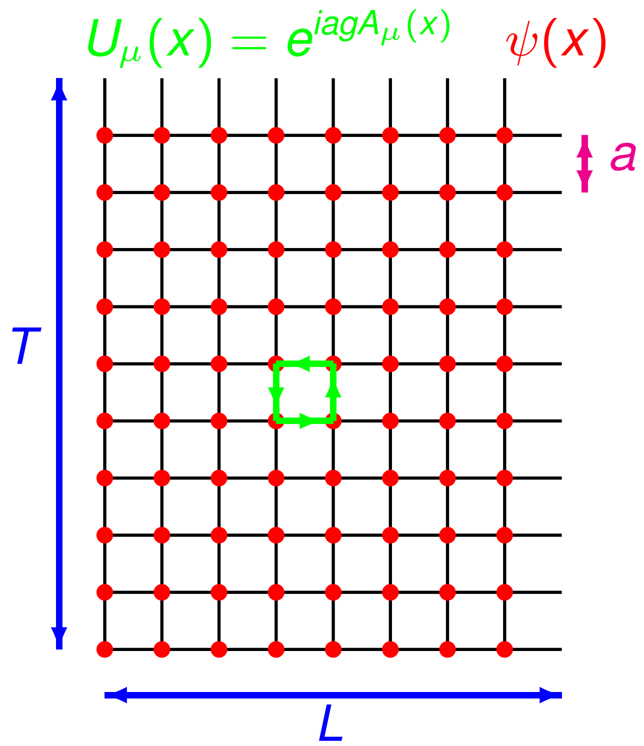
What is lattice QCD?

Lattice gauge theory $\longrightarrow$ mathematically sound definition of NP QCD:

- UV (and IR) cutoffs and a well defined path integral in Euclidean spacetime:

$$\langle O \rangle = \int \mathcal{D}U\mathcal{D}\bar\psi\mathcal{D}\psi\, e^{-S_G-\int\bar\psi D[M]\psi}\, O[U,\psi,\bar\psi]$$
$$= \int \mathcal{D}U\, e^{-S_G}\,\mathrm{det}(D[M])\, O[U]_{\mathrm{Wick}}$$

- $\mathcal{D}Ue^{-S_G}\,\mathrm{det}(D[M]) \geq 0$ and finite # of dof's $\rightarrow$ evaluate numerically using stochastic methods



NOT A MODEL: LQCD is QCD when $a \to 0$, $V \to \infty$ and stats $\to \infty$

In practice, limitations . . .

Challenge

Minimize and control all systematics

- ☞ compute hugely expensive determinant of $O(10^9 \times 10^9)$ fermion matrix
- ☞ fight fast increasing cost of simulations as:
  - $m_{ud} \searrow m_{ud}^{\mathrm{ph}} \Rightarrow$ reach physical mass point in controlled way
  - $a \searrow 0 \Rightarrow$ controlled continuum extrapolation
  - $L \to \infty \Rightarrow$ controlled infinite volume extrapolation
- ☞ nonperturbative renormalization
  $\Rightarrow$ eliminate all perturbative uncertainties

$\Rightarrow$ only then, true nonperturbative QCD predictions

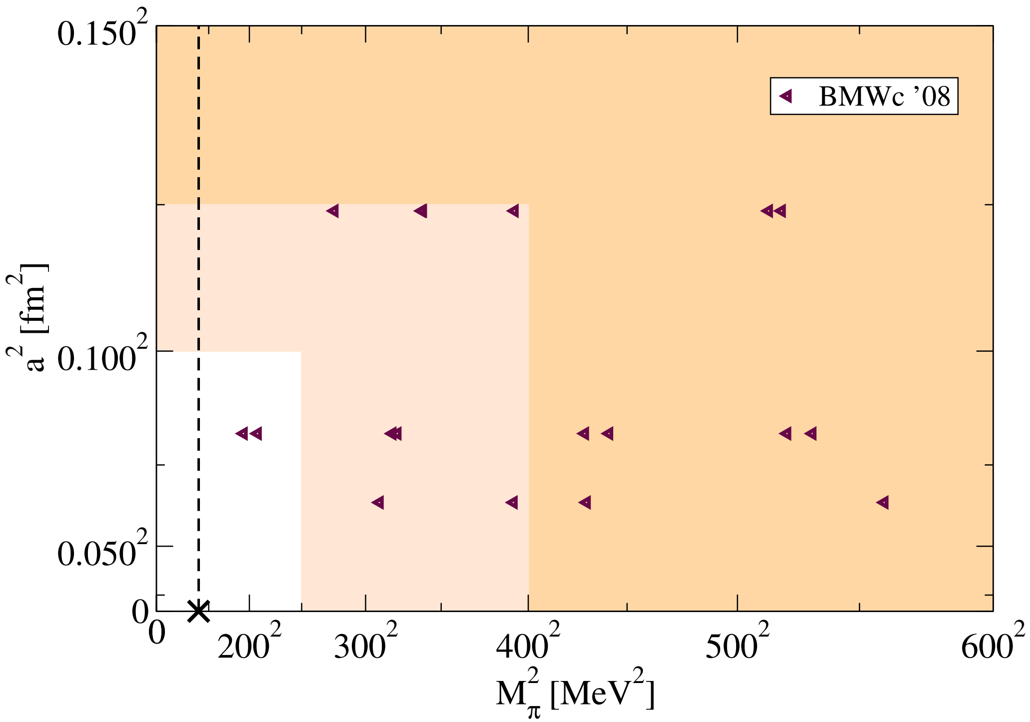
The calculation that I've been dreaming of doing

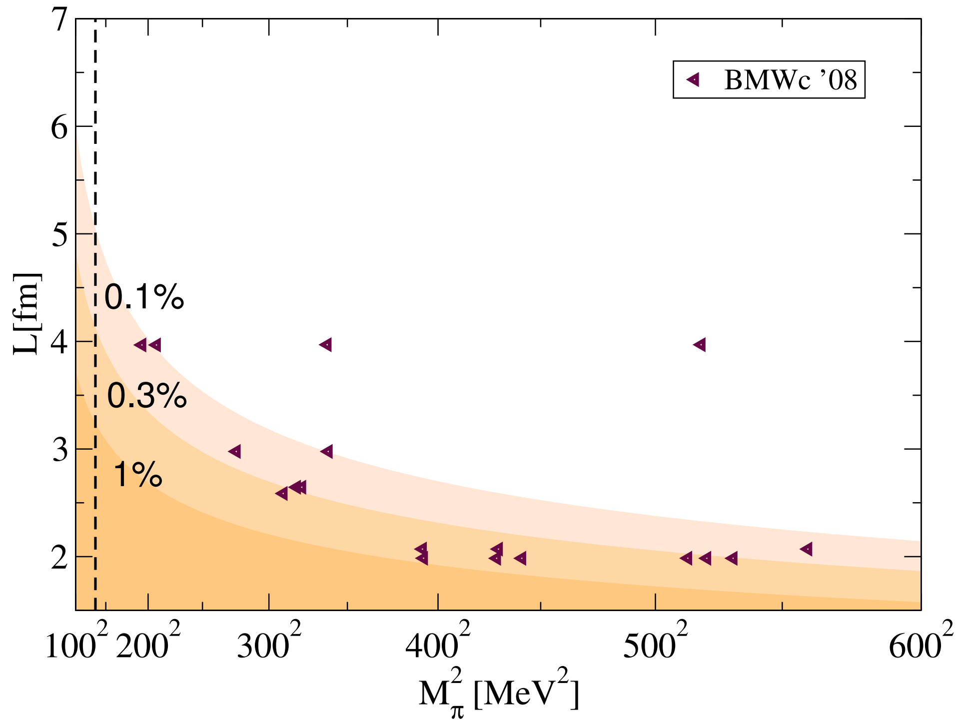
- $N_f = 2 + 1$ simulations to include u, d and s sea quark effects
- Simulations all the way down to $M_\pi \lesssim 135\,\mathrm{MeV}$ to allow small interpolation to physical mass point
- Large $L \gtrsim 5\,\mathrm{fm}$ to have sub-percent finite V errors
- At least three $a \lesssim 0.1\,\mathrm{fm}$ for controlled continuum limit
- Reliable determination of the scale w/ a well measured physical observable
- Unitary, local gauge and fermion actions
- Full nonperturbative renormalization and nonperturbative continuum running if necessary
- Complete analysis of systematic uncertainties

Dürr et al (BMWc) Science 322 '08, PRD79 '09

20 large scale $N_f = 2+1$ Wilson fermion simulations

$M_\pi \gtrsim 190\,\mathrm{MeV}$ $a \approx 0.065, 0.085, 0.125\,\mathrm{fm}$ $L \to 4\ fm$





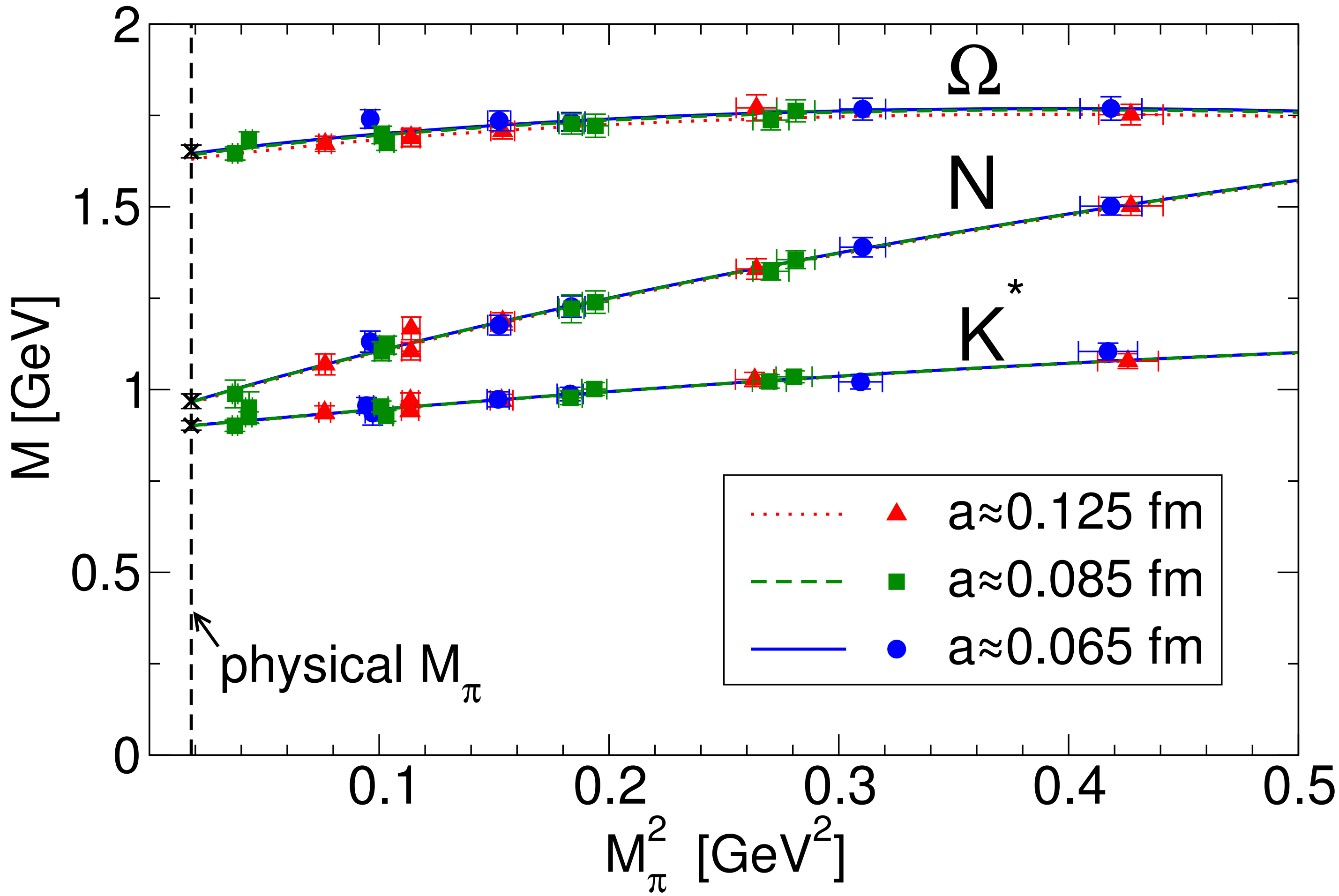
Good enough for *ab initio* calculation of light hadron masses

Computation of light hadron masses: motivations

Dürr et al (BMWc) Science 322 '08

- $>$ 99% of mass of visible universe is in the form of p & n
- Only $<$5% comes from mass of u and d constituents
- Important to verify that asymptotically free QCD generates this mass deficit in a way consistent w/ experiment
- Validate lattice QCD tools used
  $\Rightarrow$ reliable predictions
- Want QCD *not* lattice QCD results
  $\Rightarrow$ all necessary limits must be taken cleanly

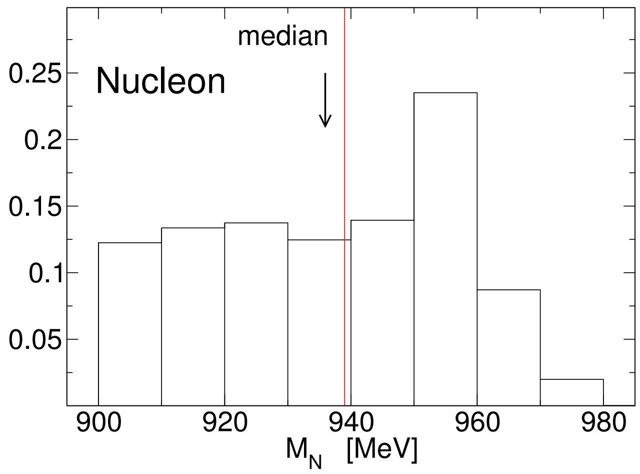
Example combined mass and continuum extrapolation

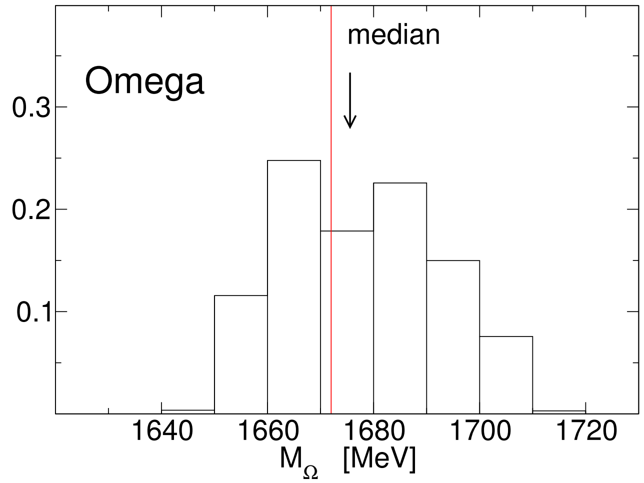


Extrapolated results very close to lightest points/small *a*-dependence
⇒ extrapolations fully controlled

Systematic and statistical error estimate

- Correct treatment of resonances (Lüscher, '85-'91)
- 432 distinct analyses for each hadron mass corresponding to different choices for: $a \searrow 0$, $M_\pi \searrow 135\,\mathrm{MeV}$, $L \nearrow \infty$, ...
- Weigh each one by fit quality $\rightarrow$ systematic error distribution
- repeat for 2000 bootstrap samples

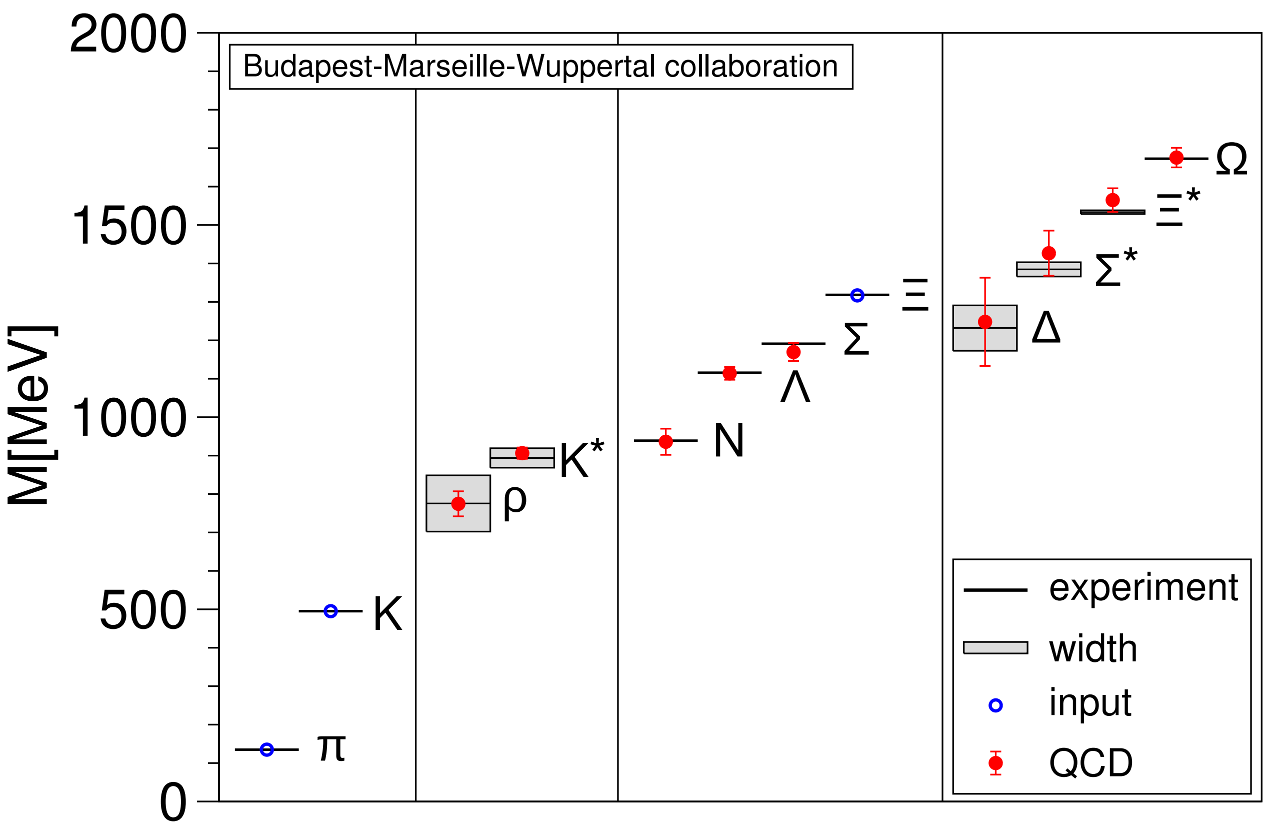




- Median $\rightarrow$ central value
- Central 68% CI $\rightarrow$ systematic error

- Central 68% CI of bootstrap distribution of medians $\rightarrow$ statistical error

Postdiction of the light hadron masses, etc.



(Partial calculations by MILC '04-'09, RBC-UKQCD '07, Del Debbio et al '07, JLQCD '07, QCDSF '07-'09, Walker-Loud et al '08, PACS-CS '08-'10, ETM '09, Gattringer et al '09, . . .)

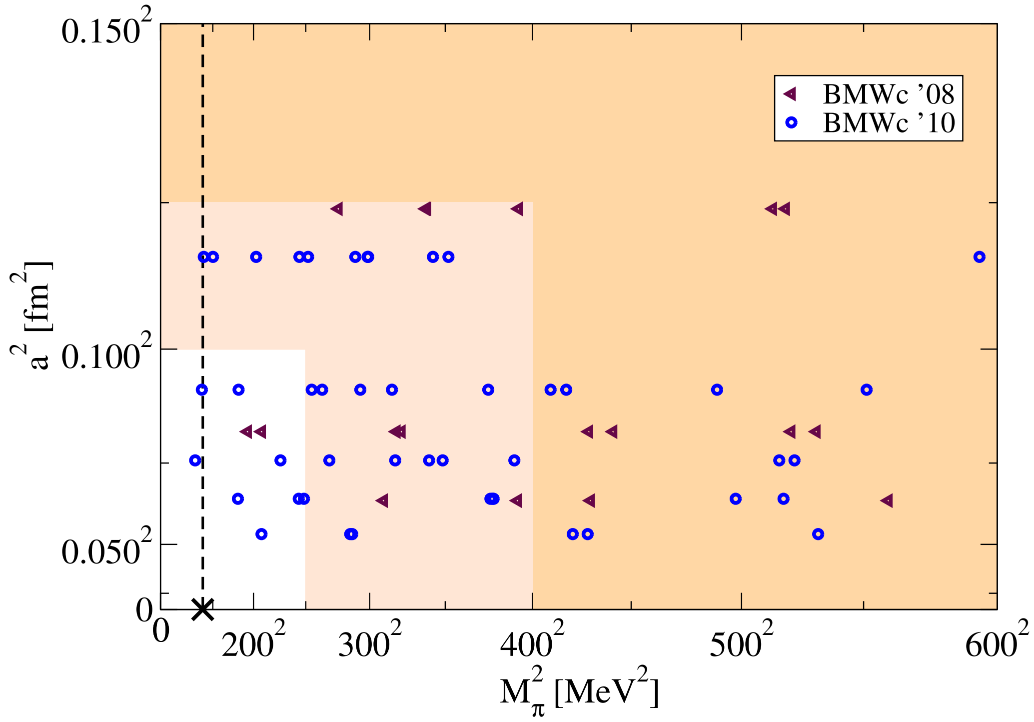
Other results on BMWc '08: F_K/F_π and 1st row CKM unitarity (BMWc, PRD81 '10), $\langle N|m_q\bar{q}q|N\rangle$, $q = u, d, s$ for dark matter (BMWc, PRD85 '12)

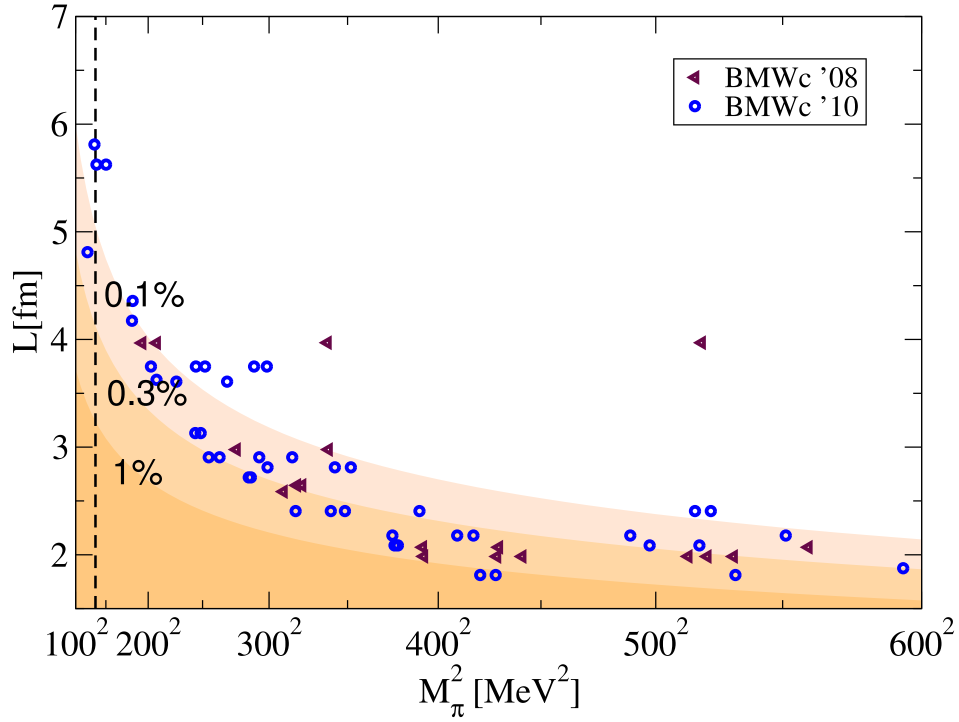
Dream comes true in 2010

Dürr et al (BMWc) PLB 701 (2011); JHEP 1108 (2011)

47 large scale $N_f = 2 + 1$ Wilson fermion simulations

$M_\pi \gtrsim 120\,\mathrm{MeV}$ $\quad$ $5a$'s $\approx 0.054 \div 0.116\,\mathrm{fm}$ $\quad$ $L \to 6\,fm$





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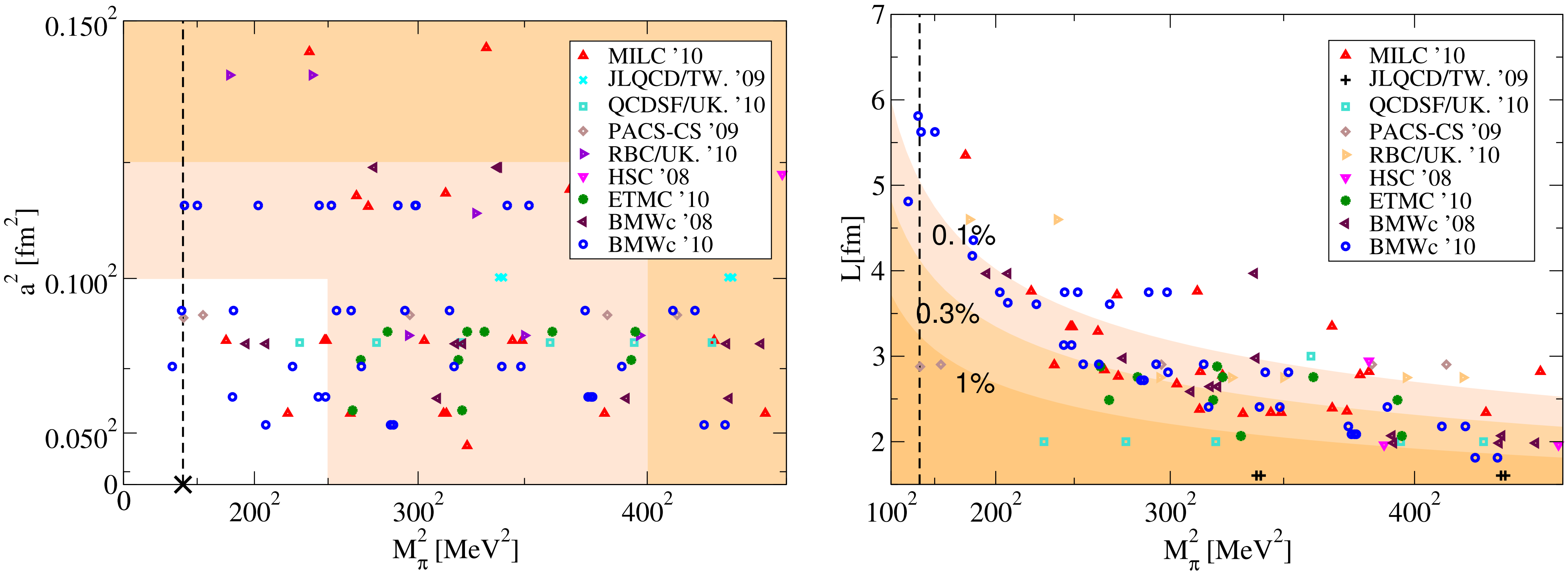


Still only ones there! (only $N_f \geq 2+1$ simulations are shown)

Action and algorithm details

Dürr et al (BMWc), PRD79 '09, Science 322 '08, JHEP 1108 '11

$N_f = 2 + 1$ QCD: degenerate u & d w/ mass m_{ud} and s quark w/ mass $m_s \sim m_s^{\rm phys}$

1) Conceptually clean discretization which balances improvement and CPU cost:

- tree-level $O(a^2)$-improved gauge action (Lüscher et al '85)
- tree-level $O(a)$-improved Wilson fermion (Sheikholeslami et al '85) w/ 2 HEX smearing (Morningstar et al '04, Hasenfratz et al '01, Capitani et al '06)
  $\Rightarrow O(\alpha_s a, a^2)$ instead of $O(a)$

2) Highly optimized algorithms (see also Urbach et al '06):

- HMC for u and d and RHMC for s
- mass preconditioning (Hasenbusch '01)
- multiple timescale integration of MD (Sexton et al '92)
- higher-order (Omelyan) integrator for MD (Takaishi et al '06)
- mixed precision acceleration of inverters via iterative refinement

3) Highly optimized codes

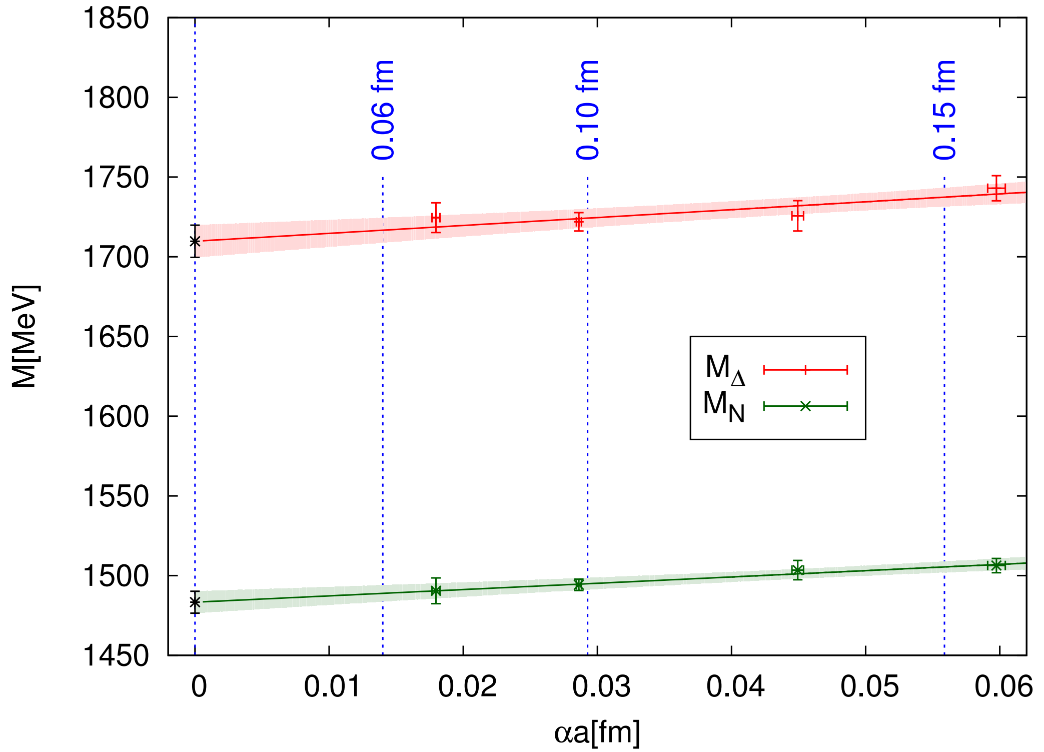
4) Rigorous battery of tests: algorithm stability, autocorrelations, scaling, finite-V, ...

Scaling study

$N_f = 3$ w/ 2 HEX action, 4 lattice spacings ($a \simeq 0.06 \div 0.15\mathrm{fm}$), $M_\pi L > 4$ fixed and

$$M_\pi / M_\rho = \sqrt{2(M_K^{ph})^2 - (M_\pi^{ph})^2} / M_\phi^{ph} \sim 0.67$$

i.e. $m_q \sim m_s^{ph}$



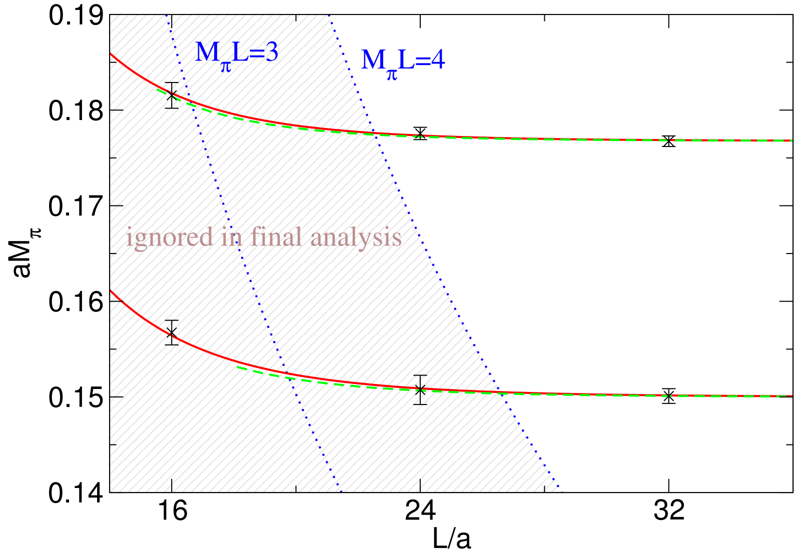
- M_N and M_Δ are linear in $\alpha_s a$ out to $a \sim 0.15\,\mathrm{fm}$

$\Rightarrow$ very good scaling: discretization errors $\lesssim 2\%$ out to $a{\sim}0.15\,\mathrm{fm}$

- Results perfectly consistent w/ analogous 6 stout analysis in BMWc PRD 79 (2009)

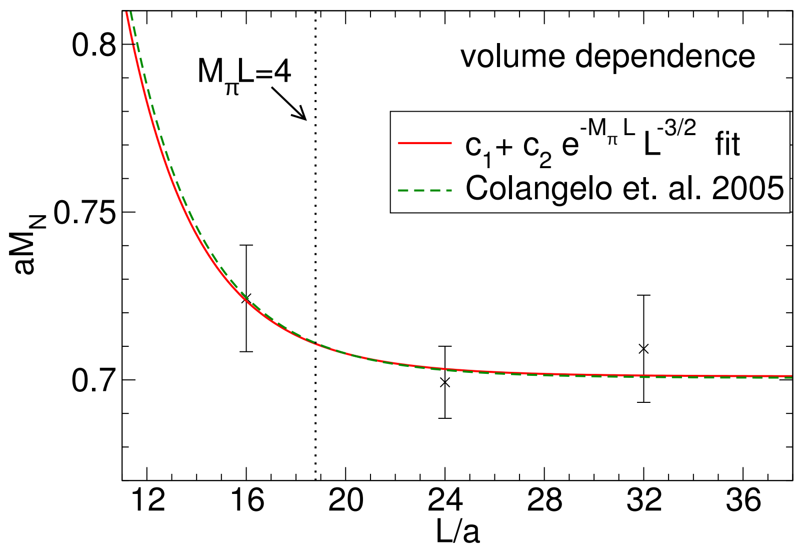
Finite volume studies

- In large volumes $\mathrm{FVE} \sim e^{-M_\pi L}$ [more complicated for resonances (Lüscher '85-'91)
- $M_\pi L \gtrsim 4$ expected to give $L \to \infty$ masses within our statistical errors



2HEX, $a \approx 0.116\,\mathrm{fm}$, $M_\pi \approx 0.25,\ 0.30\,\mathrm{GeV}$,

$M_\pi L = 2.4 \to 5.6$



6STOUT, $a \approx 0.125\,\mathrm{fm}$, $M_\pi \approx 0.33\,\mathrm{GeV}$,

$M_\pi L = 3.5 \to 7$

Well described by (and Colangelo et al, 2005)

$$\frac{M_X(L) - M_X}{M_X} = C \left(\frac{M_\pi}{\pi F_\pi}\right)^2 \frac{1}{(M_\pi L)^{3/2}} e^{-M_\pi L}$$

Very small, for the volumes that we consider

Light quark masses: motivation

Determine m_u, m_d, m_s *ab initio*

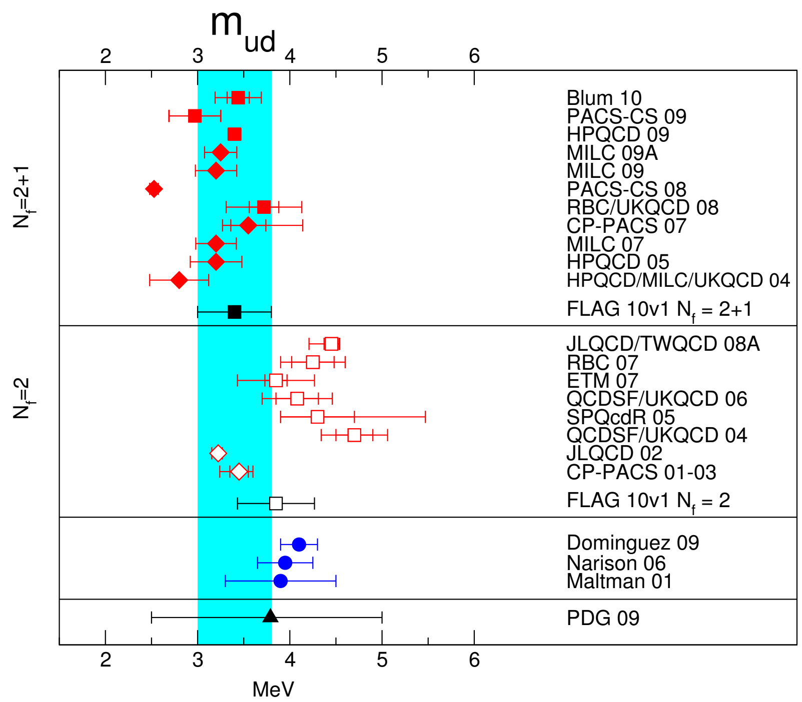
- Fundamental parameters of nature
- Precise values $\rightarrow$ stability of matter, N-N scattering lengths, presence or absence of strong CP violation, etc.
- Information about BSM: theory of fermion masses must reproduce these values
- Nonperturbative (NP) computation is required
- Would be needle in a haystack problem if not for χSB

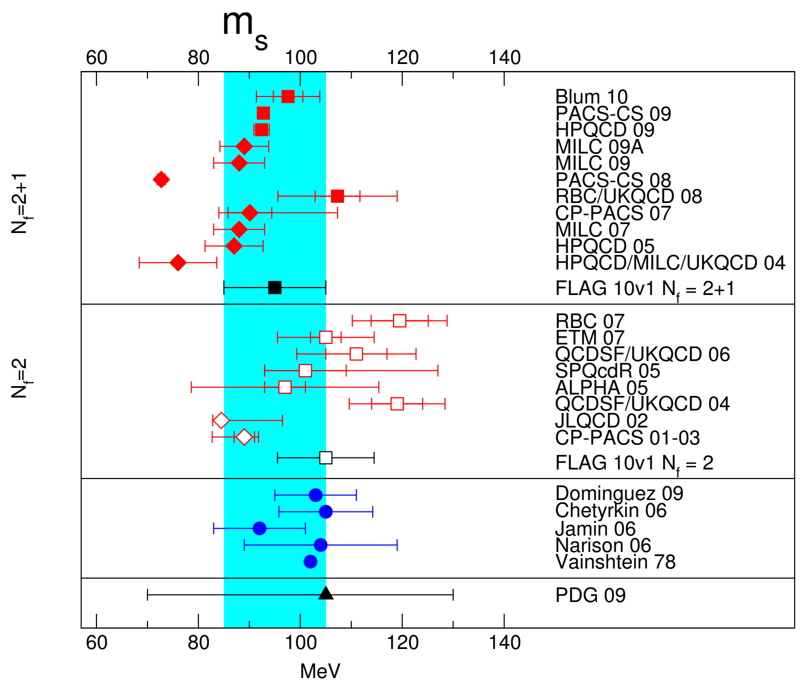
$\Rightarrow$ interesting first "measurement" w/ physical point LQCD

Light quark masses circa Aug. 2010

FLAG $\rightarrow$ analysis of unquenched lattice determinations of light quark masses

(arXiv:1011.4408v1)





$$m_{ud}^{\overline{\rm MS}}(2\,\rm GeV) = \begin{cases} 3.4(4)\,\rm MeV & [12\%] \quad \text{FLAG} \\ 2.5 \div 5.0\,\rm MeV & [30\%] \quad \text{PDG} \end{cases}$$

$$m_{s}^{\overline{\rm MS}}(2\,\rm GeV) = \begin{cases} 95.(10)\,\rm MeV & [11\%] \quad \text{FLAG} \\ 70 \div 130\,\rm MeV & [30\%] \quad \text{PDG} \end{cases}$$

Even extensive study by MILC still has:

- $M_\pi^{\rm RMS} \geq 260\,\rm MeV \quad \Rightarrow \quad m_{ud}^{\rm MILC,eff} \geq 3.7 \times m_{ud}^{\rm phys}$
- perturbative renormalization (albeit 2 loops)

Quark mass definitions

Standard

- Lagrangian mass m^{bare}
- $m^{\text{ren}} = \frac{1}{Z_S}(m^{\text{bare}} - m^{\text{crit}})$

- m^{PCAC} from $\frac{\langle \partial_0 A_0 P \rangle}{\langle P(t) P(0) \rangle}$
- $m^{\text{ren}} = \frac{Z_A}{Z_P} m^{\text{PCAC}}$

Better use ...

- $d = m_s^{\text{bare}} - m_{ud}^{\text{bare}}$
- $d^{\text{ren}} = \frac{1}{Z_S} d$

- $r = m_s^{\text{PCAC}} / m_{ud}^{\text{PCAC}}$
- $r^{\text{ren}} = r$

... and reconstruct

- $m_s^{\text{ren}} = \frac{1}{Z_S} \frac{rd}{r-1}$

- $m_{ud}^{\text{ren}} = \frac{1}{Z_S} \frac{d}{r-1}$

✓ No additive mass renormalization

✓ Only Z_S multiplicative renormalization w/ no pion poles

☞ Use $O(a)$-improved version

Renormalization strategy

Goal

- Convert bare lattice masses to finite renormalized ones . . .
- . . . fully nonperturbatively . . .
- . . . with optional accurate conversion to other schemes

Method

RI/MOM NPR (Martinelli et al '95) w/ $S(p) \to \bar{S}(p) = S(p) - \mathrm{Tr}_D[S(p)]/4$ (Becirevic et al '00)

$$m^{\mathrm{sch}}(\mu') = C^{\mathrm{RI}\to\mathrm{sch,PT}}(\mu') \left\{ [R_S^{\mathrm{RI}}(\mu',\mu) \times Z_S^{\mathrm{RI}}(a\mu, g_0)]^{-1} \times m(g_0) \right\}_{\mathrm{lat}}$$

- $\mu \ll 2\pi/a \sim 11 \div 24\,\mathrm{GeV}$ to reduce disc. errors in $Z_S(a\mu, g_0)$

  ✓ $\mu = 1.3\,\mathrm{GeV}$ ✓ $\mu = 2.1\,\mathrm{GeV}$

- $R_S^{\mathrm{RI}}(\mu',\mu)$, continuum NP running to $\mu' \gg \Lambda_{\mathrm{QCD}}$
- $C^{\mathrm{RI}\to\mathrm{sch,PT}}(\mu')$, optional conversions to other schemes in 4-loop PT
- 21 additional $N_f = 3$ RI/MOM simulations at same 5 β's

RI/MOM nonperturbative renormalization

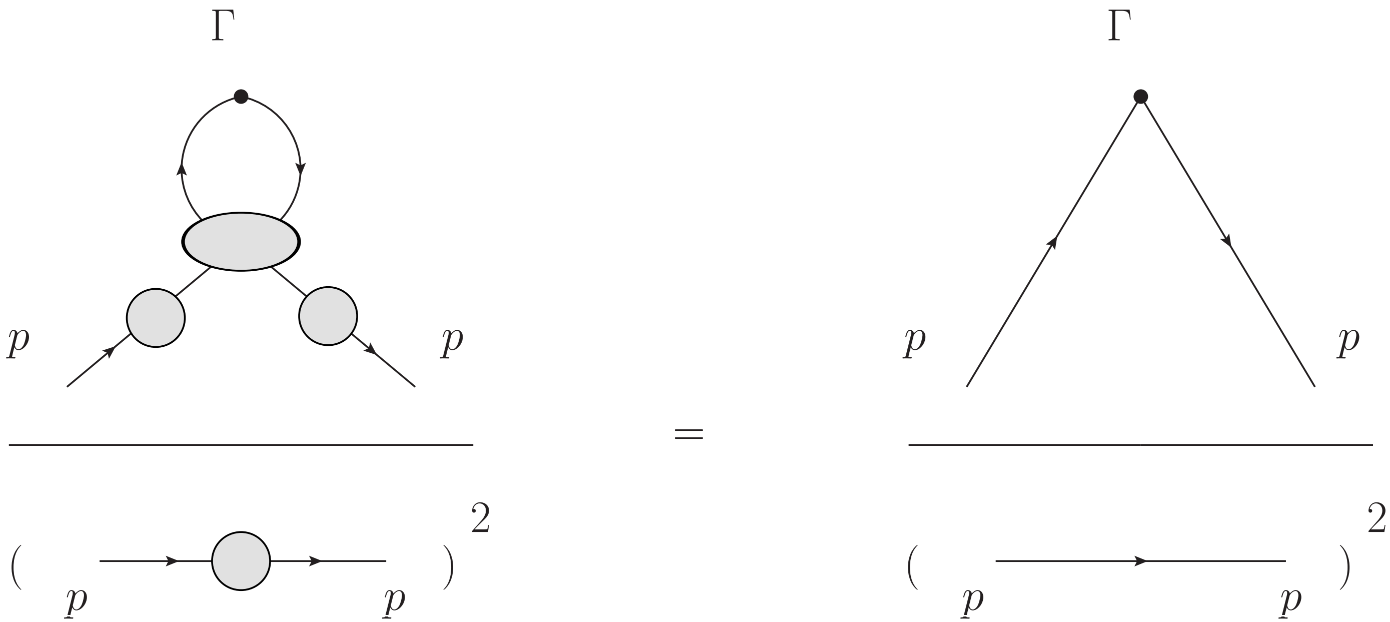
(Martinelli et al ’95)

Have

$$[\bar{q}_1 \Gamma q_2](\mu) = Z_\Gamma^{\rm RI}(a\mu, g_0)[\bar{q}_1 \Gamma q_2](a)$$

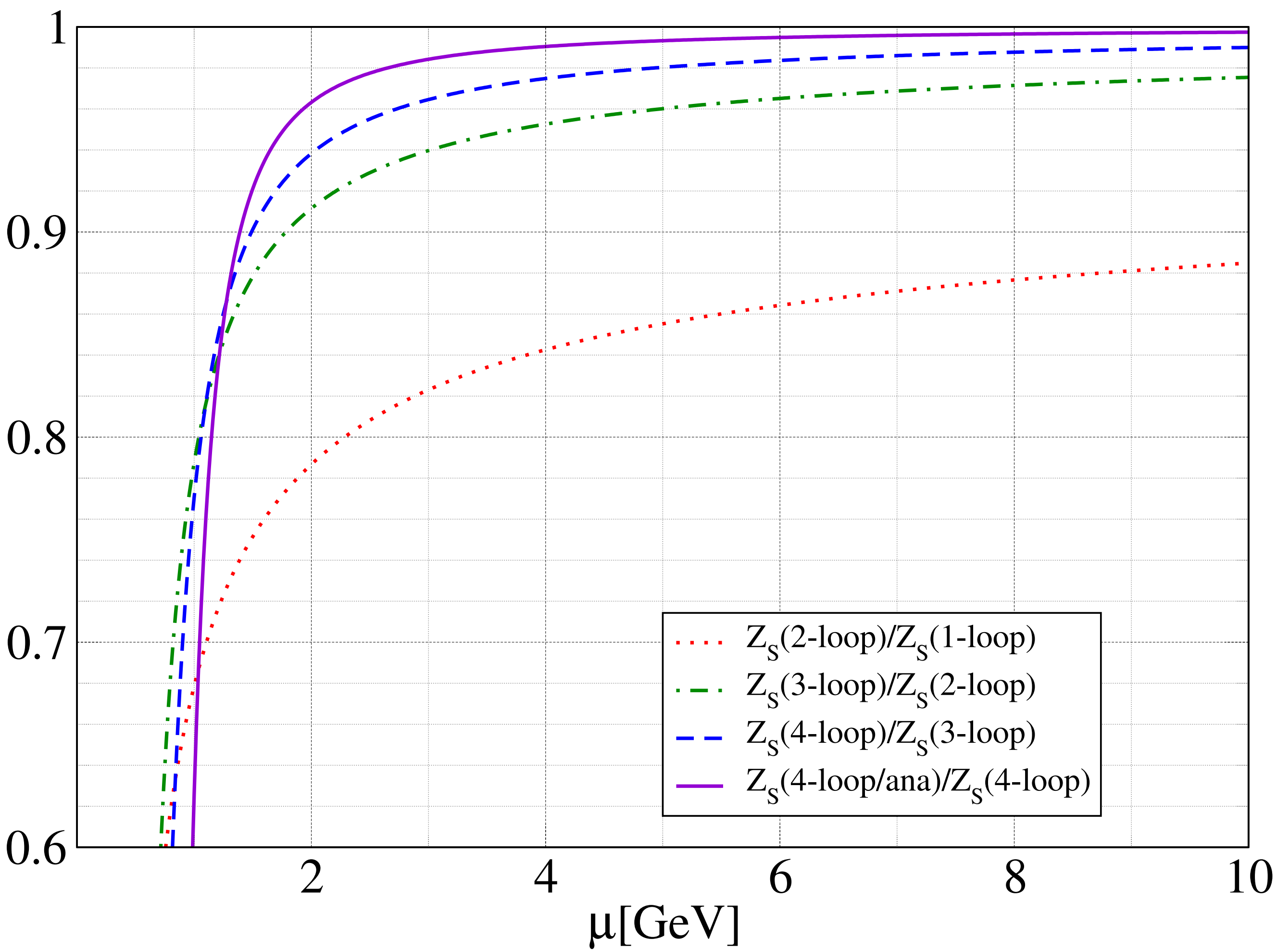
w/ $Z_\Gamma^{\rm RI}$ defined by ratio of quark Green’s fns in Landau gauge

$$\frac{Z_\Gamma^{\rm RI}(ap,g_0)}{Z_q^{\rm RI}(ap,g_0)}$$



Cancel $Z_q^{\rm RI}(a\mu, g_0)$ by normalizing with LHS of conserved vector current

Choice of target RI/MOM scale

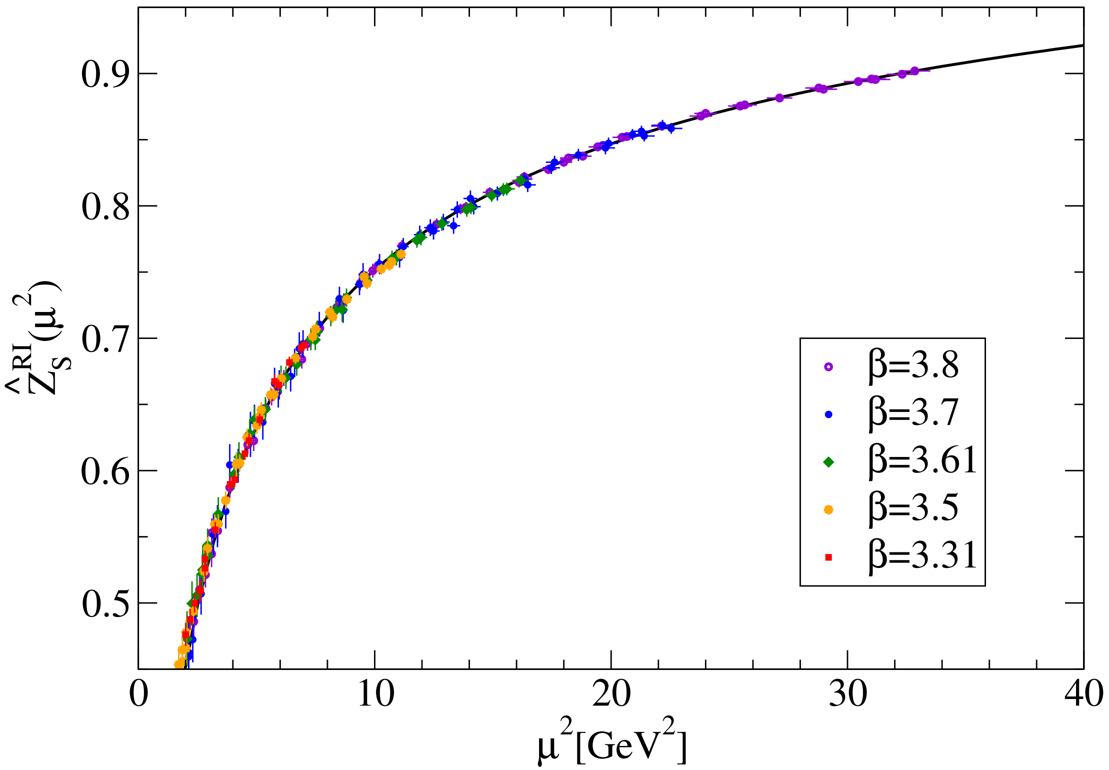


$\Rightarrow \sigma_{\mathrm{PT}} \lesssim 1\%$ for $\mu \gtrsim 4\,\mathrm{GeV}$

Nonperturbative running to 4 GeV

Determine nonperturbative running in continuum limit from (see also Constantinou et al '10, Arthur et al '10)

$$R_S^{\mathrm{RI}}(4\,\mathrm{GeV},\mu) = \lim_{a\to 0} \frac{Z_S^{\mathrm{RI}}(4\,\mathrm{GeV},a)}{Z_S^{\mathrm{RI}}(\mu,a)}$$



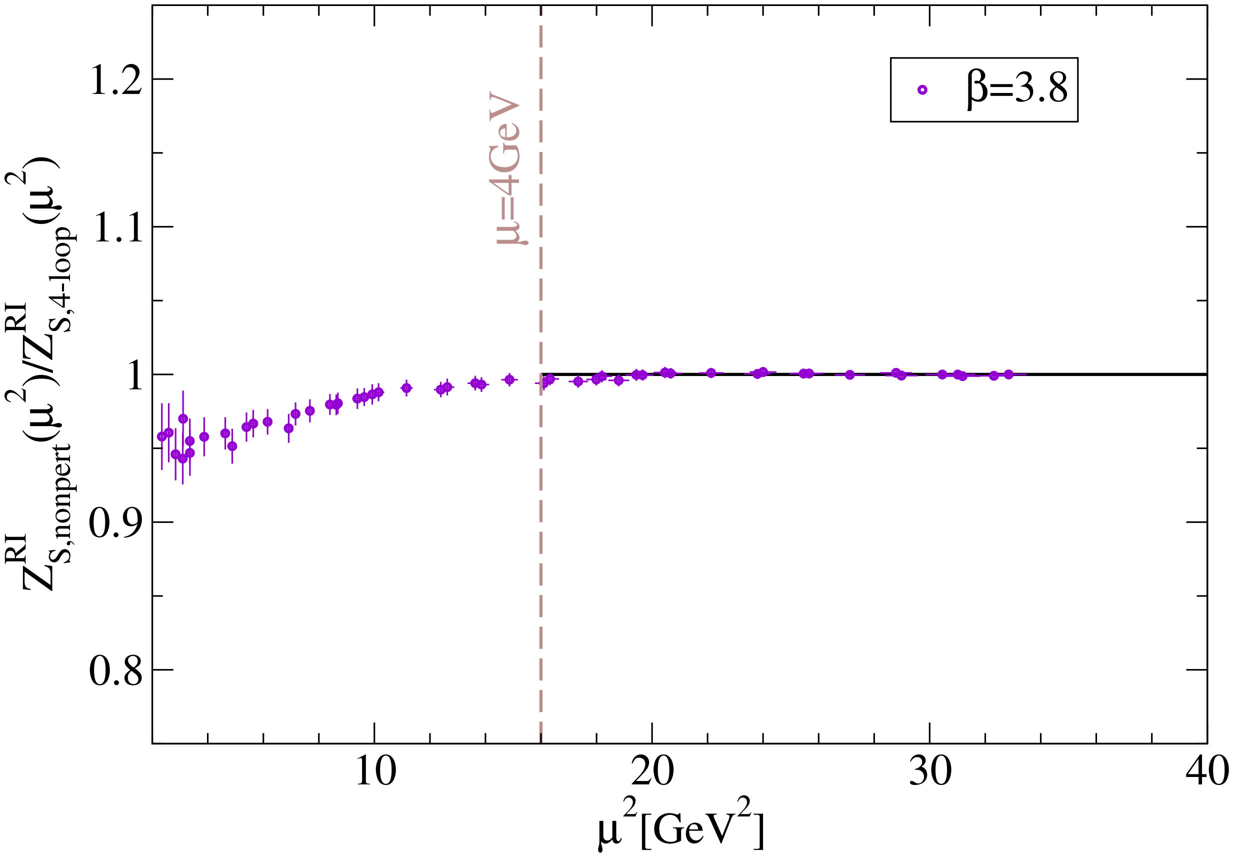
Rescaled $Z_S^{\mathrm{RI}}(a\mu,\beta)$ for $\beta < 3.8$ to $\sim$ match $Z_S^{\mathrm{RI}}(a\mu,\beta=3.8)$

- 3 β up to $\mu = 4\,\mathrm{GeV}$
- Running very similar at all 5 β

$\Rightarrow$ flat and controlled $a \to 0$ extrapolation

Running beyond 4 GeV

For $\mu > 4\,\mathrm{GeV}$, 4-loop PT and NP running agrees on finest lattice



⇒ get RGI masses w/ negligible PT error

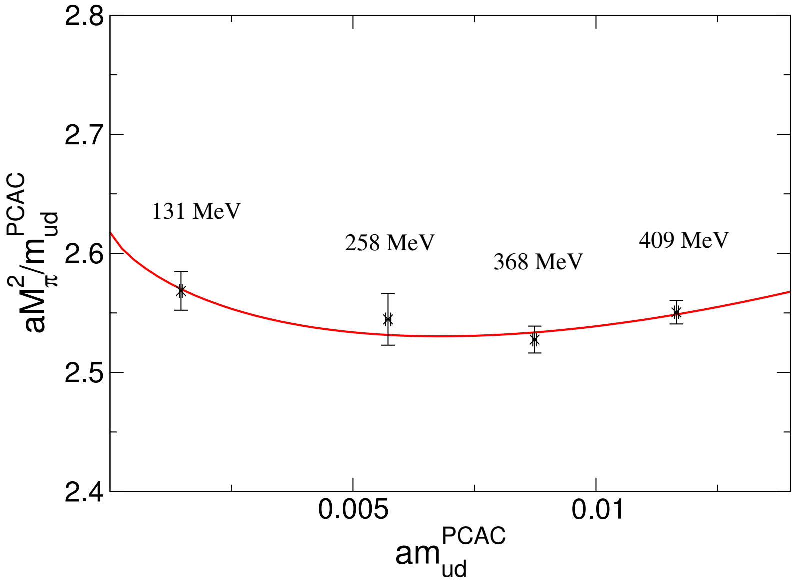
⇒ masses in other schemes w/ only errors proper to that scheme

Combined mass interp. and continuum extrap. (1)

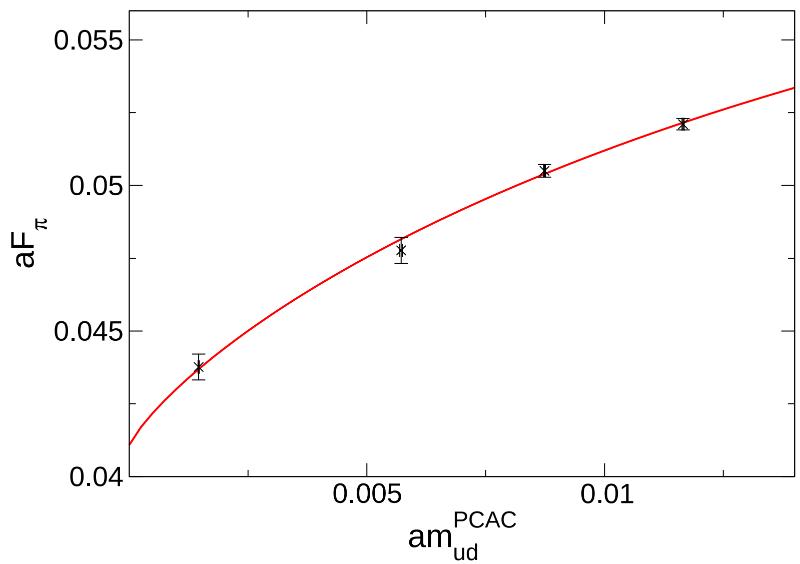
Project onto m_{ud} axis: chiral interpolation to M_π^{ph}

Illustration of chiral behavior

- Fixed $a \simeq 0.09\,\mathrm{fm}$ and $M_\pi \sim 130 \div 410\,\mathrm{MeV}$



- Fit to NLO $SU(2)$ χPT (Gasser et al '84)



✓ Consistent w/ NLO χPT for $M_\pi \lesssim 410\,\mathrm{MeV}$

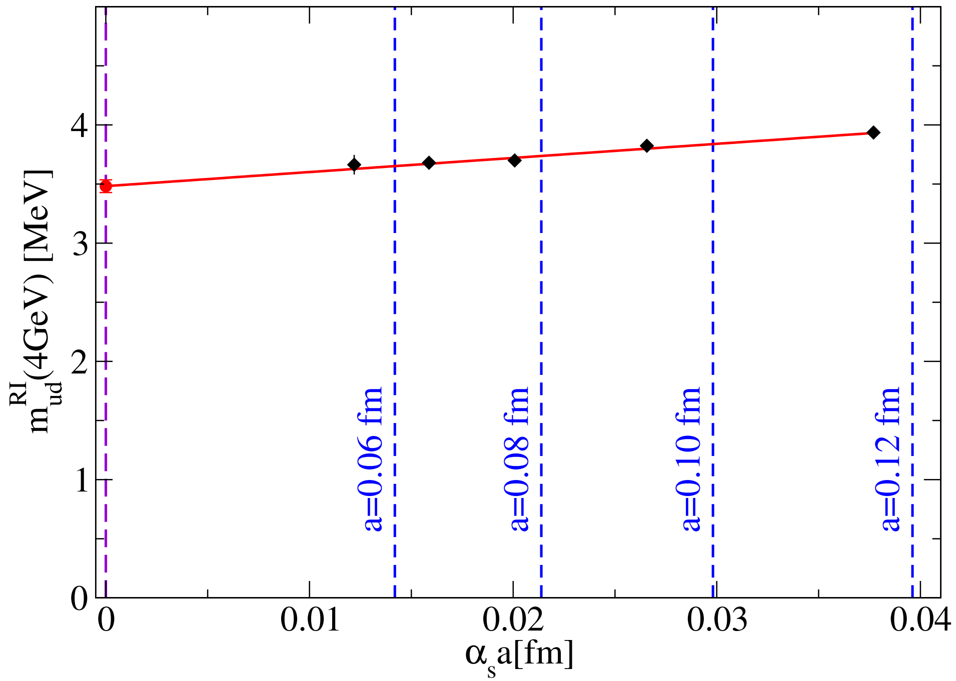
$\Rightarrow$ 2 safe interpolation ranges: $M_\pi < 340, 380\,\mathrm{MeV}$

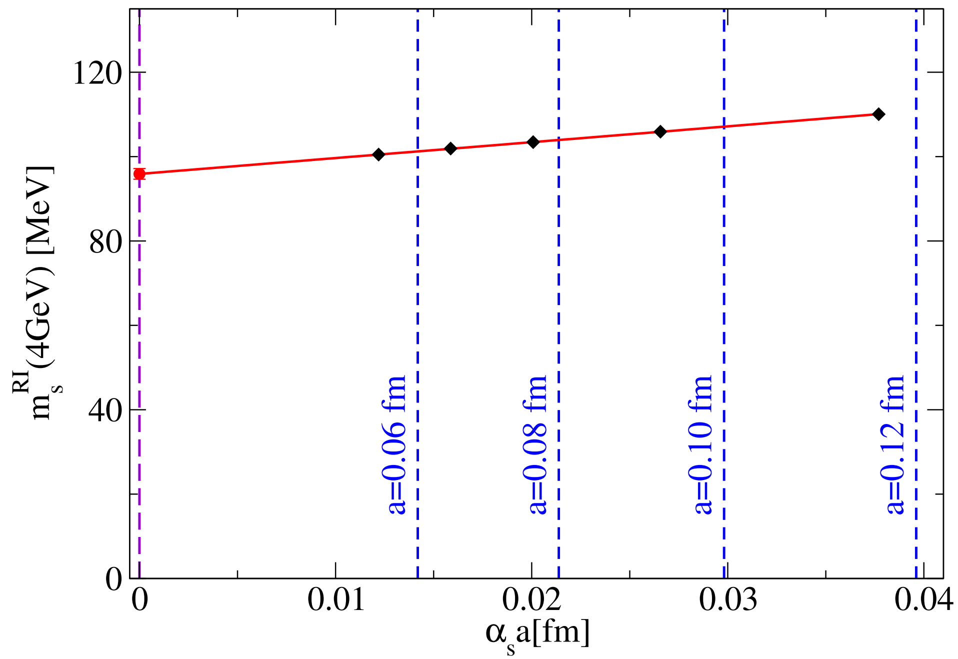
$\Rightarrow$ $SU(2)$ NLO χPT & Taylor interpolations to physical point

Combined mass interp. and continuum extrap. (2)

Project onto *a* axis: continuum extrapolation

- Leading order is $O(\alpha_s a)$
- Allow also domination of sub-leading $O(a^2)$





(continuum extrapolation examples – errors on points are statistical)

$\Rightarrow$ fully controlled continuum limit

Individual m_u and m_d

Calculation performed in isospin limit:

- $m_u = m_d$
- NO QED

$\Rightarrow$ leave *ab initio* realm

- Use dispersive Q from $\eta \to \pi\pi\pi$

$$Q^2 \equiv \frac{m_s^2 - m_{ud}^2}{m_d^2 - m_u^2}$$

☛ Precise m_{ud} and $m_s/m_{ud} \Rightarrow$

$$m_{u/d} = m_{ud}\left\{1 \mp \frac{1}{4Q^2}\left[\left(\frac{m_s}{m_{ud}}\right)^2 - 1\right]\right\}$$

- Use conservative $Q = 22.3(8)$ (Leutwyler '09)

Systematic error treatment

- 288 full analyses on 2000 boostrap samples
  - 2 correlator time fit ranges
  - 3 NPR procedures
  - 2 continuum extrap. forms for NP running
  - 3 chiral interp. forms: $2 \times SU(2)$ χPT, Taylor
  - 2 chiral interp. ranges: $M_\pi < 340, 380\,\mathrm{MeV}$
  - 2 chiral interp. ranges for scale setting channel M_Ω: $M_\pi < 340, 480\,\mathrm{MeV}$
  - 2 continuum forms
- Analyses weighted by fit quality $\Rightarrow$ systematic error distribution
  - Mean $\rightarrow$ final result
  - Std. dev. $\rightarrow$ systematic error
- Statistical error from distribution of means over 2000 samples

Results

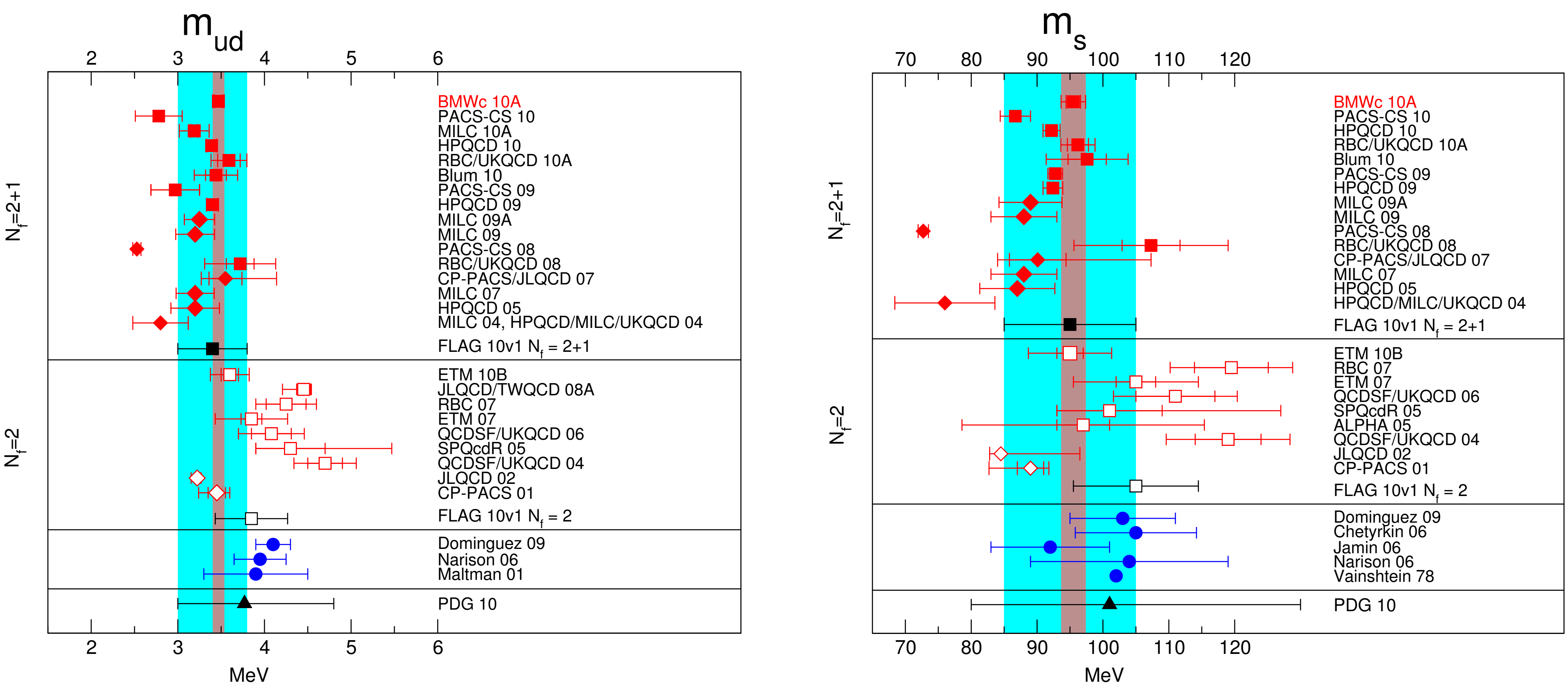
	RI 4 GeV	RGI	$\overline{\text{MS}}$ 2 GeV
m_s	96.4(1.1)(1.5)	127.3(1.5)(1.9)	95.5(1.1)(1.5)
m_{ud}	3.503(48)(49)	4.624(63)(64)	3.469(47)(48)
m_u	2.17(4)(3)(10)	2.86(5)(4)(13)	2.15(4)(3)(9)
m_d	4.84(7)(7)(10)	6.39(9)(9)(13)	4.79(7)(7)(9)

$$\frac{m_s}{m_{ud}} = 27.53(20)(8) \qquad \frac{m_u}{m_d} = 0.449(6)(2)(29)$$

Additional consistency checks

- ✓ Additional continuum, chiral and FV terms
  ☞ all compatible with 0
- ✓ Unweighted final result and systematic error
  ☞ negligible impact
- ✓ Use $m^{\rm PCAC}$ only
  ☞ compatible, slightly larger error
- ✓ Full quenched check of procedure ☞ cf. reference computation (Garden et al '00)

Comparison

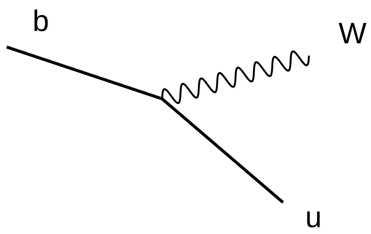


- m_{ud} and m_s are now known to 2%, m_s/m_{ud} to 0.7%
- … m_u to 5% and m_d to 3% w/ help of phenomenology

Quark flavor mixing constraints in the SM and beyond

Test SM paradigm of quark flavor mixing and CP violation and look for new physics

Unitary CKM matrix



$$\sim V_{ub} \to V = \begin{matrix} & d & s & b \\ u & 1-\frac{\lambda}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ c & -\lambda & 1-\frac{\lambda}{2} & A\lambda^2 \\ t & A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{matrix} + \mathcal{O}(\lambda^4)$$

Test CKM unitarity/quark-lepton universality and constrain NP using, e.g.

1st row unitarity:

$$\frac{G_q^2}{G_\mu^2}|V_{ud}|^2\left[1+|V_{us}/V_{ud}|^2+|V_{ub}/V_{ud}|^2\right]=1+O\left(\frac{M_W^2}{\Lambda_{NP}^2}\right)$$

Unitarity triangle:

$$\frac{G_q^2}{G_\mu^2}(V_{cd}V_{cb}^*)\left[1+\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}+\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]=O\left(\frac{M_W^2}{\Lambda_{NP}^2}\right)$$

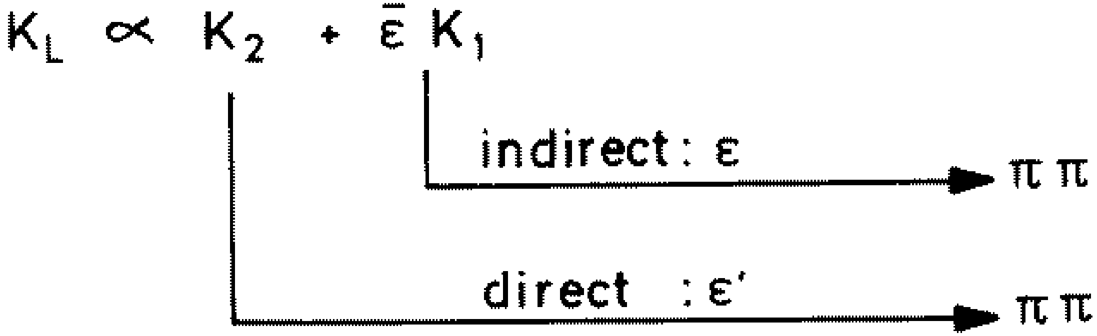
CPV and the K^0-$\bar{K}^0$ system

Two neutral kaon flavor eigenstates: $K^0(d\bar{s})$ & $\bar{K}^0(s\bar{d})$

In experiment, have predominantly:

- $K^0_S \to \pi\pi \Rightarrow K^0_S \sim K_1$, the CP even combination
- $K^0_L \to \pi\pi\pi \Rightarrow K^0_L \sim K_2$, the CP odd combination

However, CP violation $\Rightarrow K^0_L \to \pi\pi$



Experimentally:

(PDG '11)

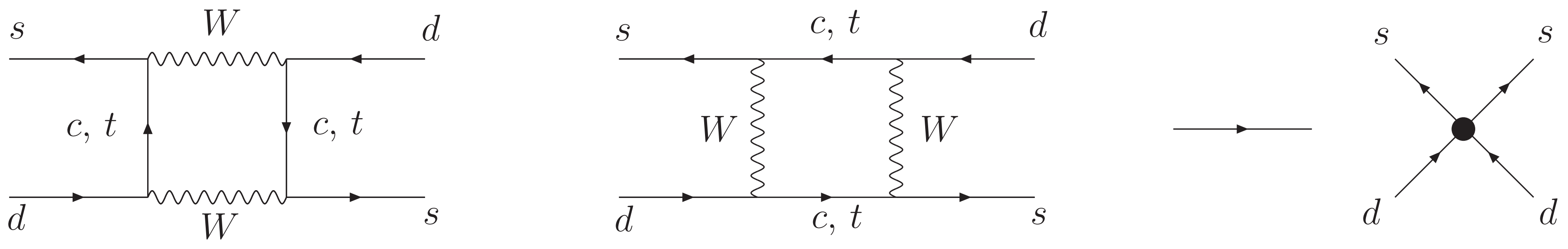
$$\Delta M_K \equiv M_{K_L} - M_{K_S} = 3.483(6) \times 10^{-12}\,\mathrm{MeV} \quad [0.2\%]$$

$$\Delta\Gamma_K \equiv \Gamma_{K_S} - \Gamma_{K_L} = 7.339(4) \times 10^{-12}\,\mathrm{MeV} \quad [0.05\%]$$

$$|\epsilon| = 2.228(11) \cdot 10^{-3} \quad [0.5\%]$$

$$\mathrm{Re}(\epsilon'/\epsilon) = 1.65(26) \cdot 10^{-3} \quad [16\%]$$

K^0-$\bar{K}^0$ mixing in the SM



$$M_{12} - \frac{i}{2}\Gamma_{12} = \frac{\langle K^0|\mathcal{H}_{\text{eff}}^{\Delta S=2}(0)|\bar{K}^0\rangle}{2M_K} - \underbrace{\frac{i}{2M_K}\int d^4x \langle K^0|\mathcal{H}_{\text{eff}}^{\Delta S=1}(x)\mathcal{H}_{\text{eff}}^{\Delta S=1}(0)|\bar{K}^0\rangle}_{\text{long-distance contributions to } M_{12}\ \&\ \Gamma_{12}} + O(G_F^3)$$

To LO in the OPE

$$2M_K M_{12}^* \stackrel{\text{LO}}{=} \langle \bar{K}^0|\mathcal{H}_{\text{eff}}^{\Delta S=2}|K^0\rangle = C_1^{\text{SM}}(\mu)\langle \bar{K}^0|O_1(\mu)|K^0\rangle$$

$$O_1 = (\bar{s}d)_{V-A}(\bar{s}d)_{V-A} \qquad \langle \bar{K}^0|O_1(\mu)|K^0\rangle = \frac{16}{3}M_K^2 F_K^2 B_K(\mu)$$

Indirect CPV in $K \to \pi\pi$

Parametrized by

$$\mathrm{Re}\,\frac{T[K_L \to (\pi\pi)_0]}{T[K_S \to (\pi\pi)_0]} = \mathrm{Re}\,\epsilon = \cos\phi_\epsilon \sin\phi_\epsilon \left[\frac{\mathrm{Im}M_{12}}{2\,\mathrm{Re}M_{12}} - \frac{\mathrm{Im}\Gamma_{12}}{2\,\mathrm{Re}\Gamma_{12}}\right]$$

w/ $\phi_\epsilon = \tan^{-1}(2\Delta M_K/\Delta\Gamma_K) = 43.51(5)^o$, $\Delta M_K \simeq 2\,\mathrm{Re}\,M_{12}$, $\Delta\Gamma_K \simeq -2\,\mathrm{Re}\,\Gamma_{12}$

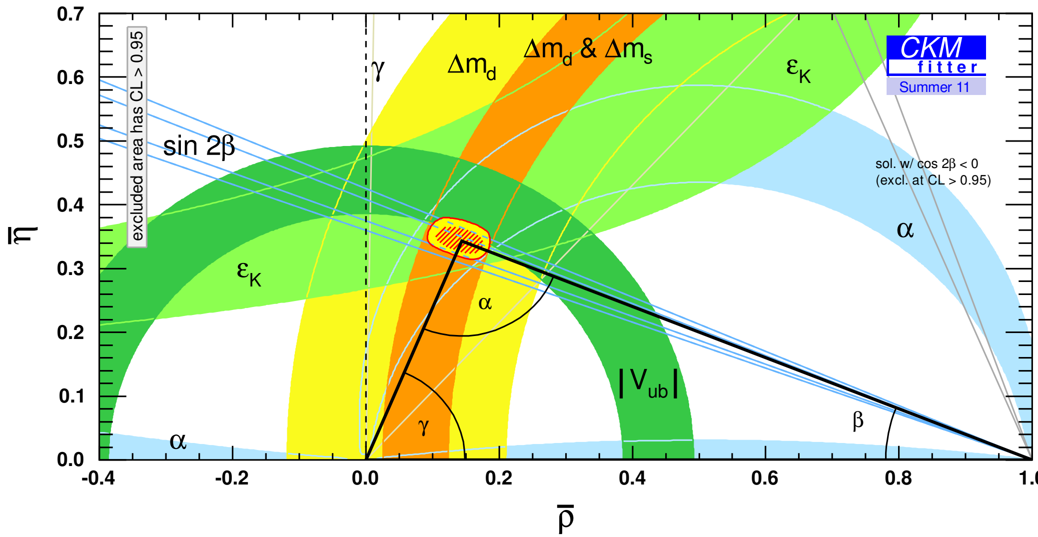
$$\longrightarrow \epsilon = \kappa_\epsilon \frac{e^{i\phi_\epsilon}}{\sqrt{2}} \frac{\mathrm{Im}M_{12}}{\Delta M_K}$$

w/ $\kappa_\epsilon = 0.94(2)$ (Buras et al. (2010))

To NLO in α_s

$$\begin{aligned} |\epsilon| &= \kappa_\epsilon C_\epsilon\,\mathrm{Im}\lambda_t \{\mathrm{Re}\lambda_c[\eta_1 S_{cc} \\ &\quad -\eta_3 S_{ct}] - \mathrm{Re}\lambda_t \eta_2 S_{tt}\}\,\hat{B}_K \\ &\propto A^2\lambda^6\bar{\eta}\,[\mathrm{cst} + \mathrm{cst} \\ &\quad \times A^2\lambda^4(1-\bar{\rho})]\,\hat{B}_K \end{aligned}$$

w/ $\lambda_q = V_{qd}V_{qs}^*$



B_K with Wilson fermions

(Dürr et al [BMWc], arXiv:1106:3230 [hep-lat])

χSB of Wilson fermions $\to$ $O_1(a)$ mixes w/ ops of different chirality:

$$\langle \bar{K}^0|O_1(\mu)|K^0\rangle = Z_{11}(g_0, a\mu)\hat{Q}_1(a)$$

w/

$$\hat{Q}_1(a) = Q_1(a) + \sum_{i=2}^{5}\Delta_{1i}(g_0)Q_i(a), \qquad Q_i(a) \equiv \langle \bar{K}^0|O_i(a)|K^0\rangle$$

and complete parity conserving basis:

$$O_1 = \gamma_\mu \otimes \gamma_\mu + \gamma_\mu\gamma_5 \otimes \gamma_\mu\gamma_5$$
$$O_2 = \gamma_\mu \otimes \gamma_\mu - \gamma_\mu\gamma_5 \otimes \gamma_\mu\gamma_5$$

$$O_3 = I \otimes I + \gamma_5 \otimes \gamma_5$$
$$O_4 = I \otimes I - \gamma_5 \otimes \gamma_5$$
$$O_5 = \sigma_{\mu\nu} \otimes \sigma_{\mu\nu}$$

w/ $\Gamma \otimes \Gamma = [\bar{s}\Gamma d][\bar{s}\Gamma d]$

Perform calculation on BMWc 2010 dataset

Multiplicative renormalization of B_K

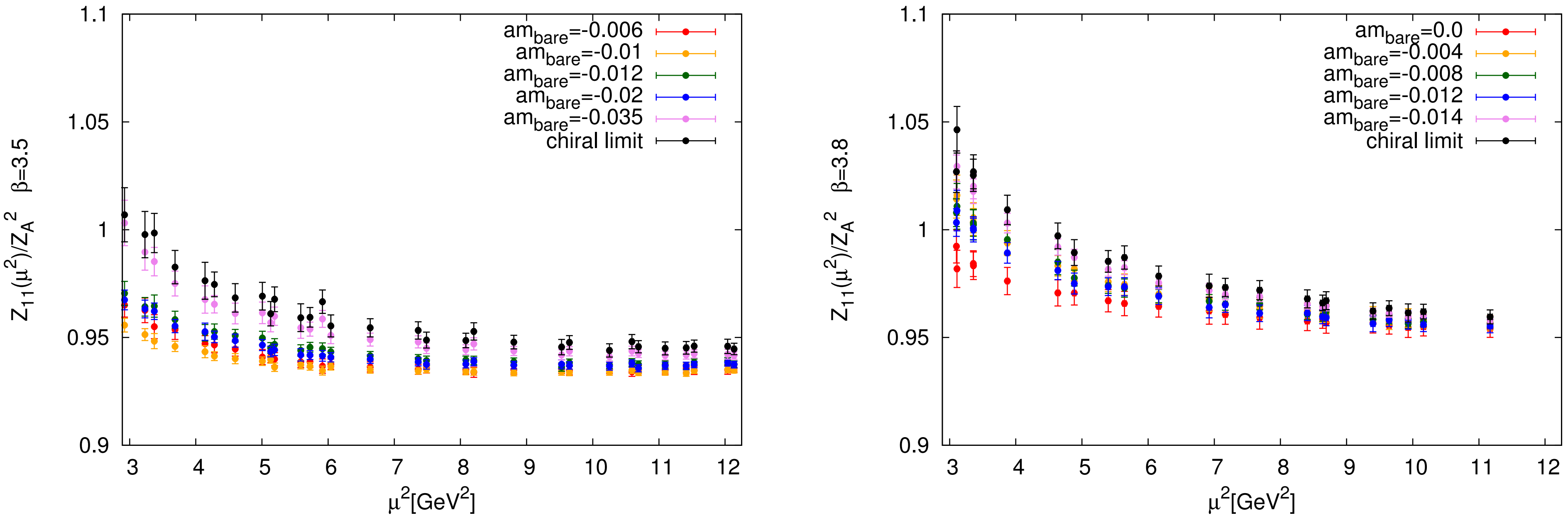
Define

$$Z_{B_K}(g_0, a\mu) \equiv Z_{11}(g_0, a\mu)/Z_A^2(g_0)$$

w/ Z_A, axial current renormalization

Use similar RI/MOM methods as for $Z_S(g_0, a\mu)$

Chiral behavior



Flat chiral extrapolation, more so at larger μ

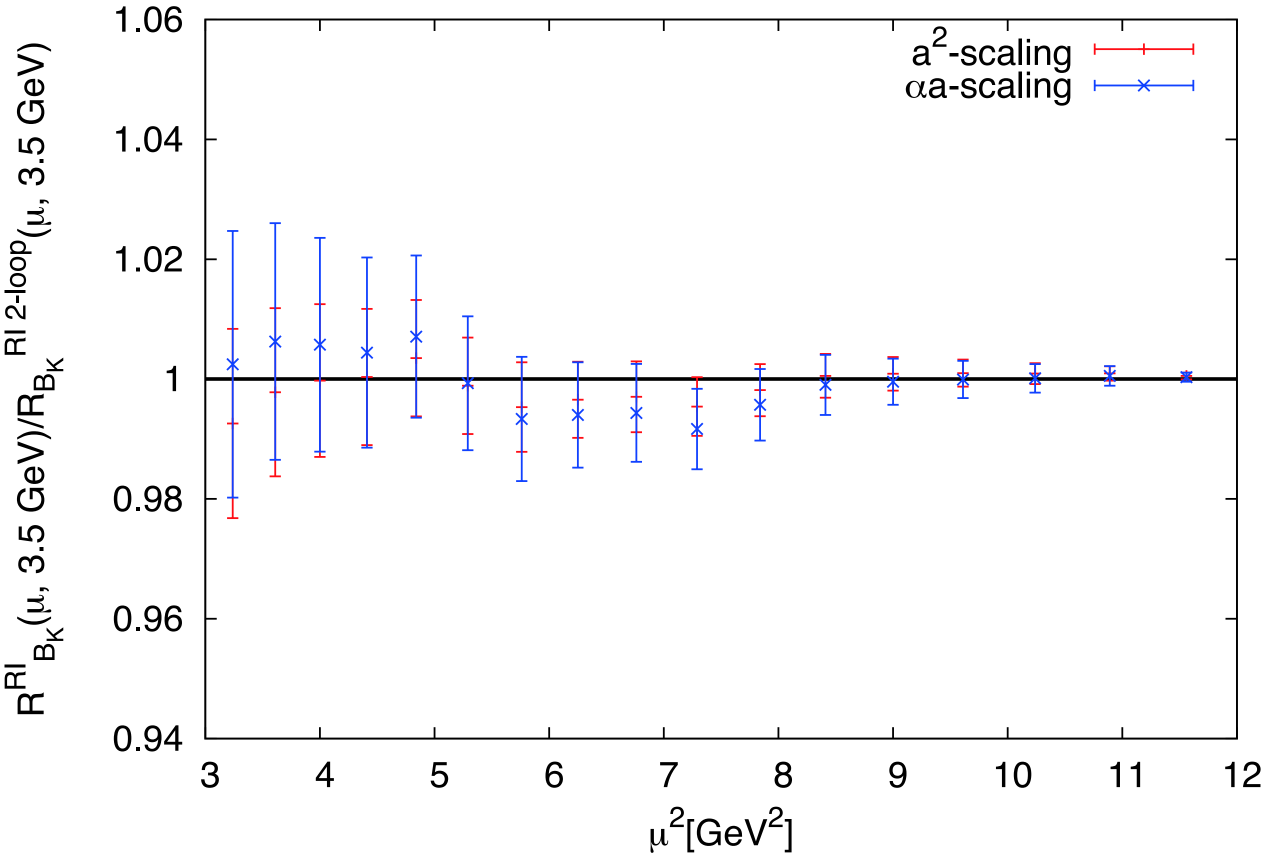
Nonperturbative continuum running of B_K

Continuum, nonperturbative running to $\mu = 3.5\,\mathrm{GeV}$ is given by

$$R^{\mathrm{RI}}_{B_K}(\mu_0, \mu) = \lim_{g_0 \to 0} Z^{\mathrm{RI}}_{B_K}(g_0, a\mu)/Z^{\mathrm{RI}}_{B_K}(g_0, a\mu_0)$$

Divided by 2-loop PT running

$\Rightarrow$ good agreement in range $1.75 \div 3.5\,\mathrm{GeV}$

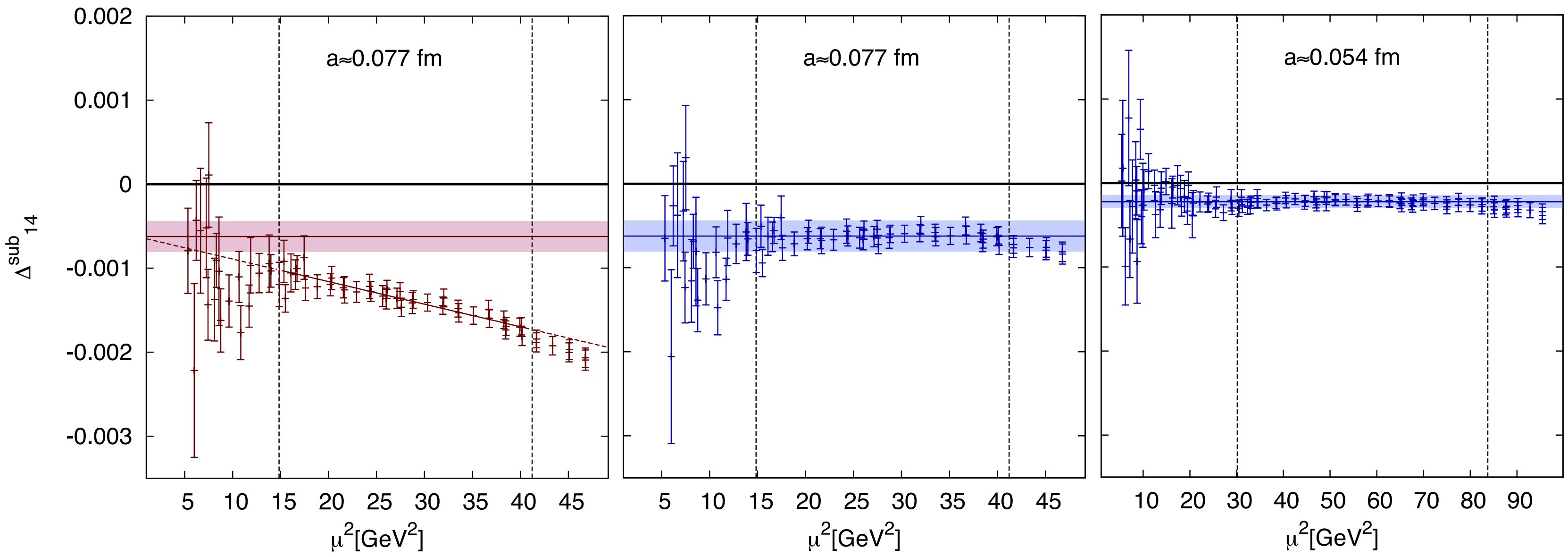


Mixing coefficients Δ_{1i}

Determined in standard way (Donini et al. (1999)), w/ RI/MOM Goldstone poles removed through (Giusti et al. (2000))

$$\Delta_{1i}^{\text{sub}} \equiv \frac{m_1 \Delta_{1i}(a, m_1) - m_2 \Delta_{1i}(a, m_2)}{m_1 - m_2}$$

Subtract remaining $1/p^4$ and $(ap)^2$ w/ fits

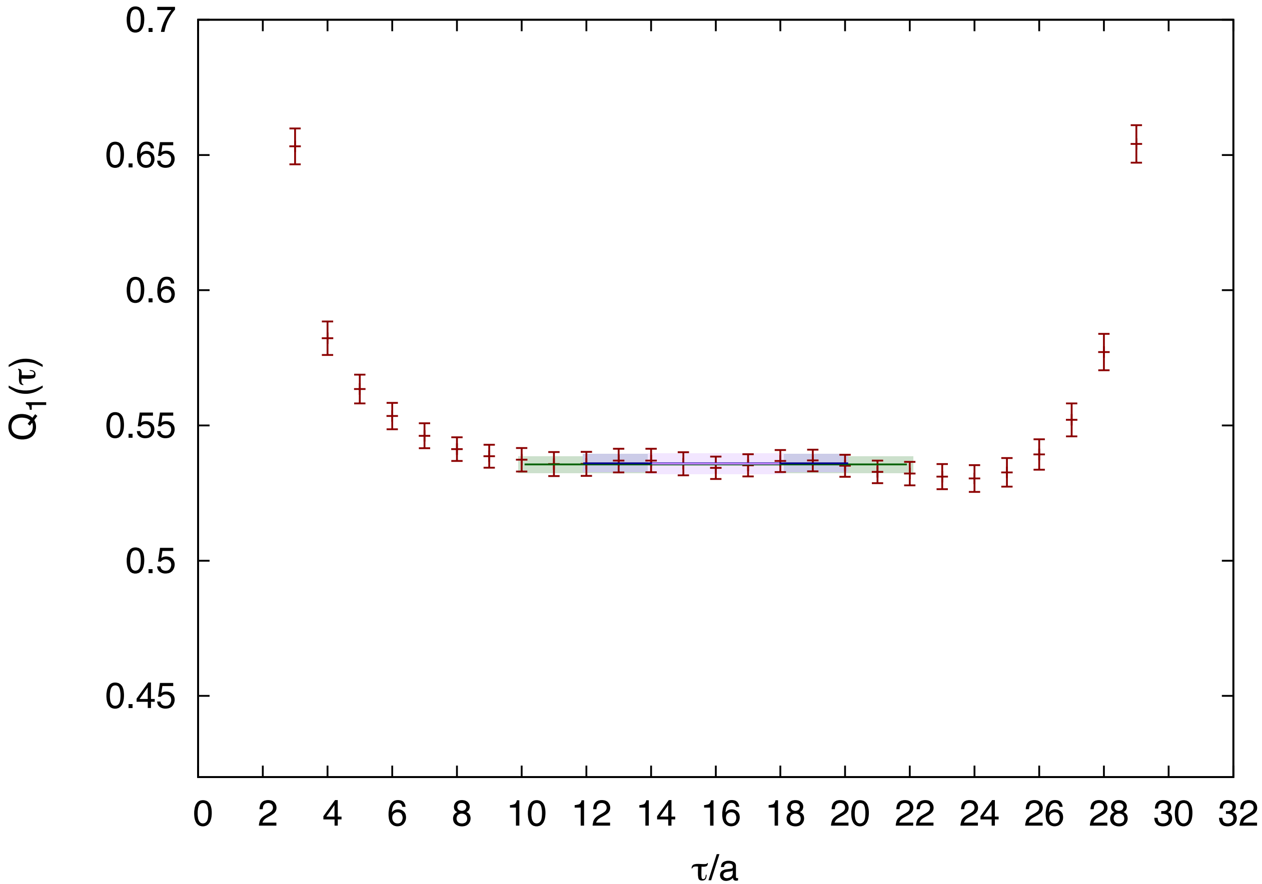


→ mixing coeffs are well determined and small thanks to smearing

→ still $O(10)\times$ mixing for DWF

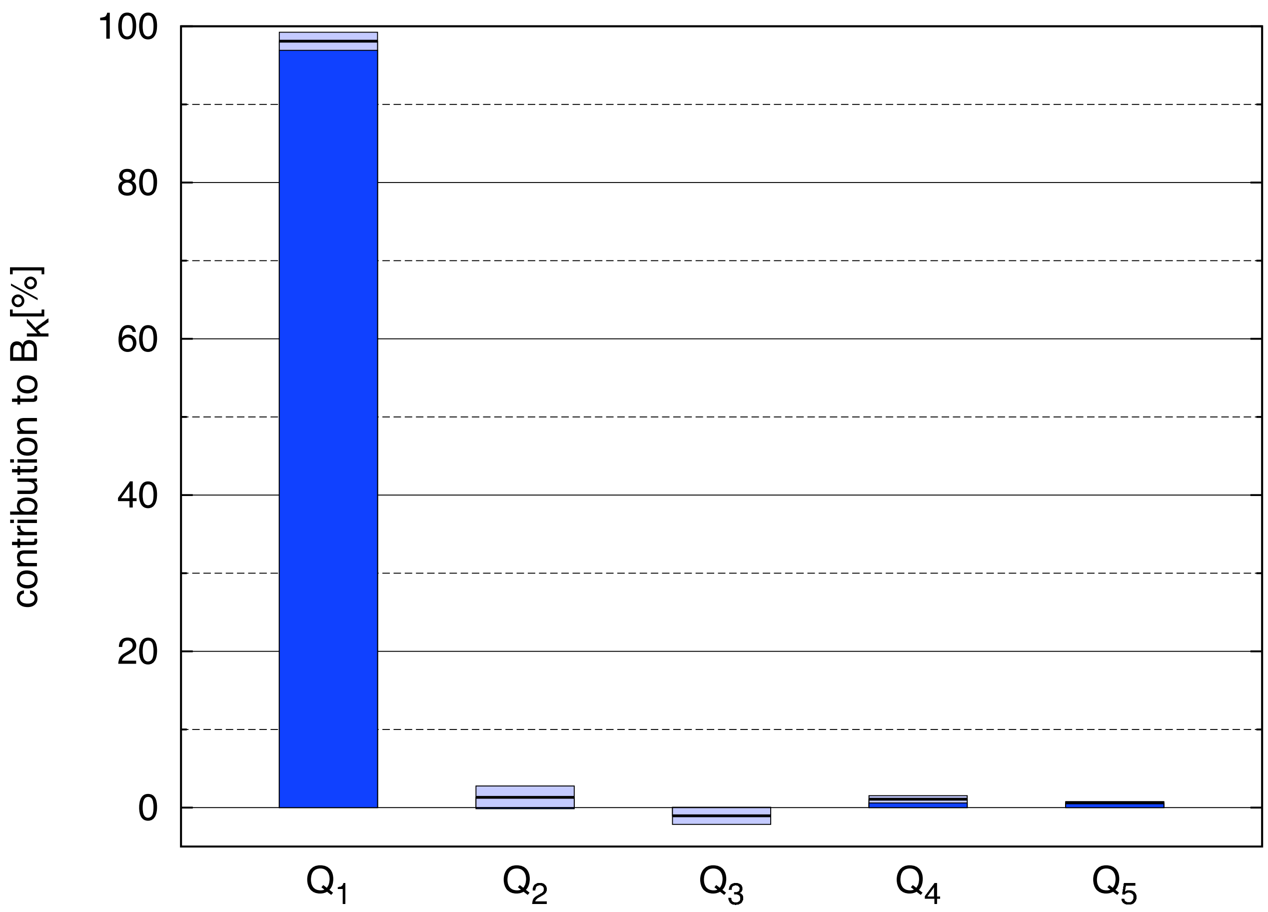
Extraction of bare matrix elements

$$B_i(g_0;t) = \frac{(L/a)^3 \sum_{\vec{x}} \langle W(T/2) O_i(g_0;t,\vec{x}) W(0)\rangle}{\frac{8}{3} \sum_{\vec{x},\vec{y}} \langle W(T/2) A_\mu(t,\vec{x})\rangle \langle A_\mu(t,\vec{y}) W(0)\rangle}$$



$(\beta = 3.7,\ M_\pi \sim 245\,\mathrm{MeV})$

Operator contributions to B_K



$(\beta = 3.61,\ M_\pi \sim 120\,\mathrm{MeV})$

→ contributions from $Q_{2,\dots,5}$ small and often consistent with zero

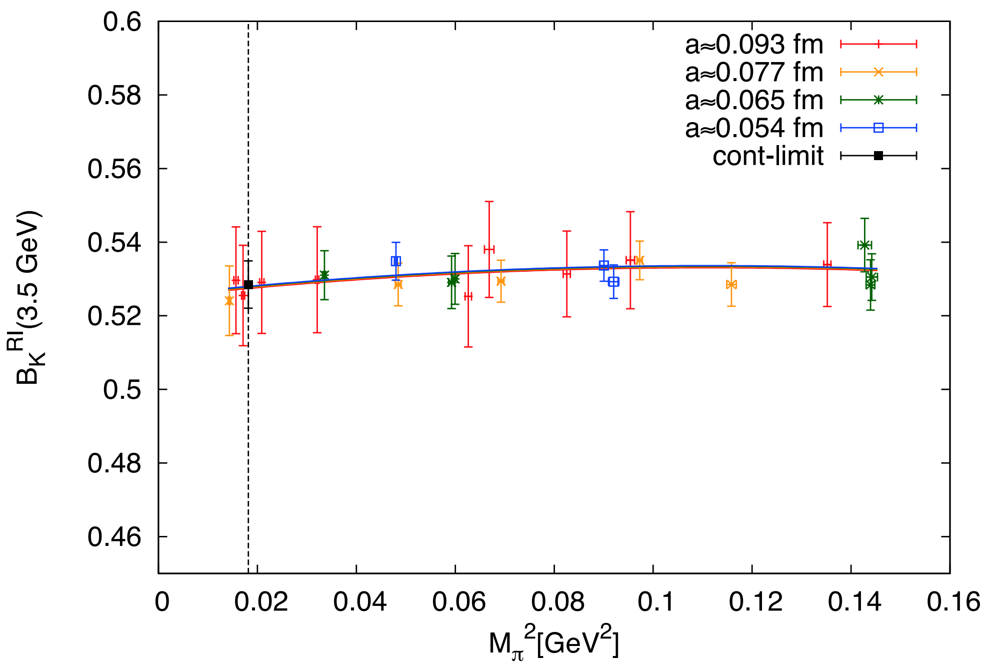
→ nevertheless, dominate systematic error

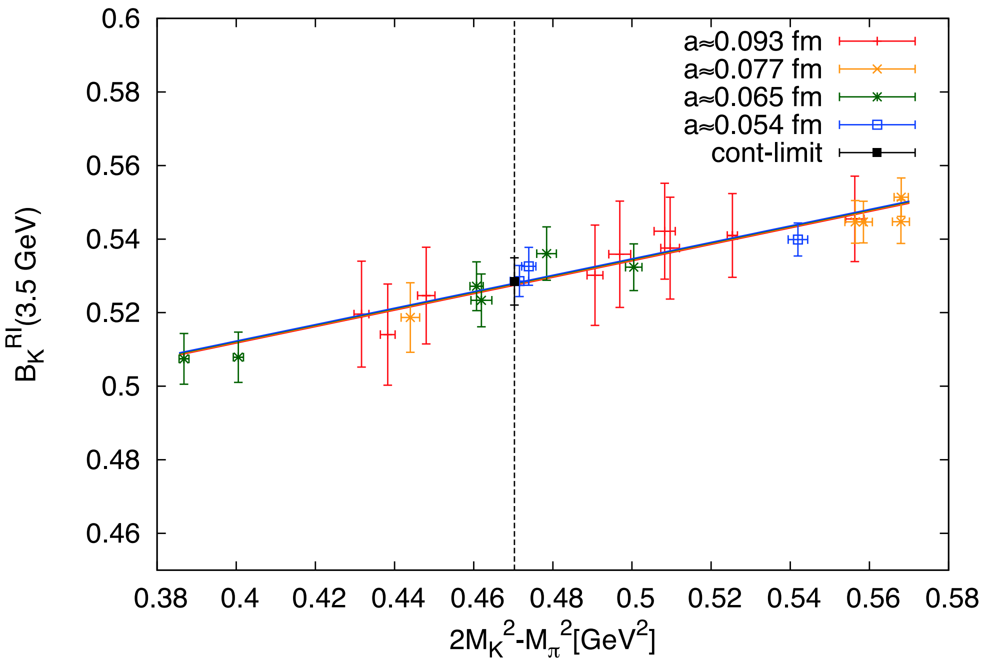
Overall strategy for B_K

- bare $Q_{1,\ldots,5}$ from $N_f = 2+1$ ensembles ($\beta = 3.5, 3.61, 3.7, 3.8$) w/ various $(M_\pi^2, 2M_K^2 - M_\pi^2)$
- RI/MOM renormalization w/ trace-subtraction from $N_f = 3$ ensembles ($\beta = 3.5, 3.61, 3.7, 3.8$) w/ various am_q
- multiplicative $Z_{B_K}(g_0, a\mu)$ w/ continuum, nonperturbative running to $\mu = 3.5\,\mathrm{GeV}$ and pole-subtracted mixing terms for $\Delta_{1,i}^{\mathrm{sub}}$
- renormalized $B_K(\mu)$ considered fn of $(M_\pi^2, 2M_K^2 - M_\pi^2, a, L)$
  $\rightarrow$ interpolated to physical mass point
  $\rightarrow$ extrapolated to continuum
- Very small FV corrections from Becirevic et al (2004) applied to data prior to analysis

Combined chiral interp. and continuum extrap. (1)

Project onto chiral axes:interpolation to $M_\pi = 134.8(3)\,\mathrm{MeV}$ and $M_K = 494.2(5)\,\mathrm{MeV}$



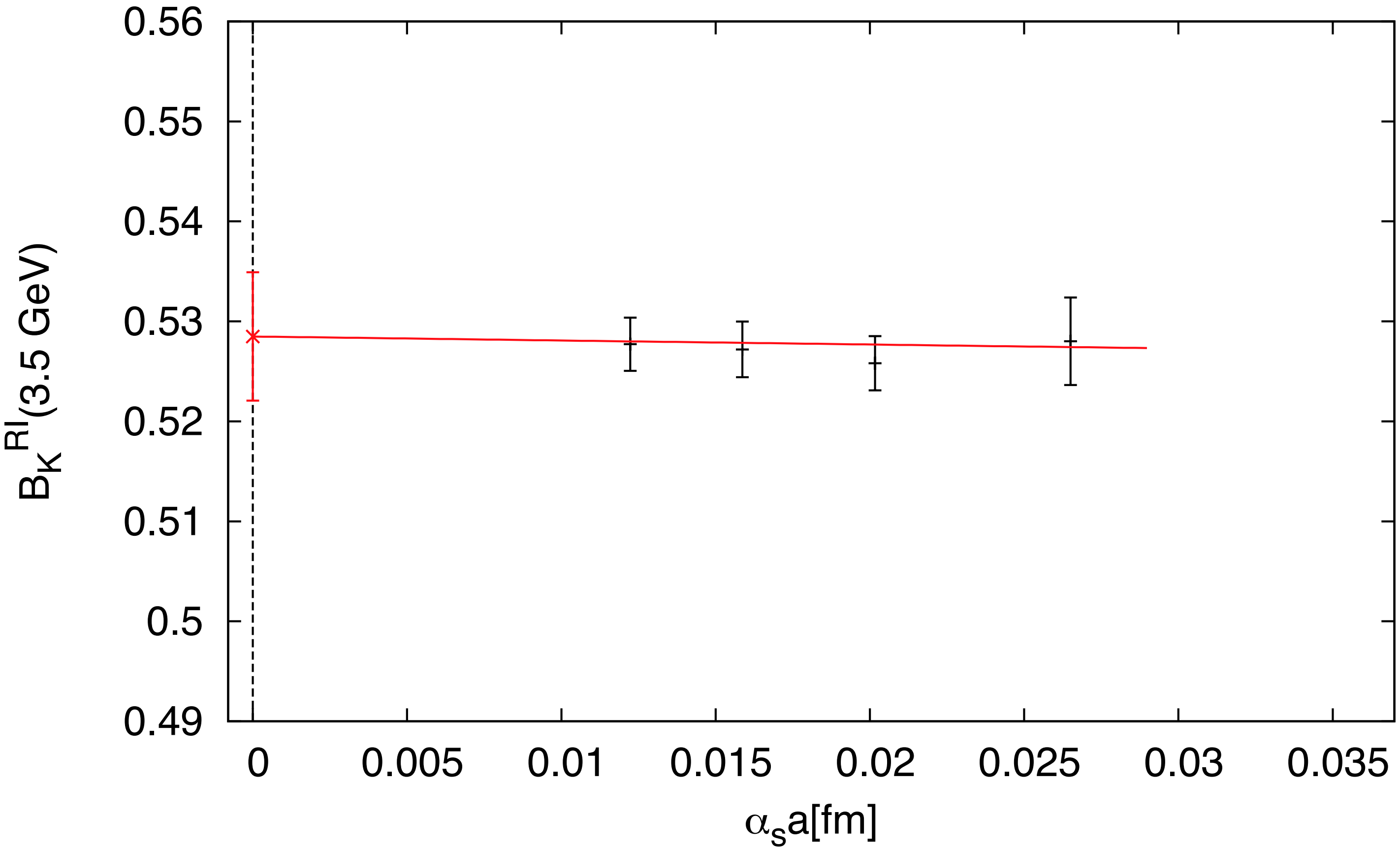


→ nearly flat m_{ud}-dependence near m_{ud}^{phys}

→ much steeper m_s-dependence near m_s^{phys}

Combined chiral interp. and continuum extrap. (2)

Project onto *a* axis: continuum extrapolation

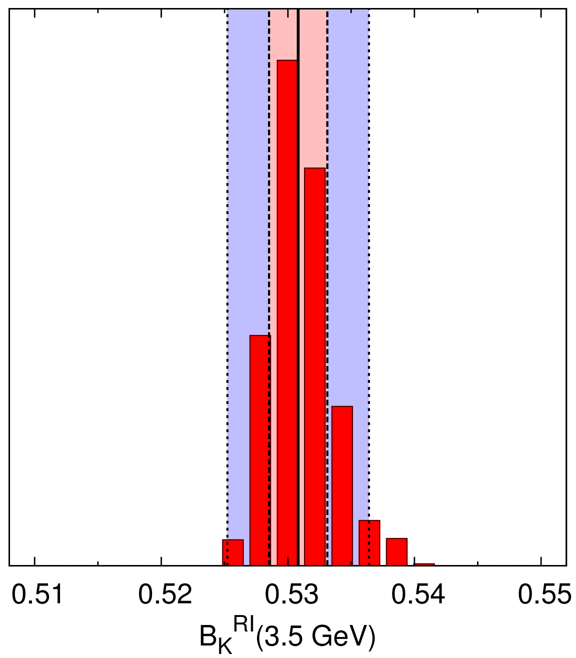


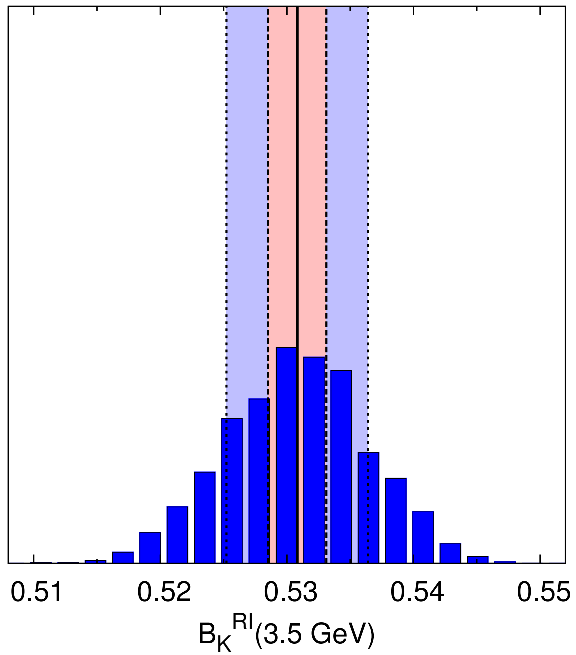
$\rightarrow$ mild extrapolation for $\beta \geq 3.5$

$\rightarrow$ $\beta \geq 3.31$ was found to be outside scaling regime

Systematic and statistical error

- 2 time-fit ranges for π and K masses
- 2 time-fit ranges for $Q_{1,\ldots,5}$
- $O(\alpha_s a)$ or $O(a^2)$ for running
- 3 intermediate renormalization scales
- 2 fit fns and 4 ranges in p^2 for Δ_{1i}
- 5 fit fns for mass interpolation
- 2 pion mass cuts $(M_\pi < 340,\ 380\,\mathrm{MeV})$





→ 5760 analyses, each of which is a reasonable choice, weighted by fit quality

→ median and central 68% give central value and systematic uncertainty

2000 bootstraps of the median give statistical error

Many cross checks

Results

Procedure gives $B_K^{\mathrm{RI}}(3.5\,\mathrm{GeV})$ fully nonperturbatively

Can convert to other schemes w/ perturbation theory (PT)
$\rightarrow$ perturbative uncertainty

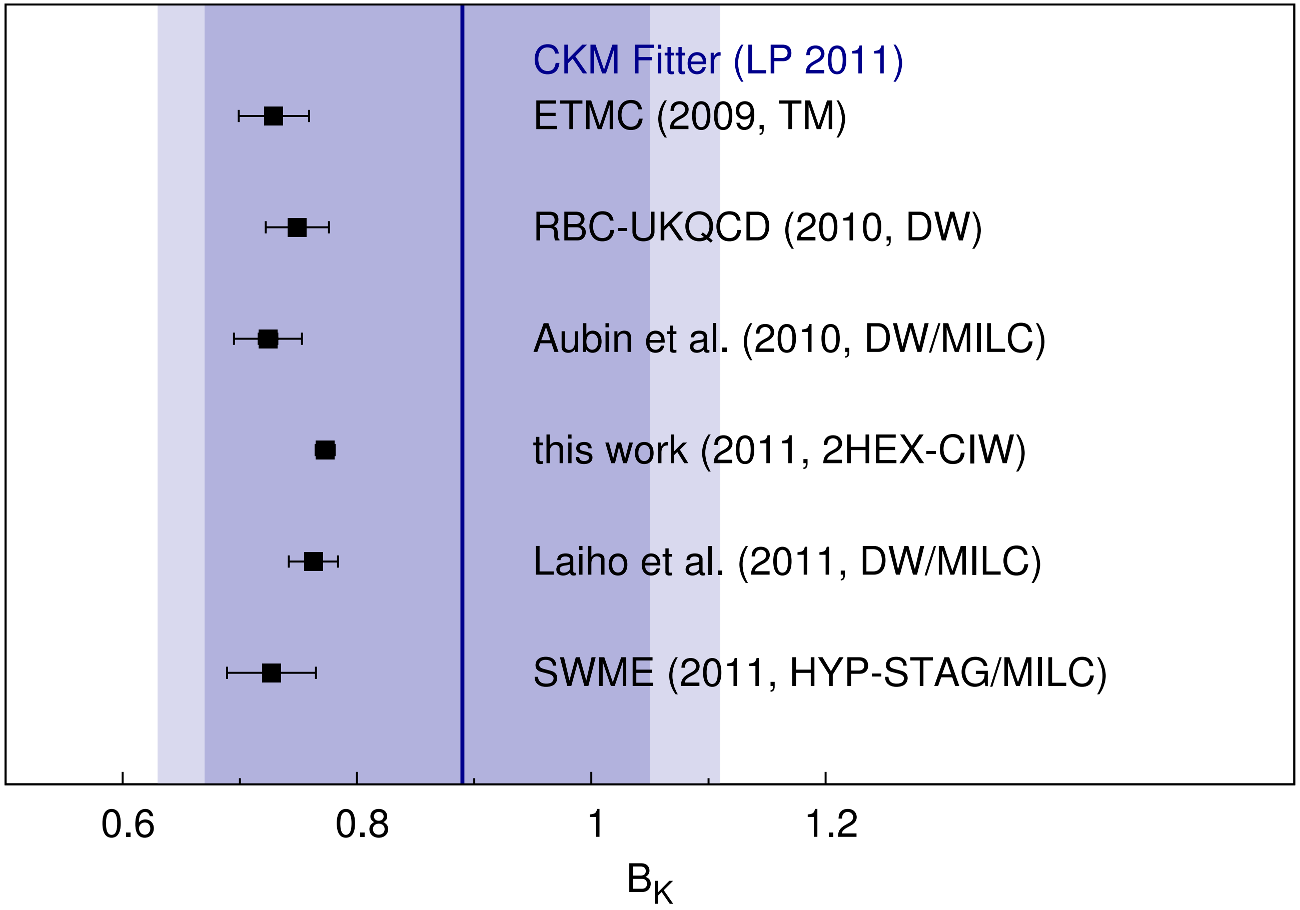
conv.	RGI	$\overline{\mathrm{MS}}$-NDR 2 GeV
4-loop β, 1-loop γ	1.427	1.047
4-loop β, 2-loop γ	1.457	1.062
ratio	1.021	1.01376

Take blanket 1% for 3 loop uncertainty

	RI @ 3.5 GeV	RGI	$\overline{\mathrm{MS}}$-NDR @ 2 GeV
B_K	0.5308(56)(23)	0.7727(81)(34)(77)	0.5644(59)(25)(56)

Total error 1.1-1.5%, statistical (and PT) dominated

B_K comparison



Dominant error in CKMfitter global fit results is $|V_{cb}|^4 \sim (A\lambda^2)^4$
$\rightarrow$ our result is an encouragement to reduce uncertainties in other parts of the calculation of ϵ

Conclusion

- After $>$ 40 years we are finally able to perform fully controlled LQCD computations all the way down to $M_\pi \leq 135\,\mathrm{MeV}$
- Presented fully controlled results for light hadron spectrum and for light quark masses and B_K w/ $\sigma_{\rm tot} < 2\%$
- Also results on BMWc 08 ensembles for light hadron masses, F_K/F_π and sigma terms, and preliminary results on BMWc 10 ensembles for E+M corrections, ρ-width, . . .
- For experts: new high-precision scale setting from Wilson/Symanzik flow on BMWc 10 ensembles (Borsányi et al, arXiv:1203.4469)

$$w_0 = 0.1755(18)(4)\,\mathrm{fm} \quad [(1.0\%)(0.2\%)]$$

Advantages:

- aw_0 is cheap, precise and reliable
- w_0 also given for non-physical quark masses
- depends weakly on quark masses
- code available on arXiv

$\Rightarrow$ modern version of Sommer scale r_0

Conclusion

- Lattice QCD is undergoing a major shift in paradigm
  - it is now possible to control and reliably quantify all systematic errors with “data” (for at most 1 initial and/or final hadron state)

  $\Rightarrow$ we are getting QCD NOT LQCD predictions
  - requires numerous simulations with $M_\pi < 200\,\mathrm{MeV}$ and preferably $\searrow 135\,\mathrm{MeV}$, more than 3 $a < 0.1\,\mathrm{fm}$ and lattice sizes $L \to 4 \div 6\,\mathrm{fm}$
  - requires trying all reasonable analyses of “data” and combining results in sensible way to obtain a reliable systematic error
- Expect many more very interesting nonperturbative QCD predictions in coming years