Pole trajectories of the $\Lambda(1380)$ and $\Lambda(1405)$ resonances from the combination of lattice and experimental data

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Problem

• Hadron Spectroscopy: Light hadrons: only few asymptotic states in QCD, Light hadron spectrum follows the pattern predicted by the quark model.
• For heavier hadrons, binding ceases, and their resonant nature can be understood only through the dynamics of QCD. $\Lambda(1405)$ is naively *uds* baryon, however lighter than $N^\star(1535)$.
• A combined analysis of lattice QCD data at unphysical pion masses and experimental results reveals evidence for a two-pole structure in the strangeness resonance near the physical point.

Basic concepts

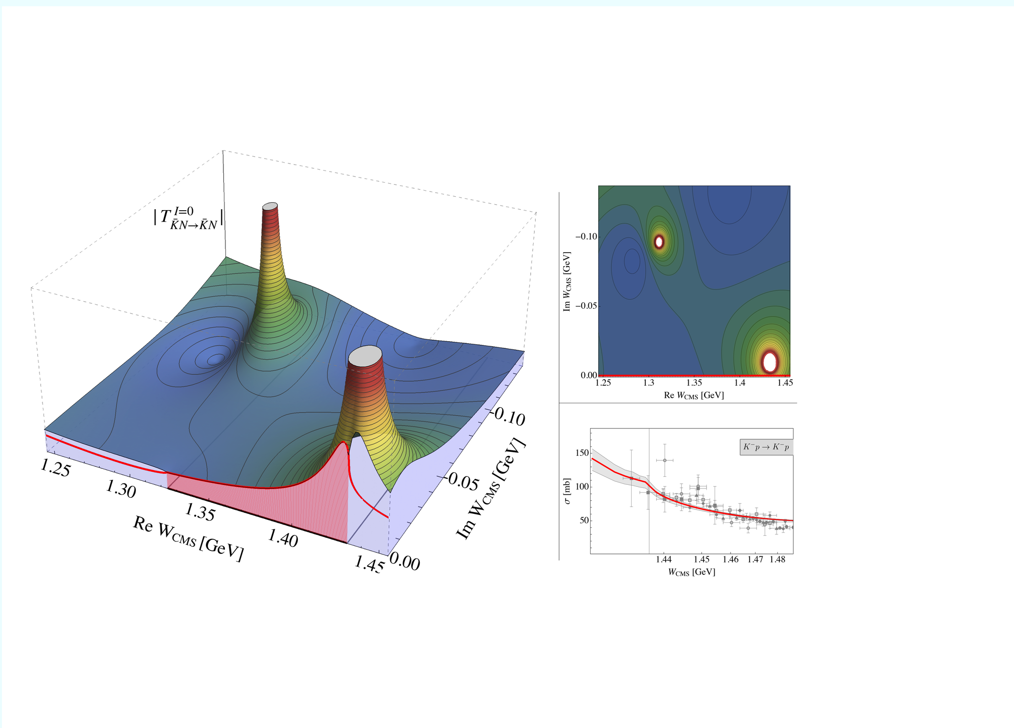


Figure 1: Left: Schematic representation of the scattering amplitude in the complex energy plane. Right: Its 2D projection and the experimental cross section data for $K^-p \to K^-p$ scattering.

Central quantity Transition amplitudes: complex analytic function

- Kinematical Constraints from S matrix (Unitarity/Analyticity/Crossing)
- Dynamical Constraints from QCD (CHPT, Lattice QCD)

Reaction-independent parameters

- Pole positions on unphysical Riemann Sheets

Chiral Unitary models

The scattering amplitude can be constructed as a sum of infinitely many diagrams originating from the the chiral meson-baryon Lagrangian:

$$\mathcal{L}_{\phi B} = \mathcal{L}^{(1)}_{\phi B} + \mathcal{L}^{(2)}_{\phi B} + \mathcal{L}^{(3)}_{\phi B} + \cdots$$

Here we apply chiral potential V and iterating it to restore two-body unitarity.
Unitarization with the Bethe Salpeter equation:

$$T(s) = -V(s) + T(s)G(s)V(s)$$

Different models with potential V:

$$\text{M1:} \quad V_{\alpha\beta}(s) = V^{\rm WT}_{\alpha\beta}(s), \tag{1}$$

$$\text{M2:} \quad V_{\alpha\beta}(s) = V^{\rm WT}_{\alpha\beta}(s) + V^{{\rm Born},s,u}_{\alpha\beta} \tag{2}$$

$$\text{M3:} \quad V_{\alpha\beta}(s) = V^{\rm WT}_{\alpha\beta}(s) + V^{{\rm Born},s,u}_{\alpha\beta} + V^{\rm NLO}_{\alpha\beta}(s). \tag{3}$$

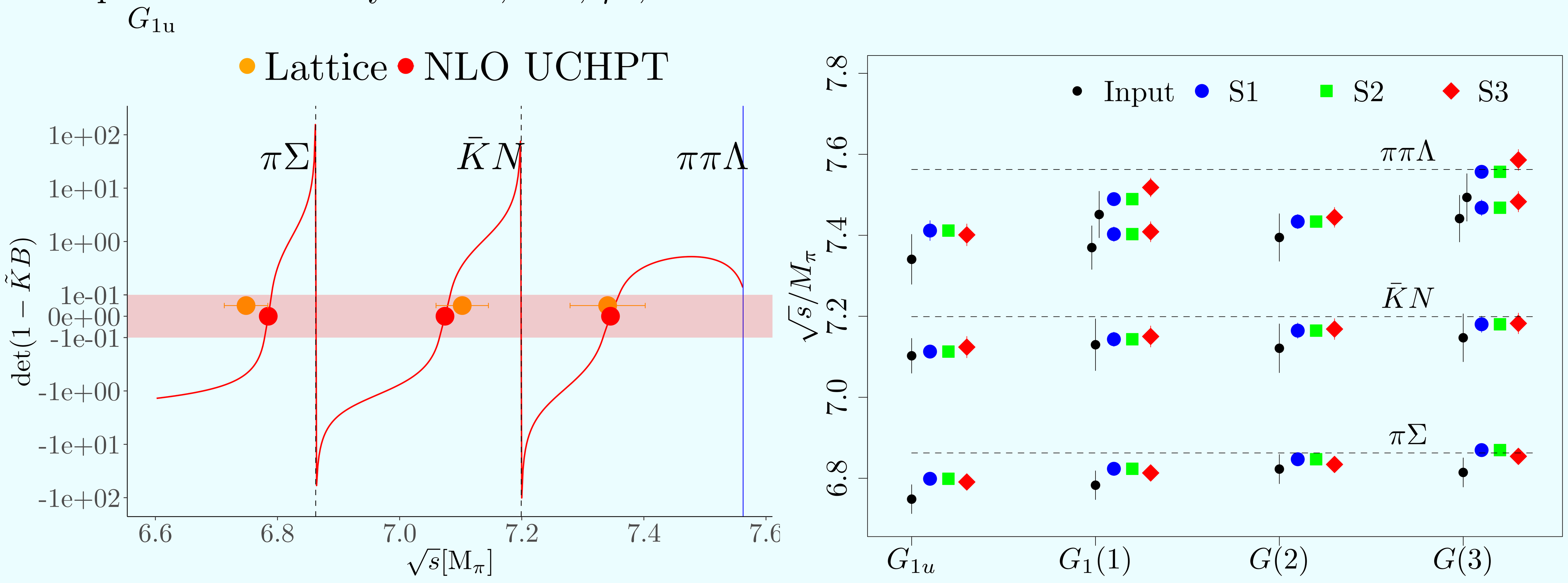
Reguralizations (S_i) with loop function G.
Parameters:
S_1: 6 subtraction constants
S_2: No parameter
S_3: One scale parameter
M_1, M_2 : LO ChPT, no additional parameter
M_3 : NLO LECs: $b_0, b_D, b_F, d_1, d_2, d_3, d_4$

Lüscher method, Quantization condition

• Gateway between finite volume lattice theory and infinite volume scattering amplitude
• In short: On shell physical 2-particle configurations $\Delta E \propto L^p$
• off shell 2-particle configurations $\Delta E \propto e^{-mL}$

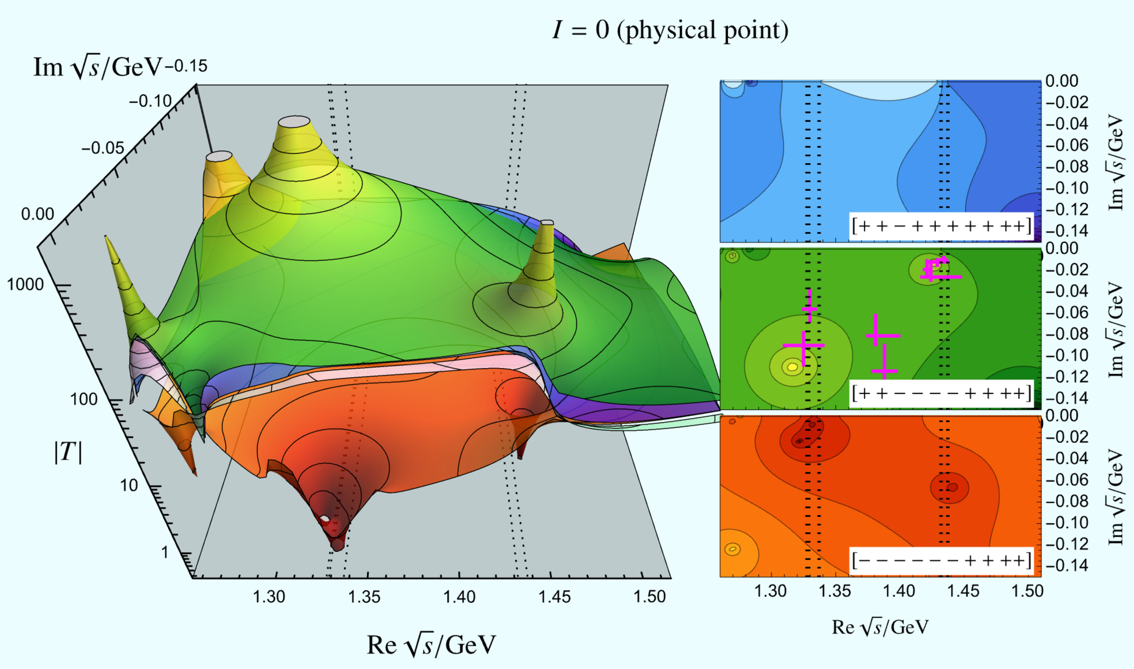
$$QC = 1 - \tilde{K}B; \mathcal{B} = diag(u_a R^a_{00}); a \in (\pi\Sigma, \bar{K}N, \eta\Lambda, K\Xi); R_{\ell m} = \frac{1}{\gamma \pi^{3/2} u_a^{\ell+1}} \mathcal{Z}_{\ell m}(s_a, \gamma, u_a^2)$$

•B is hermitian, diagonal in channel space, K describes the interaction between channels
• Input: Finite volume energy levels
• Output: Constrains on parameters of the scattering amplitude
• Coupled channel analysis: $\pi\Sigma, \bar{K}N, \eta\Lambda, K\Xi$



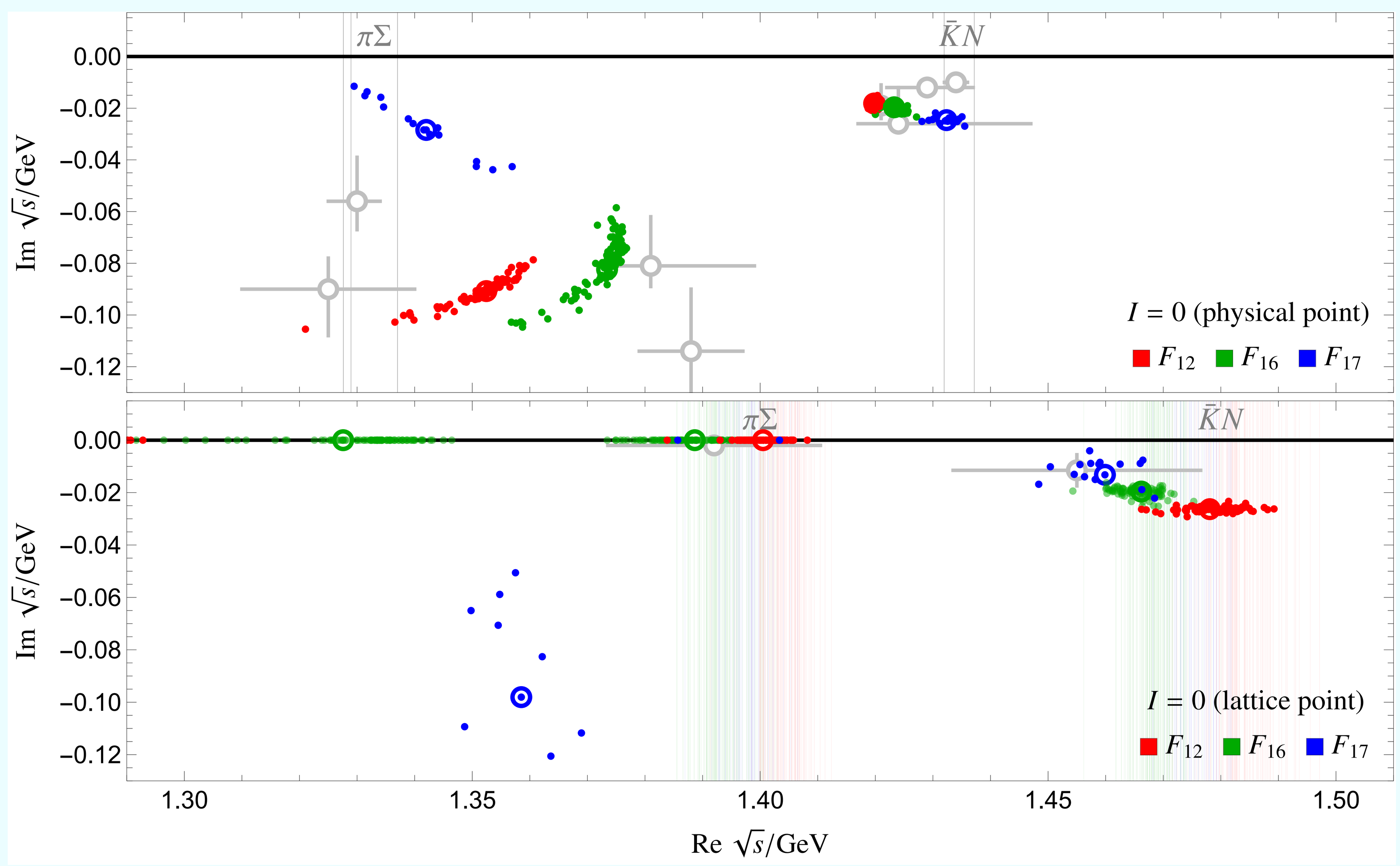
Fitting methods

$$\chi^2_{\rm dof} = \frac{\sum_a N_a}{A((\sum_a N_a) - N_{\rm par})}\chi^2_{\rm wt} \quad \text{with } \chi^2_{\rm wt} = \sum_{a=1}^{A} \frac{\chi^2_a}{N_a} \quad \text{with } \chi^2_a = \sum_{n=1,m=1}^{N_a} (f^a_n(\vec{\aleph}) - \hat{f}^a_n)\,[\hat{C}_a^{-1}]_{nm}\,(f^a_m(\vec{\aleph}) - \hat{f}^a_m)$$



• Experimental data: Cross section, SIDDHARTA, Amadeus experiment [https://github.com/maxim-mai/Experimental-Data/tree/master/Lambda1405]
• Lattice data: BaSC collaboration $M_\pi = 200$MeV [Phys.Rev.Lett. 132 (2024) 5]
• 258 experimental, 14 multihadron energy level, 4 single baryon mass
• Minimazation done with Nelder-Mead algorithm probing different initial conditions

Results



• Check compatibility of different data: Combined fit to lattice and experimental input is performed in this work for the first time
• Statistical errors are estimated with bootstrap
• All results are compatible with reference values, except lower pole M3S1.
• Results for isospin 1 is also included
• Work Accepted for publication in PRD.

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