Unpolarized GPDs on Lattice QCD at physical mass

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1 Motivation

GPDs:
- Generalization of PDFs
- Three-dimensional images of hadrons
- Access to angular moment of partons
- Non-perturbative quantities
- Experimentally hard to get $\rightarrow$ Lattice approach

2 Main Goal

- First ever extraction of light-cone GPDs from lattice at physical mass
- Extraction of GPDs moments
- Comparison with existing results

3 Methodology

GPDs cannot be extracted directly on lattice: they are defined on the light cone. Novel approaches to compute them: pseudo-GPDs [1] approach and quasi-GPDs[2,3] approach (Large Momentum Effective Theory). We use the first one to extract the moments, the second one to get the light-cone GPDs.

The two-point correlation function is defined as:

$$C_{2pt}(t_s,p) = \langle N(t_s,p)\bar{N}(0,p)\rangle$$

The three-point function is defined as:

$$C_{3pt}(t_s,p_f,p_i,\tau,z) = \langle N(t_s,p_f)\left|\bar{\psi}(z)\gamma_j \mathcal{W}(0,z)\psi(0)\right|\bar{N}(0,p_i)\,\rangle$$

where $\mathcal{W}(0,z)$ is a Wilson line.

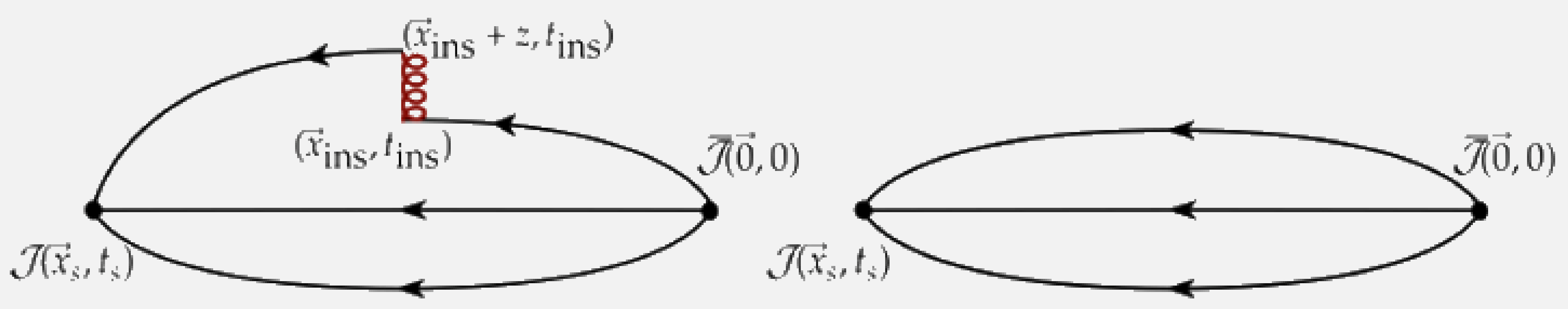

Fig.1 Three-point function (left), two-point function (right)

The quasi-GPDs are defined as [2]

$$\left\langle p_f,\lambda'\left|\bar{q}\left(-\frac{z}{2}\right)\gamma^0\mathcal{W}\left(-\frac{z}{2},\frac{z}{2}\right)q\left(\frac{z}{2}\right)\right|p_i,\lambda\right\rangle = \bar{u}(p_f,\lambda')\left[\gamma_0\mathcal{H}_0(z,P,\Delta) + \frac{i\sigma^{0\mu}\Delta_\mu}{2m}\mathcal{E}_0(z,P,\Delta)\right]u(p_i,\lambda)$$

We use a $64^3\cdot 128$ lattice with physical pion mass, 2+1+1 dynamical twisted fermions and spacing a=0.080 fm. We use only $t_s = 10a$ and $\xi = 0$ (only transversal momentum transfer). The matrix elements $\Pi_\mu(\Gamma_k)$ are extracted from

$$R_\mu(\Gamma_k,p_f,p_i;t_s,t_{ins}) = = \frac{C^{3pt}_\mu(\Gamma_k,p_f,p_i;t_s,t_{\rm ins})}{\sqrt{C^{2pt}(\Gamma_0,p_f;t_s)C^{2pt}(\Gamma_0,p_i;t_s)}}\sqrt{\frac{C^{2pt}(\Gamma_0,p_i;t_s-t_{\rm ins})C^{2pt}(\Gamma_0,p_f;t_{\rm ins})}{C^{2pt}(\Gamma_0,p_f;t_s-t_{\rm ins})C^{2pt}(\Gamma_0,p_i;t_{\rm ins})}}$$

we produced data such that $\overrightarrow{p_f} = (0,0,P_z)$; $\overrightarrow{p_i} = \overrightarrow{p_f} - \vec{\Delta} = (-\Delta_1,-\Delta_2,P_z)$; $-t^2 = (p_f - p_i)^2$, with 0.72 GeV $< P_z <$ 1.69 GeV and 0 GeV$^2 \leq -t^2 <$ 1.12 GeV^2

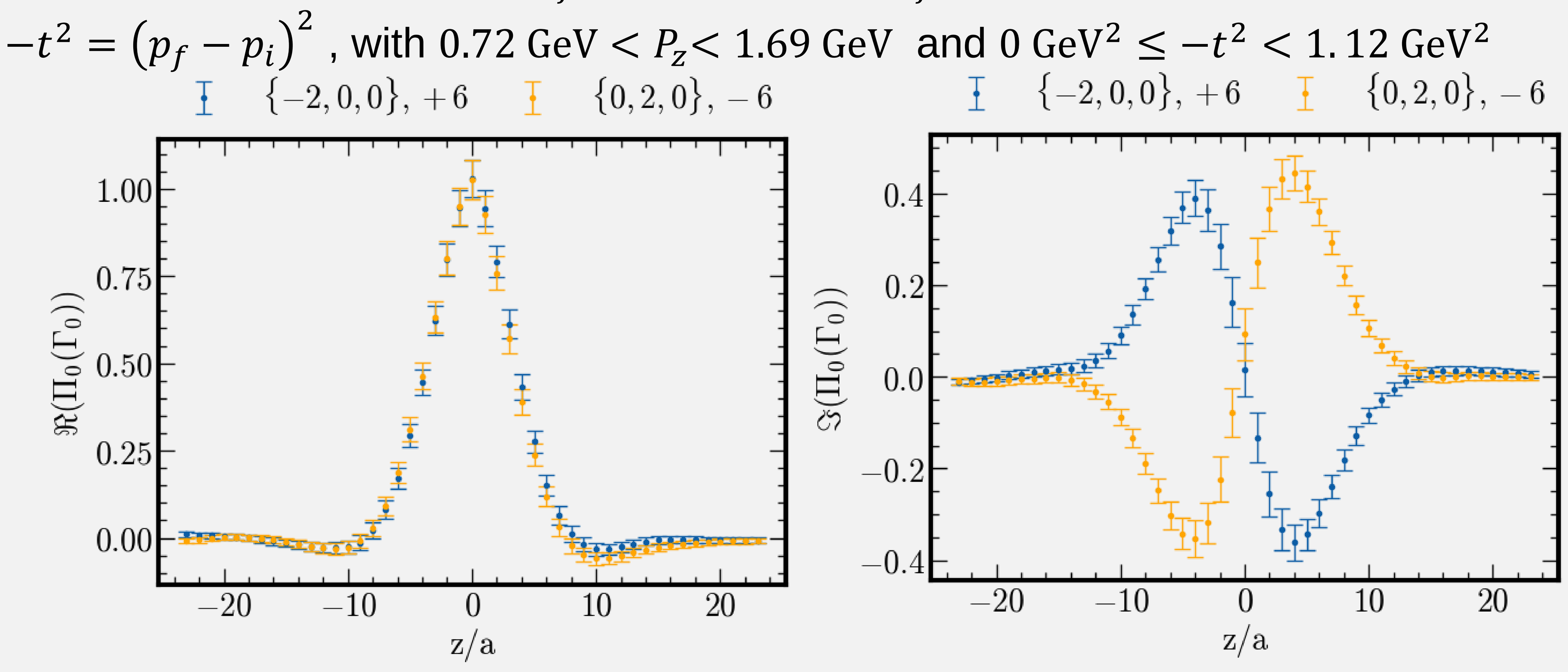

Fig. 2 Real and imaginary part of the bare matrix elements for Γ^0,γ_0. In legend is shown $\{\Delta_1,\Delta_2,0\},P_z$ (lattice units). This P_z corresponds to 1.45 GeV, $-t^2 = 0.24$ GeV²

We combine[2] the matrix elements to get the quasi-GPDs. We renormalize using RI/MOM prescription at $\mu = 2$ GeV.

$$\mathcal{G}_{ren}(z) = \mathrm{Z}(z)\cdot\mathcal{G}_{bare}(z)$$

We have to reconstruct the x-dependence of the GPDs. However, the Fourier transform from a finite set into a continuous space suffers from the so-called inverse problem. We overcome this issue with the Backus-Gilbert technique [4] to reconstruct it.

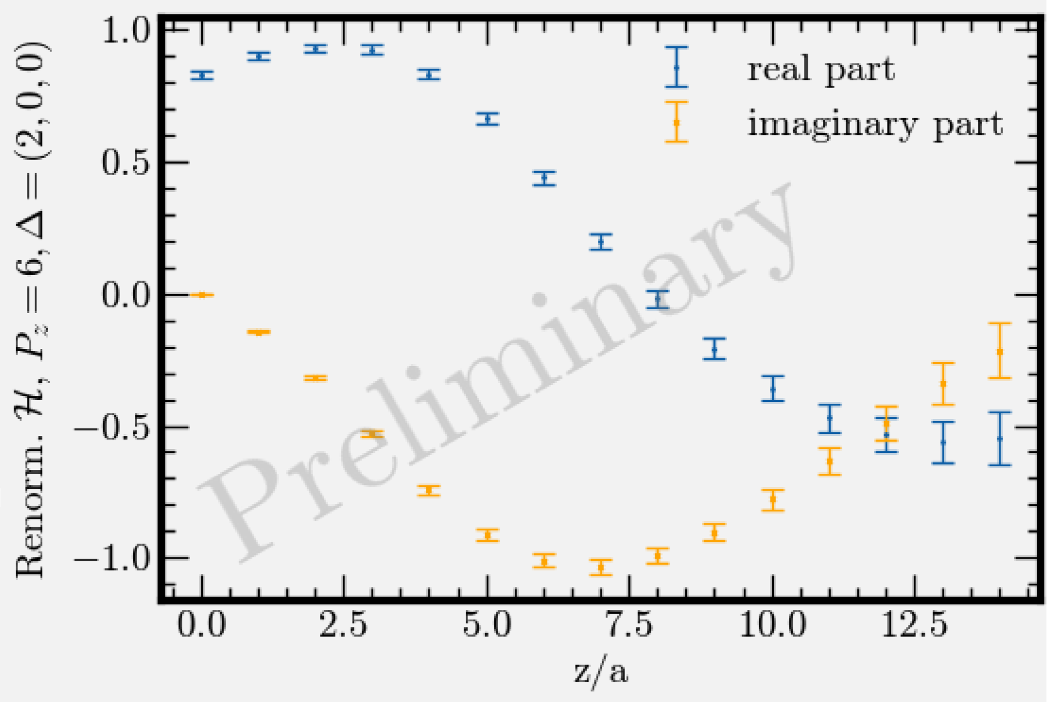

Fig.3 Renormalized quasi-H GPD. This P_z corresponds to 1.45 GeV, $-t^2 = 0.24$ GeV²

4 Results and conclusion

We match[3] the quasi-GPDs to the light cone. We do so also for the PDF, here shows in Fig. 4. For the quasi-GPDs approach, it is crucial to get convergence at large boosts: the large boosts are the light-cone limit.

We use the pseudo approach to extract the moments. We consider

$$\mathcal{M}(z,P,\Delta) = \frac{\mathcal{G}_{bare}(\vec{z},\vec{P},\vec{\Delta})}{\mathcal{G}_{bare}(\vec{z},\vec{P}=0,\vec{\Delta}=0)} = = \sum_{n=0}\frac{(-izP)^n}{n!}\frac{C_n^{\overline{MS}}(\mu^2z^2)}{C_0^{\overline{MS}}(\mu^2z^2)}\langle x^n\rangle + \mathcal{O}(\Lambda^2_{QCD}z^2)$$

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where we use the bare GPDs and the ratio scheme renormalization [1].

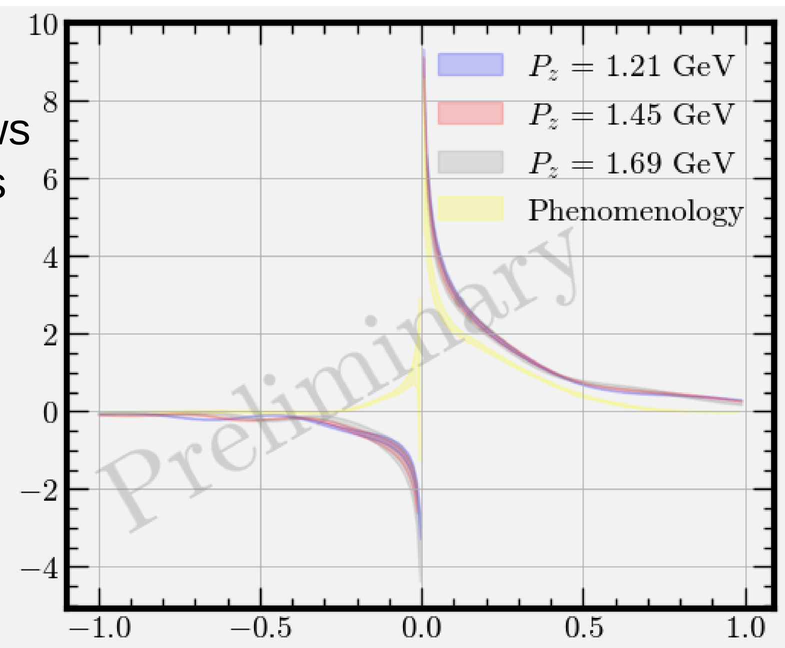

Fig. 4 Reconstructed and matched light-cone PDF, compared to phenomenological results

In conclusion:
- Good statistical results at physical mass
- Statistical error under control also for large boosts
- Convergence in the light-cone at large boosts
- Extraction of excited states and continuum limit planned to improve our results

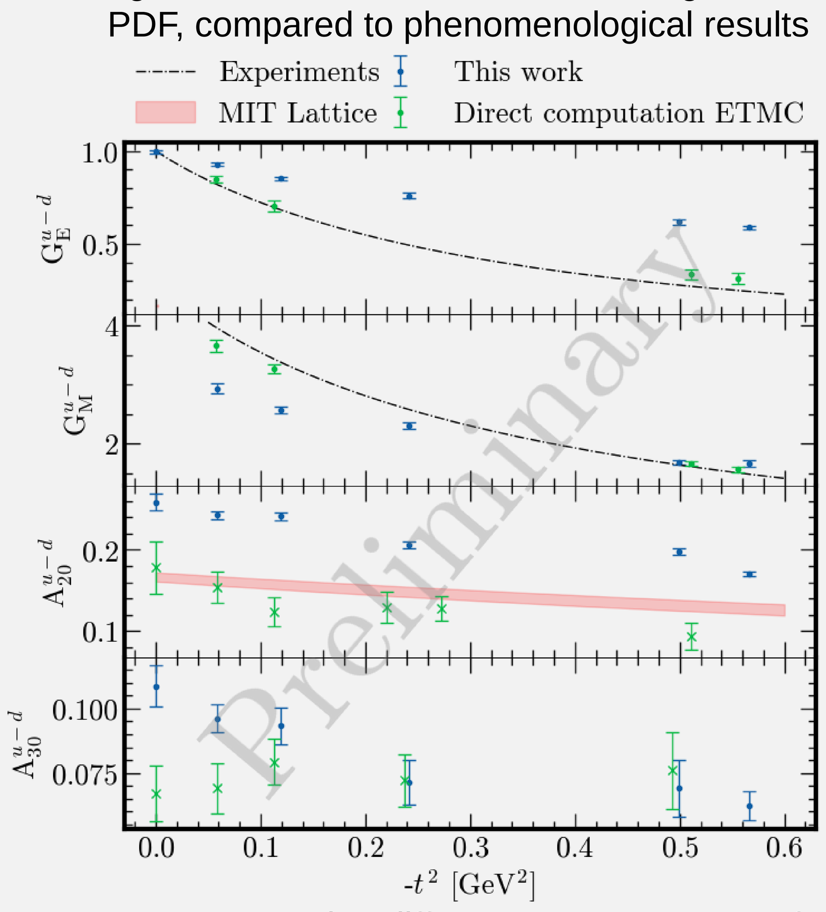

Fig. 5 Moments extracted at different momentum transfers

5 Outlook

- Computation and analysis of smaller boosts
- Implementation of the hybrid renormalization[5]
- Improvements on the Backus-Gillbert technique
- Axial and isoscalar analysis, non-zero ξ
- Excited states analysis
- Quark-disconnected and gluon GPDs
- Continuum limit

References

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Funded by the European Union

This project has received funding from the European Union's research and innovation programme under the Marie Skłodowska-Curie Doctoral Networks action and Grant Agreement No 101072344.