Coherence: a Resource for Quantum Algorithms in Quantum Computing

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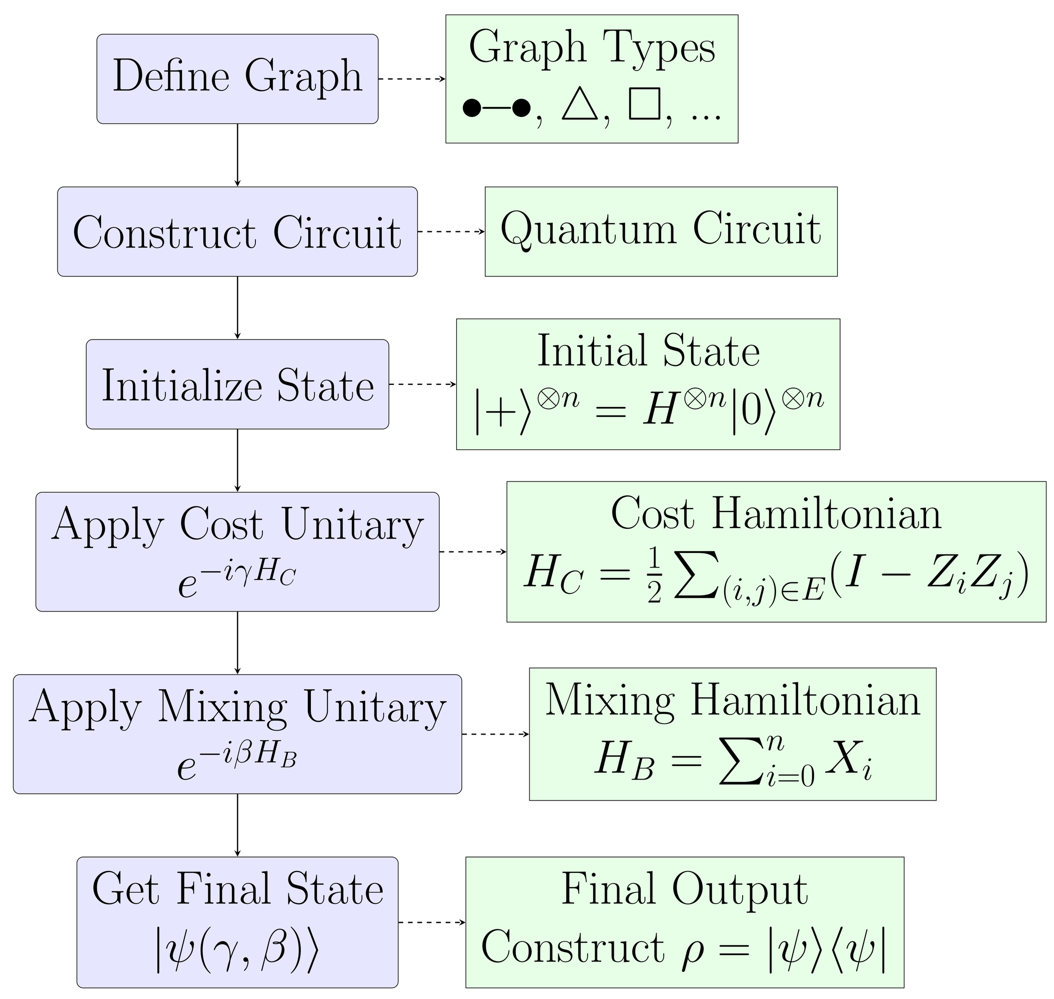
EINN2025

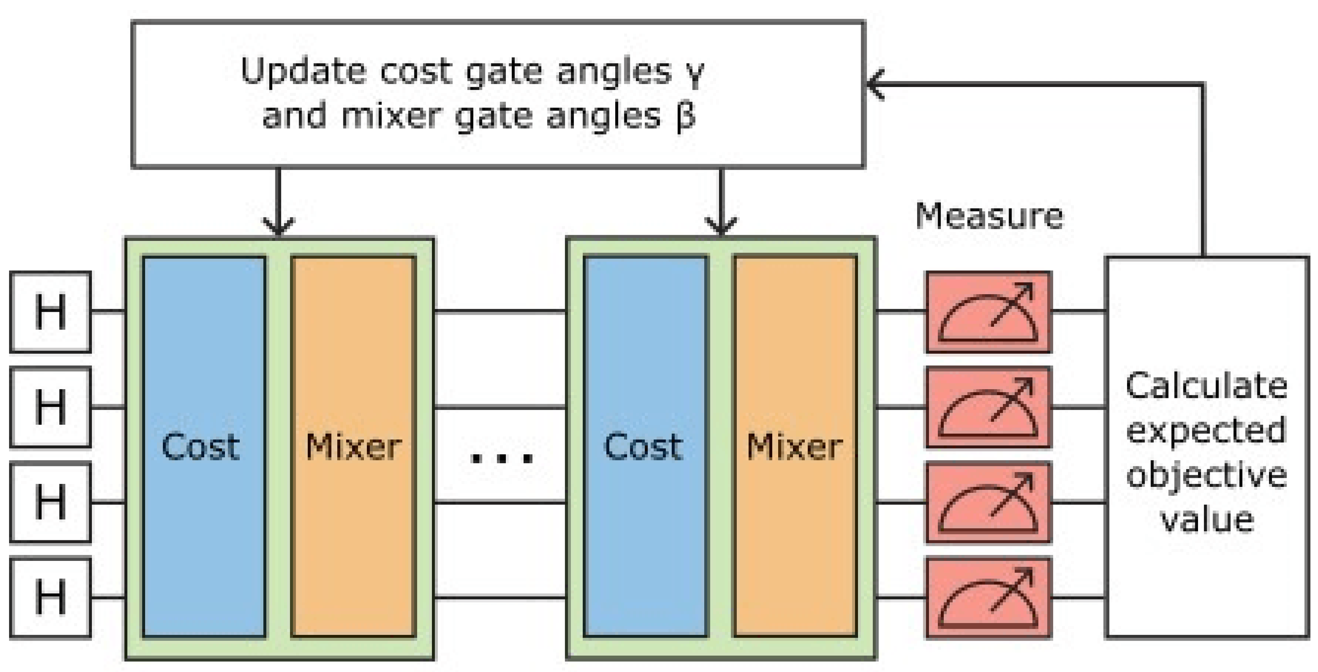
Motivation

- **Quantum computing** uses principles from quantum mechanics to potentially solve certain problems that classical computers find very hard or slow to handle.
- **Helpful in areas** like optimization, cryptography and simulating quantum systems.
- **Resource** for quantum computing is quantum coherence.
- **Role of coherence** in understanding the potential power of quantum computing.
- **Coherence** creation and detection in different quantum algorithms.
- **How** to manage and analyze coherence in quantum systems [1].
- **Goal** is to strengthen quantum computing by preserving and utilizing coherence.

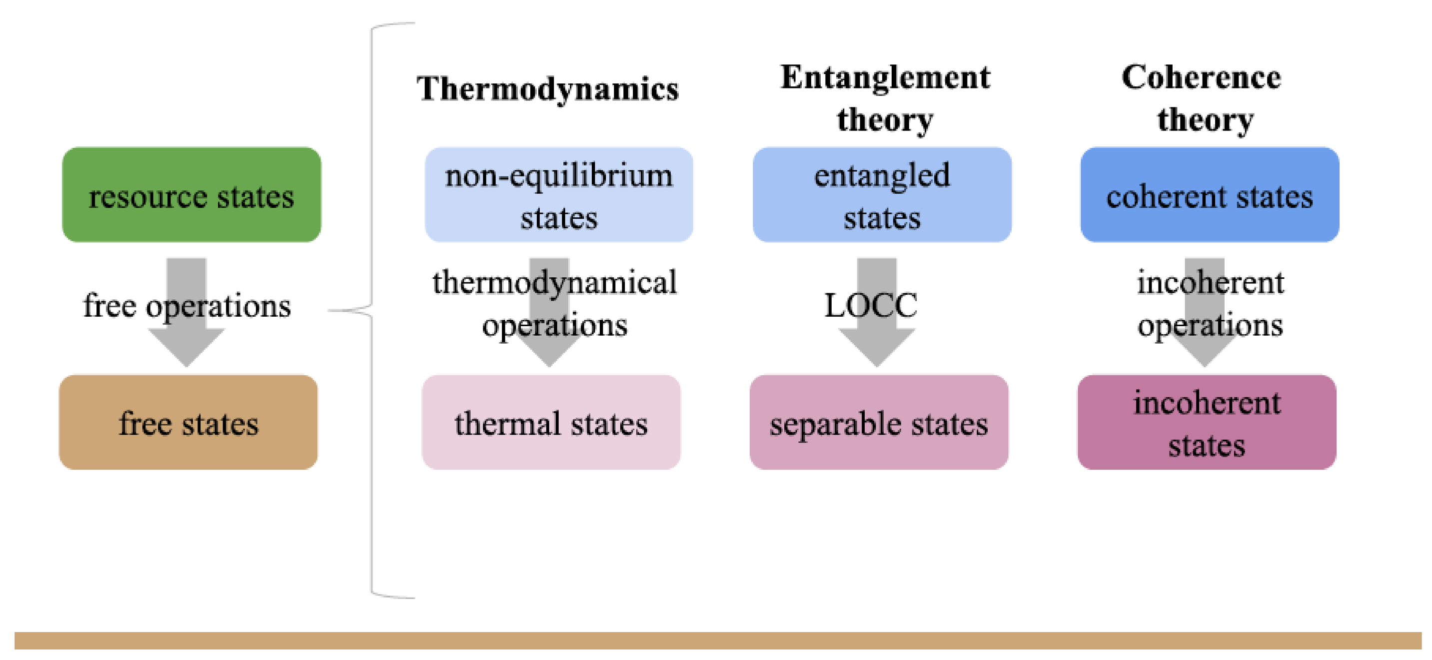
I. Quantum Approximate Optimization Algorithm (QAOA)



II. QAOA Circuit



III. Resource Theory Fundamentals



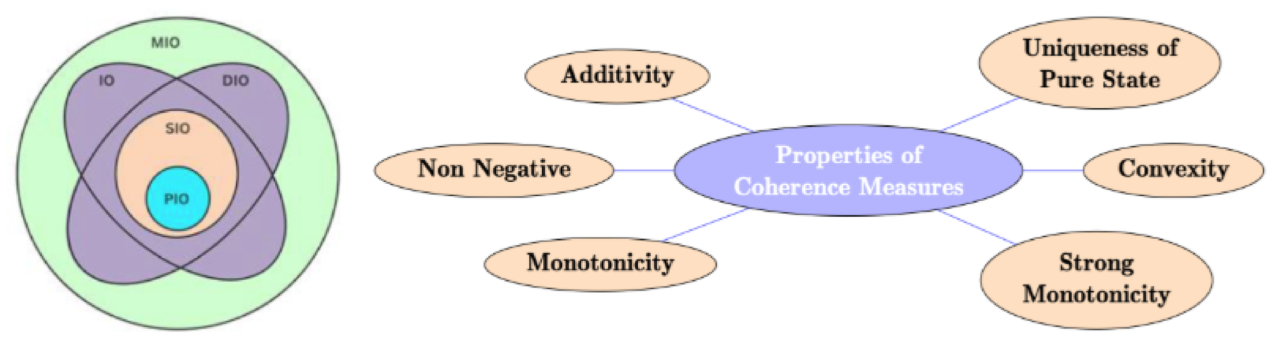
References

[1] G. Adesso A. Streltsov and B. Plenio M. "Quantum coherence as a resource". In: *Rev. Mod. Phys.* (2017).

[2] F. Ahnefeld et al. "Coherence as a Resource for Shor's Algorithm". In: *Physical Review Letters* (2022).

[3] Eric Chitambar and Gilad Gour. "Quantum resource theories". In: *Reviews of Modern Physics* (2019).

IV. Properties of Coherence Measures

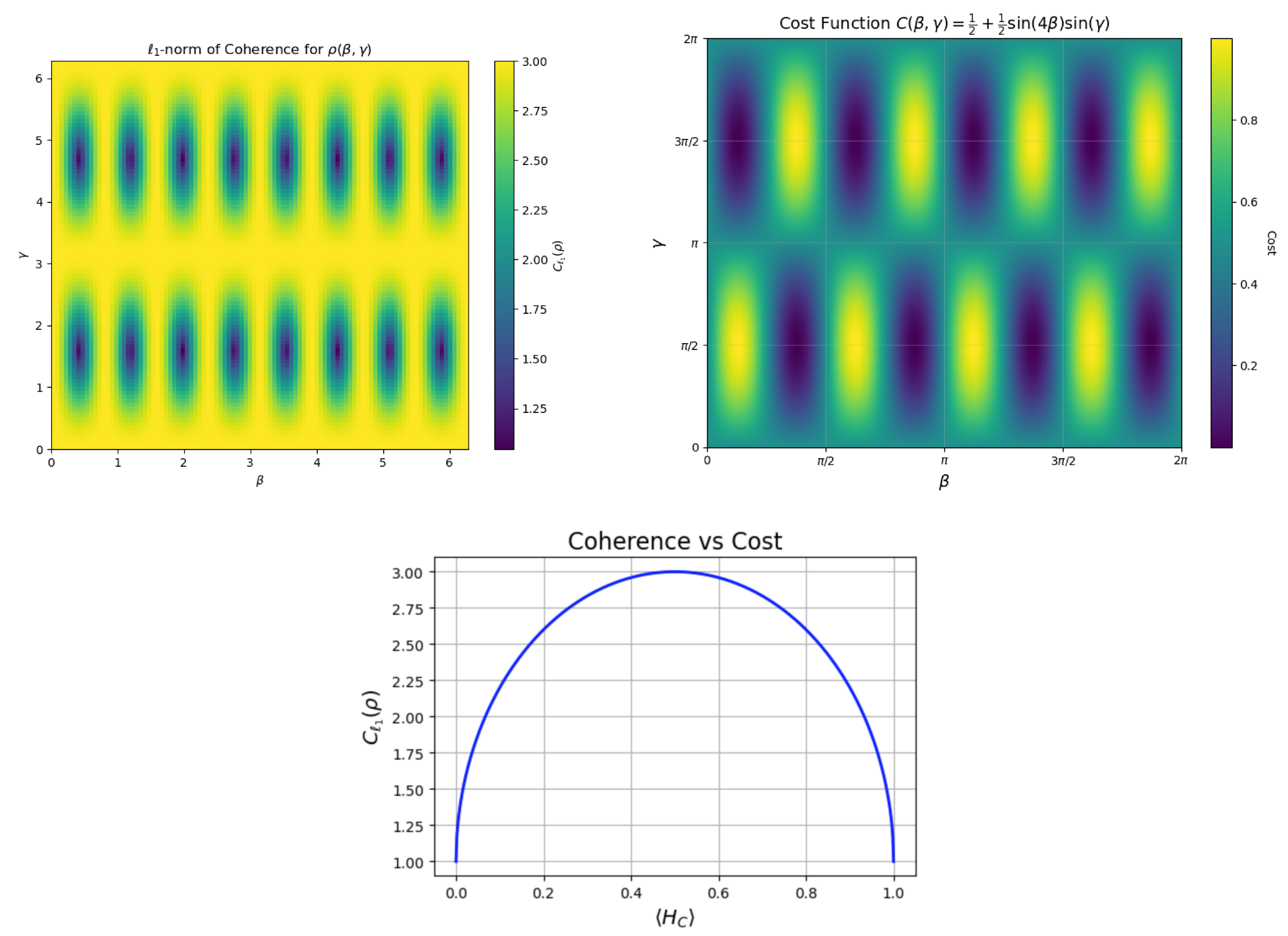


V. Coherence Measures

- **l_1 Norm of Coherence:**
For a density matrix ρ, the ℓ_1-norm of coherence [1] is the sum of the absolute values of its off-diagonal elements:

$$C_{l_1}(\rho) = \sum_{z\neq z'} |\rho_{zz'}| = \sum_{z\neq z'} |\langle z|\rho|z'\rangle| \tag{1}$$

- **Preliminary Two Qubit Results:**



- **Creation of Coherence:**
In quantum resource theory [2], coherence is treated as a resource that can be manipulated using quantum channels. To quantify coherence, one important measure is the resource generation capacity, $\mathscr{C}(\Theta)$, which quantifies how well a quantum channel Θ can generate coherence from an incoherent state:

$$\mathscr{C}(\Theta) = \max_{\tau\in\mathcal{I}} C(\Theta\tau), \tag{2}$$

based on the robustness of coherence [3]

$$C(\rho) = \min\left\{ s \geq 0 : \frac{\rho + s\tau}{1+s} \in \mathcal{I}, \tau \text{ a state} \right\}, \tag{3}$$

here, τ is an incoherent state, and $\mathcal{I}$ represents the set of incoherent states.

- **Detection of Coherence:**
To detect coherence, we use the NSID measure $\tilde{M}_\diamond(\Lambda)$, which quantifies how well a quantum channel Λ can detect coherence. The measure is defined as:

$$\tilde{M}_\diamond(\Lambda) = \min_{\Phi\in\mathcal{DI}} \max_{\sigma} \|\Delta(\Lambda - \Phi)\sigma\|_1, \tag{4}$$

where Δ is the dephasing operation. This measure captures how much coherence can be detected in the output of a quantum channel after applying an incoherent channel.

Future Directions

- Increase the system size, analyze different coherence measures for example robustness of coherence, relative entropy of coherence and the performance of the different algorithms.

Acknowledgement

This project has received funding from the European Union's HORIZON research and innovation programme HORIZON-WIDERA-2022-TALENTS-01 under grant agreement No. 101087126.

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Funded by the European Union