Subtracted dispersion relations for virtual Compton scattering off the proton

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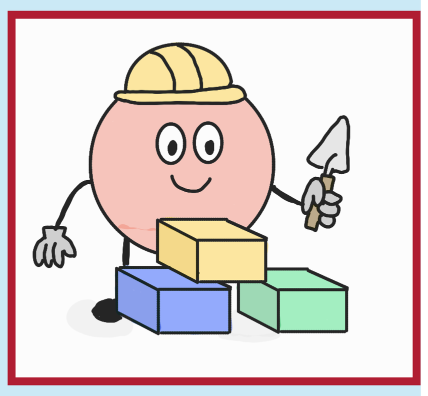
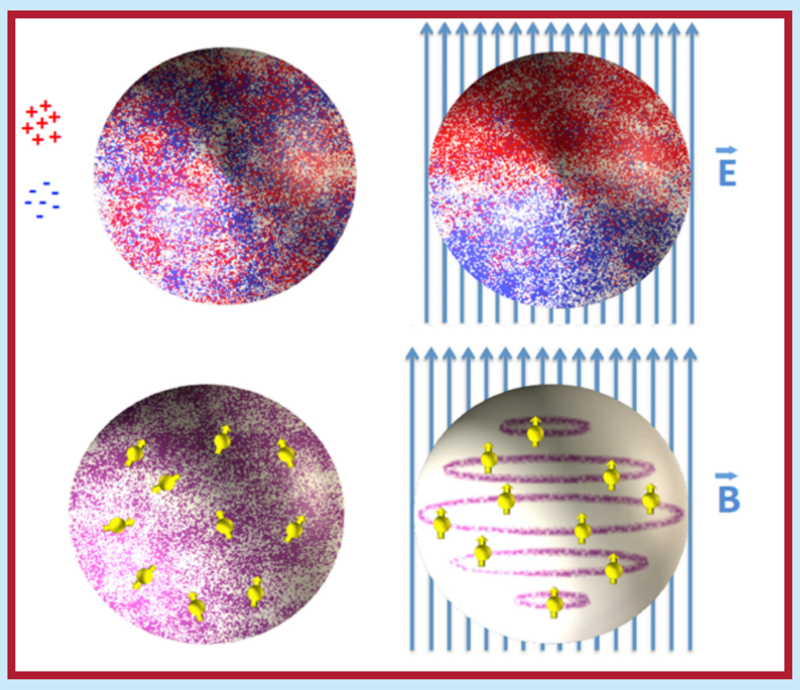
The proton: what's still to learn?

The proton is the only stable composite particle in nature and the fundamental **building block** of matter

Studying the **proton's structure** is crucial for improving our understanding of the formation of matter through the strong interaction





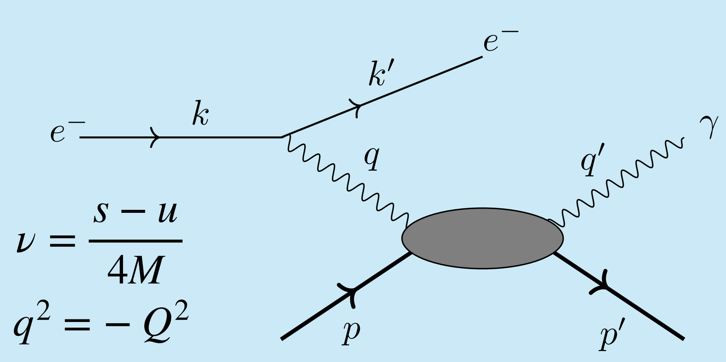
Real Compton scattering (RCS)	Virtual Compton scattering (VCS)
$\vec{D}_E \sim \alpha_{E1}\vec{E}$	$\vec{D}_E \sim \alpha_{E1}(Q^2)\vec{E}$
$\vec{D}_M \sim \beta_{M1}\vec{B}$	$\vec{D}_M \sim \beta_{M1}(Q^2)\vec{B}$

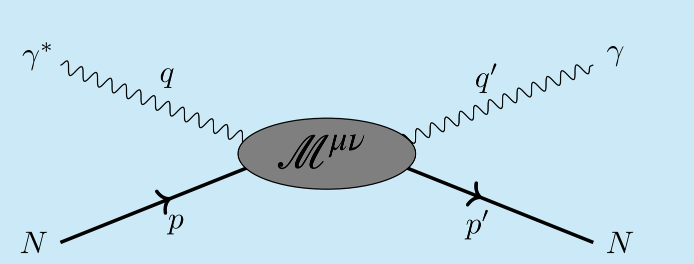
Q^2: virtuality of the initial photon

An important property is its response to an external electromagnetic field, parametrized by static polarizabilities, α_{E1} and β_{M1}.
Generalized polarizabilities (GPs), $\alpha_{E1}(Q^2)$ and $\beta_{M1}(Q^2)$, map out the spatial distribution of the induced polarization.

How can we extract GPs from VCS?

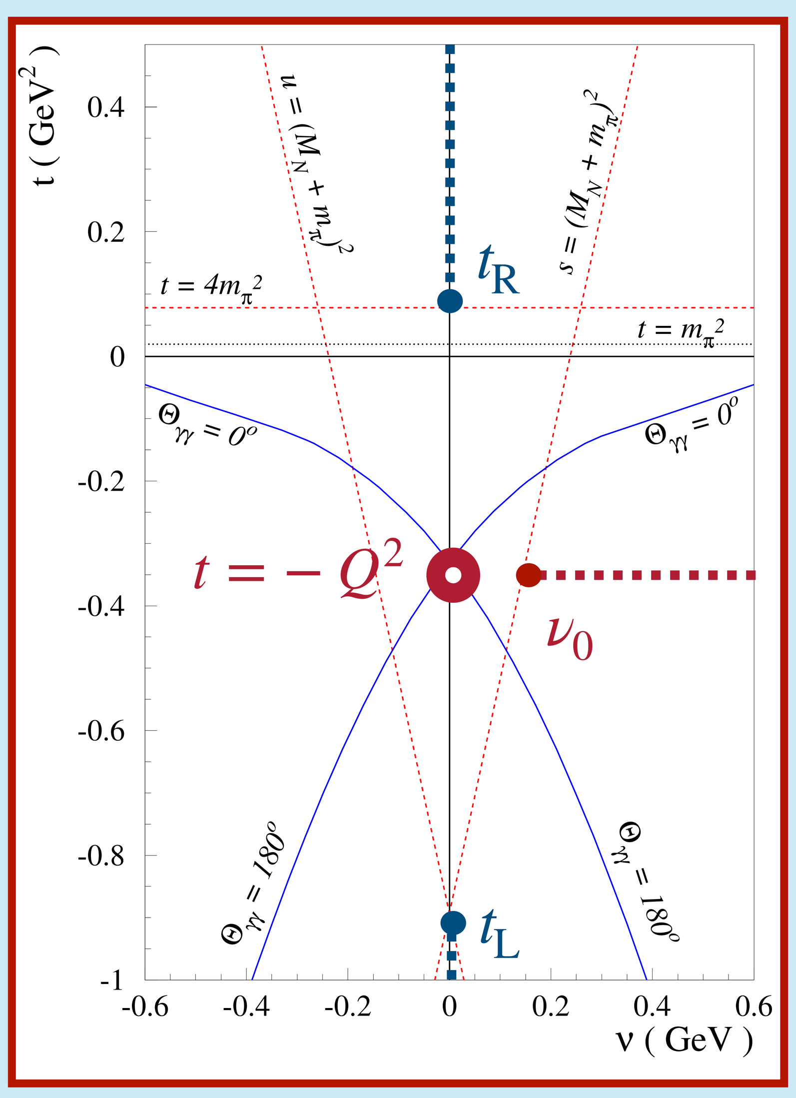
Through photon electroproduction off the proton, $ep \to ep\gamma$, we study the subprocess known as VCS ($\gamma^* p \to \gamma p$)





$$\mathcal{M}^{\mu\nu} = \sum_{i=1}^{12} F_i(Q^2,\nu,t)\,\tilde{\rho}_i^{\mu\nu} \quad \to \quad F_i(Q^2,\nu,t) = F_i(Q^2,-\nu,t), \quad \forall i$$

F_i scalar amplitudes



F_i are:
- free of kinematical singularities
- even in the variable ν

GPs are defined at the kinematical point $\nu = 0$ and $t = -Q^2$

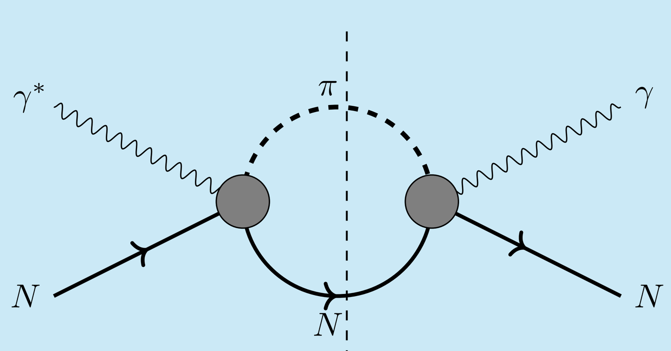
$$\boxed{F_i^{NB}(Q^2,0,-Q^2)}$$

Assuming analyticity and appropriate high-energy behavior
↓
We write unsubtracted **dispersion relations (DR)**

[B. Pasquini et al., EPJA 11, 185-208 (2001)]

$$\mathrm{Re}\,F_i^{NB}(Q^2,\nu,t) = \frac{2}{\pi}\mathscr{P}\int_{\nu_0}^{\infty} d\nu' \frac{\nu'\,\mathrm{Im}_s F_i(Q^2,\nu',t)}{\nu'^2 - \nu^2}$$

- **Low energies:** effective field theories
- **Intermediate region (~100 MeV to 1-2 GeV):** dominated by resonances, no complete theoretical description
- **High energies:** perturbative QCD



New way of extracting GPs

From Regge theory, F_1 and F_5 need a subtraction

[B. Pasquini et al., Phys. Rev. C62 (2000)]

Subtracted DR in ν at fixed t:

$$\mathrm{Re}\,F_i^{NB}(Q^2,\nu,t) = F_i^{NB}(Q^2,0,t) + \frac{2}{\pi}\nu^2\mathscr{P}\int_{\nu_0}^{\infty} d\nu' \frac{\mathrm{Im}_s F_i(Q^2,\nu',t)}{\nu'(\nu'^2 - \nu^2)}$$

"Price to pay" → subtraction function

$$F_i^{NB}(Q^2,0,t) = \boxed{F_i^{NB}(Q^2,0,-Q^2)} + \text{pole contribution}$$

GPs are free parameters

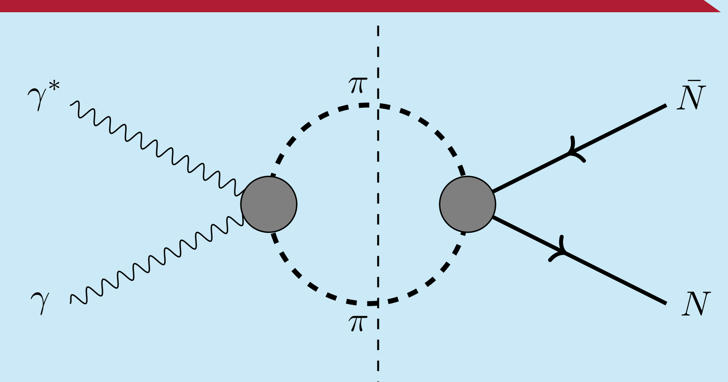
$$+\frac{(t+Q^2)}{\pi}\mathscr{P}\int_{t_R}^{\infty} dt' \frac{\mathrm{Im}_t F_i(Q^2,0,t')}{(t'+Q^2)(t'-t)} \quad :: \text{right hand cut (RHC)}$$

$$+\frac{(t+Q^2)}{\pi}\mathscr{P}\int_{-\infty}^{t_L} dt' \frac{\mathrm{Im}_t F_i(Q^2,0,t')}{(t'+Q^2)(t'-t)} \quad :: \text{left hand cut (LHC)}$$

Model dependence in the convergence of the integrals → reduced

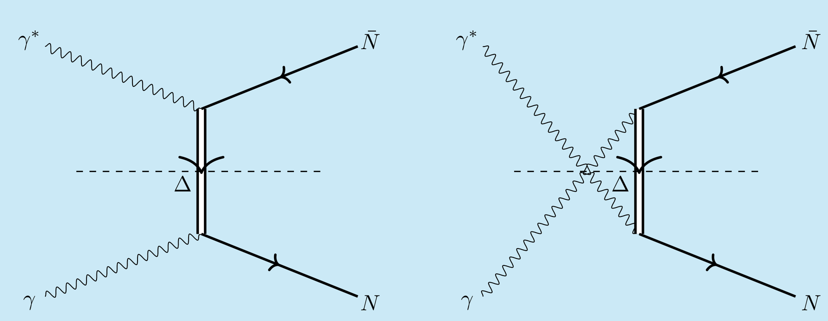
Evaluation of RHC and LHC

$$\text{RHC} :: \mathscr{P}\int_{t_R}^{\infty} dt' \frac{\mathrm{Im}_t F_i(Q^2,0,t')}{(t'-t)}$$



1) **Crossing symmetry:** integrand linked to t-channel VCS, $\gamma\gamma^* \to N\bar{N}$, helicity amplitudes
2) **Unitarity:** imaginary part expressed via the partial waves of the two subprocesses $\gamma\gamma^* \to \pi\pi$ and $\pi\pi \to N\bar{N}$

$$\text{LHC} :: \mathscr{P}\int_{-\infty}^{t_L} dt' \frac{\mathrm{Im}_t F_i(Q^2,0,t')}{(t'-t)}$$



Left-hand cut discontinuity modeled by the spectral function for the $\Delta(1232)$-resonance excitation in s- and u- channel for the VCS process

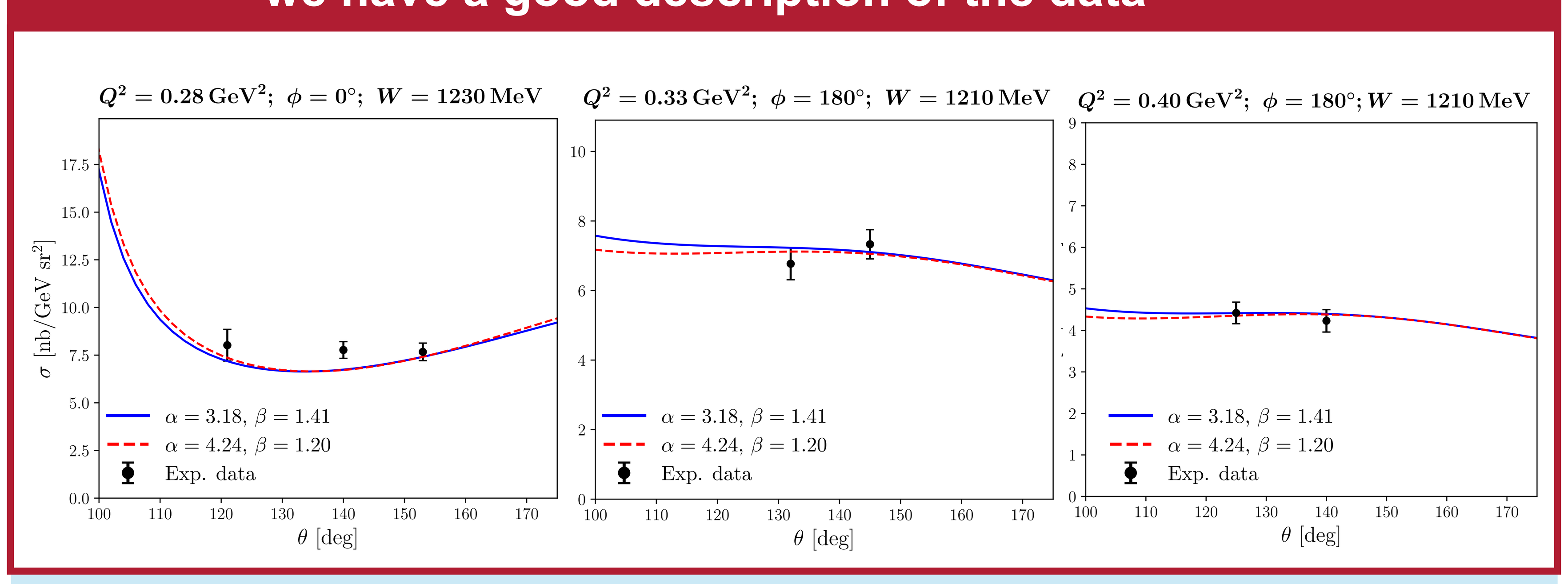
First results

We present the cross-section prediction as a function of the scattering angle θ at fixed Q^2,W and ϕ

W: total centre-of-mass energy of the $(\gamma^* p)$ system

ϕ: azimuthal angle

At low energies, below $\Delta(1232)$ resonance, we have a good description of the data



In this approach the **GPs** are input parameters that need to be obtained from external sources; in the future they will be fitted to experimental data

Values of $\alpha_{E1}(Q^2)$ and $\beta_{M1}(Q^2)$ taken from [R. Li et al., Nature 611 (2022)]

Any further questions? Please contact me: ronchima@uni-mainz.de

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