Proton and neutron electromagnetic form factors from lattice QCD in the continuum limit

Constantia Alexandrou[1,2], Simone Bacchio[1], Jacob Finkenrath[3,4], Christos Iona[1,2], Giannis Koutsou[1], Ferenc Pittler[1], Bhavna Prasad[1], and Gregoris Spanoudes[2]

[1]The Cyprus Institute, [2]University of Cyprus, [3]CERN, [4]University of Wuppertal

Motivation

- Nucleons are major contributors of the visible universe yet discrepancy in measurement exists e.g., *Proton radius puzzle*.
- Electromagnetic form factors using lattice QCD probes fundamental quantities such as moments and radii.
- Enables the computation of experimentally less-precise or unprobed fundamental observables.

Lattice setup

- We use gauge ensembles with twisted-mass fermions at multiple lattice spacings:

Table 1. N_f=2+1+1 physical point ensembles analyzed here.

Ensemble	$(\frac{L}{a})^3 \times (\frac{T}{a})$	a [fm]	m_π [MeV]	$m_\pi L$
B	$64^3 \times 128$	0.07957(13)	140.2(2)	3.62
C	$80^3 \times 160$	0.06821(13)	136.7(2)	3.78
D	$96^3 \times 192$	0.05692(12)	140.8(2)	3.90
E(only s)	$112^3 \times 224$	0.04892(11)	136.5(2)	3.79

- The form factors are given in terms of the nucleon matrix element with electromagnetic current insertion:

$$\langle N(p',s')|j_\mu|N(p,s)\rangle = \sqrt{\frac{m_N^2}{E_N(\vec{p}\,')E_N(\vec{p})}} \times \bar{u}_N(p',s')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2)\right]u_N(p,s) \quad (1)$$

- We present results on the Sachs form factors:

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_N^2}F_2(q^2)\,,$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2)\,. \quad (2)$$

- The corresponding radius can then be written as:

$$\langle r_X^2\rangle^f = \frac{-6}{G_X^f(0)}\frac{\partial G_X^f(q^2)}{\partial q^2}\Big|_{q^2=0}. \quad (3)$$

where $X = E,\ M$ and, $f =$ proton, neutron.

Two- and three-point functions

- The form factors can be accessed by obtaining the ground-state matrix element of Eq. 1.

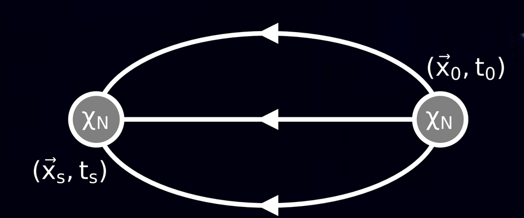

Figure 1. Nucleon two-point function.

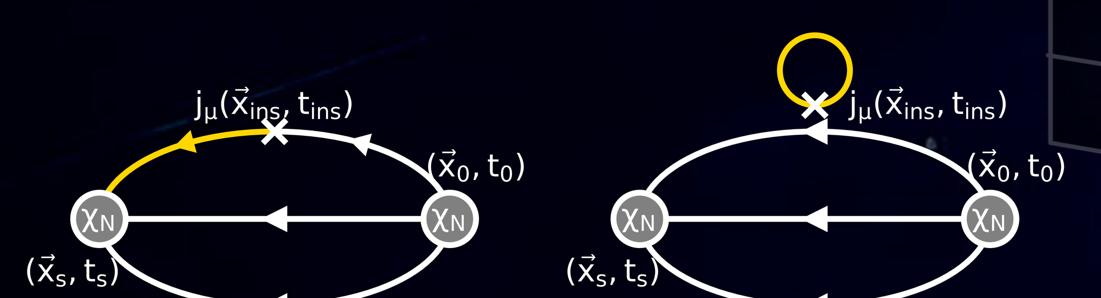

Figure 2. Connected (left) and disconnected (right) contributions to the nucleon three-point function.

- It is computed via two- and three-point correlation functions.
- Spectral decomposition of two- and three-point functions are:

$$C(\vec{p}, t_s) = \sum_n c_n(\vec{p})e^{-E_n(\vec{p})t_s}, \quad (4)$$

$$C_{\mu,\nu}(\vec{q}, \vec{p}\,'; t_s, t_{ins}) = \sum_{n,m} A^{n,m}_{\mu,\nu}(\vec{q}, \vec{p}\,')e^{-E_n(\vec{p}\,')(t_s - t_{ins}) - E_m(\vec{q})t_{ins}}.$$

- The ground-state matrix is then given by $\frac{A^{0,0}_{\mu,\nu}(\vec{q},\vec{p}\,')}{\sqrt{c_0(\vec{p}\,')c_0(\vec{q})}}$.
- We obtain $A^{0,0}_{\mu,\nu}$ and c_0 using multi-state fitting.
- Multi-state fits results are model averaged [1] for each Q^2.

Proton and Neutron results

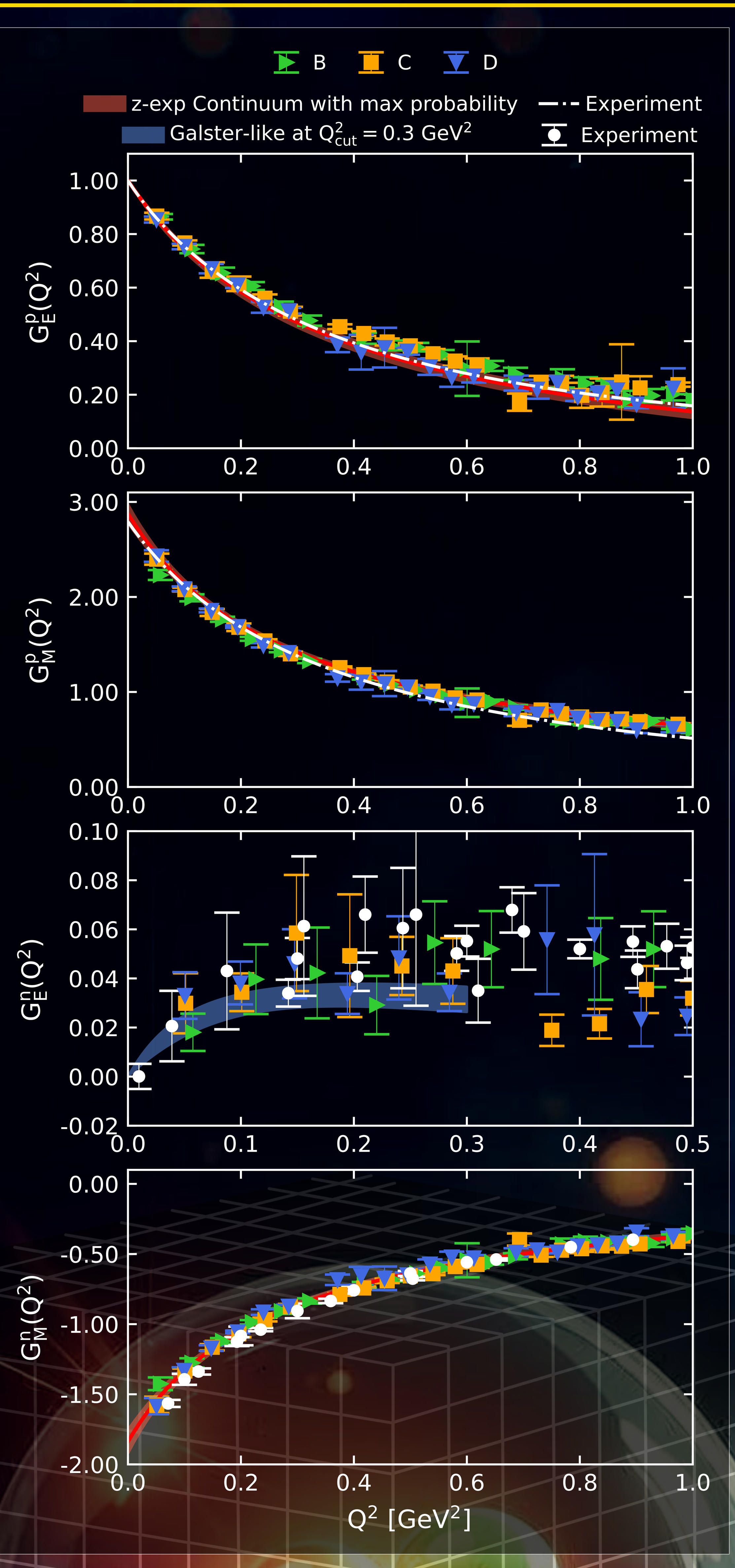

Figure 3. Results for $G_E(Q^2)$ and $G_M(Q^2)$ for proton (neutron) in first (third) and second (fourth) rows respectively with continuum limit in red except $G_E^n(Q^2)$ where no a^2 dependence is used.

- The connected and disconnected contributions are combined with appropriate renormalization to get the proton and neutron form factors.
- We use multiple Q^2 dependence fit ansaetz such as the dipole form, galster-like, and the z-expansion, all incorporating a^2 dependence in its functional form.

Comparison with other studies

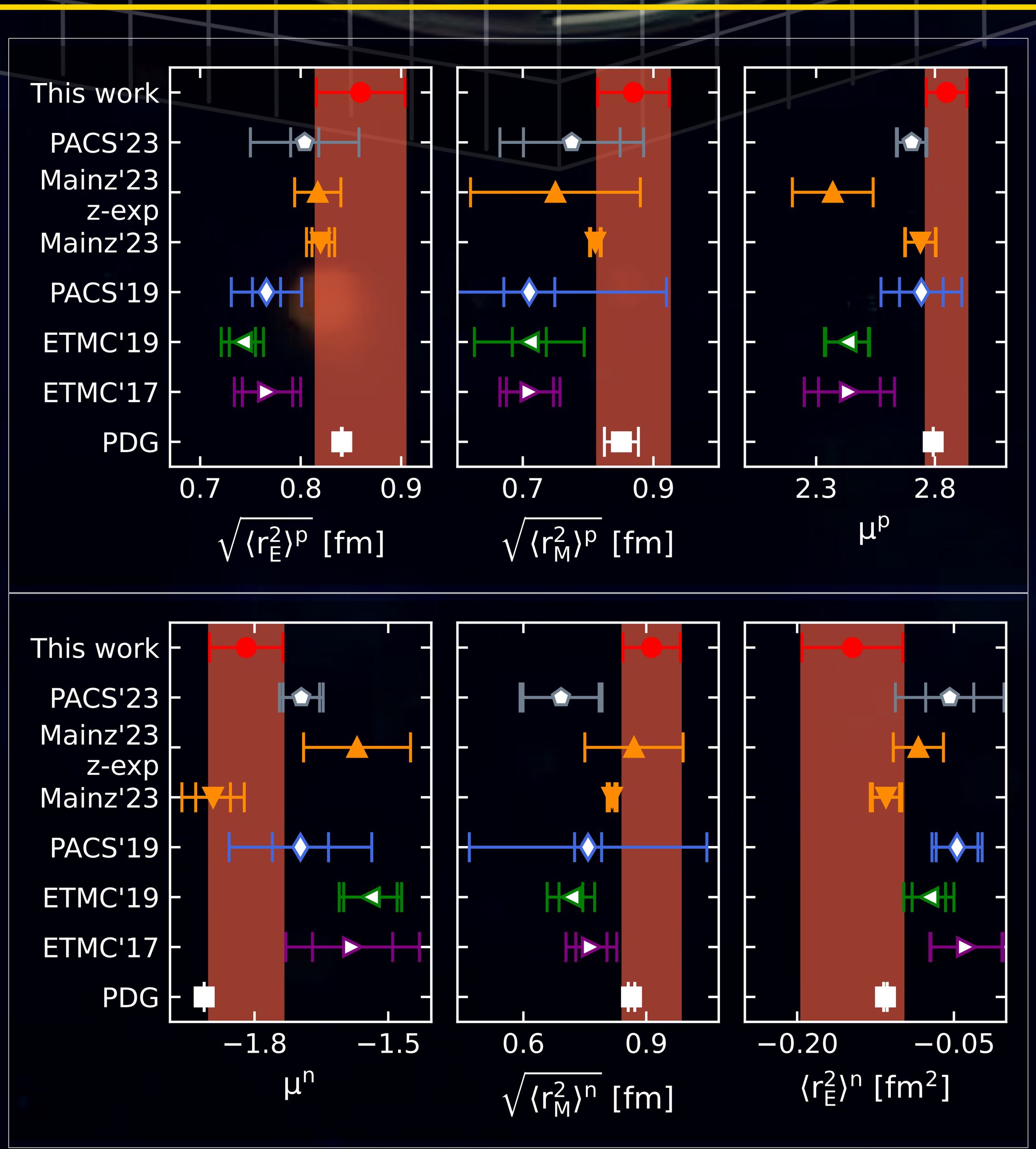

Figure 4. Comparison of radii and magnetic moments with previous works.

Strange Electromagnetic Form Factors

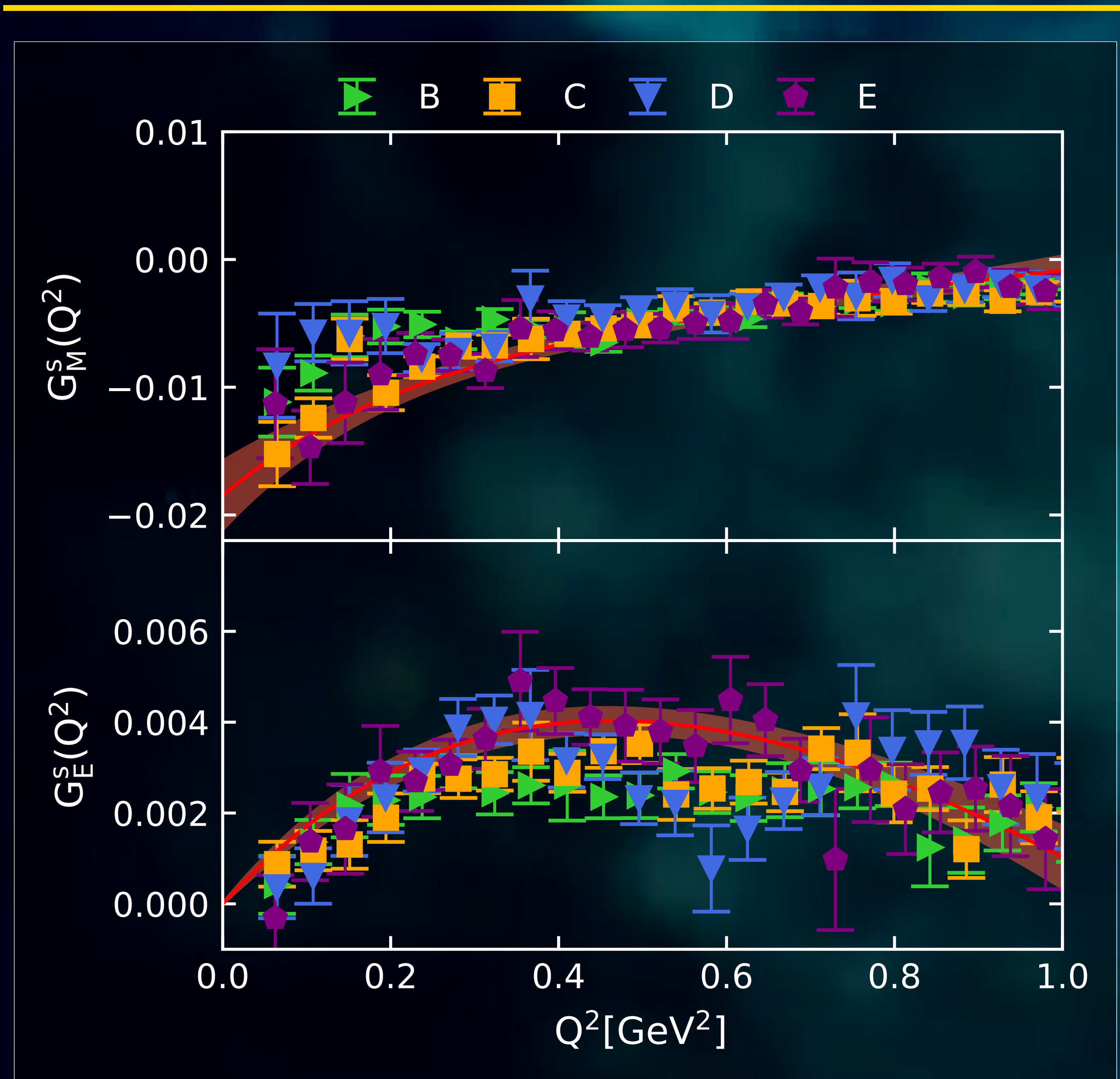

Figure 5. Results for $G_E^s(Q^2)$ and $G_M^s(Q^2)$ with red band denoting continuum limit.

- It provides an understanding of the sea quark dynamics as only disconnected three-point contributes and is a crucial input for parity-violating processes.
- We show preliminary results separately for strange as it is within errors for proton and neutron combination.
- Additional *E* ensemble at a finer lattice spacing is also used to obtain the continuum limit for these results.

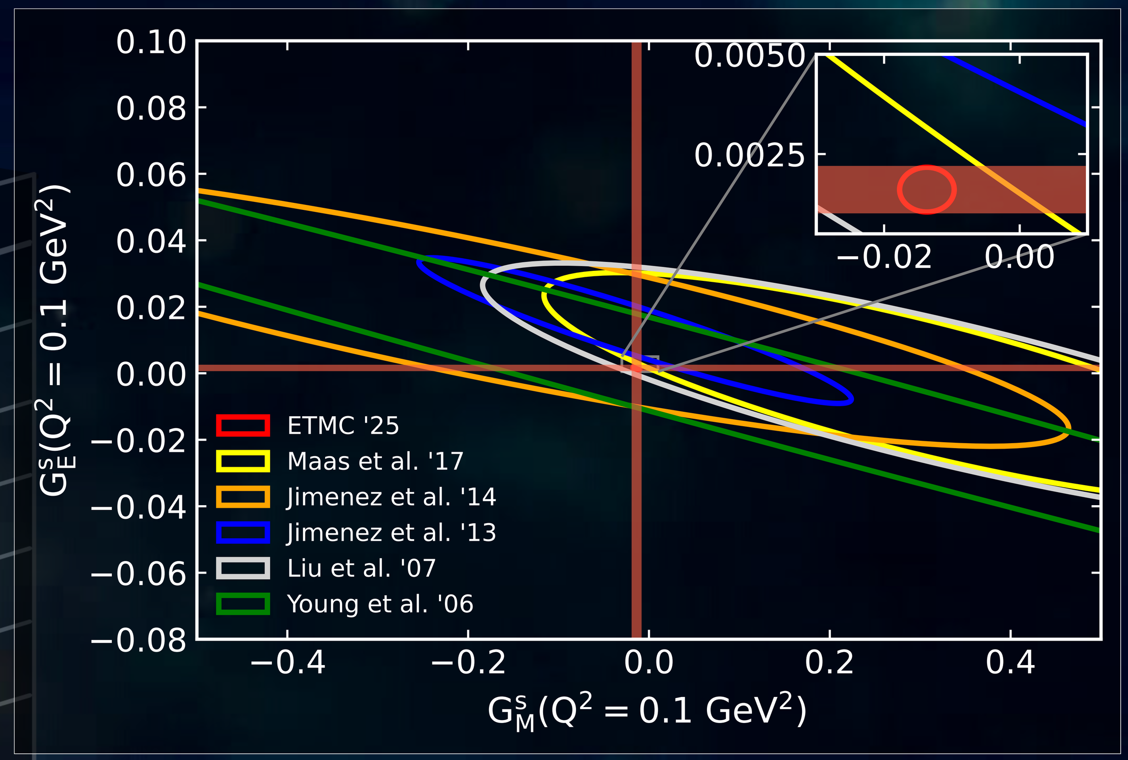

Figure 6. The red bands show the constrains arising from the values of G_E^s and G_M^s at $Q^2 = 0.1$ GeV^2 extracted in this work along with 95% confidence curves from previous works.

- We present a comparison of G_E^s and G_M^s at $Q^2 = 0.1$ GeV^2 with previous works.
- We also provide a comparison with previous lattice works.

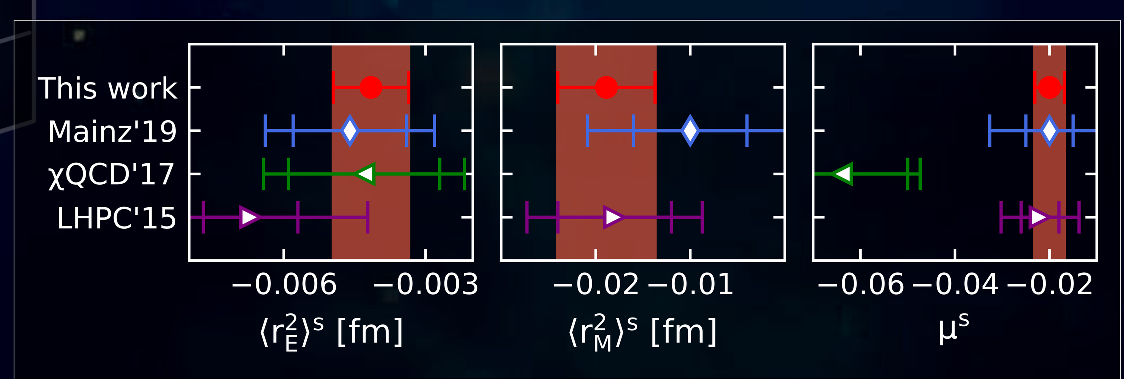

Figure 7. Comparison of current preliminary results with other lattice works.

Conclusions and Future directions

- Results on electromagnetic form factors of proton and neutron have been presented using 3 ensembles of $N_f = 2+1+1$ twisted mass fermions at physical point [2].
- Preliminary results on calculation of the strange nucleon electromagnetic form factors is also shown.
- The analysis of proton and neutron can be extended using the additional E ensemble to reduce error for all quantities.

References

[1] C. Alexandrou, S. Bacchio, M. Constantinou, J. Finkenrath, R. Frezzotti, B. Kostrzewa, G. Koutsou, G. Spanoudes, and C. Urbach. Nucleon axial and pseudoscalar form factors using twisted-mass fermion ensembles at the physical point. *Phys. Rev. D*, 109(3):034503, 2024. doi: 10.1103/PhysRevD.109.034503.

[2] C. Alexandrou, S. Bacchio, G. Koutsou, B. Prasad, and G. Spanoudes. Proton and neutron electromagnetic form factors from lattice QCD in the continuum limit. 7 2025.

Co-funded by the European Union

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 101034267. This work was also supported by the project PulseQCD co-funded by the EU within the framework of the Cohesion Policy Programme "THALIA 2021-2027".

Republic of Cyprus