Subtracted dispersion relation formalism for virtual Compton scattering off the proton

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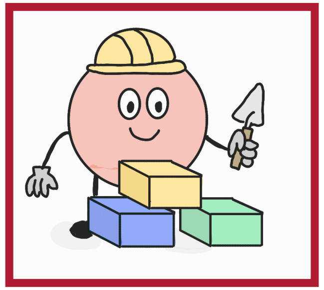
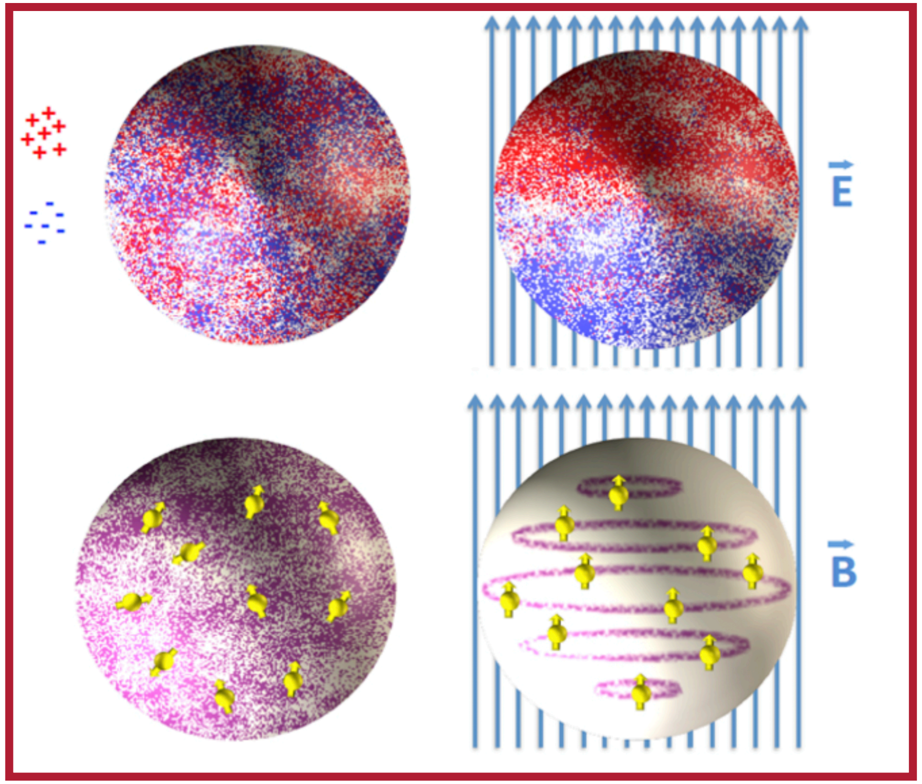
What can we still learn about the proton?



The proton is the **only stable** composite particle in nature and it is the **building block** of matter (easier to study than the neutron)



The critical electric field strength needed to induce any appreciable **polarizability** of the nucleon is $E_{\text{critic}} = 10^{23}\,\text{V/m}$

↓

Estimate of electric field strength of a $100\,\text{MeV}$ photon Compton scattering ($\gamma p \to \gamma p$) from the nucleon is approximately $10^{23}\,\text{V/m}$

[F. Hagelstein et al, Prog. Part. Nucl. Phys. 88 (2016)]

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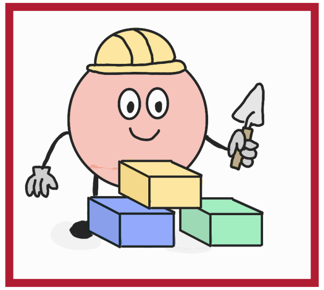

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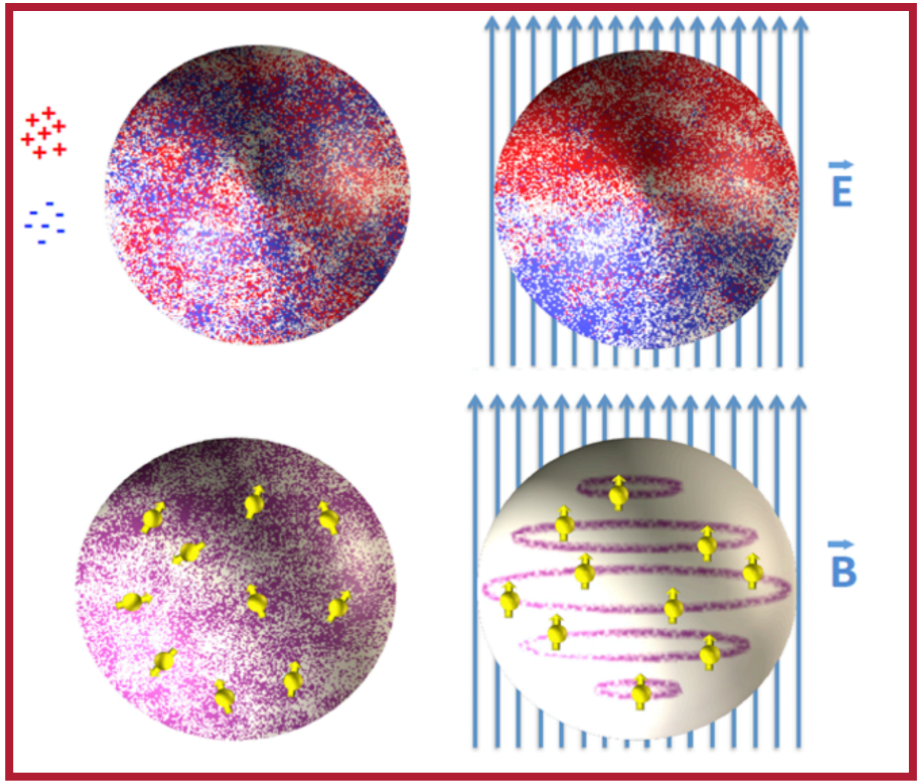


Real Compton Scattering (RCS) $\gamma p \to \gamma p$

$$\vec{D}_E \sim \alpha_{E1} \vec{E}$$

$$\vec{D}_M \sim \beta_{M1} \vec{B}$$

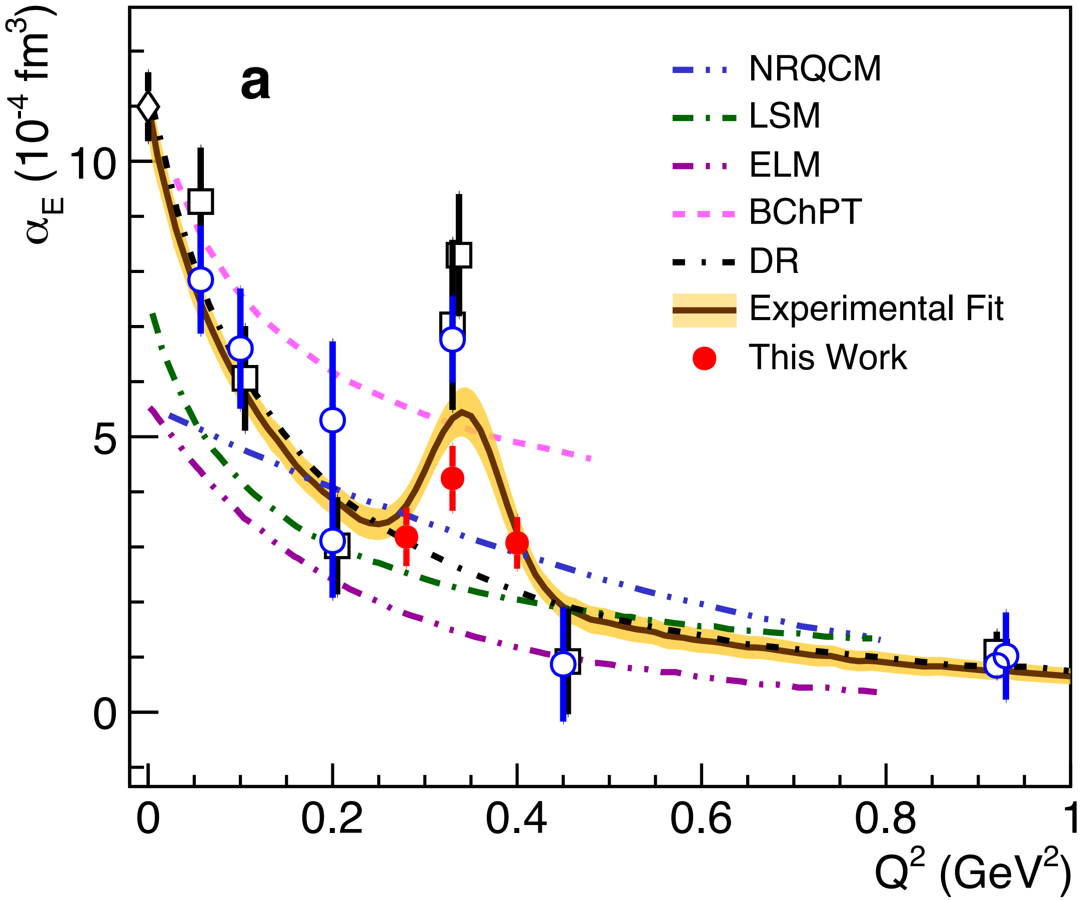
Virtual Compton Scattering (VCS) $\gamma^* p \to \gamma p$

$$\vec{D}_E \sim \alpha_{E1}(Q^2) \vec{E}$$

Q^2: virtuality of the initial photon

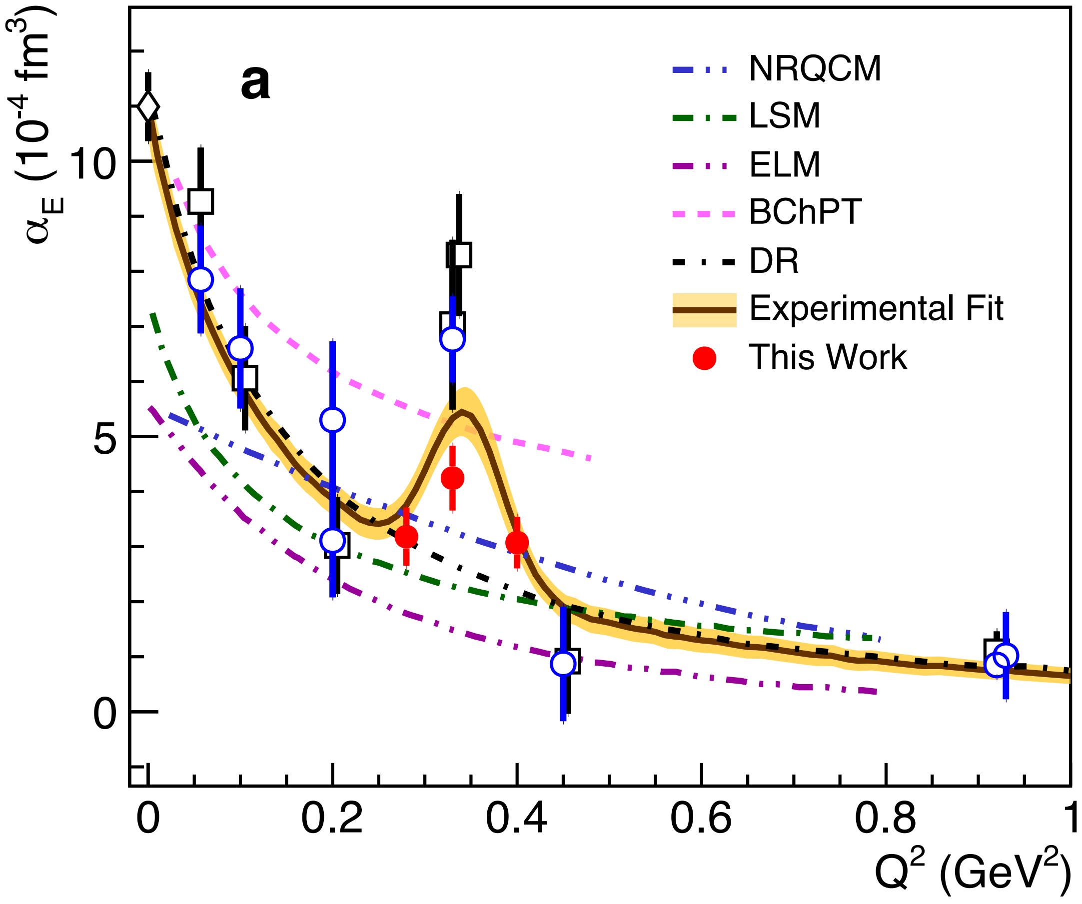
$$\vec{D}_M \sim \beta_{M1}(Q^2) \vec{B}$$

Proton polarizabilities



[R. Li et al., Nature 611, (2022)]

Proton polarizabilities



[R. Li et al., Nature 611, (2022)]

Upcoming experiments: VCS-II@JLab

They will provide measurements of the generalized polarizabilities (GPs) of unprecedented **precision** and a **fine mapping** in Q^2

[N. Sparveris talk on Wednesday]

Aim of the work

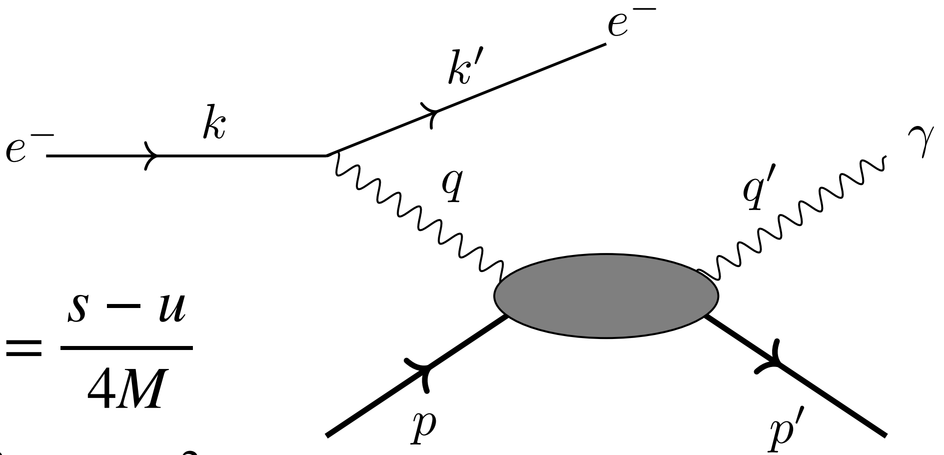
Present a VCS dispersion relations **formalism** to extract GPs up to $\Delta(1232)$ resonance energies

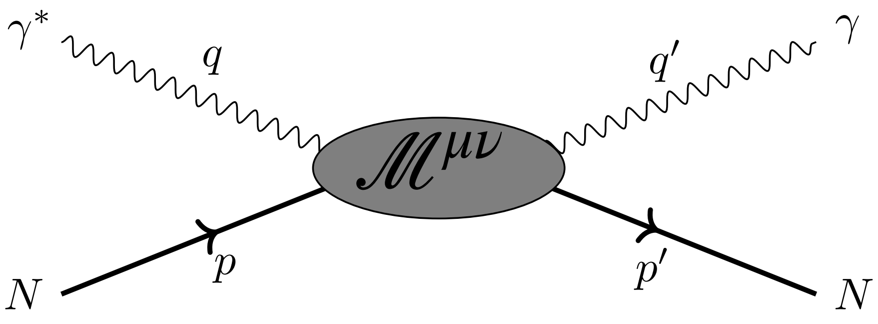
How can extract GPs?

Through photo electroproduction off the proton: $ep \to ep\gamma$, we study the subprocess known as virtual Compton scattering: $\gamma^* p \to \gamma p$ [B. Pasquini et al. EPJA, (2001)]

$$\nu = \frac{s-u}{4M}$$

$$q^2 = -Q^2$$



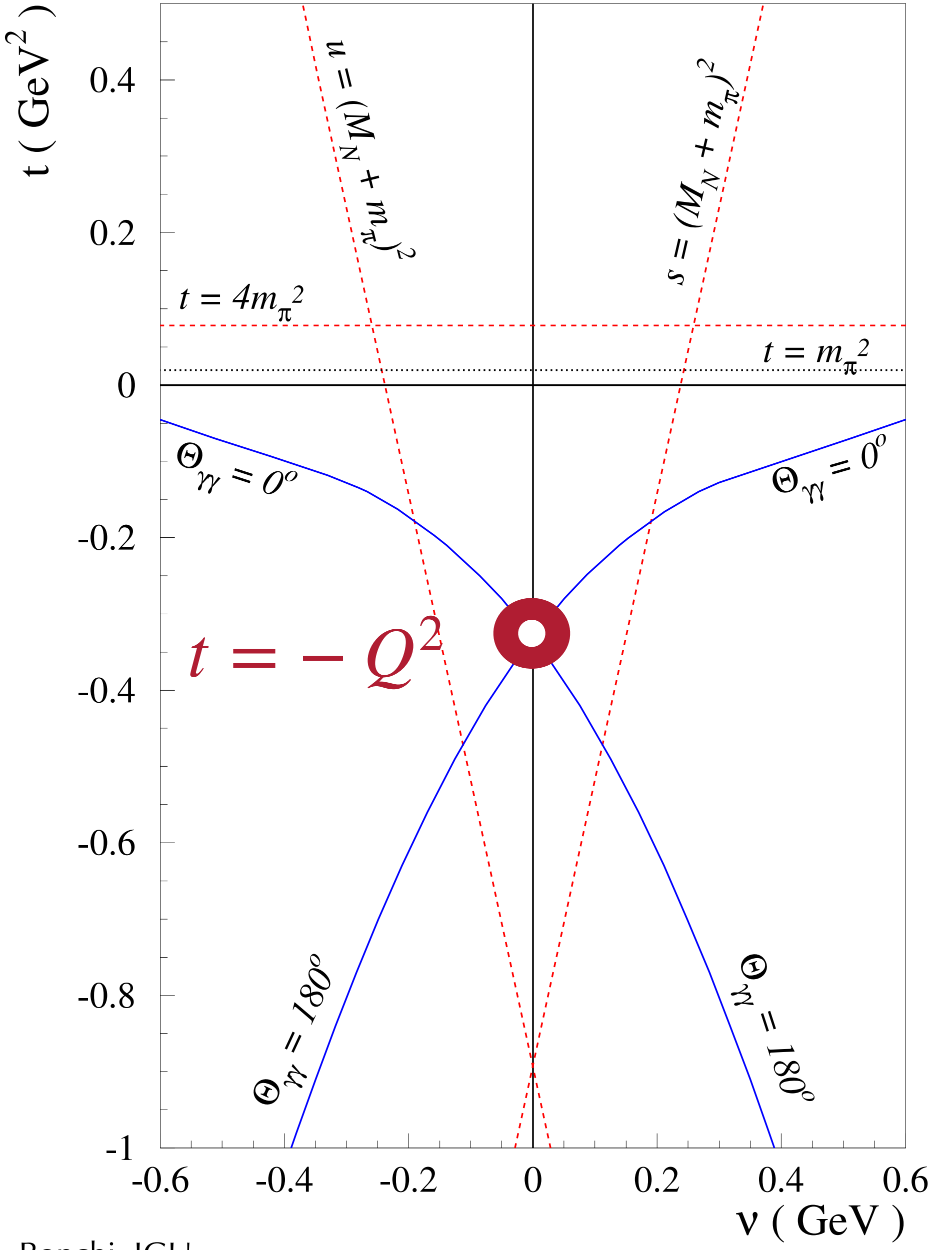


$$\mathscr{M}^{\mu\nu} = \sum_{i=1}^{12} F_i(Q^2,\nu,t)\,\tilde{\rho}_i^{\mu\nu} \quad \to \quad F_i(Q^2,\nu,t) = F_i(Q^2,-\nu,t), \ \ \forall i$$

F_i scalar amplitudes

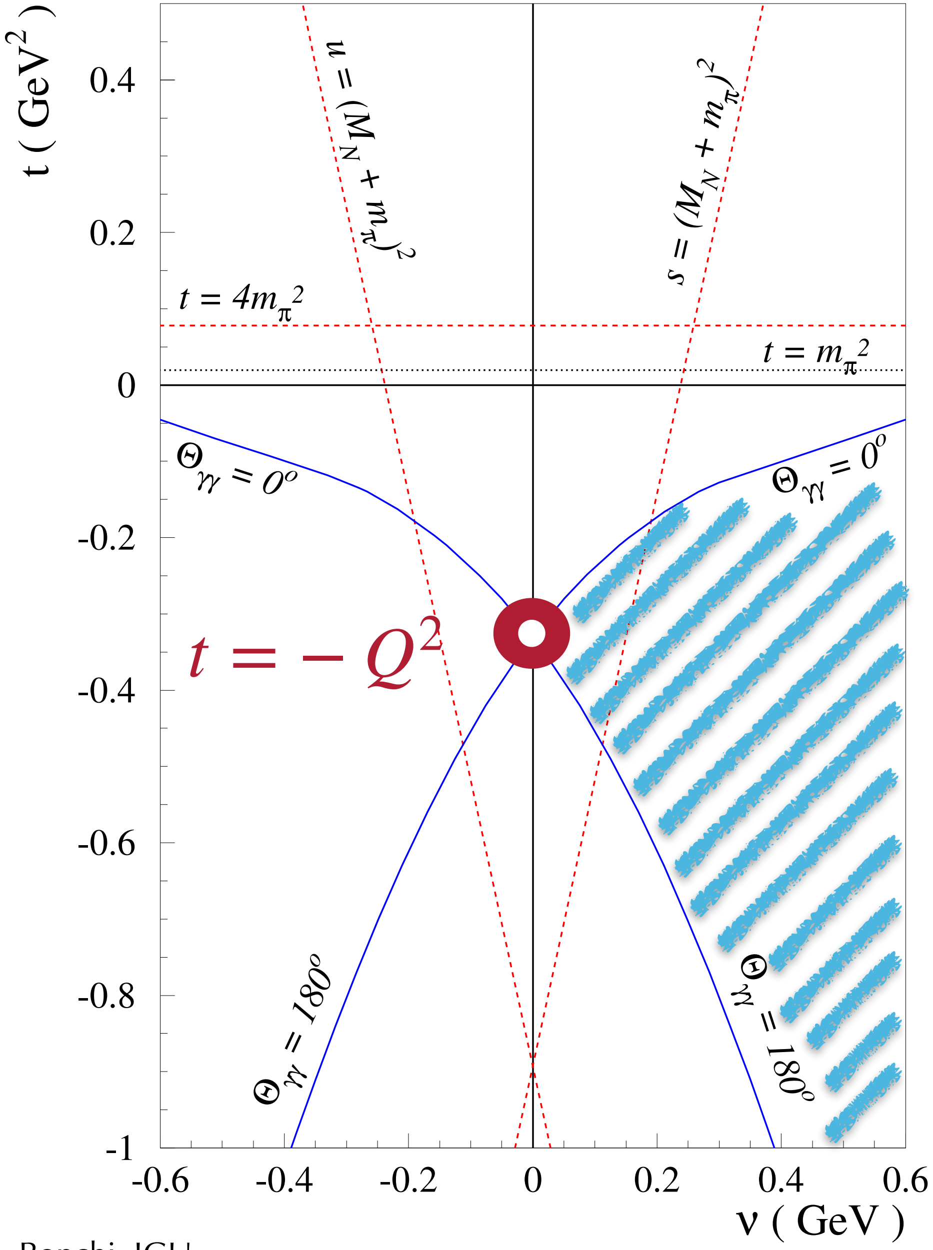
GPs can be related to: $F_i^{NB}(Q^2,\nu=0,t=-Q^2)$ $\to$ How can we obtain them?

Mandelstam plane



GPs are defined in the kinematical point $\nu = 0$ and $t = -Q^2$

Mandelstam plane



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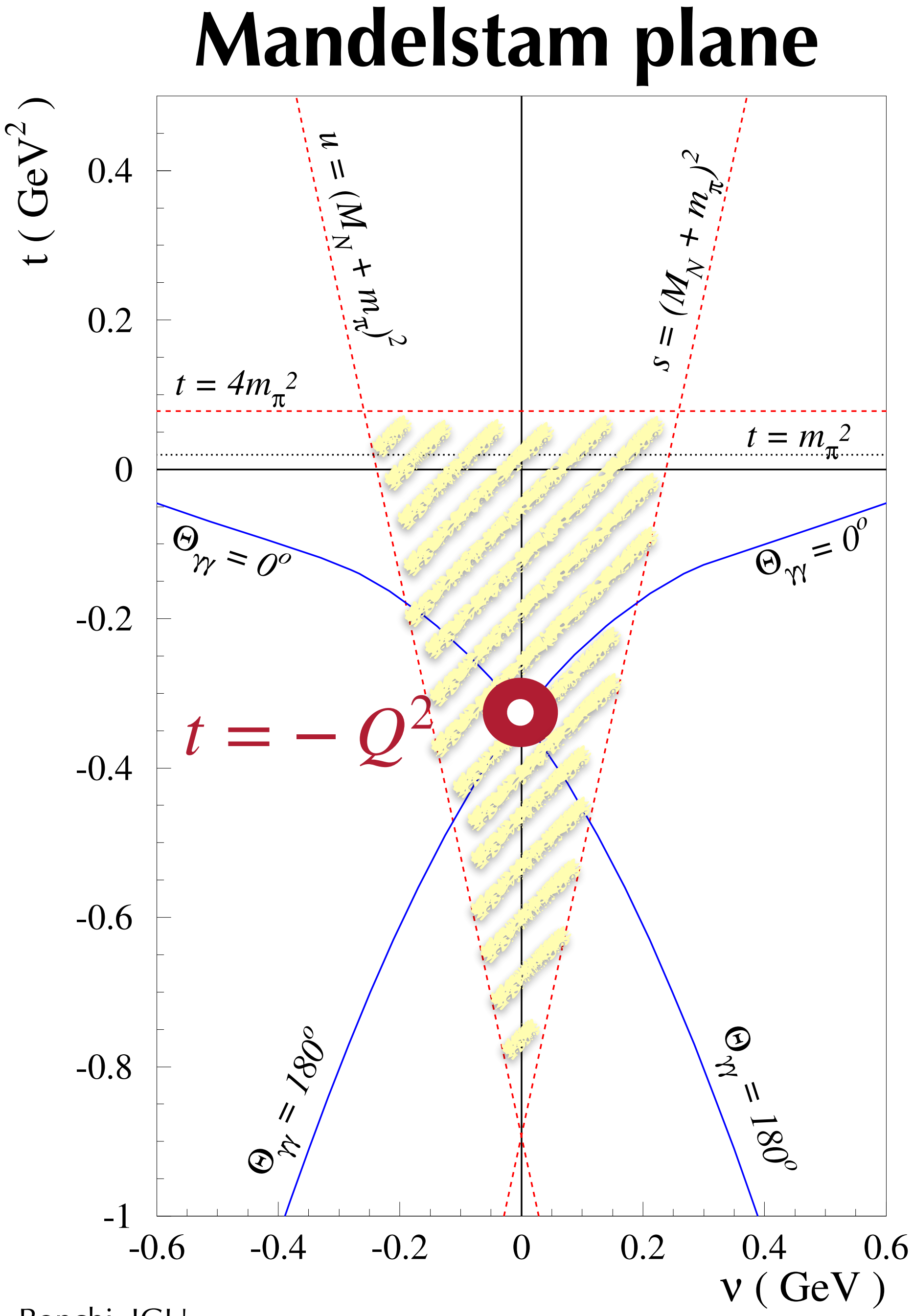
Physical region s-channel



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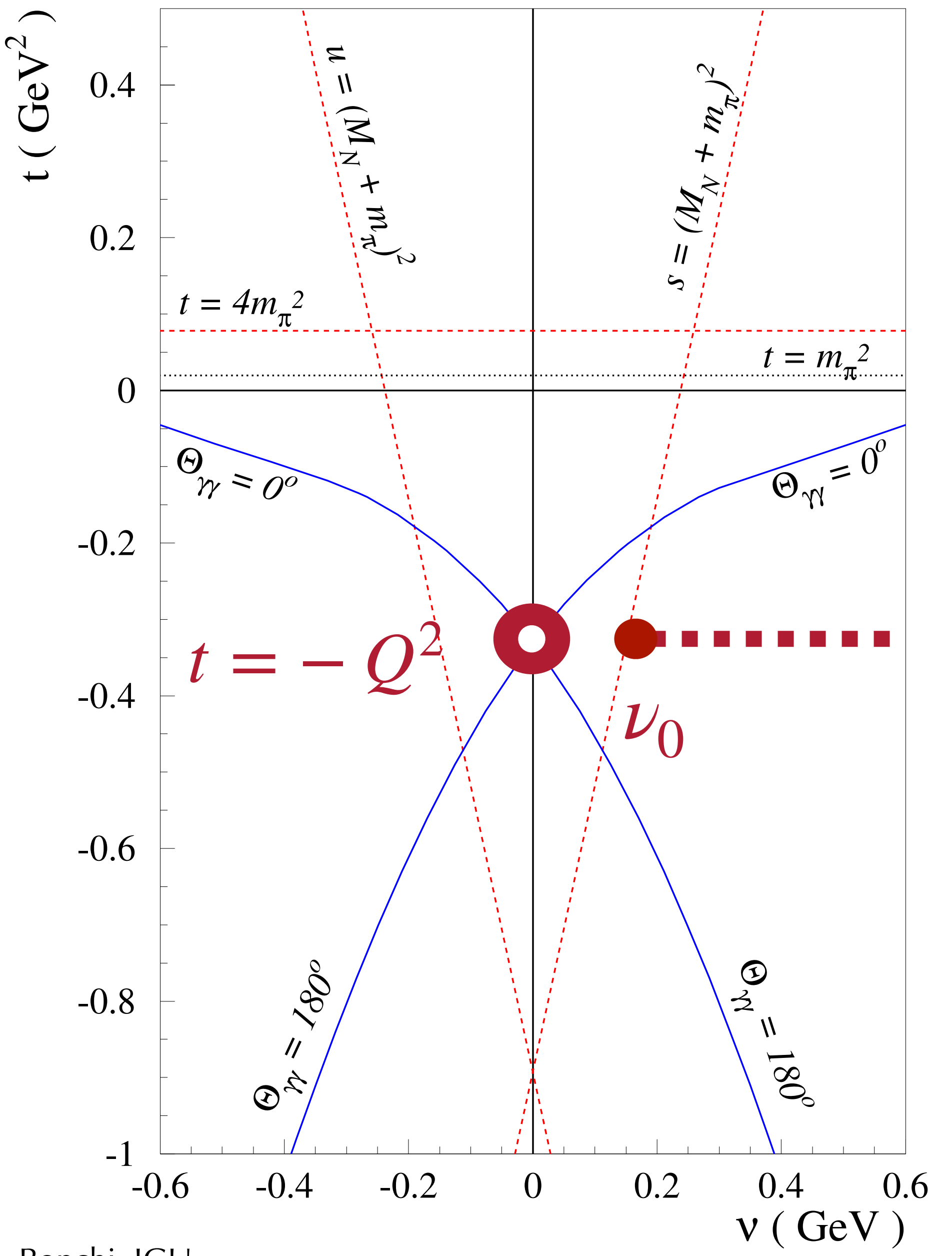
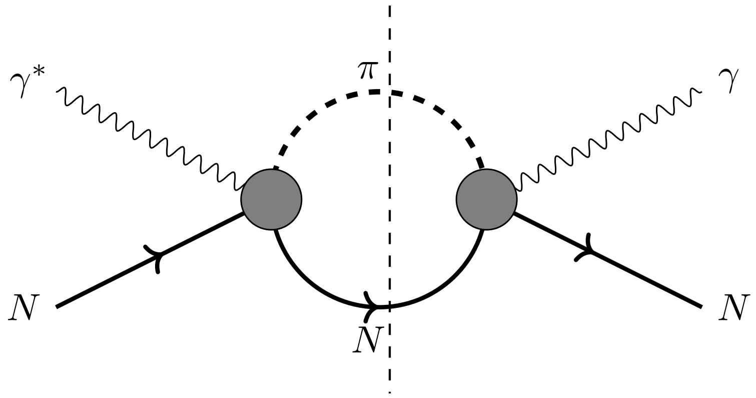
Region in which the amplitudes real



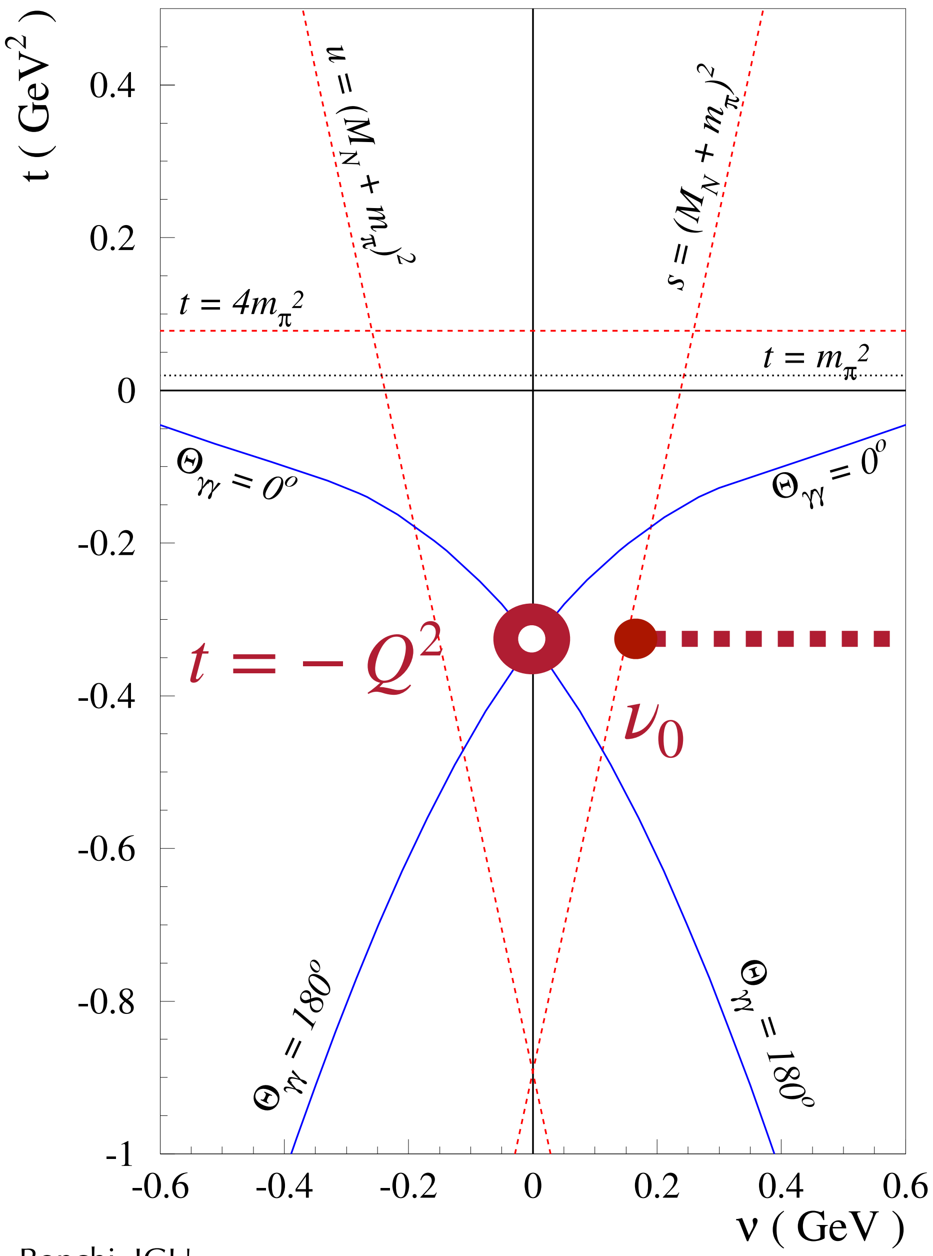
How can we extract them?

We write unsubtracted **dispersion relations (DR)** for fixed-t in the variable ν [B. Pasquini et al. EPJA, (2001)]

$$\mathrm{Re}\, F_i^{NB}(Q^2, \nu, t) = \frac{2}{\pi}\mathscr{P}\int_{\nu_0}^{\infty} d\nu' \frac{\nu' \mathrm{Im}_s F_i(Q^2, \nu', t)}{\nu'^2 - \nu^2}$$



It requires good input from πN (and $\pi\pi N$, ...) intermediate state especially at high energy

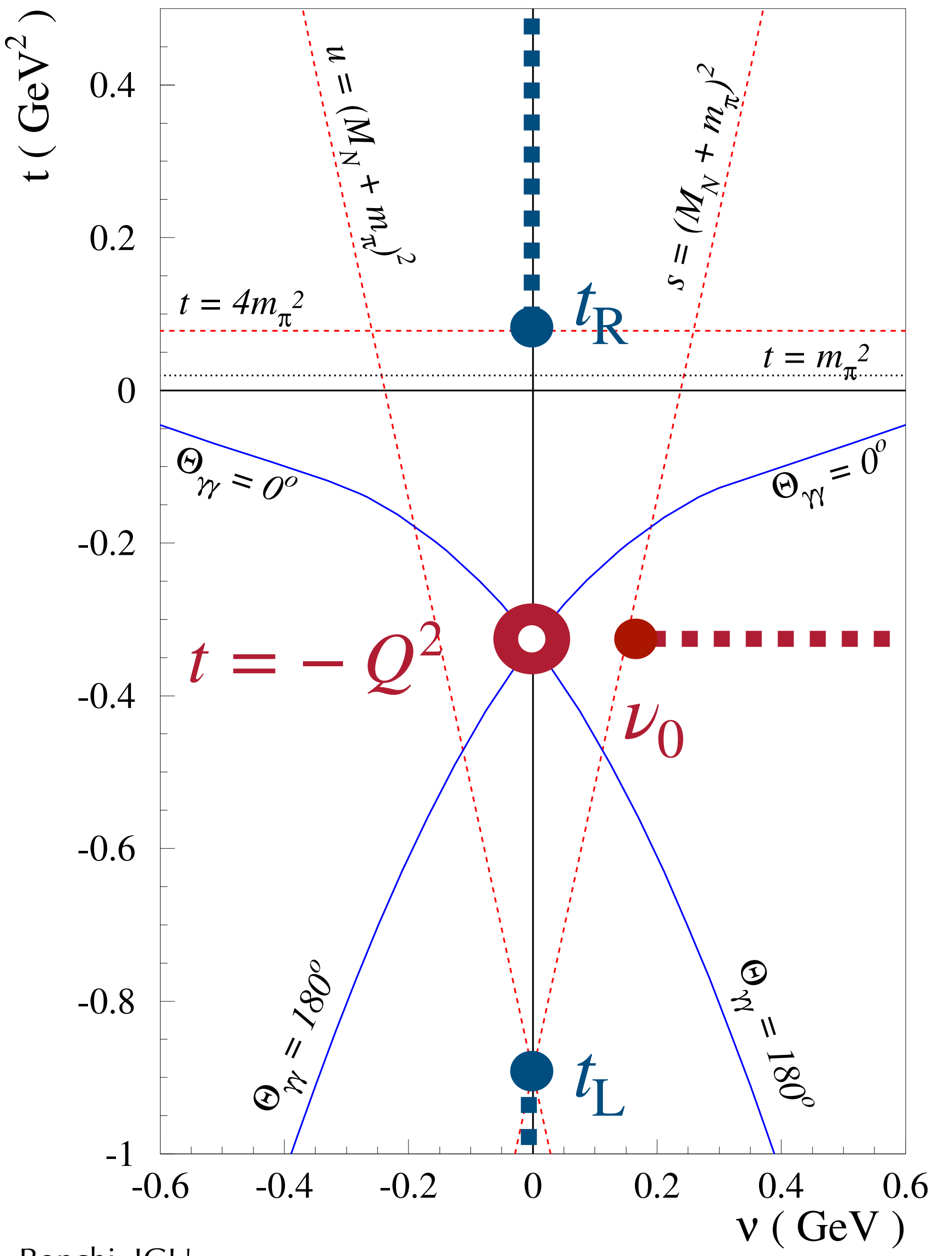


New way of extracting GPs

From Regge theory: $\mathbf{F_1}$ and $\mathbf{F_5}$ need a subtraction
[B. Pasquini et al. Phys. Rev. C62, (2000)]

Subtracted DR for fixed- t:

$$F_i^{NB}(Q^2, \nu, t) = F_i^{NB}(Q^2,0,t) + \frac{2}{\pi}\nu^2 \mathscr{P}\int_{\nu_0}^{\infty} d\nu' \frac{\mathrm{Im}_s F_i(Q^2, \nu', t)}{\nu'(\nu'^2 - \nu^2)}$$



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From Regge theory: $\mathbf{F_1}$ and $\mathbf{F_5}$ need a subtraction
[B. Pasquini et al. Phys. Rev. C62, (2000)]

Subtracted DR for fixed- t, where the subtraction function is obtained from subtracted fixed- ν DR:

$$F_i^{NB}(Q^2,\nu,t) = F_i^{NB}(Q^2,0,t) + \frac{2}{\pi}\nu^2\mathscr{P}\int_{\nu_0}^{\infty} d\nu' \frac{\mathrm{Im}_s F_i(Q^2,\nu',t)}{\nu'(\nu'^2-\nu^2)}$$

$$F_i^{NB}(Q^2,0,t) = \boxed{F_i^{NB}(Q^2,0,-Q^2)} + \text{pole contribution}$$

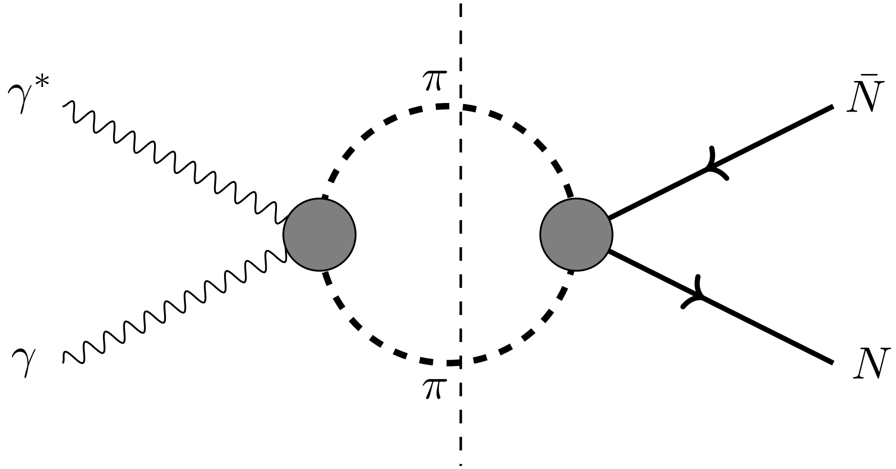
$$+\frac{(t+Q^2)}{\pi}\mathscr{P}\int_{t_R}^{\infty} dt' \frac{\mathrm{Im}_t F_i(Q^2,0,t')}{(t'+Q^2)(t'-t)} \quad :: \text{RHC}$$

$$+\frac{(t+Q^2)}{\pi}\mathscr{P}\int_{-\infty}^{t_L} dt' \frac{\mathrm{Im}_t F_i(Q^2,0,t')}{(t'+Q^2)(t'-t)} \quad :: \text{LHC}$$

Model dependence in the convergence of the integrals has been **reduced**

Evaluation of right-hand-cut (RHC)

$$\text{RHC} :: \mathscr{P}\int_{t_R}^{\infty} dt' \frac{\mathrm{Im}_t F_i(Q^2,0,t')}{(t'-t)}$$

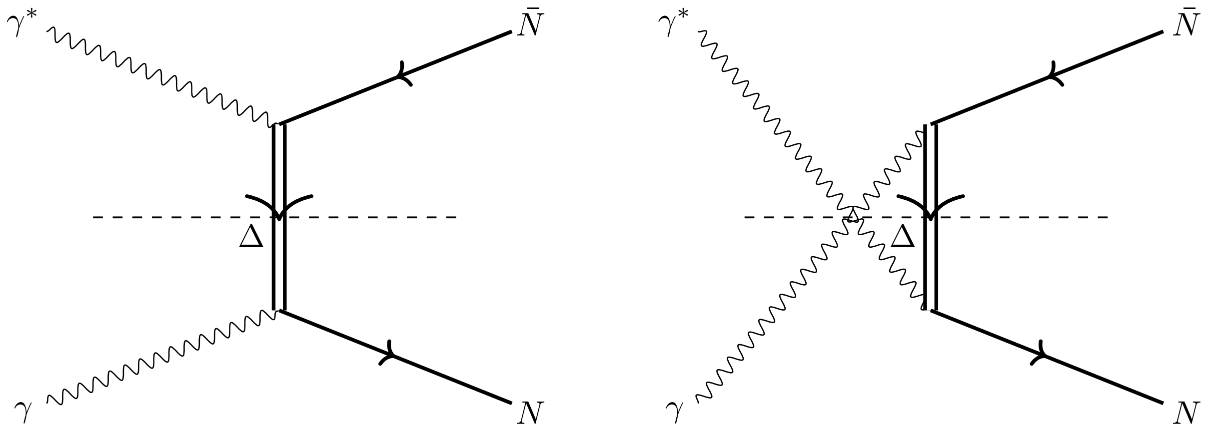


1) **Crossing symmetry:** integrand linked to t-channel VCS $\gamma\gamma^* \to N\bar{N}$ helicity amplitudes
2) Plug in **partial wave expansion** for helicity amplitudes for $\gamma\gamma^* \to N\bar{N}$
3) **Unitarity:** imaginary part expressed via the partial waves of the two subprocesses $\gamma\gamma^* \to \pi\pi$ and $\pi\pi \to N\bar{N}$

- For $\gamma\gamma^* \to \pi\pi$, we use the p.w. amplitudes from [I. Danilkin et al. Phys. Lett. B, (2018)]
- For $\pi\pi \to N\bar{N}$ available the new amplitudes from [M. Hoferichter et al. Phys. Rep., (2016)]

Evaluation of left-hand-cut (LHC)

$$\text{LHC} :: \mathscr{P}\int_{-\infty}^{t_L} dt' \frac{\text{Im}_t F_i(Q^2,0,t')}{(t'-t)}$$



Left hand cut discontinuity modeled by the spectral function for the $\Delta(1232)$-resonance excitation in s- and u- channel for the VCS process, accounting for its finite width $\Gamma_\Delta = 0.117\,\text{GeV}$

We are interested in energies of the VCS process up to Δ-region

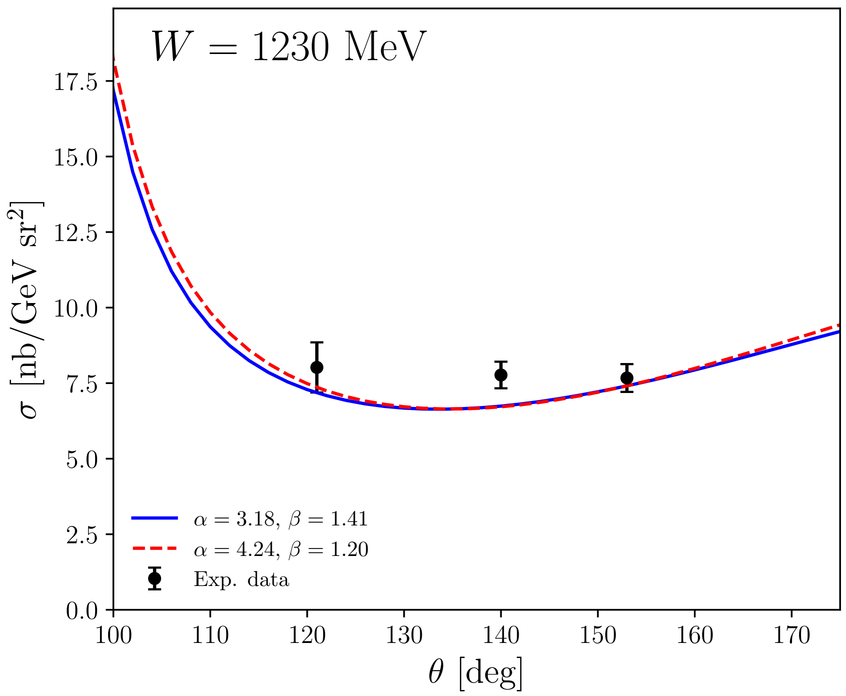
First results

In this approach the GPs are input parameters that need to be obtain from external sources, in the future they will be fitted to the experimental data

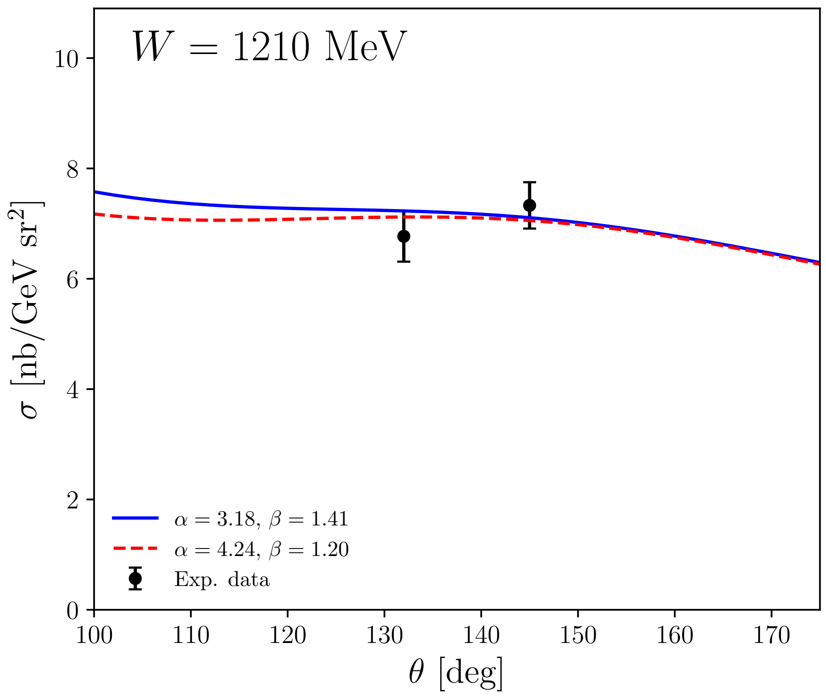
W: total centre-of-mass energy of the $(\gamma^* p)$ system

At low energies (below $\Delta(1232)$ resonance) we have a nice agreement with the data

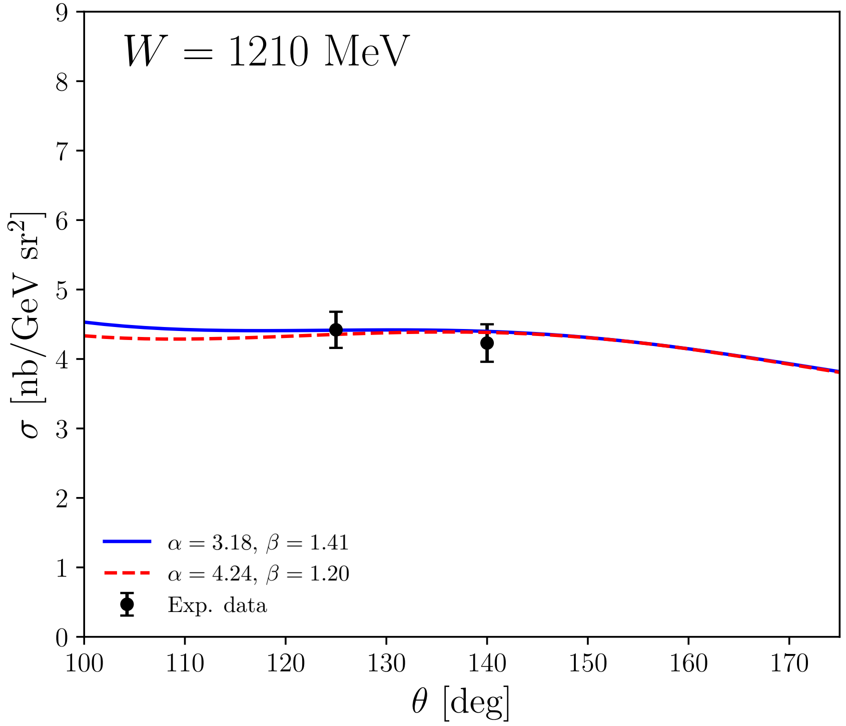
$\boldsymbol{Q^2 = 0.28\,\mathrm{GeV}^2;\ \phi = 0°}$



$\boldsymbol{Q^2 = 0.33\,\mathrm{GeV}^2;\ \phi = 180°}$



$\boldsymbol{Q^2 = 0.40\,\mathrm{GeV}^2;\ \phi = 180°}$



Values of α and β took it from [R. Li et al., Nature, (2022)]

Conclusions

- ✓ VCS dispersion relations formalism to extract the GPs up to $\Delta(1232)$ resonance energies
- ✓ GPs are input parameters: with the future experiments they will be fitted to the experimental data
- ✓ A tool to evaluate dispersion integrals in t-channel have been provided
- ✓ Using most recents measurements of $\alpha_{E1}(Q^2)$ and $\beta_{M1}(Q^2)$, we estimate the cross-section finding a good agreement with the experimental data below $1232\,\mathrm{MeV}$

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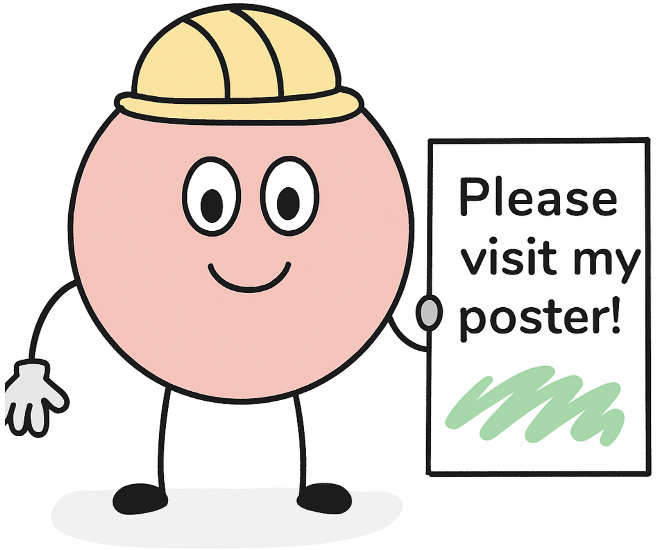

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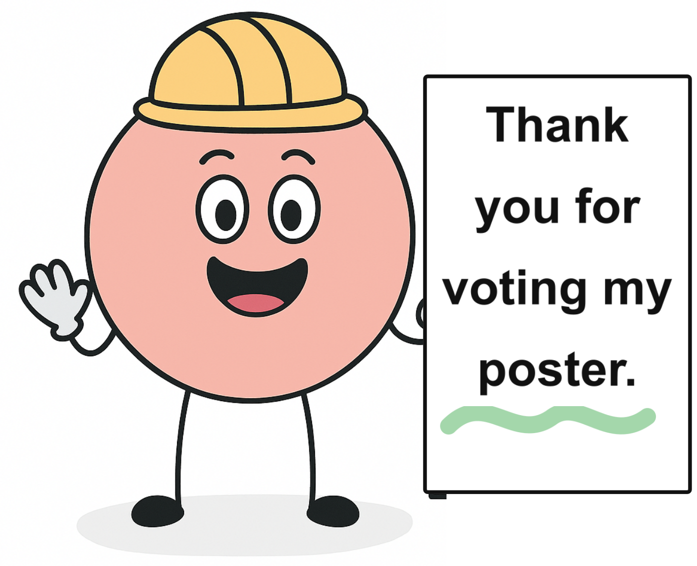
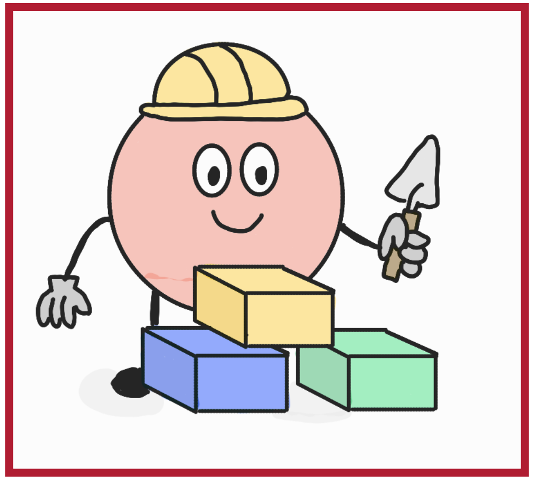
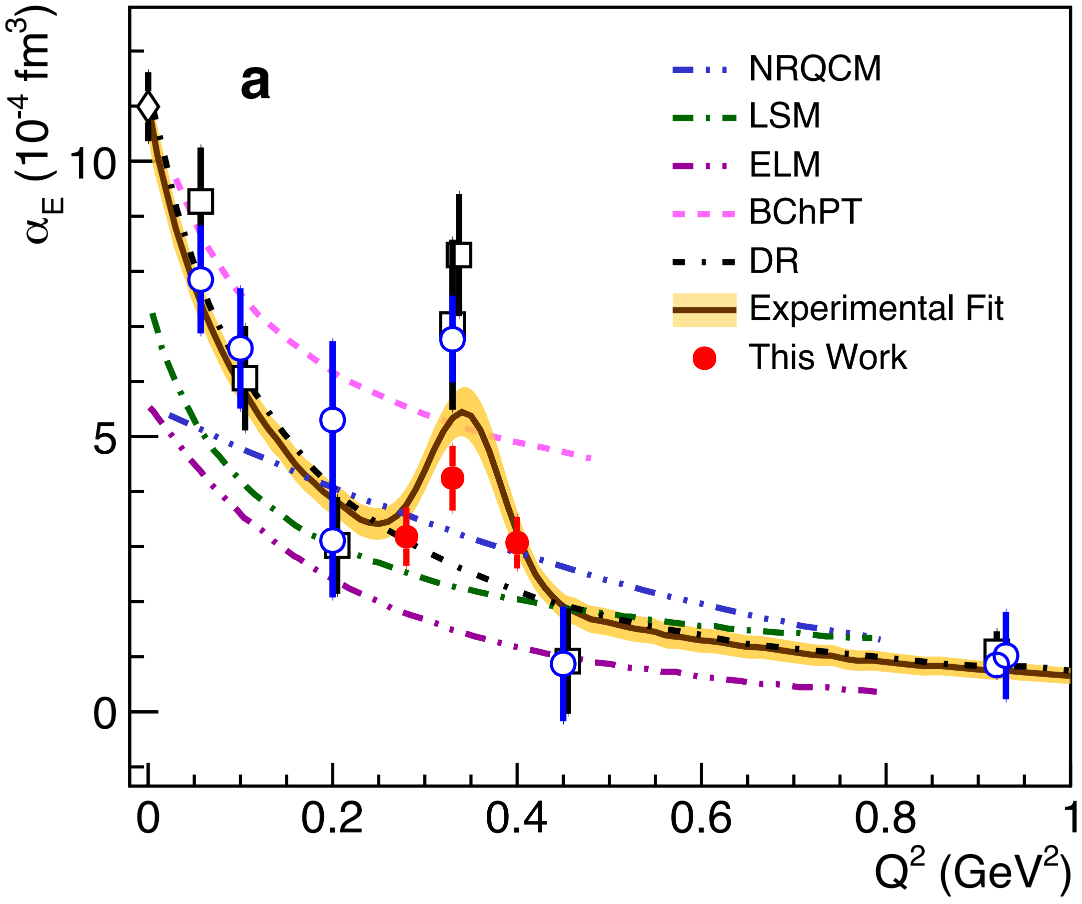




Thank you for your attention

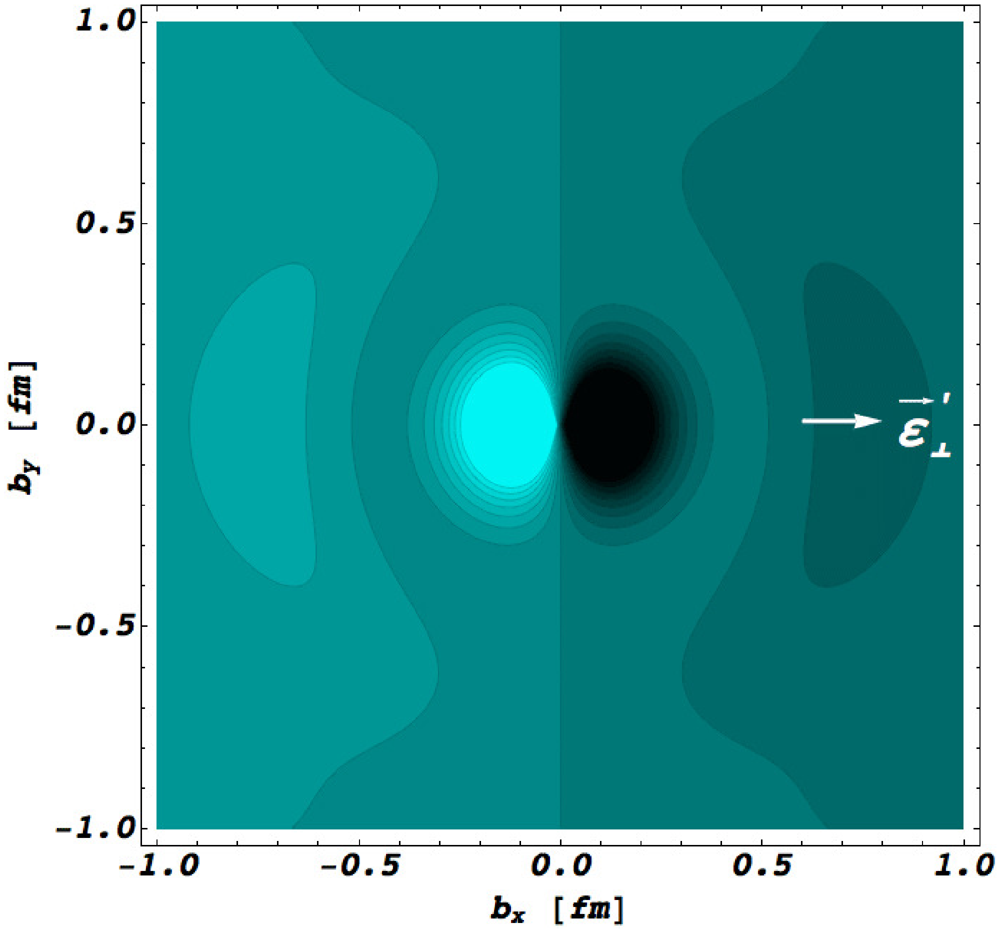
Any questions?

Proton polarizabilities



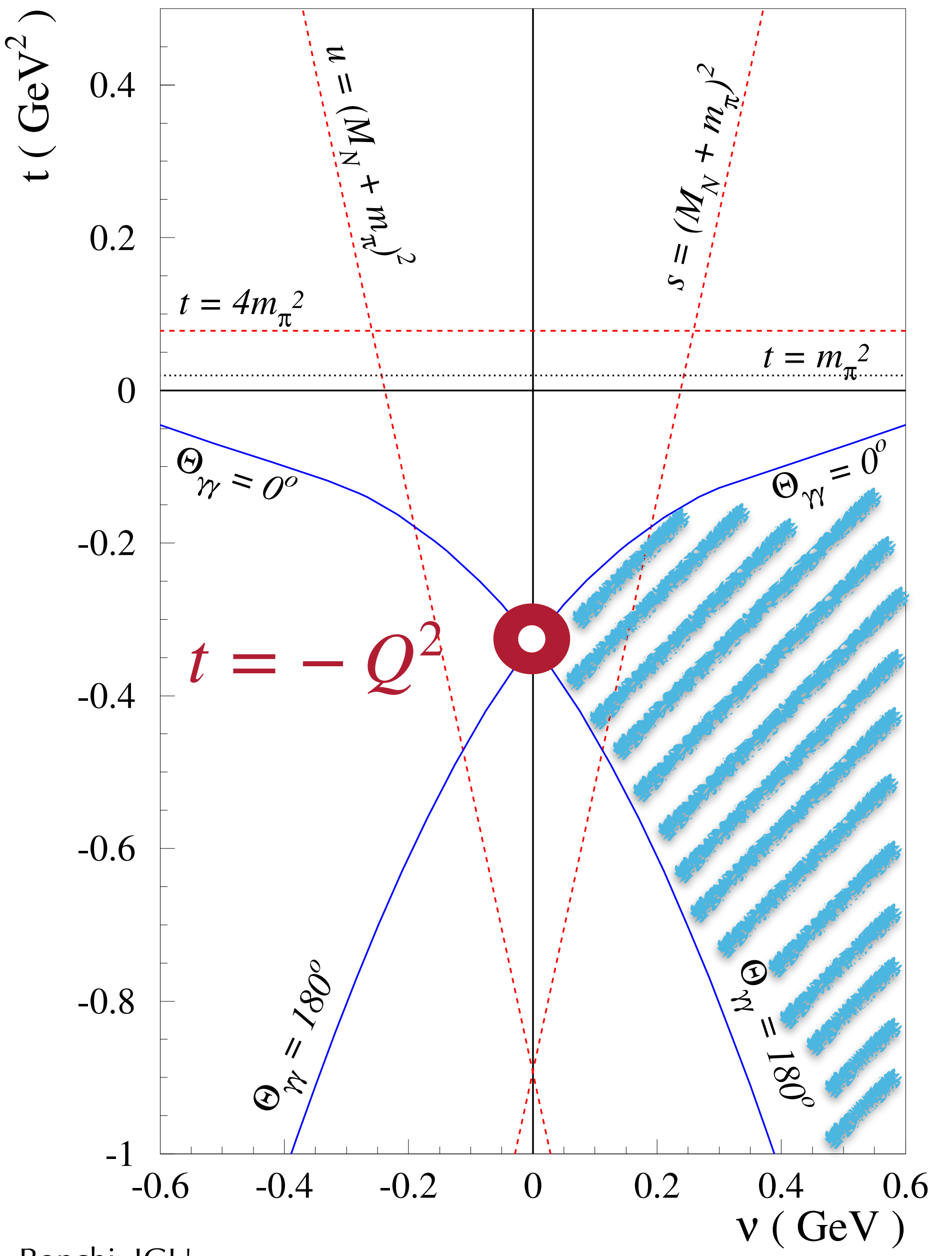
[R. Li et al., Nature 611, (2022)]

"Fourier transform" →



[M. Gorchtein et al., Phys.Rev.Lett. 104 (2010)]

Mandelstam plane



GPs are defined in the kinematical point $\nu = 0$ and $t = -Q^2$

F_i are:
- **free of kinematical singularities**
- **even** in the variable ν

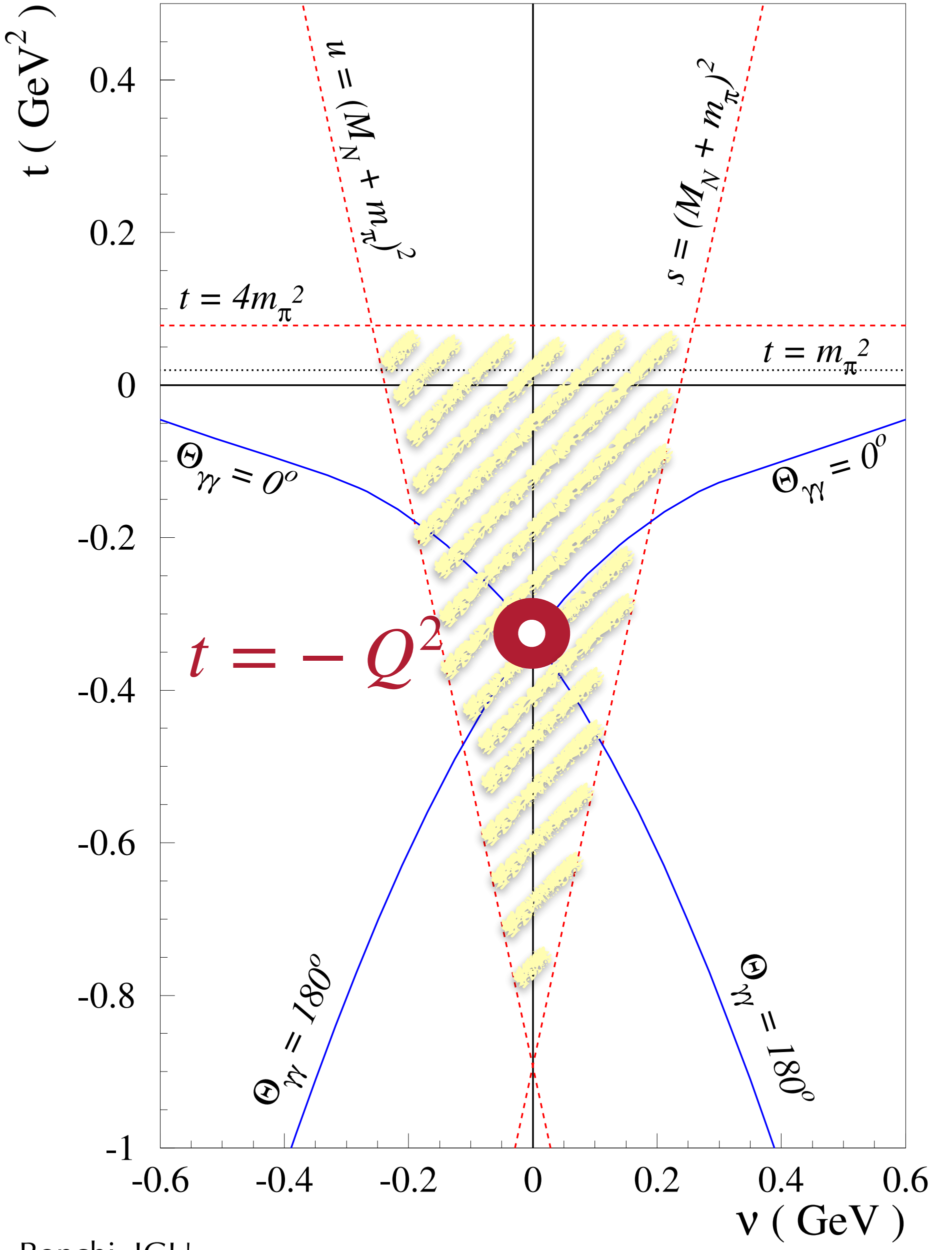
Assuming **analyticity** and **appropriate high energy behavior**

↓

We write **dispersion relations (DR)**

Physical region s-channel

Mandelstam plane



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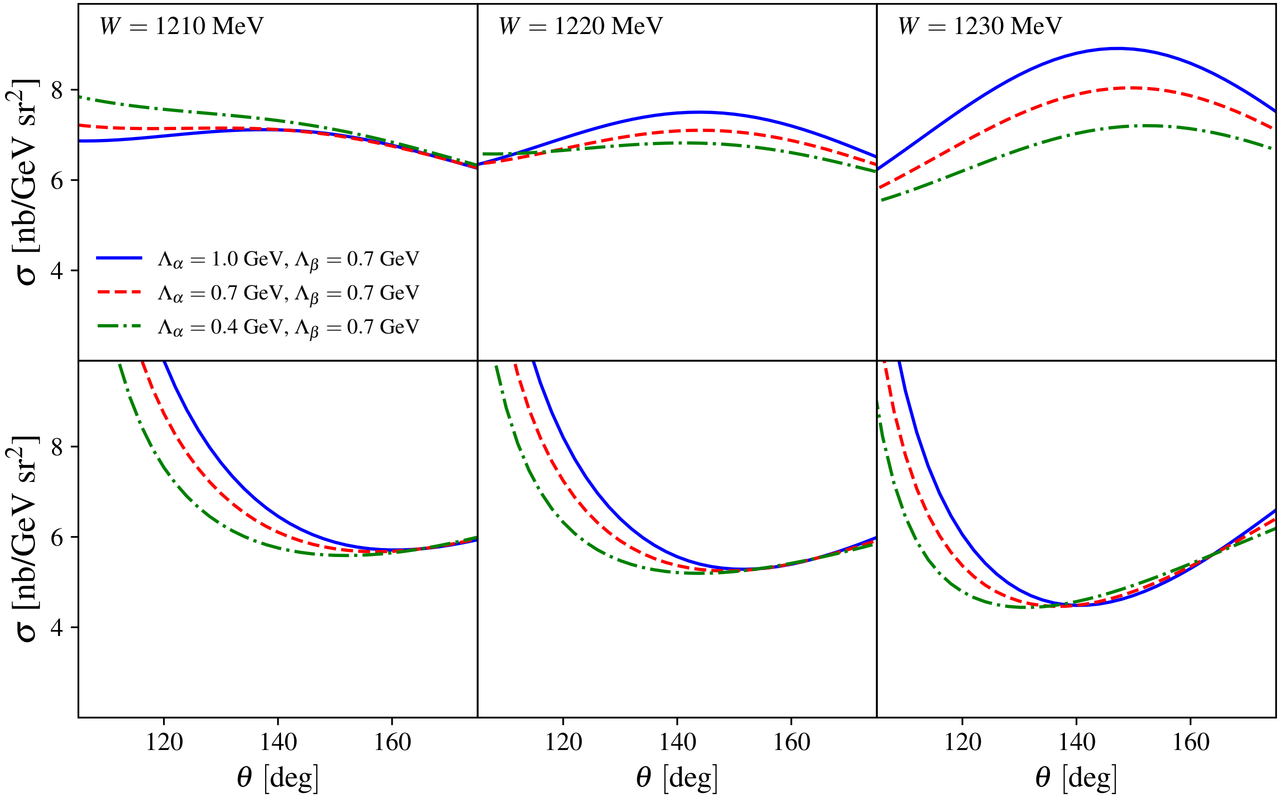
$$\alpha(Q^2) = \frac{\alpha^{\exp}(0)}{(1 + Q^2/\Lambda_\alpha^2)^2}$$

$$\beta(Q^2) = \frac{\beta^{\exp}(0)}{(1 + Q^2/\Lambda_\beta^2)^2}$$

W: total centre-of-mass energy of the $(\gamma^* p)$ system



$\phi = 180°$

$\phi = 0°$

Gauge invariant basis

$$\rho_1^{\mu\nu} = -q\cdot q' g^{\mu\nu} + q'^{\mu} q^{\nu},$$

$$\rho_2^{\mu\nu} = -(2M\nu)^2 g^{\mu\nu} - 4q\cdot q' P^{\mu} P^{\nu} + 4M\nu \left(P^{\mu} q^{\nu} + P^{\nu} q'^{\mu}\right),$$

$$\rho_3^{\mu\nu} = -2M\nu Q^2 g^{\mu\nu} - 2M\nu\, q^{\mu} q^{\nu} + 2Q^2 P^{\nu} q'^{\mu} + 2q\cdot q'\, P^{\nu} q^{\mu},$$

$$\rho_4^{\mu\nu} = 8P^{\mu} P^{\nu} \not{K} - 4M\nu \left(P^{\mu}\gamma^{\nu} + P^{\nu}\gamma^{\mu}\right) + i\, 4M\nu\, \gamma_5\, \varepsilon^{\mu\nu\alpha\beta} K_{\alpha}\gamma_{\beta},$$

$$\rho_5^{\mu\nu} = P^{\nu} q^{\mu} \not{K} - \frac{Q^2}{2}\left(P^{\mu}\gamma^{\nu} - P^{\nu}\gamma^{\mu}\right) - M\nu\, q^{\mu}\gamma^{\nu} - \frac{i}{2} Q^2\, \gamma_5\, \varepsilon^{\mu\nu\alpha\beta} K_{\alpha}\gamma_{\beta},$$

$$\rho_6^{\mu\nu} = -8q\cdot q' P^{\mu} P^{\nu} + 4M\nu \left(P^{\mu} q^{\nu} + P^{\nu} q'^{\mu}\right) + 4M q\cdot q' \left(P^{\mu}\gamma^{\nu} + P^{\nu}\gamma^{\mu}\right) - 4M^2\nu \left(q'^{\mu}\gamma^{\nu} + q^{\nu}\gamma^{\mu}\right) + i\, 4M\nu \left(q'^{\mu}\sigma^{\nu\alpha}K_{\alpha} - q^{\nu}\sigma^{\mu\alpha}K_{\alpha} + q\cdot q'\sigma^{\mu\nu}\right) + i\, 4M q\cdot q'\, \gamma_5\, \varepsilon^{\mu\nu\alpha\beta} K_{\alpha}\gamma_{\beta},$$

$$\rho_7^{\mu\nu} = \left(P^{\mu} q^{\nu} - P^{\nu} q'^{\mu}\right)\not{K} - q\cdot q' \left(P^{\mu}\gamma^{\nu} - P^{\nu}\gamma^{\mu}\right) + M\nu \left(q'^{\mu}\gamma^{\nu} - q^{\nu}\gamma^{\mu}\right),$$

$$\rho_8^{\mu\nu} = M\nu\, q^{\mu} q^{\nu} + \frac{Q^2}{2}\left(P^{\mu} q^{\nu} - P^{\nu} q'^{\mu}\right) - q\cdot q'\, P^{\nu} q^{\mu} - M q^{\mu} q^{\nu} \not{K} + M q\cdot q'\, q^{\mu}\gamma^{\nu} + \frac{M}{2} Q^2 \left(q'^{\mu}\gamma^{\nu} - q^{\nu}\gamma^{\mu}\right) - \frac{i}{2} Q^2 \left(q'^{\mu}\sigma^{\nu\alpha}K_{\alpha} - q^{\nu}\sigma^{\mu\alpha}K_{\alpha} + q\cdot q'\sigma^{\mu\nu}\right),$$

$$\rho_9^{\mu\nu} = 2M\nu \left(P^{\mu} q^{\nu} - P^{\nu} q'^{\mu}\right) - 2M q\cdot q' \left(P^{\mu}\gamma^{\nu} - P^{\nu}\gamma^{\mu}\right) + 2M^2\nu \left(q'^{\mu}\gamma^{\nu} - q^{\nu}\gamma^{\mu}\right) + i\, 2q\cdot q' \left(P^{\mu}\sigma^{\nu\alpha}K_{\alpha} + P^{\nu}\sigma^{\mu\alpha}K_{\alpha}\right) - i\, 2M\nu \left(q'^{\mu}\sigma^{\nu\alpha}K_{\alpha} + q^{\nu}\sigma^{\mu\alpha}K_{\alpha}\right),$$

$$\rho_{10}^{\mu\nu} = -4M\nu\, g^{\mu\nu} + 2\left(P^{\mu} q^{\nu} + P^{\nu} q'^{\mu}\right) + 4M\, g^{\mu\nu}\not{K} - 2M \left(q'^{\mu}\gamma^{\nu} + q^{\nu}\gamma^{\mu}\right) - 2i \left(q'^{\mu}\sigma^{\nu\alpha}K_{\alpha} - q^{\nu}\sigma^{\mu\alpha}K_{\alpha} + q\cdot q'\sigma^{\mu\nu}\right),$$

$$\rho_{11}^{\mu\nu} = 4\left(P^{\mu} q^{\nu} + P^{\nu} q'^{\mu}\right)\not{K} - 4M\nu \left(q'^{\mu}\gamma^{\nu} + q^{\nu}\gamma^{\mu}\right) + i\, 4q\cdot q'\, \gamma_5\, \varepsilon^{\mu\nu\alpha\beta} K_{\alpha}\gamma_{\beta},$$

$$\rho_{12}^{\mu\nu} = 2Q^2 P^{\mu} P^{\nu} + 2M\nu\, P^{\nu} q^{\mu} - 2M Q^2 P^{\mu}\gamma^{\nu}$$

R. Tarrach, Nuovo Cimento A 28, 409 (1975).