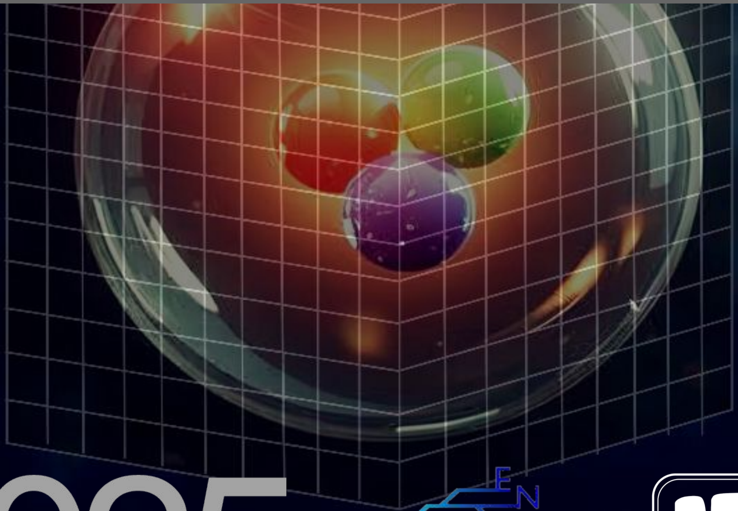
Proton and neutron electromagnetic form factors from Lattice QCD Simulations at the Physical Point

Constantia Alexandrou, Simone Bacchio, Mathis Bode, Jacob Finkenrath, Andreas Herten, Christos Iona, Giannis Koutsou, Ferenc Pittler, Bhavna Prasad, Gregoris Spanoudes.



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Why Electromagnetic Form Factors?

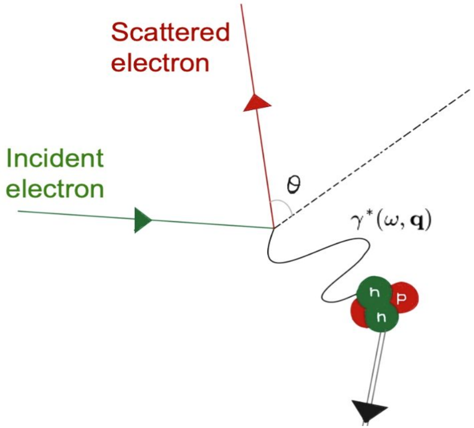
➢ Gives us insight into the structure of hadrons.

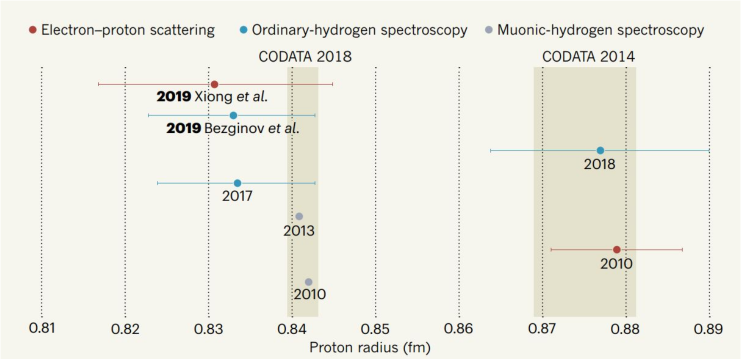
➢ Several experimental results for protons, as it is a stable hadronic bound state. Earliest 1956.

➢ Experimentally, it is essentially elastic scattering of protons with electrons.

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[\left({F_1}^2 + \tau(\kappa F_2)^2\right) + 2\tau(F_1 + \kappa F_2)^2 \tan^2\left(\frac{\theta}{2}\right)\right]$$

➢ Proton radius puzzle: A longstanding discrepancy between different experimental results for proton radii.

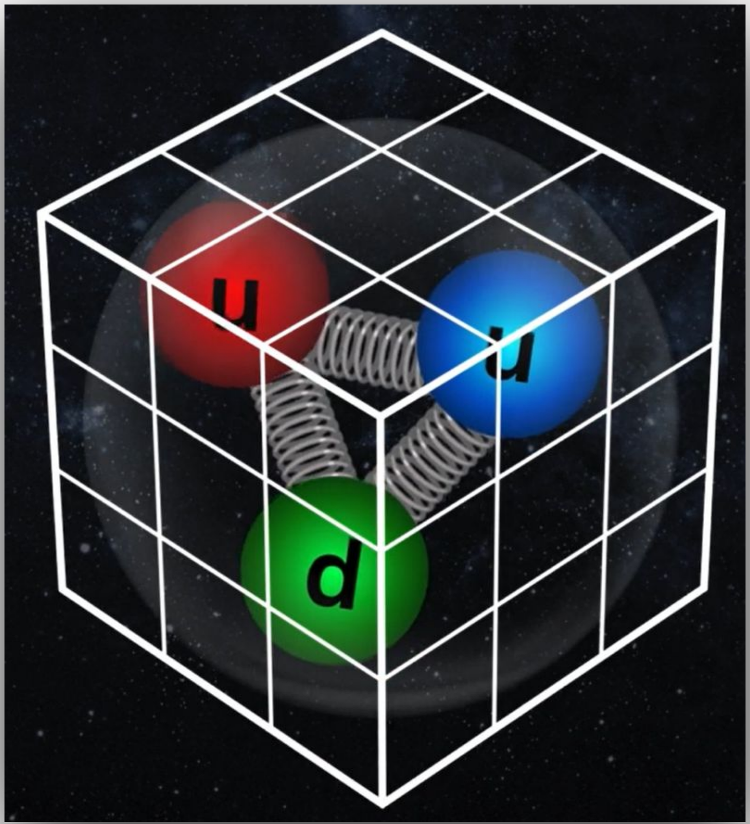




[10.1038/d41586-019-03364-z]

Electromagnetic form factors

- Interested in theoretically probing the structure of nucleons using lattice QCD.



Electromagnetic form factors

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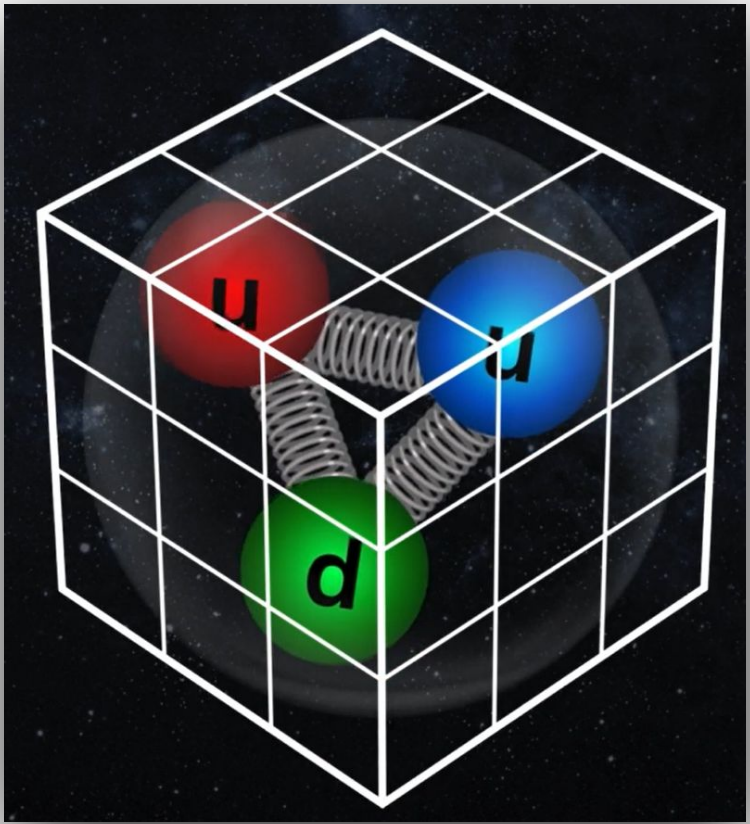
- The nucleon matrix element of for the electromagnetic current is given by:

$$\langle N(p', s')|j_\mu|N(p, s)\rangle = \sqrt{\frac{m_N^2}{E_N(\vec{p}\,')E_N(\vec{p})}} \times$$

$$\bar{u}_N(p', s')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2)\right]u_N(p, s)$$

$$j_\mu = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d - \tfrac{1}{3}\bar{s}\gamma_\mu s$$



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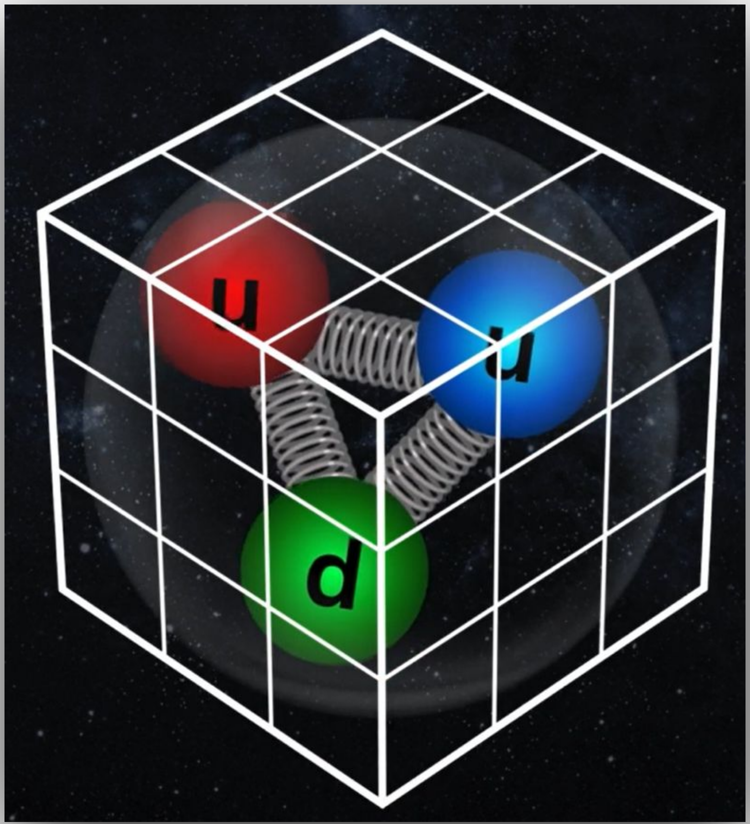
$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_N^2}F_2(q^2)\,,$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2)\,.$$



Lattice gauge theories: Introduction

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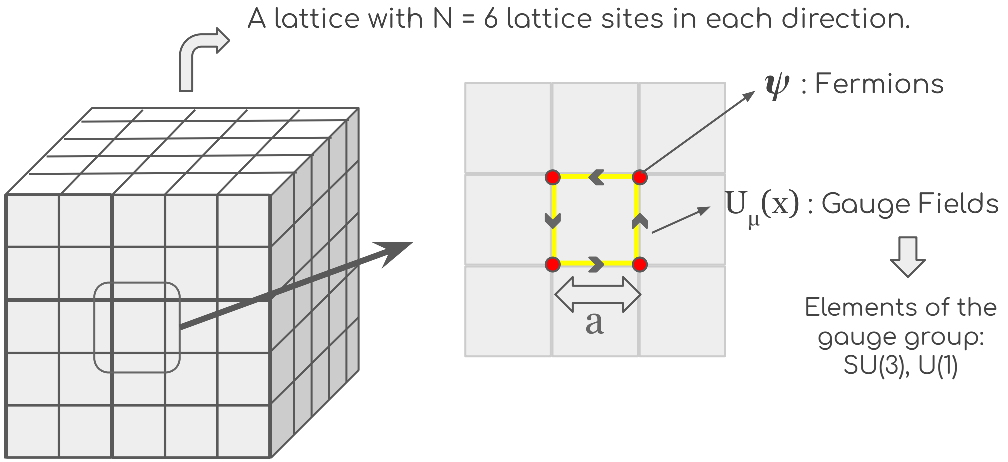

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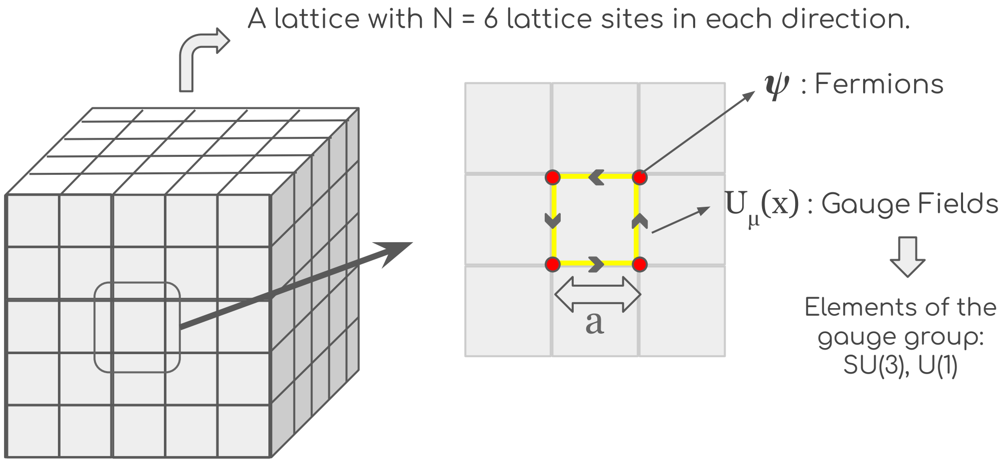

In this formulation the expectation value of an observable (without fermions) is given by:

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int D[U] D[\psi, \bar{\psi}] O(U, \psi, \bar{\psi}) e^{-S_G(U)} e^{-S_F(\psi, \bar{\psi}, U)}$$

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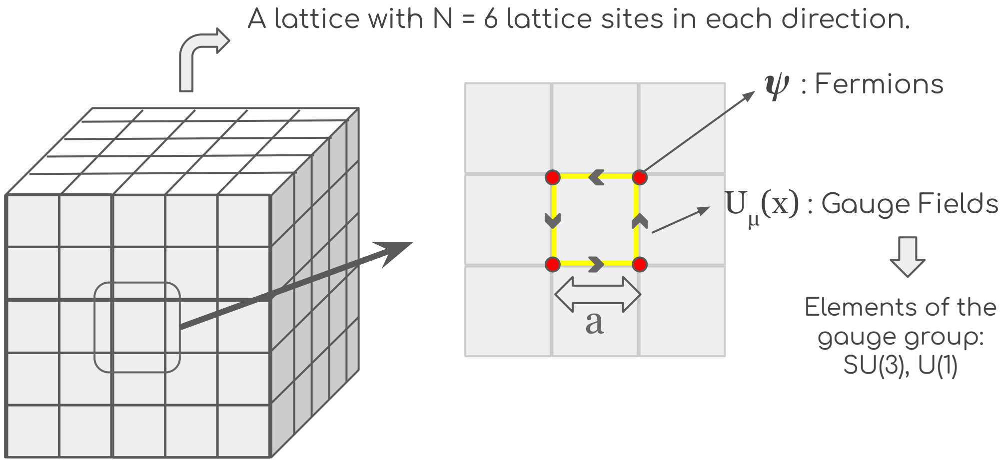

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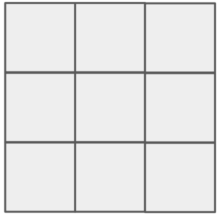
$$\Rightarrow \quad \langle O\rangle = \lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N} O[U_n]$$

Continuum limit and lattice setup

➢ Continuum limit needed to extract physical values.



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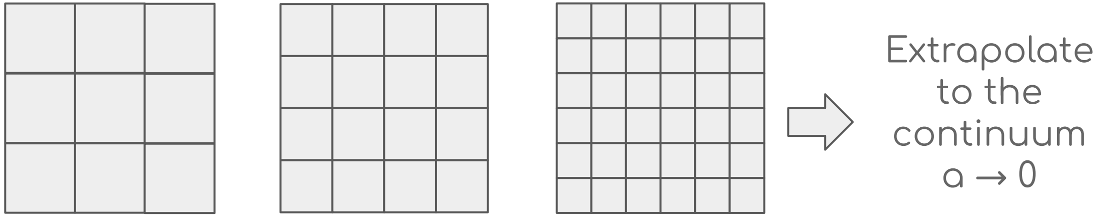

Keep physical volume $(aN)^D$ constant (D=2 above).

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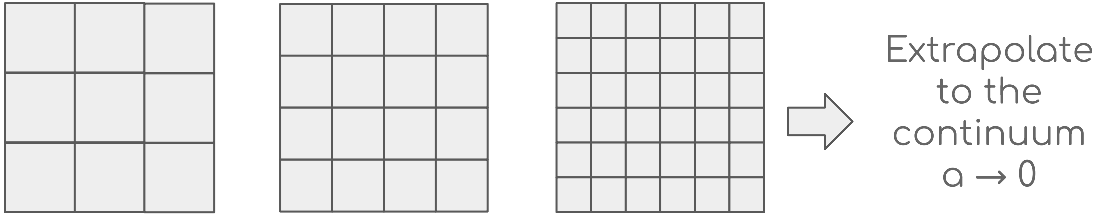

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Ensemble	$(\frac{L}{a})^3 \times (\frac{T}{a})$	a [fm]	m_π [MeV]	$m_\pi L$
cB211.072.64	$64^3 \times 128$	0.07957(13)	140.2(2)	3.62
cC211.060.80	$80^3 \times 160$	0.06821(13)	136.7(2)	3.78
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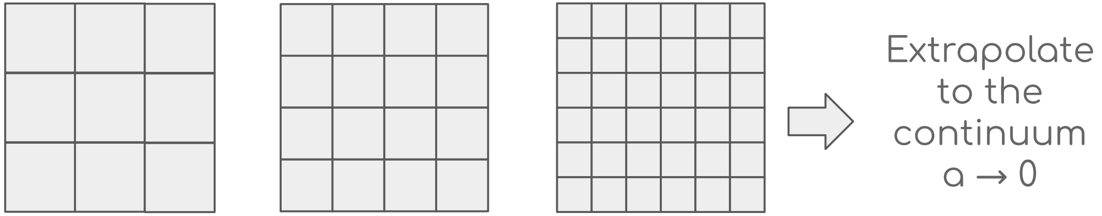

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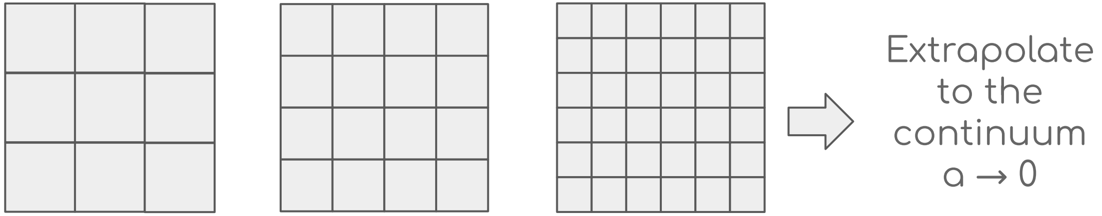

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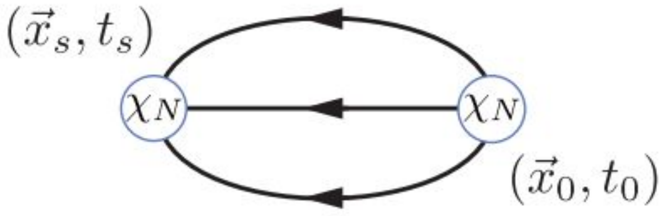
- ➢ We use clover improved, twisted-mass fermions (O(a) improved).

- ➢ At physical point, no chiral extrapolation needed.

Correlators in lattice QCD

➢ Two point correlator:

$$C(t) = \sum_{\vec{x}_s} \langle \Omega | \chi(\vec{x}_s, t_s) \bar{\chi}(\vec{x}_0, t_0) | \Omega \rangle$$



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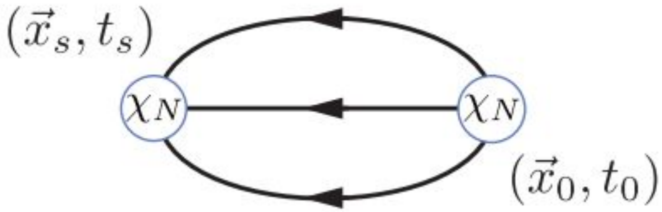

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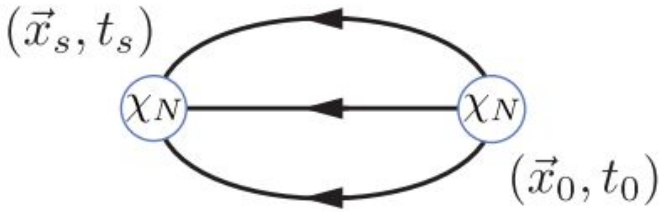

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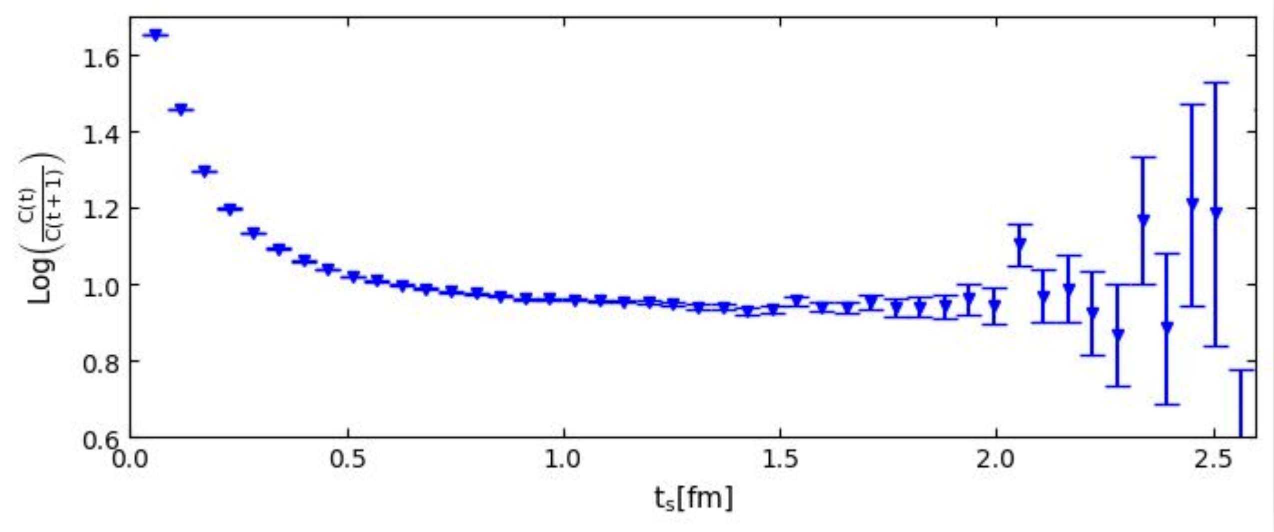

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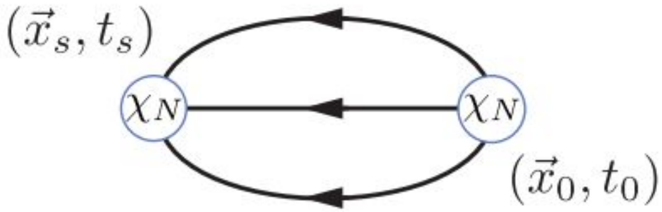

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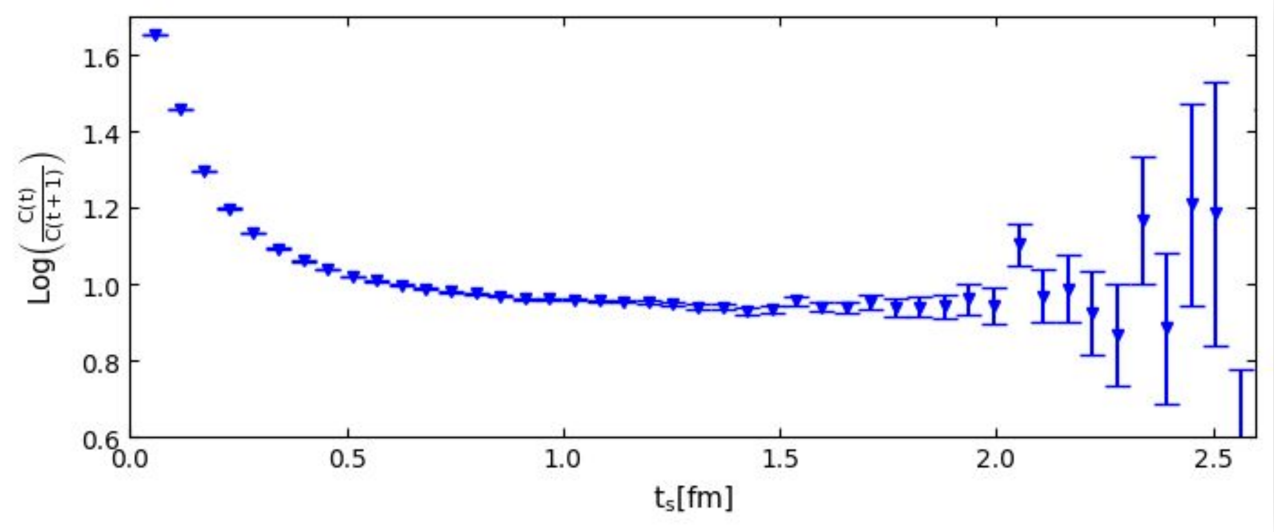

➢ Relative Error increases exponentially

$$\frac{\sigma[C_N(t)]}{C_N(t)} \propto e^{(M_N - \frac{3}{2} M_\pi) t}$$

Fitting correlators

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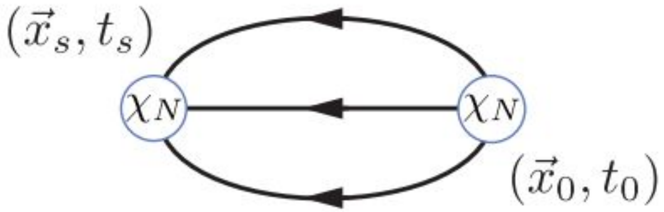

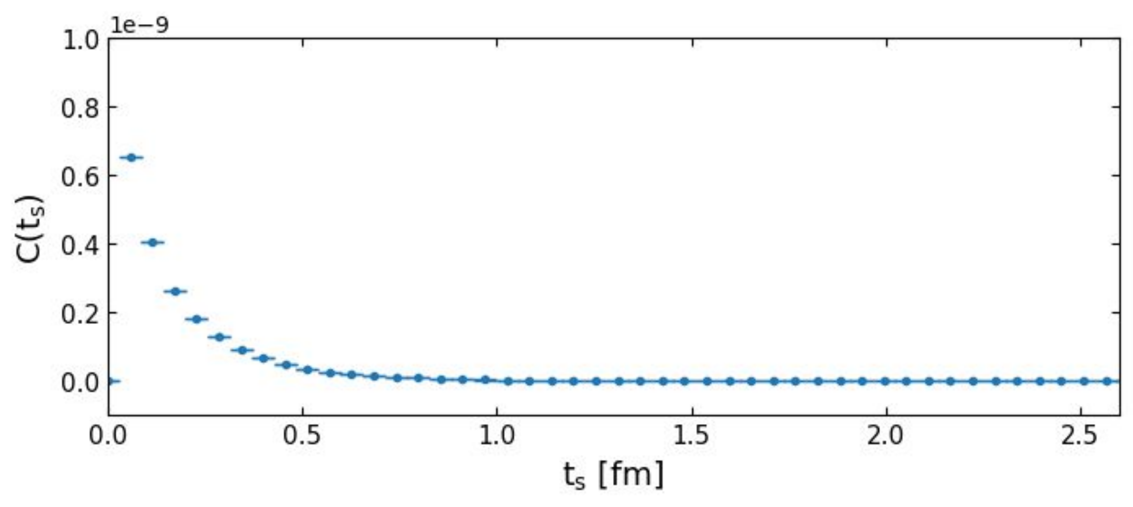

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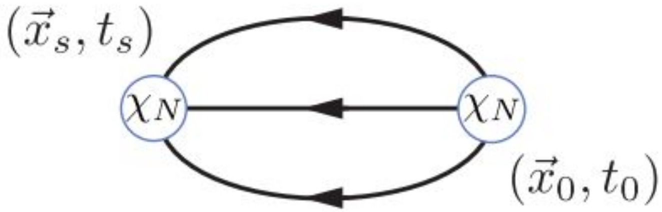

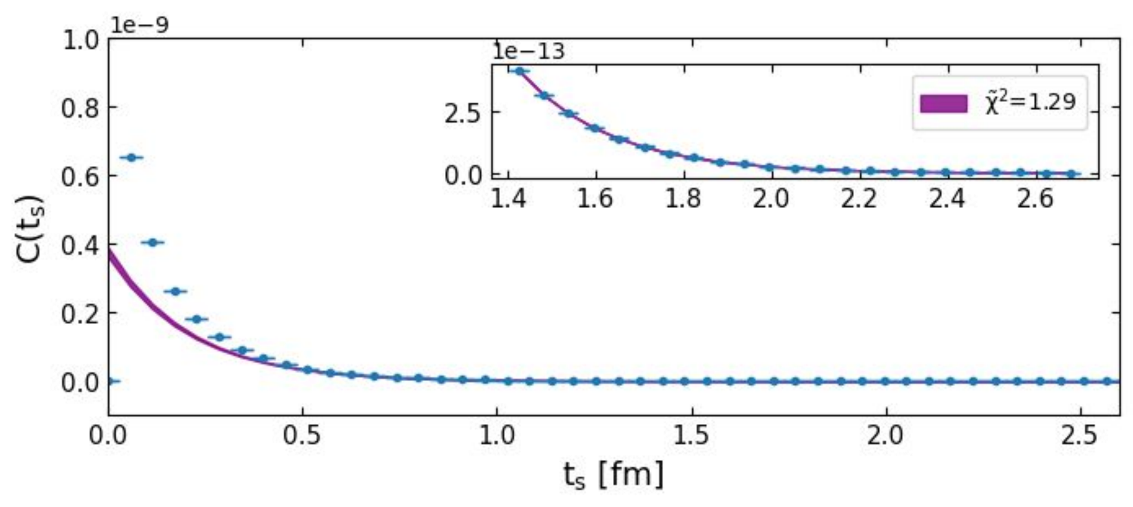

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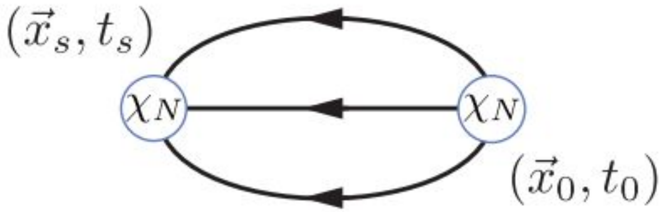

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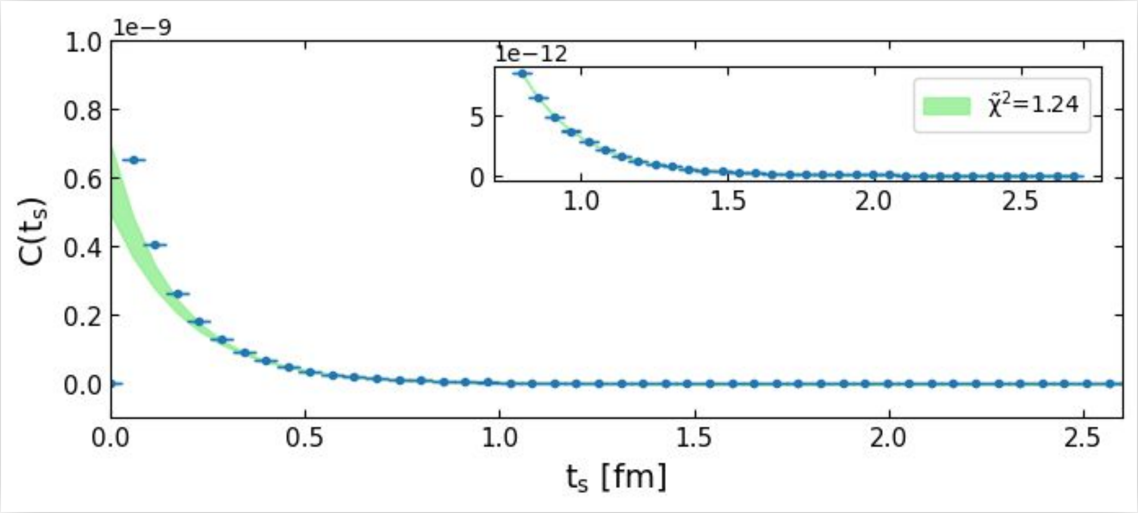

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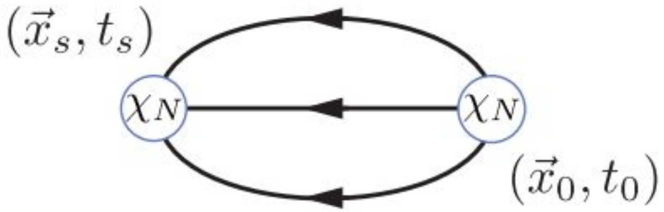

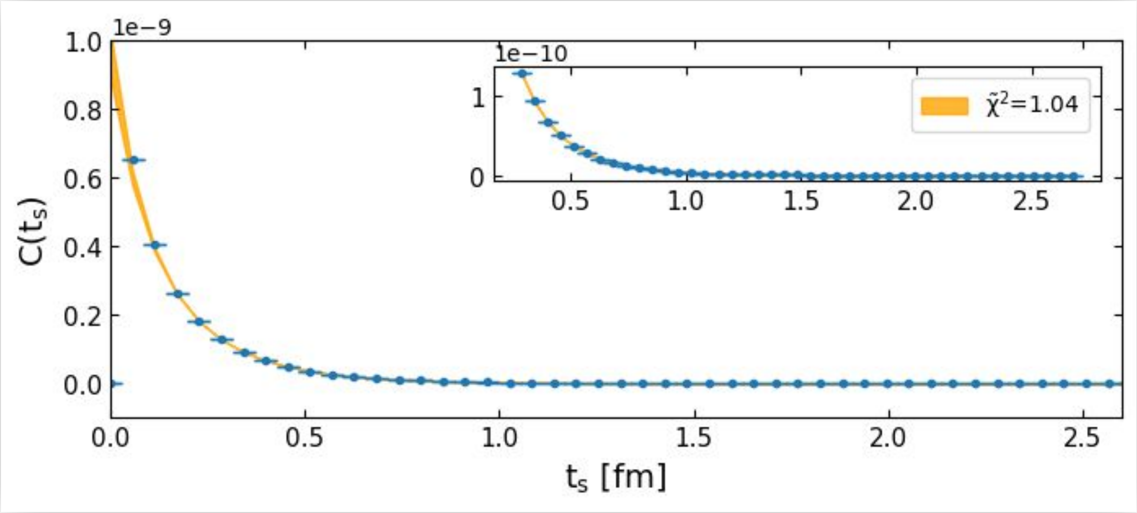

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Nucleon matrix element on lattice

➢ We take the two-point and three-point functions to momentum space.

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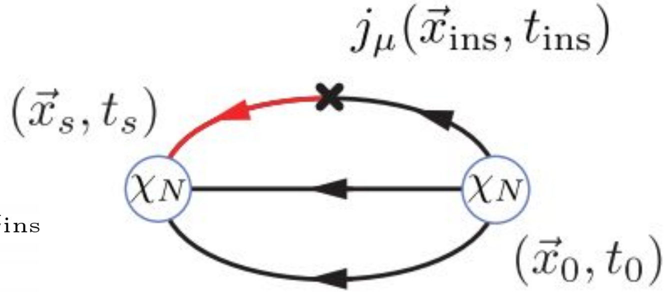

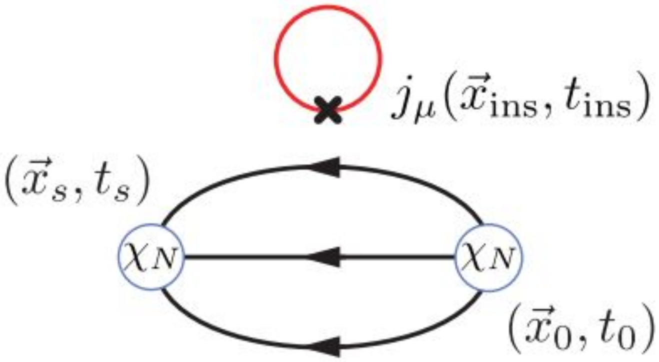

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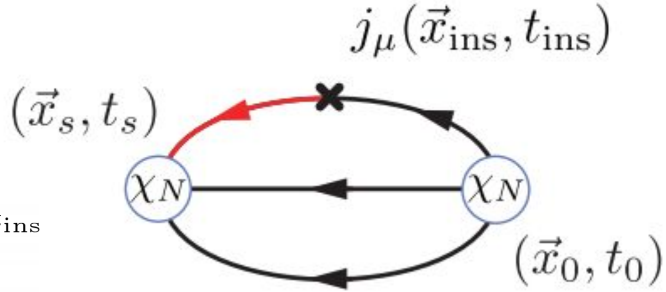
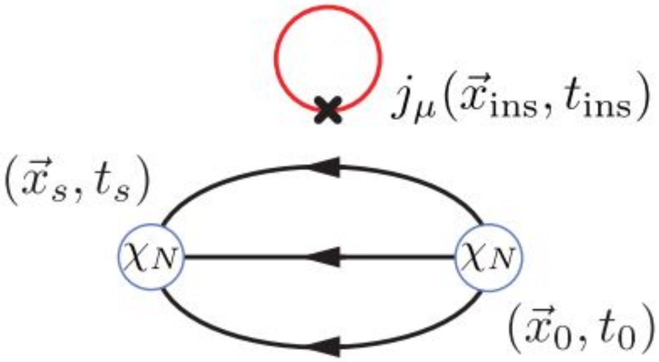
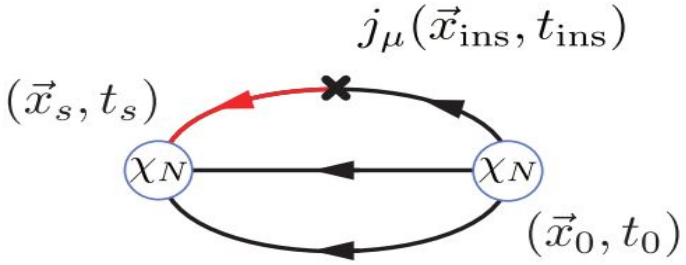


- We construct the following ratio to get rid of exponentials and overlaps.

$$\Pi_\mu(\Gamma_\nu; \vec{q}) = \frac{A_\mu^{0,0}(\Gamma_\nu, \vec{q})}{\sqrt{c_0(\vec{0}) c_0(\vec{q})}}$$



Connected contributions



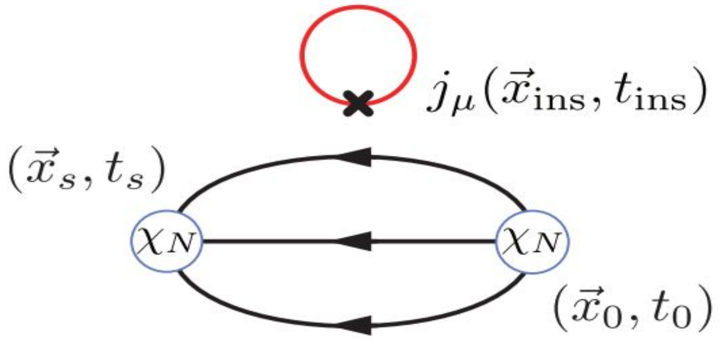
- For connected contribution, the sink momenta is set to 0.
- The number of source positions are increased for increasing t_s, to counter increase in noise.
- Lattice conserved current used, no renormalization needed.

cB211.072.64		
$n_{\rm conf}$=750		
t_s/a	t_s[fm]	n_{src}
8	0.64	1
10	0.80	2
12	0.96	5
14	1.12	10
16	1.28	32
18	1.44	112
20	1.60	128

cC211.060.80		
$n_{\rm conf}$=400		
t_s/a	t_s[fm]	n_{src}
6	0.41	1
8	0.55	2
10	0.69	4
12	0.82	10
14	0.96	22
16	1.10	48
18	1.24	45
20	1.37	116
22	1.51	246

cD211.054.96		
$n_{\rm conf}$=500		
t_s/a	t_s[fm]	n_{src}
8	0.46	1
10	0.57	2
12	0.68	4
14	0.80	8
16	0.91	16
18	1.03	32
20	1.14	64
22	1.25	16
24	1.37	32
26	1.48	64

Disconnected contributions



- ➢ Disconnected contribution is obtained from correlating high statistics two-point function with disconnected quark loop. Alexandrou et. al [1812.10311]
- ➢ Disconnected loop computed using deflation, hierarchical probing, dilution.
- ➢ Local current used, renormalization required.

Ensemble	$n_{\rm conf}$	n_{ev}	$n_{\rm src}$
cB211.072.64	750	200	477
cC211.060.80	400	450	650
cD211.054.96	500	-	480

Excited state contamination

- We are interested in the ground state matrix element of nucleons.
- For connected and disconnected, we do a multi-state fit using spectral decomposition.
- Excited state energies are kept separate between two and three-point fns [2104.00329].

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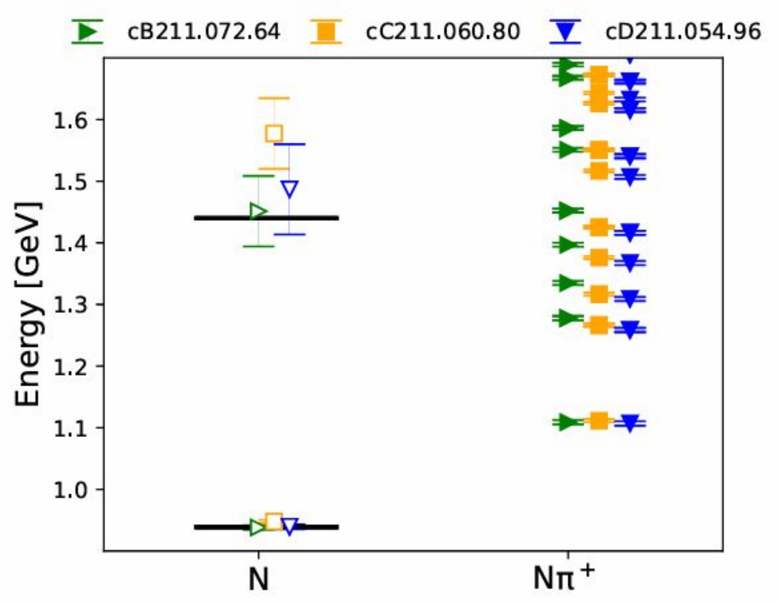

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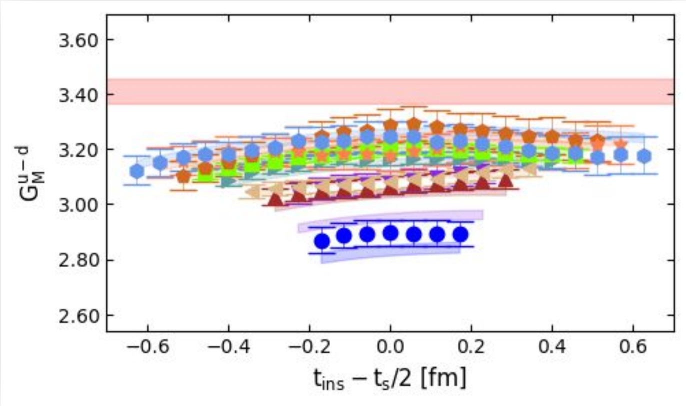

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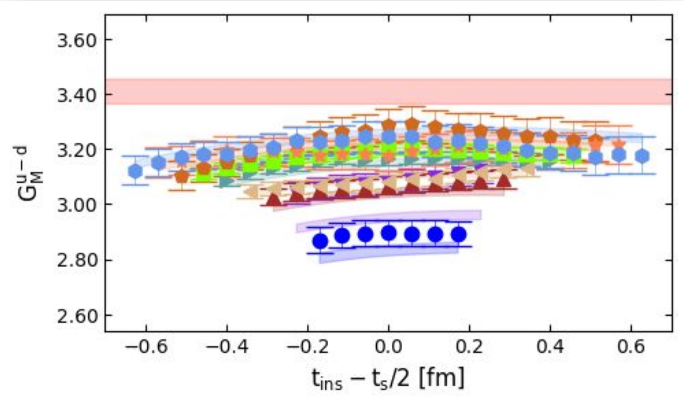
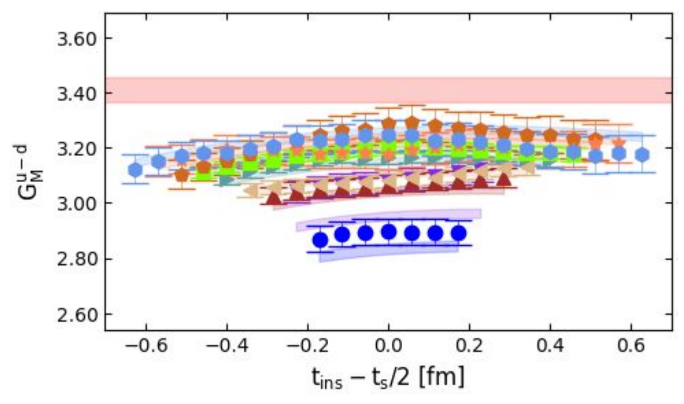


Ensemble	$t_s^{low,3pt}/a$	$t_s^{low,2pt}/a$	t_{ins}^{sink}/a
cB211.072.64	8, 10, 12, 14	1, 2, 3	2, 3, 4
cC211.060.80	8, 10, 12, 14	1, 2, 3, 4	2, 3, 4
cD211.054.96	8, 10, 12, 14	1, 2, 3, 4, 5	2, 3, 4

Extraction of Form Factors and Model Averaging

- This is done for each Q^2 value.
- We vary the ranges for two-point function $t_{s,min}$ and three-point function $t_{s,min}$ and $t_{ins,min}$.
- Results from all fits are then model averaged[2309.05774].
- For each fit we have $\chi^{2,i}$ and the $N^i_{dof} = (N_{data} - N_{params})$. Assign weight $w_i = (-0.5\chi^{2,i} + N^i_{dof})$.
- Probability = $e^{wi}/\sum_i e^{wi}$



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cD211.054.96	8, 10, 12, 14	1, 2, 3, 4, 5	2, 3, 4

Determination of radius and magnetic moment

- Once we have the parameterization of Q^2 and a^2, the radius can be obtained by:

$$\langle r_X^2 \rangle^q = \frac{-6}{G_X^q(0)} \left. \frac{\partial G_X^q(q^2)}{\partial q^2} \right|_{q^2=0}$$

- The strange moment is obtained simply by taking the value at $Q^2 = 0$:

$$G_M(Q^2 = 0) = \mu$$

Parameterization of Q^2 Dependance and continuum limit

Dipole

$$G(Q^2) = \frac{g}{\left(1 + \frac{Q^2}{12} r^2\right)^2}$$

$$G(Q^2, a^2) = \frac{g(a^2)}{\left(1 + \frac{Q^2}{12} r^2(a^2)\right)^2}$$

$$g(a^2)=g_0+a^2 g_2\,,\ \langle r^2(a^2)\rangle=\langle r\rangle_0^2+a^2\langle r^2\rangle_2$$

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z-expansion

$$G(Q^2) = \sum_{k=0}^{k_{\text{max}}} c_k z^k(Q^2)$$

$$z = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}}}}$$

$$c_k(a^2) = c_{k,0} + a^2 c_{k,2}$$

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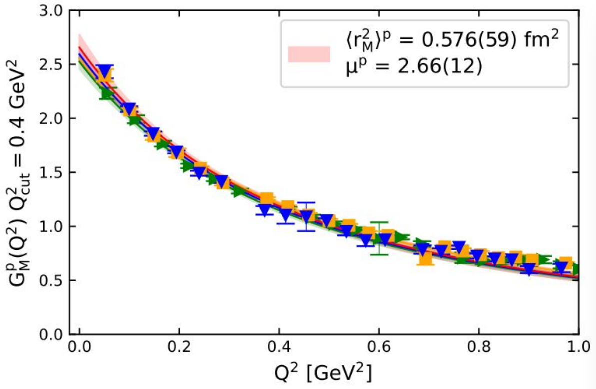
Galster-like

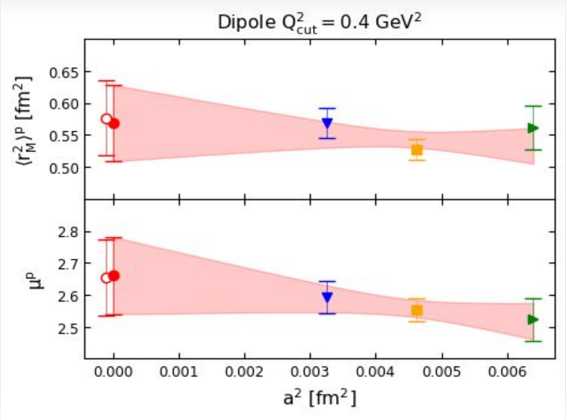
$$G(Q^2) = \frac{Q^2 A}{4m_N^2 + Q^2 B} \frac{1}{\left(1 + \frac{Q^2}{0.71\,\mathrm{GeV}^2}\right)^2}$$

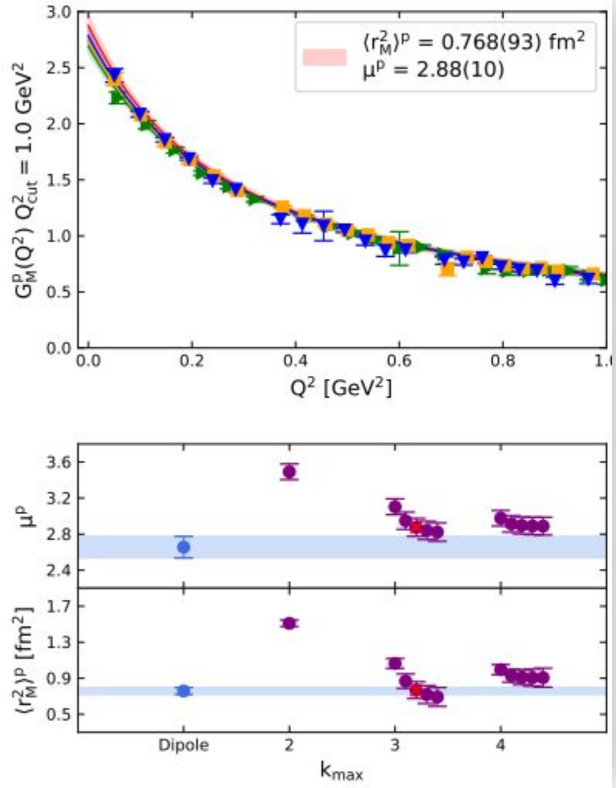
$$A(a^2) = A_0 + a^2 A_2$$

$$B(a^2) = B_0 + a^2 B_2$$

Example fits: Proton magnetic form factors





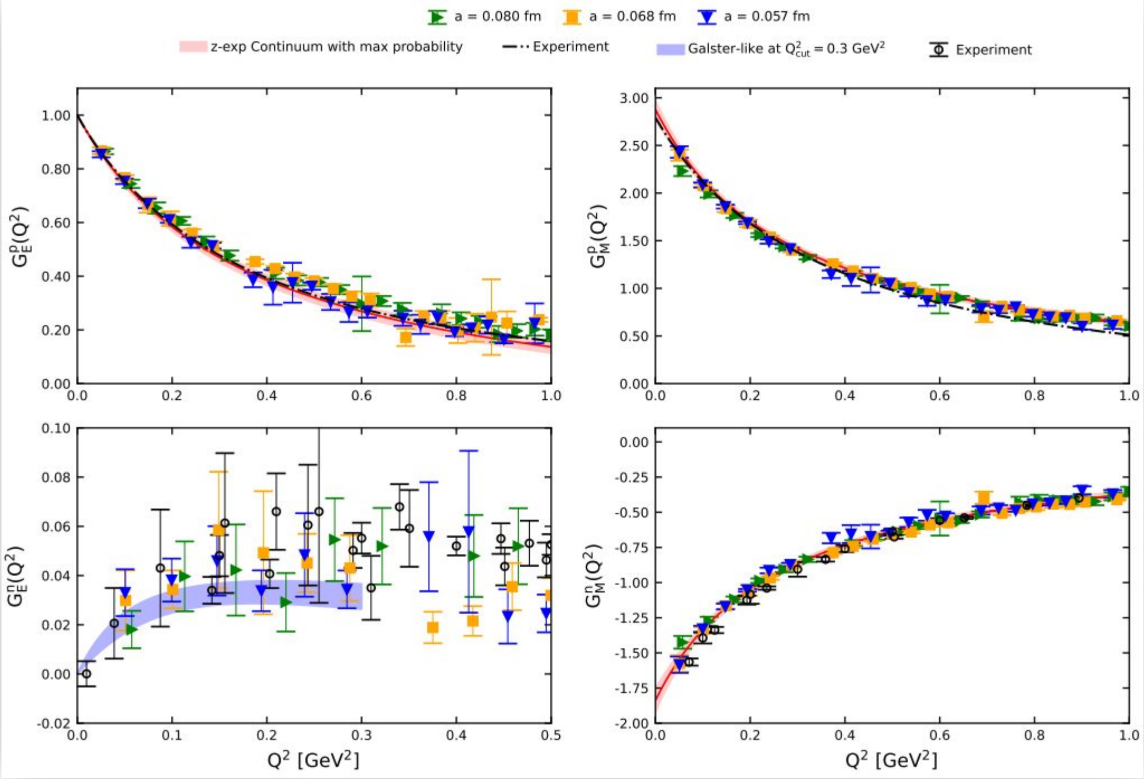


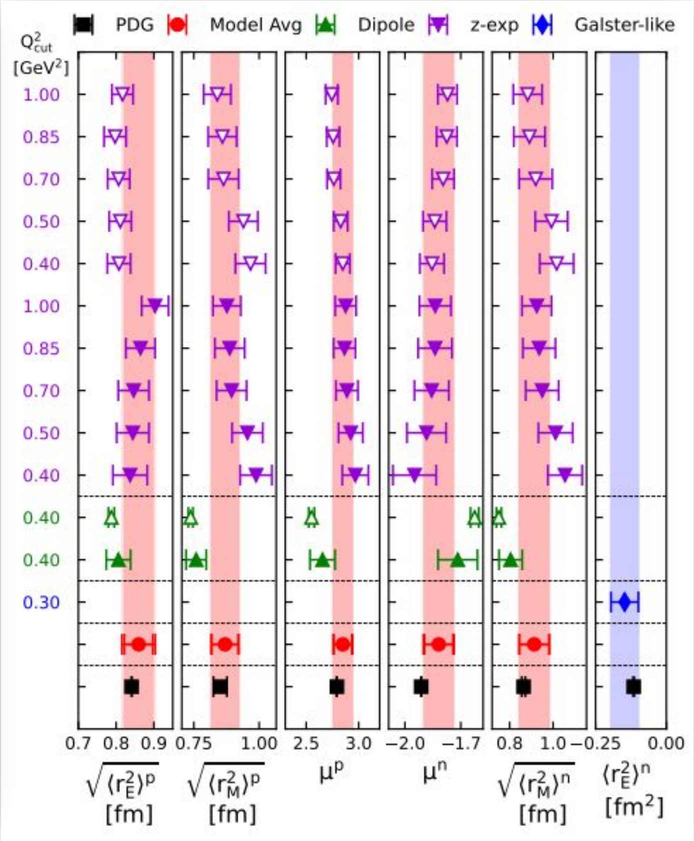
Ensemble	μ^p	$\langle r_M^2\rangle^p$ [fm^2]	$\tilde{\chi}^2$
cB211.72.64	2.524(67)	0.562(34)	1.016
cC211.60.80	2.553(37)	0.527(17)	2.230
cD211.54.96	2.592(49)	0.569(24)	2.732
$a = 0$, 1-step	2.66(12)	0.576(59)	2.326
$a = 0$, 2-step	2.66(12)	0.569(60)	-

Q^2_{cut} [GeV^2]	μ^p	$\langle r_M^2\rangle^p$ [fm^2]	$\tilde{\chi}^2$
0.40	2.97(12)	0.98(12)	1.172
0.50	2.92(12)	0.91(11)	1.007
0.70	2.89(10)	0.80(10)	1.311
0.85	2.86(10)	0.79(10)	1.338
1.00	2.88(10)	0.768(93)	1.282

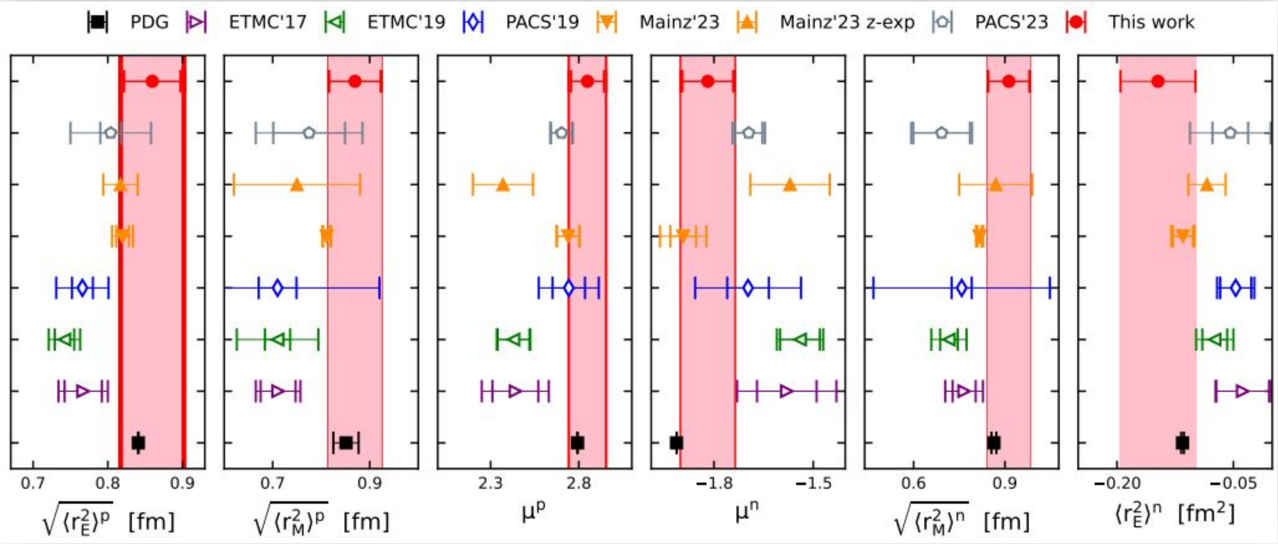
Results on Proton and Neutron

[2507.20910]





Comparison of results



Final results:
[2507.20910]

$\langle r_E^2\rangle^p$ [fm^2]	μ^p	$\langle r_M^2\rangle^p$ [fm^2]	μ^n	$\langle r_M^2\rangle^n$ [fm^2]	$\langle r_E^2\rangle^n$ [fm^2]
0.739(64)(39)	2.849(92)(52)	0.756(92)(25)	-1.819(76)(29)	0.83(12)(03)	-0.147(48)

Why Strange Electromagnetic Form Factors?

- Gives us insight into the sea quark dynamics and has a very small contribution to proton and neutron results.

- Experimentally measured through parity violating electron-proton elastic scattering.

- The difference in $\boldsymbol{\sigma}_{\mathbf{L}}$ and $\boldsymbol{\sigma}_{\mathbf{R}}$ comes from the interference of photon exchange amplitude with the amplitude of Z_0 boson exchange.

$$A^{PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

- Experimental results do not exclude zero value.

- Want to calculate it from first principle lattice calculation.

Strange electromagnetic matrix element

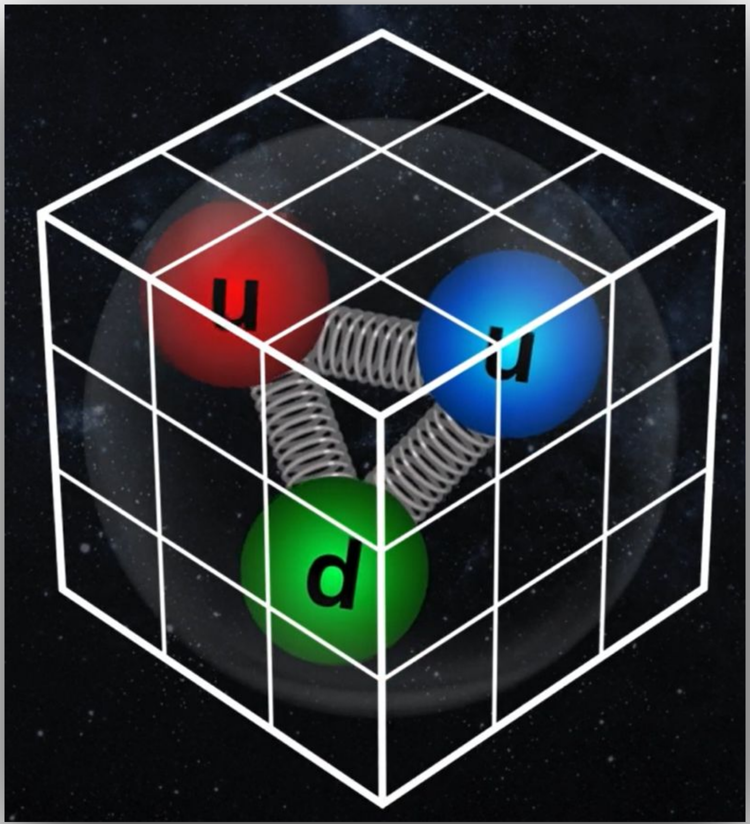
- Interested in theoretically probing the structure of nucleons using lattice QCD.

- The nucleon matrix element of for the electromagnetic current is given by:

$$\langle N(p', s')|j_\mu|N(p, s)\rangle = \sqrt{\frac{m_N^2}{E_N(\vec{p}\,')E_N(\vec{p})}} \times$$

$$\bar{u}_N(p', s')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2)\right]u_N(p, s)$$

$$j_\mu = \tfrac{2}{3}\overline{u}\gamma_\mu u - \tfrac{1}{3}\overline{d}\gamma_\mu d - \tfrac{1}{3}\overline{s}\gamma_\mu s$$



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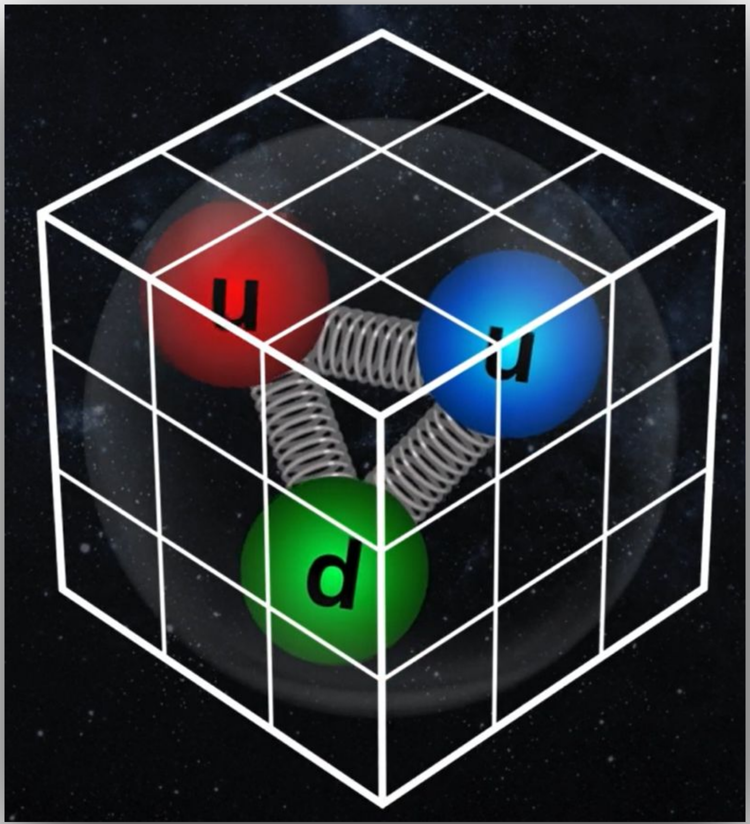
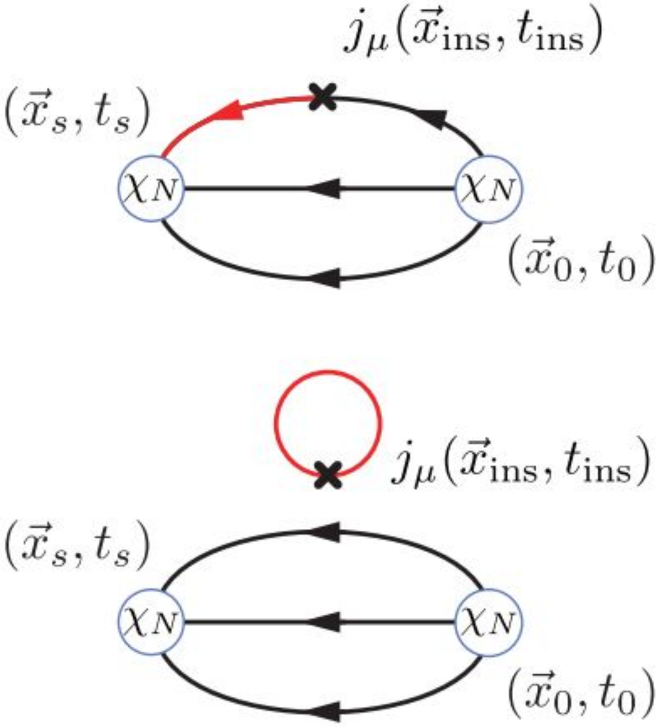


Nucleon strange matrix element on lattice

- We take the two-point and three-point functions to momentum space.

$$C(\Gamma_0, \vec{p}, t_s) = \sum_n c_n(\vec{p}) e^{-E_n(\vec{p}) t_s}$$

$$C_\mu(\Gamma_k, \vec{q}, t_s, t_{\rm ins}) = \sum_{i,j} A_\mu^{ij}(\Gamma_k, \vec{q}) e^{-E_i(\vec{p})(t_s - t_{\rm ins}) - E_j(\vec{q}) t_{\rm ins}}$$



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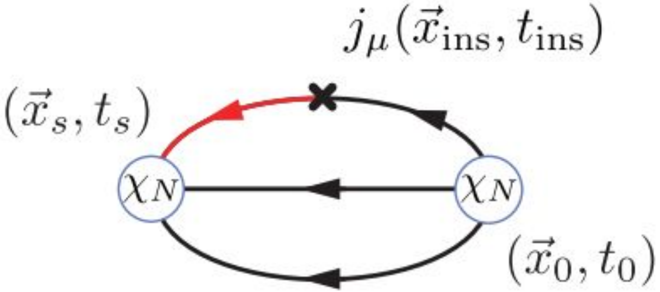
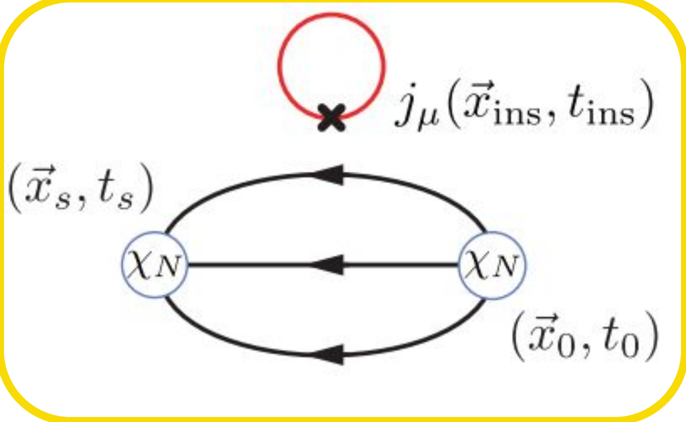


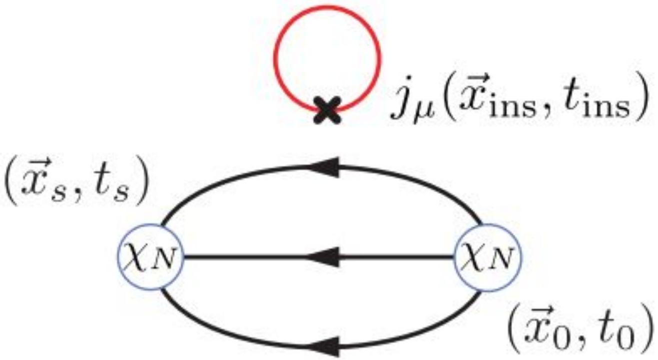


Matrix element in terms of correlators

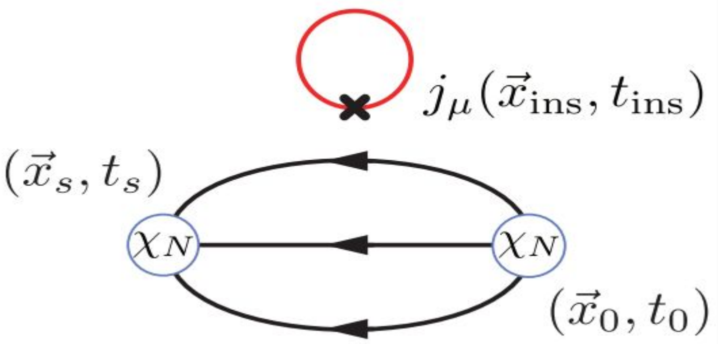
- We take an appropriate ratio of the two-point and three-point functions.

- This gives the ground state matrix element in large time separation limit upto kinematics.

$$\Pi_\mu(\Gamma_\nu, \vec{p}', \vec{p}; t_s, t_{ins}) = \frac{C_\mu(\Gamma_\nu, \vec{p}', \vec{p}; t_s, t_{ins})}{C(\Gamma_0, \vec{p}'; t_s)} \times \sqrt{\frac{C(\Gamma_0, \vec{p}; t_s - t_{ins}) C(\Gamma_0, \vec{p}'; t_{ins}) C(\Gamma_0, \vec{p}'; t_s)}{C(\Gamma_0, \vec{p}'; t_s - t_{ins}) C(\Gamma_0, \vec{p}; t_{ins}) C(\Gamma_0, \vec{p}; t_s)}}$$



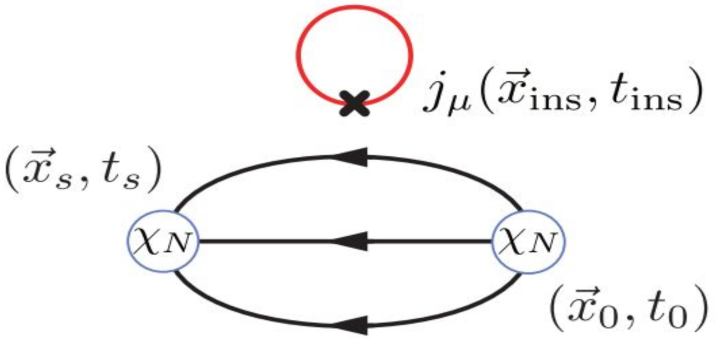
Disconnected contributions



- ➢ Disconnected contribution is obtained from correlating high statistics two-point function with disconnected quark loop (Alexandrou et. al [1812.10311, 1909.10744]).
- ➢ Disconnected loop computed using hierarchical probing, dilution.
- ➢ Local current used, renormalization required.

Ensemble	n_{conf}	n_{src}
cB211.072.64	749	349
cC211.060.80	401	650
cD211.054.96	493	368
cE211.044.112	464	311

Disconnected contributions



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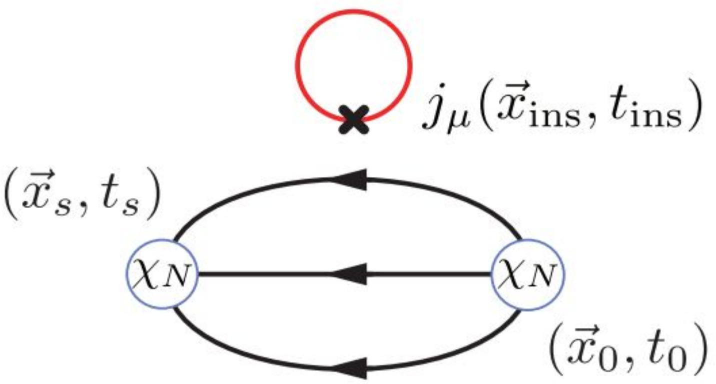
Additional sink momenta



- ➢ Once we obtain the quark loop and two-point functions we can correlate them using additional sink momenta at no additional cost.
- ➢ We thus use $p'^2=2$.
- ➢ This increases the Q^2 value to O(300) for each ensemble.
- ➢ We make use of Singular Value decomposition in obtaining results for each Q^2.

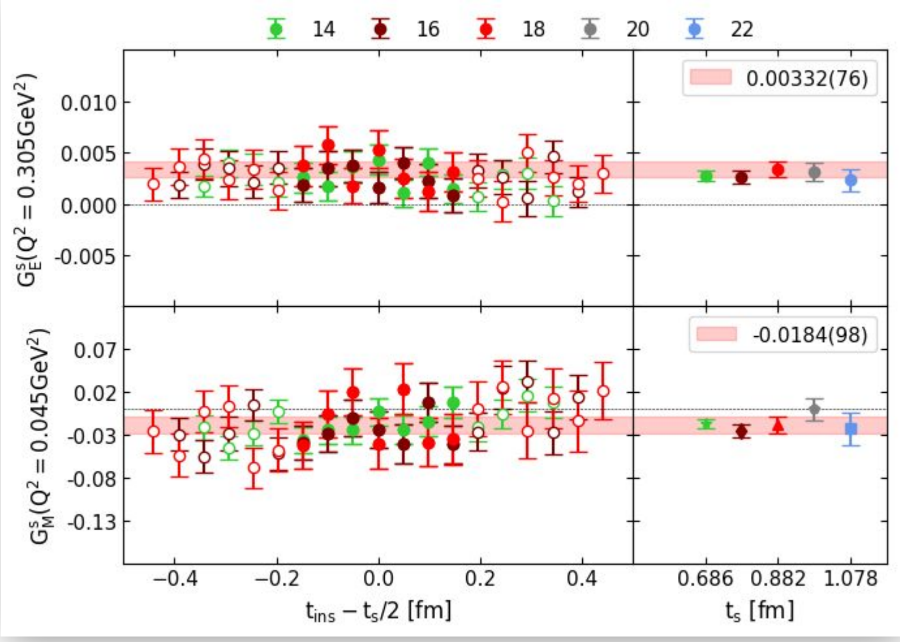
Excited state contamination

- We are interested in the ground state matrix element of nucleons.

Excited state contamination

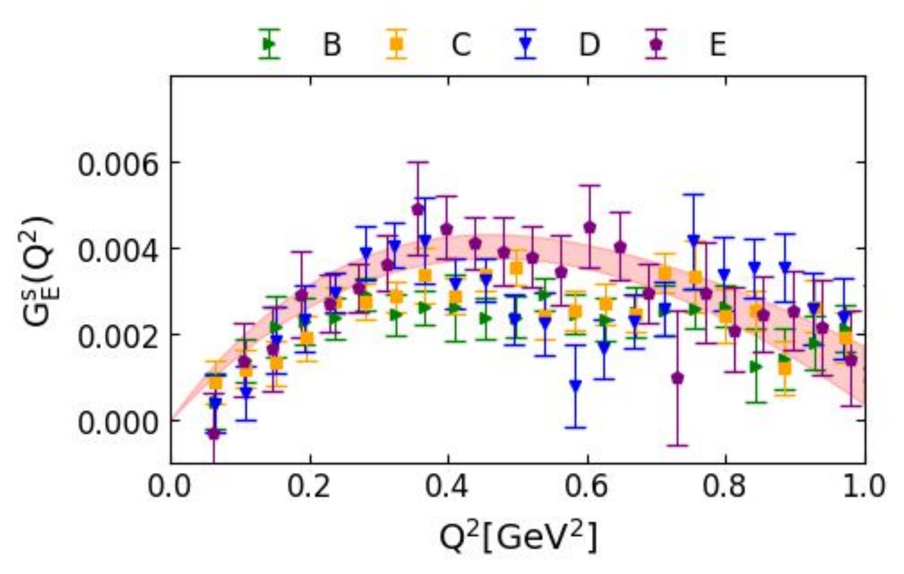
➢ We are interested in the ground state matrix element of nucleons.

➢ Given no indication of excited state with the current statistical accuracy we opt to do a plateau fit to the optimized ratio. Example for E ensemble.



Strange electric form factors

- The procedure is repeated for all Q^2 values for electric case resulting in the following.
- Results are binned into 23 bins.



Strange electric form factors

➢ The procedure is repeated for all Q^2 values for electric case resulting in the following.

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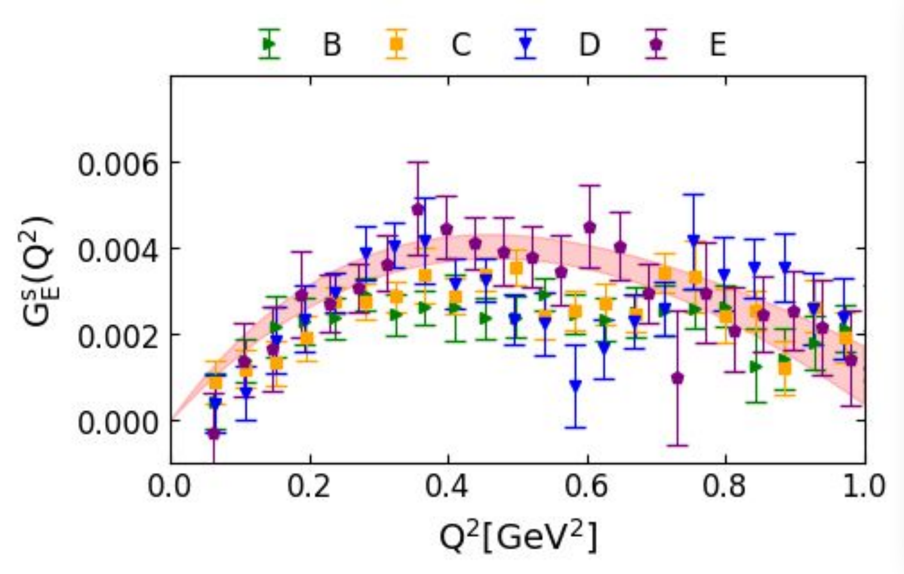
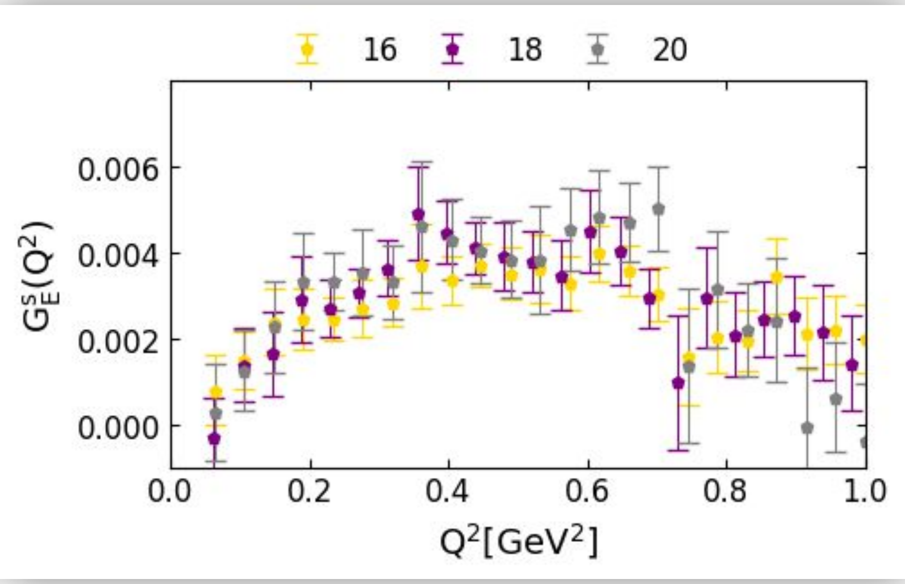
➢ Example of convergence in source-sink separation, t_s on right for E ensemble.





Strange magnetic form factors

- The procedure is repeated for all Q^2 values for magnetic case resulting in the following.
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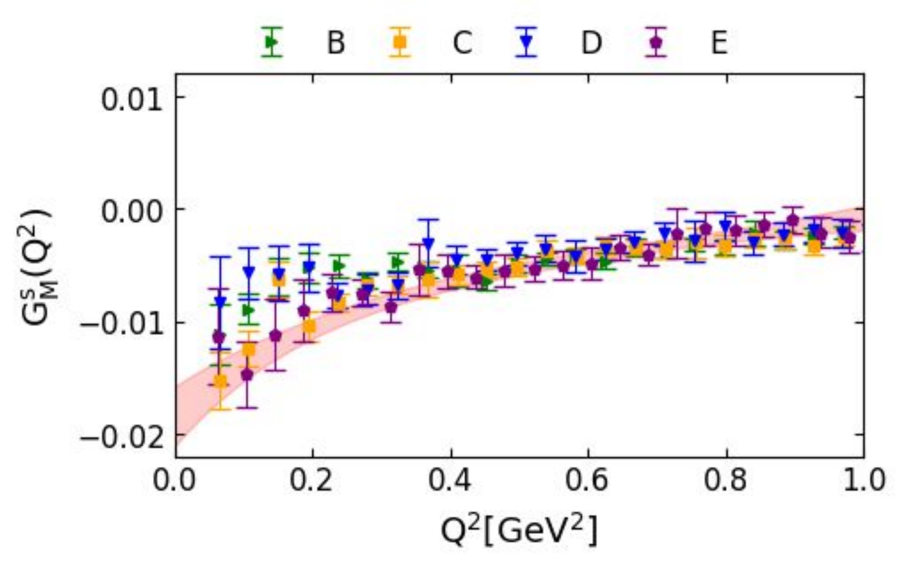

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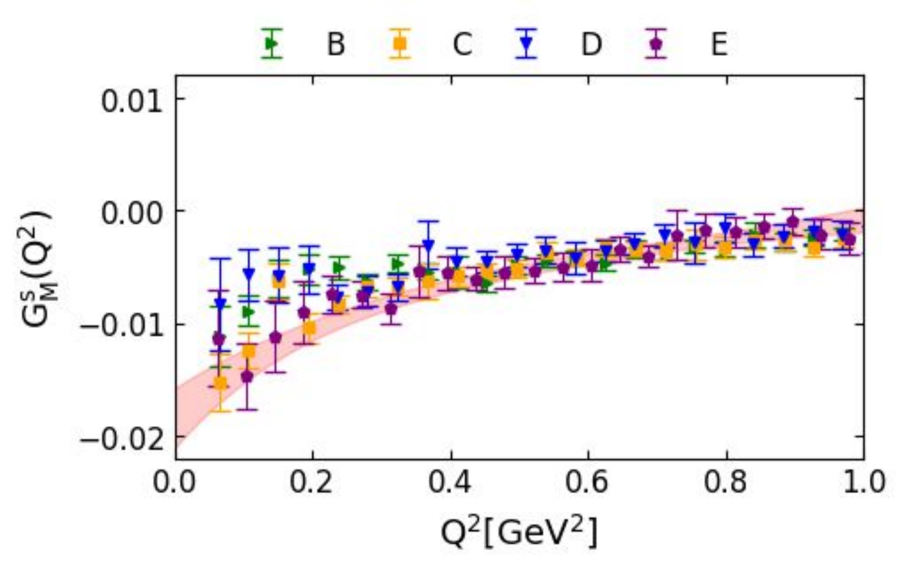
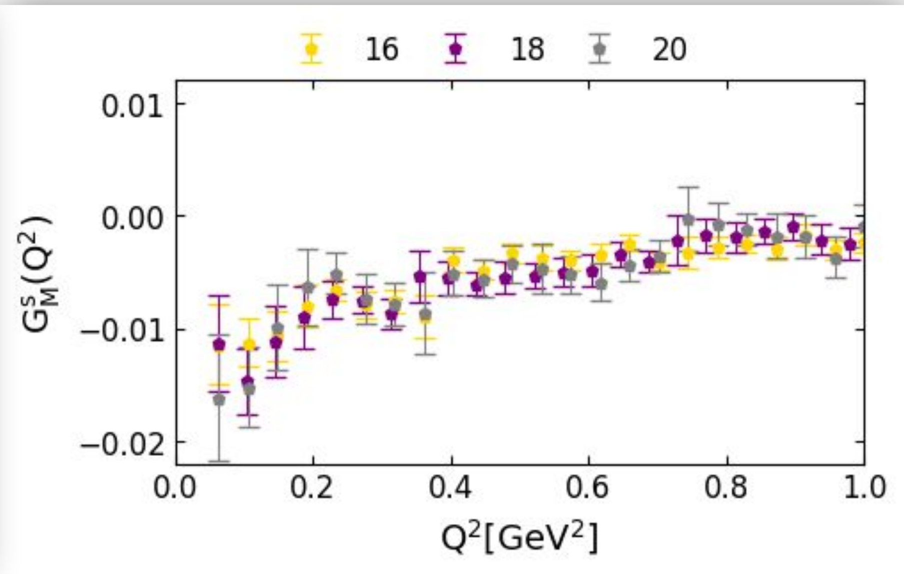
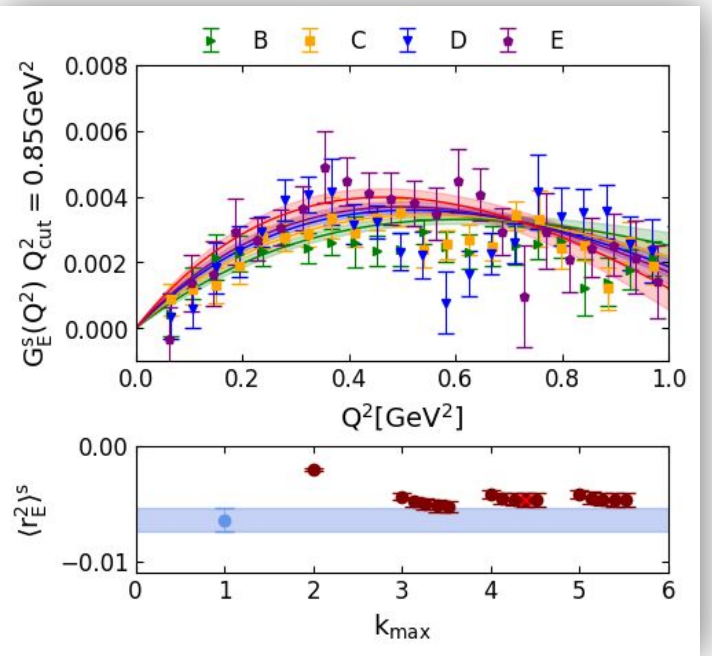


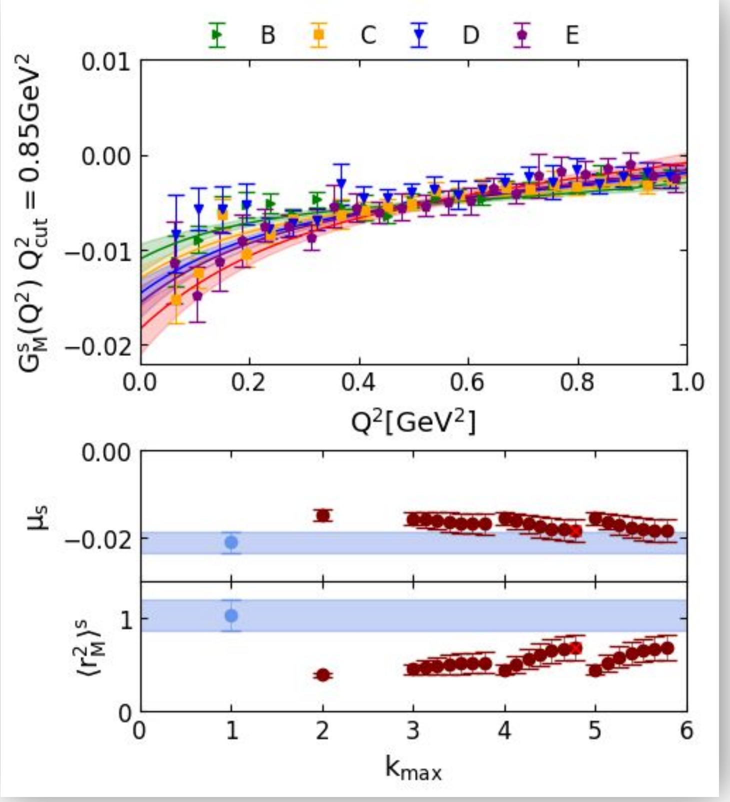


Strange electric form factors with an example fit



- ➢ Strange electric form factor with z-expansion fit.
- ➢ The bottom band shows the convergence with k_{max} and prior width as in [2507.20910].
- ➢ The blue band is result from one step Galster-like only for comparison.

Strange magnetic form factors with an example fit



- ➢ Strange magnetic form factor with z-expansion fit.
- ➢ The bottom band shows the convergence with k_{max} and prior width.
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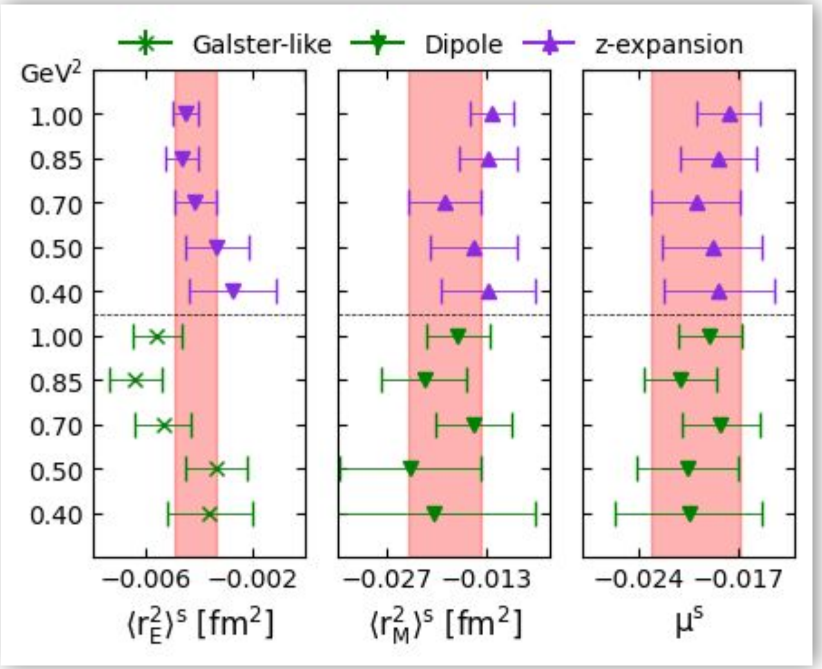
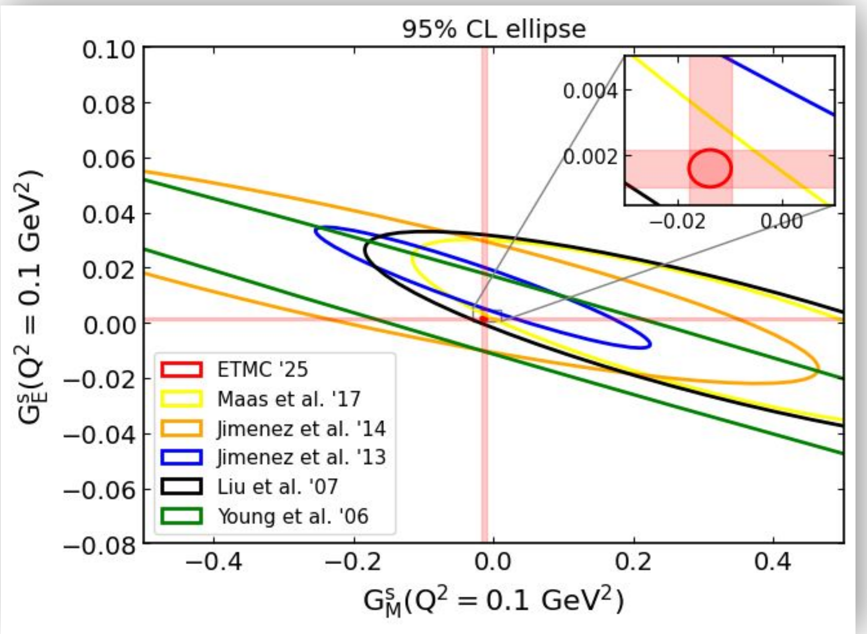
Q^2 variation



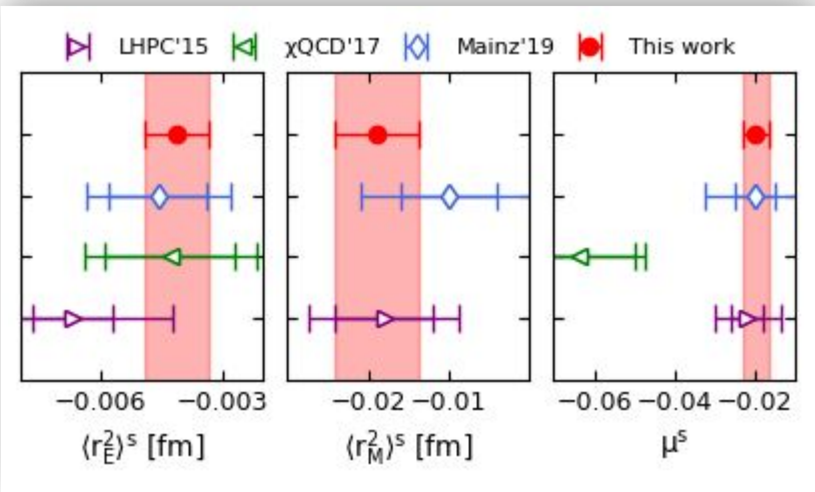
- We perform a model average using AIC over the following:
- We vary the cut in Q^2 used in fit.
- We vary over the different ansaetz.

Results with experimental comparison



➢ Strange electric form factor at Q^2=0.1 GeV^2 compared through 95% confidence curves.

Comparison with previous lattice works



➢ We present a comparison of our preliminary results with previous lattice works.

Summary

- We have results for proton, neutron and nucleon strange electromagnetic form factors at continuum limit, at physical point.
- Results include disconnected contributions with additional sink momenta.
- Multi-state fits used to ensure ground state convergence.
- Ongoing efforts to increase E ensemble statistics. We acknowledge early access to jupiter for this.

Thank you!

Co-funded by the European Union

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 101034267. This work was also supported by the project PulseQCD co-funded by the EU within the framework of the Cohesion Policy Programme "THALIA 2021-2027".

Republic of Cyprus

Backup

$$\Pi^0(\Gamma_0, \vec{q}) = C\frac{E_N + m_N}{2m_N} G_E(Q^2),$$

$$\Pi^i(\Gamma_0, \vec{q}) = C\frac{q_i}{2m_N} G_E(Q^2),$$

$$\Pi^i(\Gamma_k, \vec{q}) = C\frac{\epsilon_{ijk} q_j}{2m_N} G_M(Q^2)$$

$$C = \sqrt{\frac{2m_N^2}{E_N(E_N + m_N)}},$$

Backup

$$
\begin{aligned}
\Pi_{\mu,0} = & -iCG_E\Big[\left(p'_\mu + p_\mu\right)\left(m\left(E' + E + m\right) - p'_\rho p_\rho\right)\Big] \\
& + \frac{CG_M}{2m}\Big[\delta_{\mu 0}(4m^2 + Q^2)(m^2 + p'_\rho p_\rho) - iEQ^2 p'_\mu \\
& + 2im^2(E' - E)(p'_\mu - p_\mu) - iE'Q^2 p_\mu \\
& - imQ^2(p'_\mu + p_\mu)(2m^2 + Q^2 + 2p'_\rho p_\rho)\Big] \qquad \text{(A1)}
\end{aligned}
$$

$$
\begin{aligned}
\Pi_{\mu,k} = & \, CG_E\Big[\epsilon_{\mu k 0 \rho}(p'_\rho - p_\rho)(m^2 - p'_\sigma p_\sigma) \\
& - i\epsilon_{\mu k \rho \sigma} p'_\rho p_\sigma (E' + E) + \epsilon_{\mu 0 \rho \sigma} p'_\rho p_\sigma (p'_k + p_k)\Big] \\
& + \frac{CG_M}{2m}\Big[m\epsilon_{\mu k 0 \rho}(p'_\rho - p_\rho)(2m^2 + Q^2 + 2p'_\sigma p_\sigma) \\
& + 2im\epsilon_{\mu k \rho \sigma} p'_\rho p_\sigma (2m + E' + E + \frac{Q^2}{2m}) \\
& - 2m\epsilon_{\mu 0 \rho \sigma} p'_\rho p_\sigma (p'_k + p_k)\Big], \qquad \text{(A2)}
\end{aligned}
$$

where C is a kinematic factor given by

$$
C = \frac{m(4m^2 + Q^2)^{-1}}{E(E' + m)}\sqrt{\frac{E(E' + m)}{E'(E + m)}} \qquad \text{(A3)}
$$