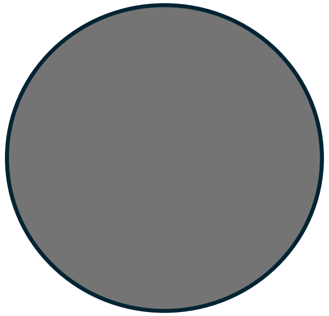
Moments of PDFs of the pion from Lattice QCD using gradient flow

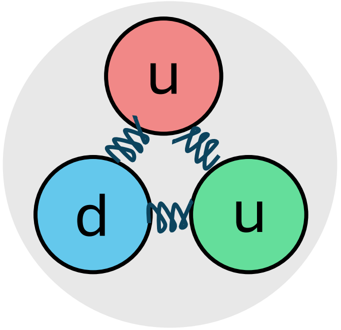
Dimitra Pefkou

in collaboration with Andrea Shindler and *OpenLat*

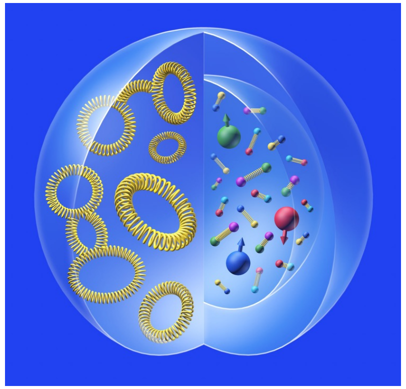
Parton picture of hadrons



proton



valence quarks



valence + sea quarks and gluons

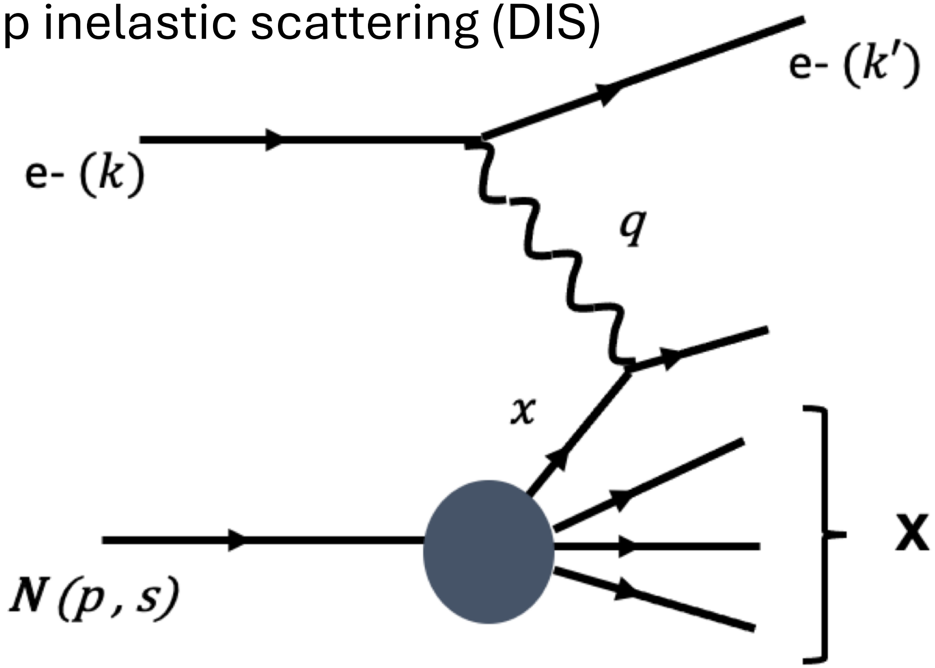
Illustration by Kent Leech for Lawrence Berkeley National Laboratory, Creative Services Office

twist = dim - spin

This talk: $f_i = f_i^{(0)}$ (only twist-2)

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} + \cdots$$

Deep inelastic scattering (DIS)



$$Q^2 = -q^2$$

$$x = \frac{-q^2}{2p \cdot q}$$

$$\sigma_{\mathrm{DIS}} = \sum_i [H^i_{DIS} \otimes f_i](x, Q^2)$$

$f_i(x)$: Parton distribution functions (PDF)
→ probability of the struck parton carrying momentum fraction x

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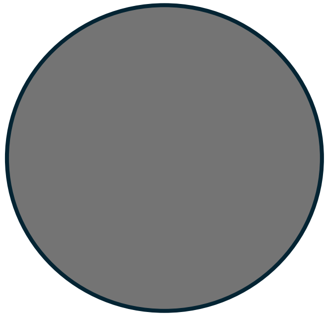

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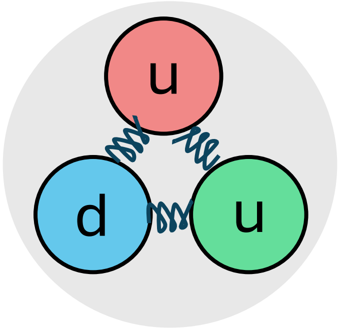

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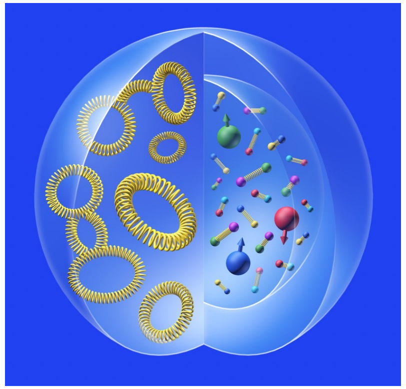

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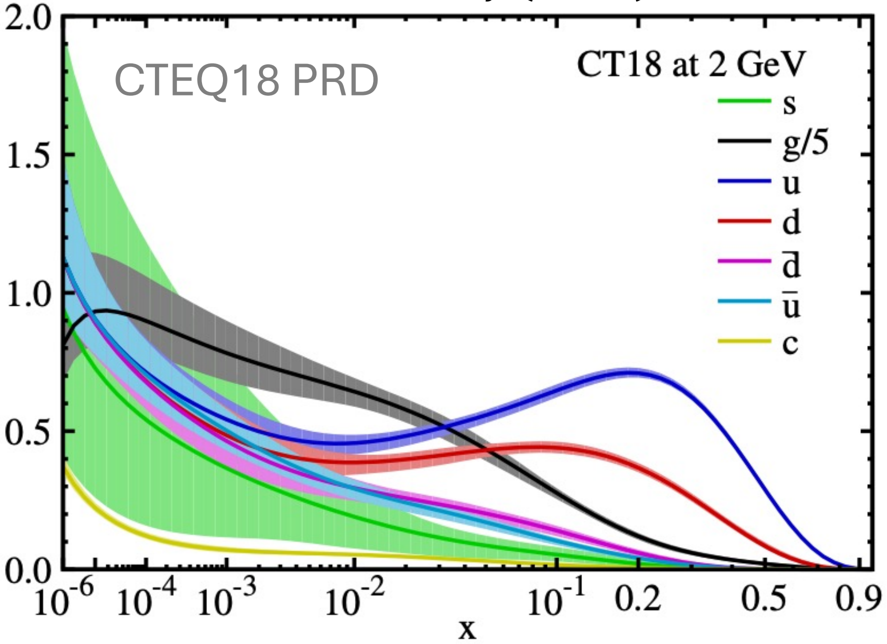
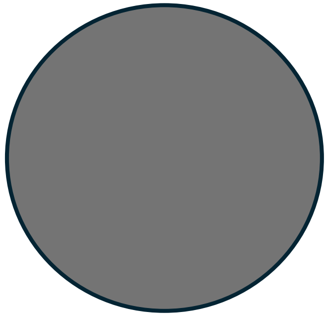
valence + sea quarks and gluons

Proton $x * f(x, Q^2)$



Proton PDFs one of dominant uncertainties for collider searches

Parton picture of hadrons



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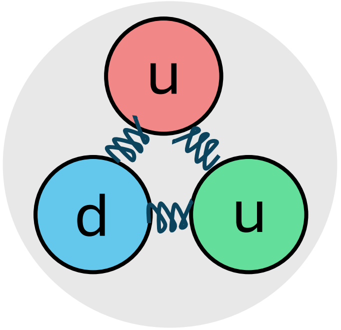

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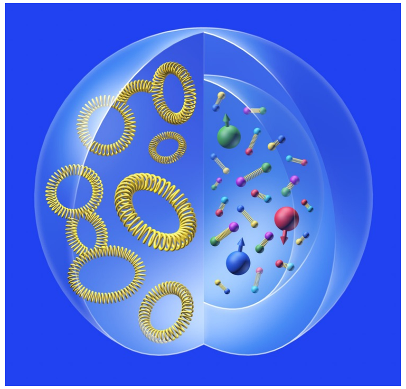

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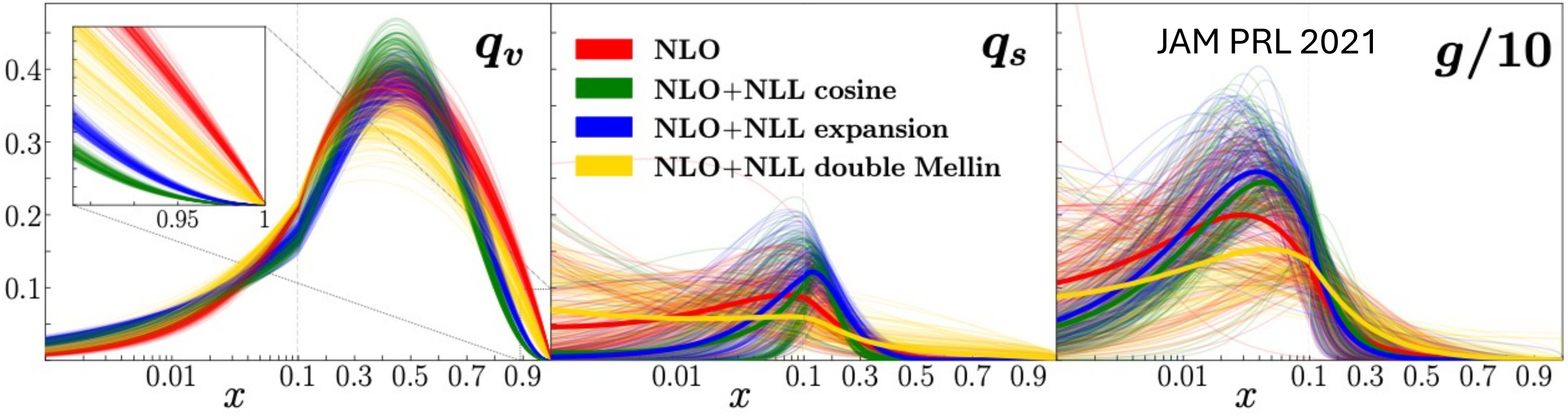


Much less constrained, from ~old experimental data.

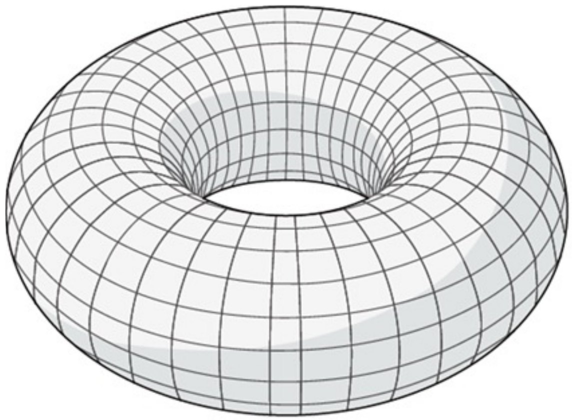
Large x behavior debated

Lattice QCD

- Wick rotation to Euclidean time: $t \to it \;\Rightarrow\; e^{iS} \to \; e^{-S}$: statistical system in 4D

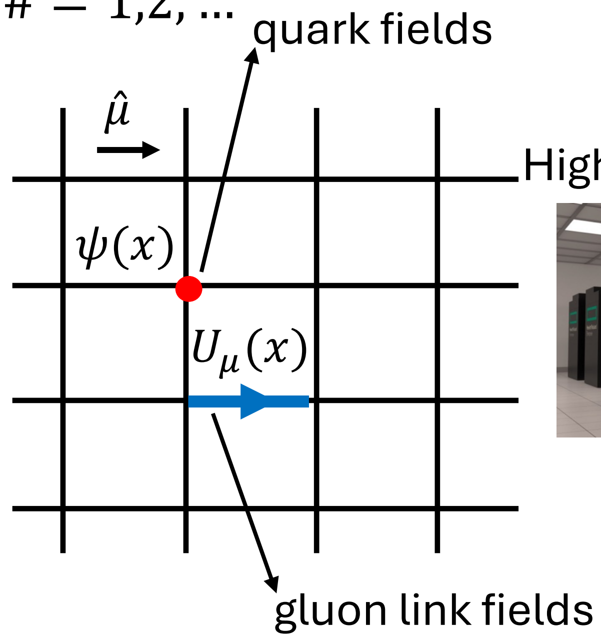
- Discretize spacetime $x_\mu \to a\, n_\mu\,, n_\mu \;\in \{1, 2, \ldots, L_\mu\}$
  lattice spacing a: UV cutoff

$L{\times}L{\times}L{\times}T$



- Multiple ways to choose action $S^{\text{latt}} = S^{\text{QCD}} + \mathcal{O}\left(a^{\#}\right)$, $\# = 1{,}2, \ldots$
  such that $S^{\text{latt}} \to S^{\text{QCD}}$ as $a \to 0$

- Connect to physical world:
  → volume $L^3 \to \infty$
  → lattice spacing $a \to 0$
  → hadron masses e.g. $m_\pi, m_K, \ldots$ →physical
  (cheaper when heavier)



High Performance Computing

Perlmutter at NERSC, LBL

PDFs in lattice QCD

- Unpolarized quark PDFs defined via matrix elements of *non-local* operator
  $O(-\frac{n}{2},\frac{n}{2}) = \bar{q}\left(-\frac{n}{2}\right)\gamma^{\mu}\mathcal{U}_{\left[-\frac{n}{2},+\frac{n}{2}\right]}q\left(+\frac{n}{2}\right)$, n lightlike 4-vector

- Euclidean $t \to it$: lightlike separation collapses to a single point

- Proposals to obtain x-dependence in lattice QCD:
  → hadronic tensor method [Liu Dong hep-ph/9306299]
  → auxiliary scalar quark [Aglieti et al hep-ph/9804416]
  → auxiliary heavy quark [Detmold Lin hep-lat/0507007]
  → auxiliary light quark [Braun Müller 0709.1348]
  → quasi-distributions [Ji 1305.1539]
  → current-current approach [Ma Qiu 1709.03018]
  → pseudo-distributions [Radyuskin 1612.05170]
  → OPE-based method [Chambers et al (QCDSF) 1703.01153]

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Huge progress on x-dependence, see e.g talks by Huey-Wen Lin, Joshua Miller, Joseph Delmar, Pawel Sznajder, Krzysztof Cichy, posters by Joseph Torsiello, Gabriele Pierini

Mellin moments of PDFs

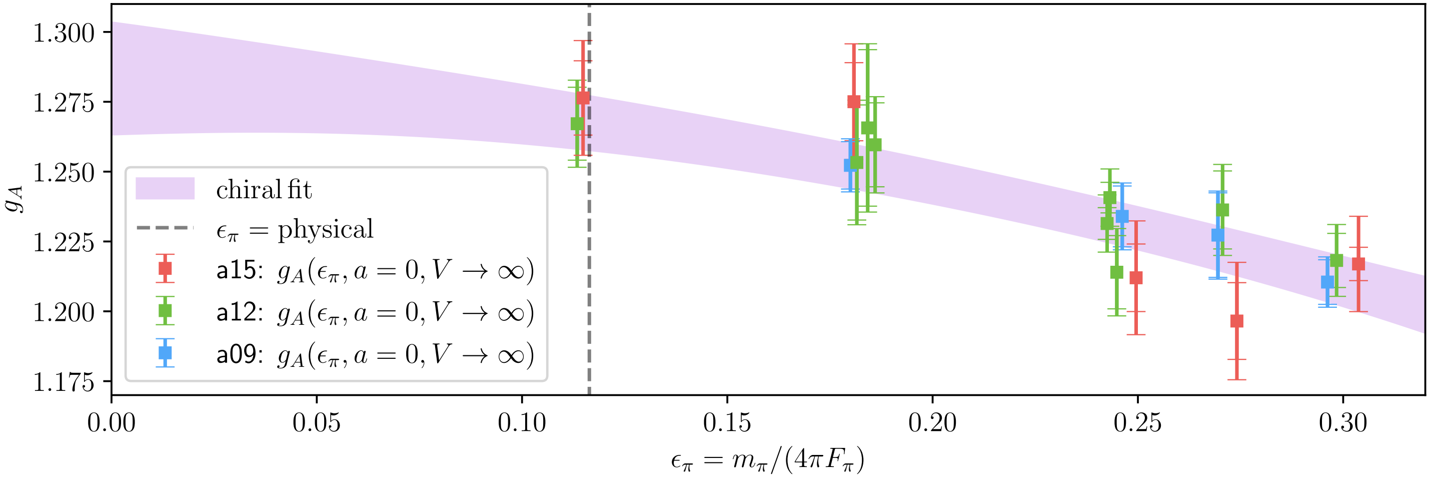
- Operator product expansion: $O(-\frac{n}{2},\frac{n}{2}) \rightarrow \mathcal{O}^{\mu_1\cdots\mu_n} = i^{n-1}\bar{\psi}\gamma^{\{\mu_1}\overleftrightarrow{D}^{\mu_2}\ldots\overleftrightarrow{D}^{\mu_n\}}\psi - \text{Traces}$

- $\mathcal{O}^{\mu_1\cdots\mu_n}$: → twist-2 local operators
  → transform irreducibly under Lorentz group

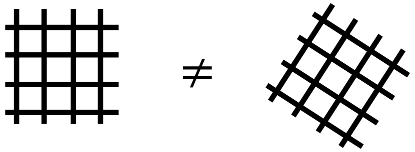
- Matrix elements with respect to hadron h → Mellin moments of PDFs $\langle x^{n-1}\rangle$:

$$\langle h(p)|\mathcal{O}^{\mu_1\cdots\mu_n}|h(p)\rangle = 2\langle x^{n-1}\rangle p^{\mu_1}p^{\mu_2}\ldots p^{\mu_n} - \text{Traces}$$

- Lattice QCD great at controlled high precision calculations of **local** operator matrix elements.
  e.g. nucleon axial charge g_A at sub-percent level (Hall **Pefkou** et al (Callat) 2503.09891)



Moments of PDFs in lattice QCD



(a lattice spacing)

- Discrete lattice: Lorentz symmetry reduces to hypercubic symmetry
  20 irreducible representations named $\tau_1^{(1)}, \tau_1^{(3)}, \ldots$

$$\langle h(p)|\mathcal{O}^{\mu_1 \cdots \mu_n}|h(p)\rangle = 2\langle x^{n-1}\rangle p^{\mu_1} p^{\mu_2} \ldots p^{\mu_n} + \cdots$$

Mandula Zweig Govaerts 1983
Göckeler et al, hep-lat/9602029

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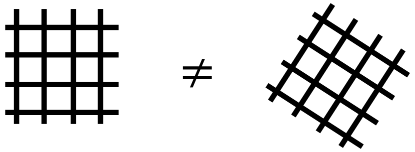

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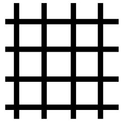
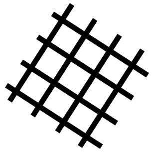
- $n = 2: \ \tau_1^{(3)} \ \rightarrow \mathcal{O}_{44} - \text{Traces}, \ldots$
  $\tau_3^{(6)} \ \rightarrow \mathcal{O}_{\{14\}}, \ldots$ → requires boost in 1 direction (more noisy than 44)

Moments of PDFs in lattice QCD

 $\neq$  (a lattice spacing)

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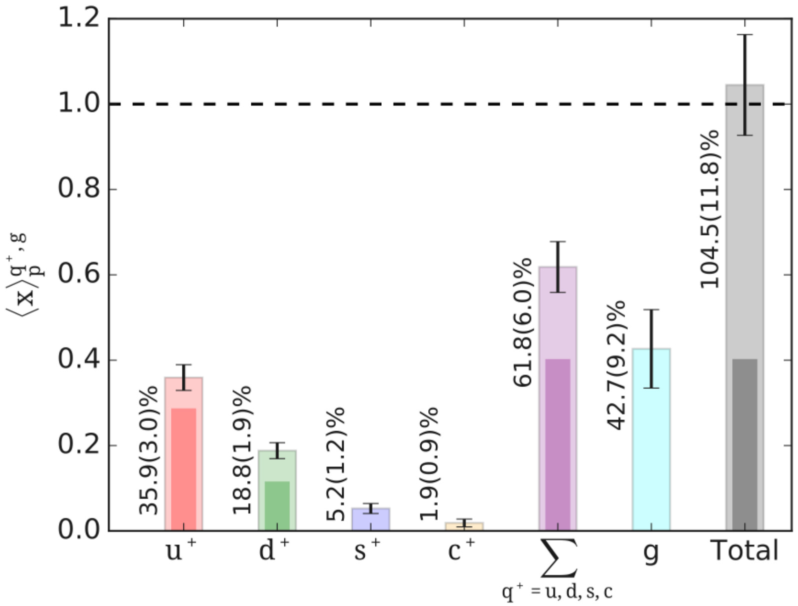
Momentum fraction $\langle x\rangle$ well determined by multiple collaborations, e.g.
Flavor-vector $(u-d)$: NME 2011.12787
Flavor-singlet $(u+d, g)$: MSU 2208.00980, χQCD 1808.08677, ETMC 2003.08486 (no continuum limit for full flavor decomposition yet, but underway (see poster by Yan Li))



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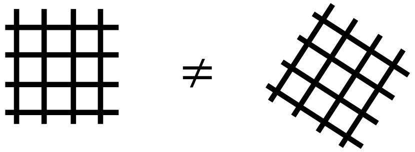

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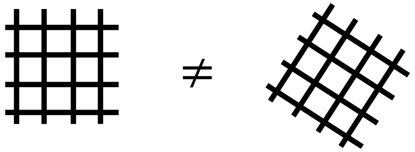
- $n = 3:\ \tau_1^{(4)} \ \to \mathcal{O}_{444} - \text{Traces}, \ldots \to \frac{1}{a^{\#}}$ mixing (power divergent as $a \to 0$)
  $\tau_2^{(4)} \ \to \mathcal{O}_{\{412\}}, \ldots$ → requires boost in 2 directions (even more noisy!)
  $\ldots$

Moments of PDFs in lattice QCD



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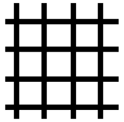
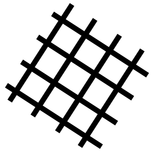
 $\neq$  (a lattice spacing)

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ETMC 2010.03495, 2104.02247,
EINN25 posters by Luis Alberto Rodriguez Chacon, Christian Kummer

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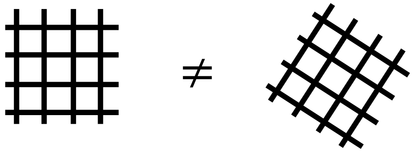

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- $n > 4$: no irrep safe from power-divergent mixing

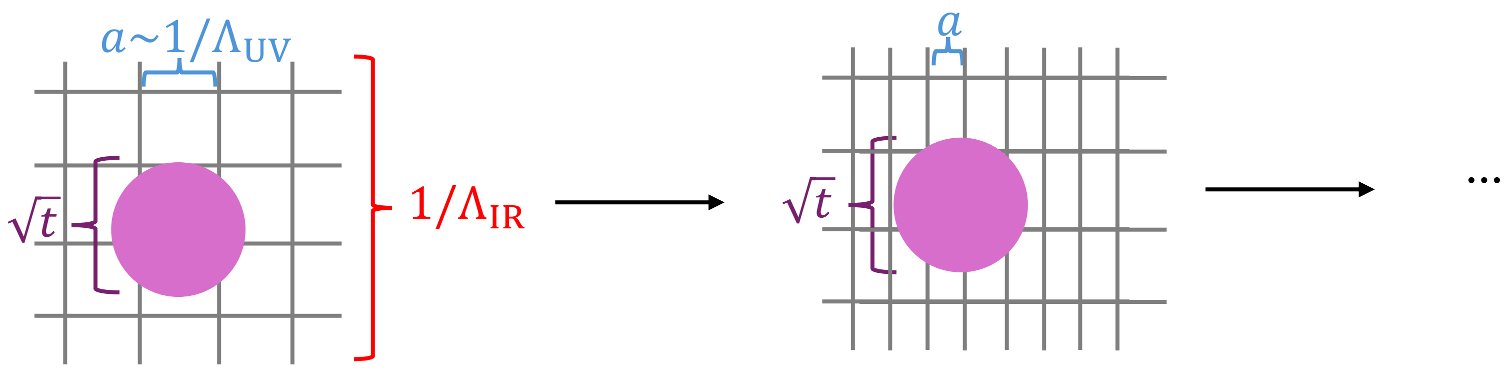
Kronfeld Photiadis 1985

Gradient flow to the rescue

- Introduce fictitious "flow-time" t (units of length^2)
- Yang-Mills flow: "flow" the gauge field $A_\mu \to A_\mu(t)$ in the direction of steepest decent of the YM action:

$$\partial_t A_\mu(t) = D_\nu(t) F_{\nu\mu}(t)$$
$$F_{\mu\nu}(t) = \partial_\mu A_\nu(t) - \partial_\nu A_\mu(t) + [A_\mu(t), A_\nu(t)]$$

- It smooths out the field over region $\sim\sqrt{8t}$
- Lattice QCD formulations, e.g. gradient flow, for gauge link and fermion fields [Lüscher 1006.4518, 1302.5246]
- Flowed gauge field does not need to be renormalized [Lüscher Weisz 1101.0963], quark multiplicatively
- Flow time t $[\mathrm{fm}^2]$: <u>regulator for UV singularities</u> → can take $a \to 0$ at fixed t



Gradient flow for moments of PDFs

Shindler
2311.18704

- Proposal: calculate *flowed moments* from matrix elements of flowed twist-2 operators:
$$\langle h(p)|\mathcal{O}^{\mu_1\dots\mu_n}(t)|h(p)\rangle$$

- Still requires the renormalization of the flowed fermion fields:
→ cancels for ratios of matrix elements with the same quark content
→ Wilson-type fermions: $\mathcal{O}(a^2)$ discretization artifacts for ratios (n-independent $\mathcal{O}(am)$ cancel)

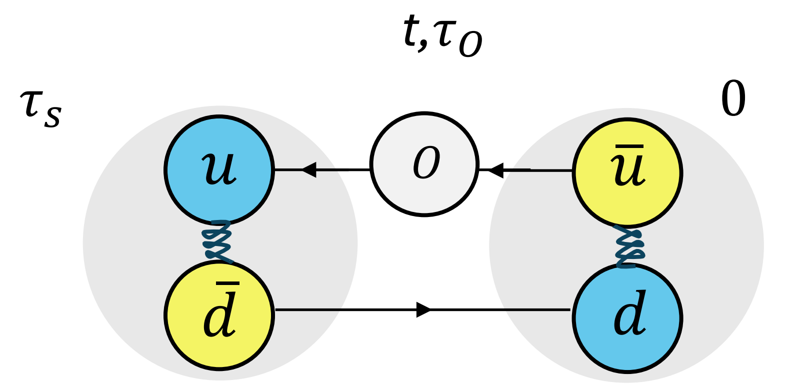
$$\left\{ \frac{\langle x^{n-1}\rangle(\mu)}{\langle x^{m-1}\rangle(\mu)} = \frac{\zeta_m(t,\mu)}{\zeta_n(t,\mu)} \frac{\langle h(p)|\mathcal{O}_{\mu_1\dots\mu_n}(t)|h(p)\rangle}{\langle h(p)|\mathcal{O}_{\mu_1\dots\mu_m}(t)|h(p)\rangle} \right]$$

→ divergence free! continuum limit well defined
→ No boosting ($\boldsymbol{p} = 0$) required when $\mu_i = 4$
$\langle h(p)|\mathcal{O}^{\mu_1\dots\mu_n}|h(p)\rangle = 2\langle x^{n-1}\rangle p^{\mu_1}p^{\mu_2}\dots p^{\mu_n}$ − Traces
→ best signal-to-noise properties

→Matches flowed quantity to $\overline{\mathrm{MS}}$
→Perturbatively calculated in *short flowed time expansion* (SFTX)
→NLO by Shindler 2311.18704

- Can normalize ratios using $\langle x\rangle$ from standard lattice QCD methods or phenomenology

$\dfrac{\langle h|\mathcal{O}^n(t)|h\rangle}{\langle h|\mathcal{O}^m(t)|h\rangle}$ from lattice QCD



- 3-point correlation function projected to zero momentum

t: flow time
τ_s: sink time
τ_O: operator insertion time

$$C^{3\mathrm{pt}}_{\mu_1\mu_2\ldots\mu_n}(\tau_s,\tau_O,t) = \sum_{\boldsymbol{x},\boldsymbol{y}} \langle \chi(\boldsymbol{x},t_s)\bar{\psi}(\boldsymbol{y},\tau,t)\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\ldots\overleftrightarrow{D}_{\mu_n}(\boldsymbol{y},\tau,t)\psi\,(\boldsymbol{y},\tau,t)\bar{\chi}(0)\rangle$$

$\bar{\chi}(x)$: operator with quantum numbers of hadron state

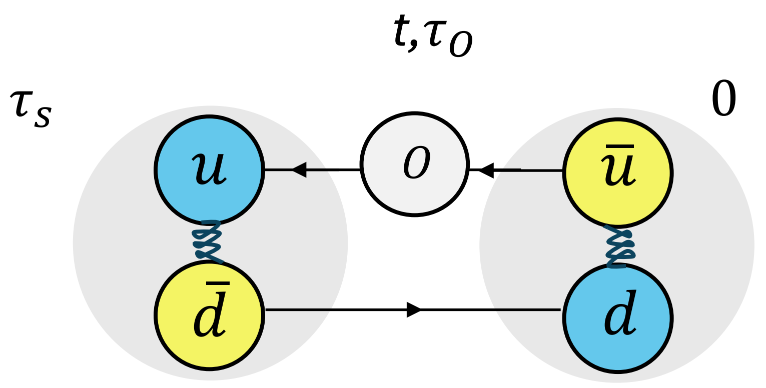
spectral decomposition:

$$C^{3\mathrm{pt}}_{\mu_1\mu_2\ldots\mu_n}(\tau_s,\tau_O,t) = \sum_{n,m} \langle\Omega|\eta|n\rangle\,\langle m|\bar{\eta}|\Omega\rangle e^{-E_n t_s - E_m(t_s-\tau)}\langle n|\mathcal{O}_{\mu_1\ldots\mu_n}(t)|m\rangle$$

take ratio:

$$\frac{C^{3\mathrm{pt}}_{\mu_1\mu_2\ldots\mu_n}(t_s,\tau,t)}{C^{3\mathrm{pt}}_{\mu_1\mu_2\ldots\mu_m}(t_s,\tau,t)} \propto \frac{\langle h|\mathcal{O}^n(t)|h\rangle}{\langle h|\mathcal{O}^m(t)|h\rangle} \longrightarrow$$ *plateaus* to ground state for large τ_s, τ_O

$\frac{\langle h|\mathcal{O}^n(t)|h\rangle}{\langle h|\mathcal{O}^m(t)|h\rangle}$ from lattice QCD



- 3-point correlation function projected to zero momentum

$$C^{3\mathrm{pt}}_{\mu_1\mu_2\ldots\mu_n}(\tau_s,\tau_O,t) = \sum_{\boldsymbol{x},\boldsymbol{y}}\langle \chi(\boldsymbol{x},t_s)\bar{\psi}(\boldsymbol{y},\tau,t)\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\ldots\overleftrightarrow{D}_{\mu_n}(\boldsymbol{y},\tau,t)\psi\,(\boldsymbol{y},\tau,t)\bar{\chi}(0)\rangle$$

t: flow time
τ_s: sink time
τ_O: operator insertion time

$\bar{\chi}(x)$: operator with quantum numbers of hadron state

spectral decomposition:

$$C^{3\mathrm{pt}}_{\mu_1\mu_2\ldots\mu_n}(\tau_s,\tau_O,t) = \sum_{n,m}\langle\Omega|\eta|n\rangle\,\langle m|\bar{\eta}|\Omega\rangle e^{-E_n t_s - E_m(t_s-\tau)}\langle n|\mathcal{O}_{\mu_1\ldots\mu_n}(t)|m\rangle$$

take ratio:

$$\frac{C^{3\mathrm{pt}}_{\mu_1\mu_2\ldots\mu_n}(t_s,\tau,t)}{C^{3\mathrm{pt}}_{\mu_1\mu_2\ldots\mu_m}(t_s,\tau,t)} \propto \frac{\langle h|\mathcal{O}^n(t)|h\rangle}{\langle h|\mathcal{O}^m(t)|h\rangle} \longrightarrow$$ *plateaus* to ground state for large τ_s, τ_O

Computational challenges:
→ no boost for $\mu_i = 4$, but still need to subtract traces from $\mathcal{O}_{4\ldots4}$ (3 for $n=2$, 39 for $n=4$, 543 for $n=6$, ...)
→ each discretized derivative $\overleftrightarrow{D}_\mu$ contains 4 contributions $\longrightarrow$ 4^{n-1} for each term
To address both at once, wrote a depth-first search contraction algorithm that employs symmetries to reduce to $2^{\text{temporal indices }\#}$

Contractions algorithm

Contractions :

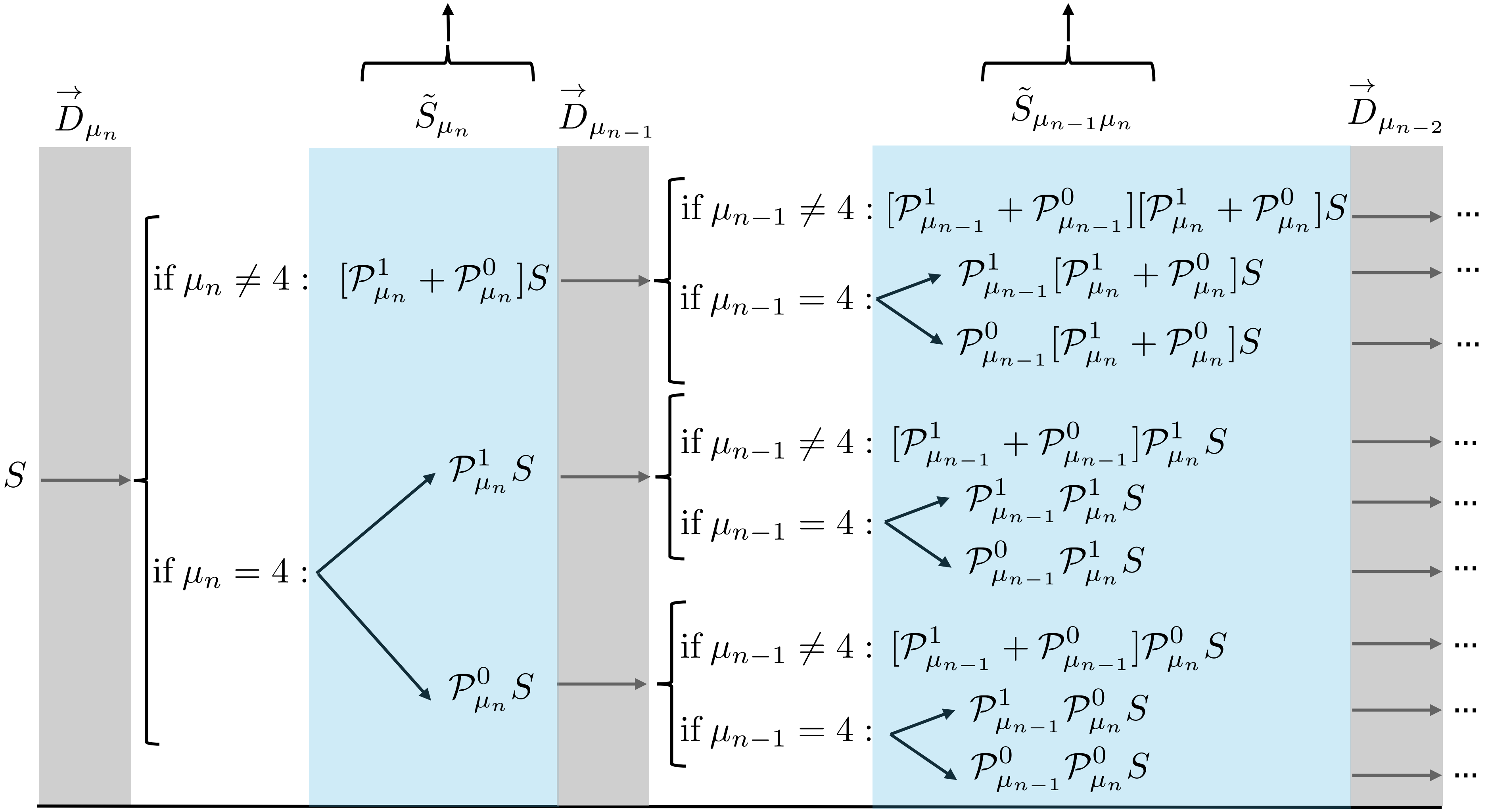
$n = 2$

$$\mathcal{C}_{\mu_1\mu_2} = \sum \mathrm{Tr}\left[\Sigma^\dagger \gamma_5 \gamma_{\mu_1} \tilde{S}_{\mu_2}\right]$$

$n = 3$

$$\mathcal{C}_{\mu_1\mu_2\mu_3} = \sum \mathrm{Tr}\left[\Sigma^\dagger \gamma_5 \gamma_{\mu_1} \tilde{S}_{\mu_2\mu_3}\right]$$

...



Analysis steps

- Compute 3-point functions $\mathrm{C}_n^{\mathrm{3pt}}(\tau_s, \tau_O, t)$ of $\mathcal{O}^n$ for increasing flow time t

- For each t, form ratio $\frac{\mathrm{C}_n^{\mathrm{3pt}}(\tau_s,\tau_O,t)}{\mathrm{C}_2^{\mathrm{3pt}}(\tau_s,\tau_O,t)}$ to obtain ratio of matrix elements $\frac{\left\langle h \middle| \mathcal{O}^n(t) \middle| h \right\rangle}{\left\langle h \middle| \mathcal{O}_{44}(t) \middle| h \right\rangle}$

  no advantage from considering different denominators

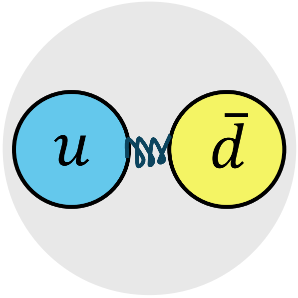
- Extrapolate to $a \to 0$ (multiple ensembles needed) at fixed flow time t

- Multiply by matching factors $\frac{\zeta_2(t,\mu)}{\zeta_n(t,\mu)}$ perturbatively computed

- Examine any remnant flow-time dependence (e.g. linear from mixing with *higher* dimensional operators that goes to zero at $t = 0$, log from matching factors,...)

- Fit and extrapolate to $t \to 0$ to obtain $\frac{\langle x^{n-1}(\mu) \rangle}{\langle x(\mu) \rangle}$ at $\overline{\mathrm{MS}}$

Test case: Heavy 410 MeV pion at SU(3) flavor point



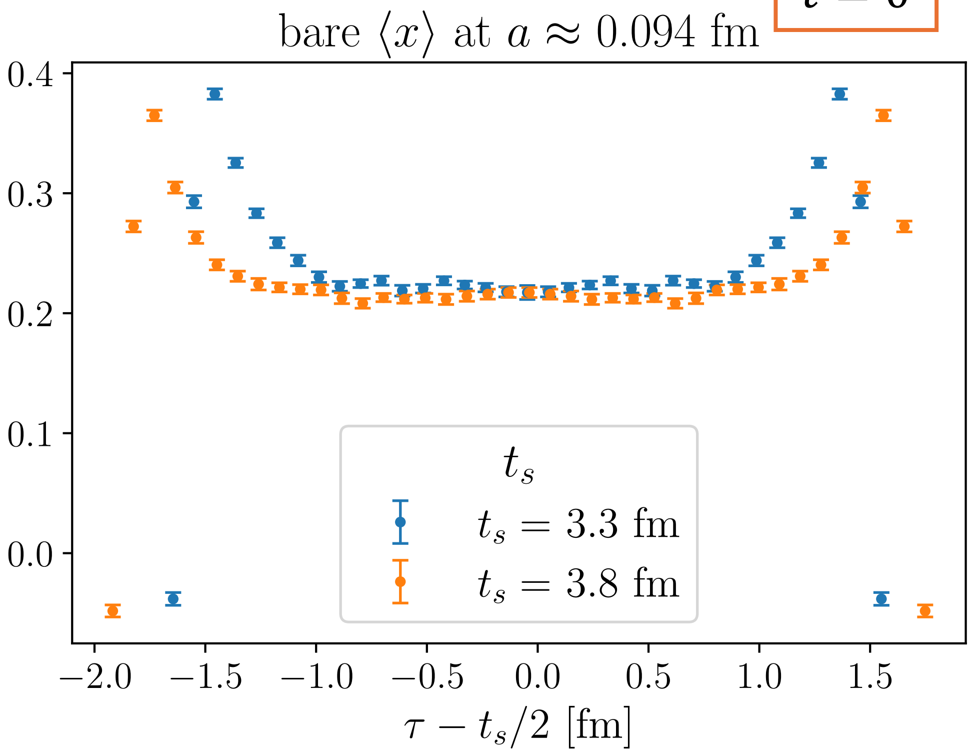
- Consider purely connected light-quark contribution u^+
  → if $m_u = m_d = m_s$, corresponds to non-singlet $\bar{u}u + \bar{d}d - 2\bar{s}s$

- Very moderate statistical sample needed:
  $a \approx 0.12$ fm: $N_{cfg} = 850, \frac{\tau_s}{a} \in \{35,40\}$
  $a \approx 0.094$ fm: $N_{cfg} = 700, \frac{\tau_s}{a} \in \{35,40\}$
  $a \approx 0.077$ fm: $N_{cfg} = 400, \frac{\tau_s}{a} \in \{40,42\}$
  $a \approx 0.064$ fm: $N_{cfg} = 500, \frac{\tau_s}{a} \in \{40,42\}$

  Measurements up to flow radius $\sqrt{8t} \approx 0.6$ fm, $t/t_0 \approx 2.5$

- Stabilized Wilson fermion (SWF) ensembles generated by *OpenLat*

$\boxed{t = 0}$

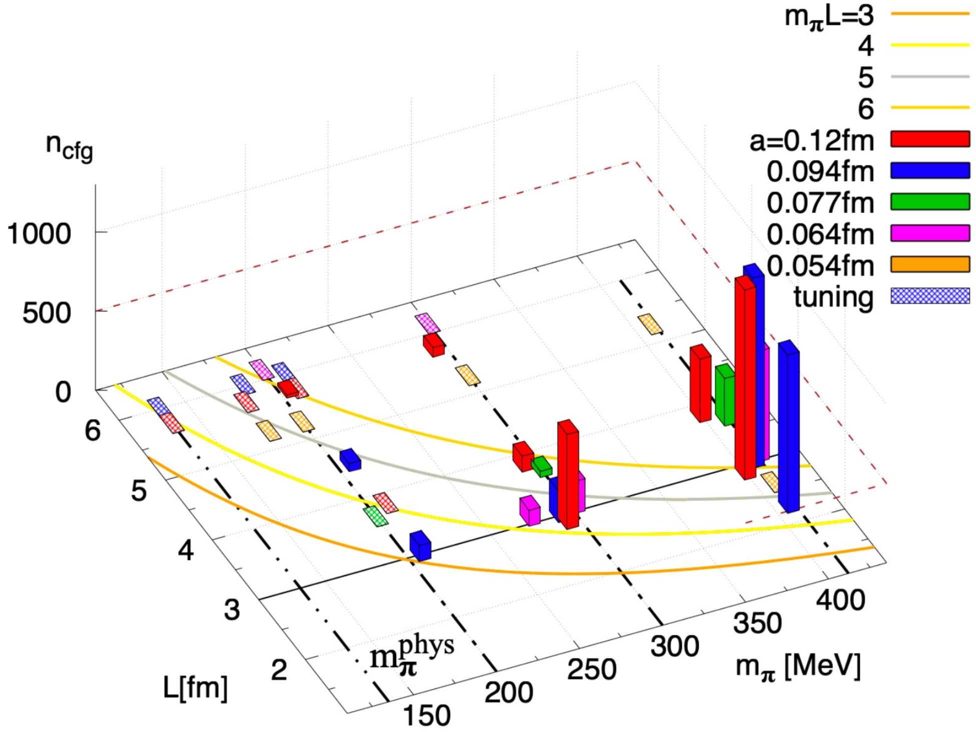


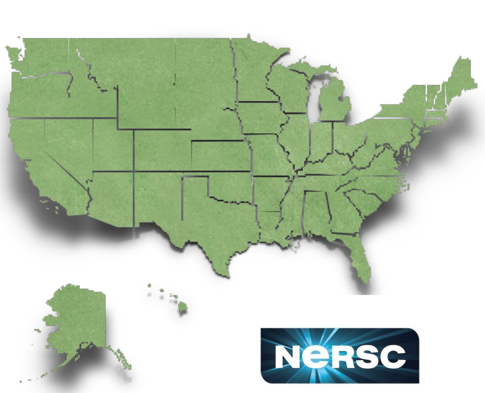
- Main results:
  Francis Fritzsch Harlander Karur Kim Kohnen Pederiva **Pefkou** Rago Shindler Walker-Loud Zafeiropoulos 2509.02472
  Longer paper with details:
  Francis Fritzsch Karur Kim Pederiva **Pefkou** Rago Shindler Walker-Loud Zafeiropoulos 2510.XXXXX

OpenLat Initiative

[https://openlat1.gitlab.io]

OpenLat: open science initiative.
Gauges with SWF open to the whole community





Jangho Kim

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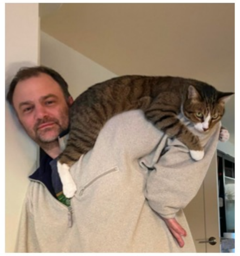

André
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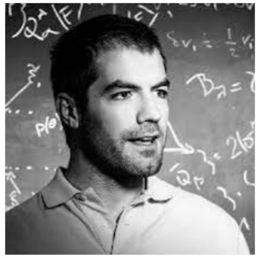
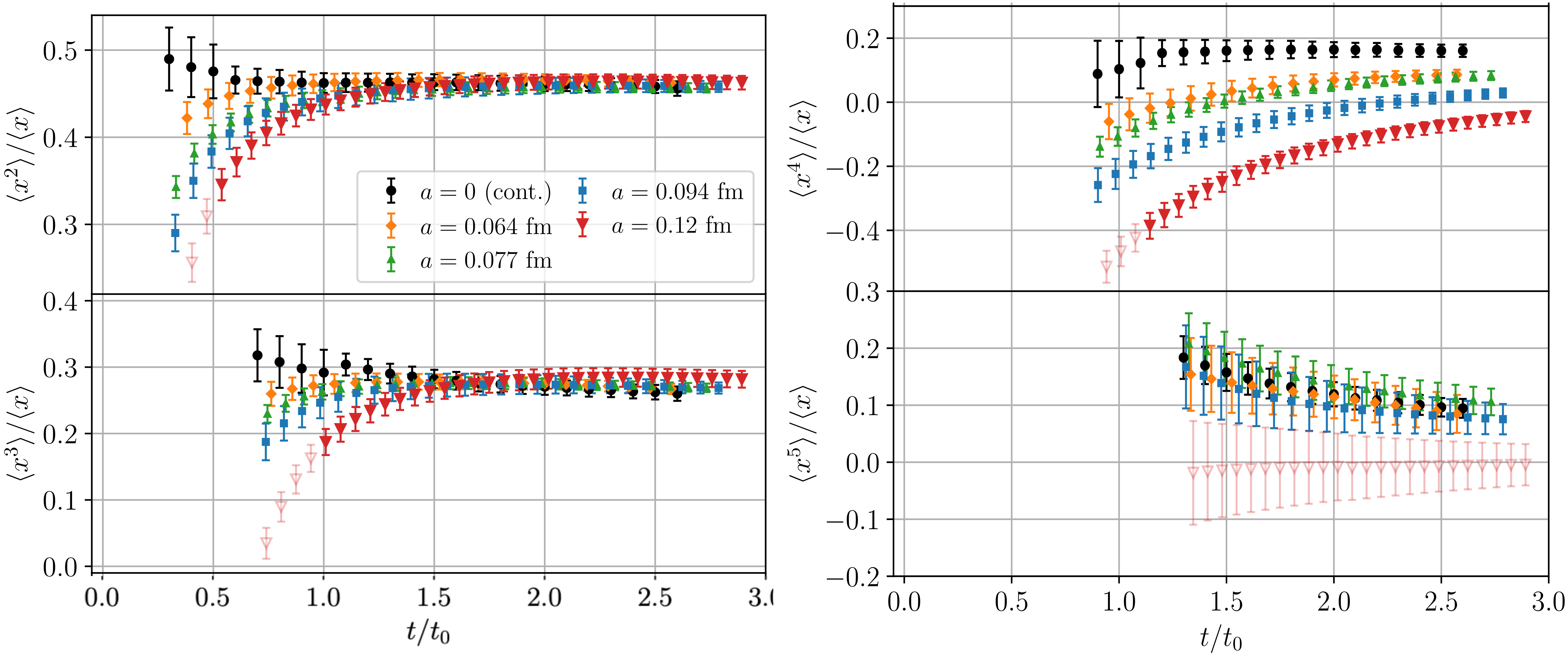
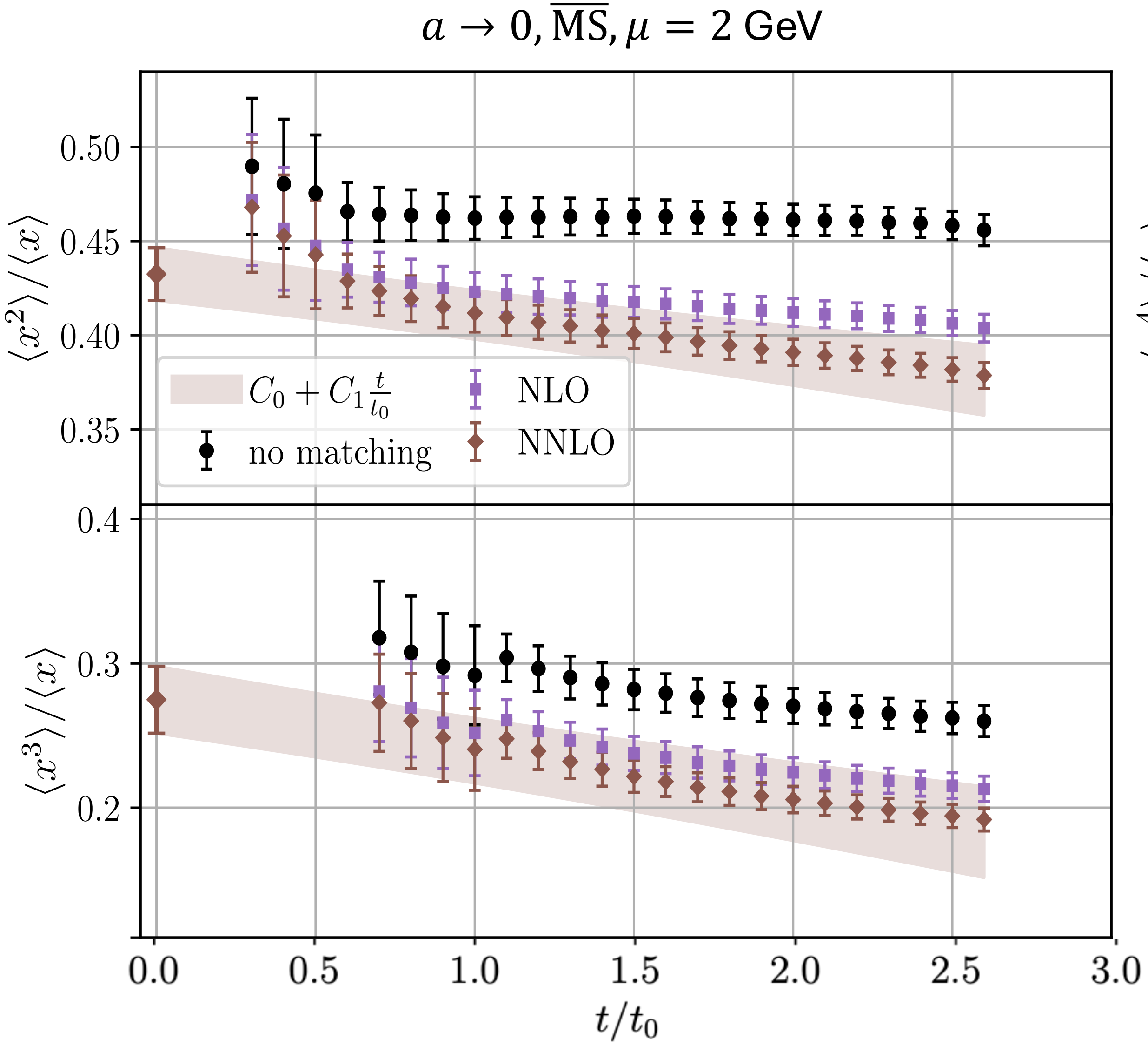


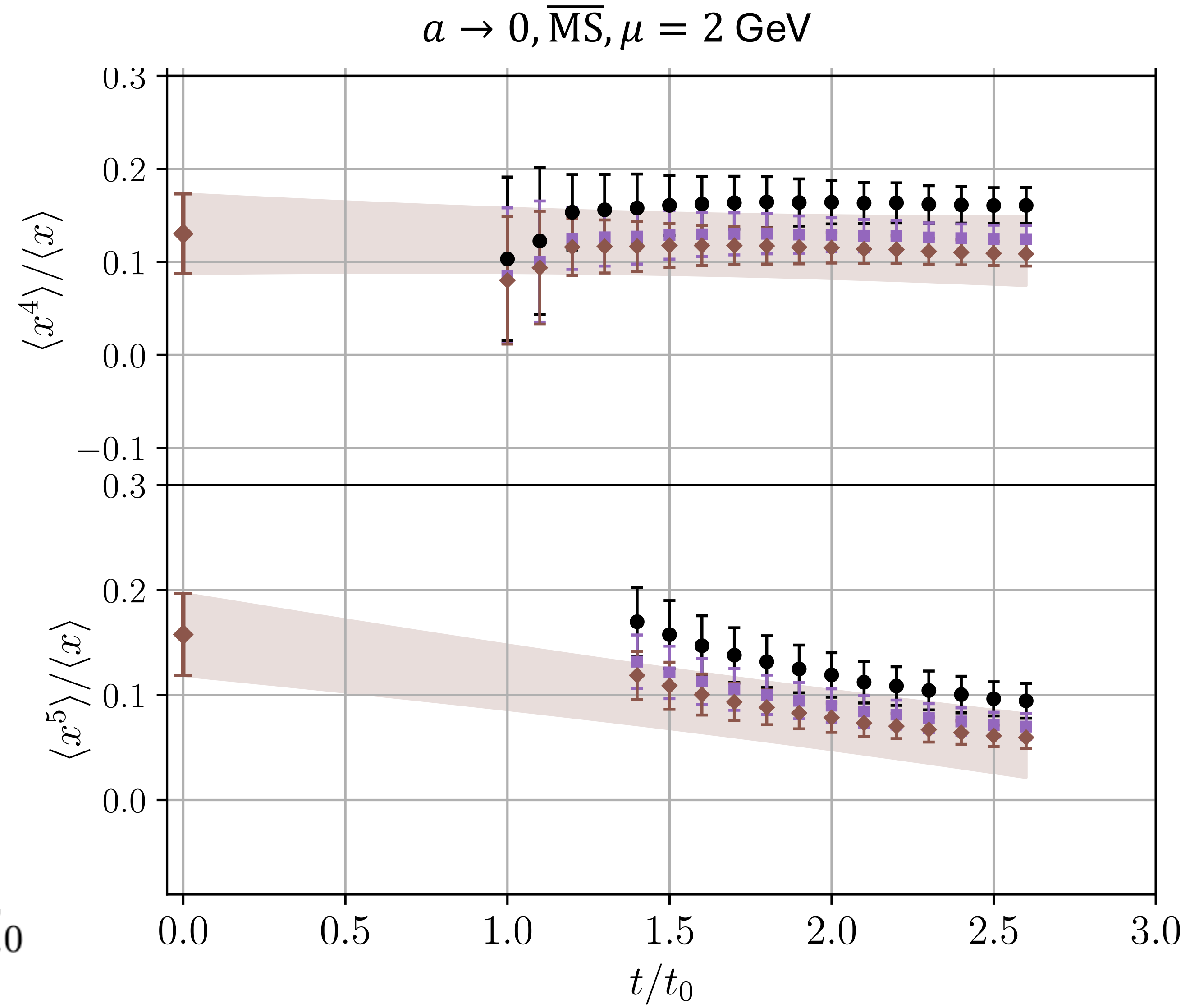
Savvas
Zafeiropoulos

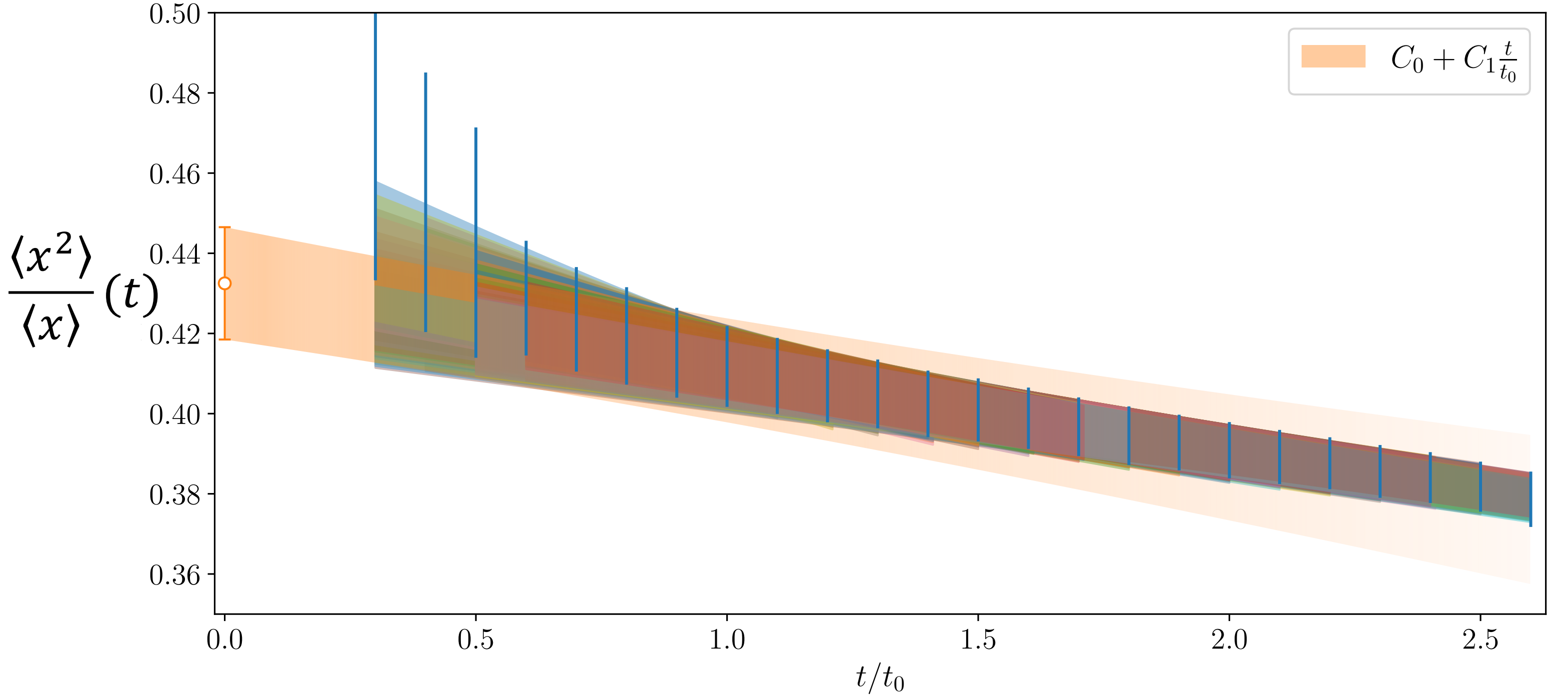
Continuum $a \to 0$ extrapolation

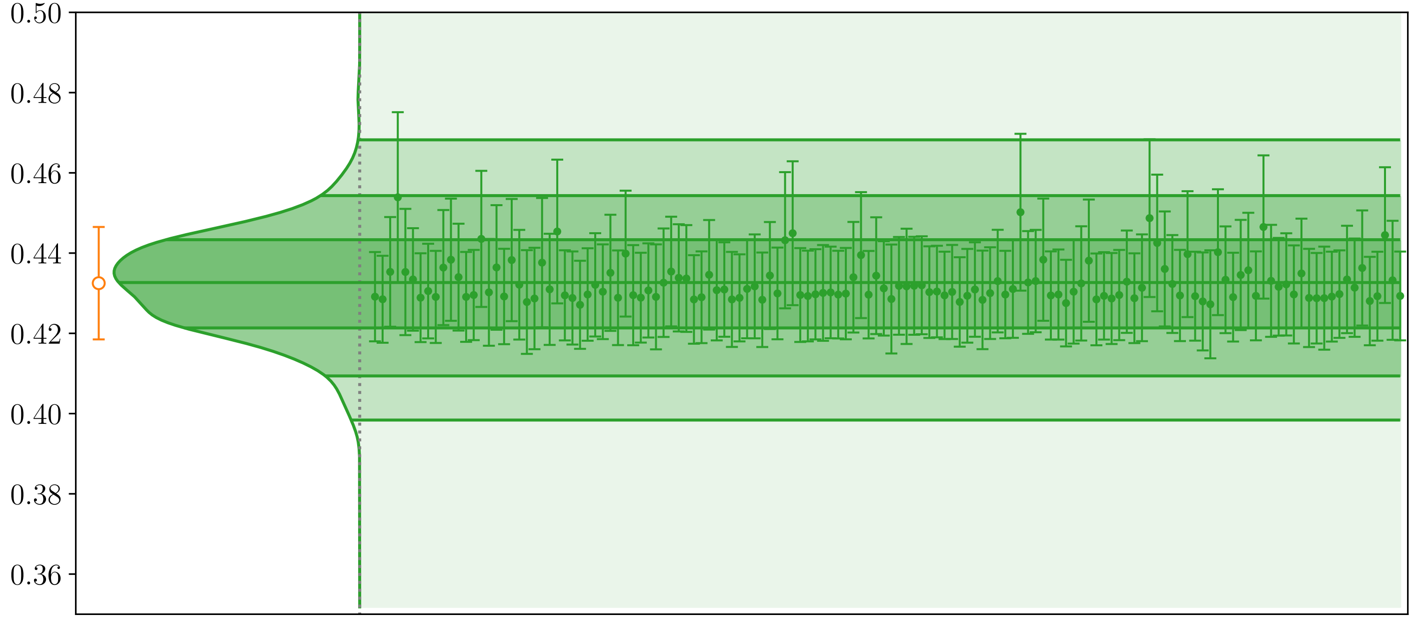


Matching and flow time $t \to 0$ extrapolation





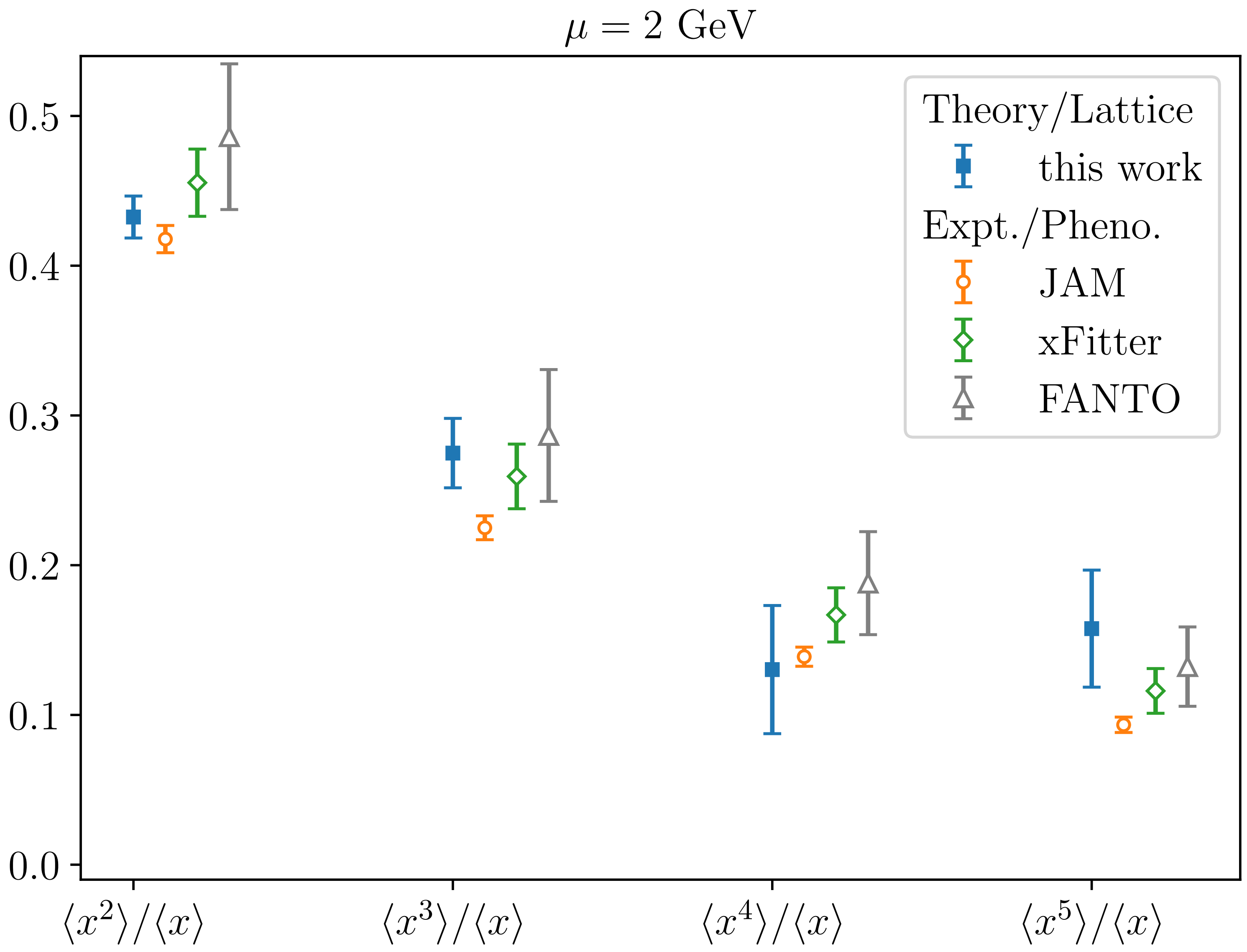




Flow time $t \to 0$ extrapolation example

- Perform every linear fit of flow time range ~ 0.8
- Take a flat average of all with p-value > 0.1
- Add shifts of central values from mean as systematic error

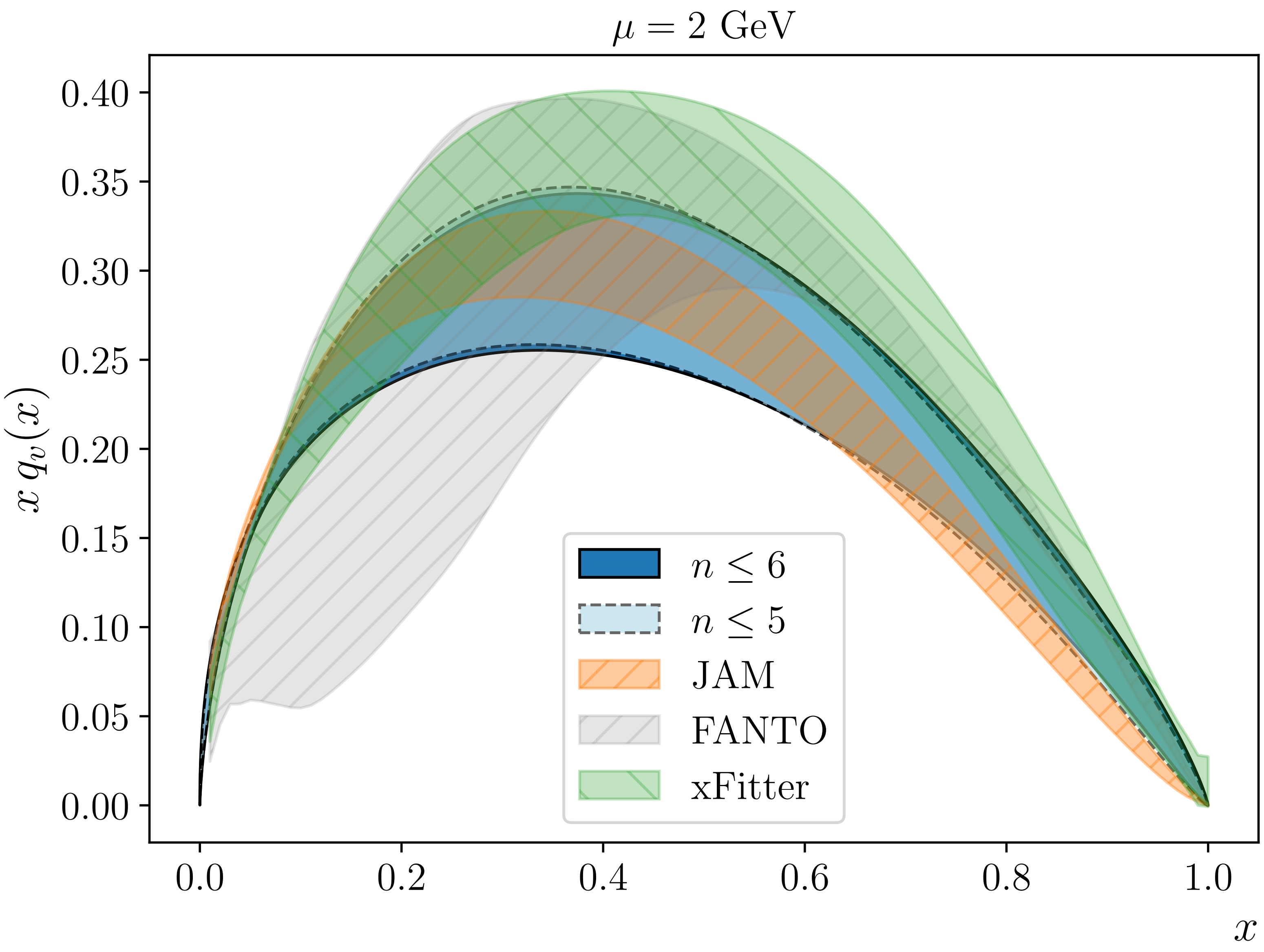
Comparison with phenomenology



* this work $m_\pi \approx 410$ MeV, $\mathrm{SU}(3)_f$

JAM: 2108.05822
xFitter: 2002.02902
FANTO: 2505.13594

PDF reconstruction



$$q_v \propto x^{\alpha}(1-x)^{\beta}$$

Non-trivial, use of priors to stabilize fit

Difficulties in constraining α but large x parameter $\beta{\sim}1$ across all fit variations

Plans to use Gaussian Processes (underway by UC Berkeley PhD student Rohith Karur)

Summary and remarks

- First direct lattice QCD extraction of flavor non-singlet pion PDF moment ratios using novel method that employs the gradient flow

- With a modest statistical sample, moments up to $\langle x^5 \rangle$ and resulting PDF show good agreement and competitive precision with phenomenological extractions

- Resolution to the long-standing problem of unattainable higher moments due to power divergent mixing in lattice QCD

- High precision moments offer complementary information on PDFs that can be used in global analyses of experimental and/or lattice data

- Future extensions: → chiral extrapolation to physical pion
  → proton
  → quark singlet + gluon PDFs
  → helicity + transversity PDFs
  → generalized form factors for GPDs
  → higher twist

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THANK YOU!