

What's next for the theory of muon $g-2$?

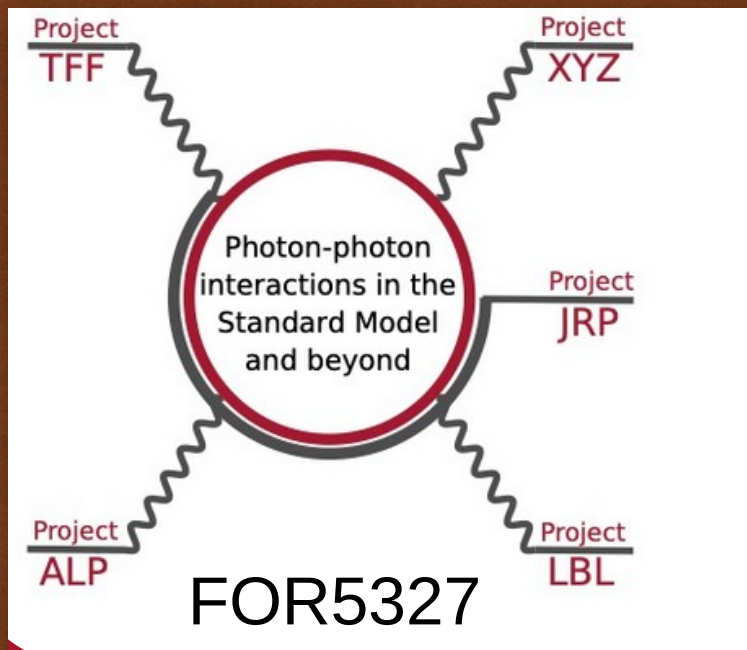
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University of Mainz, Germany**



@ EINN, Cyprus
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Project LbL



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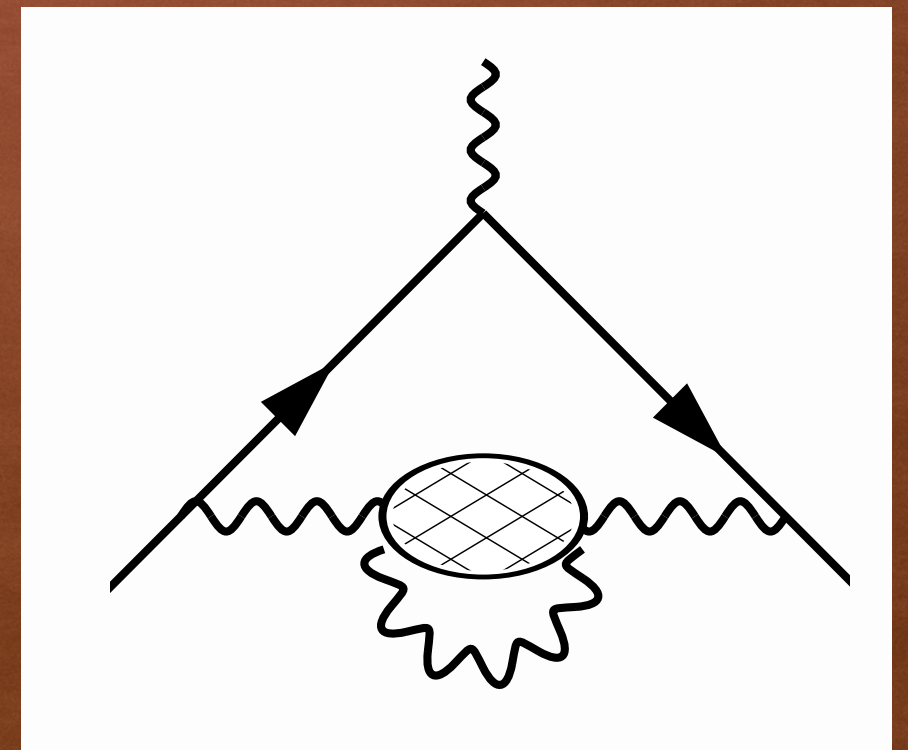
[2509.08115](#) [hep-ph]

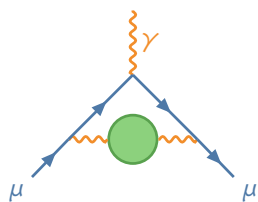
[2507.18234](#) [hep-ph], *Phys. Lett. B*

[2505.24344](#) [hep-lat], *JHEP*

[2501.03192](#) [hep-lat], *JHEP*

[2209.02149](#) [hep-lat], *JHEP*

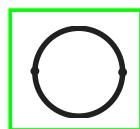




The different contributions

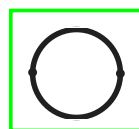
Figure from Borsanyi et al. Nature 2021

Isospin-symmetric



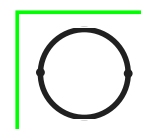
Connected light

$$633.7(2.1)_{\text{stat}}(4.2)_{\text{syst}}$$



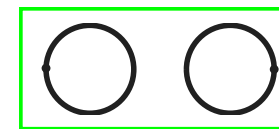
Connected strange

$$53.393(89)_{\text{stat}}(68)_{\text{syst}}$$



Connected charm

$$14.6(0)_{\text{stat}}(1)_{\text{syst}}$$



Disconnected

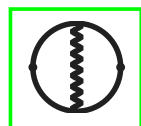
$$-13.36(1.18)_{\text{stat}}(1.36)_{\text{syst}}$$

QED isospin breaking: valence



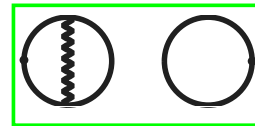
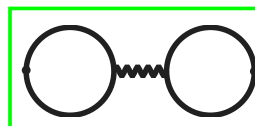
Connected

$$-1.23(40)_{\text{stat}}(31)_{\text{syst}}$$

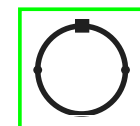


Disconnected

$$-0.55(15)_{\text{stat}}(10)_{\text{syst}}$$

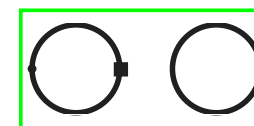


Strong-isospin breaking



Connected

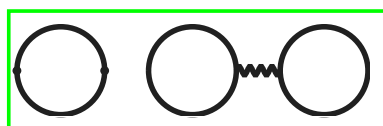
$$6.60(63)_{\text{stat}}(53)_{\text{syst}}$$



Disconnected

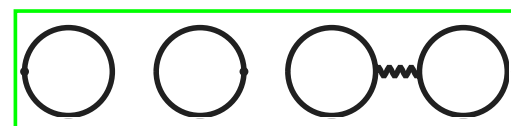
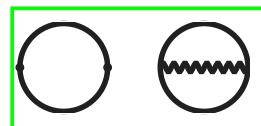
$$-4.67(54)_{\text{stat}}(69)_{\text{syst}}$$

QED isospin breaking: sea



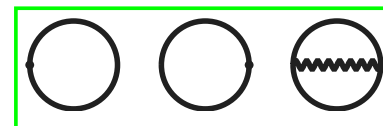
Connected

$$0.37(21)_{\text{stat}}(24)_{\text{syst}}$$



Disconnected

$$-0.040(33)_{\text{stat}}(21)_{\text{syst}}$$

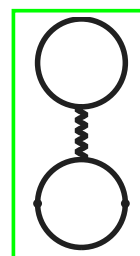


Other

Bottom; higher-order;
perturbative

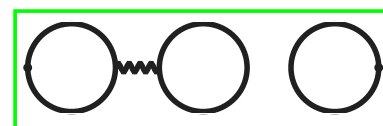
$$0.11(4)_{\text{tot}}$$

QED isospin breaking: mixed



Connected

$$-0.0093(86)_{\text{stat}}(95)_{\text{syst}}$$



Disconnected

$$0.011(24)_{\text{stat}}(14)_{\text{syst}}$$

Finite-size effects

Isospin-symmetric

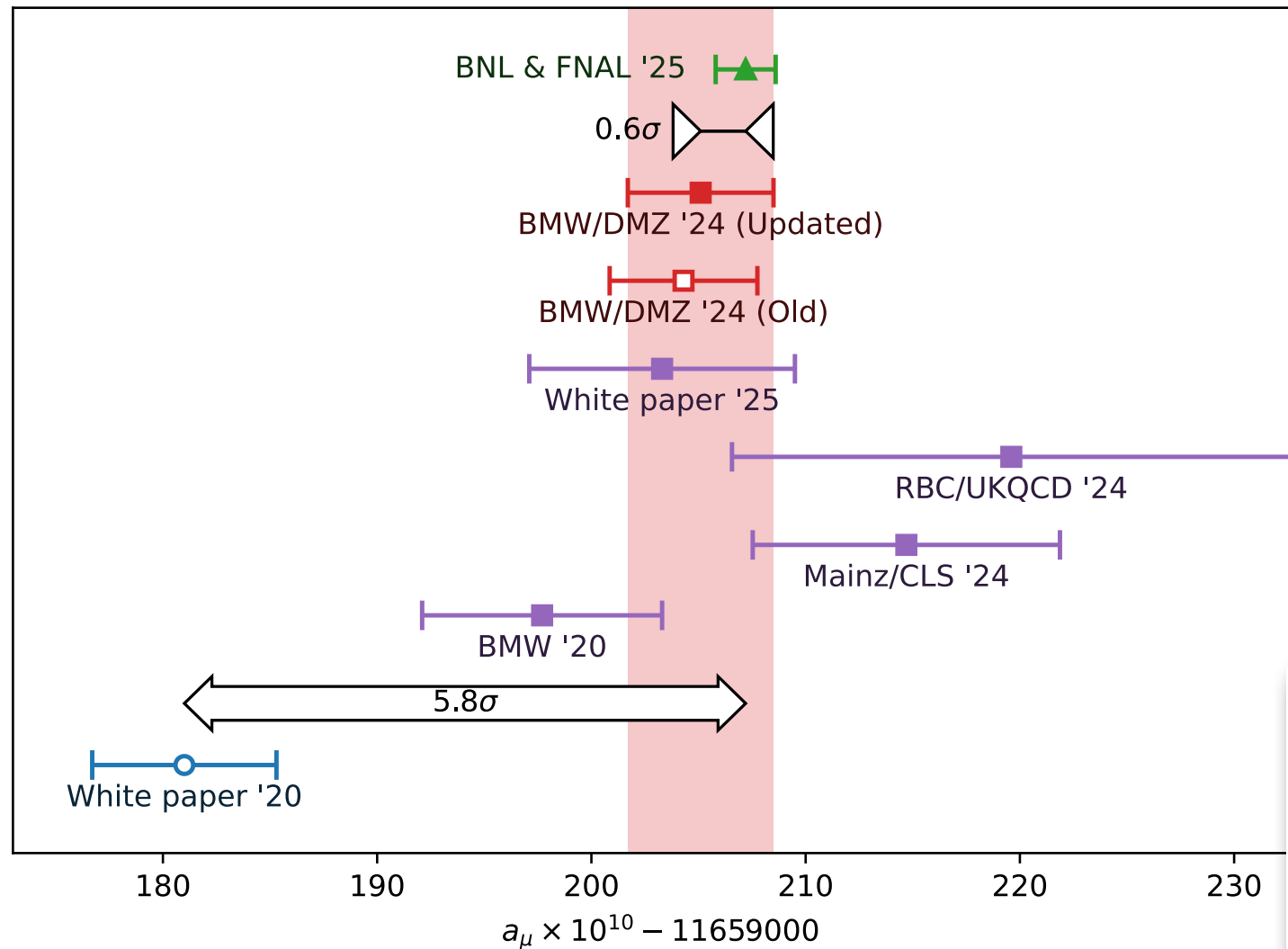
$$18.7(2.5)_{\text{tot}}$$

Isospin-breaking

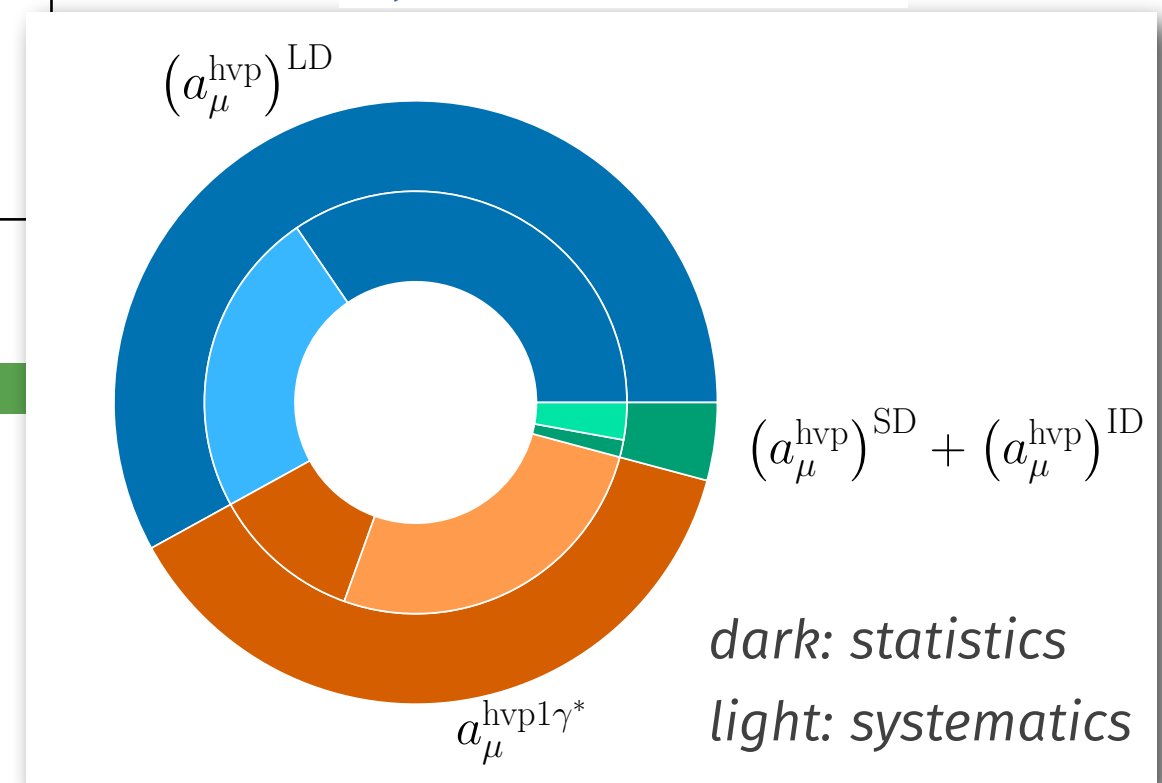
$$0.0(0.1)_{\text{tot}}$$

$$a_{\mu}^{\text{LO-HVP}} (\times 10^{10}) = 707.5(2.3)_{\text{stat}}(5.0)_{\text{syst}}(5.5)_{\text{tot}}$$

BMW update & Mainz error budget



[Djukanovic et al., 2411.07969]



Finn M. Stokes

BMWc HVP status

Theory Initiative 2025

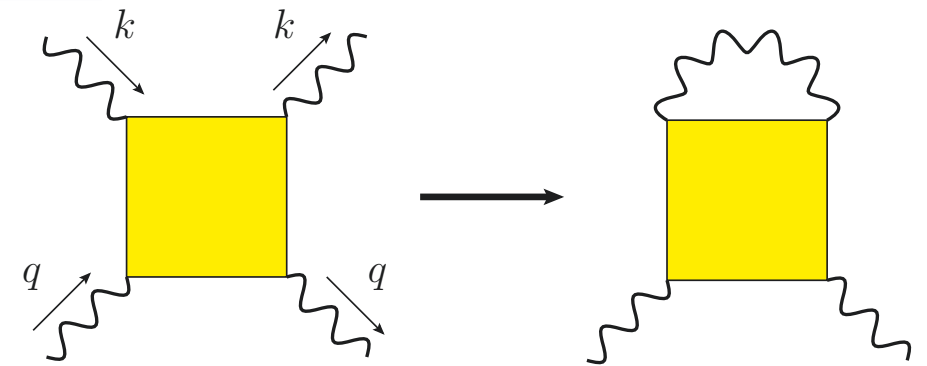
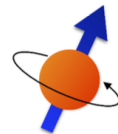
[arXiv:2407.10913v2]

From idea to framework (5 years)

COTTINGHAM-TYPE FORMULA FOR THE LBL CONTRIBUTION TO HVP

Vladimir Pascalutsa
with
F. Hagelstein, J. Green, H. Meyer, A. Gerardin, ...

@ 1st Workshop on
Muon $g-2$ Theory Initiative
Fermilab, USA
June 2-6, 2017



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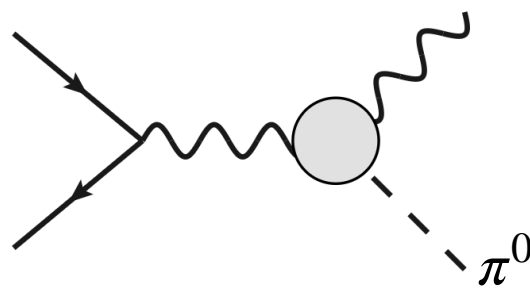
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Forward light-by-light scattering and electromagnetic correction to hadronic vacuum polarization

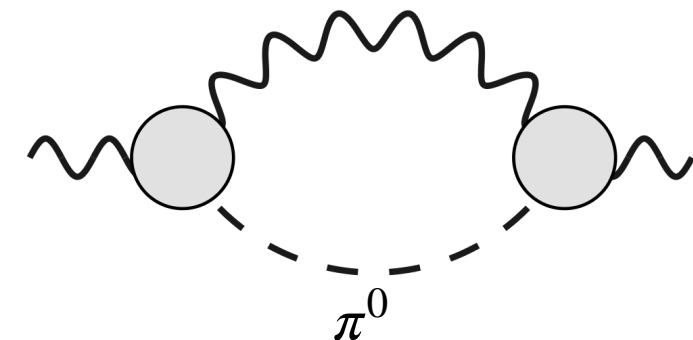
Volodymyr Biloshytskyi,^a En-Hung Chao,^a Antoine Gérardin,^e Jeremy R. Green,^b
Franziska Hagelstein,^{a,c} Harvey B. Meyer,^{a,d} Julian Parrino^a and Vladimir Pascalutsa^a

Data-driven (timelike) vs field-theoretic (spacelike)



- data-driven evaluation:
[Hoid et al., EPJC (2020)]

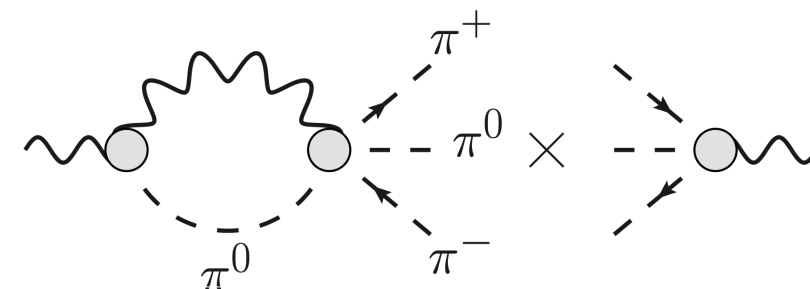
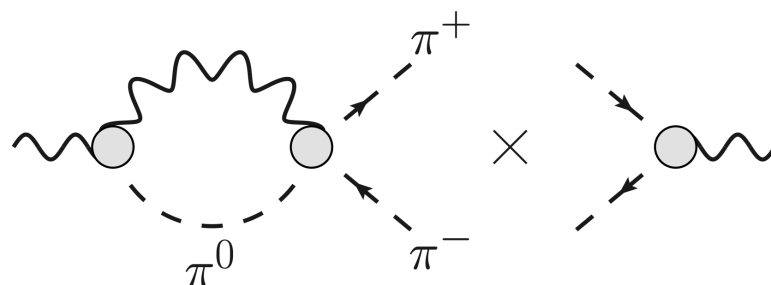
$$a_{\mu}^{(\pi^0\gamma)}(\text{timelike}) = 4.38(6) \times 10^{-10}$$



- simple VMD-based model
[Blokland et al., PRL (2002)]

$$a_{\mu}^{(\pi^0\gamma)}(\text{spacelike}) \simeq 0.37 \times 10^{-10}$$

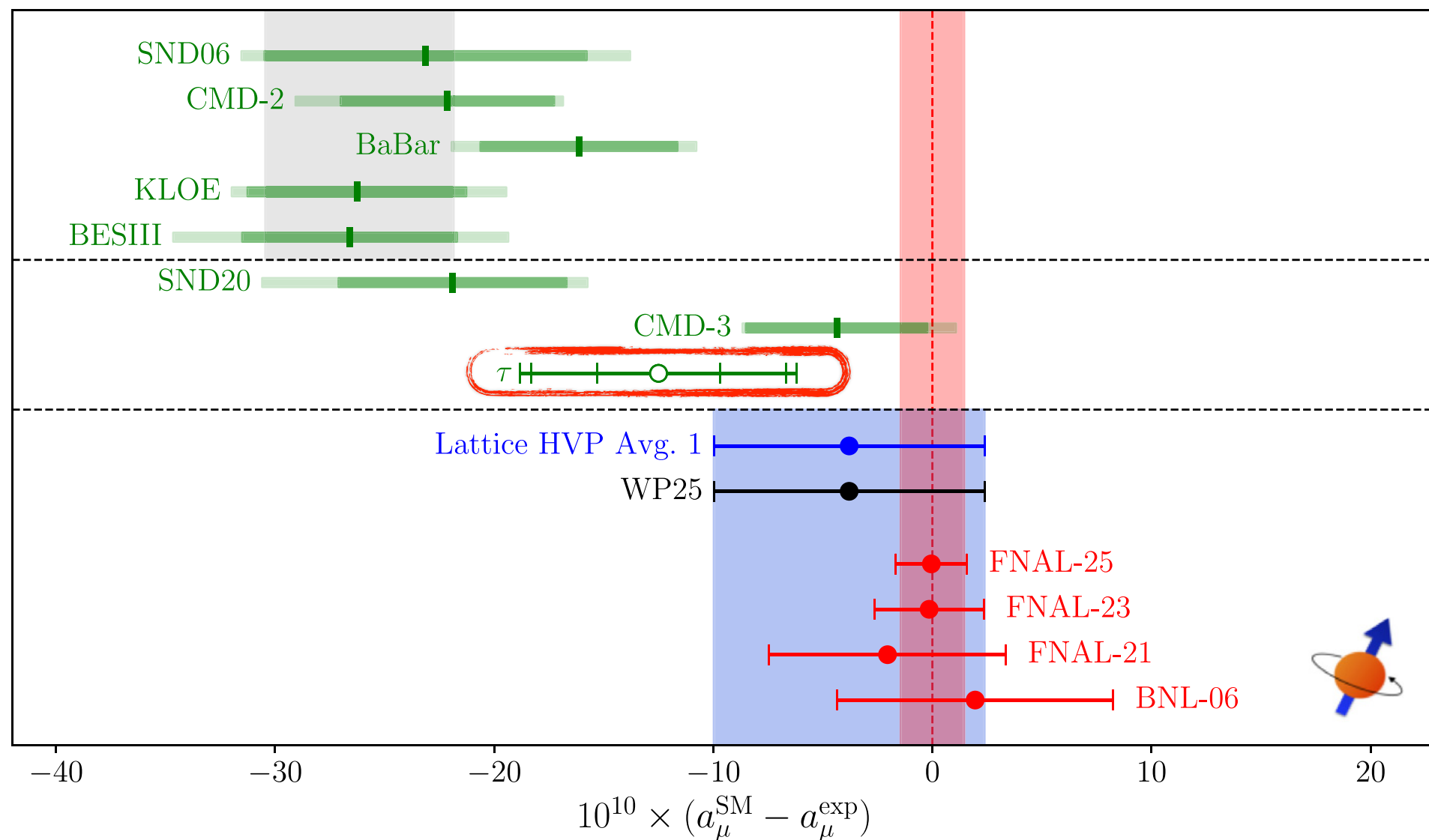
- $\mathcal{O}(\alpha_{\text{em}})$ interference terms from hadronic channels reconcile the discrepancy



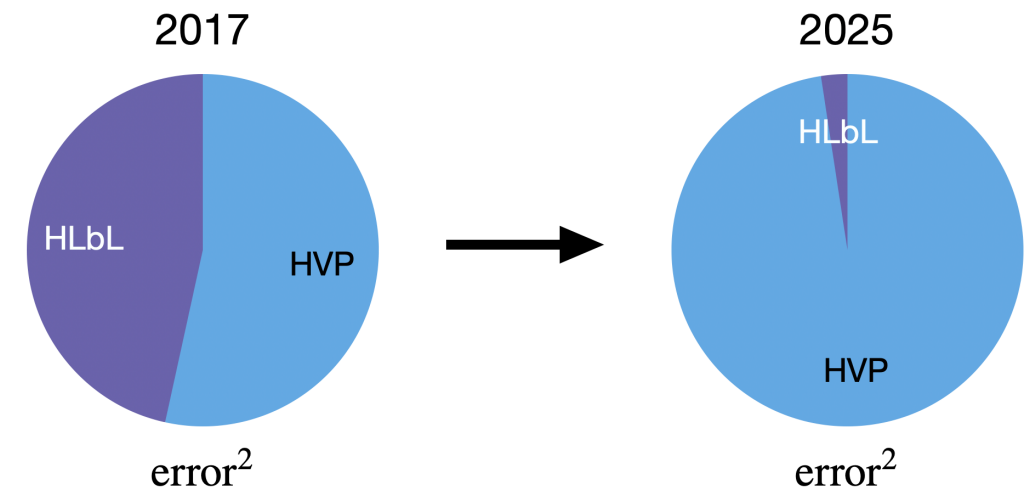
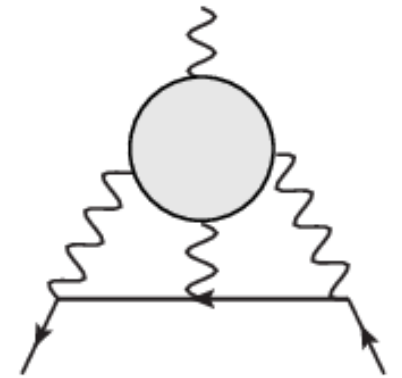
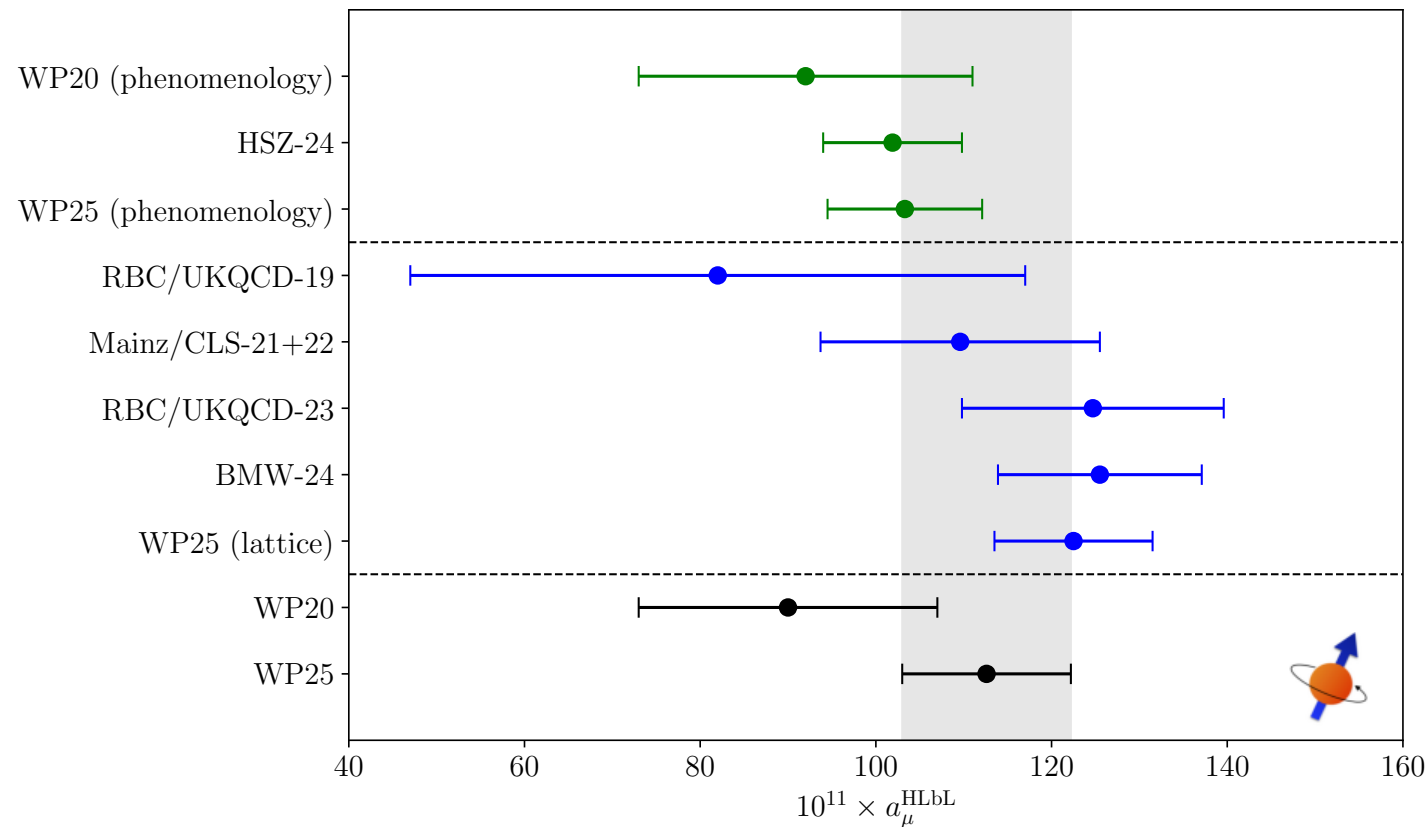
Affects the isospin-breaking in τ decays?

τ data as an independent and precise measurement to clarify the situation in dominant $\pi\pi$ channel

[R. Alemany, M. Davier, and A Hocker '98]



Hadronic light-by-light (HLbL)



- ▶ Three independent lattice determinations ($\neq$ actions) :
→ Mainz group, RBC/UKQCD and BMW collaborations
- ▶ Dispersive and Lattice results with comparable precision - good agreement
- ▶ Combination phenomenology and lattice :

$$a_\mu^{\text{HLbL}} = 112.6(9.6) \times 10^{-11}$$

$$\rightarrow \delta a_\mu^{\text{HLbL}} = 9.6 \times 10^{-11} \text{ to be compared with } \delta a_\mu^{\text{exp}} = 14.5 \times 10^{-11}.$$

Dispersion relations

Not defined via a single observable ($\neq$ HVP)

Dispersive framework ('21) $a_\mu \times 10^{11}$

π^0, η, η'	93.8 ± 4
pion/kaon loops	-16.4 ± 0.2
S-wave $\pi\pi$	-8 ± 1
axial vector	6 ± 6
scalar + tensor	-1 ± 3
q-loops / short. dist. cstr	15 ± 10
charm + heavy quarks	3 ± 1
sum	92 ± 19

- Use cut singularities / pole to identify contributions
- input : form factors

$$a_\mu^{\text{HLbL}} = \text{[Diagram: Triangle with a blob]} \approx \text{[Diagram: Triangle with blob and } \pi^0, \eta, \eta' \text{ exchange]} + \text{[Diagram: Triangle with blob and } \pi \text{ exchange]} + \dots$$



Universal dispersive formula: Schwinger Sum Rule

J. S. Schwinger, Proc. Nat. Acad. Sci. 72, 1 (1975); ibid. 72, 1559 (1975).

A. M. Harun ar-Rashid, Nuovo Cim. A 33, 447 (1976).

anomalous
magnetic moment
 $a = \frac{1}{2}(g-2) \rightarrow$

$$a = \frac{m^2}{\pi^2 \alpha} \int_{\nu_0}^{\infty} d\nu \left[\frac{\sigma_{LT}(\nu, Q^2)}{Q} \right]_{Q^2=0}$$

muon mass m ↓

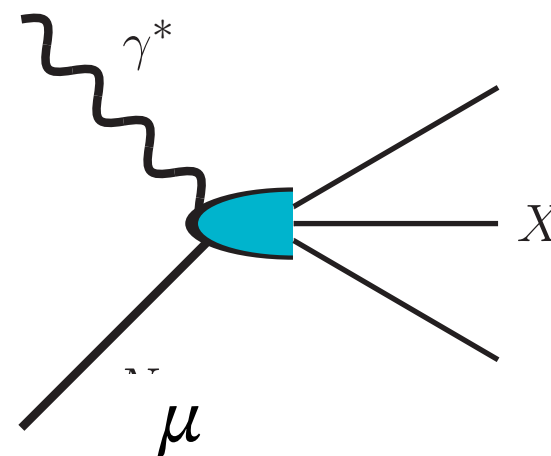
photon lab-frame energy ν
and virtuality $Q^2 \approx -q^2$ ↓

fine-structure constant $\alpha \approx 1/137$ ↑

photo-absorption threshold ν_0 ↑

longitudinal-transverse photo-absorption cross section σ_{LT} ↑

- σ_{LT} inclusive cross section of polarized photo-absorption on muon:



$= \gamma\mu, \pi^0\mu, \dots$

HVP from Schwinger sum rule

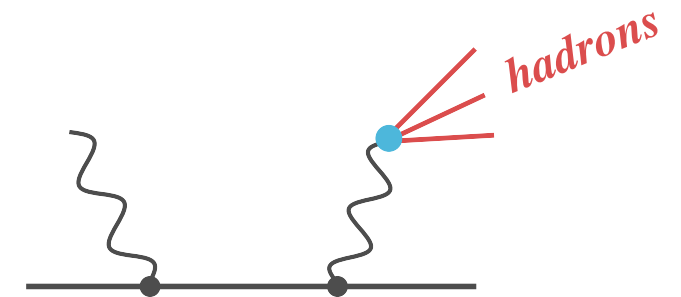
Hagelstein & VP, PRL 120, 072002 (2018).

$$a = \frac{m^2}{\pi^2 \alpha} \int_{4m_\pi^2}^{\infty} dM_X^2 \int_{\nu_0}^{\infty} d\nu \left[\frac{1}{Q} \frac{d\sigma_{LT}^{\gamma\mu \rightarrow \mu X}(\nu, Q^2)}{dM_X^2} \right]_{Q^2=0}$$

$$= \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} dM_X^2 \frac{\text{Im } \Pi^{\text{had}}(M_X^2)}{M_X^2} \left[\frac{m^2}{\pi^2 \alpha} \int_{\nu_0}^{\infty} d\nu \left[\frac{\sigma_{LT}^{\gamma\mu \rightarrow \gamma^* \mu}(\nu, Q^2)}{Q} \right]_{Q^2=0} \right]$$

kernel function: $\uparrow = \frac{\alpha}{\pi} K(M_X^2/m^2) \equiv \frac{\alpha}{\pi} \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(M_X^2/m^2)}$

for $M_X=0$, we find $K(0)=1/2$, and therefore
the Schwinger term: $a^{(1)} = \alpha/2\pi$



- reproduces the HVP **standard dispersion formula**

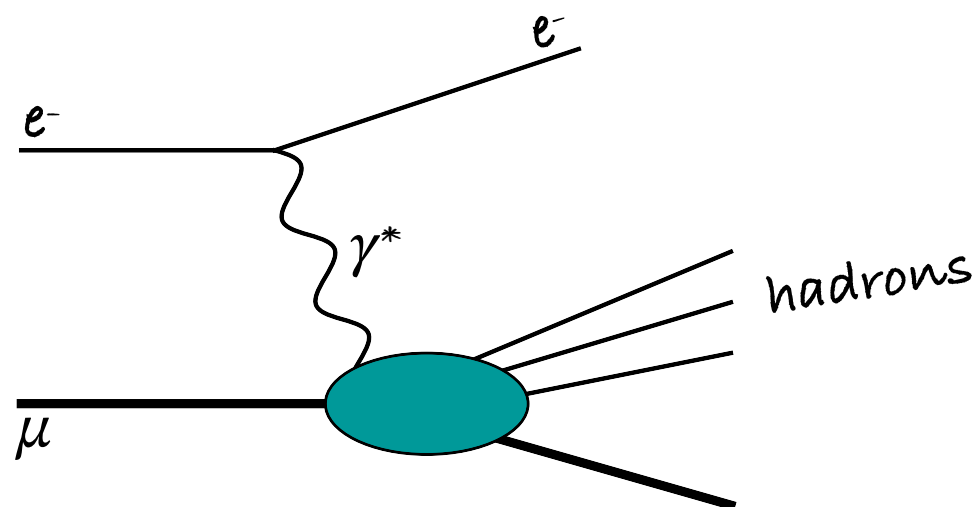
$$a^{\text{HVP}} = \frac{\alpha}{\pi^2} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} \text{Im } \Pi^{\text{had}}(s) \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)}$$

Expression in terms of spin structure functions

$$a = \frac{m^2}{\pi^2 \alpha} \int_{\nu_0}^{\infty} d\nu \left[\frac{\sigma_{LT}(\nu, Q^2)}{Q} \right]_{Q^2=0}$$

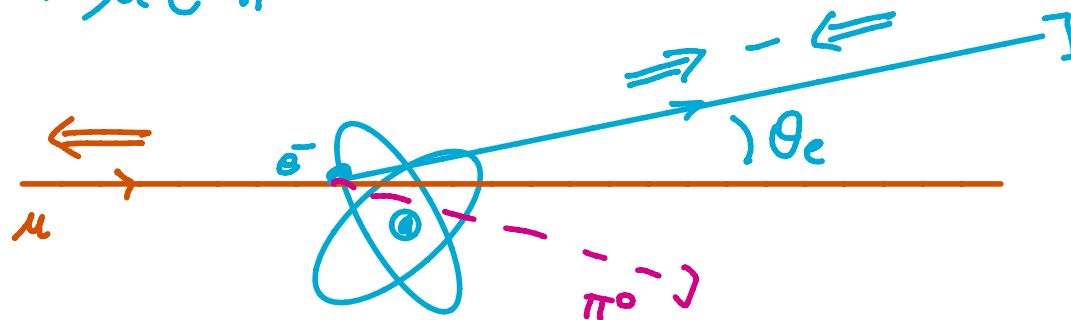
$$= \lim_{Q^2 \rightarrow 0} \frac{8m^2}{Q^2} \int_0^{x_0} dx [\bar{g}_1 + \bar{g}_2](x, Q^2)$$

muon spin structure functions
 $\bar{g}_1$ and $\bar{g}_2$



Prospects for neutral-pion production measurement (Along the MUonE experiment)

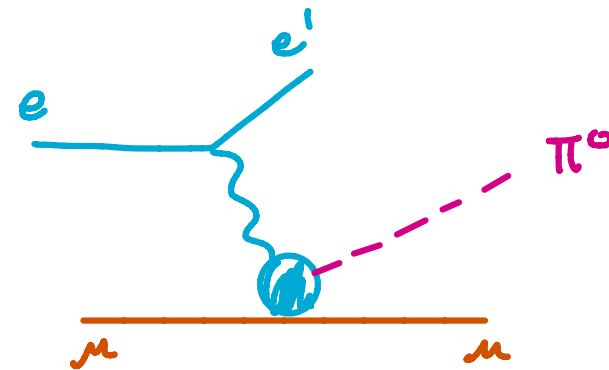
$$\mu e \rightarrow \mu e \pi^0$$



$$E_\mu = 150, 200 \text{ GeV}$$

$$E'_e \simeq 1 \text{ GeV}$$

$$\theta_e \simeq 10 \text{ mrad}$$

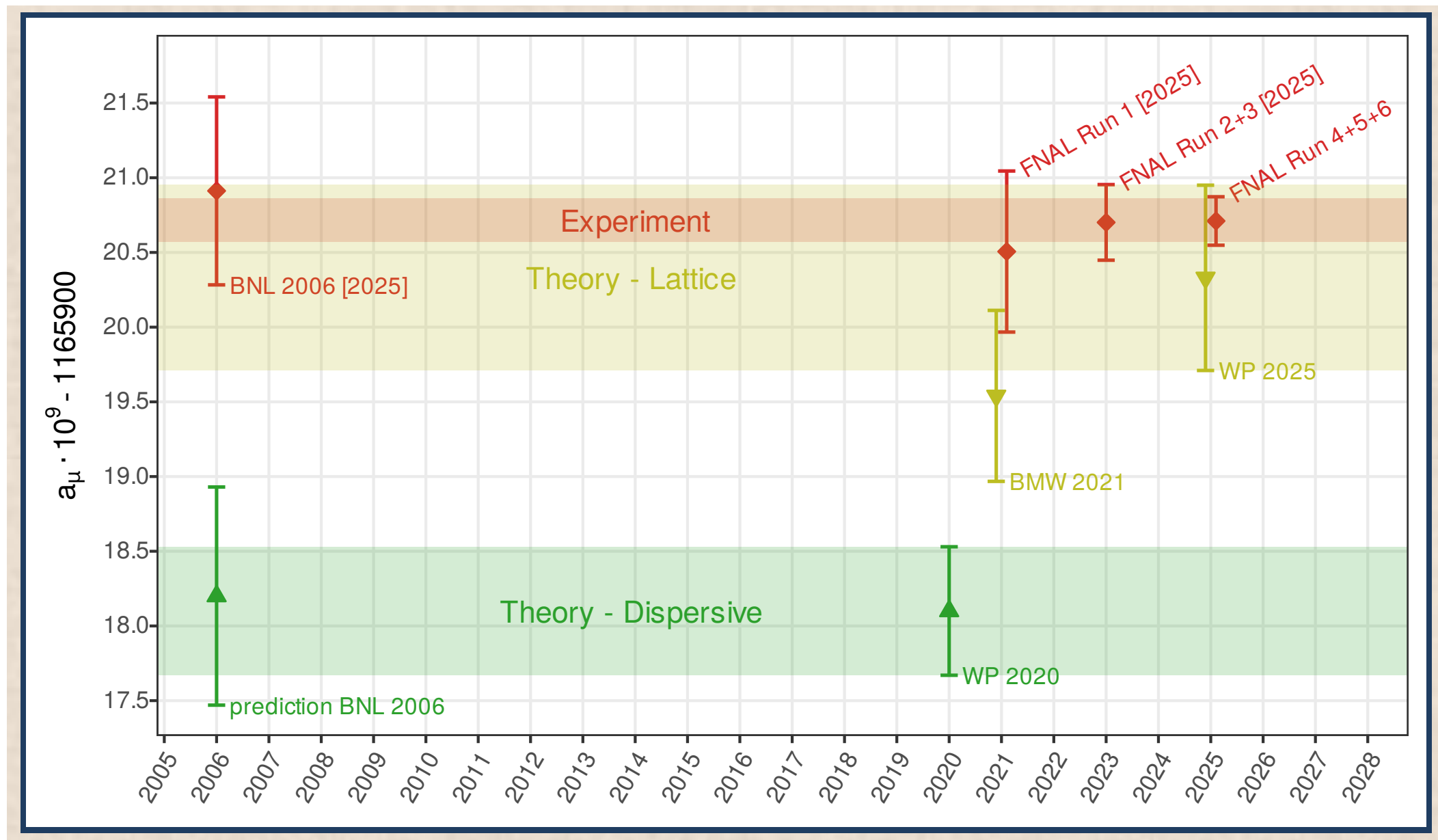


$$Q^2 \simeq 2m_e E'_e \simeq 10^{-3} \text{ GeV}^2$$

$$\gamma \simeq \frac{m_e E_\mu}{m_\mu} \left(1 - 2 \frac{E'_e}{m_e} \sin^2 \frac{\theta}{2} \right) = (\gamma_{\pi^0}, 1 \text{ GeV})$$

$$\gamma_{\pi^0} = \frac{m_{\pi^0}}{m_\mu} \left(\frac{1}{2} m_{\pi^0} + m_\mu \right) \simeq 230 \text{ MeV}$$

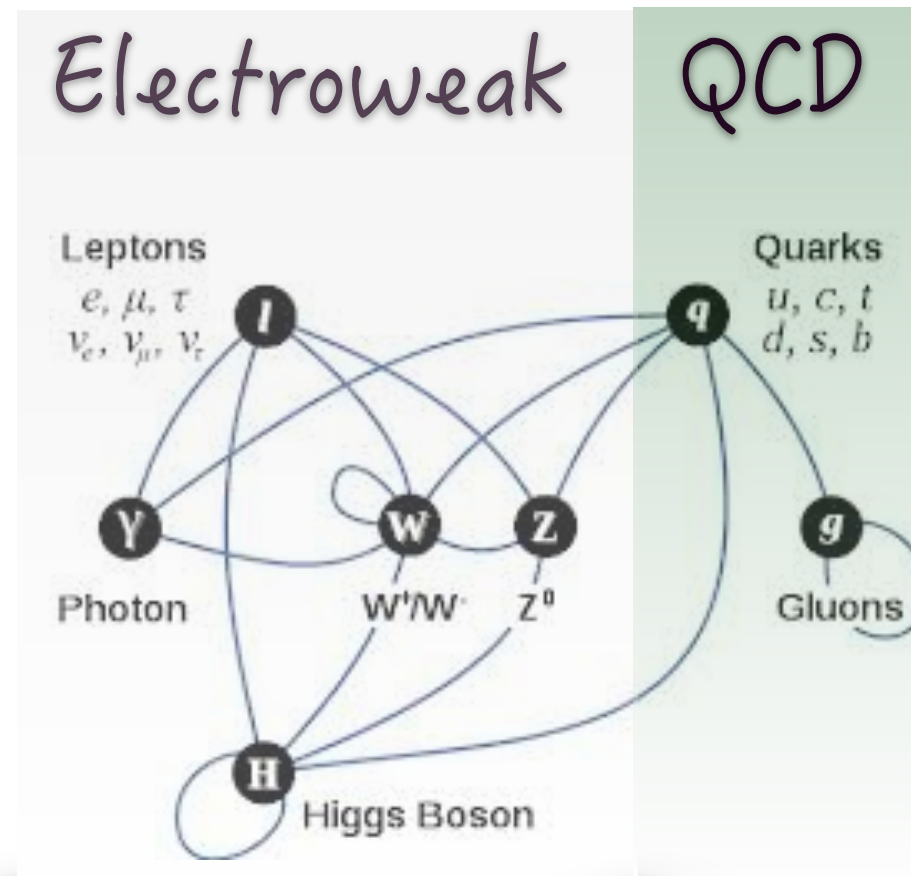
Current situation — disappointing?



$$a_\mu \text{ FNAL} = 0.001165920705 \text{ (148) [127 ppb]}$$

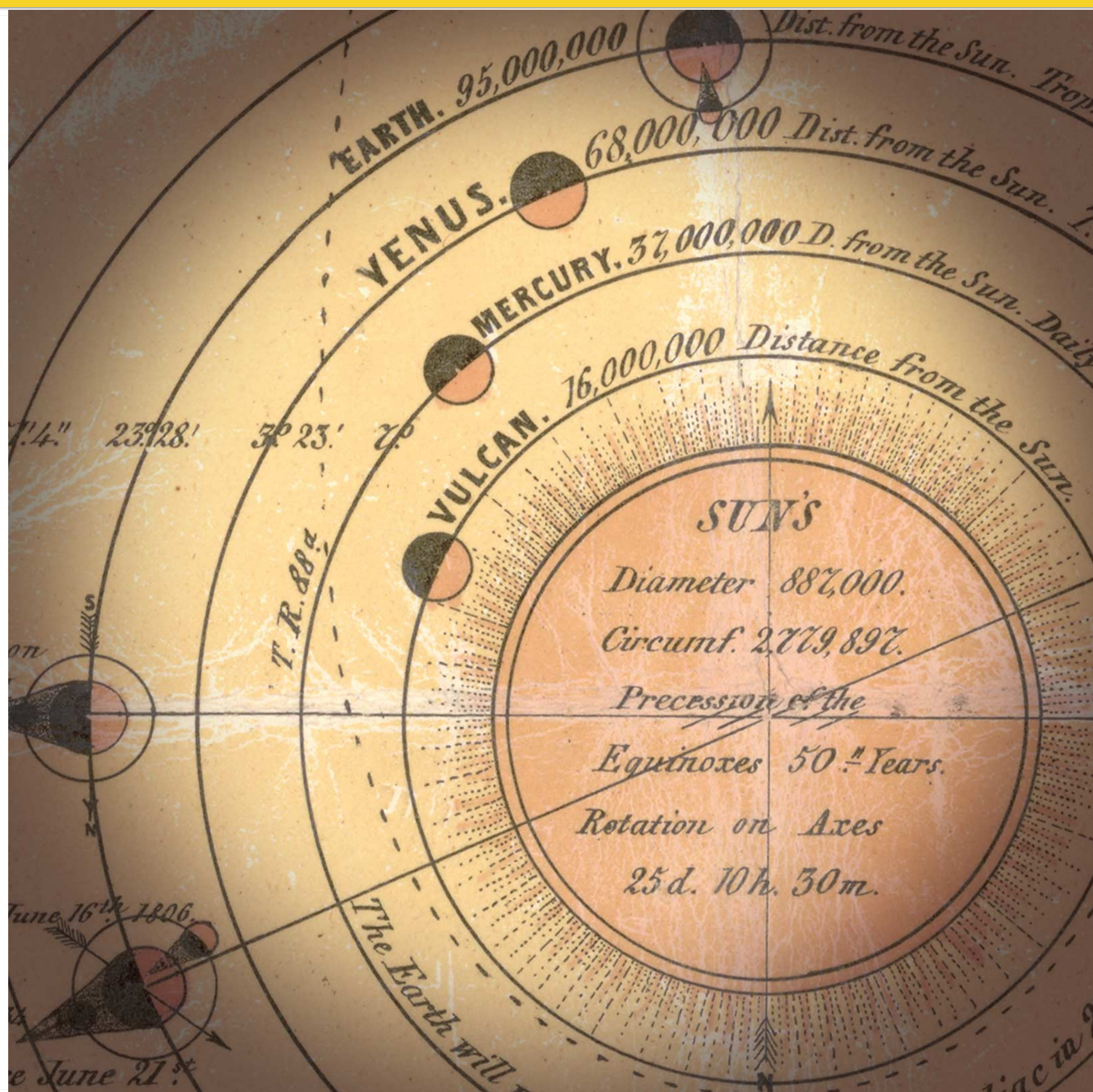
$$a_\mu \text{ EXP} = 0.001165920715 \text{ (145) [124 ppb]}$$

Standard Model



presently the best theory of (nearly) everything





What's next?

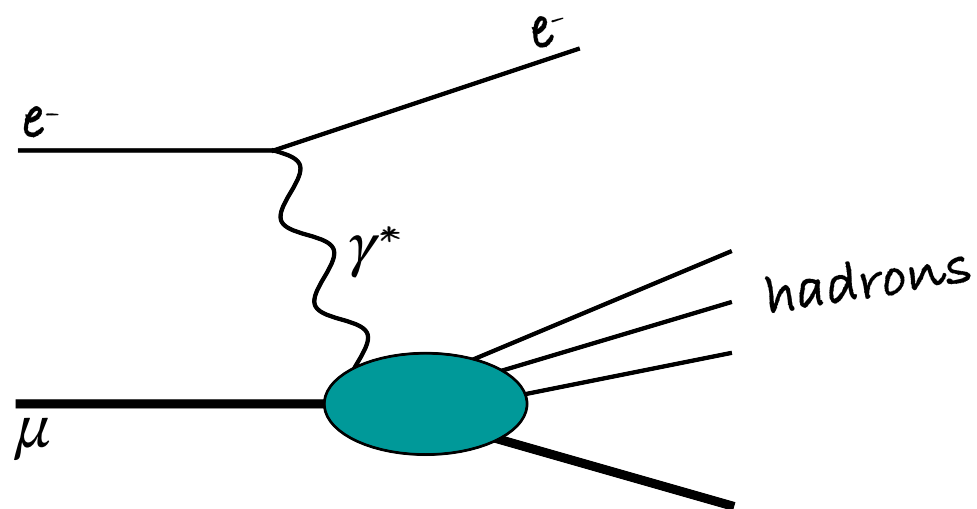
- Reach the experimental precision
 - **data-driven HVP**: the discrepancy, IB in τ decays
 - **lattice HVP**: IB effects (via LbL and Cottingham formula)
 - **data-driven LbL**: Schwinger sum rule
 - **lattice LbL**: nearly there
- Look for a theory superseding the SM

Backup

Spin structure functions

$$\mathbf{a} = \frac{m^2}{\pi^2 \alpha} \int_{\nu_0}^{\infty} d\nu \left[\frac{\sigma_{LT}(\nu, Q^2)}{Q} \right]_{Q^2=0}$$

$$= \lim_{Q^2 \rightarrow 0} \frac{8m^2}{Q^2} \int_0^{x_0} dx [\bar{g}_1 + \bar{g}_2](x, Q^2)$$



muon spin structure functions
 g_1 and g_2

- Spin-dependent forward doubly-virtual Compton scattering:

$$T_A^{\mu\nu}(q, p) = -\frac{1}{M} \gamma^{\mu\nu\alpha} q_\alpha S_1(\nu, Q^2) + \frac{Q^2}{M^2} \gamma^{\mu\nu} S_2(\nu, Q^2)$$

$$\text{Im} \left[\text{diagram} \right] \propto \left| \text{diagram} \right|^2$$

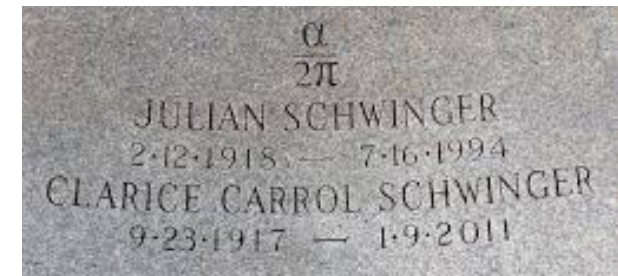
- Optical theorem:

$$\text{Im } S_1(\nu, Q^2) = \frac{4\pi^2 \alpha}{\nu} g_1(x, Q^2) = \frac{M\nu^2}{\nu^2 + Q^2} \left[\frac{Q}{\nu} \sigma_{LT} + \sigma_{TT} \right] (\nu, Q^2)$$

$$\text{Im } S_2(\nu, Q^2) = \frac{4\pi^2 \alpha M}{\nu^2} g_2(x, Q^2) = \frac{M^2 \nu}{\nu^2 + Q^2} \left[\frac{\nu}{Q} \sigma_{LT} - \sigma_{TT} \right] (\nu, Q^2)$$

Verifying the Schwinger sum rule in QED

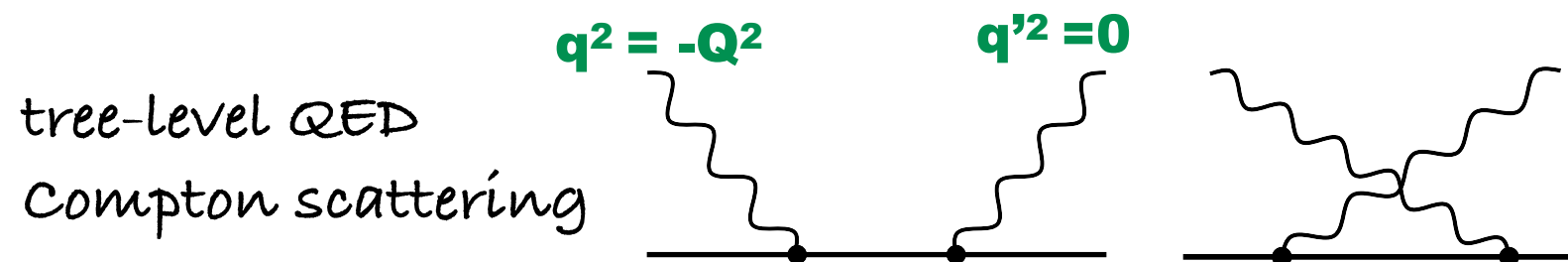
$$\begin{array}{c}
 \text{---} \bigcirc \text{---} \\
 F_2(0) = a
 \end{array}
 =
 \begin{array}{c}
 \text{---} \text{---} \\
 a(0) = 0
 \end{array}
 +
 \begin{array}{c}
 \text{---} \text{---} \\
 a^{(1)} = \alpha/2\pi
 \end{array}$$



Schwinger term —
the leading QED result

■ Schwinger sum rule: $a = \frac{m^2}{\pi^2 \alpha} \int_{\nu_0}^{\infty} d\nu \left[\frac{\sigma_{LT}(\nu, Q^2)}{Q} \right]_{Q^2=0}$

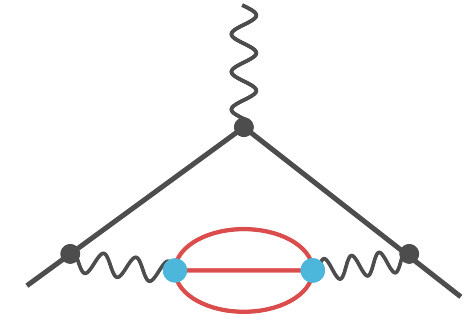
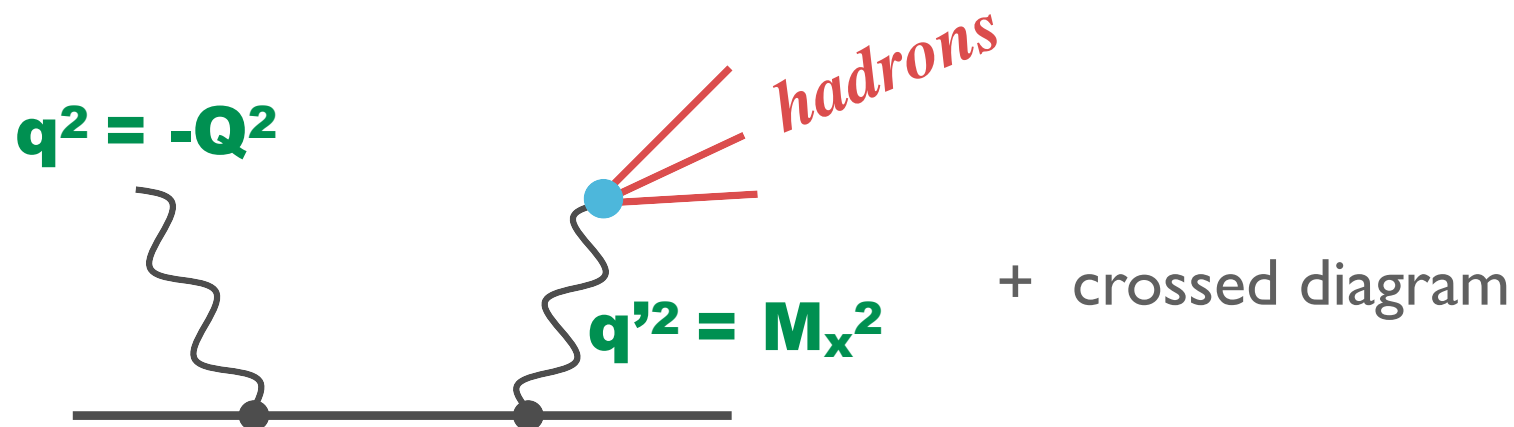
■ Input:



$$\sigma_{LT}^{\gamma^* \mu \rightarrow \gamma \mu}(\nu, Q^2) = \frac{\pi \alpha^2 Q (s - m^2)^2}{4m^3 \nu^2 (\nu^2 + Q^2)} \left(-2 - \frac{m(m + \nu)}{s} + \frac{3m + 2\nu}{\sqrt{\nu^2 + Q^2}} \operatorname{arccoth} \frac{m + \nu}{\sqrt{\nu^2 + Q^2}} \right)$$

with $s = m^2 + 2m\nu - Q^2$

HVP from Schwinger sum rule



- Cross section of hadron production through timelike Compton scattering:

factorizes as:

$$\sigma(\gamma\mu \rightarrow \mu X) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{dM_X^2}{M_X^2} \sigma(\gamma\mu \rightarrow \gamma^*\mu) \text{Im} \Pi_X(M_X^2)$$

↑ timelike Compton scattering ↑ virtual-photon decay into hadrons

- Timelike Compton scattering cross section:

$$\left[\frac{\sigma_{LT}^{\gamma\mu \rightarrow \gamma^*\mu}(\nu, Q^2)}{Q} \right]_{Q^2=0} = \frac{\pi\alpha^2}{2m^2\nu^3} \left[-(5s + m^2 + M_X^2)\lambda + (s + 2m^2 - 2M_X^2) \log \frac{\beta + \lambda}{\beta - \lambda} \right]$$

$$\beta = (s + m^2 - M_X^2)/2s \quad s = m^2 + 2m\nu$$

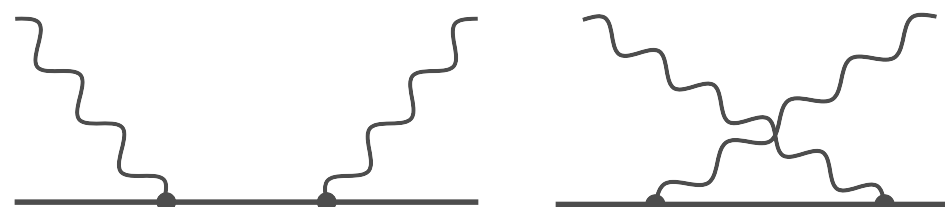
$$\lambda = (1/2s) \sqrt{[s - (m + M_X)^2][s - (m - M_X)^2]}$$

Reproducing the leading QED result

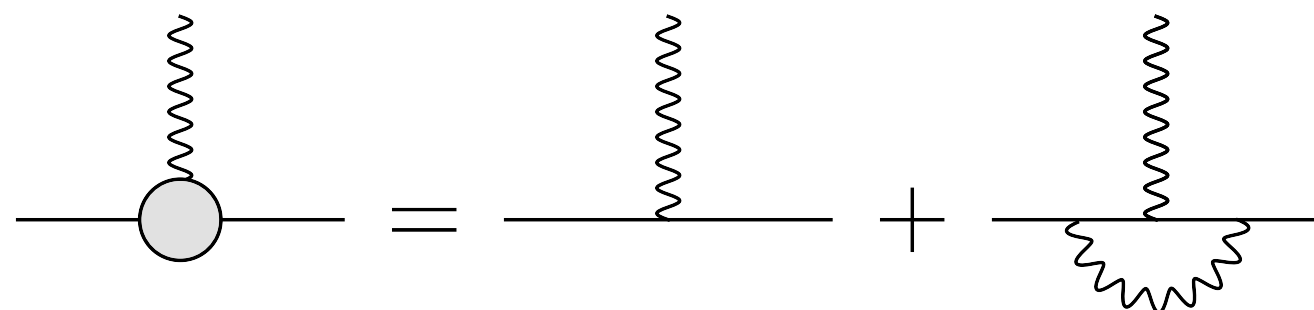
- Schwinger sum rule: $a = \frac{m^2}{\pi^2 \alpha} \int_{\nu_0}^{\infty} d\nu \left[\frac{\sigma_{LT}(\nu, Q^2)}{Q} \right]_{Q^2=0}$
- Input: longitudinal-transverse photo-absorption cross section

tree-level QED

Compton scattering



$$\sigma_{LT}^{\gamma^* \mu \rightarrow \gamma \mu}(\nu, Q^2) = \frac{\pi \alpha^2 Q (s - m^2)^2}{4m^3 \nu^2 (\nu^2 + Q^2)} \left(-2 - \frac{m(m + \nu)}{s} + \frac{3m + 2\nu}{\sqrt{\nu^2 + Q^2}} \operatorname{arccoth} \frac{m + \nu}{\sqrt{\nu^2 + Q^2}} \right)$$



$$F_2(0) = a$$

$$a(0) = 0$$

$$a(1) = \alpha/2\pi$$



Summary and Conclusions

1. Schwinger sum rule — dispersive formula applying equally to HVP and HLbL

2. Reproduces $\alpha/2\pi$ and HVP formula:

$$\begin{aligned}
 \text{Triangle Diagram} &= \frac{m_\mu^2}{d\pi^2} \int d\nu \left| \text{Diagram 1} + \text{Diagram 2} \right|^2 \\
 a_\mu^{\text{HVP}} &= \frac{m_\mu^2}{d\pi^2} \int d\nu \int dM_x^2 \sigma_{LT}(\gamma\mu \rightarrow \gamma_x^* \mu) \Gamma(\gamma_x^* \rightarrow \text{hadrons})
 \end{aligned}$$

3. Splits contributions into hadron production and e.m. (LbL) channels



measurable

spin structure functions



LQCD ?



direct LbL scattering (ATLAS)

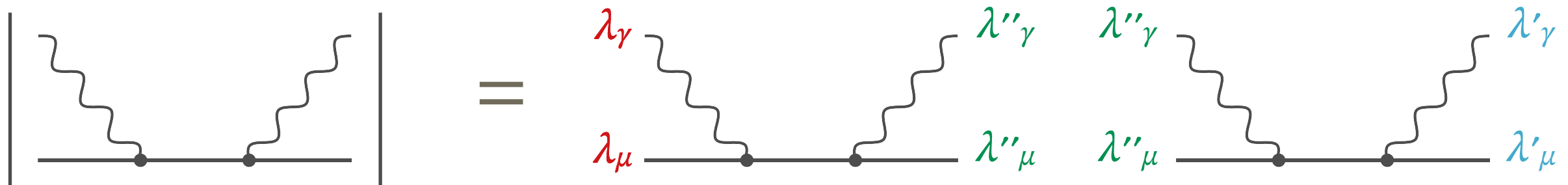
4. **Partial** calculation of PS-meson contributions: a factor of 2 to 3 larger than the conventional model calculations.

The Cross section σ_{LT}

- Example: tree-level QED Compton scattering cross section

$$d\sigma_{\lambda'_\gamma \lambda'_\mu \lambda_\gamma \lambda_\mu} = (2\pi)^4 \delta^{(4)}(p_f - p_i) \sum_{\lambda''_\gamma, \lambda''_\mu} \frac{\mathcal{M}_{\lambda'_\gamma \lambda'_\mu \lambda''_\gamma \lambda''_\mu}^\dagger \mathcal{M}_{\lambda''_\gamma \lambda''_\mu \lambda_\gamma \lambda_\mu}}{4I} \prod_a \frac{d^3 p'_a}{(2\pi)^3 2E'_a},$$

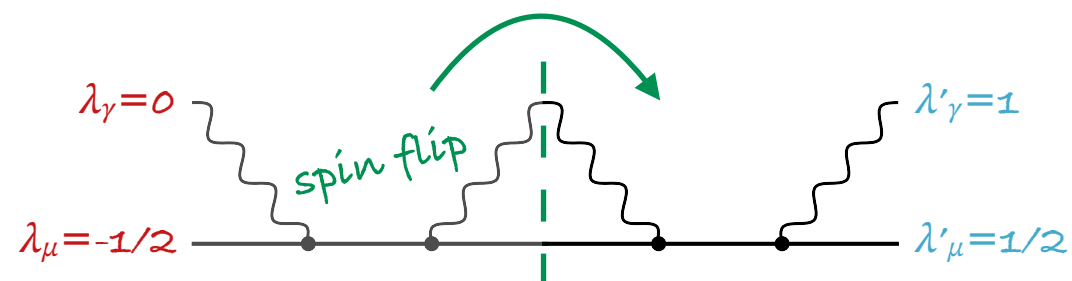
with conserved helicity: $\hbar = \lambda'_\gamma - \lambda'_\mu = \lambda_\gamma - \lambda_\mu$



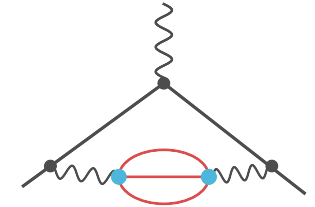
- helicity difference photo-absorption cross section: $\sigma_{TT} = 1/2 (\sigma_{1/2} - \sigma_{3/2})$

- longitudinal-transverse photo-absorption cross section:

$$\gamma^*(\lambda_\gamma=0) + \mu(\lambda_\mu=-1/2) \rightarrow \gamma(\lambda'_\gamma=1) + \mu(\lambda'_\mu=1/2)$$



Timelike CS mechanism



$q^2 = -Q^2$
 $q'^2 = M_X^2$
+ crossed diagram

$\sigma(\gamma\mu \rightarrow \mu X) = \frac{(2\pi)^4}{4I} \int d^4q' \int \prod_i \frac{d^3k_i}{2E_{k_i}(2\pi)^3} \int \frac{d^3p'}{2E_{p'}(2\pi)^3} \left[\frac{\Lambda^{\dagger\mu} \Lambda^\nu \rho_{\mu\nu}}{(-q'^2)^2} \right] \delta^4(q' - \sum_i k_i) \delta^4(p + q - p' - q')$

initial flux factor
 $I = (p \cdot q)^2 - p^2 q^2$

Λ^ν : virtual-photon decay vertex ↓

phase space of the final state

$\rho_{\mu\nu}$: squared matrix element of timelike CS

- Virtual-photon decay width into hadronic state X:

$$[\Gamma(\gamma^* \rightarrow X)]^{\mu\nu} = \int \prod_i \frac{d^3k_i}{2E_{k_i}(2\pi)^3} \frac{\Lambda^{\dagger\mu} \Lambda^\nu}{2E_{q'}} (2\pi)^4 \delta^4(q' - \sum_i k_i)$$

$$= -\frac{1}{\sqrt{q'^2}} (q'^2 g^{\mu\nu} - q'^\mu q'^\nu) \text{Im} \Pi_X(q'^2)$$

↑
 $\text{Im} \Pi_X$: contribution of state X to the VP

- Combine into: $\sigma(\gamma\mu \rightarrow \mu X) = -\frac{1}{2I} \int d^4q' \int \frac{d^3p'}{2E_{p'}(2\pi)^3} \rho_\mu^\mu \frac{\text{Im} \Pi_X(q'^2)}{q'^2} \delta^4(p + q - p' - q')$

- Final factorized cross section:

$$\sigma(\gamma\mu \rightarrow \mu X) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{dM_X^2}{M_X^2} \sigma(\gamma\mu \rightarrow \gamma^* \mu) \text{Im} \Pi_X(M_X^2)$$