From exclusive to inclusive decays with Lattice QCD

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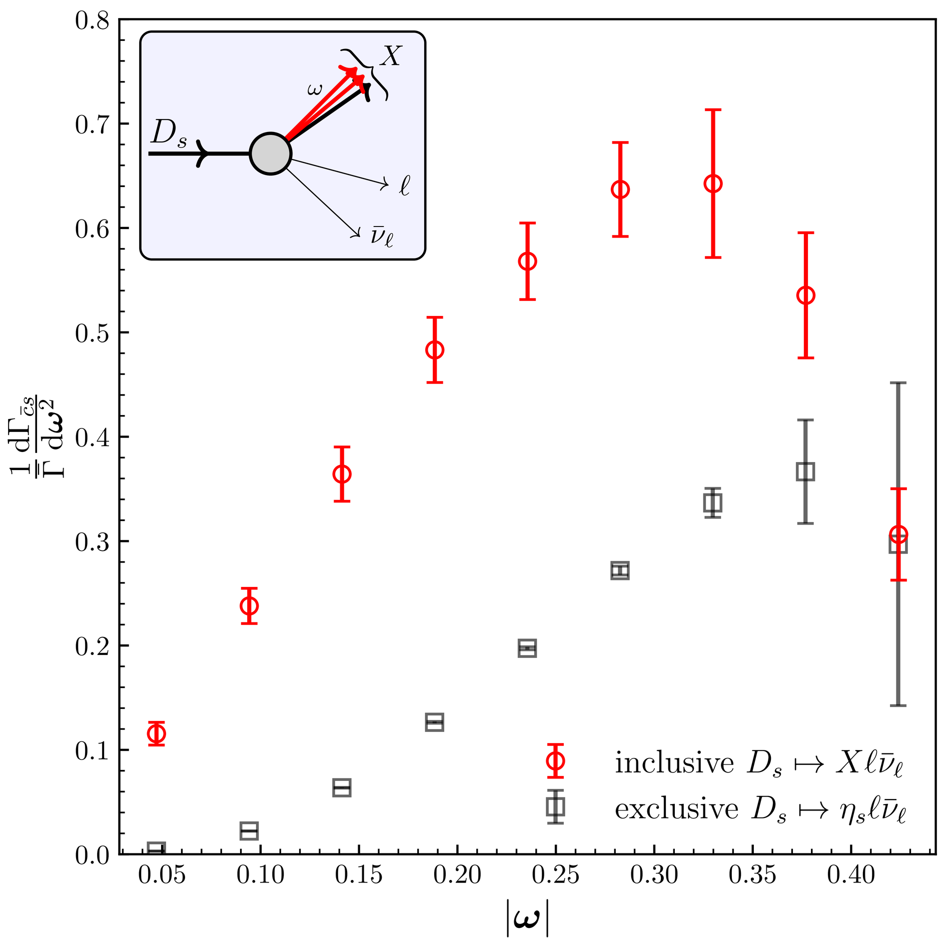
based on Refs. [1, 2]

Overview

Computing inclusive decay rates from first principles has **long challenged lattice QCD** because of the need to access fully summed multi-hadron final states. Using **spectral density reconstruction techniques**, we perform the first lattice QCD study of the **inclusive semileptonic decays of the D_s meson** [1, 2], obtaining precise determinations of the total decay rate and the first two lepton-energy moments. This represents a **new solid approach** to heavy mesons decay, alternative to exclusive computations, opening the way to the computation of **inclusive semileptonic decays of $B_{(s)}$ mesons**.



Analysis

The **weak current correlators** are computed on gauge ensembles, close to the **pion physical point**, by the ETM Collaboration employing the Iwasaki gluon action and $N_f = 2+1+1$ flavours of Wilson–Clover twisted-mass fermions at maximal twist [5, 6].

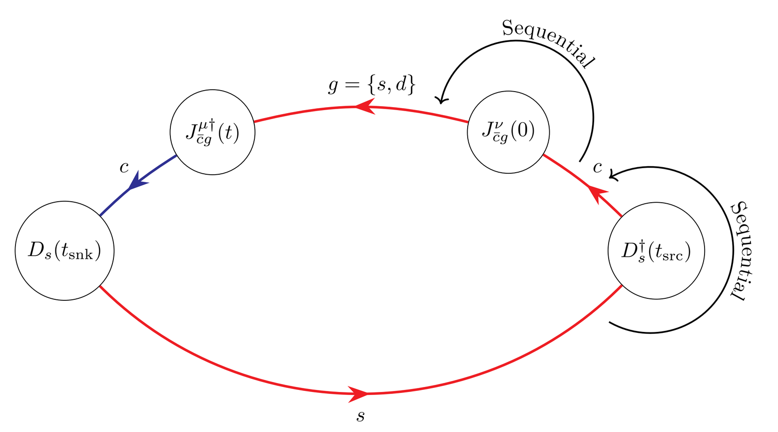


Figure 1: Diagrammatic representation of the $C_{\mu\nu}(t,\omega)$ correlators used in the analysis.

a [fm]	ID	$(L^3 \times T)/a^4$	L [fm]
	B48	$48^3 \times 96$	3.8
0.080	B64	$64^3 \times 128$	5.1
	B96	$96^3 \times 192$	7.6
0.068	C80	$80^3 \times 160$	5.4
0.057	D96	$96^3 \times 192$	5.5
0.050	E112	$112^3 \times 224$	5.6

To **recover the physical differential decay rate**:

1) spectral densities reconstruction at fixed σ and ω;
2) ∞-volume limit with B ensembles;
3) continuum limit extrapolation;
4) $\sigma \to 0$ limit via theory-based ansatz [7].

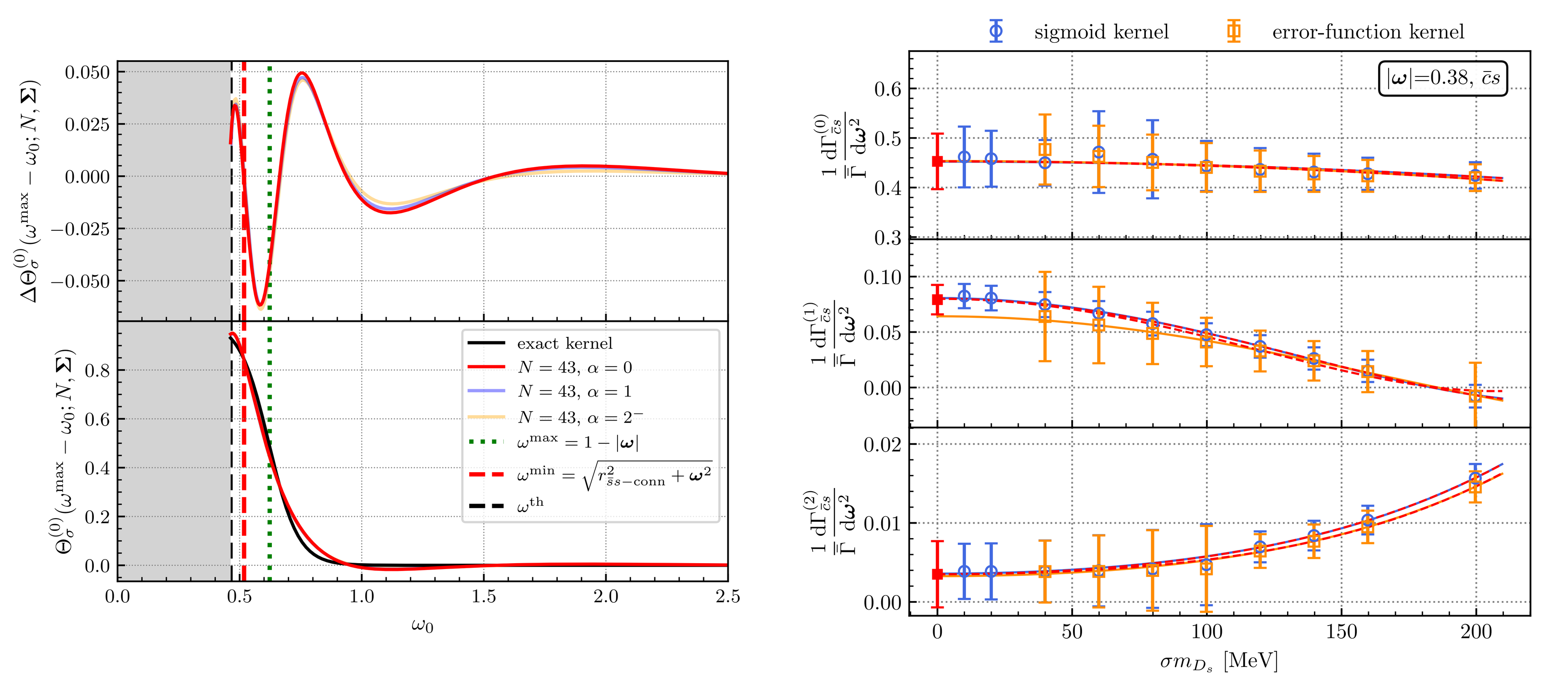


Figure 2: Left: Comparison of the approximated and exact kernels at varying the HLT algorithmic parameters. Right: $\sigma \mapsto 0$ extrapolation at fixed momenta $|\omega| = 0.38$.

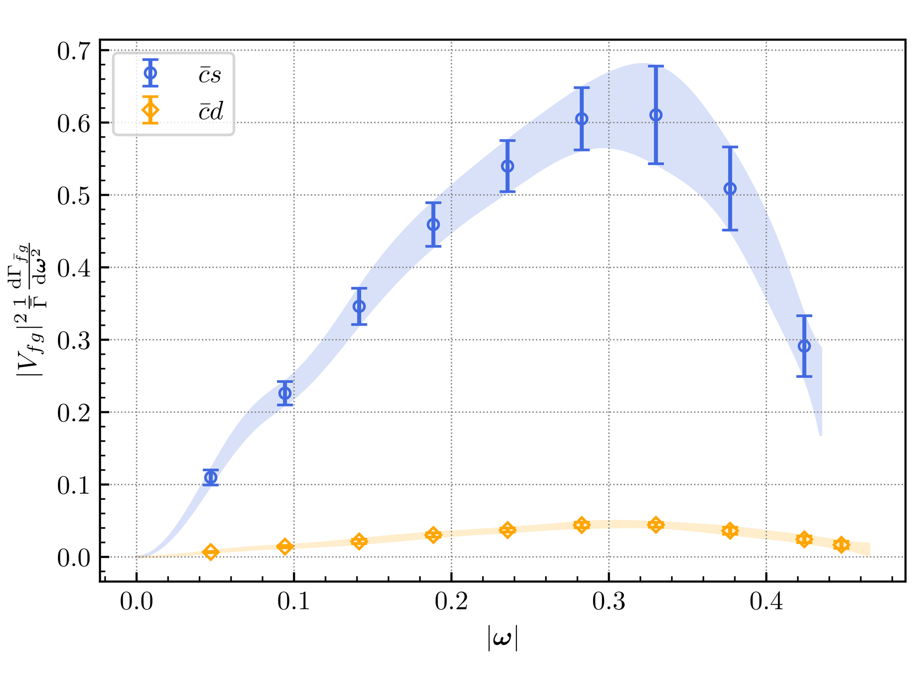


Figure 3: The differential decay rate $\mathrm{d}\Gamma/\mathrm{d}\omega^2$ for the $\bar{c}s$ and $\bar{c}d$ channels.

Combined and **separate extrapolations** of the data from two smearing-θ representations, treating the resulting differences as a source of systematic uncertainty. Using **spline interpolation**, we obtained a smooth differential decay rate, which we compared with experimental results after integrating over the phase space.

The inclusive decay rate

The fully inclusive decay rate $\Gamma \equiv \Gamma\left(D_s \mapsto X\ell\bar{\nu}_\ell\right)$ can be obtained from

$$\Gamma \propto \int \mathrm{d}\Phi_\ell\, \mathrm{d}\Phi_\nu\, K_{\mu\nu}(p_\ell, p_\nu)\, H_{\mu\nu}(p, p - p_\ell - p_\nu)$$

where $K_{\mu\nu}(p_\ell, p_\nu)$ is an analytical kernel arising from the leptonic part and

$$\begin{aligned} H_{\mu\nu}(p,\omega) &= \frac{1}{2m_{D_s}} \sum_X \langle D_s(p)|\, J_\mu^\dagger(0)\, |X(\omega)\rangle \langle X(\omega)|\, J_\nu(0)\, |D_s(p)\rangle \\ &= \frac{(2\pi)^4}{2m_{D_s}} \langle D_s(p)|\, J_\mu^\dagger(0)\, \delta^4(\mathbb{P} - \omega)\, J_\nu(0)\, |D_s(p)\rangle \ , \end{aligned}$$

is the hadronic tensor. Typically, on the lattice, it is straightforward to isolate the ground state (P) entering in the hadronic tensor $H_{\mu\nu}(p,\omega)$. The **exclusive decay rate** is then extracted from

$$\frac{d\Gamma}{d\boldsymbol{\omega}^2} \propto |\boldsymbol{\omega}|^3\, f_+^2(\boldsymbol{\omega}^2) \qquad \langle D_s(p)|\, J_\mu^\dagger(0)\, |P(\omega)\rangle = (p+\omega)f_+ + (p-\omega)f_-$$

The **inclusive decay rate** can be computed by using the **HLT method for extracting spectral densities** [3, 4]. Indeed, by working out the phase space

$$\begin{aligned} \frac{\mathrm{d}\Gamma(\sigma)}{\mathrm{d}\boldsymbol{\omega}^2} &\propto \sum_{p=0}^{2} |\boldsymbol{\omega}|^{3-p} \lim_{\sigma\to 0} \int_{\omega^{\min}}^{\infty} \mathrm{d}\omega_0\, \Theta_\sigma^{(p)}(\omega^{\max} - \omega_0)\, Z^{(p)}(\omega_0, \boldsymbol{\omega}^2) \\ &\propto \sum_{p=0}^{2} |\boldsymbol{\omega}|^{3-p} \lim_{\sigma\to 0} \lim_{N\mapsto\infty} \sum_{n=1}^{N} g^{(p)}(n,\sigma)\, \hat{Z}^{(p)}(an, \boldsymbol{\omega}^2) \end{aligned}$$

where $\Theta_\sigma^{(p)}$ is a smearing analytical kernel arising from the leptonic part and phase space, and $Z^{(p)}$ is a suitable combination of the hadronic tensor elements, which is related to the four-point Euclidean correlator through

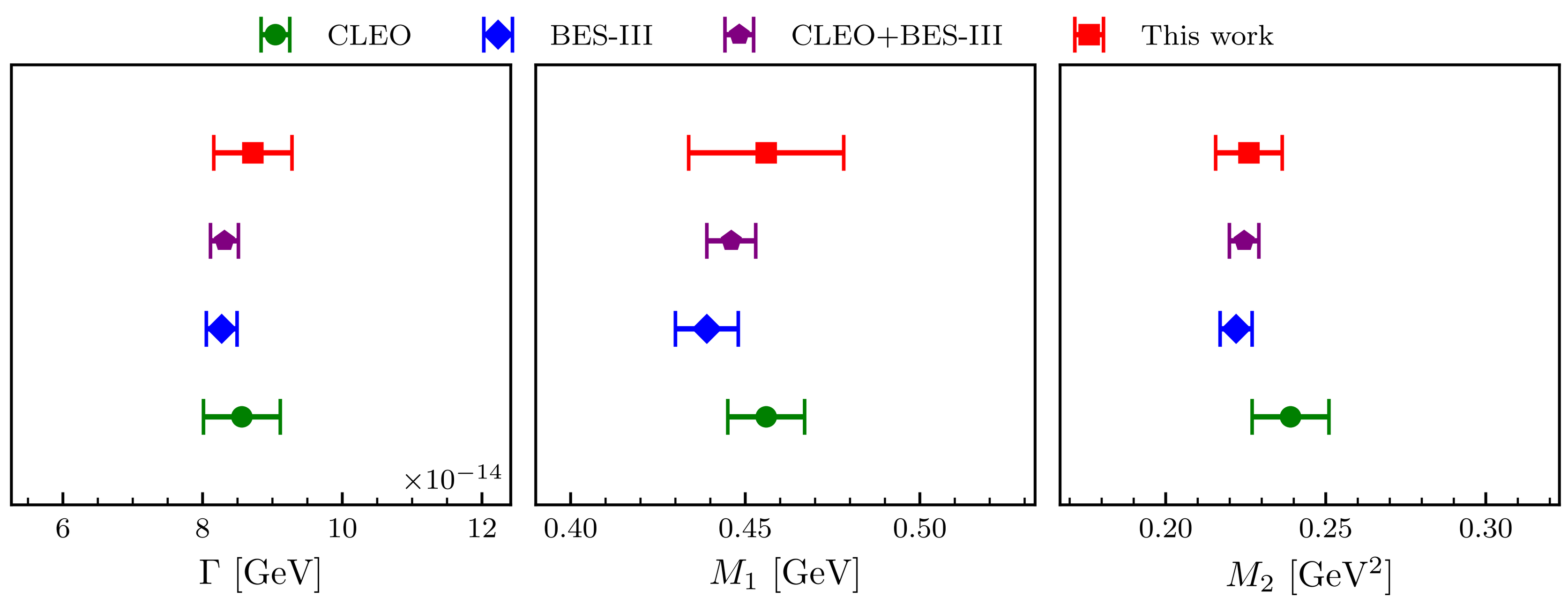
$$C_{\mu\nu}(t,\boldsymbol{\omega}) = \int_{\omega^{\min}}^{\infty} \mathrm{d}\omega_0\, e^{-m_{D_s}\omega_0 t} H_{\mu\nu}(\omega_0, \omega)$$

Results

Using the latest PDG values for the CKM matrix elements, our final result for the total decay rate is

$$\Gamma\left(D_s \to X\ell\nu_\ell\right) = 8.72\,(56) \times 10^{-14}\ \mathrm{GeV}\,,$$

which is in excellent agreement with the experimental measurements from CLEO and BESIII [8, 9], as well our results for the first two lepton-energy moments.



Aknowlegments

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