Towards continuum limit of Meson Charge Radii using large volume configuration at physical point in Nf=2+1 lattice QCD

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Takeshi Yamazaki (University of Tsukuba)
for PACS Collaboration

SEIKEI

EINN 2025
Paphos, Cyprus, October 28th, 2025

Target: π^+, K^+, K^0 charge radii

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Data characteristics: Large volume and physical point

+

Analysis Method : without fit ansatz

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Rough sketch

π^+ meson

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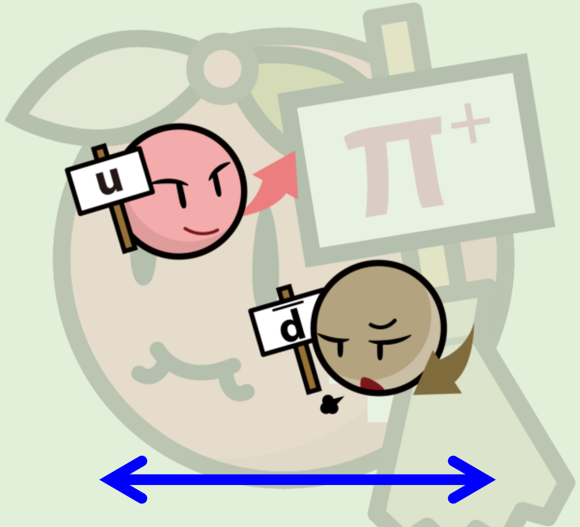

electrical spread out

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electromagnetic current: $V_\mu = \sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f$

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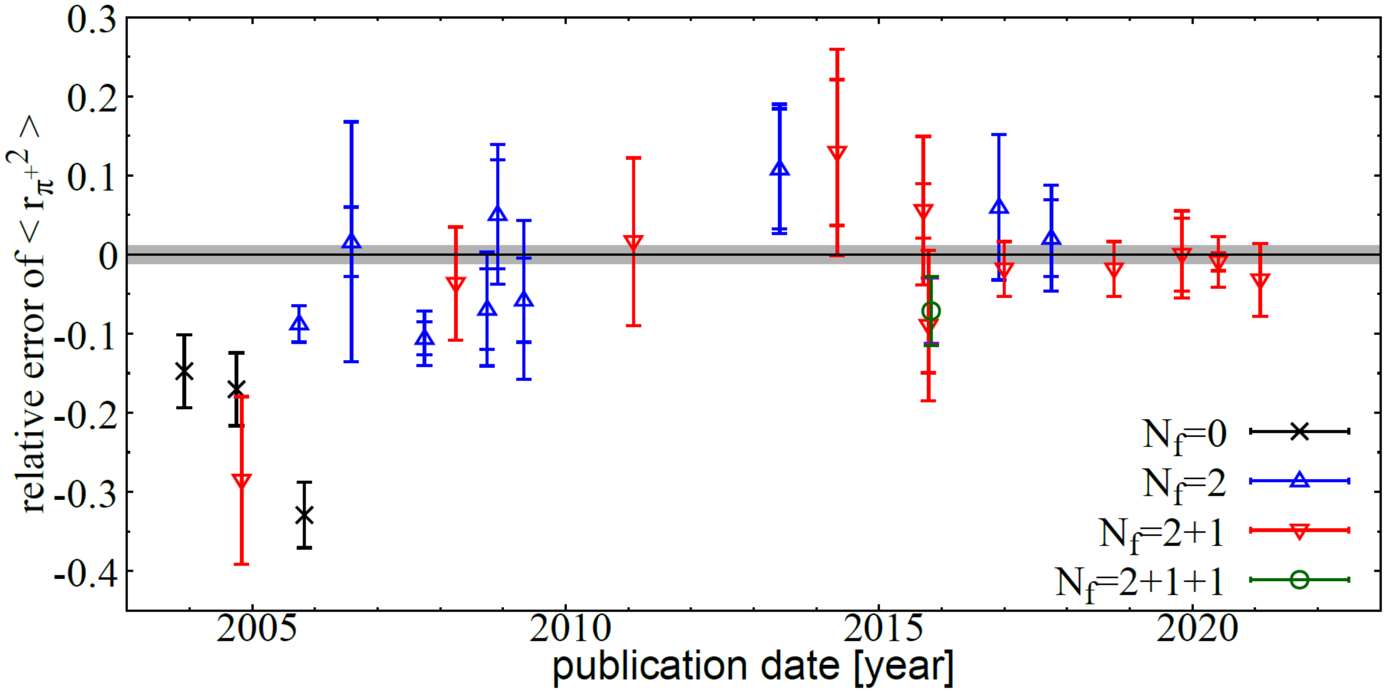

- ✓ Initially : large difference
  Recently : consistent
- ✓ Error : Lattice > Experimental
  There are 4 main systematic errors
  - ➢ Chiral extrapolation
  - ➢ Continuum extrapolation
  - ➢ Finite volume effect
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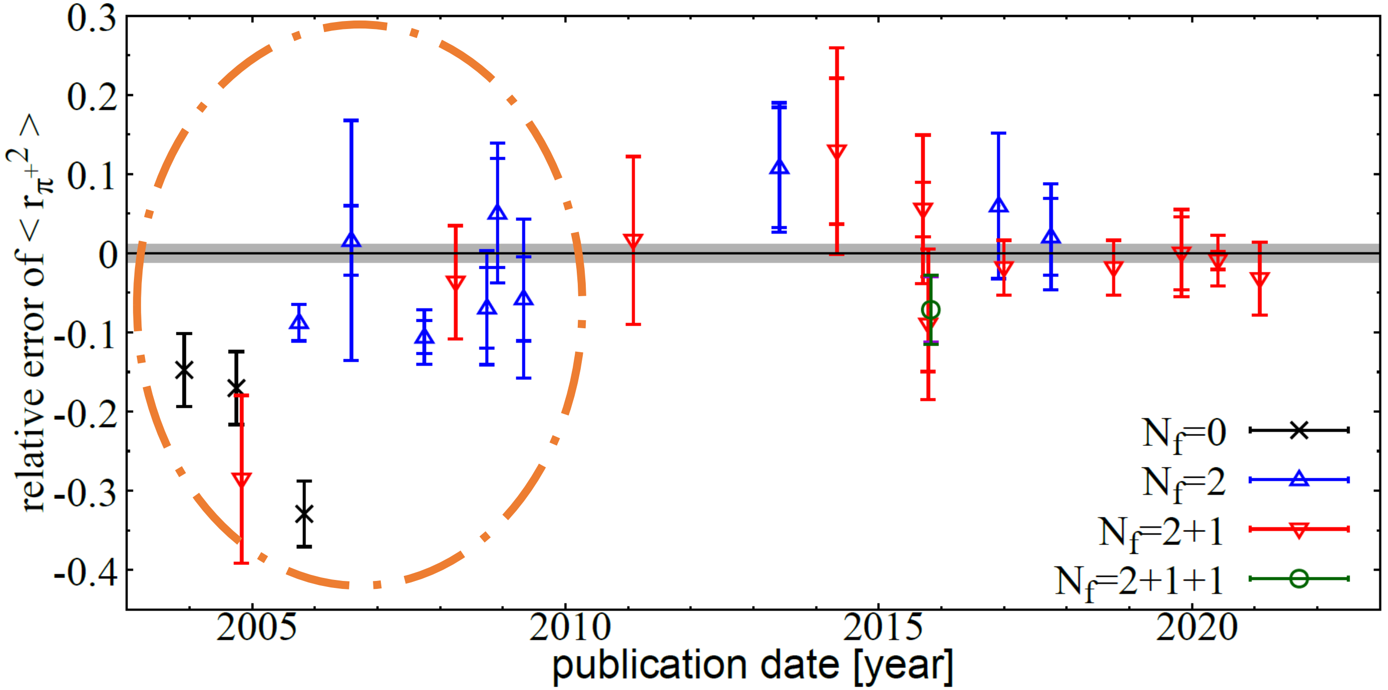

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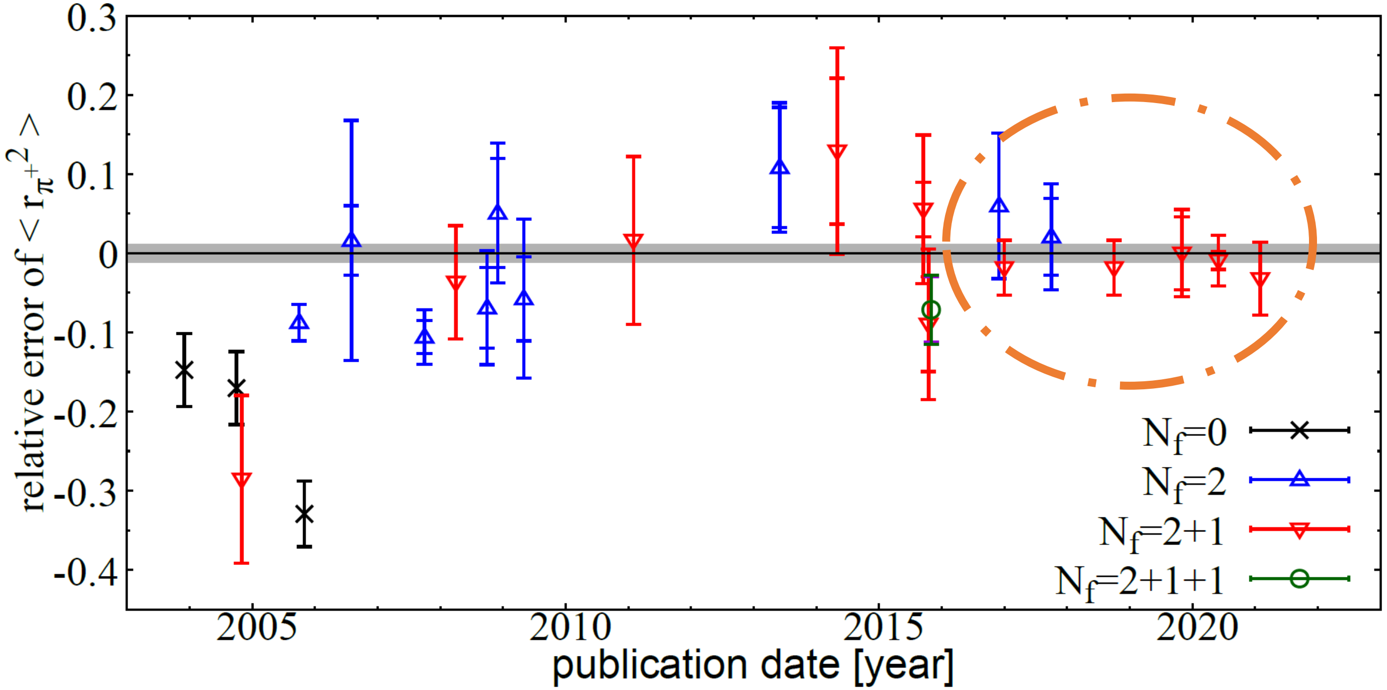

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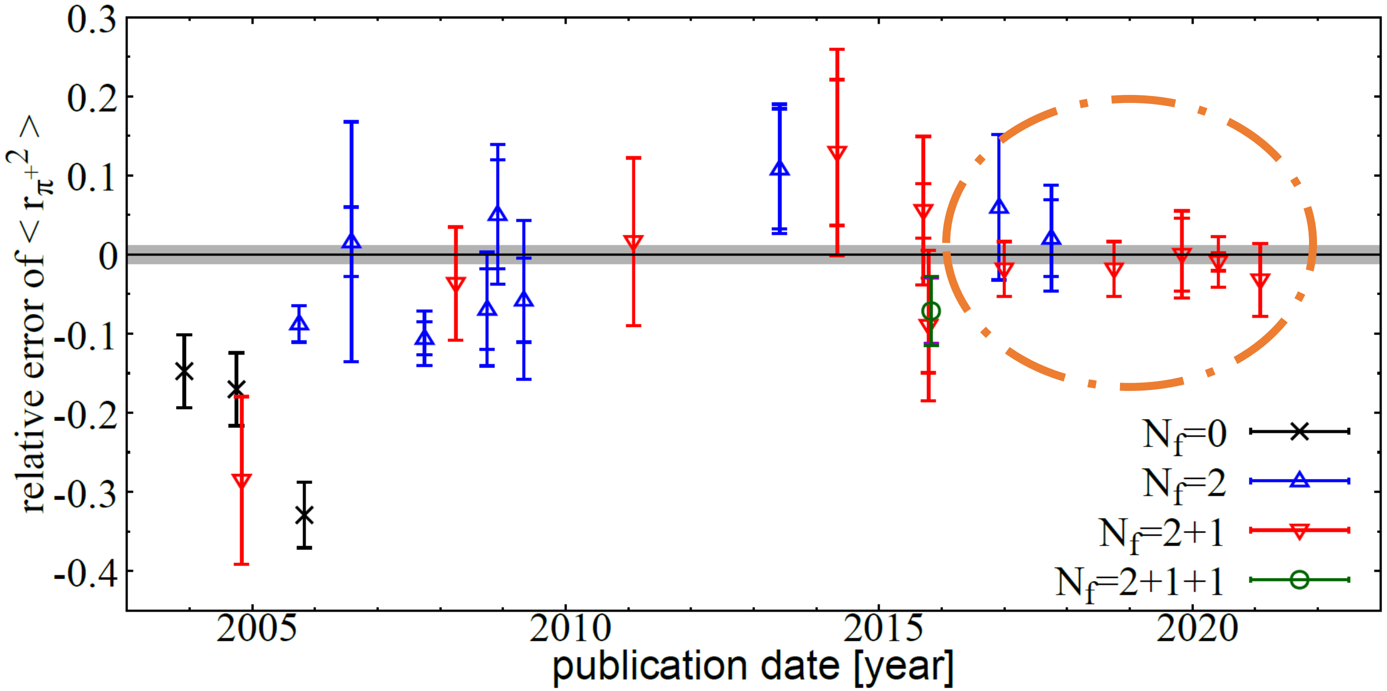

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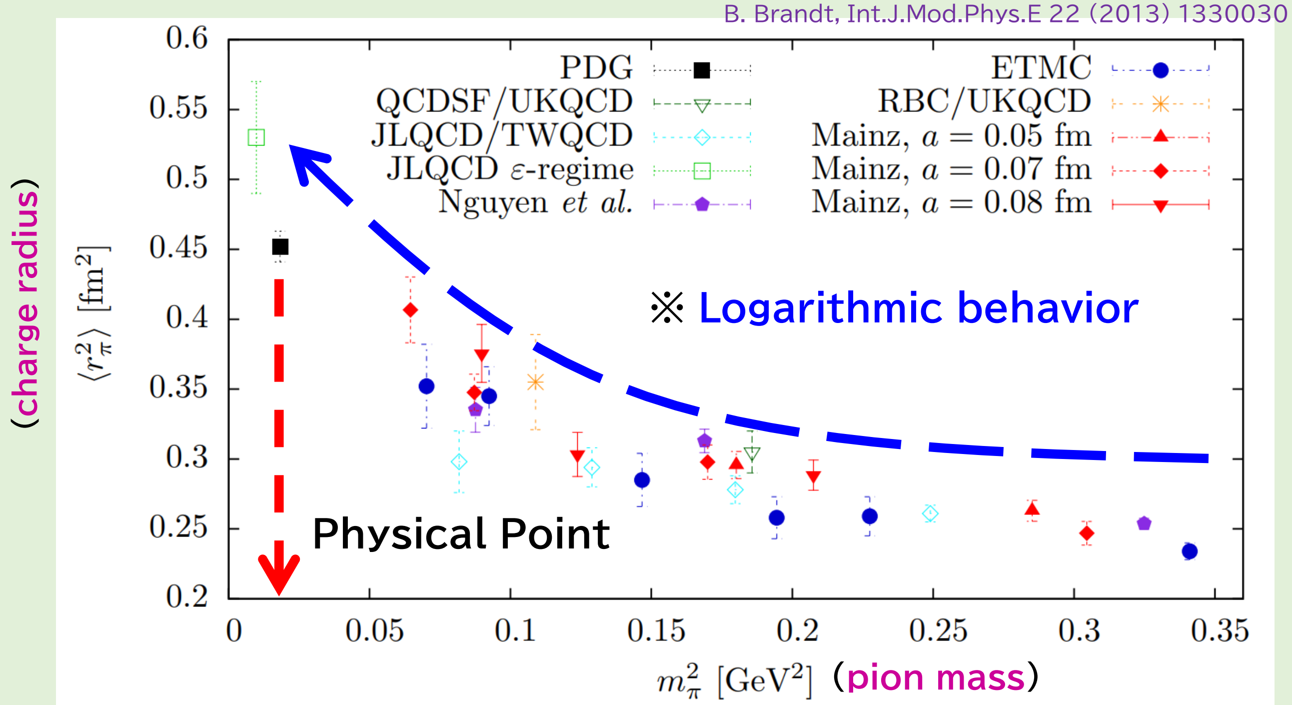
By changing theoretical parameters, we can compute physics in various worlds.

B. Brandt, Int.J.Mod.Phys.E 22 (2013) 1330030



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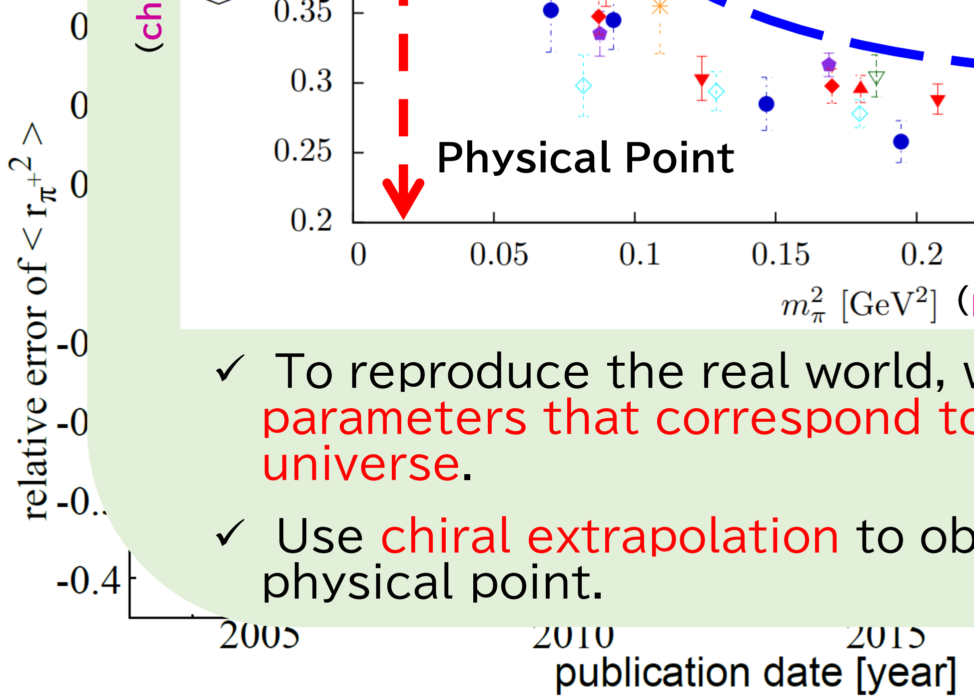
- ✓ Use chiral extrapolation to obtain values at the physical point.



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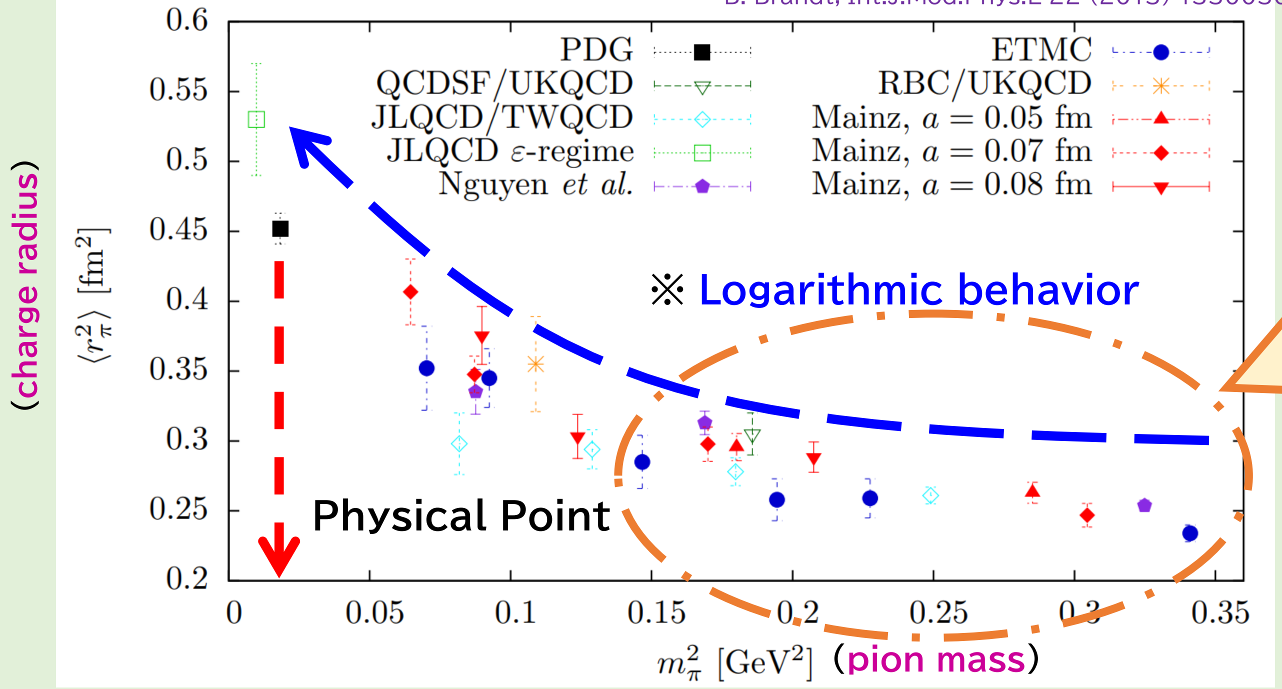

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Earlier lattice QCD calculations

Many simulations were performed at heavier pion masses

We needed to extrapolate these results to the physical point using chiral extrapolation

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relative error of $< r_{\pi^+}^2 >$

-0.4

2005 2010 2015 2020

publication date [year]

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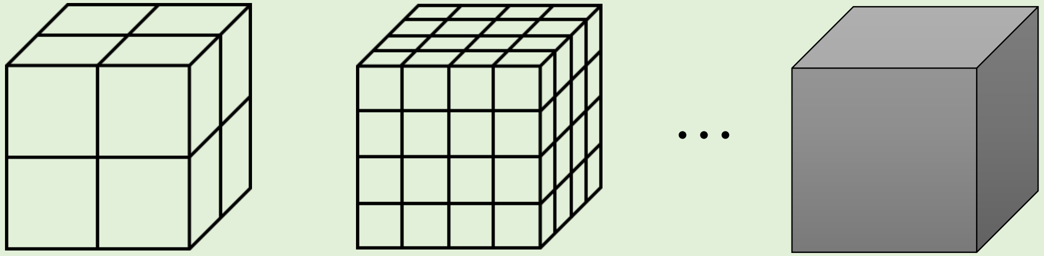

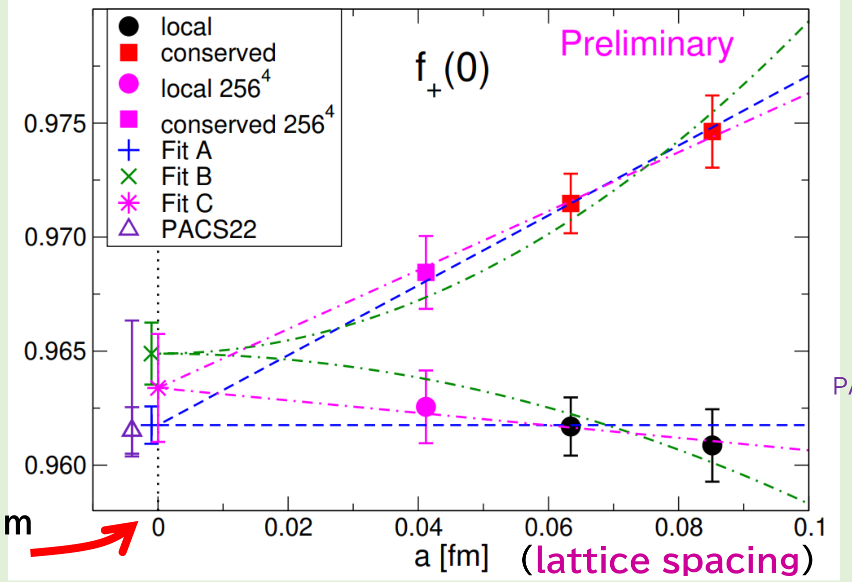

PACS, PoS LATTICE2023 (2024) 276

✓ Continuum extrapolation derives real values from discrete data.

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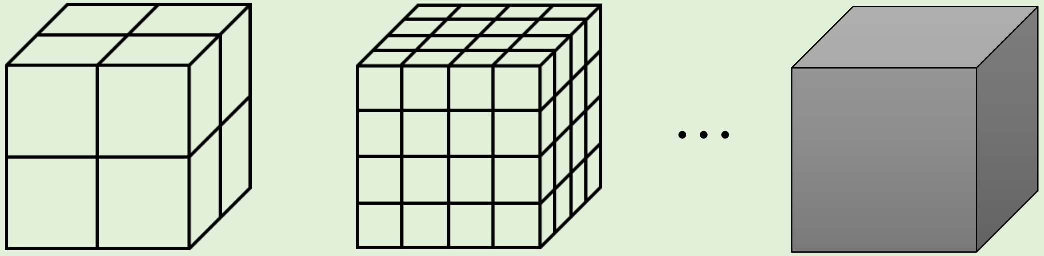

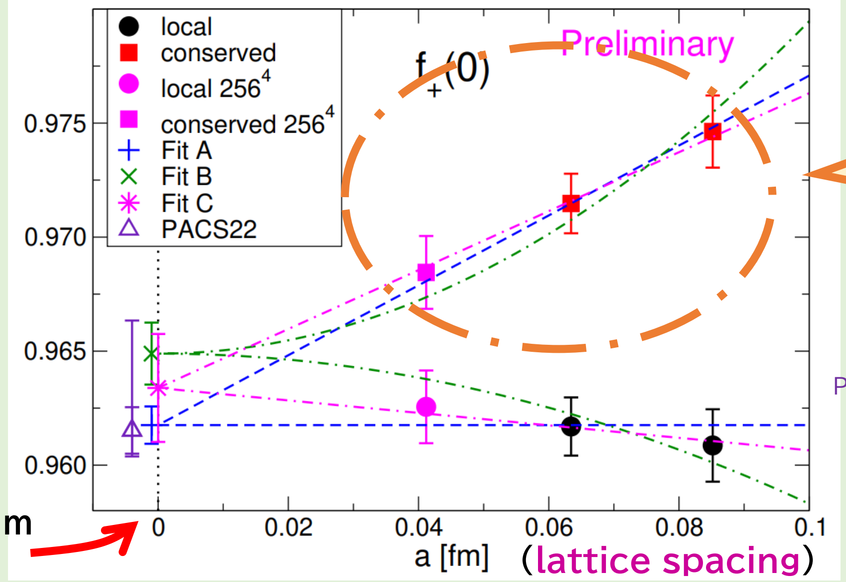

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By the discretization of space-time

Values change depending on the lattice spacing

We need to calculate at multiple lattice spacings and then extrapolate

✓ Continuum extrapolation derives real values from discrete data.

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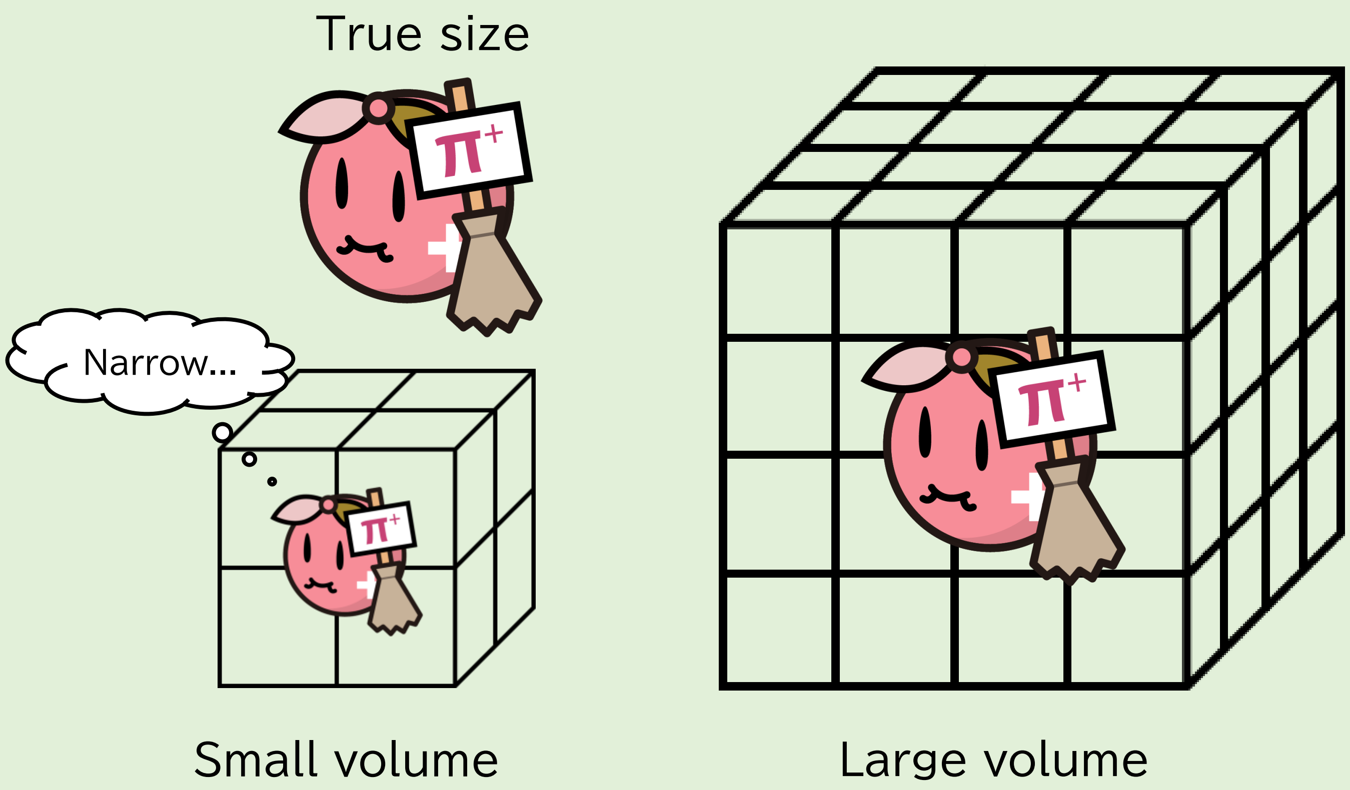
Lattice calculations use finite spacetime volume.



✓ Finite-volume effects from small volume.

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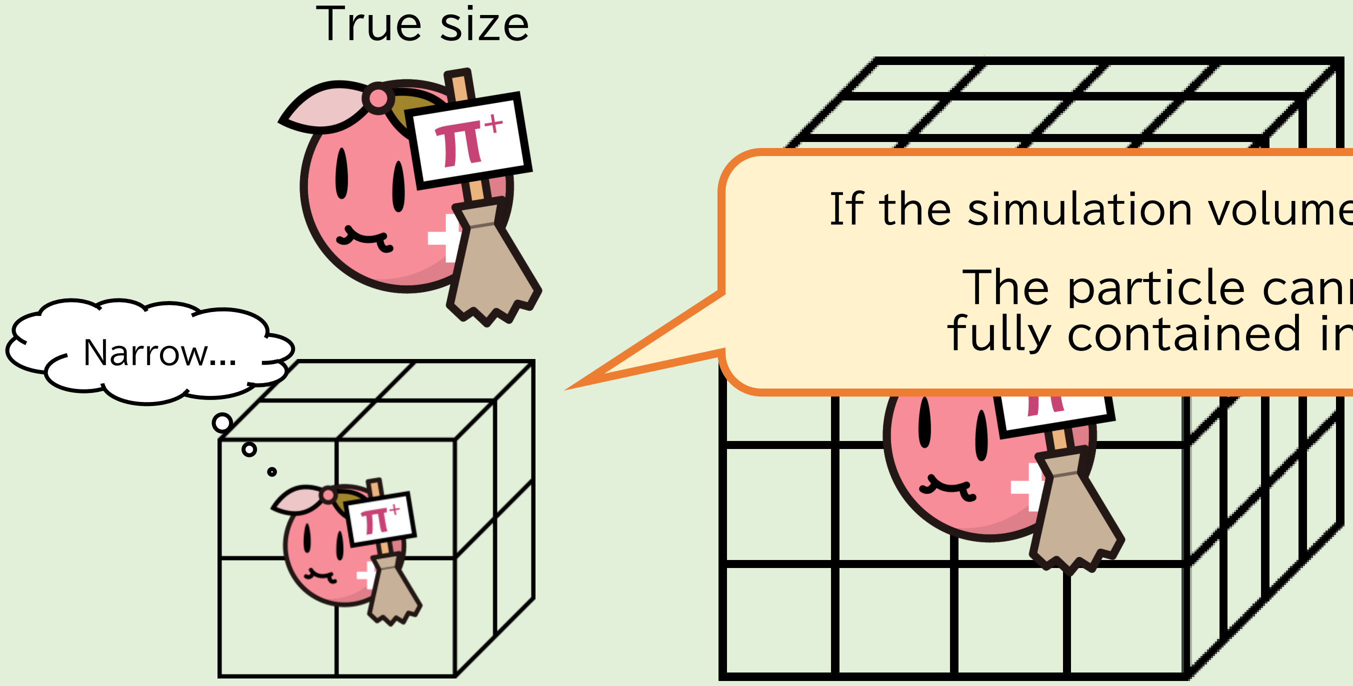
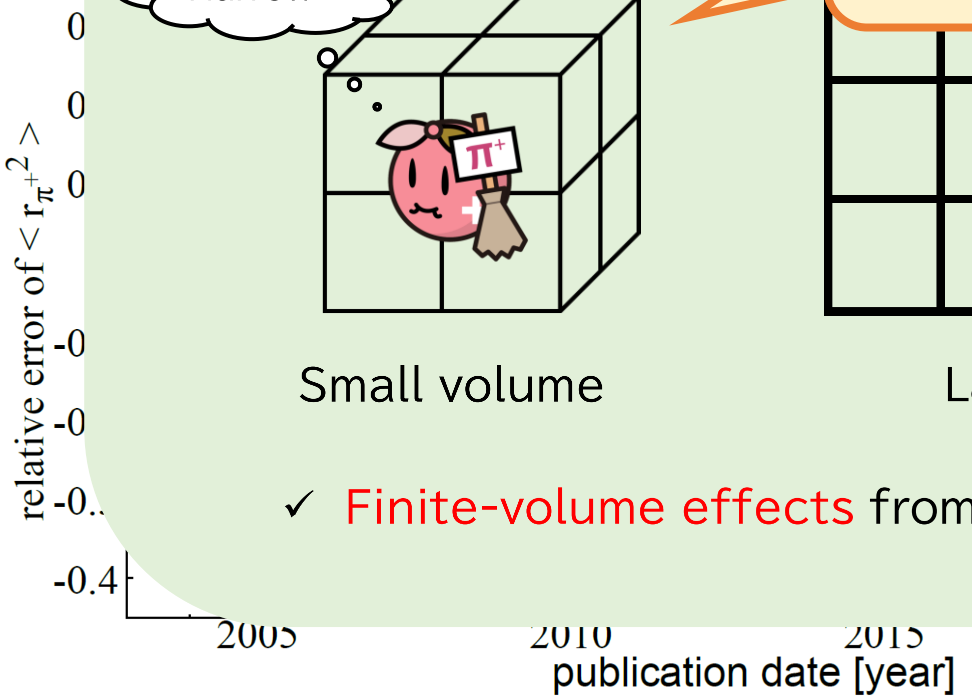


If the simulation volume is too small

The particle cannot be fully contained in space

✓ Finite-volume effects from small volume.



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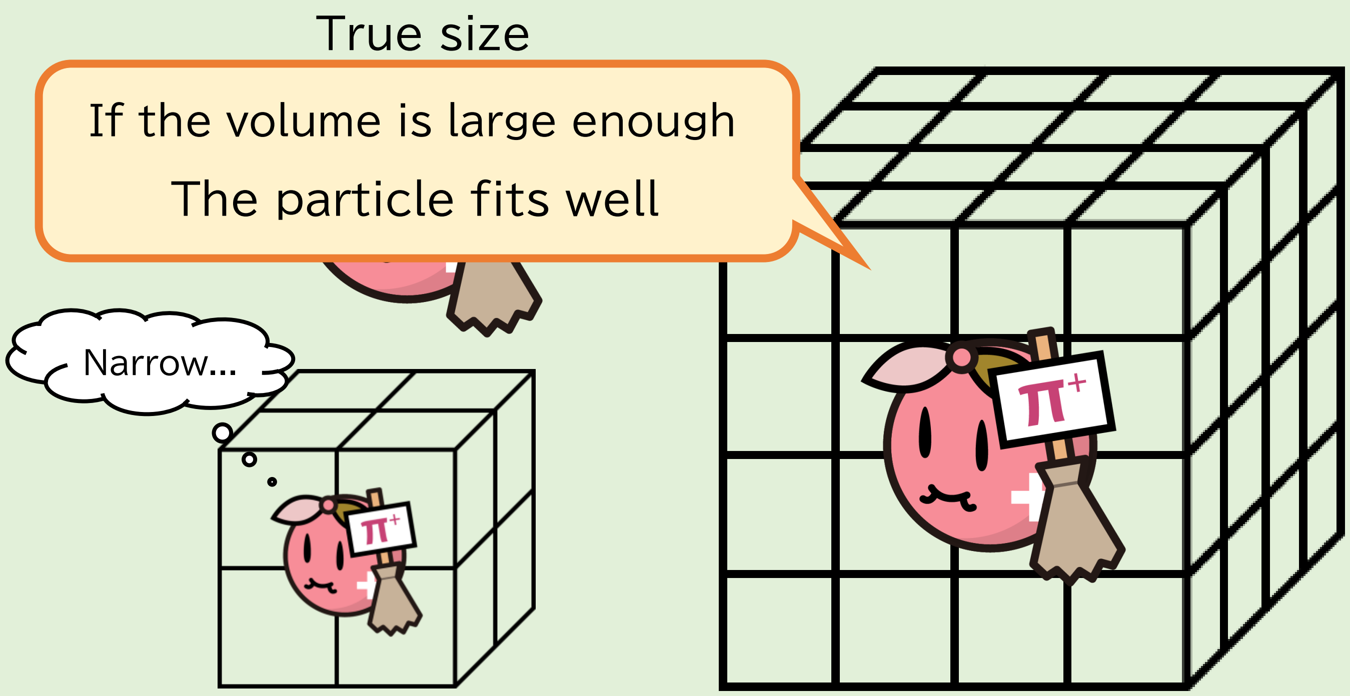

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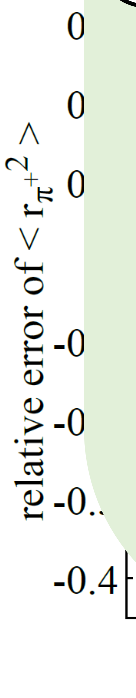

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PACS10 configurations

…Large-volume configurations at physical point (3 lattice spacings).

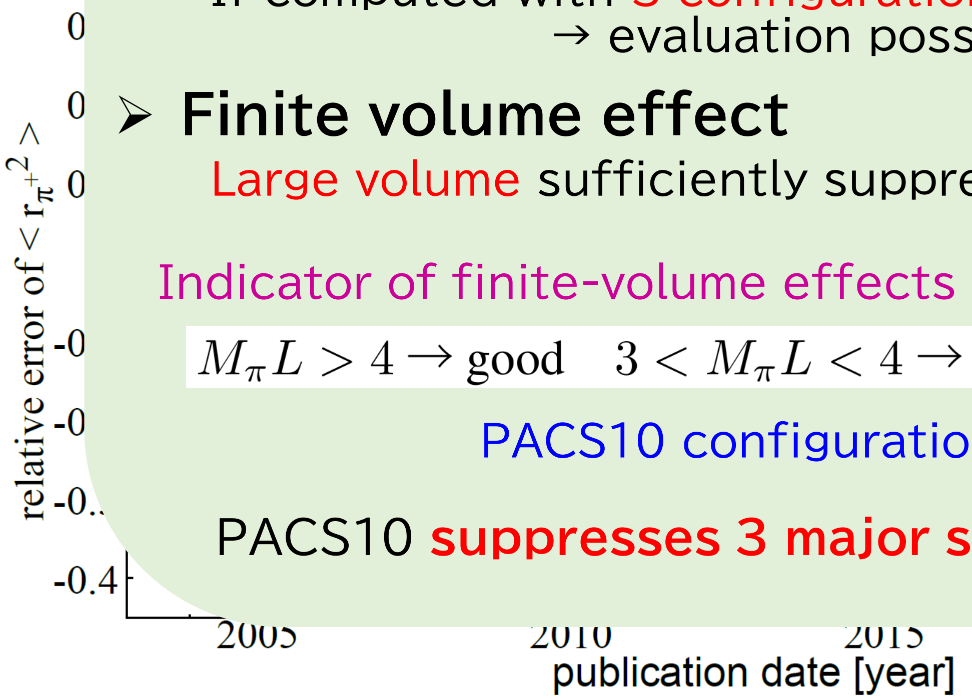
- **Chiral extrapolation**
  At physical point → almost unnecessary → good!!
  Hadron masses reproduced within 5% error
- **Continuum extrapolation**
  If computed with 3 configurations
  → evaluation possible → good!!
- **Finite volume effect**
  Large volume sufficiently suppresses effects → good!!

Indicator of finite-volume effects

$M_\pi L > 4 \to \text{good} \quad 3 < M_\pi L < 4 \to \text{soso} \quad M_\pi L < 3 \to \text{bad}$

PACS10 configurations ~ 7

PACS10 **suppresses 3 major systematic errors.**



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2005, 2010, 2015, 2020

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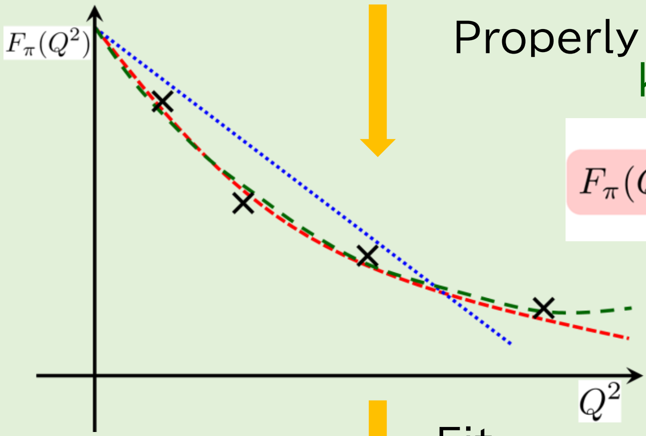
Systematic errors from traditional analysis methods

3-pt functions (lattice QCD)

Known function

$$\frac{\sum_{\vec{x}_{\text{sink}},\vec{x}} \langle \pi^+(\vec{x}_{\text{sink}}, t_{\text{sink}}) V_4(\vec{x},t) \pi^+(0)^\dagger \rangle e^{-ipx_1}}{\sum_{\vec{x}_{\text{sink}},\vec{x}} \langle \pi^+(\vec{x}_{\text{sink}}, t_{\text{sink}}) V_4(\vec{x},t) \pi^+(0)^\dagger \rangle} = \frac{E_\pi(p) + m_\pi}{2E_\pi(p)} e^{-(E_\pi(p)-m_\pi)t} F_\pi(Q^2)$$

Form factors (desired quantity)



Properly combine lattice data and known functions.

$$F_\pi(Q^2) = \frac{\text{Input from lattice calculation}}{\text{Known function}}$$

Fit

$$\langle r_\pi^2 \rangle = -6 \frac{\mathrm{d}}{\mathrm{d}Q^2} F_\pi(Q^2) \Big|_{Q^2=0}$$

Fit ansatz

$$F_\pi(Q^2) = \begin{cases} \dfrac{1}{1+Q^2/M_{\text{pole}}^2} \\ 1 + \sum_k f_k (Q^2)^k \\ \sum_k a_k z^k \quad \left(z := \dfrac{\sqrt{4m_\pi^2 + Q^2} - \sqrt{4m_\pi^2}}{\sqrt{4m_\pi^2 + Q^2} + \sqrt{4m_\pi^2}} \right) \\ 1 + \dfrac{1}{f_\pi^2}\left[2 l_6^r(\mu) Q^2 + 4H(m_\pi^2, Q^2, \mu^2)\right] \end{cases}$$

✓ The fit ansatz error arises from the choice of the fitting function and the fitting range.

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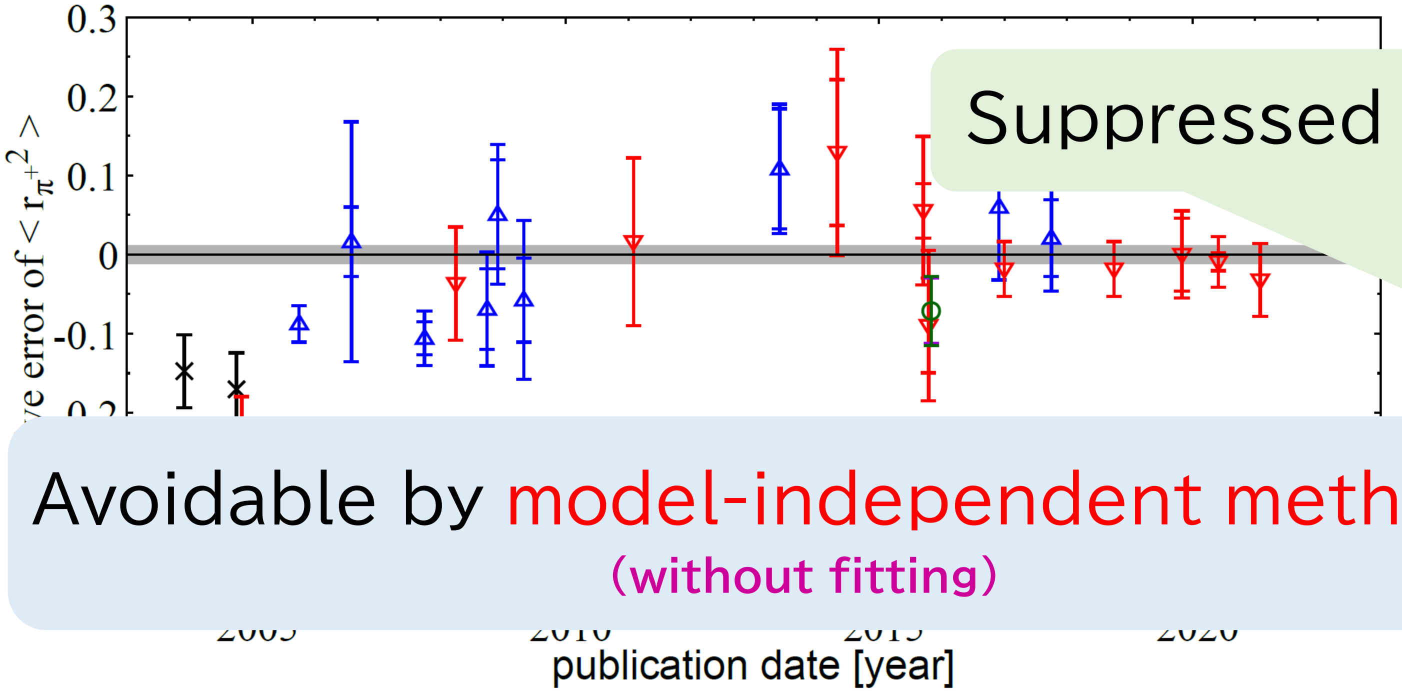

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Suppressed with PACS10 configurations

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Avoidable by model-independent method
(without fitting)

Outline

✓ ~~Introduction~~

- ~~Charge radius~~
- ~~Main systematic errors~~

✓ Overview of model-independent method

- Reducing contamination using spatial moment

✓ Application to PACS10 configurations

- Analysis of π^+, K^+, and K^0 charge radii using model-independent method

✓ Summary

Overview of model-independent method

$F_\pi(Q^2)$

$$(\text{3-point function}) = (\text{Known function}) \times (1 + f_1 Q^2 + f_2 Q^4 + \cdots)$$

Example: difference $\left.\frac{\mathrm{d}}{\mathrm{d}Q^2} F_\pi(Q^2)\right|_{Q^2=0} = \underbrace{\left[\frac{\text{3-point function}}{(\text{Known function})} - 1\right] \Big/ Q^2}_{\text{can be exactly evaluated}} + \underbrace{(\text{Higher-order contamination})}_{\text{cannot be exactly evaluated}}$ (If Q^2 is sufficiently small)

request：

✓ Is there a way to extract the first derivative of the form factor from the three-point function with less contamination?

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✓ Key idea：**Calculate spatial moment**

U. Aglietti et al., Phys.Lett.B324,85(1994); UKQCD, Nucl.Phys.B444,401(1995) ;
C. Bouchard et al., PoS LATTICE2016,170(2016); PACS, Phys.Rev.D104, 074514(2021)

➢ Differentiation of Fourier transform

$$\left.\frac{\mathrm{d}\tilde{C}(p)}{\mathrm{d}p^2}\right|_{p^2=0} = \left.\frac{\mathrm{d}}{\mathrm{d}p^2}\sum_x C(x)e^{-ipx}\right|_{p^2=0} = -\frac{1}{2}\sum_x x^2 C(x)$$

($N_{space} \to \infty$, a :finite ; $L = N_{space}a \to \infty$)

infinite volume limit

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- Differentiation of Fourier transform

$$\left.\frac{\mathrm{d}\tilde{C}(p)}{\mathrm{d}p^2}\right|_{p^2=0} = \left.\frac{\mathrm{d}}{\mathrm{d}p^2}\sum_x C(x) e^{-ipx}\right|_{p^2=0} = -\frac{1}{2}\sum_x x^2 C(x)$$

($N_{space} \to \infty$, a :finite ; $L = N_{space} a \to \infty$)

infinite volume limit

We can suppress the contamination at large volume.

$$\sum_x x^2 C_{3pt}(x,t) \sim f_1 + (f_2 + f_3 + \cdots)$$

Overview of model-independent method

$$F_\pi(Q^2)$$
$$(\text{3-point function}) = (\text{Known function}) \times (1 + f_1 Q^2 + f_2 Q^4 + \cdots)$$

Example: difference $\left.\frac{\mathrm{d}}{\mathrm{d}Q^2} F_\pi(Q^2)\right|_{Q^2=0} = \underbrace{\left[\frac{\text{3-point function}}{\text{(Known function)}} - 1\right] \Big/ Q^2}_{\text{can be exactly evaluated}} + \underbrace{(\text{Higher-order contamination})}_{\text{cannot be exactly evaluated}}$ (If Q^2 is sufficiently small)

request：

✓ Is there a way to extract the first derivative of the form factor from the three-point function with less contamination?

✓ Key idea：**Calculate spatial moment**

U. Aglietti et al., Phys.Lett.B324,85(1994); UKQCD, Nucl.Phys.B444,401(1995) ;
C. Bouchard et al., PoS LATTICE2016,170(2016); PACS, Phys.Rev.D104, 074514(2021)

➢ Differentiation of Fourier transform

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($N_{space} \to \infty$, a :finite ; $L = N_{space}a \to \infty$)

infinite volume limit

We can suppress the contamination at large volume.

$$\sum_x x^2 C_{3pt}(x,t) \sim f_1 + (f_2 + f_3 + \cdots)$$

✓ Method：**Combining x^2 and x^4 moments**

Xu Feng et al., Phys.Rev.D101,051502(R)(2020)

$$\sum_x x^4 C_{3pt}(x,t) \sim f_1 + (f_2 + f_3 + \cdots)$$

We can cleverly add these higher-order moments to reduce the contamination.

$$R(t) = \alpha_1 (x^2\text{moment}) + \alpha_2 (x^4\text{moment})$$
$$= \left.\frac{\mathrm{d}}{\mathrm{d}Q^2} F_\pi(Q^2)\right|_{Q^2=0} + (f_3 + f_4 + \cdots)$$

α_1, α_2: parameters which set to cancel out the contamination

Our Improvement : K. S. et al., PoS LAT22,122(2022) ; PoS LAT23,312(2023)
-> Effective at small lattice size

Simulation parameters

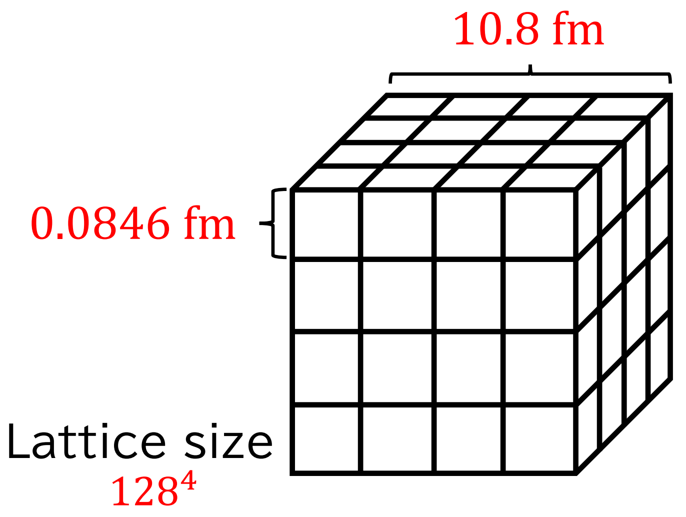
✓ Gauge configuration (PACS, PRD 99, 014504 (2019); PACS, PRD 106, 094505 (2022); PACS, PRD 109, 094505 (2024))

PACS10 configuration

$N_f = 2+1$ six-stout-smeared non-perturbative $O(a)$-improved Wilson action+ Iwasaki gauge action

β	$L^3 \cdot T$	L[fm]	a[fm]	a^{-1}[GeV]	m_π[MeV]	m_K[MeV]	$N_{\rm conf}$
2.20	256^4	10.5	0.041	4.792	142	514	20
2.00	160^4	10.2	0.063	3.111	137	501	20
1.82	128^4	10.9	0.085	2.316	135	497	20

All preliminary results are obtained on the 128 and 160 lattice



- Chiral extrapolation
  → Physical point
- Continuum extrapolation
  → 3 lattice spacings
- Finite volume effects
  → Large volume

✓ Measurement parameter

- 20 config. , 2304~13824 meas. Per config.
- Periodic + anti-periodic correlation functions in time direction
- Forward + Backward correlation functions in time direction
- $|t_{sink} - t_{source}| = 36, 42, 48$(L128); $50, 58, 64$(L160)

→ ↑ Statistics
Reduce the wrapping-around effect
Systematic errors can be evaluated

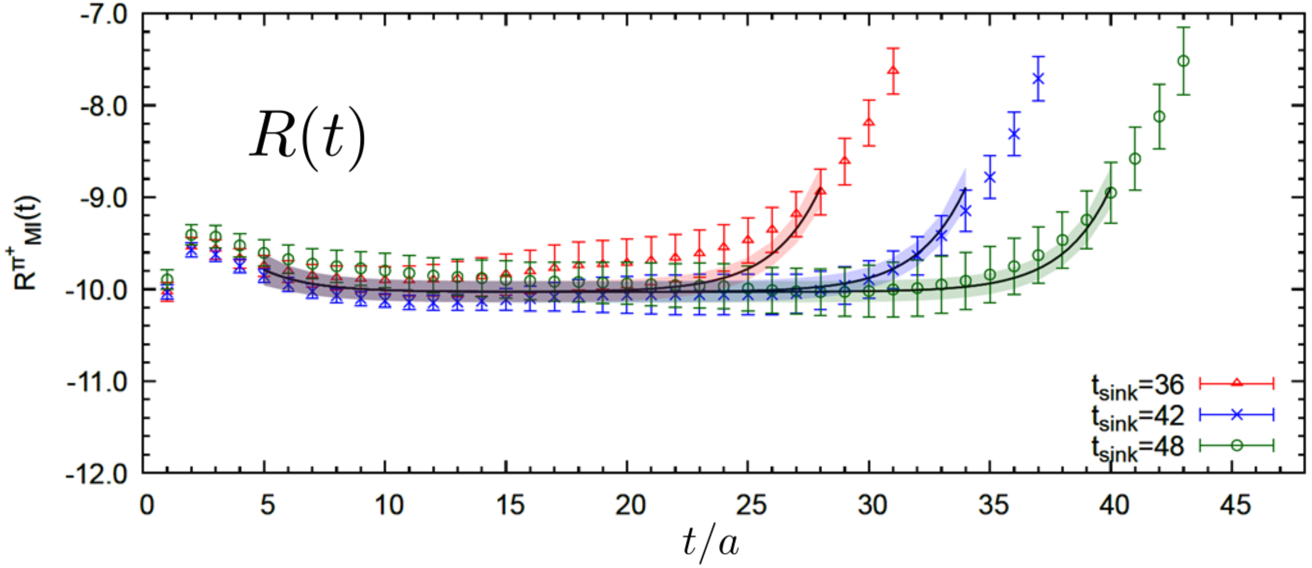
Result

✓ Model-independent method
(without fit ansatz)

$$\boxed{\sum_x x^2 C_{3pt}(x,t) \sim f_1 + (f_2 + f_3 + \cdots)}$$

$$\boxed{\sum_x x^4 C_{3pt}(x,t) \sim f_1 + (f_2 + f_3 + \cdots)}$$

$$R(t) = \alpha_1\,(x^2\text{moment}) + \alpha_2\,(x^4\text{moment})$$
$$= \frac{\mathrm{d}}{\mathrm{d}Q^2}F_\pi(Q^2)\Big|_{Q^2=0} + (f_3 + f_4 + \cdots)$$



Result

✓ Model-independent method (without fit ansatz)

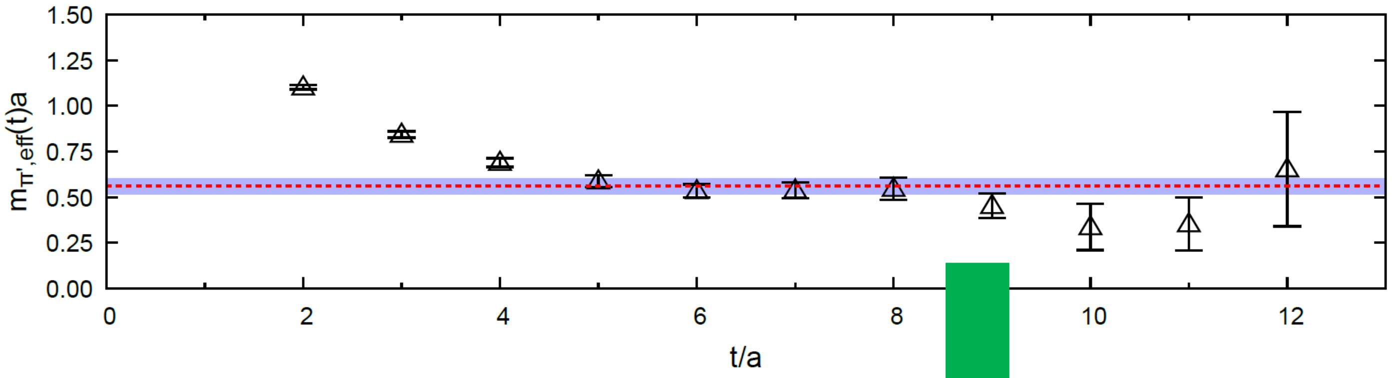
$$\sum_x x^2 C_{3pt}(x,t) \sim f_1 + (f_2 + f_3 + \cdots)$$

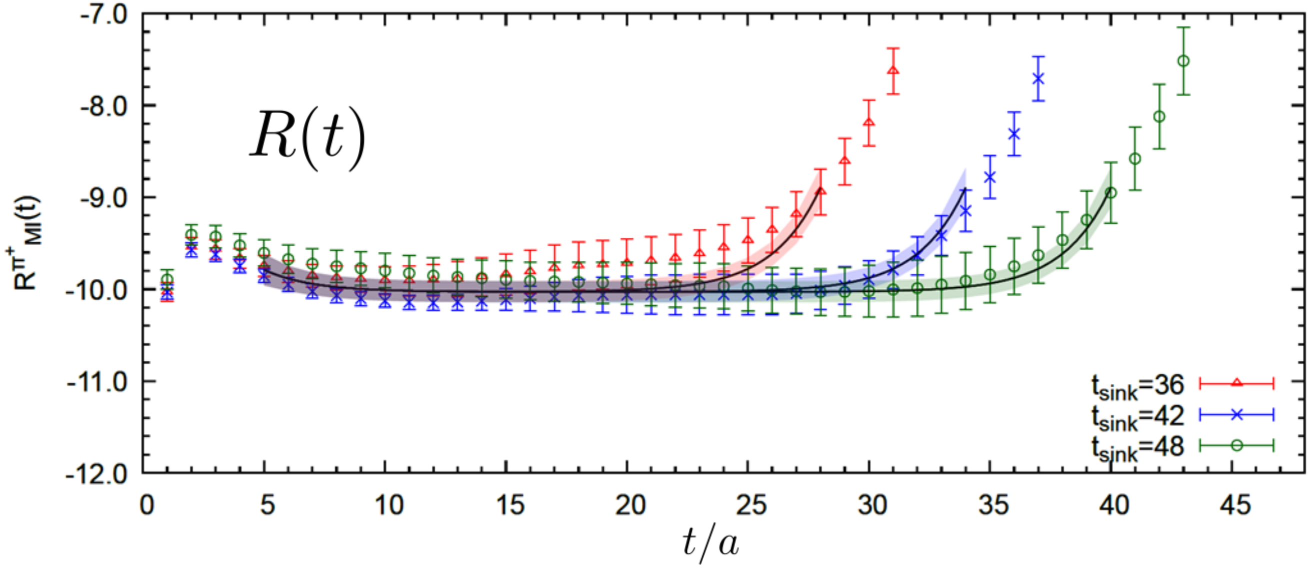
$$R(t) = \alpha_1 \ (x^2\text{moment}) + \alpha_2 \ (x^4\text{m}\ \ \text{men}$$

$$= \frac{\mathrm{d}}{\mathrm{d}Q^2} F_\pi(Q^2)\Big|_{Q^2=0} + (f_3 + f_4 + \cdots)$$

✓ Extraction including the first excited state

$$f(t; t_{\text{sink}}) = \frac{\mathrm{d}}{\mathrm{d}Q^2} F_{\pi^+}(Q^2)\Big|_{Q^2=0} + A\, e^{-(m_{X'} - m_X)t} + B\, e^{-(m_{X'} - m_X)(t_{\text{sink}} - t)}$$





✓ From the plateau, we can see that the PACS10 configurations reproduce the experimental mass of the first excited state.

✓ We use experimental value as the mass of the first excited state

Result

✓ Model-independent method (without fit ansatz)

- Central value : result using all source–sink separations
- Statistic error : Jackknife error of the central value
- Systematic error : Maximum difference between the central value and each separation

$$\sum_x x^2 C_{3pt}(x,t) \sim f_1 + (f_2 + f_3 + \cdots)$$

$$\sum_x x^4 C_{3pt}(x,t) \sim f_1 + (f_2 + f_3 + \cdots)$$

$$R(t) = \alpha_1 (x^2\text{moment}) + \alpha_2 (x^4\text{moment})$$
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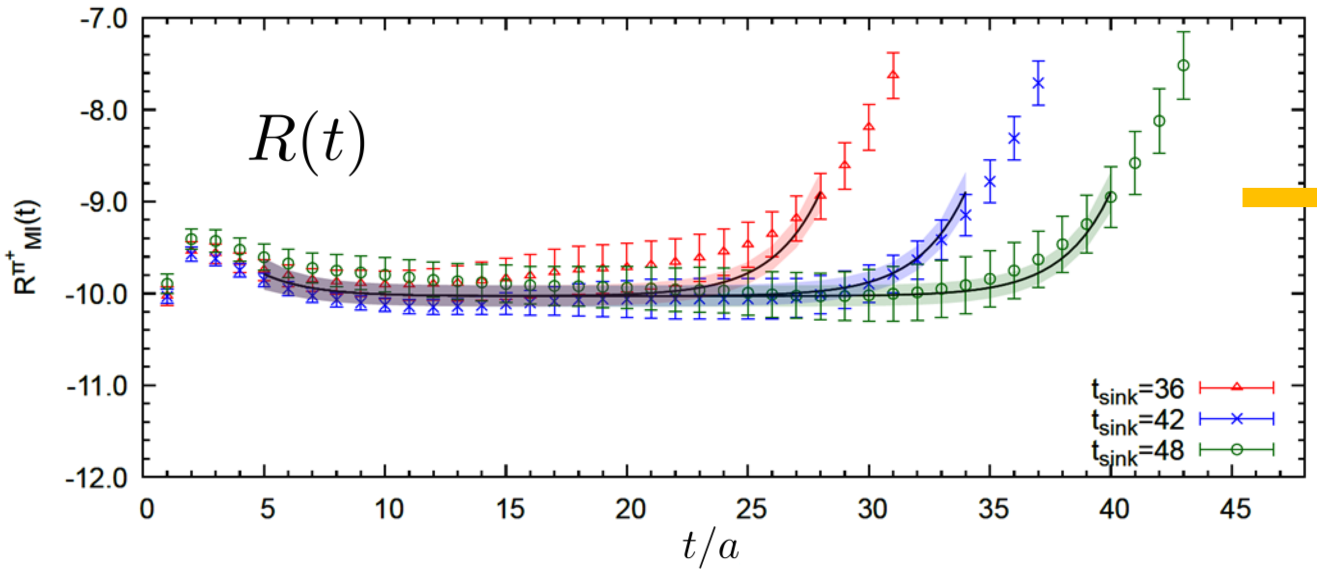

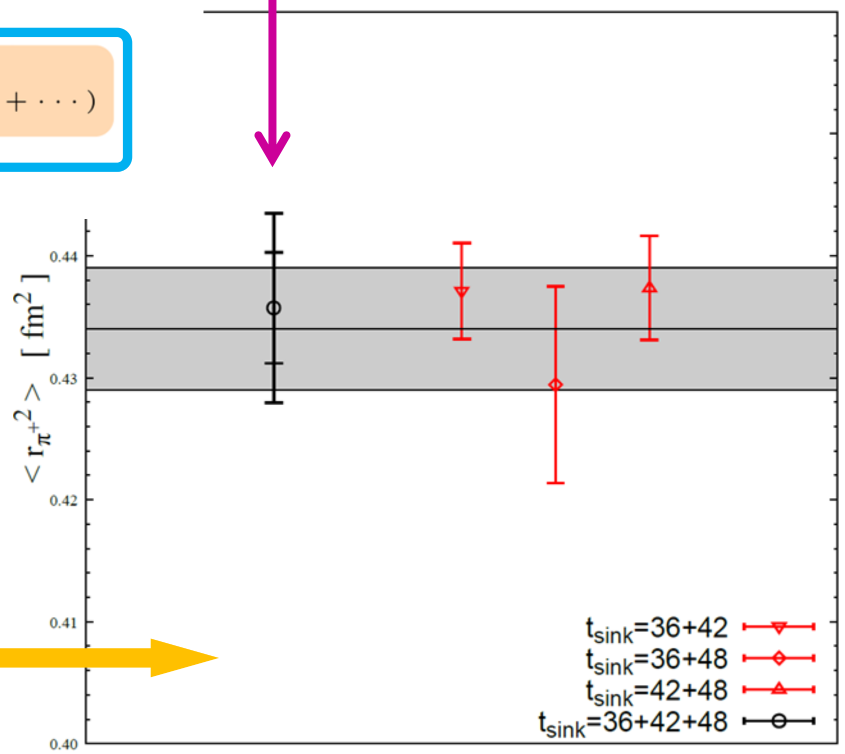

Result

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- Central value : result using all source–sink separations
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$$\sum_x x^4 C_{3pt}(x,t) \sim f_1 + (f_2 + f_3 + \cdots)$$

$$R(t) = \alpha_1 \,(x^2\text{moment}) + \alpha_2\,(x^4\text{moment})$$

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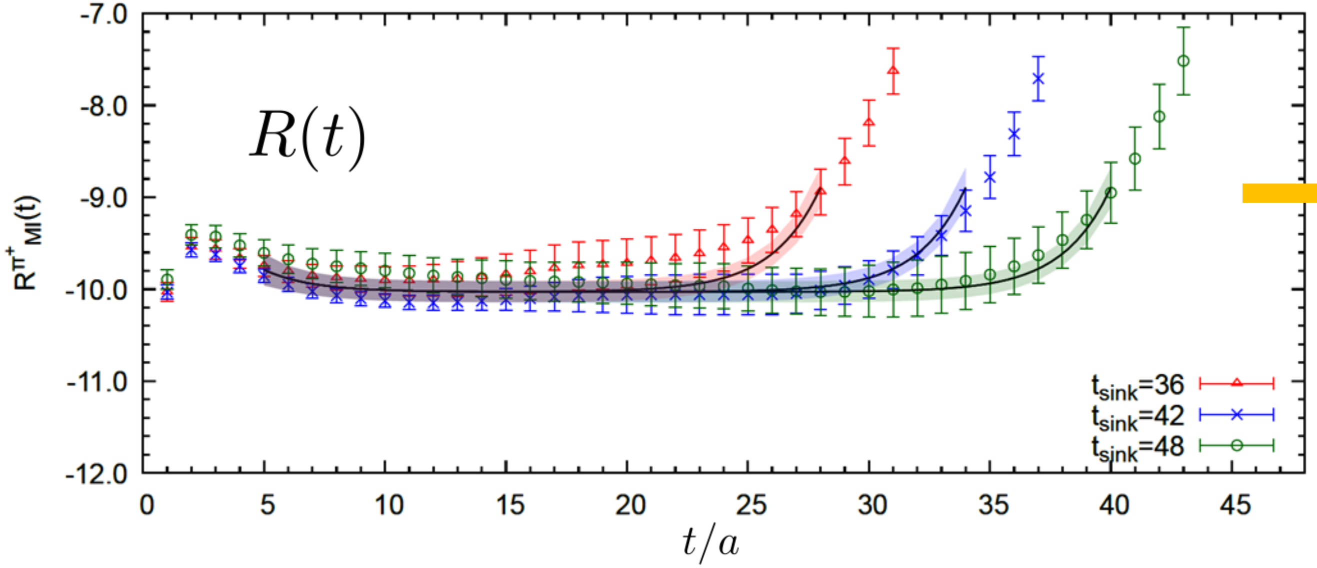
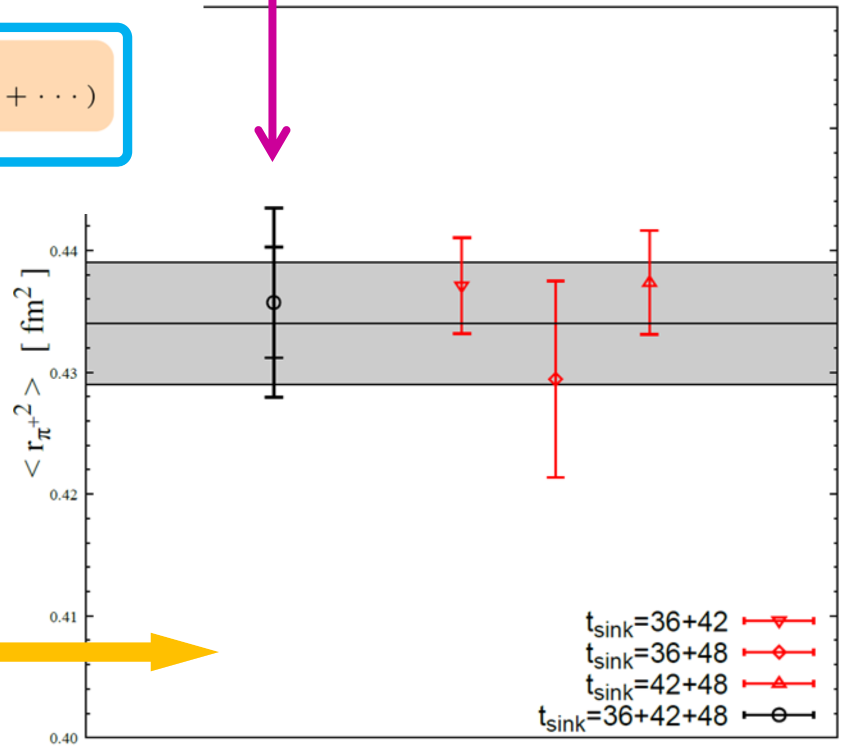
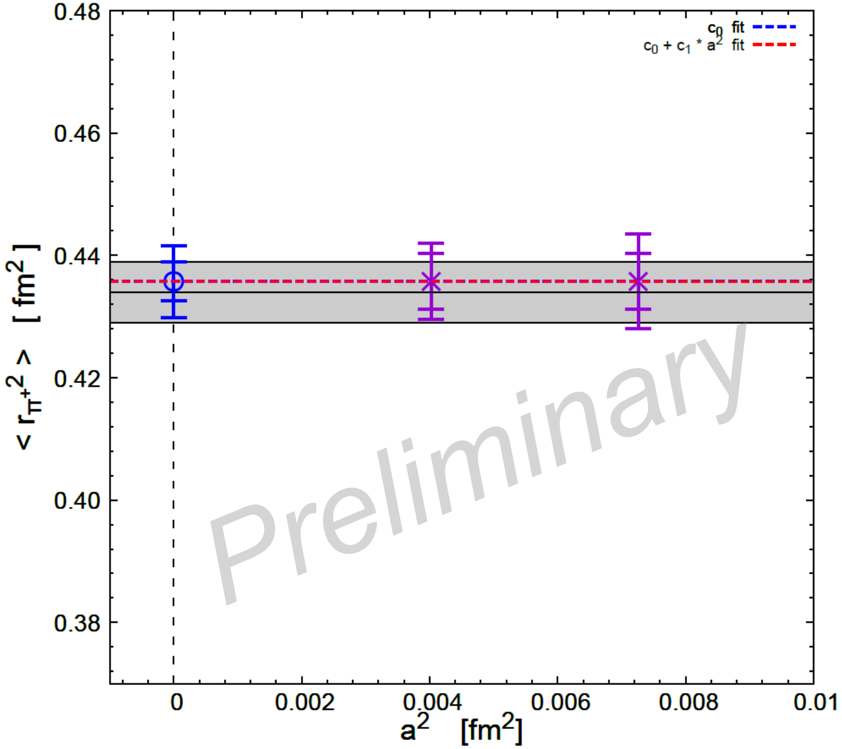




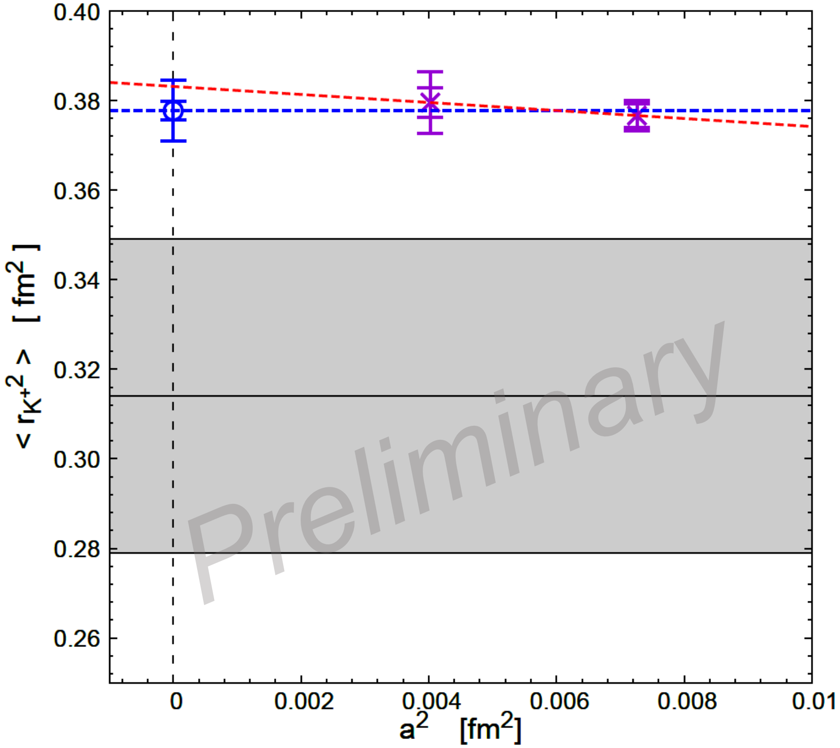
✓ Also perform the same calculation for K mesons and L160 configuration.

Preliminary results

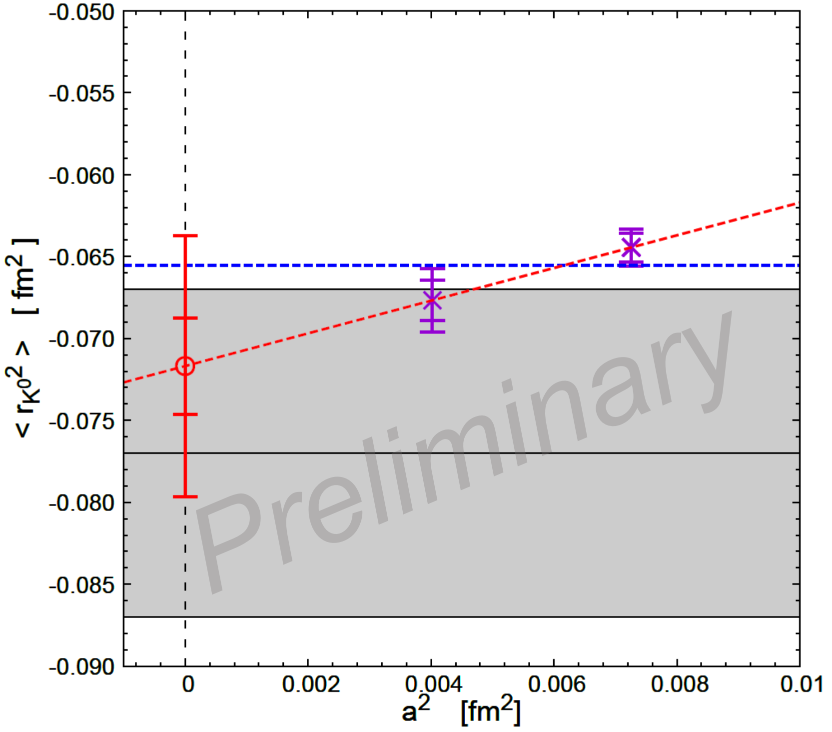
✓ π^+ charge radius



✓ K^+ charge radius



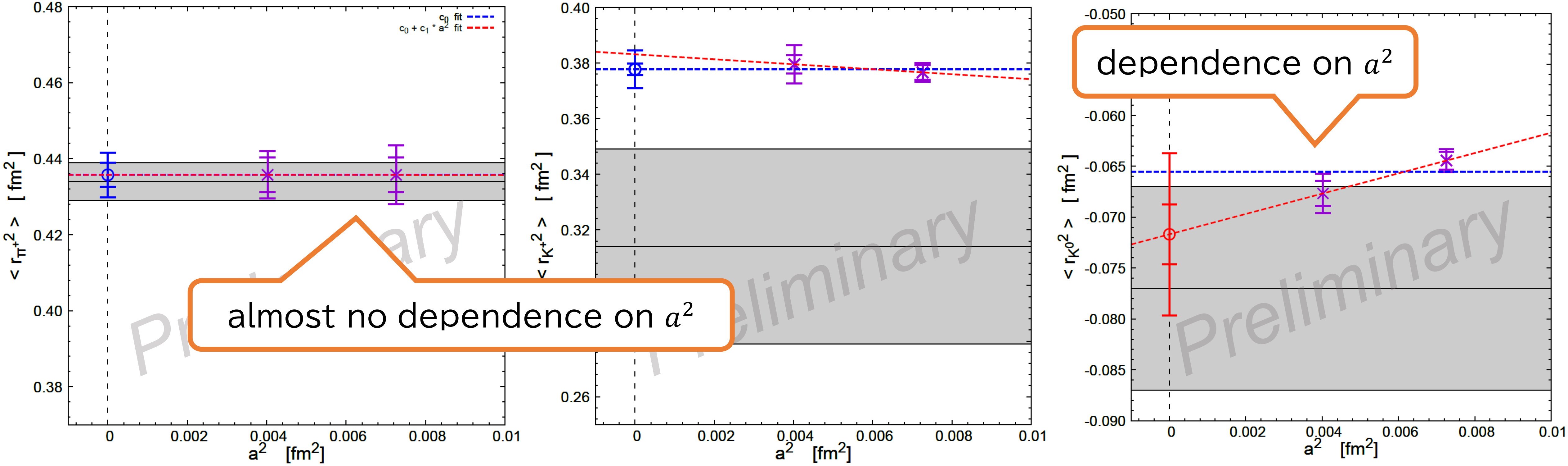
✓ K^0 charge radius



- ✓ As a preliminary extrapolation, we perform both a constant and a linear fit in a^2.
- ✓ We take the constant extrapolation as the central value for π^+ and K^+, and the linear extrapolation in a^2 as the central value for K^0.
- ✓ For the continuum limit values, results agree with PDG within $\sim 2\sigma$.
- ✓ π^+ and K^0 achieve the same level of experimental error, while K⁺ achieves precision exceeding the experiment.

Preliminary results

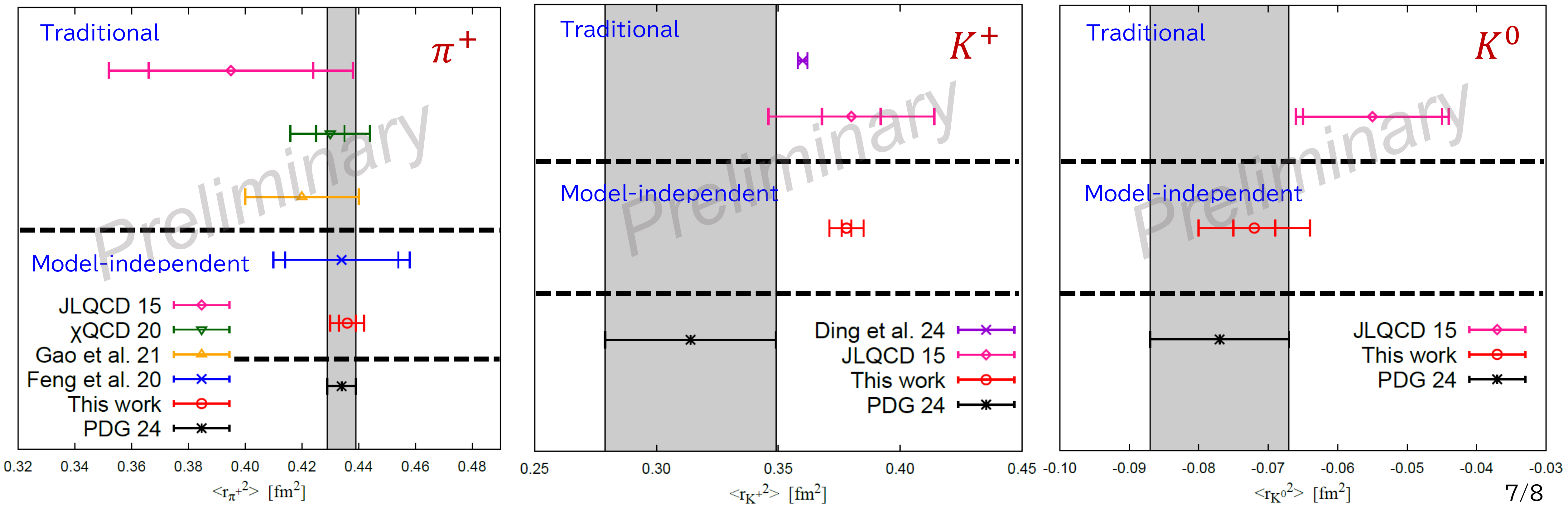
✓ π^+ charge radius ✓ K^+ charge radius ✓ K^0 charge radius



- ✓ As a preliminary extrapolation, we perform both a constant and a linear fit in a^2.
- ✓ We take the constant extrapolation as the central value for π^+ and K^+, and the linear extrapolation in a^2 as the central value for K^0.
- ✓ For the continuum limit values, results agree with PDG within $\sim 2\sigma$.
- ✓ π^+ and K^0 achieve the same level of experimental error, while K⁺ achieves precision exceeding the experiment.

Summary

- ✓ Calculated charge radii of π^+, K^+, and K^0 on PACS10/L128 and L160 configurations.
- ✓ Used the model-independent method.
- ✓ Although preliminary, the results are consistent with the experimental value(PDG) and the results of the previous lattice calculations.
- ✓ Some of our results exceed their precision.

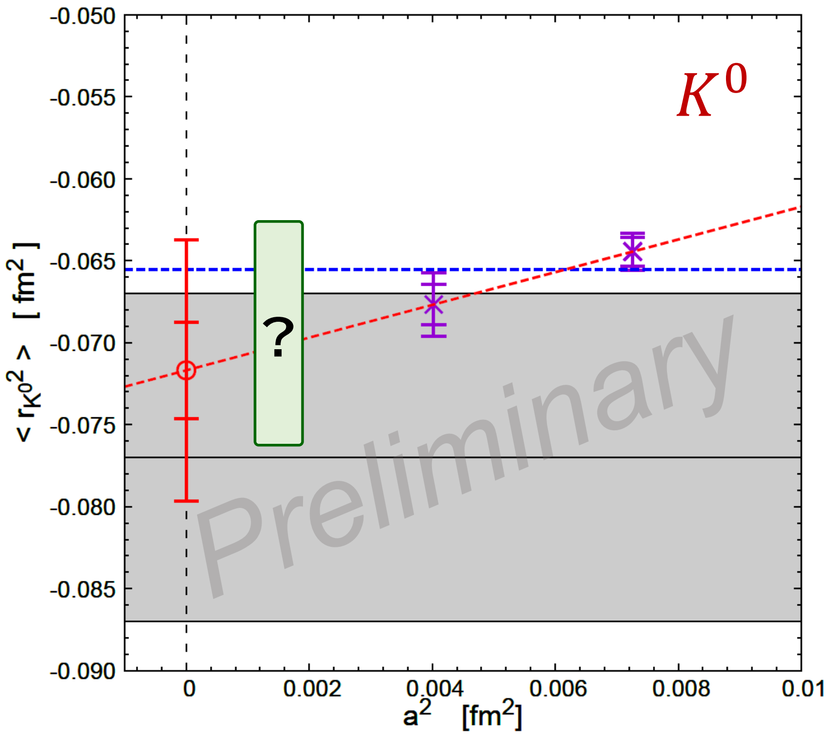


Summary

Future works :

- ✓ Increase statistics for PACS/L160
- ✓ Analyze data using conserved currents
- ✓ Perform calculations on PACS/L256

=> The precision of our continuous extrapolation will improve.



Expectations for the Experiment:

The experimental error for K mesons is large.

- ✓ An experiment called AMBER is currently being conducted at CERN.
- ✓ Phase 2 plans to conduct a high-precision measurement of the K meson's charge radius.

We are looking forward to seeing the results of these experiments.