

Towards continuum limit of Meson Charge Radii using large volume configuration at physical point in $N_f=2+1$ lattice QCD

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Takeshi Yamazaki (University of Tsukuba)

for PACS Collaboration



Target: π^+ , K^+ , K^0 charge radii

Towards continuum limit of **Meson Charge Radii**
using **large volume configuration**
at physical point in $N_f=2+1$ lattice QCD

Data characteristics: **Large volume** and **physical point**
+
Analysis Method : **without fit ansatz**

for PACS Collaboration



Introduction

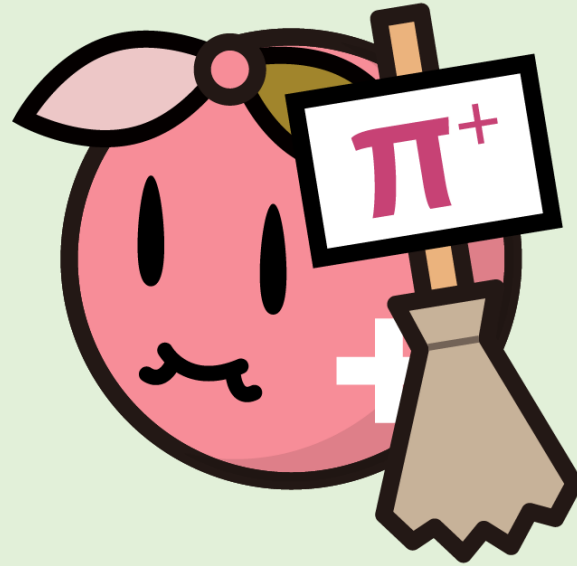
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It represents the spread of the charge distribution.

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Rough sketch

π^+ meson

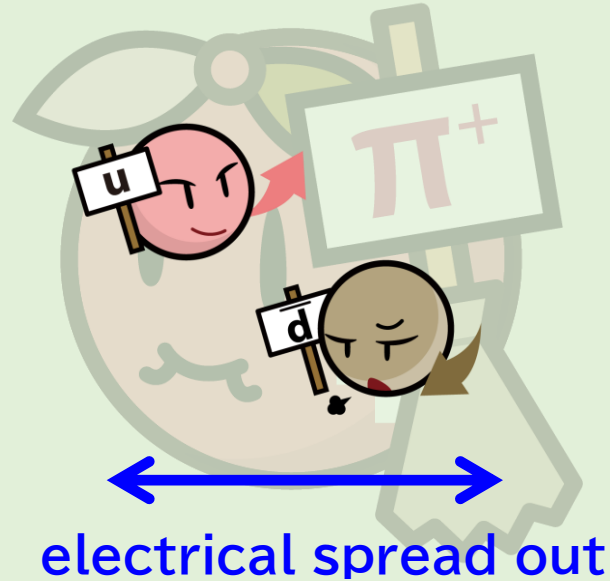


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(mean-square) charge radius ... a quantity that characterizes the structure of hadrons.
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$$\langle r_\pi^2 \rangle = -6 \frac{d}{dQ^2} \underbrace{F_\pi(Q^2)}_{\text{electromagnetic form factor}} \Big|_{Q^2=0}$$

$$\left[\begin{array}{ll} \langle \pi^+(p_f) | V_\mu | \pi^+(p_i) \rangle = (p_f + p_i)_\mu F_\pi(Q^2) \\ \text{electromagnetic} & \text{Momentum} \\ \text{current:} & \text{transfer :} \\ V_\mu = \sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f & Q^2 = -(p_f - p_i)^2 \geq 0 \end{array} \right]$$

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3pt function
(input from lattice QCD)

form factor(output)

(mean-square) charge radius

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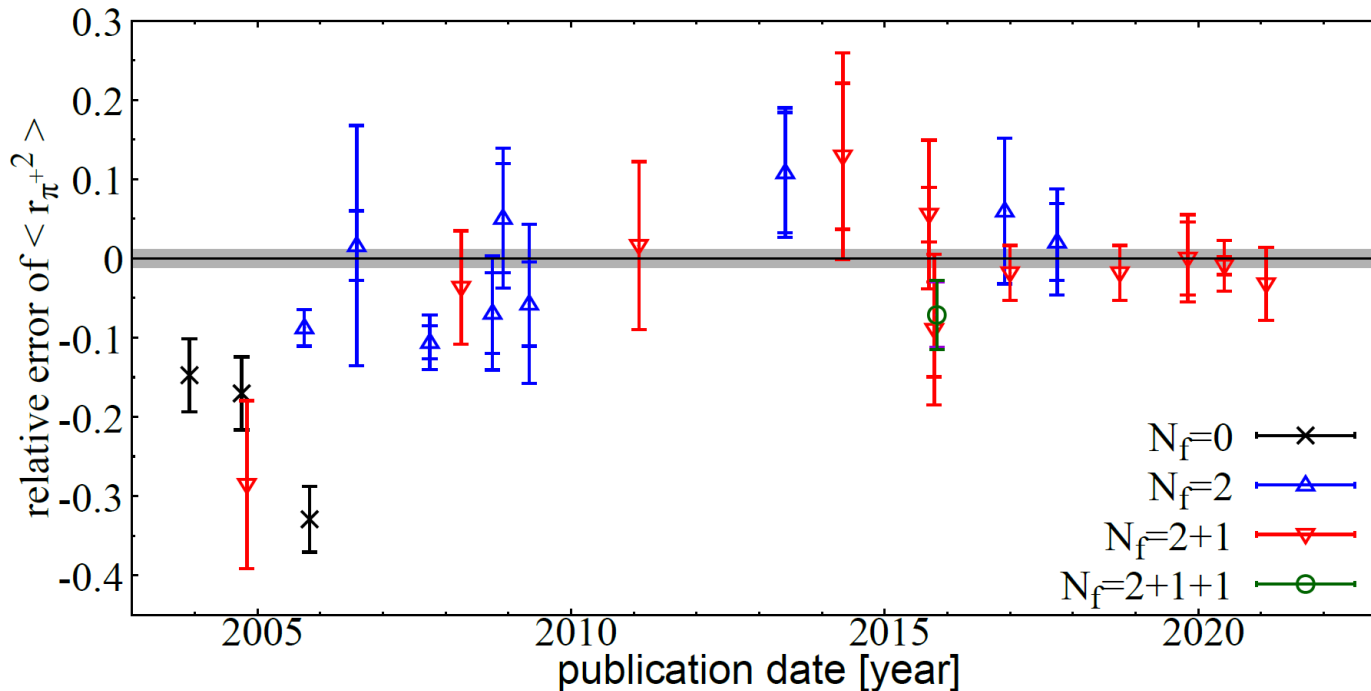
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Recently : consistent
- ✓ Error : Lattice > Experimental
- There are 4 main systematic errors
 - Chiral extrapolation
 - Continuum extrapolation
 - Finite volume effect
 - Fit ansatz

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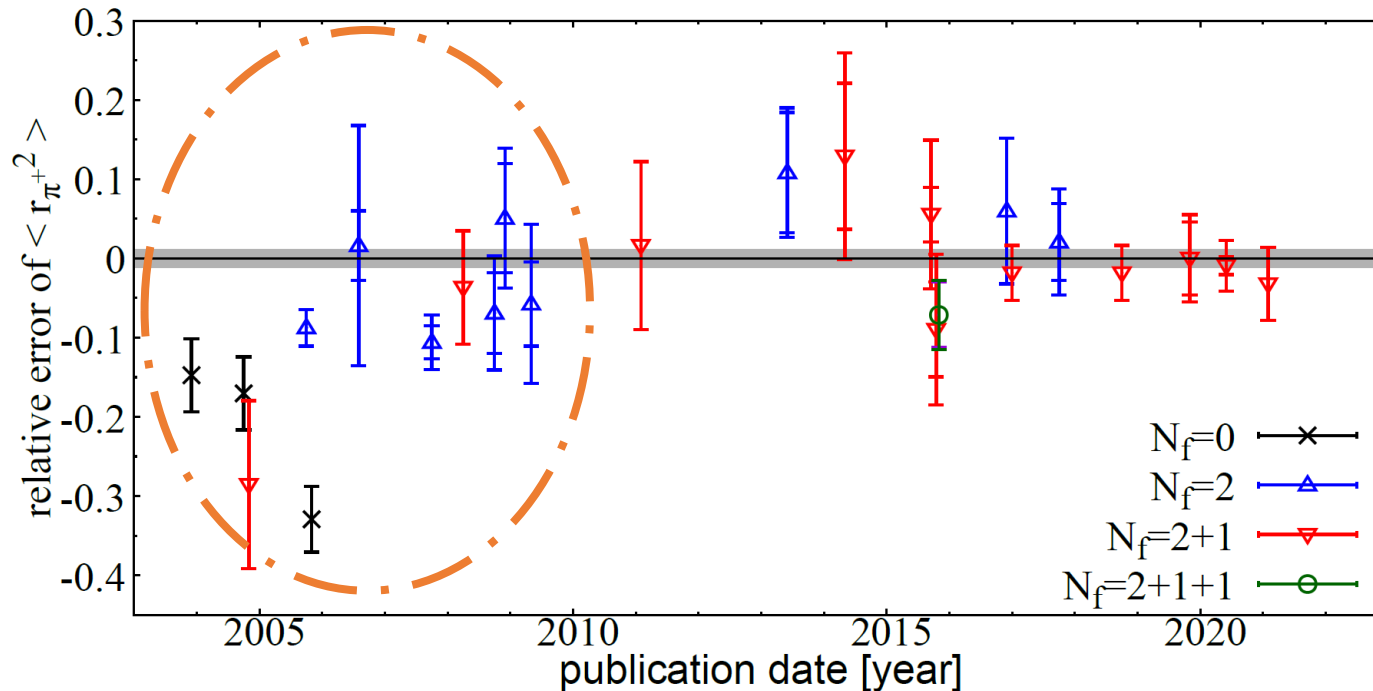
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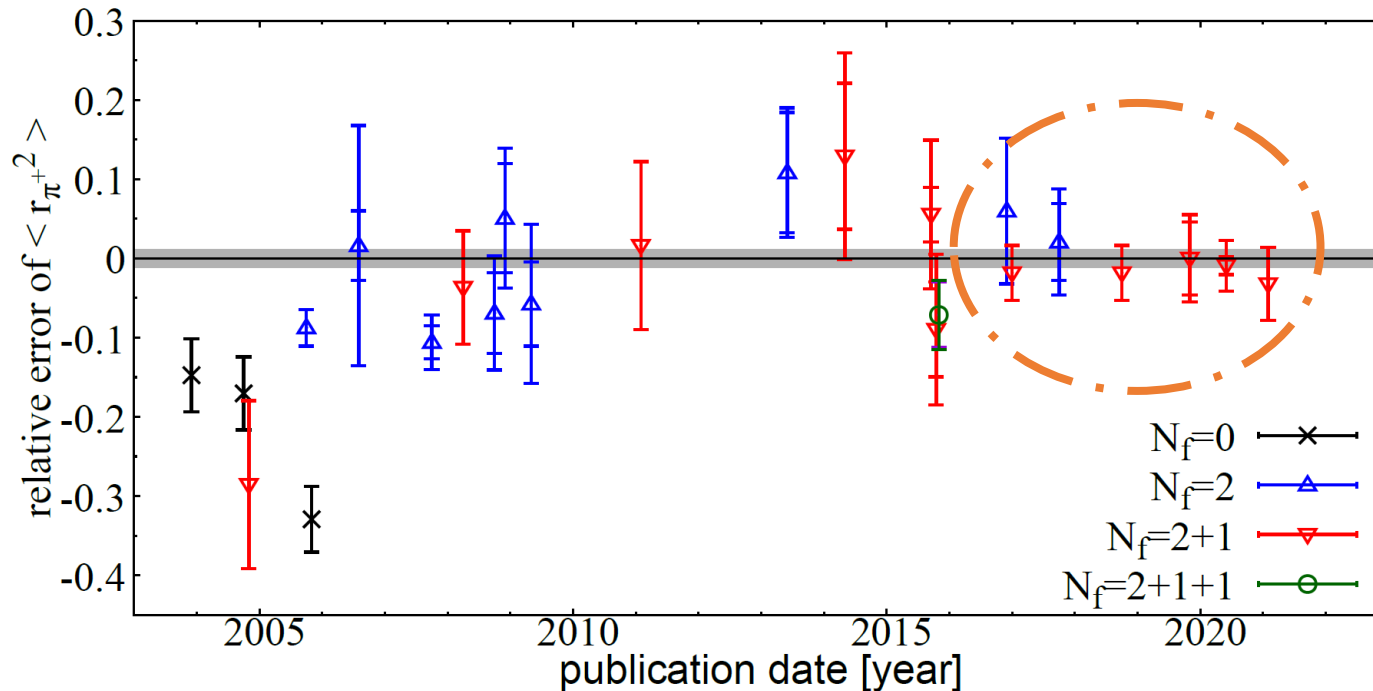
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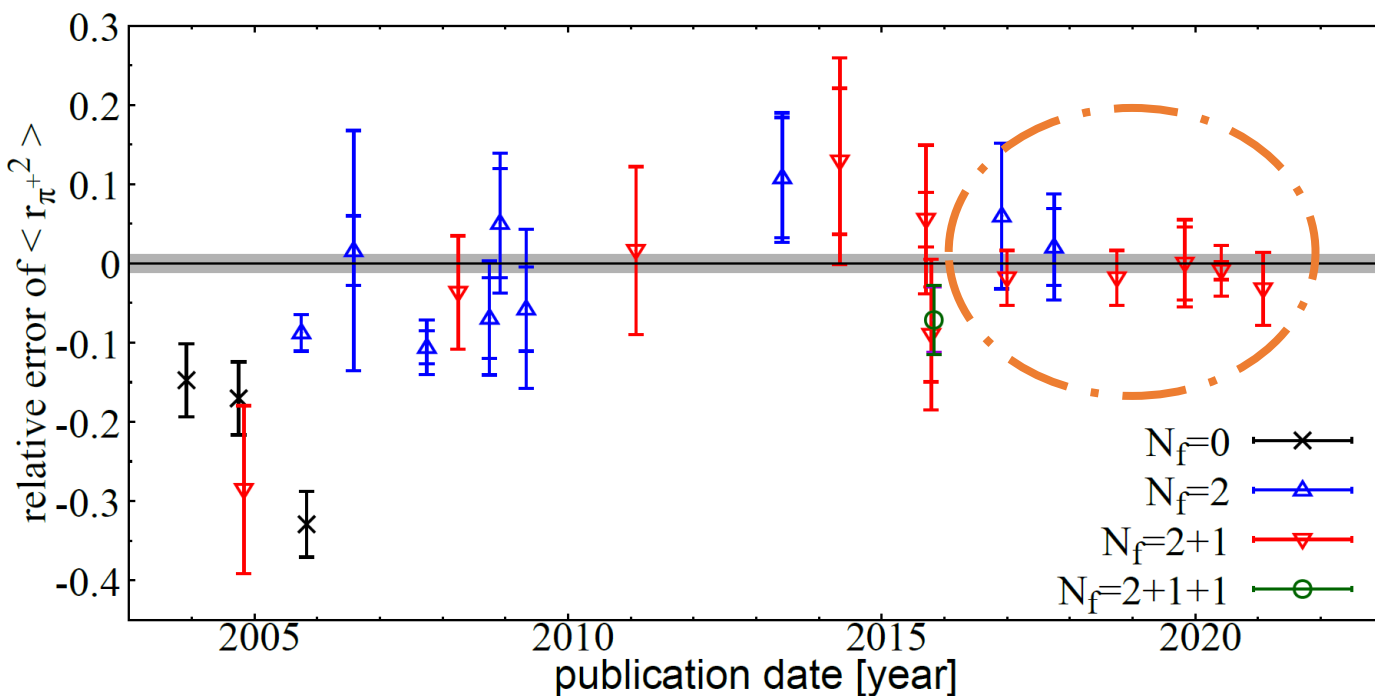
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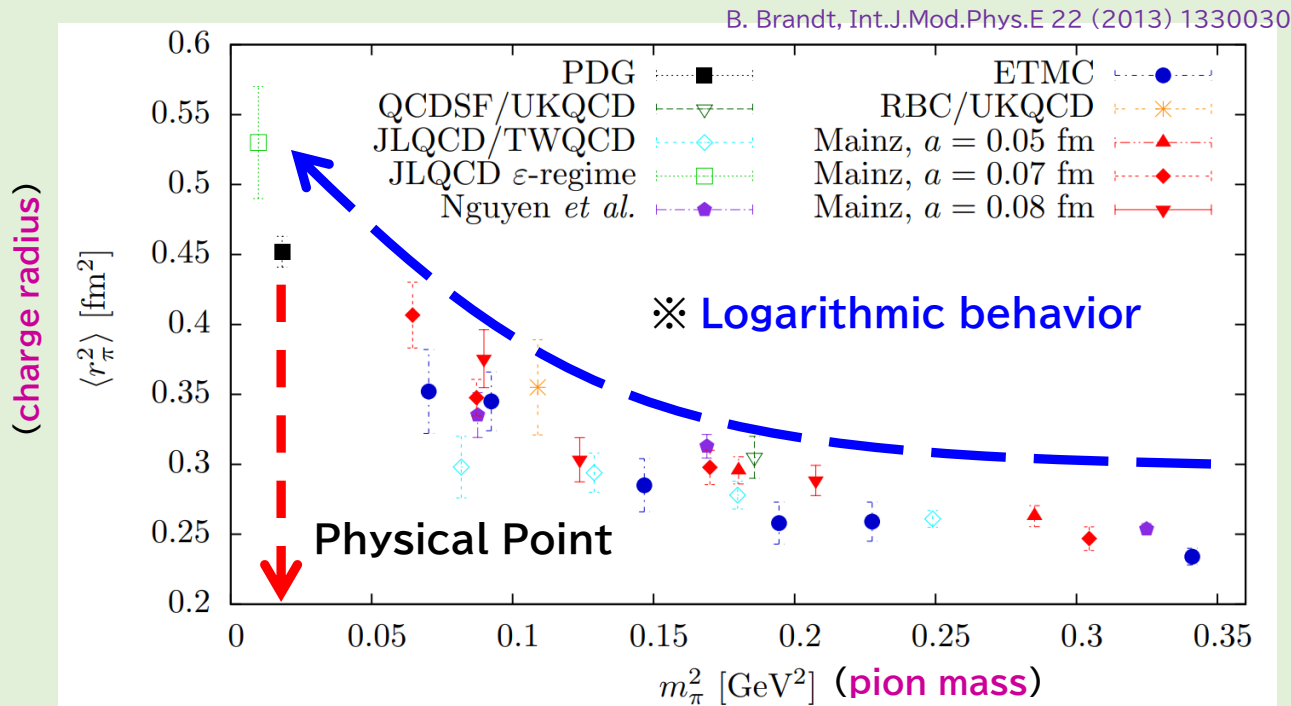
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(m) By **changing theoretical parameters**, we can compute **physics in various worlds**.



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- ✓ To reproduce the real world, we must use the **parameters that correspond to our physical universe**.
- ✓ Use **chiral extrapolation** to obtain values at the physical point.

2005 2010 2015 2020
publication date [year]

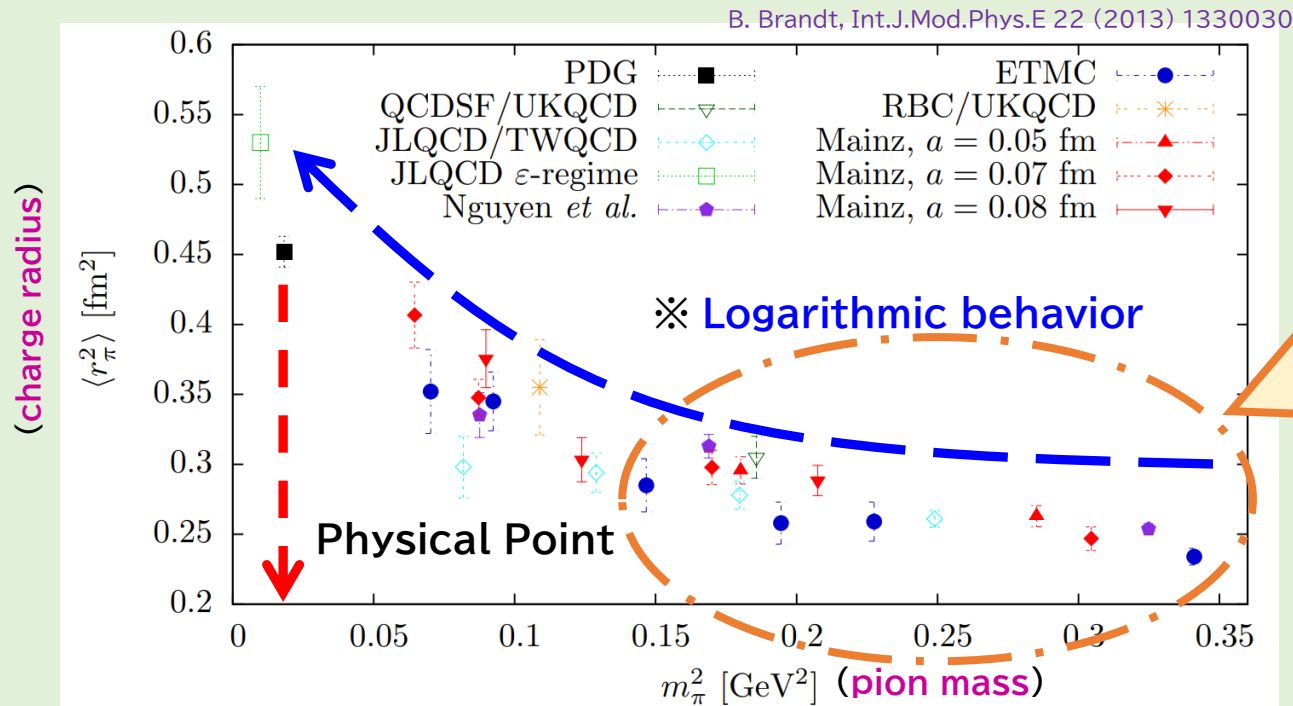
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characterizes the structure of hadrons.



Earlier lattice QCD calculations

Many simulations were performed at **heavier pion masses**



We needed to extrapolate these results to the physical point using **chiral extrapolation**

recently : consistent

or : **Lattice > Experimental**

are 4 main systematic errors

➤ **Chiral extrapolation**

➤ Continuum extrapolation

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2010

2015

2020

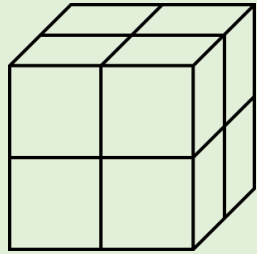
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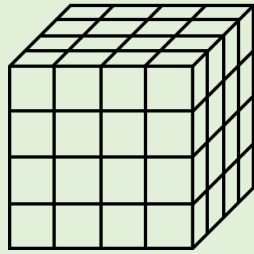
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Lattice calculations discretize spacetime.

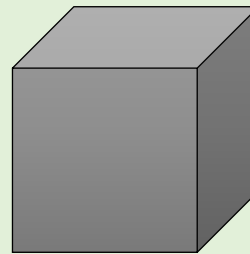


Coarse lattice



Fine lattice

...



Continuum
(real world)

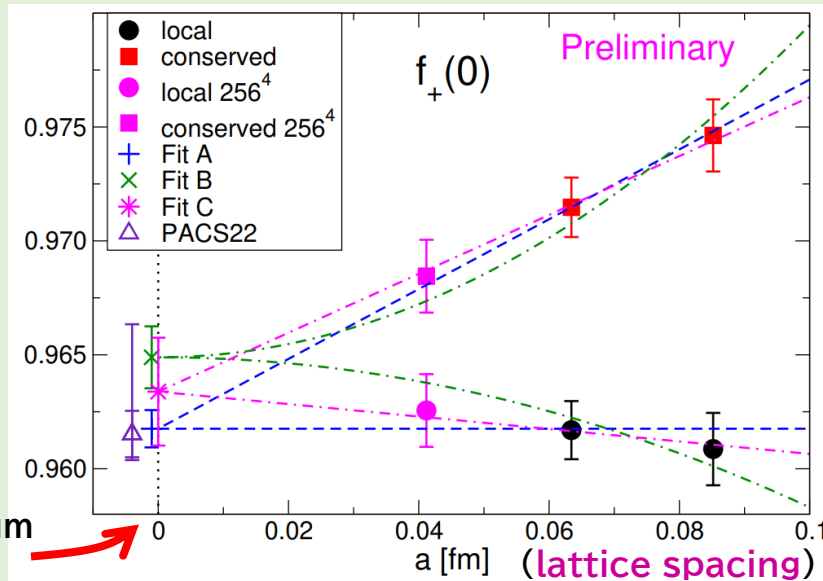
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Form factor
at zero momentum



continuum
limit

✓ Continuum extrapolation derives real values from discrete data.

Initially : large difference

recently : consistent

for : Lattice > Experimental

There are 4 main systematic errors

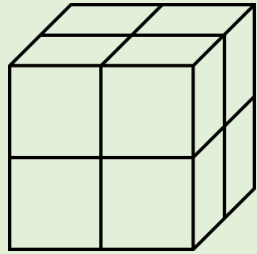
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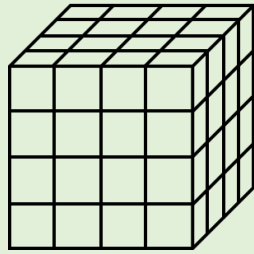
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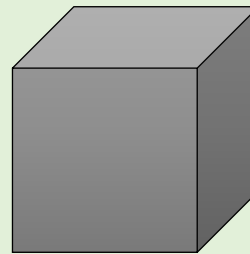


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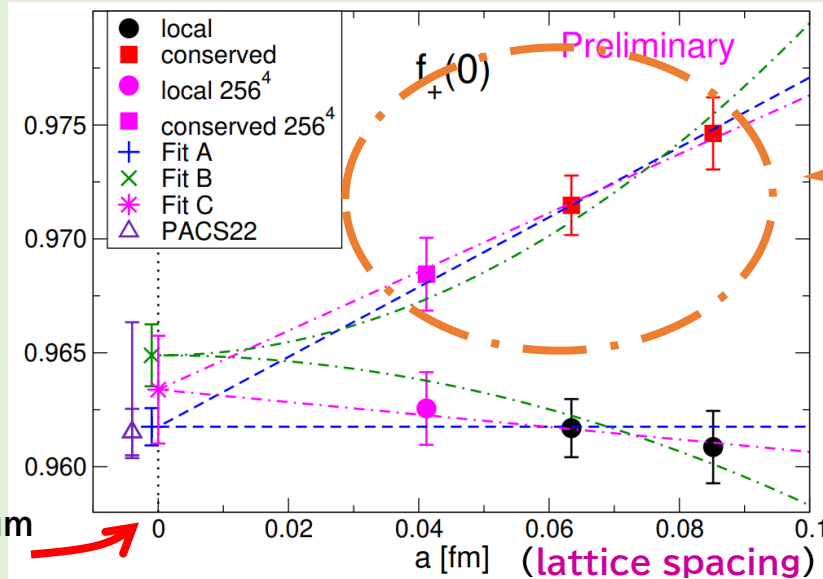
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By the discretization of space-time
Values change depending on the
lattice spacing

We need to calculate at multiple lattice
spacings and then extrapolate

Form factor
at zero momentum



PACS, PoS LATTICE2023
(2024) 276

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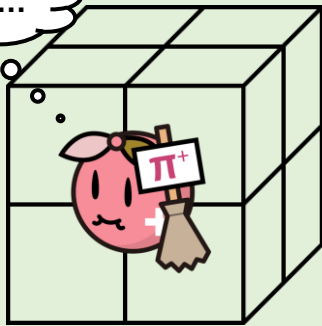
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Lattice calculations use **finite spacetime volume**.

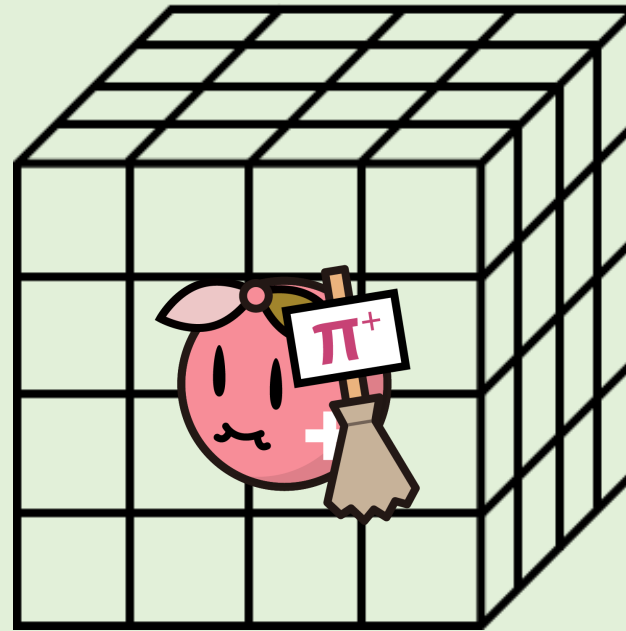
True size



Narrow...



Small volume



Large volume

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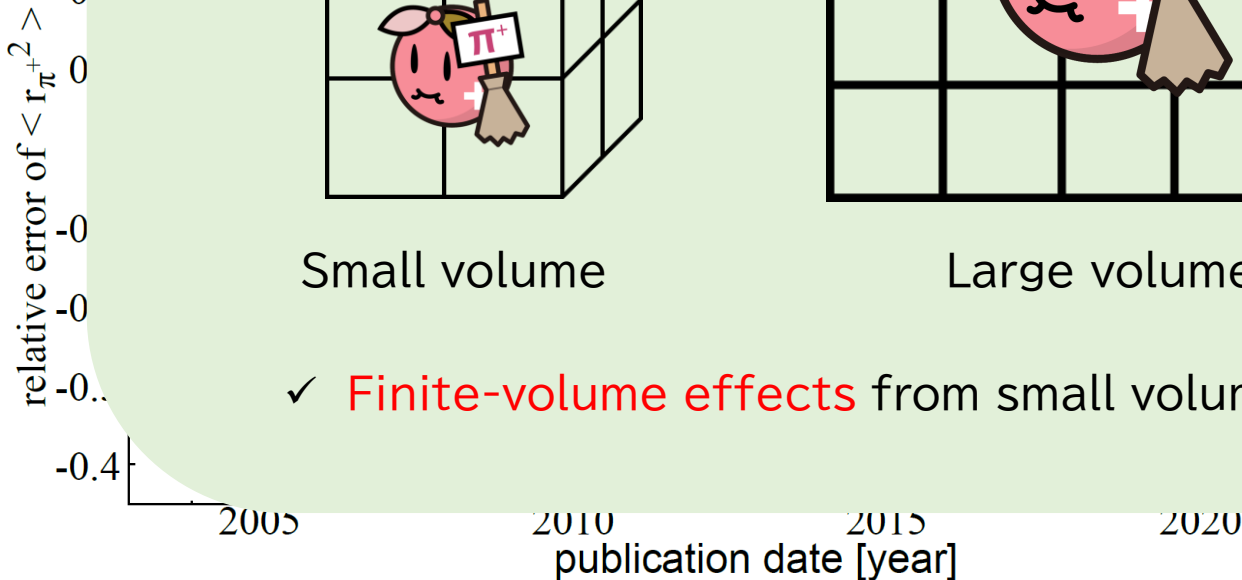
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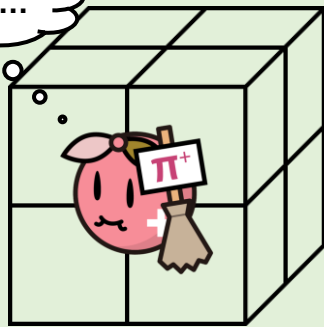
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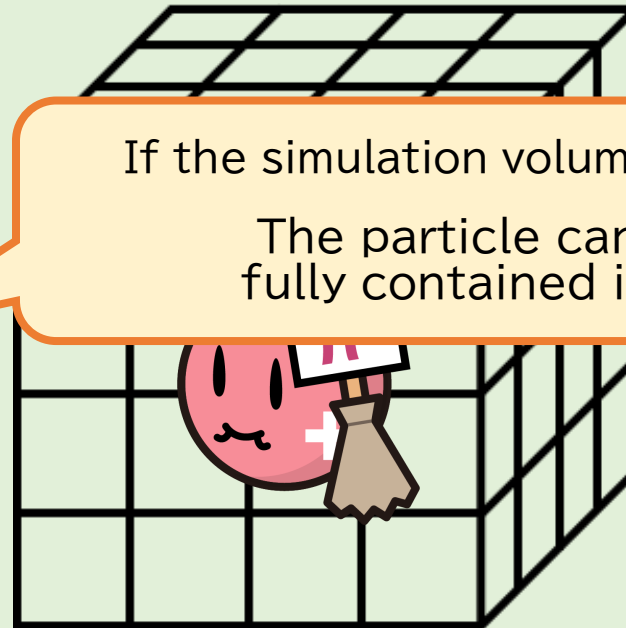


Narrow...



Small volume

If the simulation volume is too small
The particle cannot be
fully contained in space



Large volume

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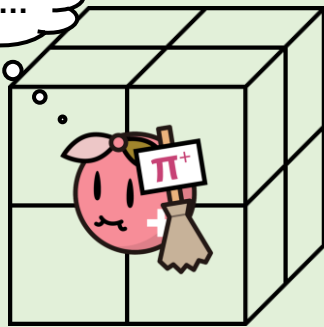
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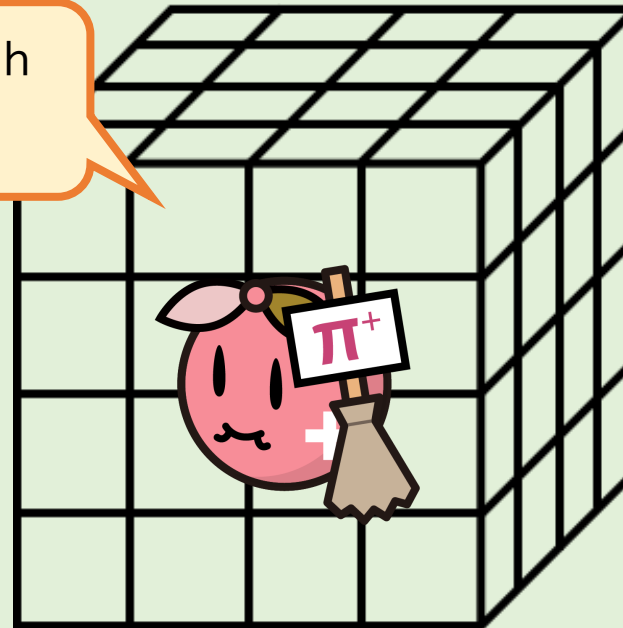
True size

If the volume is large enough
The particle fits well

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PACS10 configurations

... Large-volume configurations at physical point (3 lattice spacings).

➤ Chiral extrapolation

At physical point → almost unnecessary → good!!

Hadron masses reproduced within 5% error

➤ Continuum extrapolation

If computed with 3 configurations
→ evaluation possible → good!!

➤ Finite volume effect

Large volume sufficiently suppresses effects → good!!

Indicator of finite-volume effects

$M_\pi L > 4 \rightarrow \text{good}$ $3 < M_\pi L < 4 \rightarrow \text{so so}$ $M_\pi L < 3 \rightarrow \text{bad}$

PACS10 configurations ~ 7

PACS10 suppresses 3 major systematic errors.

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(m) Systematic errors from traditional analysis methods

3-pt functions (lattice QCD)

Known function

$$\frac{\sum_{\vec{x}_{\text{sink}}, \vec{x}} \langle \pi^+(\vec{x}_{\text{sink}}, t_{\text{sink}}) V_4(\vec{x}, t) \pi^+(0)^\dagger \rangle e^{-ipx_1}}{\sum_{\vec{x}_{\text{sink}}, \vec{x}} \langle \pi^+(\vec{x}_{\text{sink}}, t_{\text{sink}}) V_4(\vec{x}, t) \pi^+(0)^\dagger \rangle} = \frac{E_\pi(p) + m_\pi}{2E_\pi(p)} e^{-(E_\pi(p) - m_\pi)t} F_\pi(Q^2)$$

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Form factors (desired quantity)

Properly combine lattice data and
known functions.

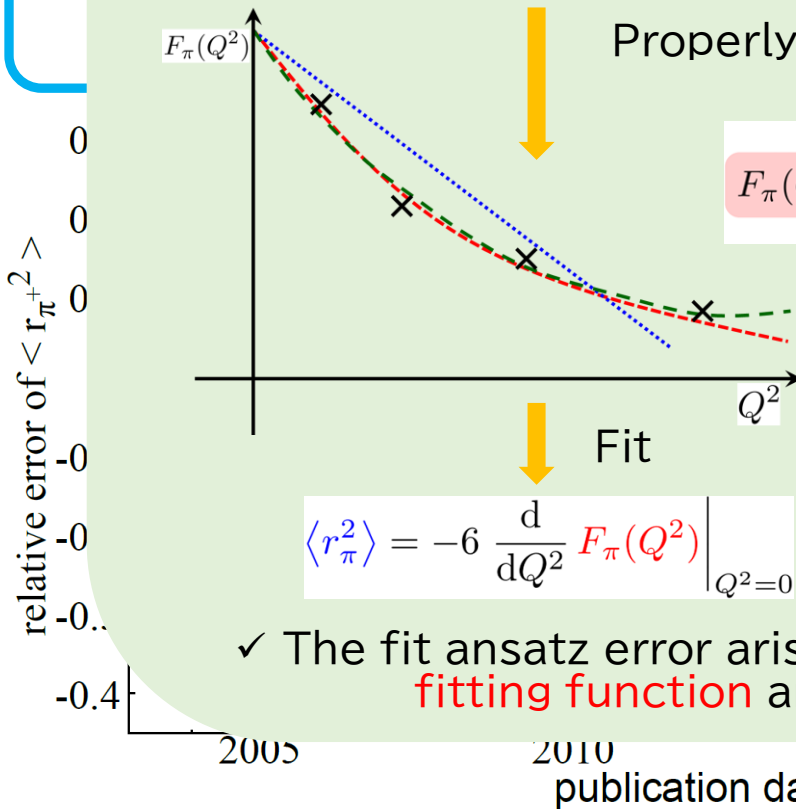
$$F_\pi(Q^2) = \frac{\text{Input from lattice calculation}}{\text{Known function}}$$

Fit ansatz

$$F_\pi(Q^2) = \begin{cases} \frac{1}{1 + Q^2/M_{\text{pole}}^2} \\ 1 + \sum_k f_k(Q^2)^k \\ \sum_k a_k z^k \left(z := \frac{\sqrt{4m_\pi^2 + Q^2} - \sqrt{4m_\pi^2}}{\sqrt{4m_\pi^2 + Q^2} + \sqrt{4m_\pi^2}} \right) \\ 1 + \frac{1}{f_\pi^2} [2l_6^r(\mu)Q^2 + 4H(m_\pi^2, Q^2, \mu^2)] \end{cases}$$

$$\langle r_\pi^2 \rangle = -6 \left. \frac{d}{dQ^2} F_\pi(Q^2) \right|_{Q^2=0}$$

✓ The fit ansatz error arises from the choice of the
fitting function and the fitting range.



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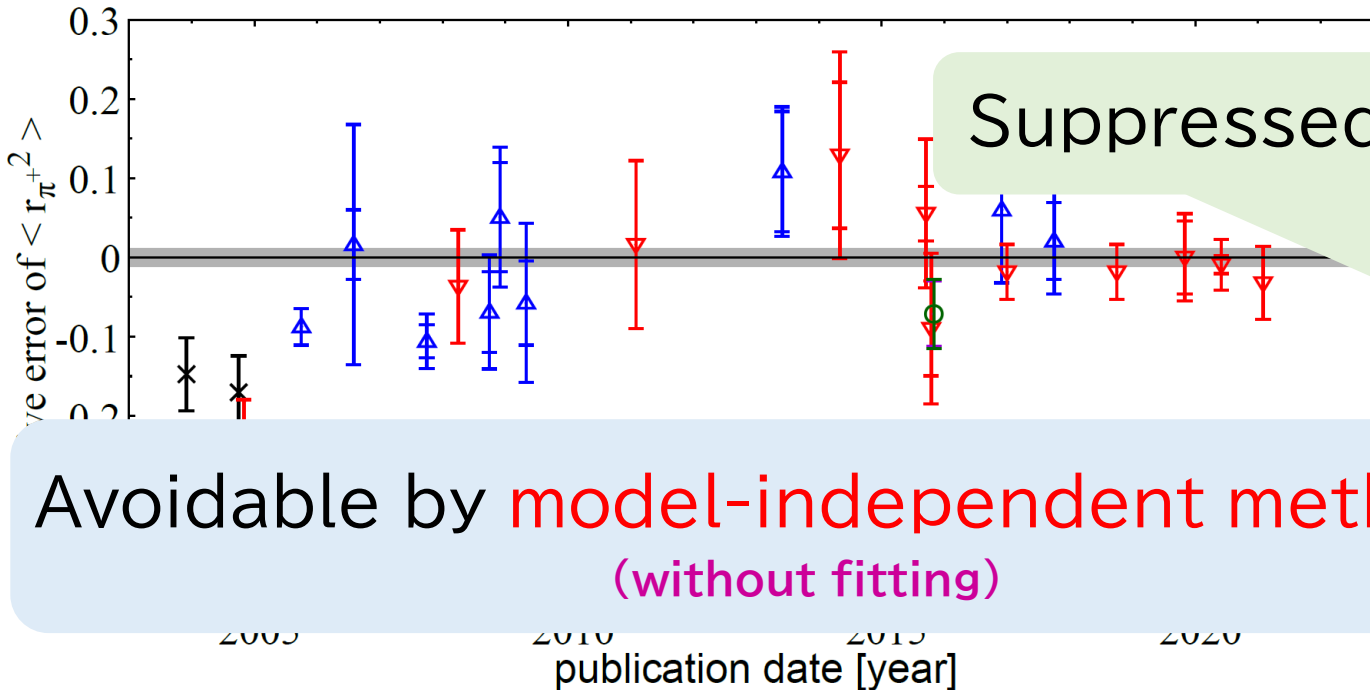
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Suppressed with PACS10 configurations

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Outline

- ✓ Introduction
 - Charge radius
 - Main systematic errors
- ✓ Overview of model-independent method
 - Reducing contamination using spatial moment
- ✓ Application to PACS10 configurations
 - Analysis of π^+ , K^+ , and K^0 charge radii using model-independent method
- ✓ Summary

Overview of model-independent method

$$F_\pi(Q^2)$$

$$(\text{3-point function}) = (\text{Known function}) \times (1 + f_1 Q^2 + f_2 Q^4 + \dots)$$

Example: difference $\left. \frac{d}{dQ^2} F_\pi(Q^2) \right|_{Q^2=0} = \underbrace{\left[\frac{\text{3-point function}}{(\text{Known function})} - 1 \right]}_{\text{can be exactly evaluated}} \bigg/ Q^2 + \underbrace{(\text{Higher-order contamination})}_{\text{cannot be exactly evaluated}}$ (If Q^2 is sufficiently small)

request :

- ✓ Is there a way to extract the **first derivative of the form factor** from the **three-point function** with **less contamination**?

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U. Aglietti et al., Phys.Lett.B324,85(1994); UKQCD, Nucl.Phys.B444,401(1995) ;
C. Bouchard et al., PoS LATTICE2016,170(2016); PACS, Phys.Rev.D104, 074514(2021)

➤ Differentiation of Fourier transform

$$\left. \frac{d\tilde{C}(p)}{dp^2} \right|_{p^2=0} = \frac{d}{dp^2} \sum_x C(x) e^{-ipx} \bigg|_{p^2=0} = -\frac{1}{2} \sum_x x^2 C(x)$$

($N_{space} \rightarrow \infty$, a :finite ; $L = N_{space} a \rightarrow \infty$)

infinite volume limit

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($N_{space} \rightarrow \infty$, a :finite ; $L = N_{space} a \rightarrow \infty$)

infinite volume limit

We can **suppress the contamination at large volume**.

$$\sum_x x^2 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

Overview of model-independent method

$$F_\pi(Q^2)$$

$$(3\text{-point function}) = (\text{Known function}) \times (1 + f_1 Q^2 + f_2 Q^4 + \dots)$$

Example: difference $\left. \frac{d}{dQ^2} F_\pi(Q^2) \right|_{Q^2=0} = \underbrace{\left[\frac{\text{3-point function}}{(\text{Known function})} - 1 \right]}_{\text{can be exactly evaluated}} \bigg/ Q^2 + \underbrace{(\text{Higher-order contamination})}_{\text{cannot be exactly evaluated}}$ (If Q^2 is sufficiently small)

request :

- ✓ Is there a way to extract the **first derivative of the form factor** from the **three-point function** with **less contamination**?

✓ **Key idea: Calculate spatial moment**

U. Aglietti et al., Phys.Lett.B324,85(1994); UKQCD, Nucl.Phys.B444,401(1995);
C. Bouchard et al., PoS LATTICE2016,170(2016); PACS, Phys.Rev.D104, 074514(2021)

➤ **Differentiation of Fourier transform**

$$\left. \frac{d\tilde{C}(p)}{dp^2} \right|_{p^2=0} = \frac{d}{dp^2} \sum_x C(x) e^{-ipx} \bigg|_{p^2=0} = -\frac{1}{2} \sum_x x^2 C(x)$$

($N_{space} \rightarrow \infty$, a :finite ; $L = N_{space} a \rightarrow \infty$)

infinite volume limit

We can **suppress the contamination at large volume**.

$$\sum_x x^2 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

✓ **Method: Combining x^2 and x^4 moments**

Xu Feng et al., Phys.Rev.D101,051502(R)(2020)

$$\sum_x x^4 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

We can cleverly **add these higher-order moments** to reduce the contamination.

$$R(t) = \alpha_1 (x^2 \text{moment}) + \alpha_2 (x^4 \text{moment})$$

$$= \left. \frac{d}{dQ^2} F_\pi(Q^2) \right|_{Q^2=0} + (f_3 + f_4 + \dots)$$

α_1, α_2 : parameters which **set to cancel out the contamination**

Our Improvement : K. S. et al., PoS LAT22,122(2022) ; PoS LAT23,312(2023)

-> **Effective at small lattice size**

Simulation parameters

- ✓ Gauge configuration (PACS, PRD 99, 014504 (2019);
PACS, PRD 106, 094505 (2022);
PACS, PRD 109, 094505 (2024))

PACS10 configuration

$N_f = 2 + 1$ six-stout-smeared non-perturbative
 $O(a)$ -improved Wilson action+ Iwasaki gauge action

β	$L^3 \cdot T$	$L[\text{fm}]$	$a[\text{fm}]$	$a^{-1}[\text{GeV}]$	$m_\pi[\text{MeV}]$	$m_K[\text{MeV}]$	N_{conf}
2.20	256^4	10.5	0.041	4.792	142	514	20
2.00	160^4	10.2	0.063	3.111	137	501	20
1.82	128^4	10.9	0.085	2.316	135	497	20

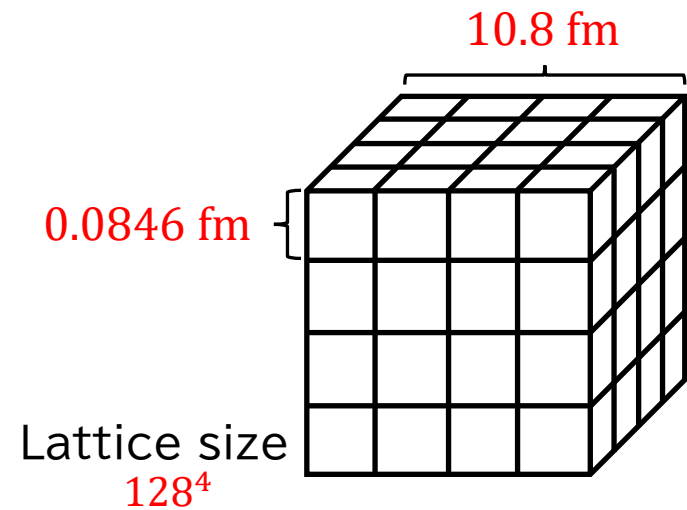
All preliminary results are obtained on the 128 and 160 lattice

✓ Measurement parameter

- 20 config. , 2304~13824 meas. Per config.
- **Periodic** + **anti-periodic** correlation functions in time direction
- **Forward** + **Backward** correlation functions in time direction
- $|t_{\text{sink}} - t_{\text{source}}| = 36, 42, 48(\text{L128}); 50, 58, 64(\text{L160})$



↑ Statistics
 Reduce the wrapping-around effect
 Systematic errors can be evaluated



- Chiral extrapolation
 → Physical point
- Continuum extrapolation
 → 3 lattice spacings
- Finite volume effects
 → Large volume

Result

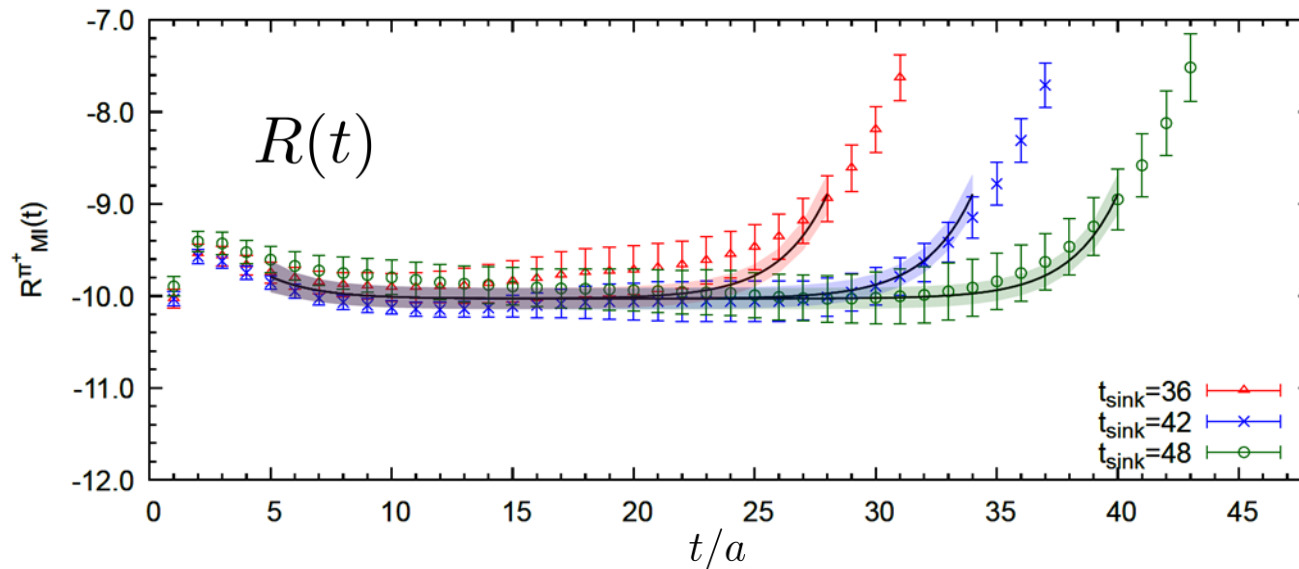
- ✓ Model-independent method
(without fit ansatz)

$$\sum_x x^2 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

$$\sum_x x^4 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

$$R(t) = \alpha_1 (x^2 \text{moment}) + \alpha_2 (x^4 \text{moment})$$

$$= \left. \frac{d}{dQ^2} F_\pi(Q^2) \right|_{Q^2=0} + (f_3 + f_4 + \dots)$$



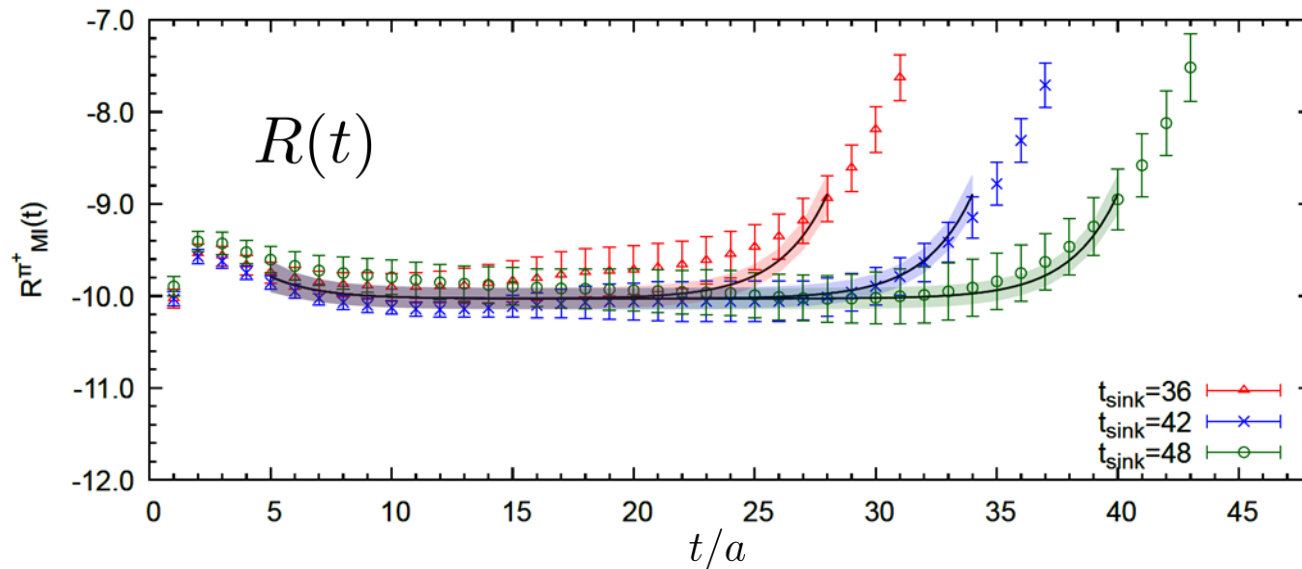
Result

- ✓ Model-independent method (without fit ansatz)

$$\sum_x x^2 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

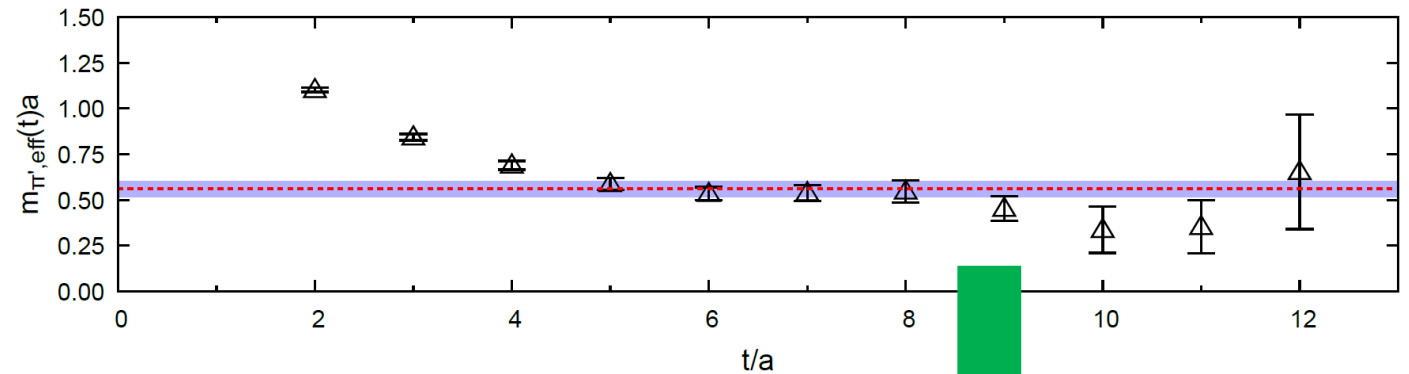
$$R(t) = \alpha_1 (x^2 \text{moment}) + \alpha_2 (x^4 \text{moment}) + \dots$$

$$= \left. \frac{d}{dQ^2} F_\pi(Q^2) \right|_{Q^2=0} + (f_3 + f_4 + \dots)$$



- ✓ Extraction including the first excited state

$$f(t; t_{\text{sink}}) = \left. \frac{d}{dQ^2} F_\pi(Q^2) \right|_{Q^2=0} + A e^{-(m_{X'} - m_X)t} + B e^{-(m_{X'} - m_X)(t_{\text{sink}} - t)}$$



- ✓ From the plateau, we can see that the PACS10 configurations reproduce the experimental mass of the first excited state.

- ✓ We use experimental value as the mass of the first excited state

Result

✓ Model-independent method
(without fit ansatz)

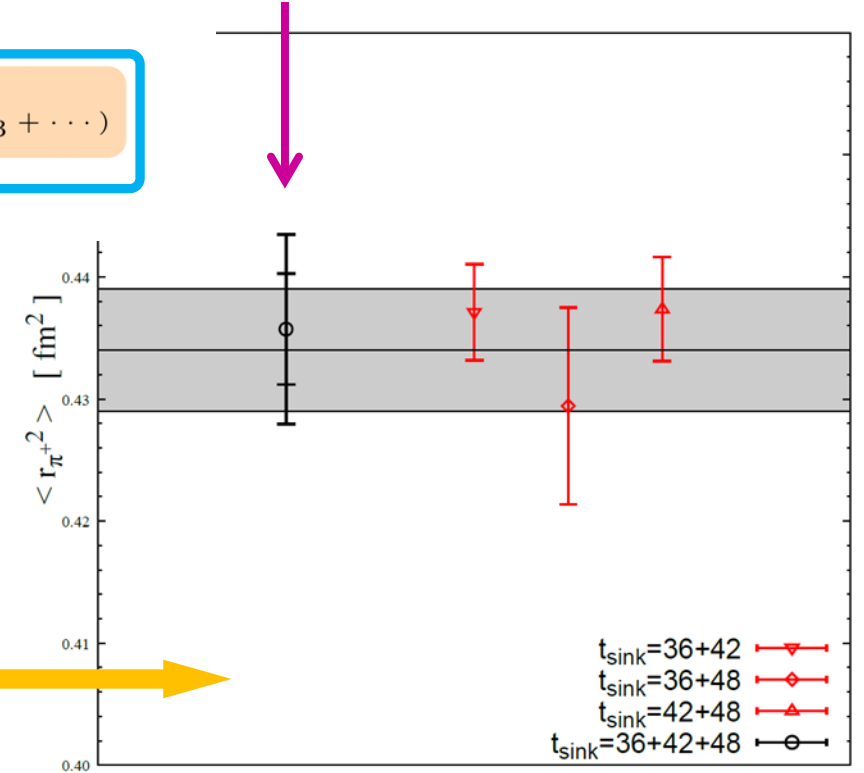
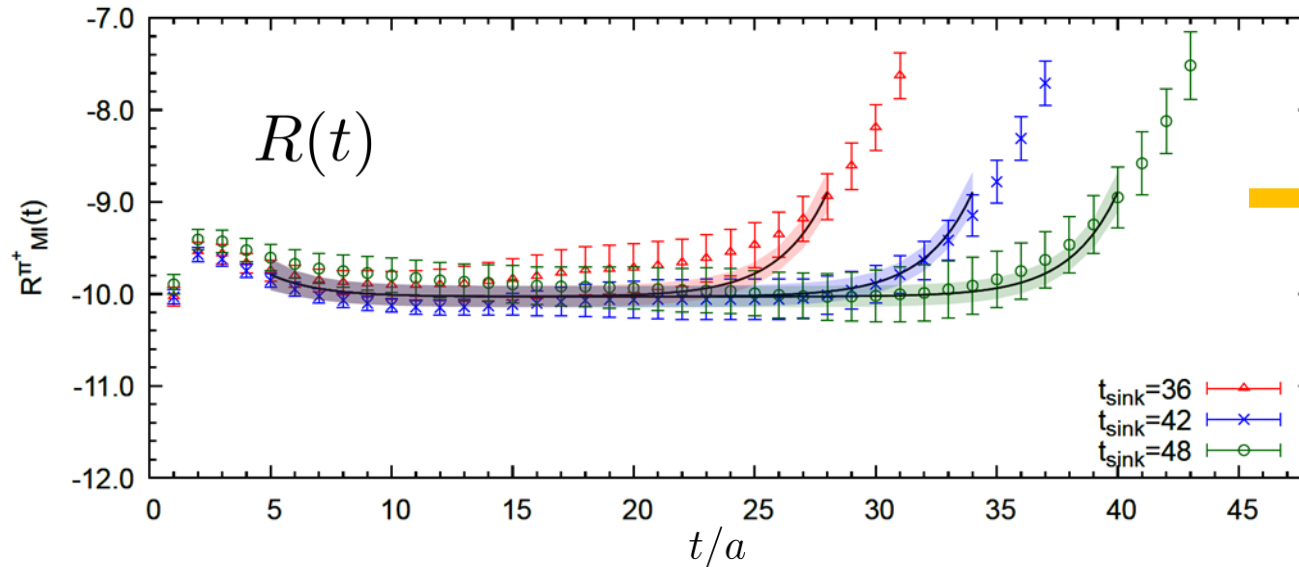
- Central value : result using all source-sink separations
- Statistic error : Jackknife error of the central value
- Systematic error : Maximum difference between the central value and each separation

$$\sum_x x^2 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

$$\sum_x x^4 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

$$R(t) = \alpha_1 (x^2 \text{moment}) + \alpha_2 (x^4 \text{moment})$$

$$= \left. \frac{d}{dQ^2} F_\pi(Q^2) \right|_{Q^2=0} + (f_3 + f_4 + \dots)$$



Result

✓ **Model-independent method**
(without fit ansatz)

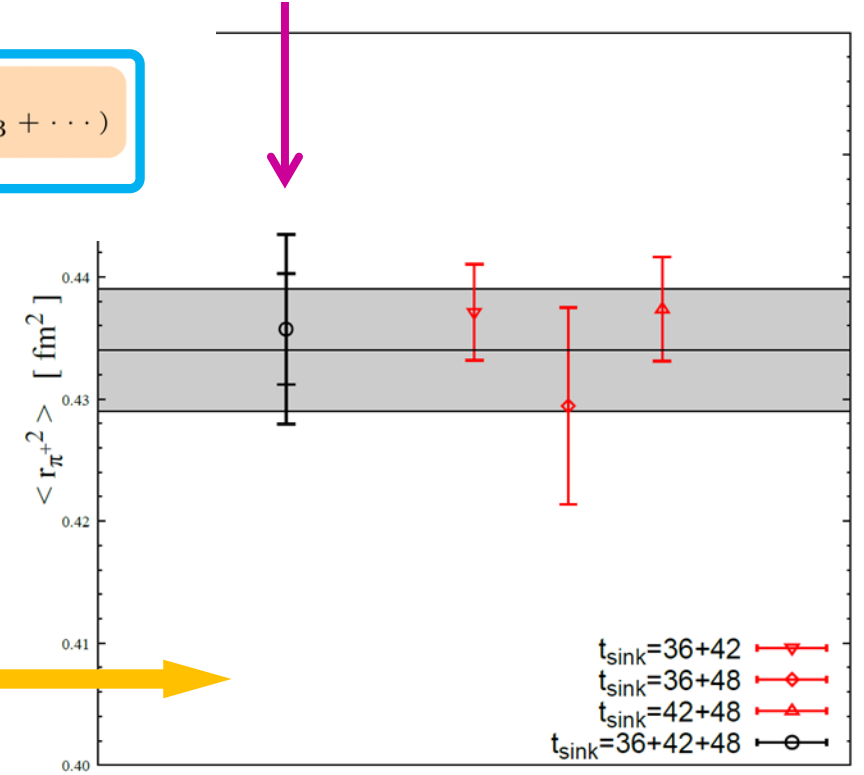
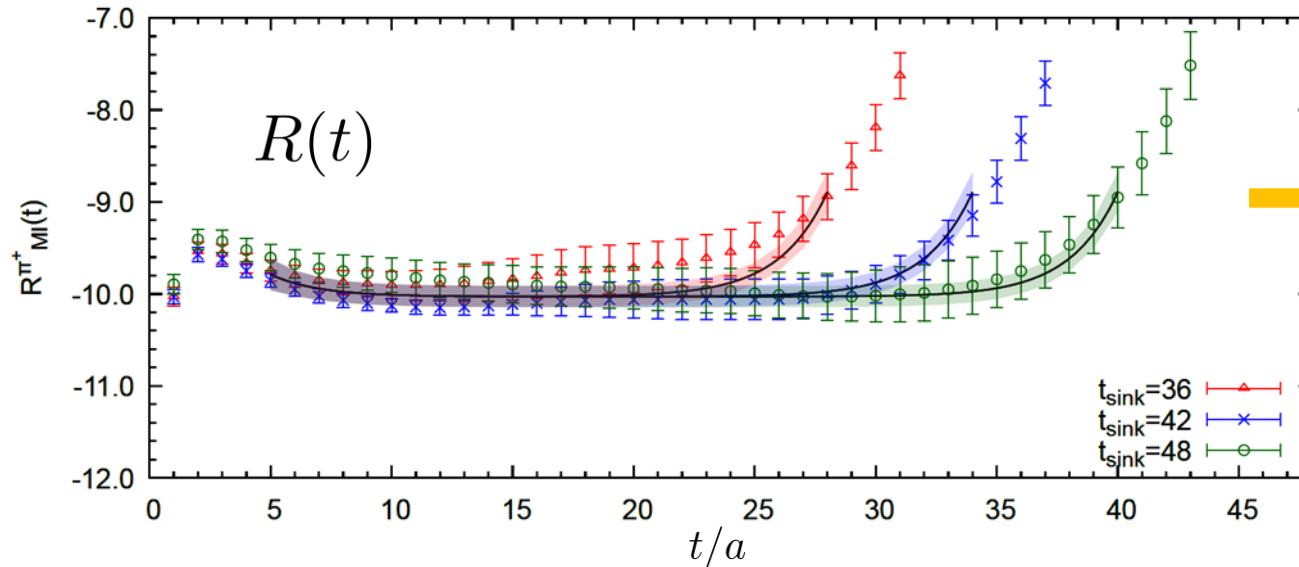
- Central value : result using **all source-sink separations**
- Statistic error : **Jackknife error** of the **central value**
- Systematic error : **Maximum difference** between the **central value** and **each separation**

$$\sum_x x^2 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

$$\sum_x x^4 C_{3pt}(x, t) \sim f_1 + (f_2 + f_3 + \dots)$$

$$R(t) = \alpha_1 (x^2 \text{moment}) + \alpha_2 (x^4 \text{moment})$$

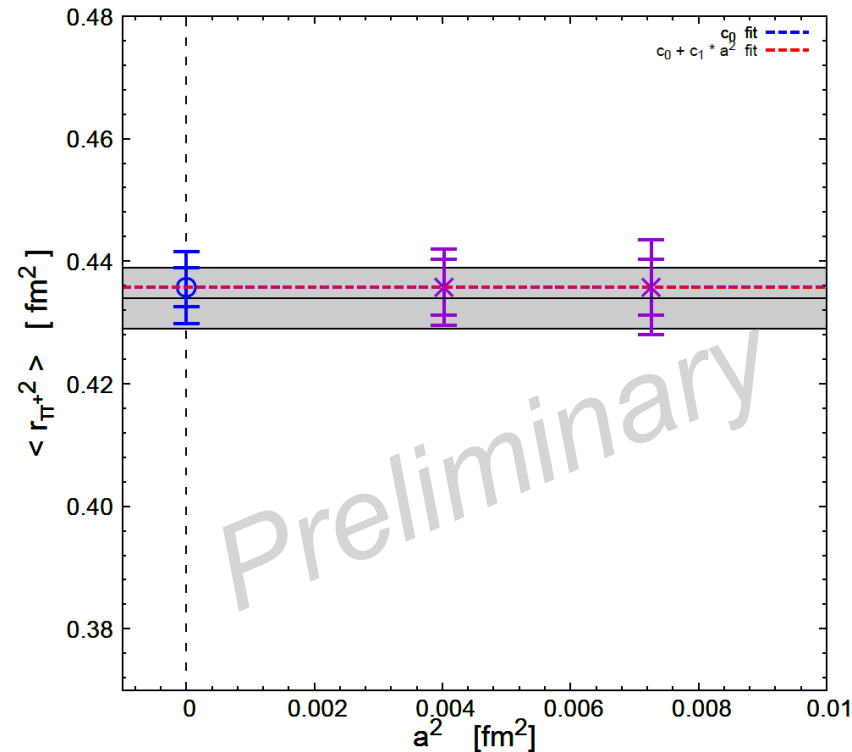
$$= \left. \frac{d}{dQ^2} F_\pi(Q^2) \right|_{Q^2=0} + (f_3 + f_4 + \dots)$$



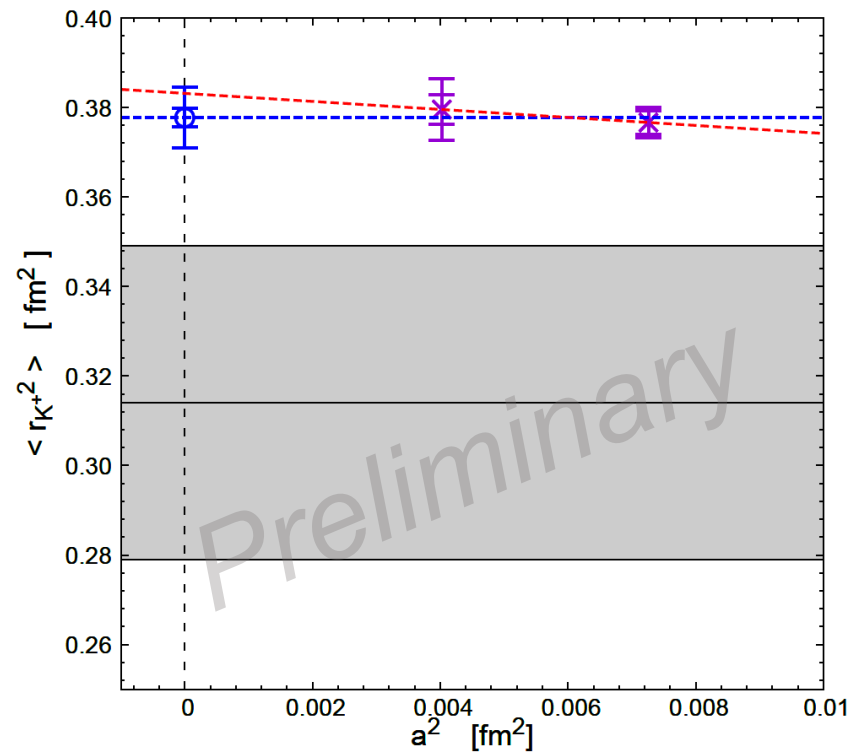
✓ Also perform the same calculation
for **K mesons** and **L160 configuration**.

Preliminary results

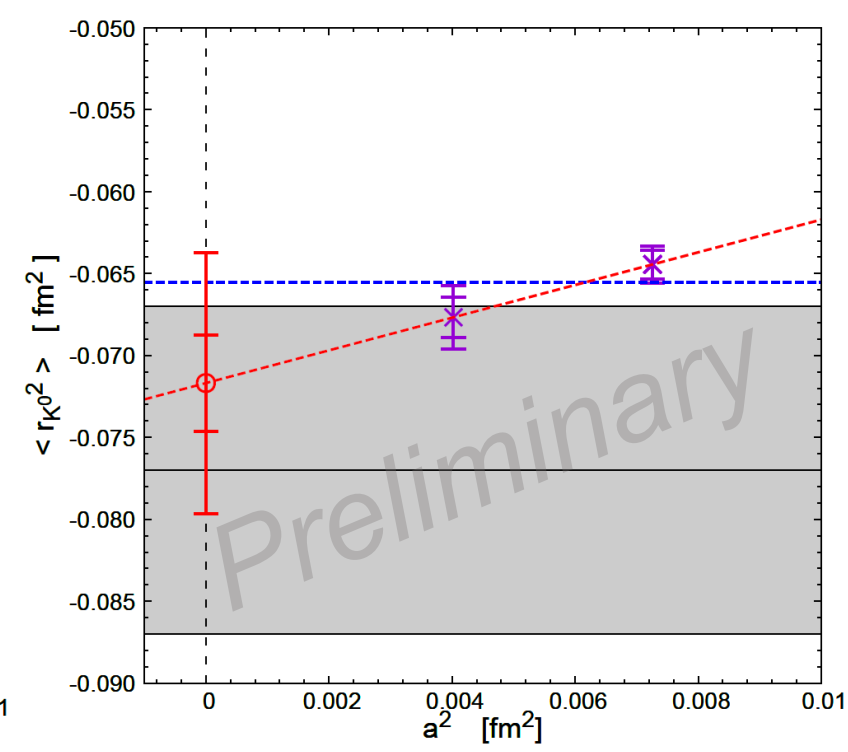
✓ π^+ charge radius



✓ K^+ charge radius



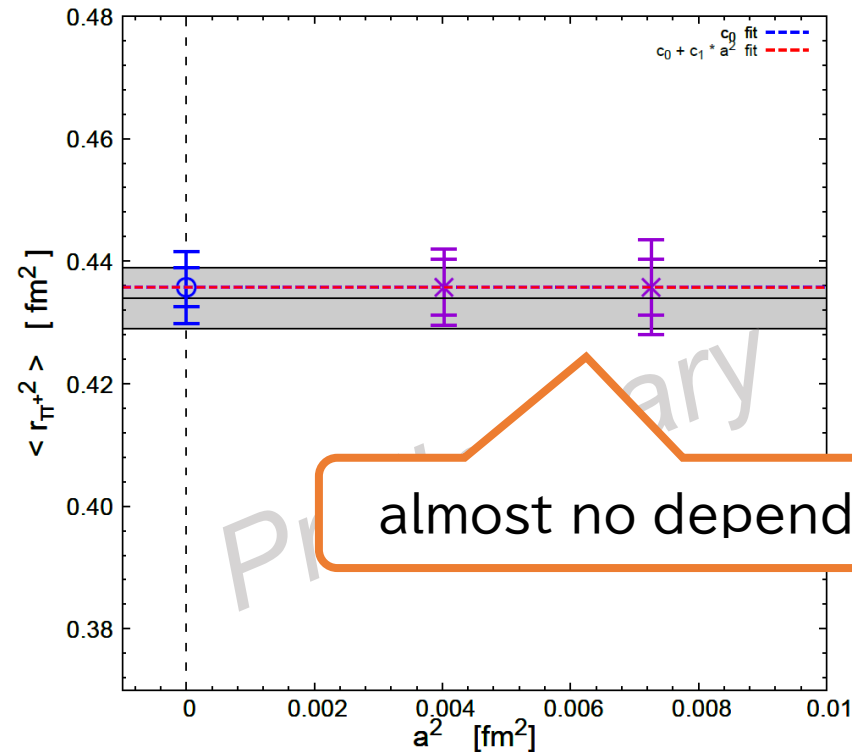
✓ K^0 charge radius



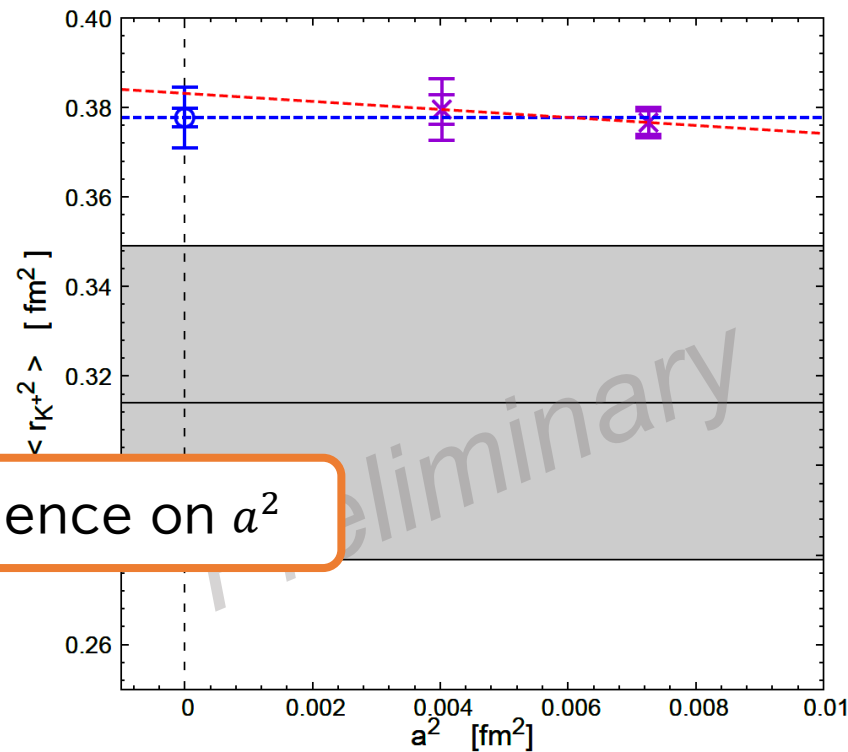
- ✓ As a preliminary extrapolation, we perform both a **constant** and a **linear fit in a^2** .
- ✓ We take the constant extrapolation as the central value for π^+ and K^+ , and the linear extrapolation in a^2 as the central value for K^0 .
- ✓ For the continuum limit values, results **agree with PDG** within $\sim 2\sigma$.
- ✓ π^+ and K^0 achieve the **same level of experimental error**, while K^+ achieves precision **exceeding the experiment**.

Preliminary results

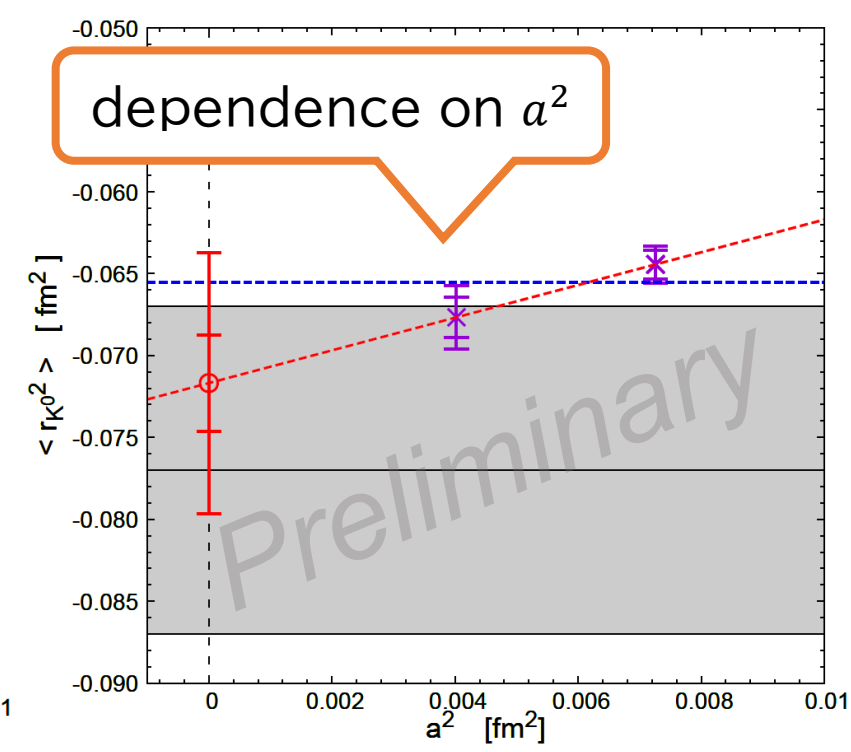
✓ π^+ charge radius



✓ K^+ charge radius



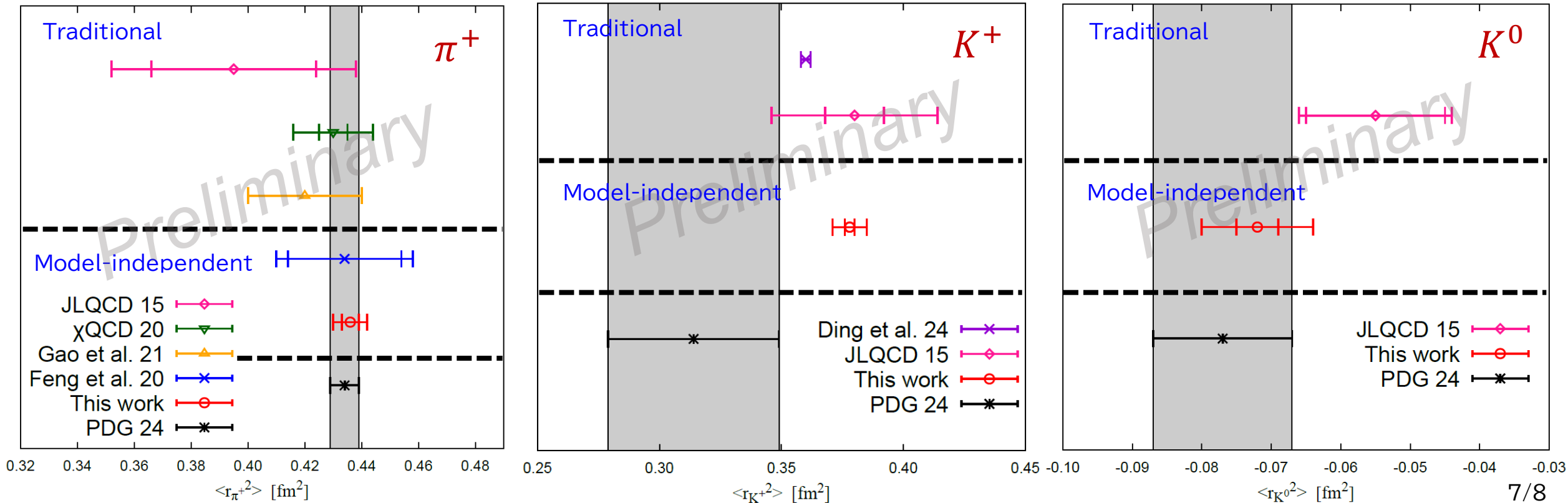
✓ K^0 charge radius



- ✓ As a preliminary extrapolation, we perform both a constant and a linear fit in a^2 .
- ✓ We take the constant extrapolation as the central value for π^+ and K^+ , and the linear extrapolation in a^2 as the central value for K^0 .
- ✓ For the continuum limit values, results agree with PDG within $\sim 2\sigma$.
- ✓ π^+ and K^0 achieve the same level of experimental error, while K^+ achieves precision exceeding the experiment.

Summary

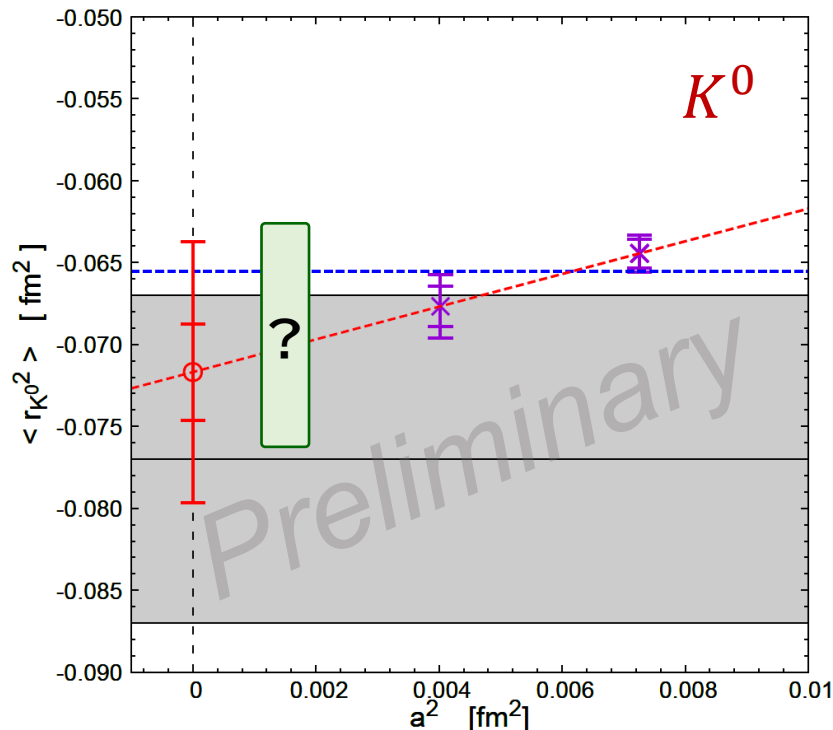
- ✓ Calculated charge radii of π^+ , K^+ , and K^0 on PACS10/L128 and L160 configurations.
- ✓ Used the model-independent method.
- ✓ Although preliminary, the results are consistent with the experimental value(PDG) and the results of the previous lattice calculations.
- ✓ Some of our results exceed their precision.



Summary

Future works :

- ✓ Increase statistics for PACS/L160
 - ✓ Analyze data using conserved currents
 - ✓ Perform calculations on PACS/L256
- => The precision of our continuous extrapolation will improve.



Expectations for the Experiment:

The experimental error for K mesons is large.

- ✓ An experiment called AMBER is currently being conducted at CERN.
- ✓ Phase 2 plans to conduct a high-precision measurement of the K meson's charge radius.

We are looking forward to seeing the results of these experiments.