Kaon semileptonic form factors at the physical quark masses on large volumes in lattice QCD

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Outline

- PACS10 project
- Nucleon form factor
- Kaon semileptonic decay form factor
- Summary

PACS10 project

Lattice QCD: nonperturbative calculation of QCD

understanding strong interaction at least quantitatively

PACS Collaboration

Univ. of Tsukuba : N. Ishizuka, Y. Kuramashi, E. Shintani,

T. Taniguchi, N. Ukita, T. Yamazaki, T. Yoshié

Hiroshima Univ. : K.-I. Ishikawa; Tohoku Univ. : S. Sasaki, M. Nagatsuka

Riken-CCS : Y. Aoki, Y. Nakamura; Fukuyama Univ. : N. Namekawa

KEK : R. Tsuji; Keio Univ. : H. Watanabe; Seikei Univ. : K. Sato

Larger than $(10 \text{ fm})^4$ volume

Purpose of PACS10 project

Removing main three uncertainties in $N_f = 2+1$ lattice QCD

1. large m_π, 2. finite V, 3. finite a

using three ensembles at physical m_π on $(10 \text{ fm})^4$ volume

$\Rightarrow$ PACS10 configuration

PACS10 project since 2016

	PACS10 configuration		
$L^3 \cdot T$	128^4	160^4	256^4
L [fm]	10.9	10.2	10.5
a [fm]	0.08	0.06	0.04
m_π [GeV]	0.135	0.137	0.142
m_K [GeV]	0.497	0.501	0.514
Machine	OFP	OFP	OFP→Fugaku
Node	512	512	2048→16384

OFP: Oakforest-PACS (KNL machine); Fugaku: since 2020

PACS10 configuration: More than (10 fm)^4 at the physical point

$N_f = 2+1$ nonperturbatively $O(a)$ improved Wilson clover quark action with 6-stout smeared link + Iwasaki gauge action

same actions as HPCI Field 5 project using K computer [PoS LATTICE2015 (2016) 075]

a^{-1} determined from Ξ baryon mass

Fugaku co-design outcome: [Ishikawa *et al.*:CPC(2023)]

QCD Wide SIMD (QWS) Library for Fugaku

Resources: Fugaku in HPCI System Research Project
hp200062, hp200167, hp210112, hp220079, hp230199

From PACS10 to PACS10$_c$ since 2023

	PACS10$_c$ configuration		
$L^3 \cdot T$	128^4	168^4	256^4
L [fm]	10.7	10.5	10.5
a [fm]	0.08	0.06	0.04
m_π [GeV]	0.137	0.137	$\sim$0.137
m_K [GeV]	0.498	0.497	$\sim$0.497
Machine	Fugaku	Fugaku	Fugaku
Node	1024	2744	16384
Status	done	done	running

	PACS10 configuration		
$L^3 \cdot T$	128^4	160^4	256^4
L [fm]	10.9	10.2	10.5
a [fm]	0.08	0.06	0.04
m_π [GeV]	0.135	0.138	0.141
m_K [GeV]	0.497	0.505	0.514

PACS10$_c$ configuration: More than (10 fm)^4 at the physical m_{ud}, m_s, m_c

$N_f = 2+1+1$ nonperturbatively $O(a)$ improved Wilson clover quark action

with 6-stout smeared link + Iwasaki gauge action

a^{-1} determined from Ξ baryon mass

Fugaku co-design outcome: [Ishikawa *et al.*:CPC(2023)]

QCD Wide SIMD (QWS) Library for Fugaku

Resources: Fugaku in HPCI System Research Project: hp230199, hp240207, hp250218

PACS10 project

precise determination of physical quantity
w/o main systematic uncertainties in lattice QCD

I. quantitatively understand property of hadrons
reproduce experimental values in high accuracy

- Hadron spectrum
- Light meson charge radii — Kohei Sato's talk
- Nucleon form factor — Tsuji, Nagatsuka, Sasaki, Shintani

Next three slides

II. search for new physics beyond the standard model
discrepancy between theoretical calculation and experiment

- Proton decay matrix element
- Hadron vacuum polarization
- Kaon semileptonic decay form factor — This talk

Nucleon form factor

Nucleon form factor with PACS10 confs.

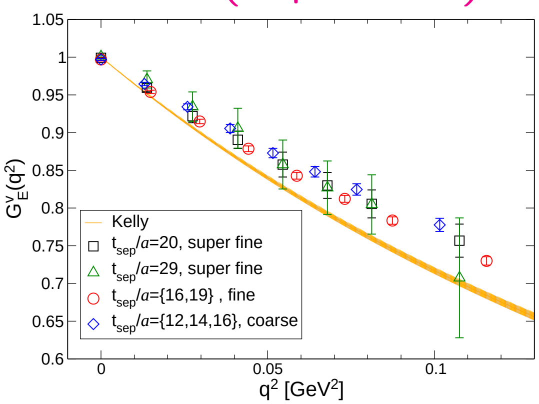
PACS10/L128 [PRD99:1,014510(2020)]; PACS10/L160 [PRD109:9,094505(2024)]

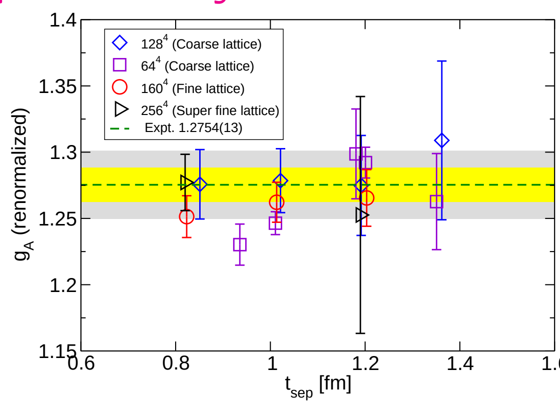
PACS10/L256 in progress, presented by Nagatsuka @ Lat25

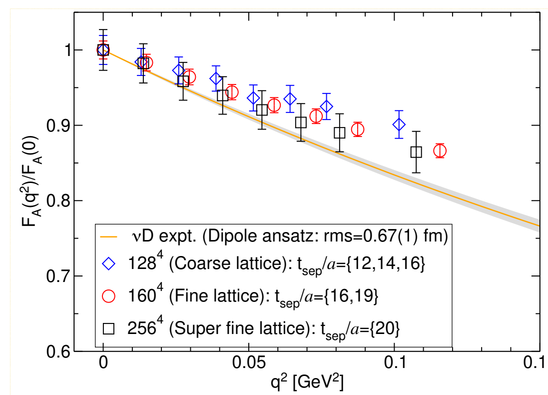
Isovector form factors : G_E, G_M, g_A, F_A, F_P, G_P

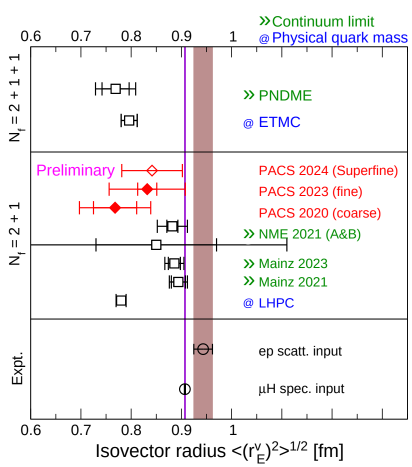
related to Proton size puzzle and νN scattering

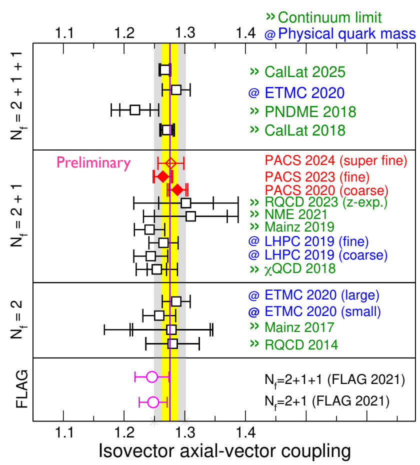
256^4 lattice (super fine) : preliminary results

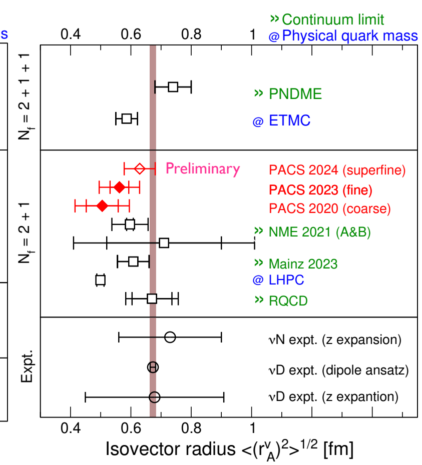










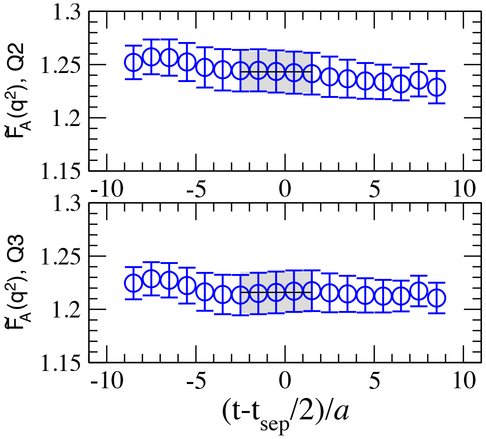


Reasonably consistent with results from other groups

Reduction of leading πN contribution in F_P, G_P [PRD112:7,074510(2025)]

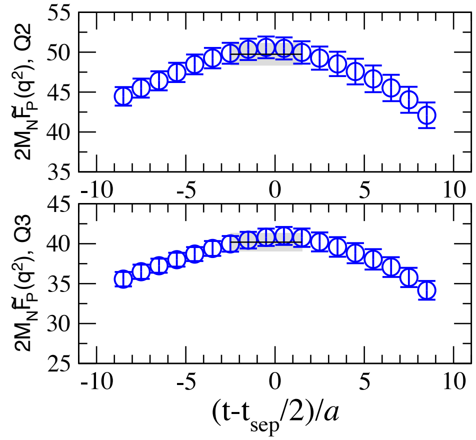
Large πN contribution in F_P, G_P

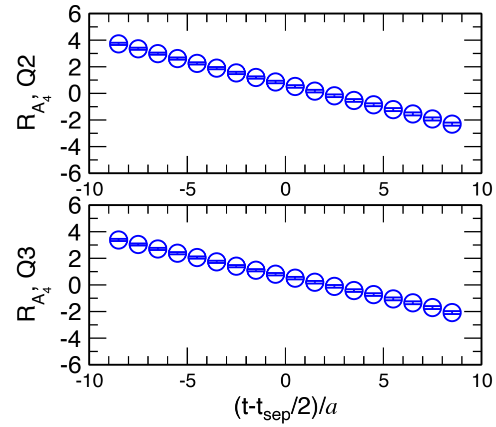
[Bär, PRD99:054506(2019); Meyer *et al.*, PoS(LAT2018)062; RQCD, JHEP5:126(2020), $\cdots$]



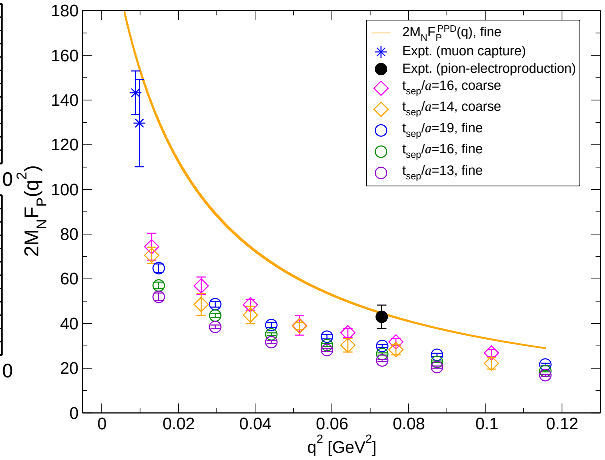
reasonably flat

w/ optimized operator





clear t dependence = large excited state



F_P: largely underestimated and $t_{\rm sep}$ dependent

Simple assumption of leading πN contribution inspired by Bär's results

$$R_{A_i}(t,\vec{q}) = K^{-1}[(E_N+M_N)F_A(q^2)\delta_{i3} - q_3 q_i(F_P(q^2) - \Delta_+(t,\vec{q}))]$$

$$R_{A_4}(t,\vec{q}) = i q_3 K^{-1}[F_A(q^2) - \Delta E_N F_P(q^2) + E_\pi \Delta_-(t,\vec{q})]$$

$$\Delta_\pm(t,\vec{q}) = B(q)e^{-(E_\pi-\Delta E_N)t} \pm C(q)e^{-(E_\pi+\Delta E_N)(t_{\rm sep}-t)}$$

R_A: ratio of 3-pt function with A current to 2-pt function, $K=\sqrt{2E_N(E_N+M_N)}$, $\Delta E_N = E_N - M_N$,

$t_{\rm sep}$: source-sink separation

well describe t dependence of our data

Reduction of leading πN contribution in F_P, G_P [PRD112:7,074510(2025)]

Simple assumption of leading πN contribution inspired by Bär's results

$$R_{A_i}(t,\vec{q}) = K^{-1}[(E_N+M_N)F_A(q^2)\delta_{i3} - q_3 q_i(F_P(q^2) - \Delta_+(t,\vec{q}))]$$

$$R_{A_4}(t,\vec{q}) = iq_3 K^{-1}[F_A(q^2) - \Delta E_N F_P(q^2) + E_\pi \Delta_-(t,\vec{q})]$$

$$\Delta_\pm(t,\vec{q}) = B(q)e^{-(E_\pi - \Delta E_N)t} \pm C(q)e^{-(E_\pi + \Delta E_N)(t_{\rm sep}-t)}$$

R_A: ratio of 3-pt function with A current to 2-pt function, $K = \sqrt{2E_N(E_N+M_N)}$, $\Delta E_N = E_N - M_N$,

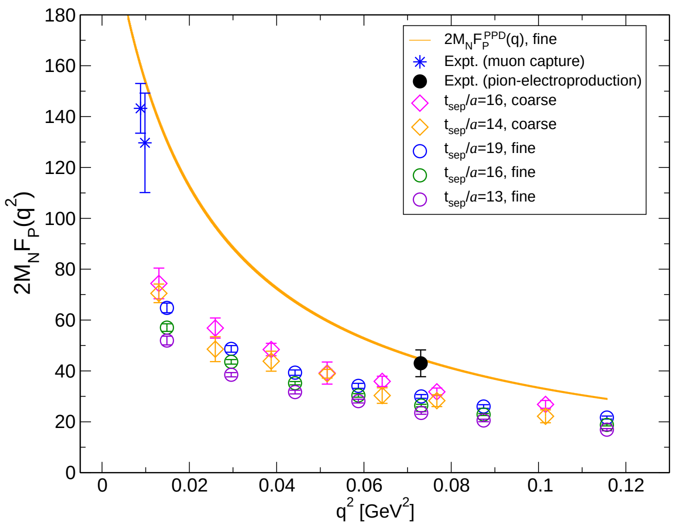
$t_{\rm sep}$: source-sink separation

$\Delta_\pm(t,\vec{q})$ can be removed by linear combination with R_A and $\partial_t R_A$

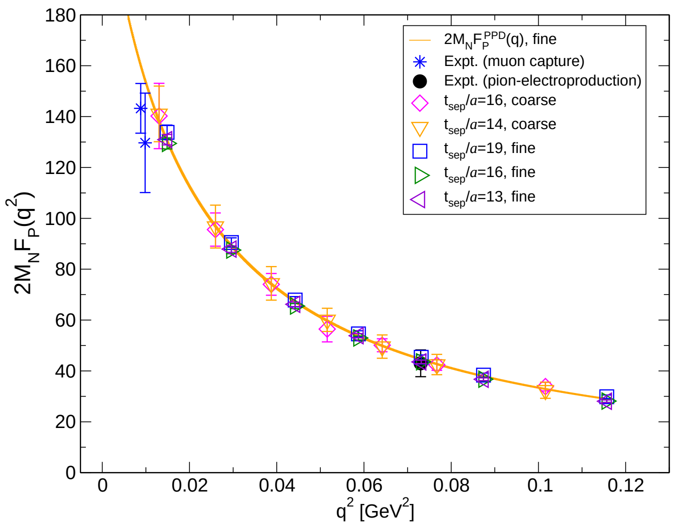
$$F_P(q^2) = \alpha R_{A_i} + \beta\partial_t R_{A_i} + \gamma\partial_t R_{A_4}$$

α, β, γ: appropriate coefficients

standard



new



independent of $t_{\rm sep}$, consistent with pion-pole dominance model

Similar analysis performed with G_P data

Kaon semileptonic ($K_{\ell 3}$) decay form factor

Introduction

Urgent task in particl physics
Search for signal beyond standard model (BSM)

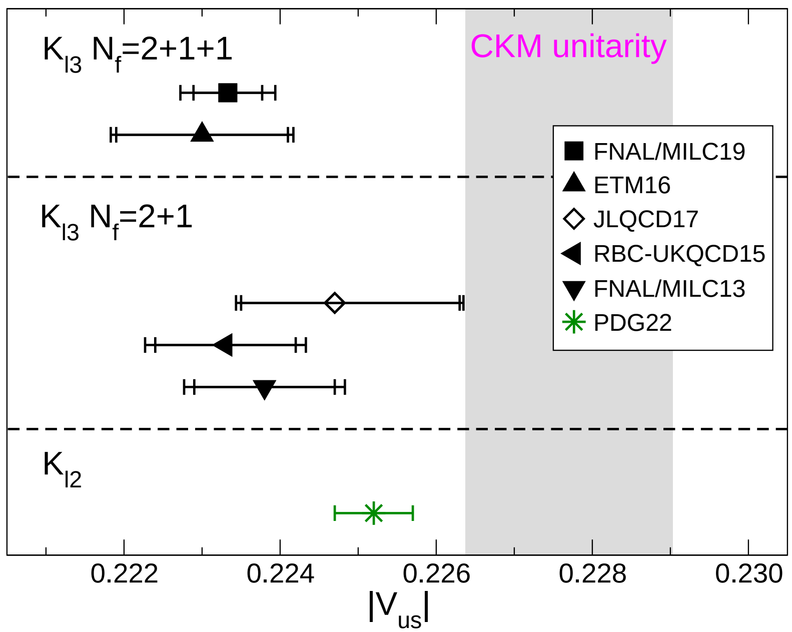
Muon $g-2$ @ FNAL, White paper 2025 : ~~5σ away from SM~~(?)

important contribution from lattice QCD

$|V_{us}|$: a candidate of BSM signal

Three determinations of $|V_{us}|$

1. CKM unitarity $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
   constraint in SM (grey band)
   $|V_{us}| \approx \sqrt{1-|V_{ud}|^2}$ w/ new $|V_{ud}|$
   ['18 Seng *et al.*, '20 Hardy, Towner]
2. $K_{\ell 2}$ $K \to \ell\nu$ decay
3. $K_{\ell 3}$ $K \to \pi\ell\nu$ decay
   Most accurate value ['19 FNAL/MILC]



$|V_{us}|^{1.} \neq |V_{us}|^{2.} \neq |V_{us}|^{3.} \lesssim 3\sigma$ differences

Important to confirm by several independent calculations

$K_{\ell 3}$ calcualtion with PACS10$_{(c)}$ configuration

$K_{\ell 3}$ form factors with PACS10/L128,L160 configurations

PACS10/L128[PRD101,9,094504(2020)]; PACS10/L160[PRD106,9,094501(2022)]

PACS10 configuration: $N_f = 2+1, L \gtrsim 10$[fm] at physical point

Negligible finite L effect, tiny q^2 region, without chiral extrapolation

Largest uncertainty form finite a effect

This talk: Preliminary result with PACS10/L256 configuration

PACS10$_c$ configuration: $N_f = 2+1+1, L \gtrsim 10$[fm] at physical point

Dynamical charm quark effect in addition to PACS10 conf.

less than $\mathcal{O}(1)\%$ effect, but maybe important in indirect BSM search

This talk:

Preliminary result with PACS10$_c$/L128,L168 configurations

Simulation parameters

	PACS10			PACS10$_c$	
$L^3 \cdot T$	128^4	160^4	256^4	128^4	168^4
L [fm]	10.9	10.2	10.5	10.7	10.5
a [fm]	0.08	0.06	0.04	0.08	0.06
m_π [GeV]	0.135	0.138	0.141	0.137	0.137
m_K [GeV]	0.497	0.505	0.514	0.498	0.497
N_{conf}	20	20	20	40	40
N_{meas}	< 2560		1280	5120	

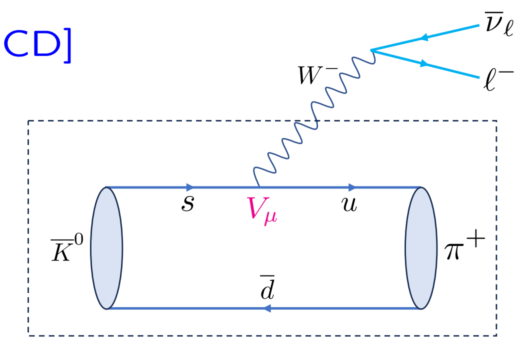
PACS10/L256, PACS10$_c$/L128,L168 are preliminary.

- 2-, 3-point functions w/ $Z(2) \otimes Z(2)$ random source ['08 RBC-UKQCD]

$$C_{\pi,K}(t,p) = \langle 0 | O_{\pi,K}(t,\mathbf{p}) O^\dagger_{\pi,K}(0,\mathbf{p}) | 0 \rangle$$

$$C_{V_\mu}(t,t_{\text{sep}},p) = \langle 0 | O_K(t_{\text{sep}},\mathbf{0}) V_\mu(t,\mathbf{p}) O^\dagger_\pi(0,\mathbf{p}) | 0 \rangle$$

V_μ: Local vector current renormalized by Z_V
Conserved vector current



- finite $p^2 = (2\pi/L)^2 n$, $n = 0$–6, for q^2 interpolation
- several t_{sep} for investigation of excited state contributions

Calculation method w/ local vector current

Details in PACS:PRD101,9,094504(2020)

3-point function* $(0 \ll t \ll t_{\text{sep}})$

$$C_{V_\mu}(t, t_{\text{sep}}, p) = \langle 0 | O_K(t_{\text{sep}}, \mathbf{0}) V_\mu(t, \mathbf{p}) O_\pi^\dagger(0, \mathbf{p}) | 0 \rangle$$

$$= \frac{Z_\pi Z_K}{Z_V} \frac{M_\mu(p)}{4 E_\pi M_K} e^{-E_\pi t} e^{-M_K(t_{\text{sep}}-t)} + \cdots$$

$p^2 \equiv |\mathbf{p}|^2 = \left(\frac{2\pi}{L}\right)^2 n, \ n = 0\text{–}6$

with periodic boundary

$\cdots$ corresponds to excited state contributions

2-point function* $X = \pi, K$ $(0 \ll t)$

$$C_X(t, p) = \langle 0 | O_X(t, \mathbf{p}) O_X^\dagger(0, \mathbf{p}) | 0 \rangle = \frac{Z_X^2}{2E_X} \left(e^{-E_X t} + e^{-E_X(2T-t)} \right) + \cdots$$

*Averaging ones with periodic, anti-periodic temporal boundary conditions reducing wrapping around effect in 3pt, and doubling periodicity in 2pt

$Z_V = 1/\sqrt{F_\pi^{\text{bare}}(0) F_K^{\text{bare}}(0)}$ determined w/ electromagnetic form factor $F_{\pi,K}(0) = 1$

Ratio $(0 \ll t \ll t_{\text{sep}})$

$$\frac{Z_\pi Z_K Z_V C_{V_\mu}(t, t_{\text{sep}}, p)}{C_\pi(t, p) C_K(t_{\text{sep}} - t, 0)} = M_\mu(p) + \underbrace{A(p) e^{-\Delta_\pi(p) t} + B(p) e^{-\Delta_K(t_{\text{sep}}-t)}}_{\text{1st excited state, } \pi', K', \text{ contributions}} + \cdots$$

$\Delta_\pi(p) = E_{\pi'}(p) - E_\pi(p), \Delta_K = M_{K'} - M_K$

Extract $M_\mu(p) = \langle \pi(p) | V_\mu | K(0) \rangle$ w/ fit including 1st excited states

Conserved current case: $V_\mu \to \widetilde{V}_\mu$ and $Z_V = 1$

Calculation method

Details in PACS:PRD101,9,094504(2020)

$M_\mu(p) = \langle \pi(p) | V_\mu | K(0) \rangle$ extracted from ratio

$K_{\ell 3}$ form factors $f_+(q^2), f_0(q^2)$

$$\langle \pi(p) | V_\mu | K(0) \rangle = (p_K + p_\pi)_\mu f_+(q^2) + (p_K - p_\pi)_\mu f_-(q^2)$$

$$f_0(q^2) = f_+(q^2) - \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2)$$

$p_K = (M_K, \mathbf{0}), p_\pi = (E_\pi, \vec{p})$

$q^2 = -(M_K - E_\pi)^2 + p^2$

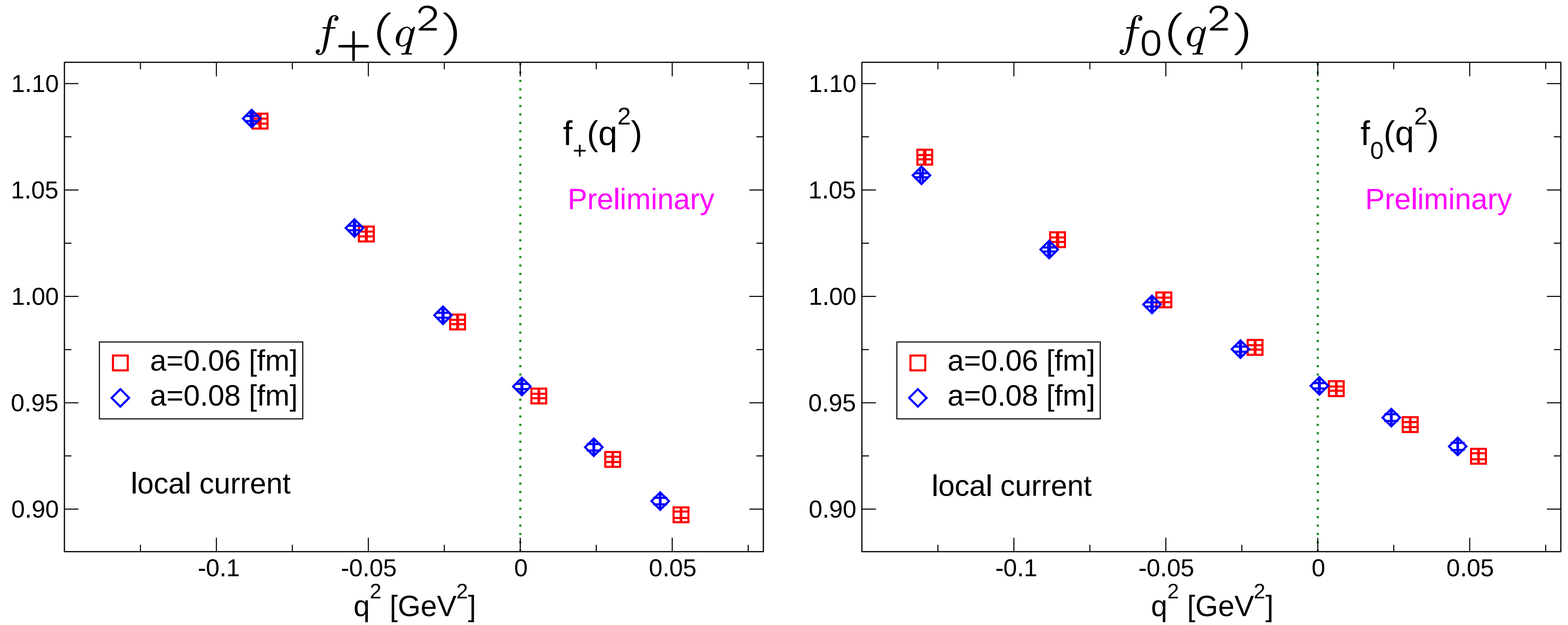
Determination of $|V_{us}|$

$M_4(p), M_i(p) \to f_+(q^2), f_0(q^2)$ at each q^2 except for $p = 0$, where only $f_0(q^2)$

$\to q^2$ interpolation for $f_+(q^2), f_0(q^2) \to f_+(0)\,(= f_0(0))$

$\to |V_{us}|$ through $|V_{us}| f_+(0) = 0.21654(41)$ [Moulson:PoS(CKM2016)033(2017)]

$f_+(q^2)$ and $f_0(q^2)$ with PACS10$_c$/L128,L168 (local current)



Clear signal at both lattice spacings

Several data in tiny q^2 region thanks to huge volume

Little a dependence in small q^2 region, but small a effect in $q^2 \sim 0$

Similar results obtained with conserved current

q^2 interpolation

Fit based on SU(3) NLO ChPT with $f_+(0) = f_0(0)$ [PACS:PRD106,9,094501(2022)]

$$f_+(q^2) = 1 - \frac{4}{F_0^2} L_9(\mu) q^2 + K_+(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_0 + c_2^+ q^4$$

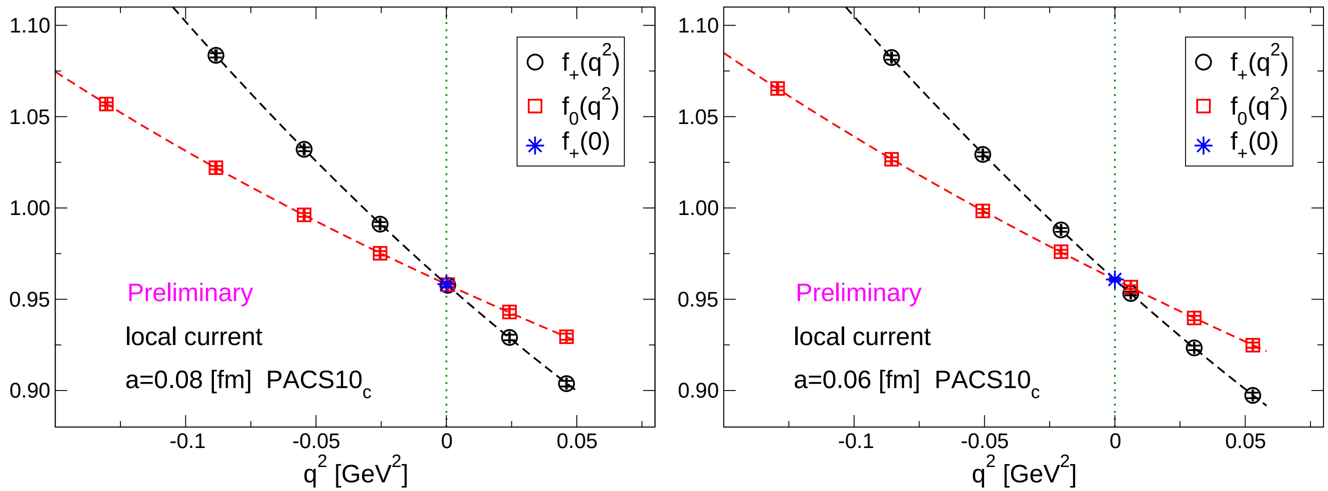
$$f_0(q^2) = 1 - \frac{8}{F_0^2} L_5(\mu) q^2 + K_0(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_0 + c_2^0 q^4$$

K_+, K_0: known functions ['85 Gasser, Leutwyler]

free parameters: $L_5(\mu), L_9(\mu), c_2^+, c_2^0, c_0$

fixed parameters: $\mu = 0.77$ GeV, $F_0 = 0.11205$ GeV

F_0 estimated from FLAG $F^{SU(2)}/F_0$ w/ $F^{SU(2)} = 0.129$ GeV



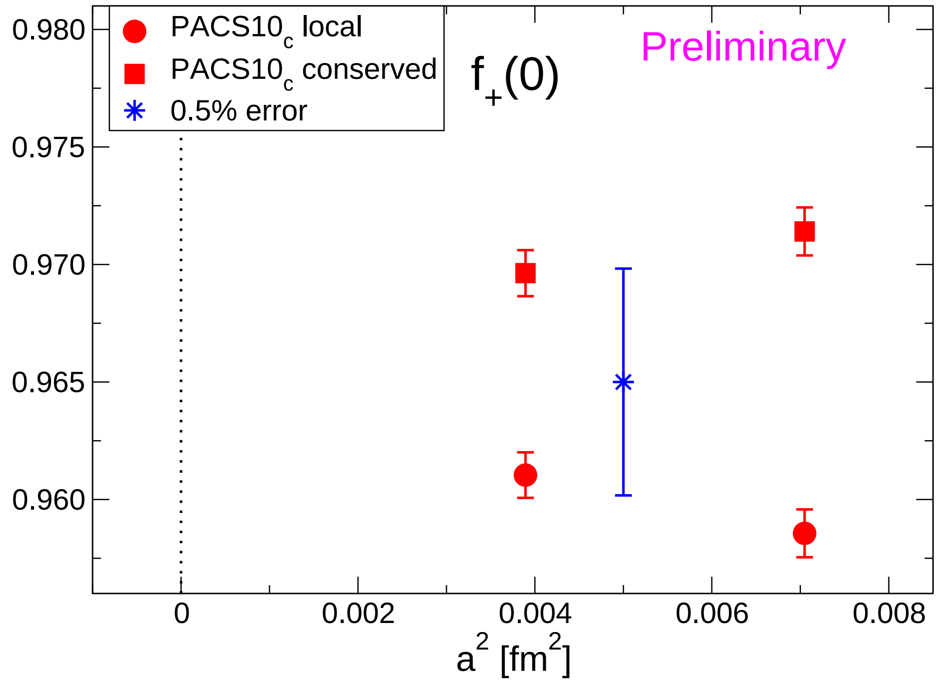
Simultaneous fit for (f_+, f_0) works well.

Tiny extrapolation to physical M_{π^-} and M_{K^0} using same formulas

a[fm]	M_π[MeV]	M_K[MeV]
0.04	137	498
0.06	137	497

$m_{\pi^-} = 139.57061$ MeV, $m_{K^0} = 497.611$ MeV

a dependence of $f_+(0)$ from PACS10$_c$ conf.



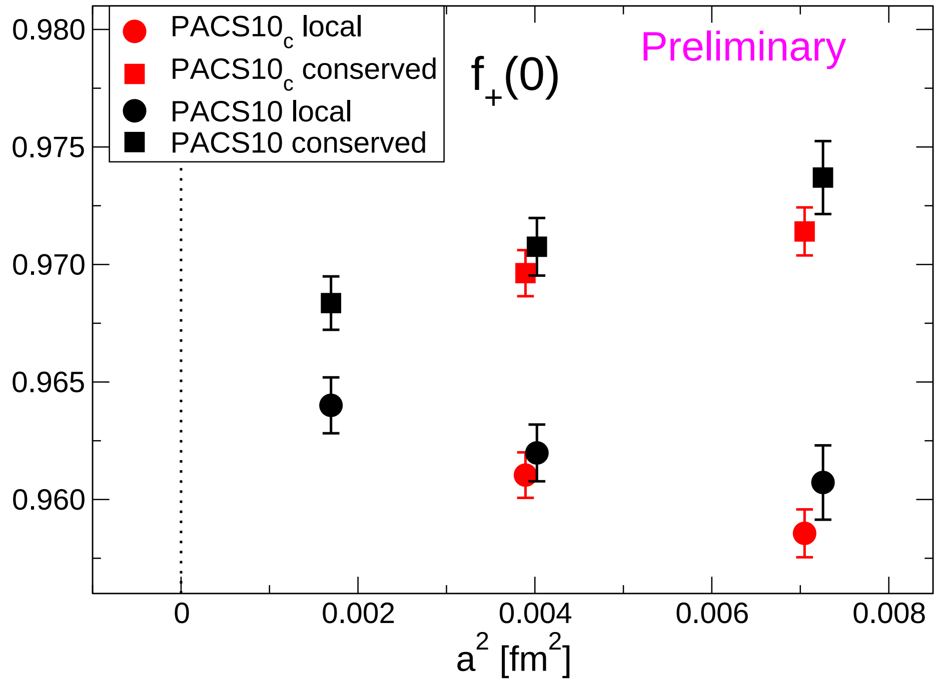
Deviation between local and conserved currents

Due to different definition of vector current on lattce
deviation decreases with a as expected

Difference is tiny $\rightarrow f_+(0)$ with about 0.5% error

Calculation with PACS10c/L256 will be started soon.

Continnum extrapolation of $f_+(0)$ from PACS10 conf.

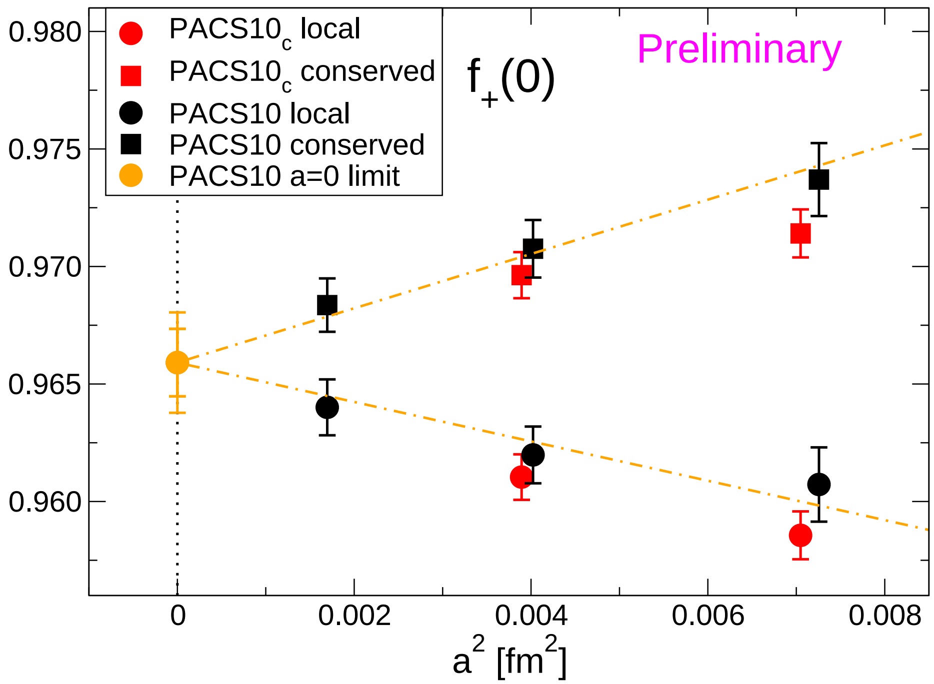


Same calculations with PACS10 configuraions at 3 lattice spacings

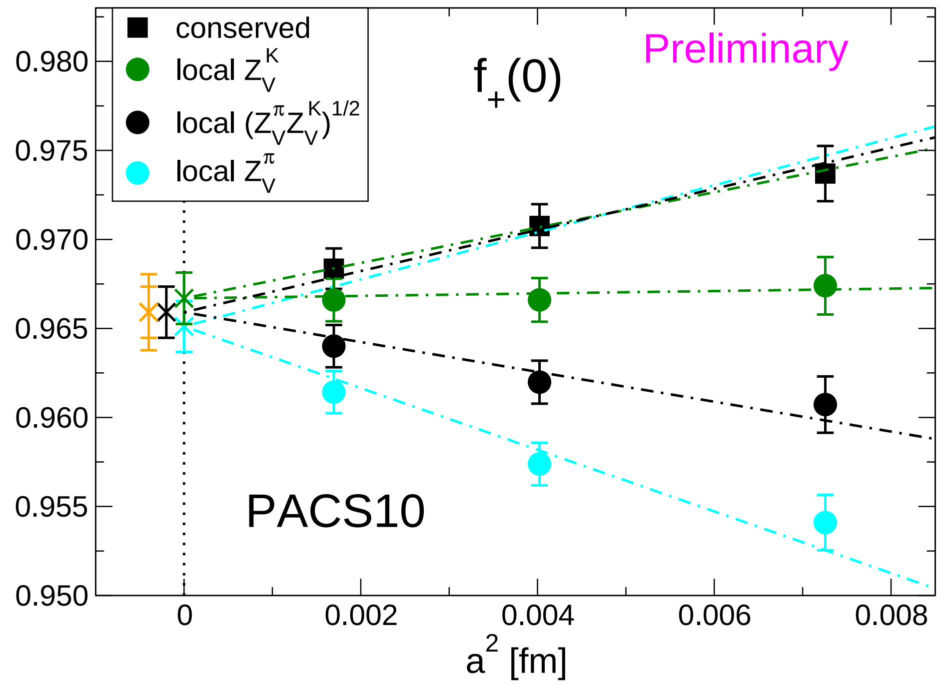
Similar trends observed to PACS10$_c$ results even at smaller a

charm effect might be smaller than statistical error

Continnum extrapolation of $f_+(0)$ from PACS10 conf.



closed (open) symbol: physical (simulation) point data



example of systematic error estimate

$Z_V^K = 1/F_K^{\text{bare}}(0), Z_V^\pi = 1/F_\pi^{\text{bare}}(0)$

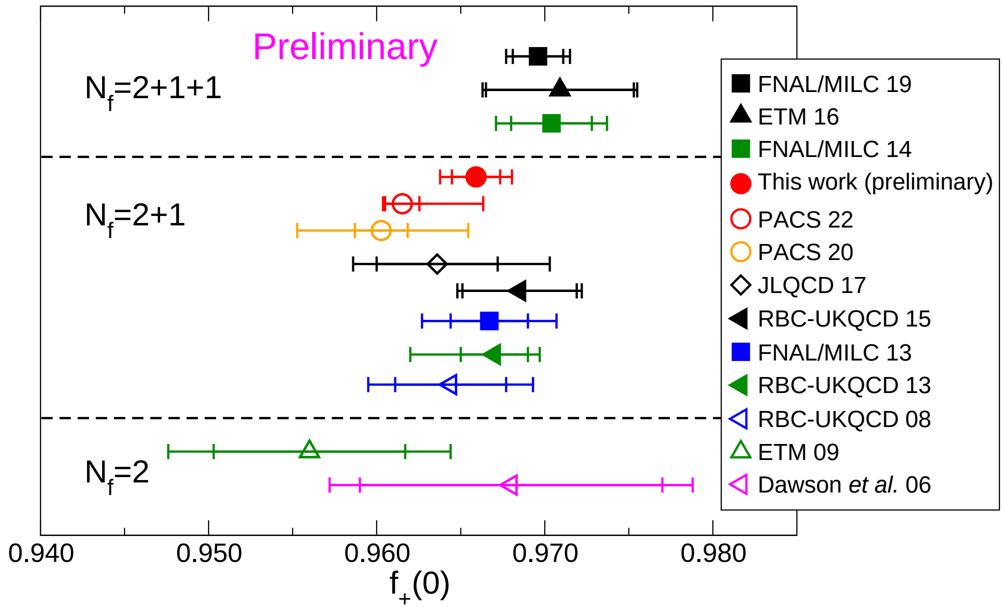
Same calculations with PACS10 configuraions at 3 lattice spacings

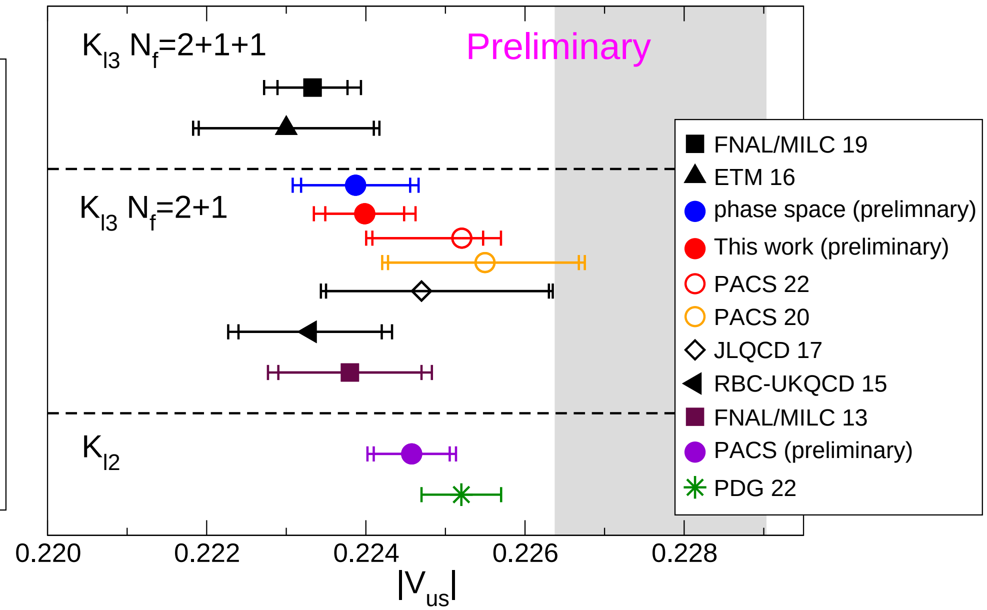
Similar trends observed to PACS10$_c$ results even at smaller a

charm effect might be smaller than statistical error

Continuum extrapolation with only PACS10 results

$f_+(0)$ and $|V_{us}|$ from PACS10 confs.





inner, outer = statistical, total(stat.+sys.) inner, outer = lattice, total(lat.+exp.)

Standard model (SM) prediction using $|V_{us}| \approx \sqrt{1 - |V_{ud}|^2}$, grey band: ['20 Hardy, Towner]

$f_+(0)$: Reasonably agree with previous lattice calculations $\lesssim 2\sigma$

$|V_{us}|$ using $|V_{us}|f_+(0) = 0.21654(41)$ ['17 Moulson]

agree with $|V_{us}|$ from K_{l2} using f_K/f_π $\quad \dfrac{|V_{us}|}{|V_{ud}|}\dfrac{f_K}{f_\pi} = 0.27683(35)$ ['19 Di Carlo *et al.*]

$2 \sim 3\sigma$ difference from CKM unitarity (grey band)

Future work: $a = 0$ limit with PACS10$_c$ results + IB correction

Summary

PACS10 Project

calculation w/o three main systematic uncertainties in lattice QCD

PACS10/PACS10$_c$ configuration

$V \gtrsim (10\ \mathrm{fm})^4$ in physical point at three lattice spacings

Various calculations

Hadron spectrum, Nucleon form factor, Light meson charge radii, Proton decay matrix element, Hadron vacuum polarization

Kaon semileptonic decay form factor with PACS10/PACS10$_c$ confs.

Precise calculation for $f_+(q^2)$ and $f_0(q^2)$ to determine $|V_{us}|$

Comparable result of $|V_{us}|$ with those from other groups

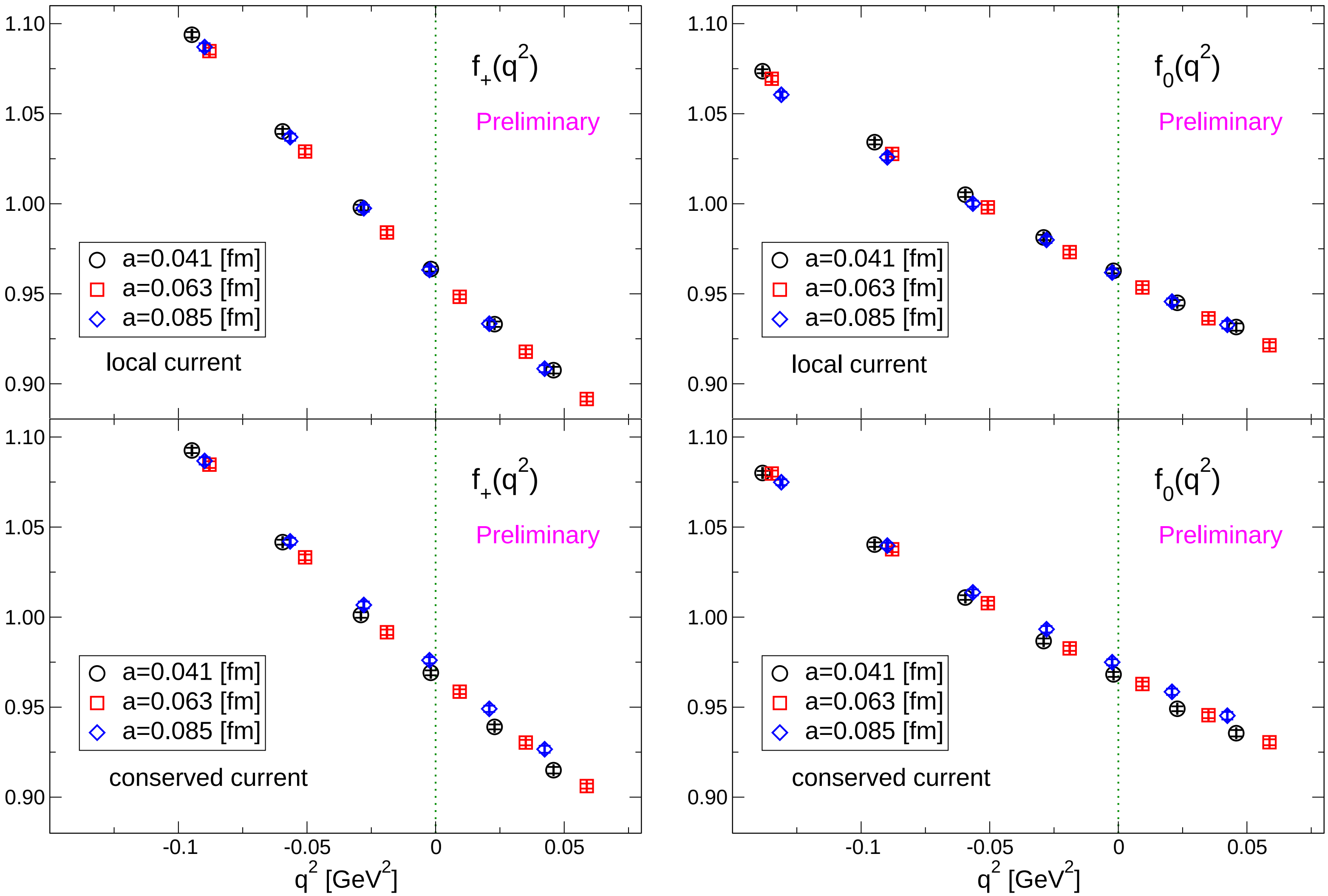
Future work for $K_{\ell 3}$ calculation

Calculation with PACS10$_c$/L256 configuration for $a \to 0$ extrapolation

Estimate systematic uncertainties *e.g.*, isospin breaking

Back up

$f_+(q^2)$ and $f_0(q^2)$ with PACS10 confs.



Clear signal at three lattice spacings

Several data in tiny q^2 region thanks to huge volume

Update from Lat23: Removing $\mathcal{O}(a^2)$ factor in conserved $f_+(q^2), f_0(q^2)$ due to point-splitting current

q^2 interpolation + $a \to 0$ extrapolation with PACS10 confs.

Fit based on SU(3) NLO ChPT with $f_+(0) = f_0(0)$ c.f.) PACS:PRD106,9,094501(2022)

$$f_+(q^2) = 1 - \frac{4}{F_0^2} L_9(\mu) q^2 + K_+(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_M \Delta M + c_2^+ q^4 + g_+^{\text{cur}}(a, q^2)$$

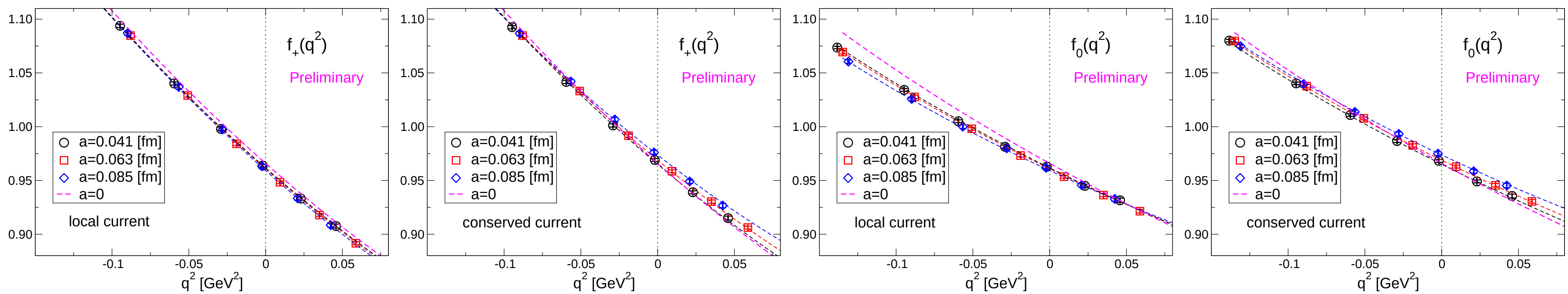
$$f_0(q^2) = 1 - \frac{8}{F_0^2} L_5(\mu) q^2 + K_0(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_M \Delta M + c_2^0 q^4 + g_0^{\text{cur}}(a, q^2)$$

K_+, K_0: known functions ['85 Gasser, Leutwyler], $\Delta M = (M_K^2 - M_\pi^2)^2$

$g_{+,0}^{\text{cur}} = \sum_{n,m} e_{+,0}^{\text{cur},nm} a^n q^{2m}$, cur = local, conserved: a^2, a extrapolation investigated

free parameters: $L_5(\mu), L_9(\mu), c_0, c_2^+, c_2^0 + e_{+,0}^{\text{cur},nm}$

fixed parameters: $\mu = 0.77$ GeV, $F_0 = 0.11205$ GeV $\quad$ F_0 estimated from FLAG $F^{\text{SU(2)}}/F_0$ w/ $F^{\text{SU(2)}} = 0.129$ GeV



magenta dashed-line : $a = 0$ @ physical point

Simultaneous fit for (f_+, f_0) with (local,conserved)

Tiny extrapolation to physical M_{π^-} and M_{K^0} using same formulas

a[fm]	M_π[MeV]	M_K[MeV]
0.041	142	514
0.063	137	501
0.085	135	497

$m_{\pi^-} = 139.57061$ MeV, $m_{K^0} = 497.611$ MeV

Phase space integral I_K^ℓ

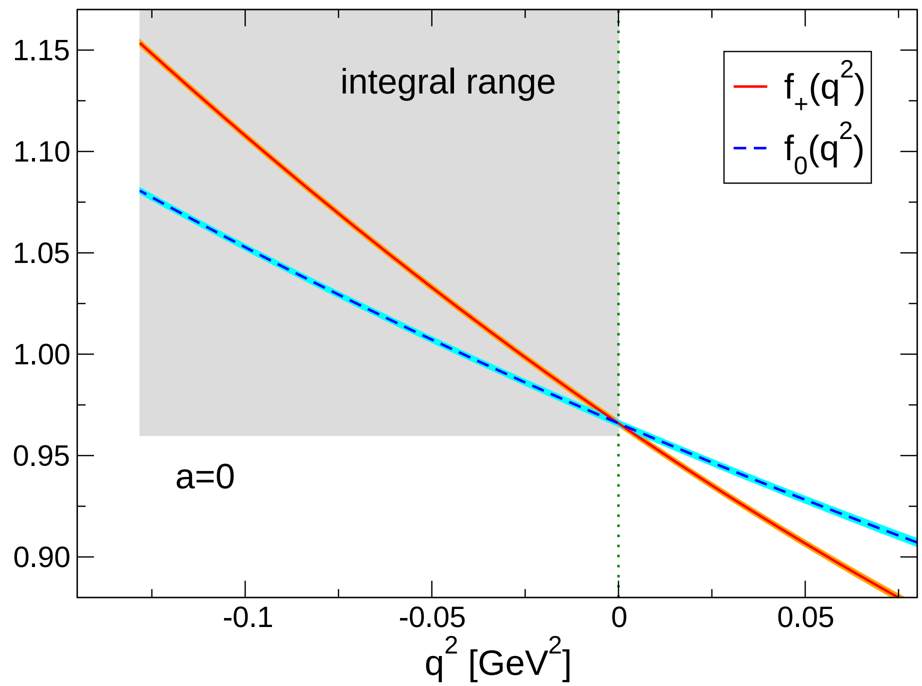
$\Gamma_{K_{\ell 3}} = C_{K_{\ell 3}} (|V_{us}| f_+(0))^2 I_K^\ell$ $\quad$ $\Gamma_{K_{\ell 3}}$: decay width, $C_{K_{\ell 3}}$: known factor, $\ell = e, \mu$

$|V_{us}| f_+(0) = 0.21654(41)$ ['17 Moulson]

$\leftarrow$ I_K^ℓ from dispersive representation of experimental $\overline{F}_{+,0}(t)$

$$I_K^\ell = \int_{m_\ell^2}^{(M_K - M_\pi)^2} dt \left(J_+(t) \overline{F}_+^2(t) + J_0(t) \overline{F}_0^2(t) \right), \quad \overline{F}_{+,0}(t) = \frac{f_{+,0}(-t)}{f_+(0)}$$

$J_{+,0}(t)$: known function ['84 Leutwyler, Roos]



Phase space integral I_K^ℓ (old result)

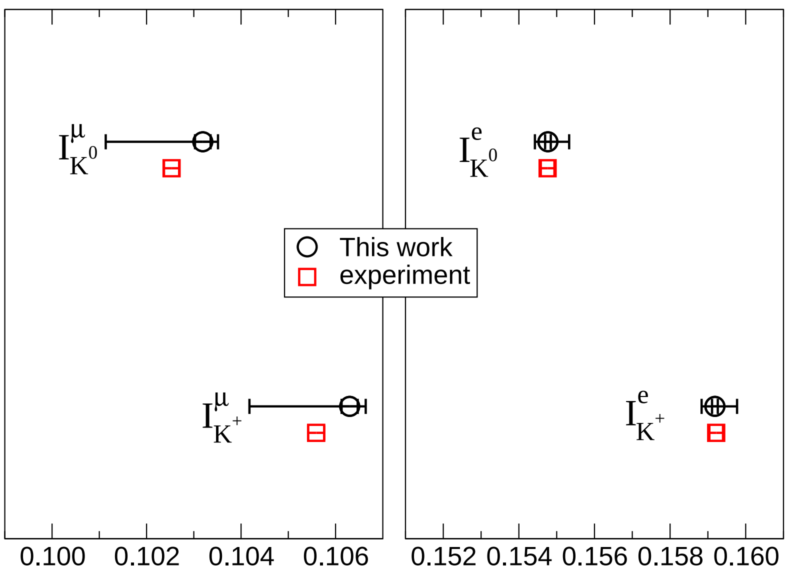
$\Gamma_{K_{\ell 3}} = C_{K_{\ell 3}} (|V_{us}| f_+(0))^2 I_K^\ell$ $\quad$ $\Gamma_{K_{\ell 3}}$: decay width, $C_{K_{\ell 3}}$: known factor, $\ell = e, \mu$

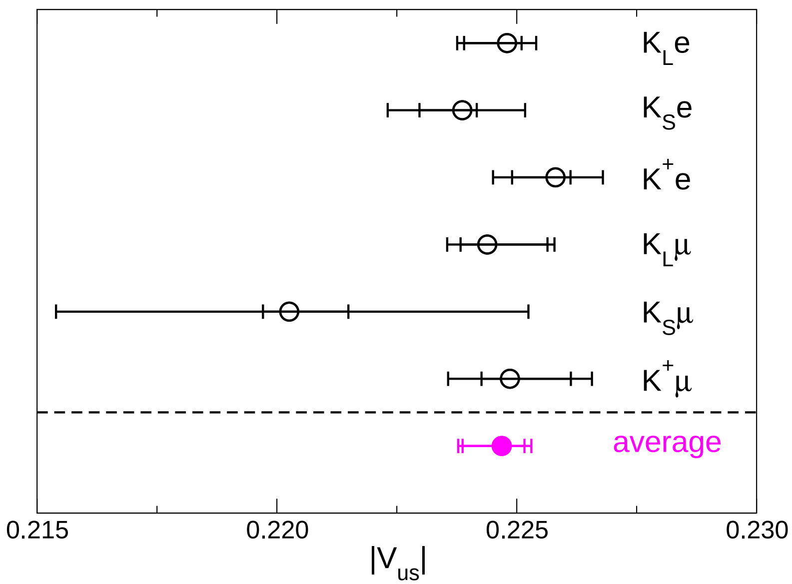
$|V_{us}| f_+(0) = 0.21654(41)$ ['17 Moulson]

$\leftarrow$ I_K^ℓ from dispersive representation of experimental $\overline{F}_{+,0}(t)$

$$I_K^\ell = \int_{m_\ell^2}^{(M_K - M_\pi)^2} dt \left(J_+(t) \overline{F}_+^2(t) + J_0(t) \overline{F}_0^2(t) \right), \quad \overline{F}_{+,0}(t) = \frac{f_{+,0}(-t)}{f_+(0)}$$

$J_{+,0}(t)$: known function ['84 Leutwyler, Roos]





inner: stat. error; outer: (stat.+sys.) error

Reasonably agree with experimental values ['10 Antonelli *et al.*]

Large uncertainty from fit form of $a \to 0$

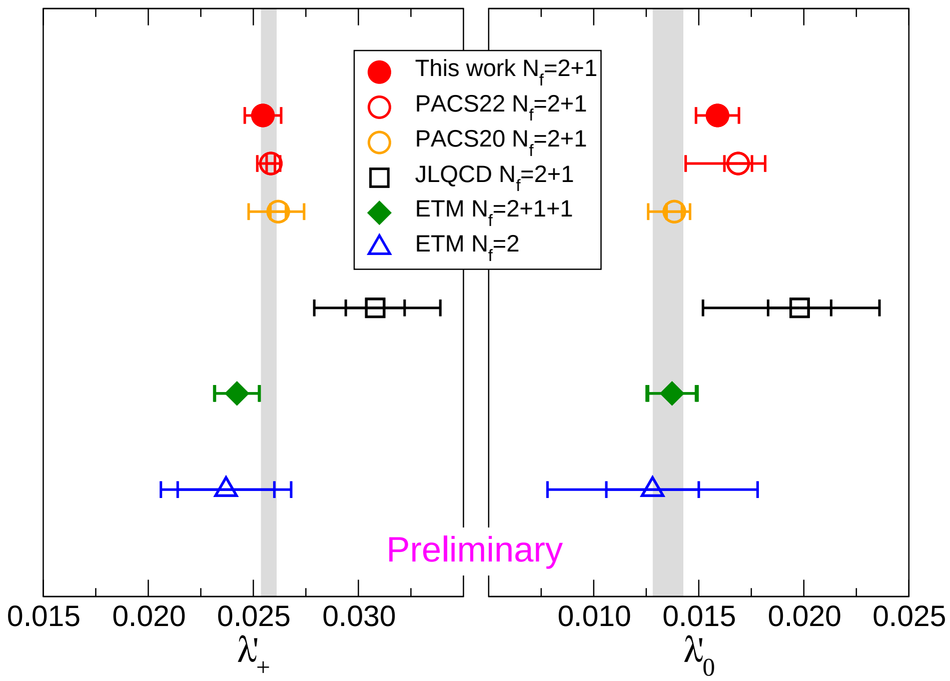
Slope and curvature for $f_+(q^2)$ and $f_0(q^2)$

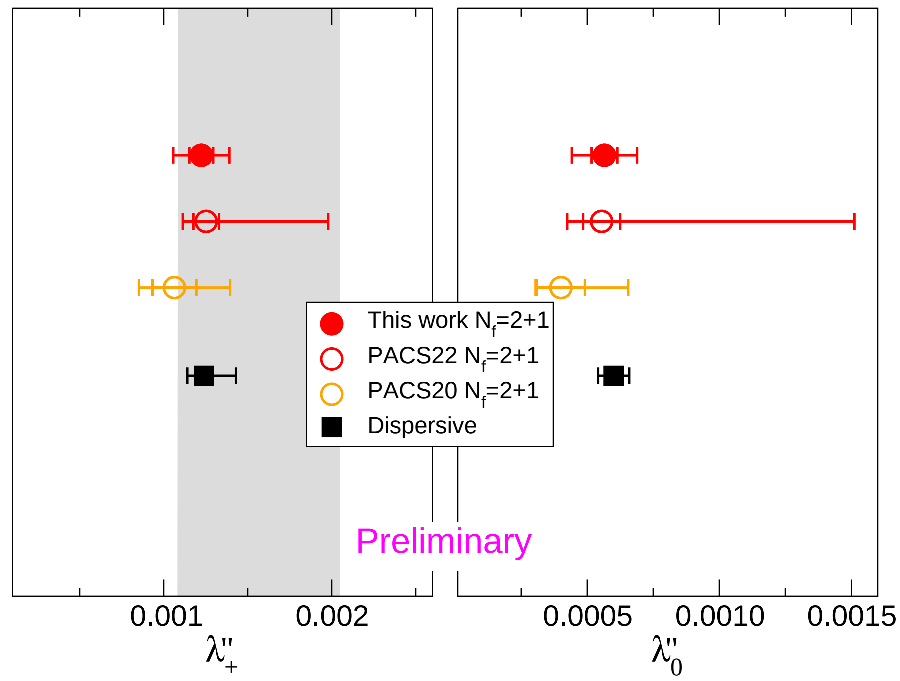
slope

$$\lambda'_{+,0} = \frac{M_{\pi^-}^2}{f_+(0)} \frac{df_{+,0}(q^2)}{d(-q^2)}$$

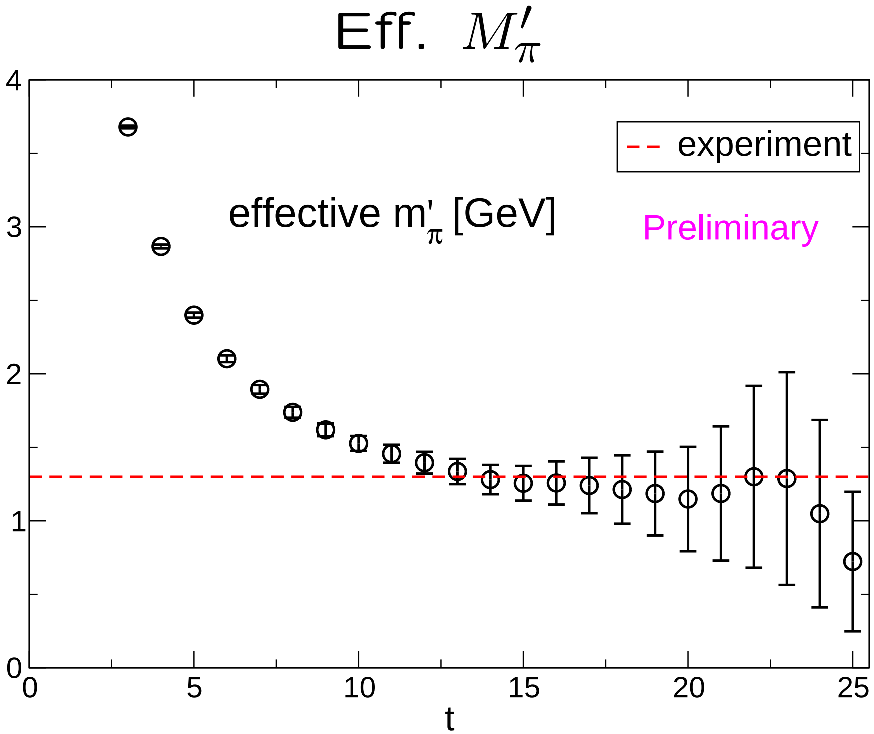
curvature

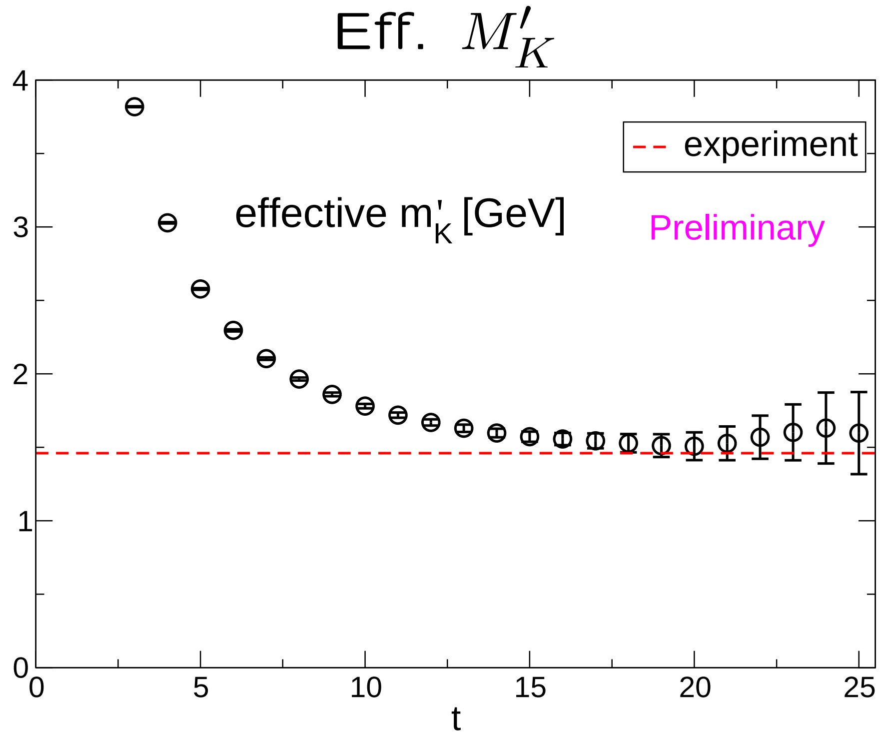
$$\lambda''_{+,0} = \frac{M_{\pi^-}^4}{f_+(0)} \frac{d^2 f_{+,0}(q^2)}{d(-q^2)^2}$$





Effective mass for 1st excited states π', K' on PACS10/L256





Effective mass of 1st excited state (local source)

$$M'(t) = \log\left(\frac{C'(t)}{C'(t+1)}\right),\ C'(t) = C(t) - \underline{A_0 e^{-M_0 t}}$$

ground state contribution from fit in $t \gg 1$