Electromagnetic Form Factors of the Nucleon at Large Momentum

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(Lattice Hadron Physics collaboration)

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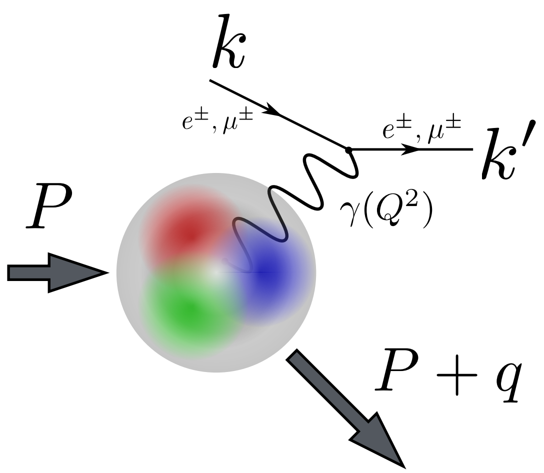
Outline

- Large-Q^2 Nucleon vector form factors
  - *Motivation*
  - *Methodology & challenges on a lattice*
- Preview of results
  - *Form factors and ratios*
  - *Light-Flavor contributions*
- Examining systematic effects
  - *G_E/G_M ratio*
  - *Disconnected diagrams*
- Summary & Outlook

Nucleon Elastic E&M Form Factors



$$Q^2 = -(k-k')^2 > 0$$

Elastic e^-p amplitude

$$\langle P+q|\, \bar{q}\gamma^\mu q\, |P\rangle = \bar{U}_{P+q}\Big[\underbrace{F_1(Q^2)}_{\text{(Dirac)}}\gamma^\mu + \underbrace{F_2(Q^2)}_{\text{(Pauli)}}\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]U_P$$

Sachs Electric $\quad G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)$

Magnetic $\quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$

Elastic e^-p cross-section

- $G_{E,M}$ from ϵ-dep. at fixed $\tau(Q^2)$ (***"Rosenbluth separation"***)
- dominated by G_M at large Q^2
- 2γ corrections at $Q^2 \gtrsim 1$ GeV^2

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\mathrm{Mott}}}{1+\tau}\left[G_E^2 + \frac{\tau}{\epsilon}G_M^2\right]$$

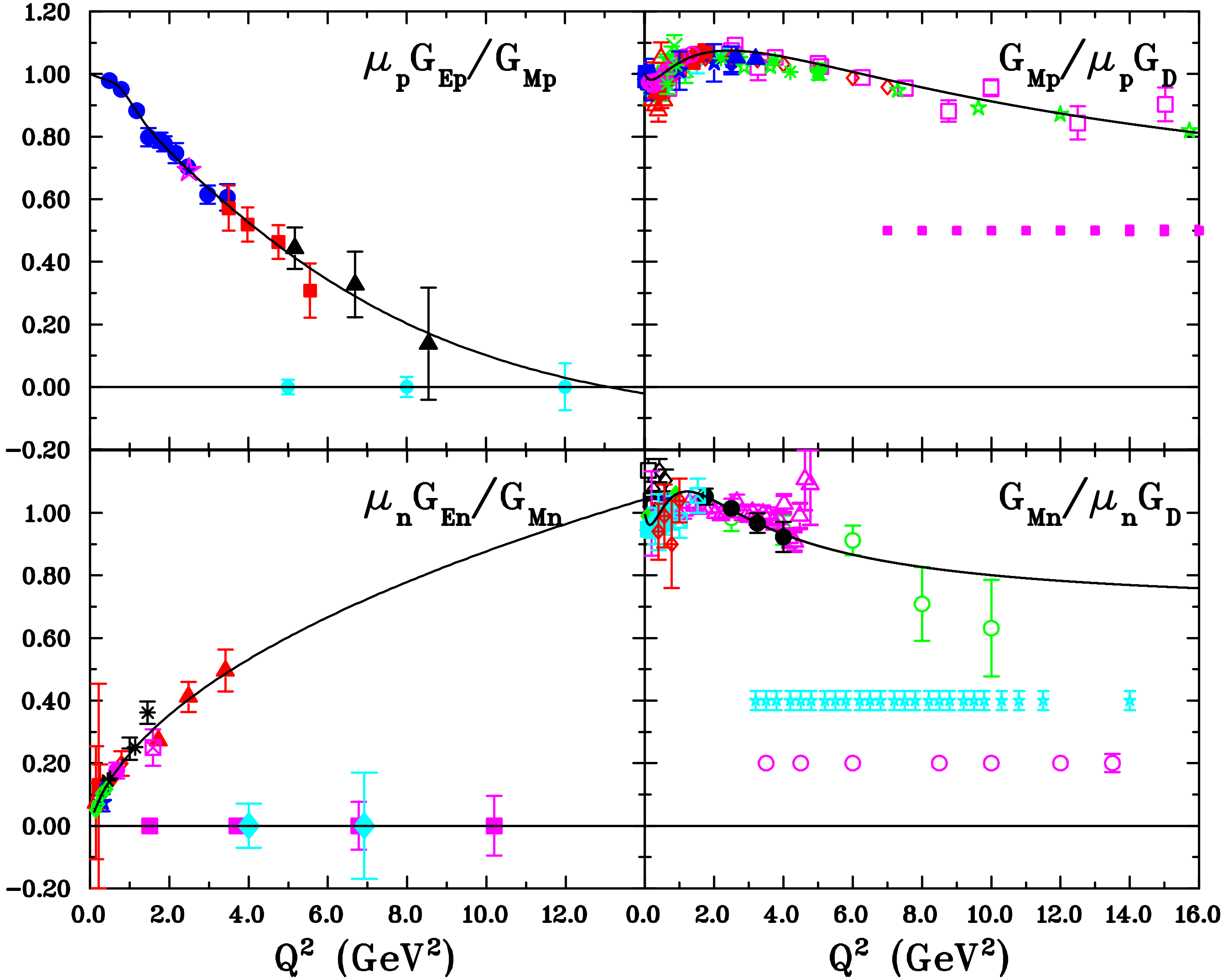
$$\tau = \frac{Q^2}{4M_N^2} \qquad \epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1}$$

Polarization transfer: polarized e^-beam
+ detect polarization of recoil nucleon
(alt.: transverse asymmetry on pol. target)

- G_E/G_M *ratio* (only small radiative corrections)

$$P_t/P_l \propto G_E/G_M$$

Recent Experiments @JLab : Projections

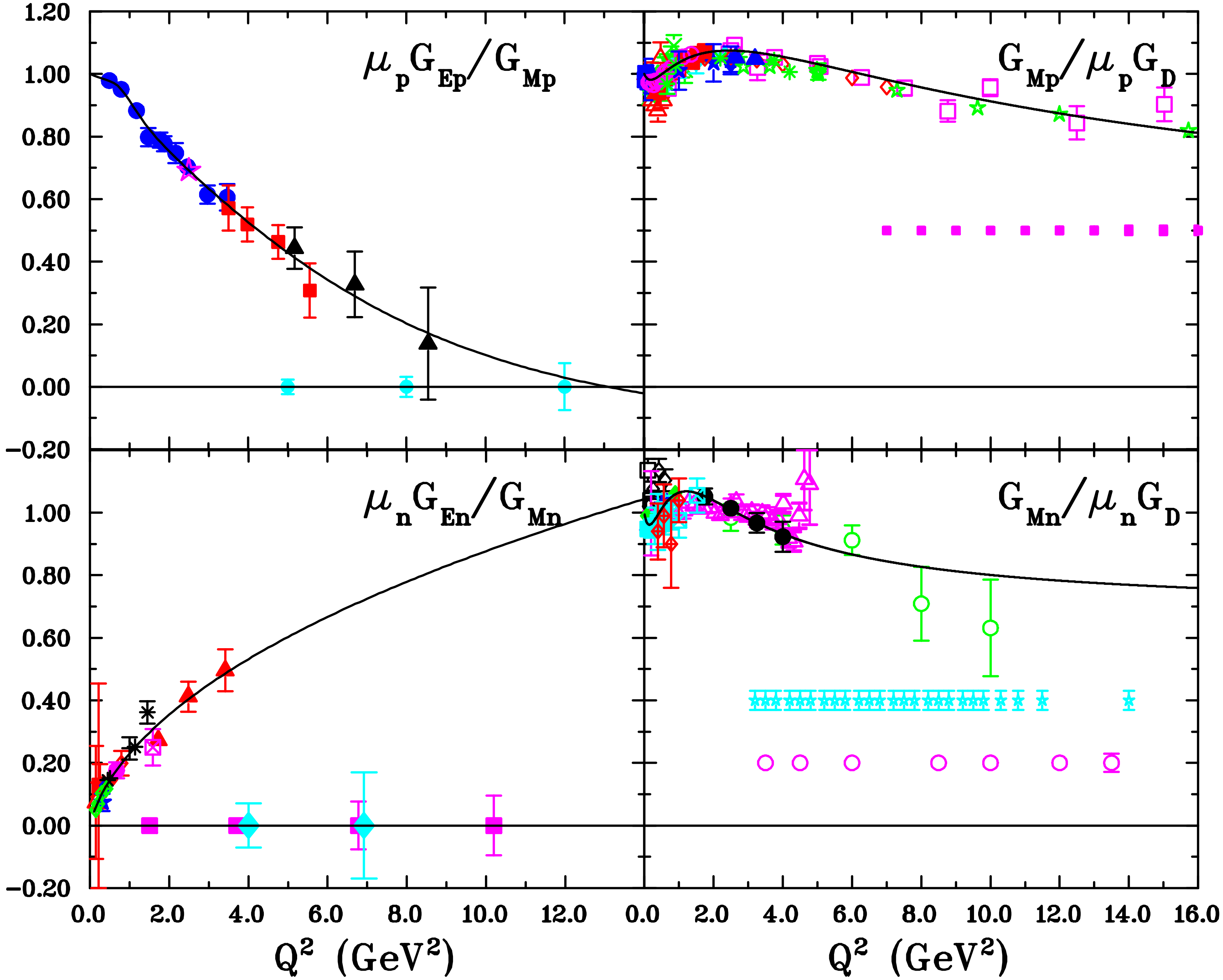


Projected new precision on proton & neutron form factors
[V. Punjabi et al, EPJ A51: 79 (2015); arXiv: 1503.01452]

Experiments at JLab@12GeV

- Hall A (HRS, SBS):
  G_{Mp} @ $Q^2 \lesssim 17.5$ GeV^2
  G_{Ep}/G_{Mp} @ $Q^2 \lesssim 15$ GeV^2 ;
  G_{Mn} @ $Q^2 \lesssim 18$ GeV^2
  G_{En}/G_{Mn} @ $Q^2 \lesssim 10.2$ GeV^2 ;
- Hall B (CLAS12):
  G_{Mn} @ $Q^2 \lesssim 14$ GeV^2
- Hall C :
  G_{En}/G_{Mn} @ $Q^2 \lesssim 6.9$ GeV^2

Recent Experiments: Projections + G_{Mp} Result



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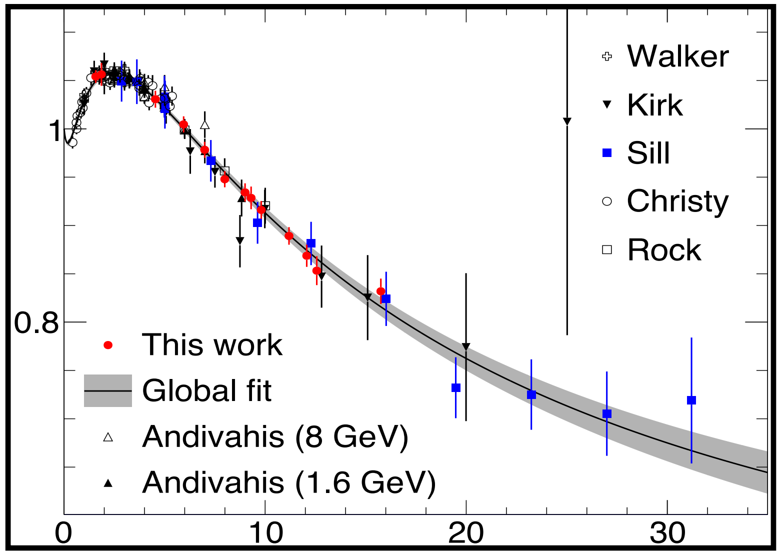


New G_{Mp} results (JLab Hall A)
[Christy et al, PRL'22]

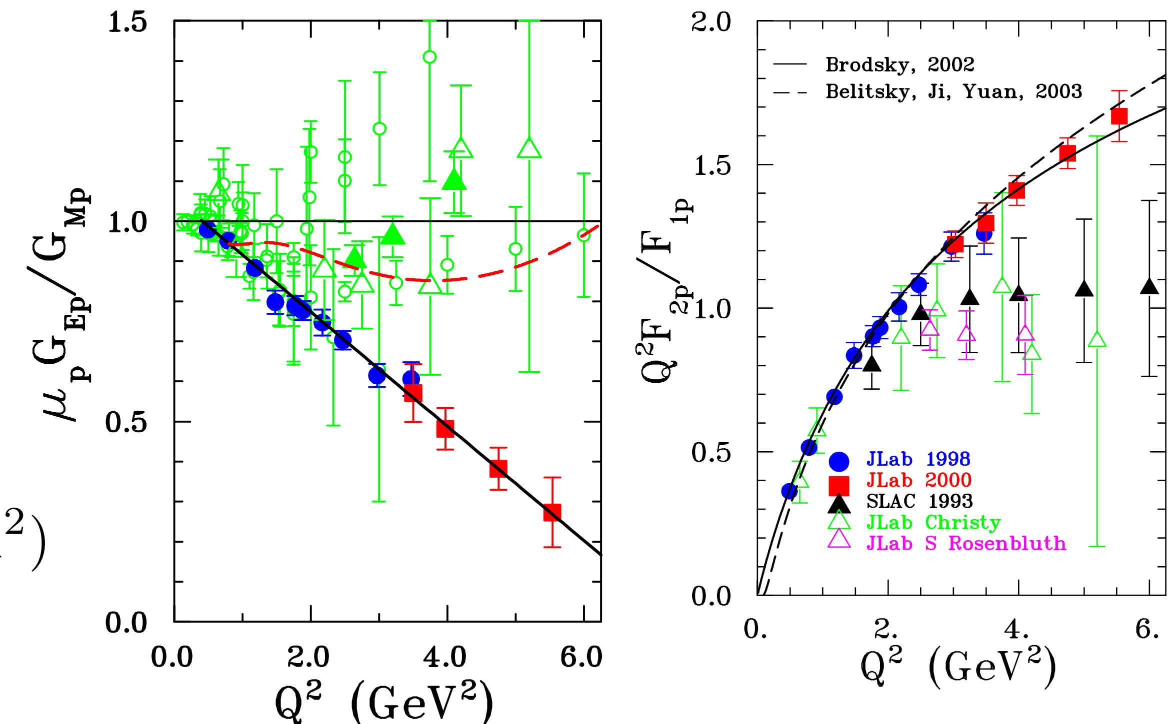
Why Large-Q^2 Nucleon Form Factors?

- *Rosenbluth / Pol.transfer discrepancy in G_E/G_M ?*
  Input from lattice QCD may help

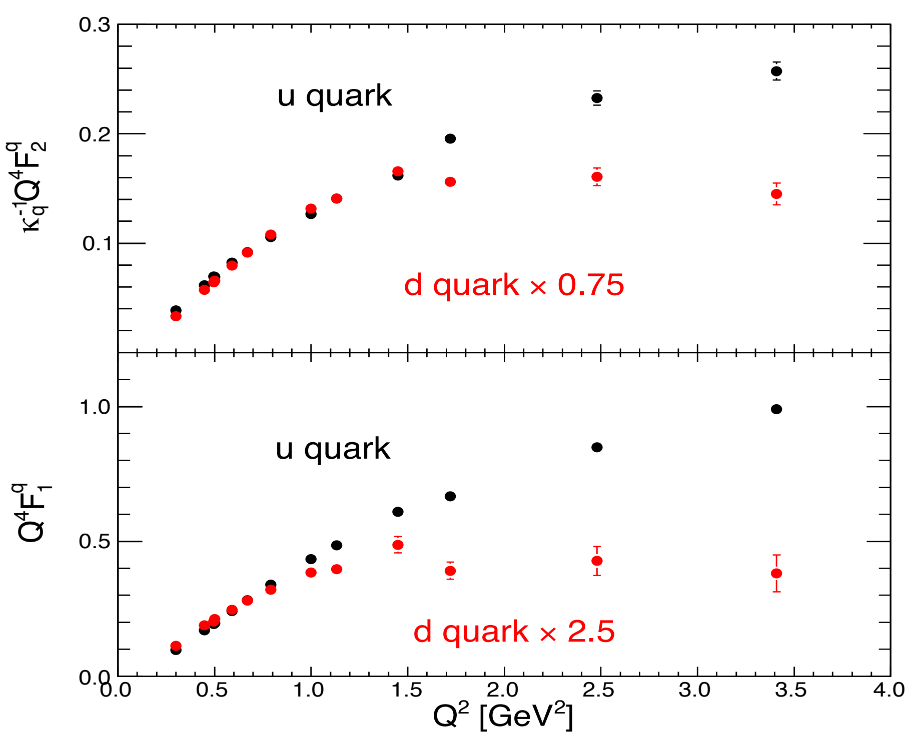
- *Role of diquark correlations in elastic scattering ?*
  Neutron & proton G_E/G_M at/above $Q^2 = 8$ GeV^2

- *Scale of transition to perturbative QCD ?*
  (F_2/F_1) scaling at large Q^2 : $\quad Q^2 F_{2p}/F_{1p} \overset{?}{\propto} \log^2(Q^2/\Lambda^2)$

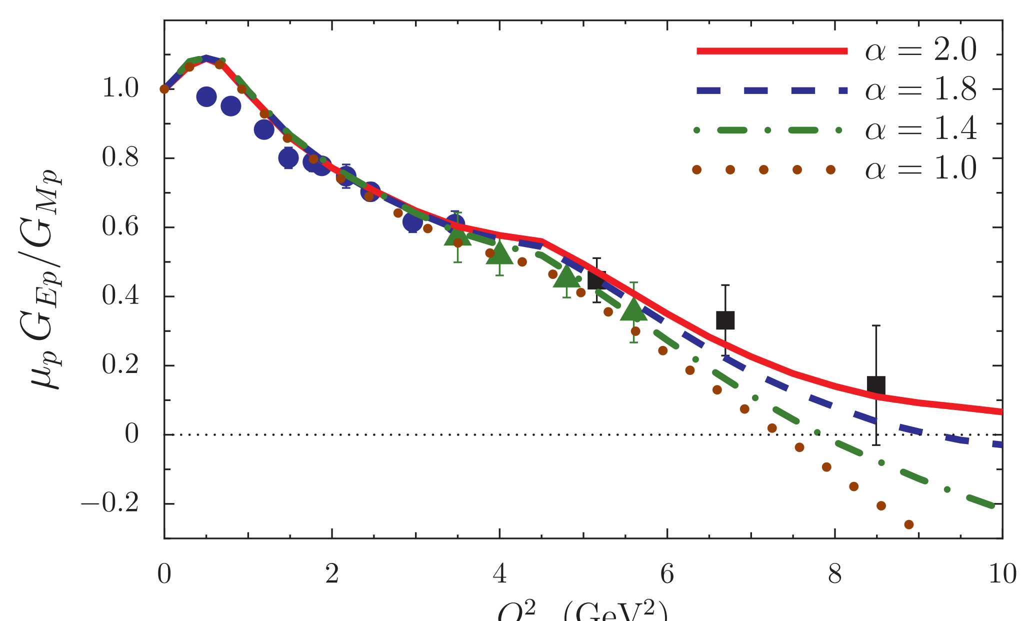
- *What are contributions from u and d flavors?*
  Proton <u>and</u> neutron data needed in wide Q^2 range



[Research Mgmt. Plan for SBS(JLab Hall A)]



[G.D.Cates, C.W.de Jager, S.Riordan, B.Wojtsekhovski, PRL106:252003, arXiv:1103.1808]



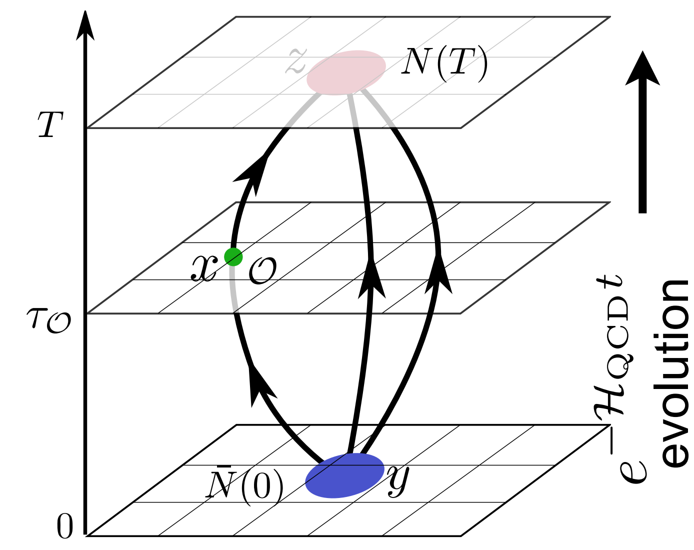
Dyson-Schwinger Eqns : quarks & 0^+, 1^+ diquarks
($\alpha \approx$ rate of transition const.quarks → pQCD with Q^2)
[Cloet, Roberts, Prog.Part.Nucl.Phys 77:1 (2014)]

Nucleon Structure from Lattice QCD (in a Nutshell)

- Compute nucleon correlation functions

<u>Hadron Spectrum:</u>

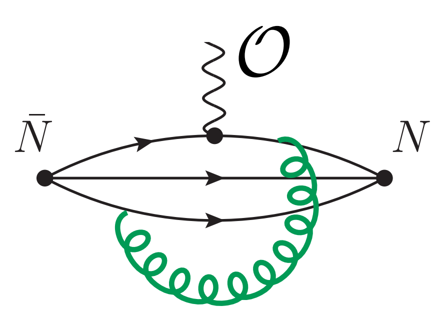
$$\langle N(\vec{p},T)\,\bar{N}(0)\rangle = \sum_{\vec{y}} \langle N(\vec{y},T)\,\bar{N}(0)\rangle$$
$$\sim |Z_0|^2 e^{-E_0 T}\left[1 + e^{-\Delta E_{10} T} + \ldots\right]$$



Quark lines = $(\not{D} + m)^{-1} \cdot \psi$

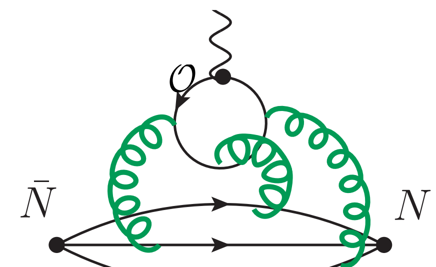
<u>Hadron Matrix Elements:</u>

$$\langle N(T)\mathcal{O}(\tau)\bar{N}(0)\rangle =$$



quark-connected

+



quark-disconnected

- Extract ground state matrix elements

$$\langle N(T)\mathcal{O}(\tau)N(0)\rangle = \sum_{n,m} Z_m e^{-E_n(T-\tau)} {\color{red}\langle n|\mathcal{O}|m\rangle} e^{-E_m \tau} Z_n^*$$

$$\xrightarrow{T\to\infty} Z_0 Z_{0'}^* e^{-E_0\tau - E_{0'}(T-\tau)}\left[{\color{red}\langle 0'|\mathcal{O}|0\rangle} + \underbrace{O\left(e^{-\Delta E\{T,\tau,(T-\tau)\}}\right)}_{\text{excited states}}\right]$$

Ground state form factors

$$[\bar{u}'\gamma^\mu u]{\color{red}F_1} + [\bar{u}'\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}u]{\color{red}F_2}$$

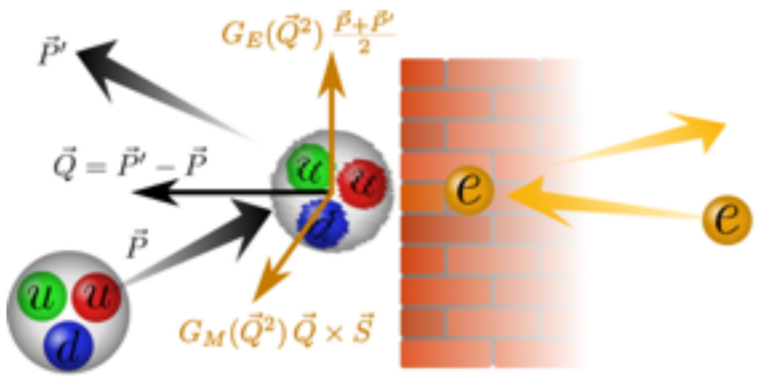
Fit and discard

Systematic uncertainties

- discretization effects
- unphysical-heavy pion mass
- finite volume
- excited state contributions
- quark-disconnected diagrams

Challenges at Large Q^2

- Excited-states gaps shrink
  - $E_1 - E_0 = \sqrt{M_1^2 + \vec{p}^2} - \sqrt{M_2^2 + \vec{p}^2} < M_1 - M_0$
  - N(~1500): $p_N \to 1.5$ GeV ⇒ ΔE = 500 → 300 MeV



- Stochastic noise grows faster with T [Lepage'89]:

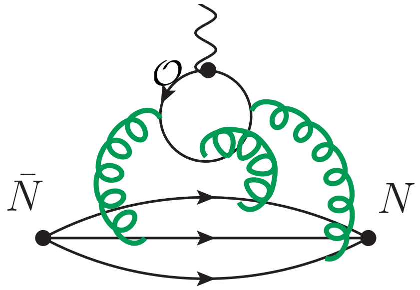
$$\begin{array}{lll} \text{Signal} & \langle N(T)\bar{N}(0)\rangle & \sim e^{-E_N T} \\ \text{Noise} & \langle |N(T)\bar{N}(0)|^2\rangle - |\langle N(T)\bar{N}(0)\rangle|^2 & \sim e^{-3m_\pi T} \\ \text{Signal/Noise} & & \sim e^{-(E_N - \frac{3}{2}m_\pi)T} \end{array}$$

SNR decay ~ O(10^{–4})
at 1 fm/c
(phys.quarks, $Q^2 \approx 12$ GeV^2)

- Discretization effects:
  O(a) Correction to current operator

$$(V_\mu)_I = [\bar{q}\gamma_\mu q] + c_V\, a\, \underbrace{\partial_\nu[\bar{q}i\sigma_{\mu\nu}q]}_{\propto Q}$$

- Quark-disconnected contributions:
  negligible (≲1%) at $Q^2 \leq 1$ GeV^2, unknown at large Q^2



- No reliable EFT/ChPT for m_π-, lattice-volume extrapolation at large-$p_{Nucleon}$

Many of the same challenges as for parton physics on a lattice
(LaMET , Quasi-/Pseudo- PDFs/DAs/TMDs)

Present QCD Calculation Parameters

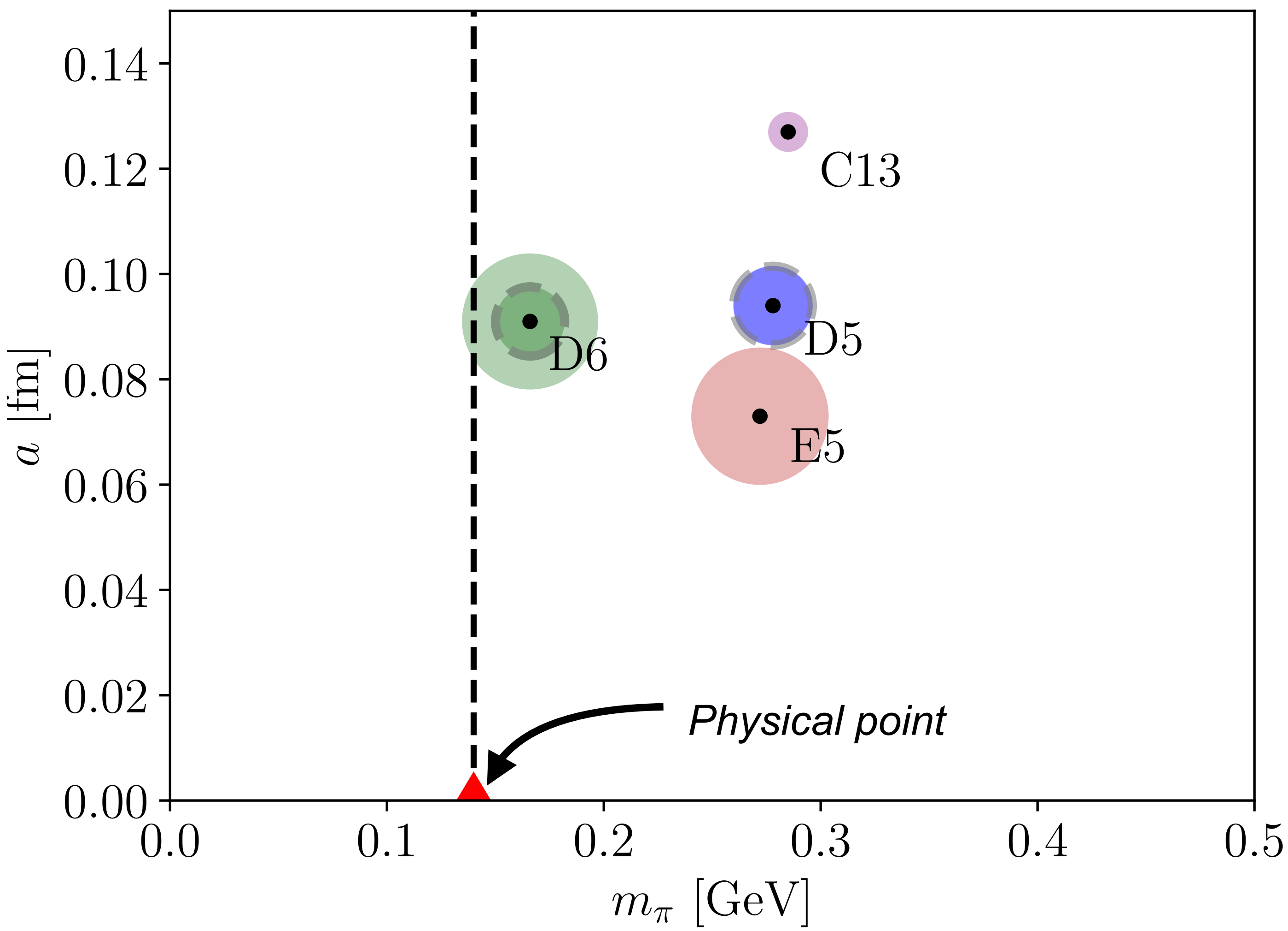
QCD on lattice with $N_F = 2{+}1$ O(a^2)-improved dynamic quarks

Many thanks to [JLab / W&M / LANL / MIT]

- Lattice spacing $a \approx 0.073 \div 0.091$ fm
- SU(2)$_f$ -symmetric + strange quarks
- Pion mass $m\pi = 170 \div 280$ MeV
- Physical volume $L \gtrsim 3.7\,(m\pi)^{-1}$
- Euclidean time t_{sep}= 0.5 ÷ 1.1 fm

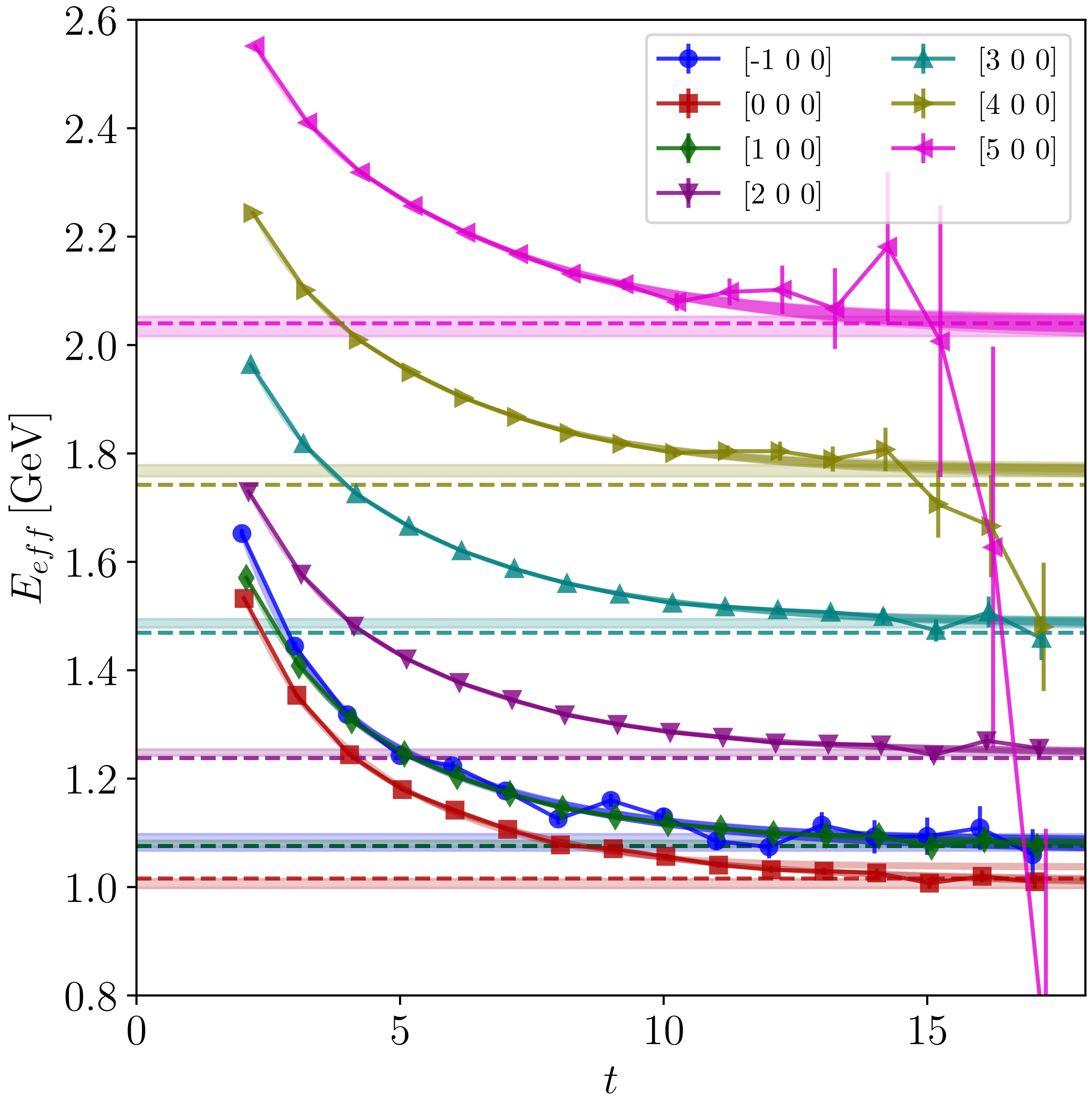
2022/25:

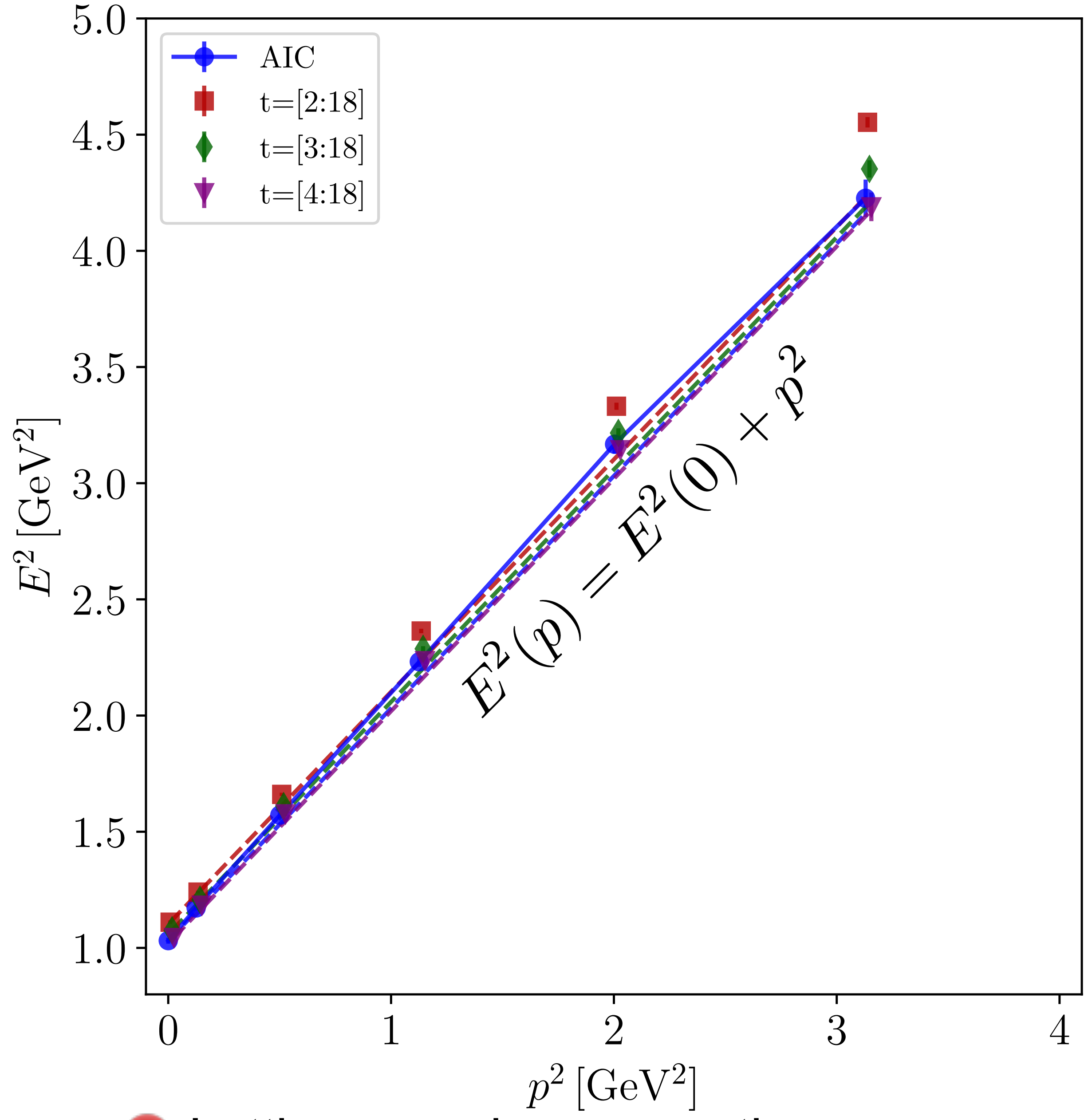
- MC Statistics ≳250k on D6 (48^3 x 96), E5 (48^3 x 128)
- Multitude of kinematic combos.
- Disconnected contractions on D5, D6



Lattice Nucleon Energy & Dispersion Relation (E5)

- E5 : $m\pi =$ 272 MeV , spacing $a = 0.073$ fm , 266k MC samples





- 2-state fits compared to Effective energy

$$E_{eff} = \frac{1}{a} \log \frac{C_{N\bar{N}}(t)}{C_{N\bar{N}}(t+a)}$$

- Lattice energies vs. continuum dispersion relation (dashed)

Analysis Minutiae

- Filter outliers: large deviation from median (~2% at a=0.09 fm, ~0.15% at a=0.07 fm)
  - *suppress noise in covariance estimates*

  [A.Agadjanov et al, Phys.Rev.Lett. 131 (2023) 26, 26]

$$|y - y_{median}| \geq 25 \cdot \underbrace{\Delta y}_{\text{(median abs. deviation)}}$$

- *Averages of fits with Akaike inf.crit.*

  [W.Jay, E.Neil PRD 103:114502(2021); E.Neil, J.W.Sitison, 109:014510(2024)].

$$w_i \;\propto \exp\left(-\chi_i^2 + N_i^{\rm dof}\right)$$

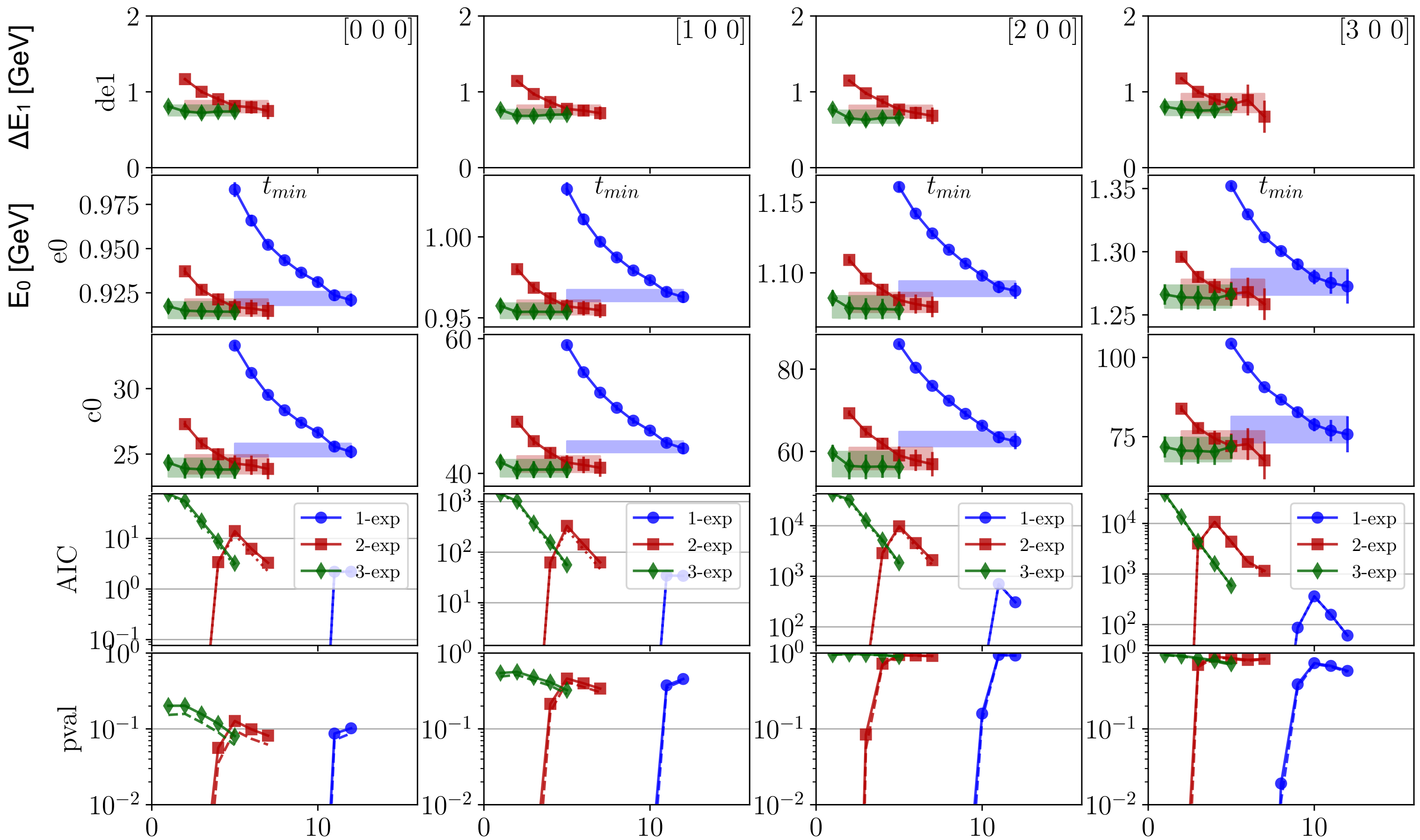
current matrix elements from $\langle N\, J\, \bar{N}\rangle$:

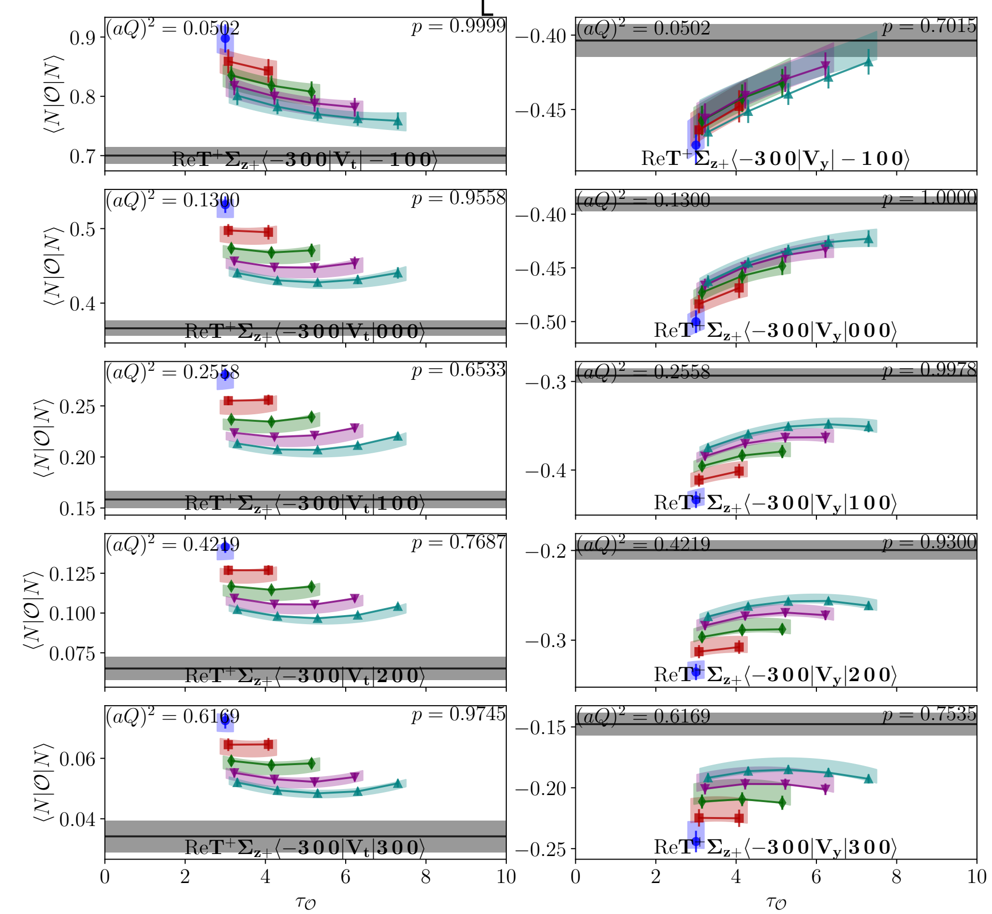
$$\begin{cases} \langle J_t\rangle & \propto G_E \\ \langle J_{\vec{q}\times\vec{S}}\rangle & \propto G_M \end{cases}$$

energies and g.s. amplitudes from $\langle N\, \bar{N}\rangle$:

$$\langle N(t)\,\bar{N}(0)\rangle \sim c_0 e^{-E_0 t} + c_1 e^{-(E_0+\Delta E_1)t} + \ldots$$

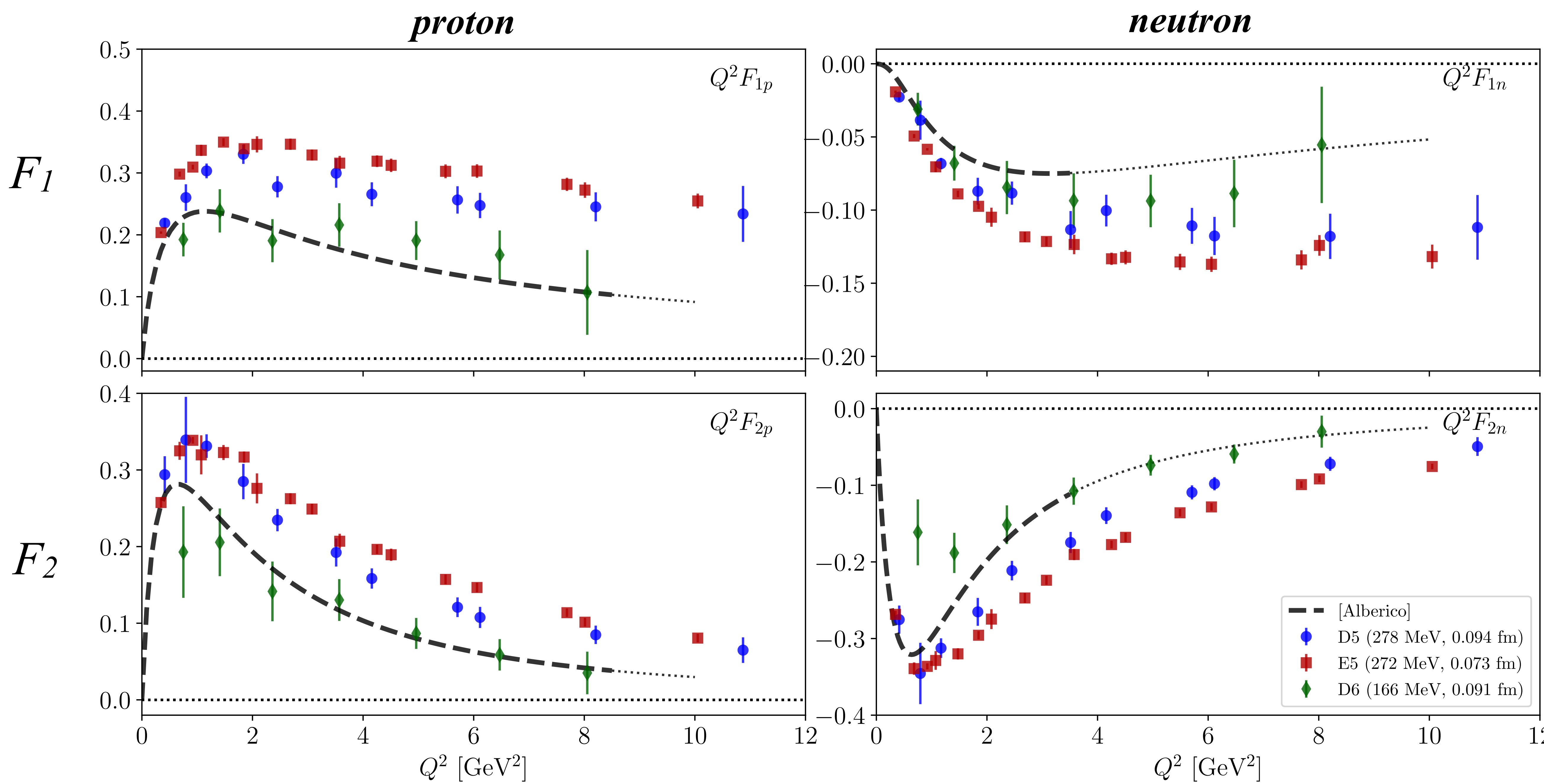
$$\langle N'|\,J(\tau)\,|N\rangle \sim \mathcal{A}_{00}\left[1 + O(e^{-\Delta E_1^{(\prime)} t} + \ldots)_{\rm exc}\right]$$





Nucleon Form Factors: Dirac and Pauli

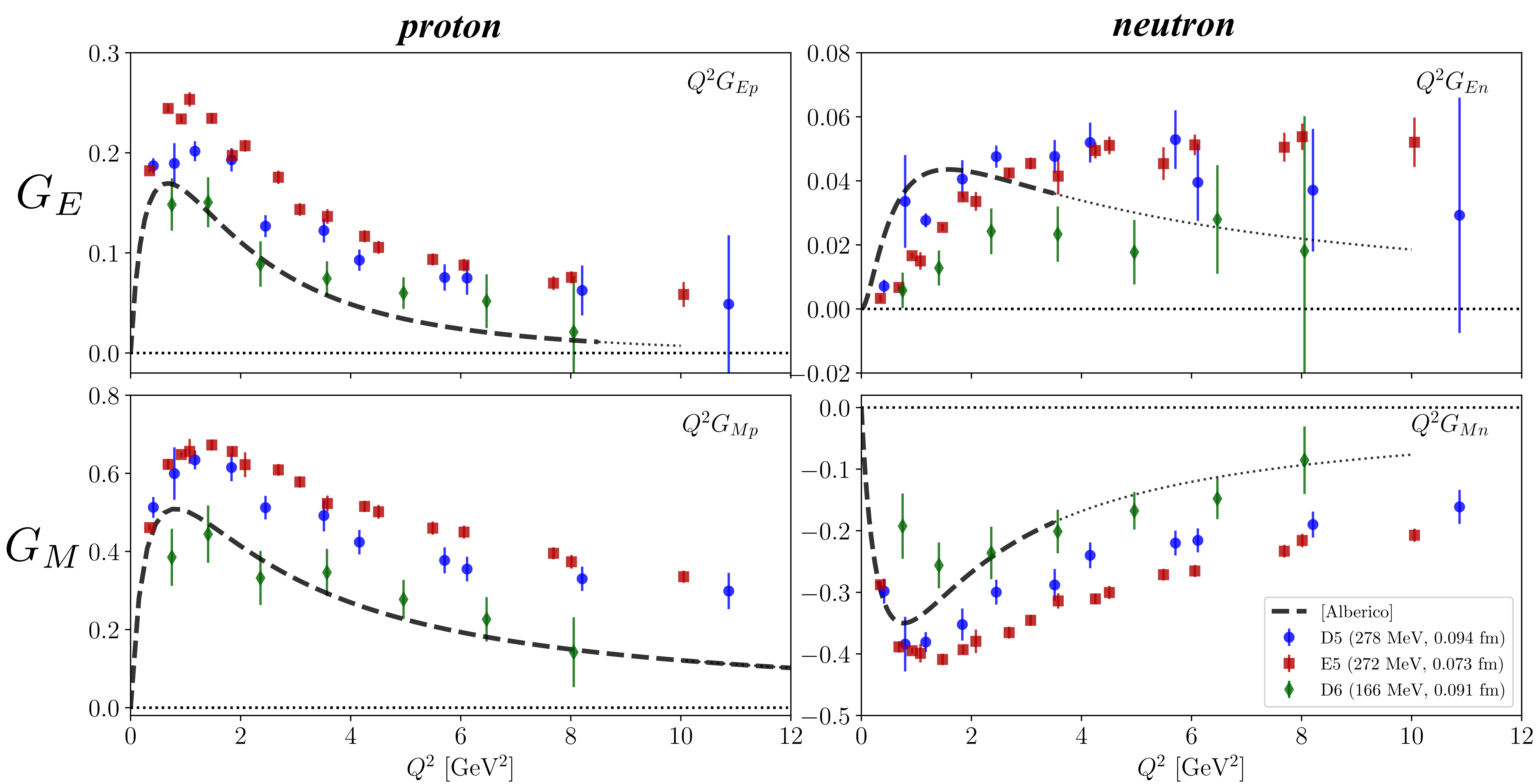
- Comparison of 3 ensembles (**D5** : 86k, **E5** : 266k, **D6** : 261k samples)
- *(dashed line)* Phenomenological fit [Alberico et al, PRC79:065204 (2008)]



- *No disconnected diagrams*

Nucleon Form Factors: Sachs Electric and Magn.

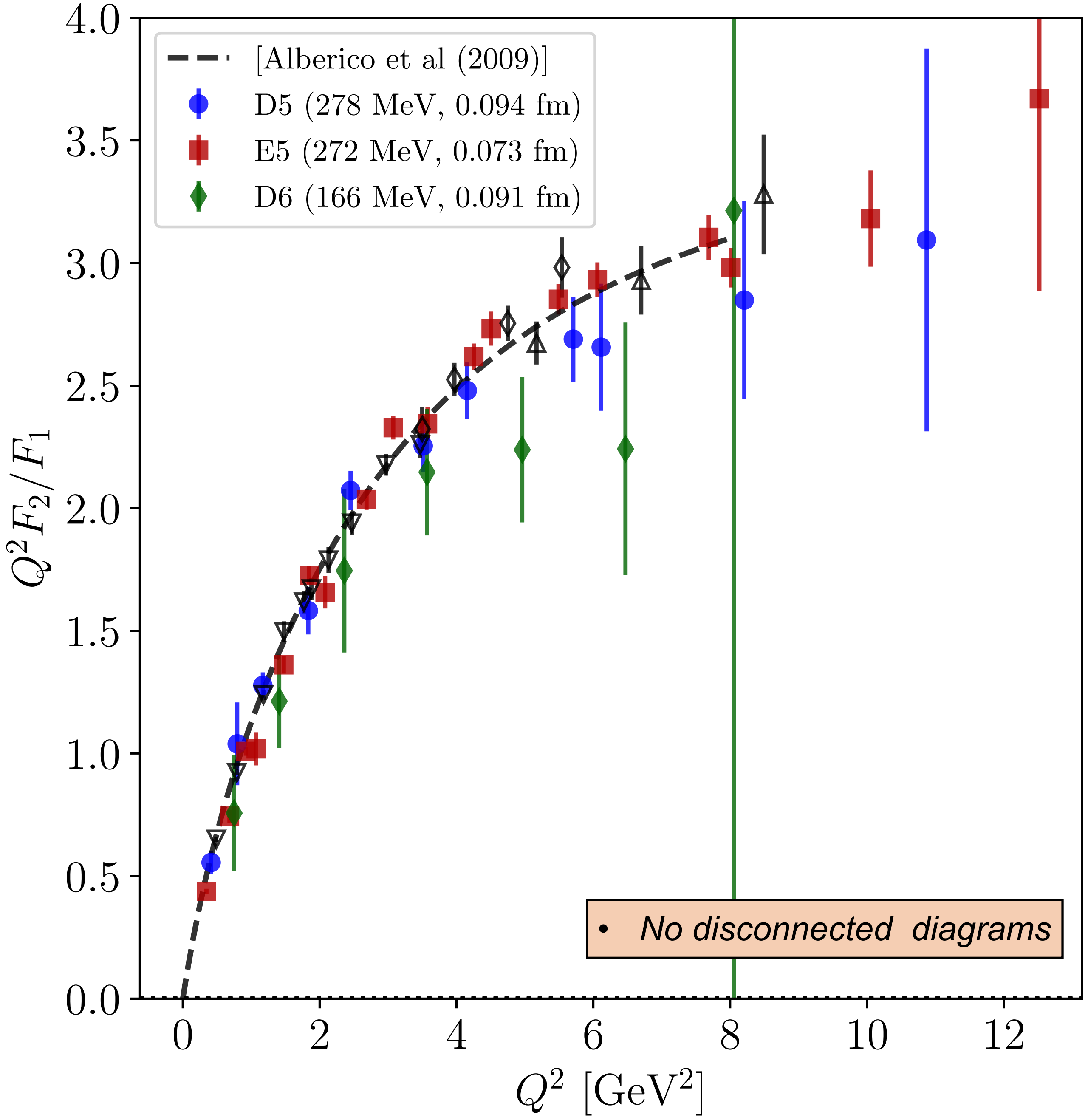
- Comparison of 3 ensembles (**D5** : 86k, **E5** : 266k, **D6** : 261k samples)
- *(dashed line)* Phenomenological fit [Alberico et al, PRC79:065204 (2008)]



- *No disconnected diagrams*

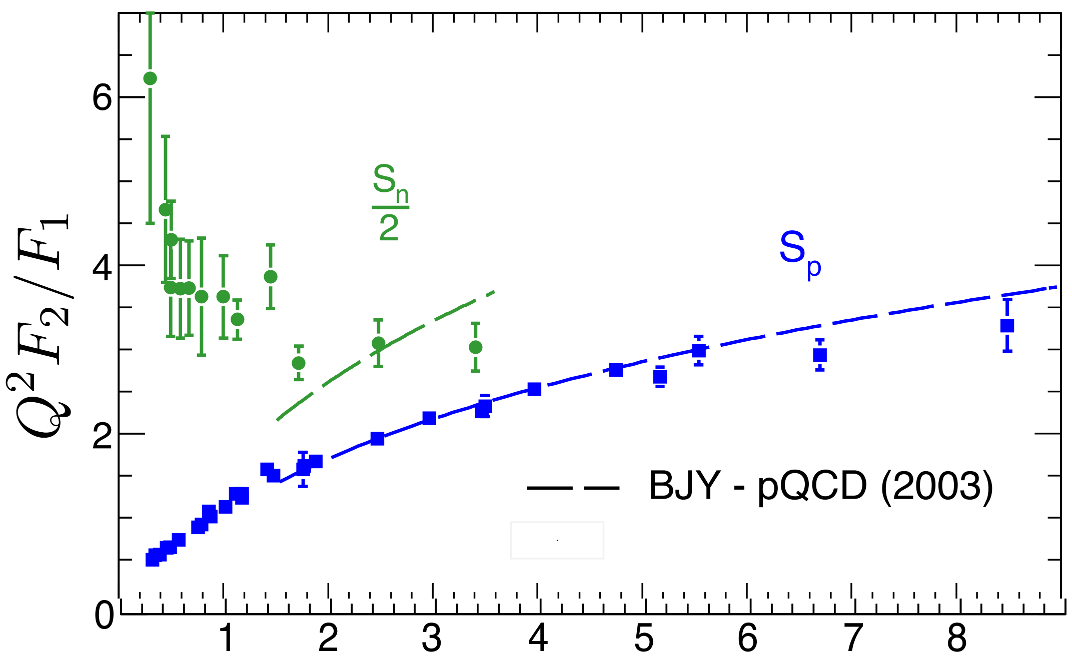
Proton F_2/F_1 Ratio

- Comparison of 3 ensembles (**D5** : 86k, **E5** : 266k, **D6** : 261k samples)
- (*dashed line*) Phenomenological fit [Alberico et al, PRC79:065204 (2008)]
- *(black points)* Experimental data for proton ($Q^2 \lesssim 8.5$ GeV^2)



- Prediction from pQCD + quark OAM [Balitsky, Ji, Yuan (2003)]

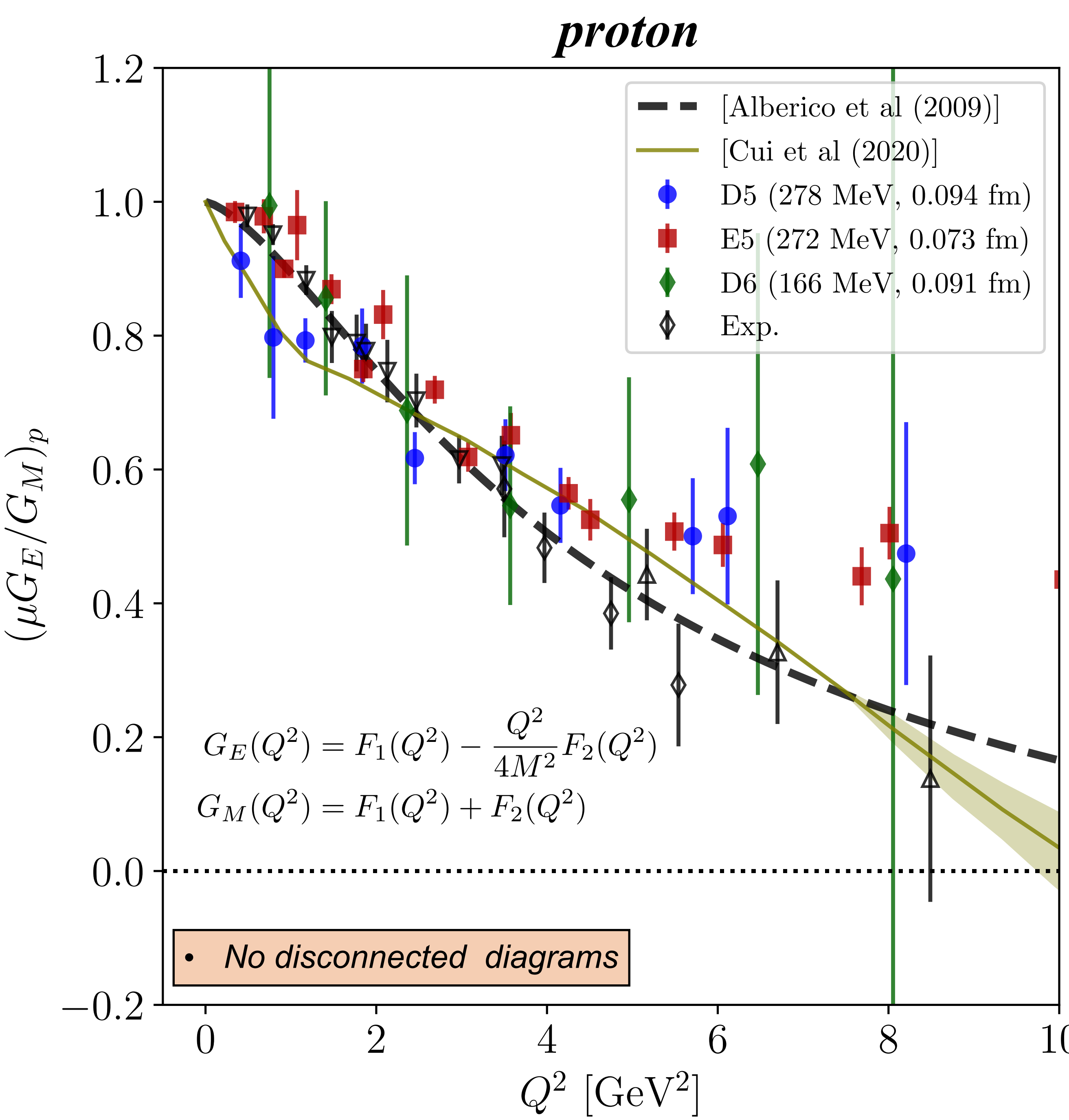
$$Q^2 F_{2p}/F_{1p} \overset{?}{\propto} \log^2(Q^2/\Lambda^2)$$

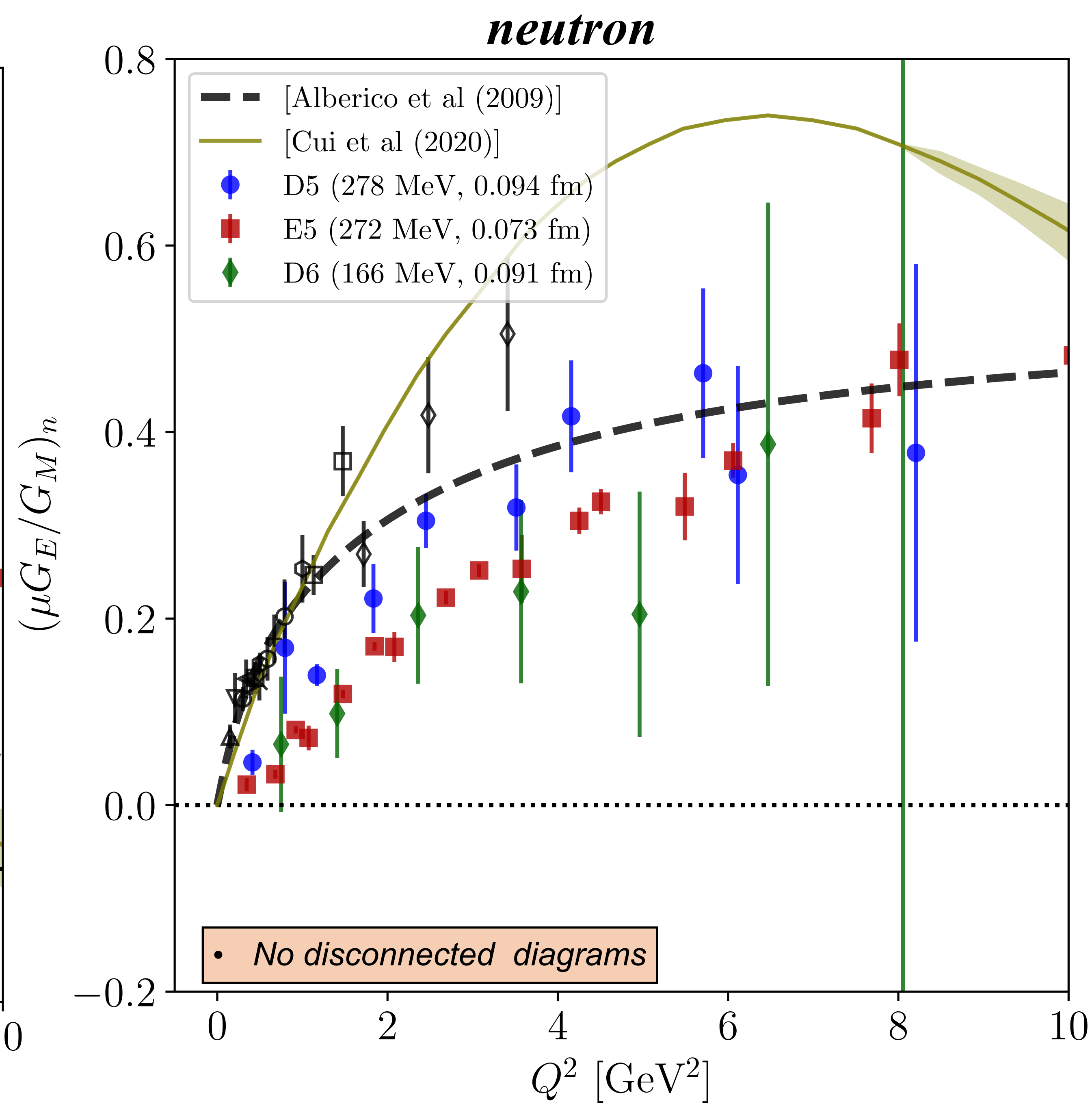


[G.D.Cates, et al, PRL106:252003 (2011)]

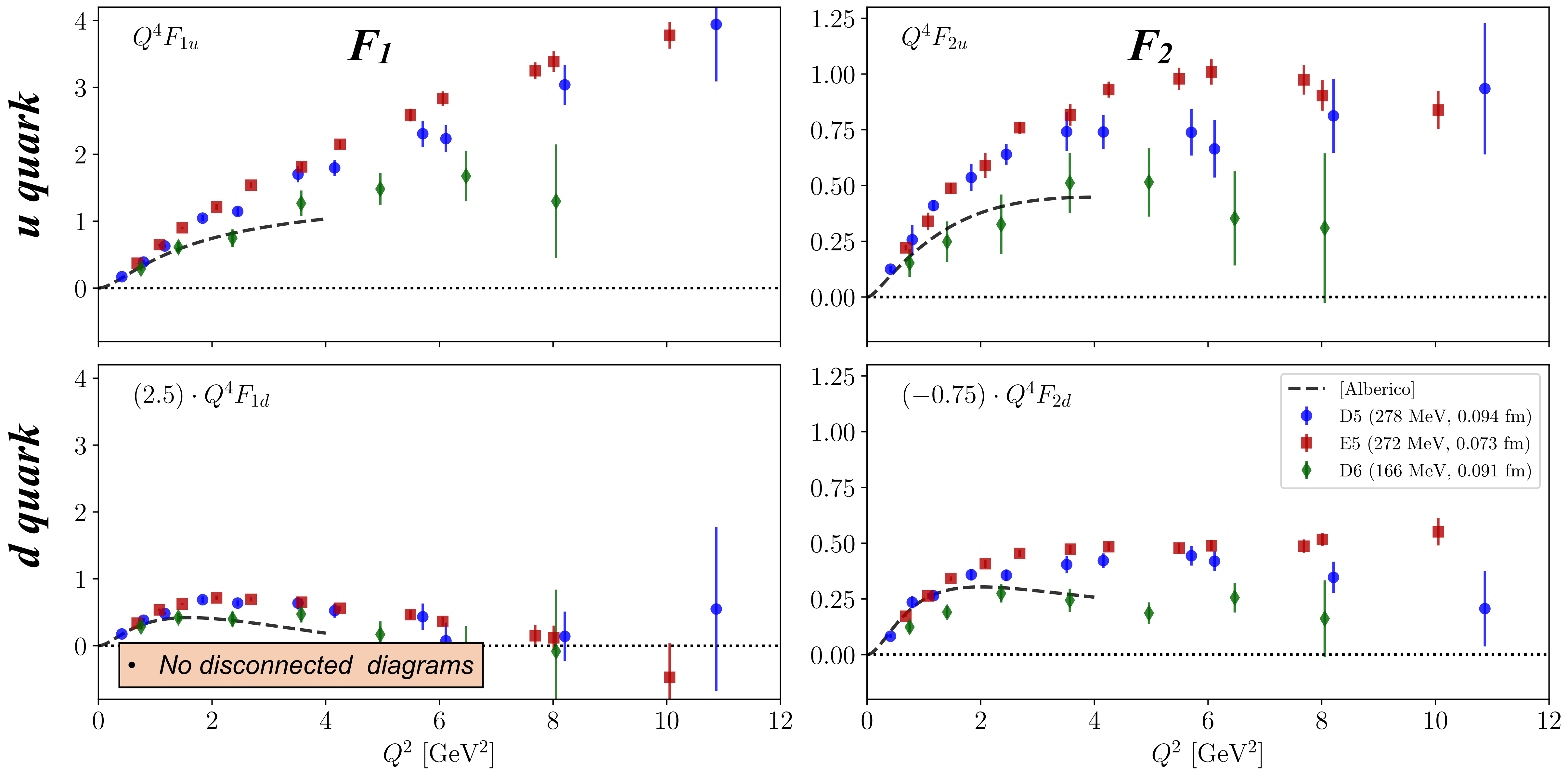
Proton & Neutron G_E/G_M Ratio (min. t_{sep}= 0.5 fm)

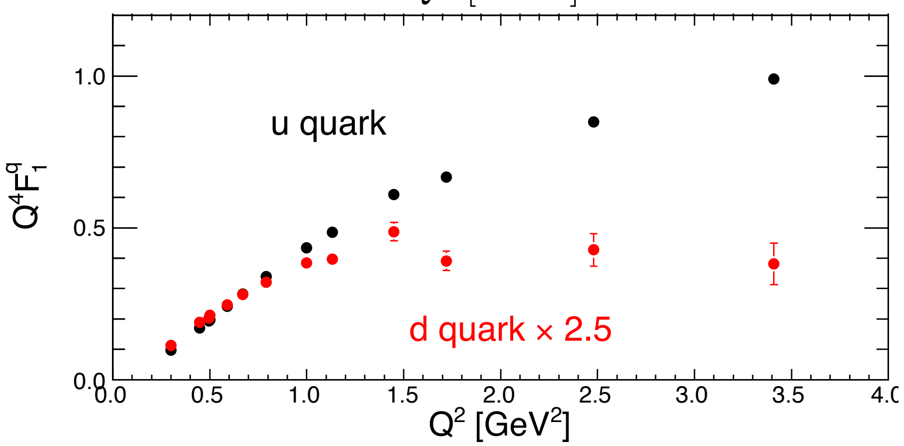
- Comparison of 3 ensembles (**D5** : 86k, **E5** : 266k, **D6** : 261k samples)
- *(dashed line)* Phenomenological fit [Alberico et al, PRC79:065204 (2008)]
- *(black points)* Experimental data for proton ($Q^2 \lesssim 8.5$ GeV^2) and neutron ($Q^2 \lesssim 3.4$ GeV^2)

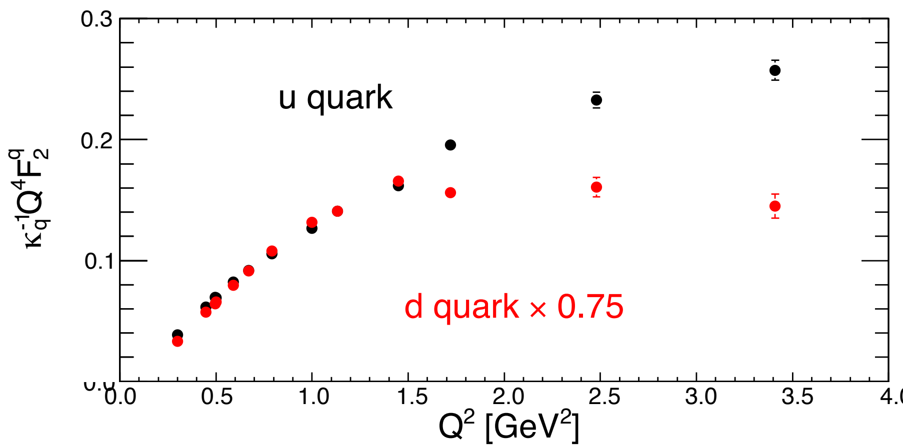




Light-flavor Contributions to (proton) Form Factors







- Reproduce qual.features of flavor dependence [G.D.Cates, et al, PRL106:252003(2011)]

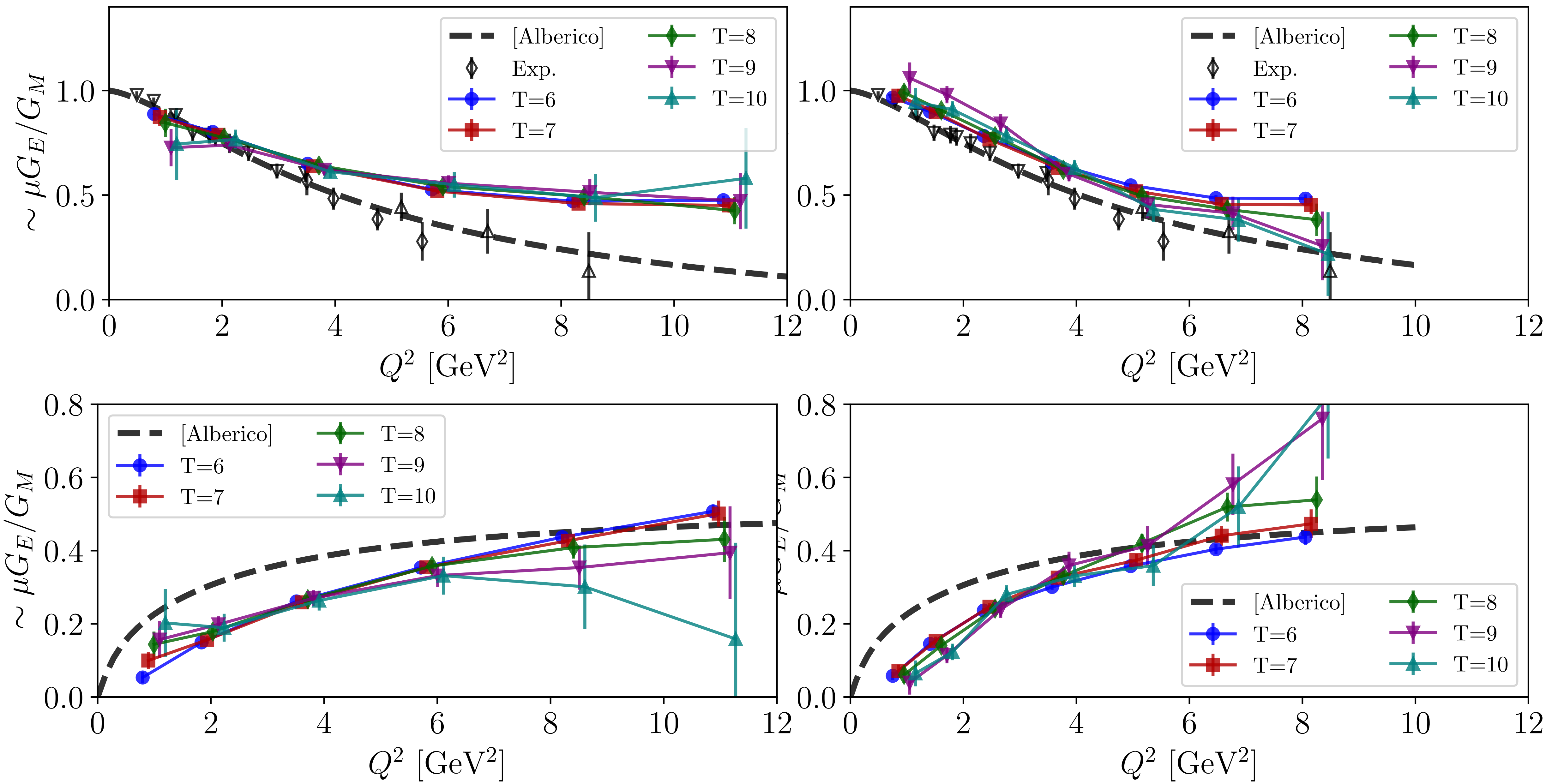
Proton&Neutron G_E/G_M from $\langle N J \bar{N} \rangle$ Correlators

$$\left(\frac{\sinh\frac{\lambda'-\lambda}{2}}{\cosh\frac{\lambda'+\lambda}{2}}\right)\frac{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_t(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle}{\mathrm{Re}\langle N_\uparrow(p'_x,T)\,J_y(T/2)\,\bar{N}_\uparrow(p_x,0)\rangle} \overset{T\to\infty}{=} G_E/G_M \quad \text{with} \begin{pmatrix} p^{(\prime)} & = m_N \sinh\lambda^{(\prime)} \\ E^{(\prime)} & = m_N \cosh\lambda^{(\prime)} \end{pmatrix}$$

D5($m\pi$ =278 MeV, a = 0.094 fm) **D6($m\pi$ =170 MeV, a = 0.094 fm)**

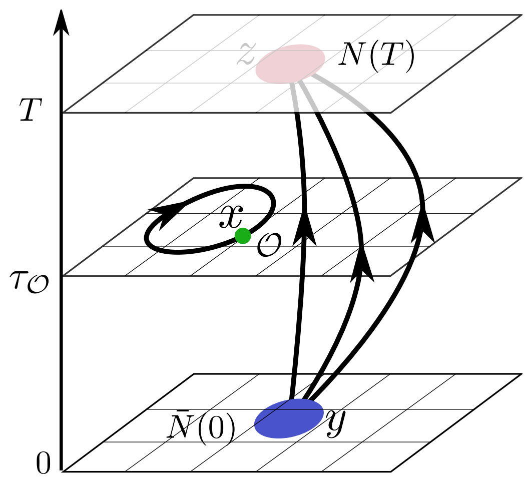
proton

neutron



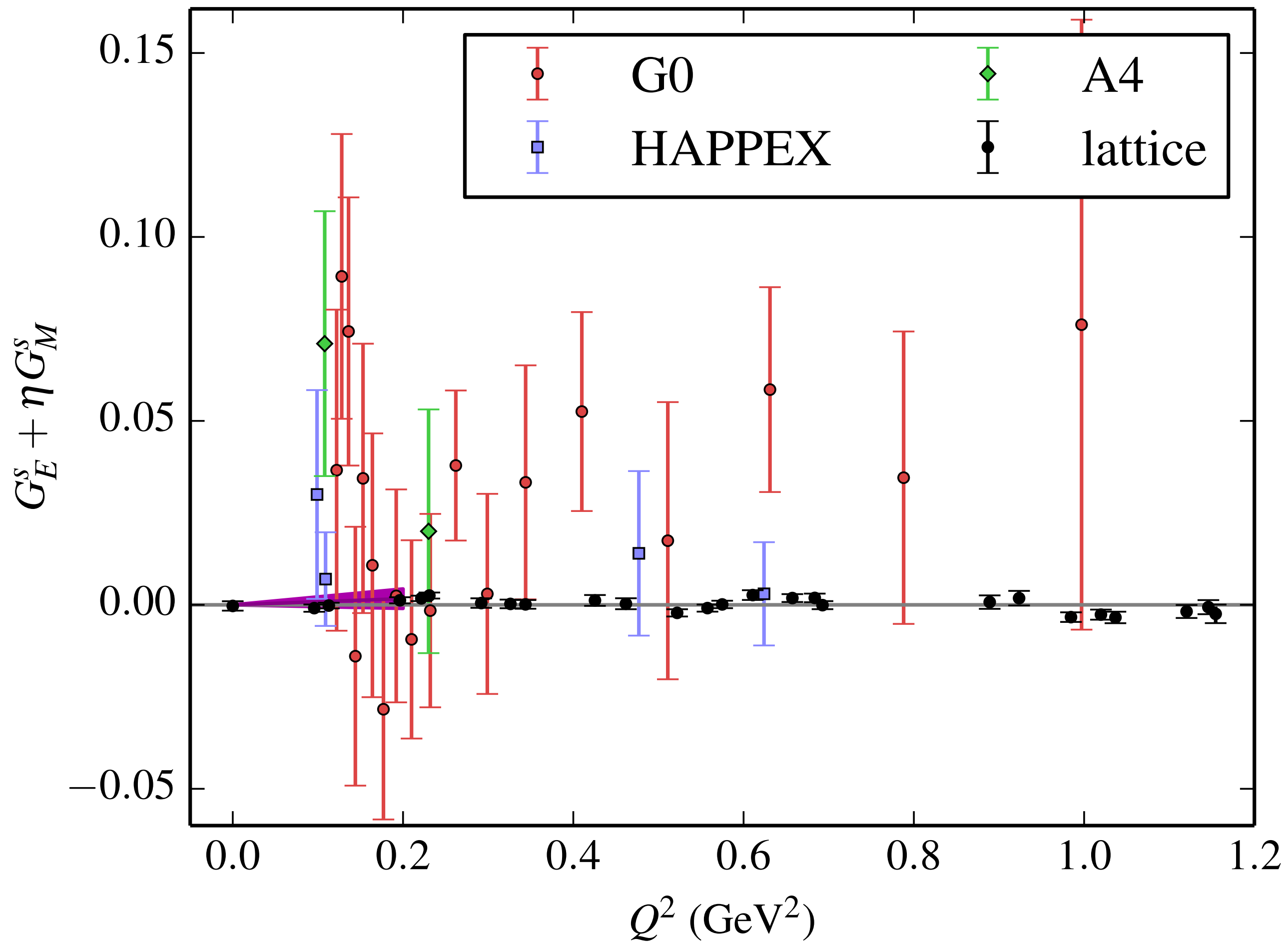
Nucleon excited state(s) have very similar G_E/G_M at large Q^2?

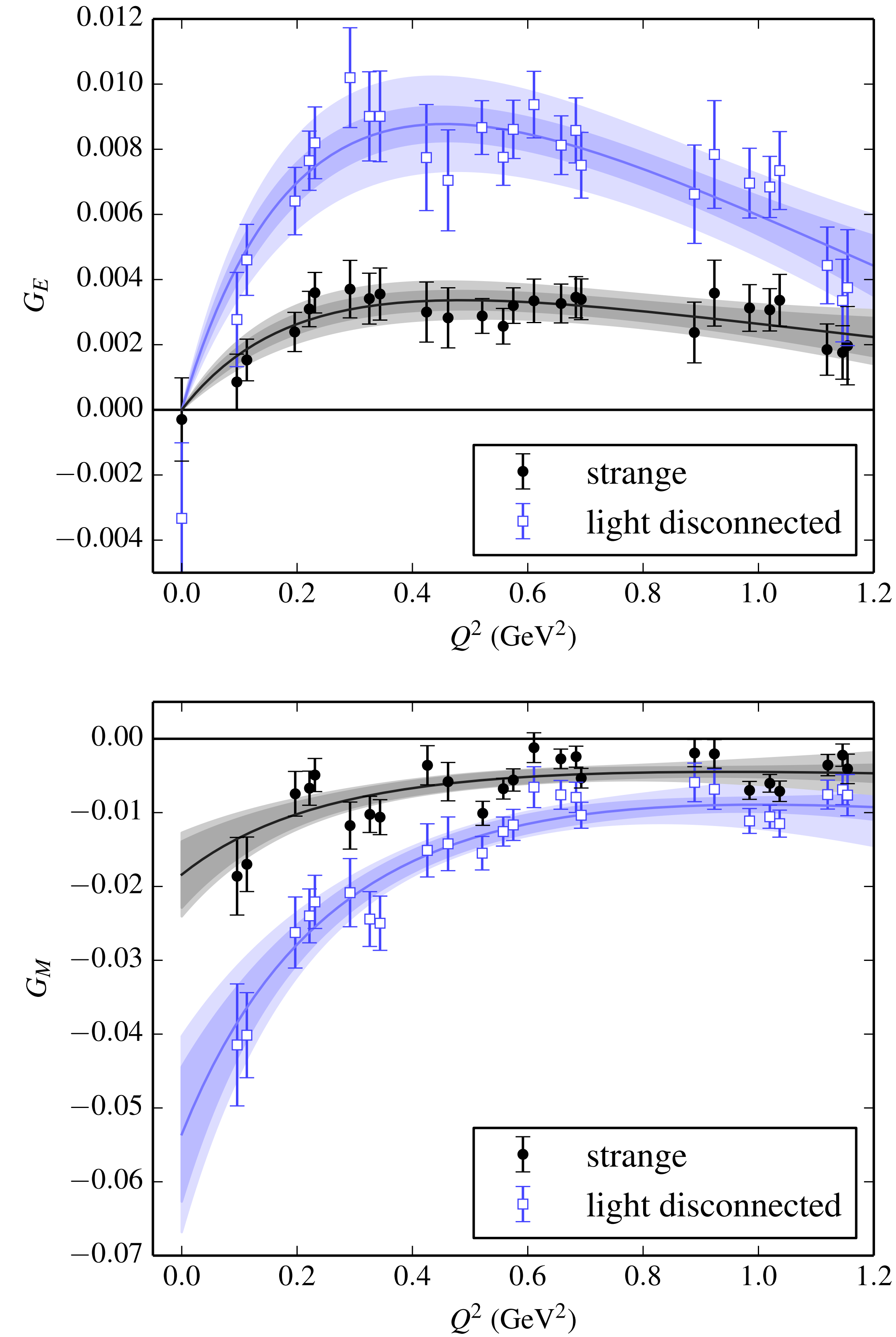
Disconnected Contributions to Vector FFs?



Disconnected-U,D and S ≲ 1% of nucleon FFs at Q^2≲1 GeV^2

[J. Green, S. Meinel, S.S. et al; PRD92:031501 (2015)]

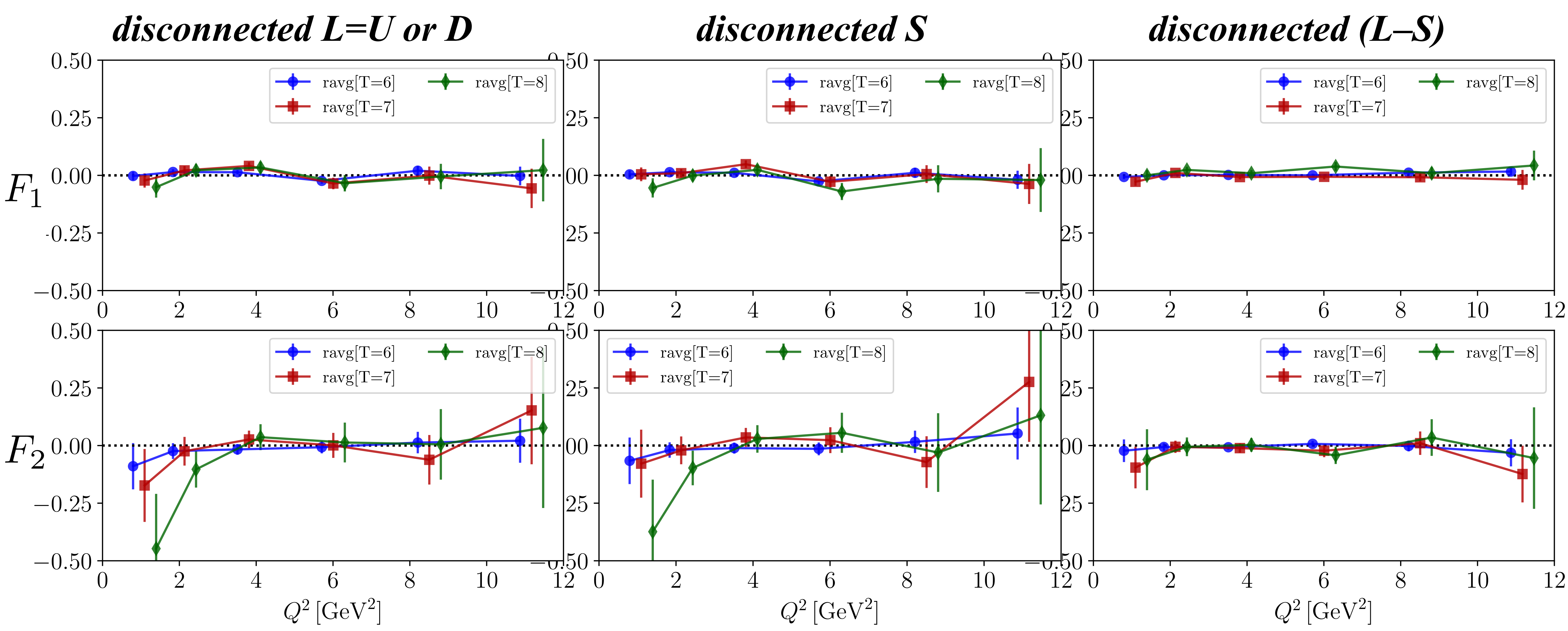




Disconnected-U,D & Strange vs. Connected (D5)

- *relative correction* $|\delta F_{1,2}^{disc} / F_{1,2}^{conn}| \lesssim 10\%$ *but quickly grows with Euclidean time*
- partial noise cancellation in (L–S) contributing to proton & neutron

$$P = \frac{1}{3}\left[2U - D\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$
$$N = \frac{1}{3}\left[2D - U\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$



D5 ensemble (m_π=280 MeV, a=0.094 fm), 1346 configs ⊗ 64 samples of $\langle N\bar{N}\rangle$

G_E/G_M : Systematics from Disconnected U,D & S

- partial noise cancellation in (L–S) contributing to proton & neutron

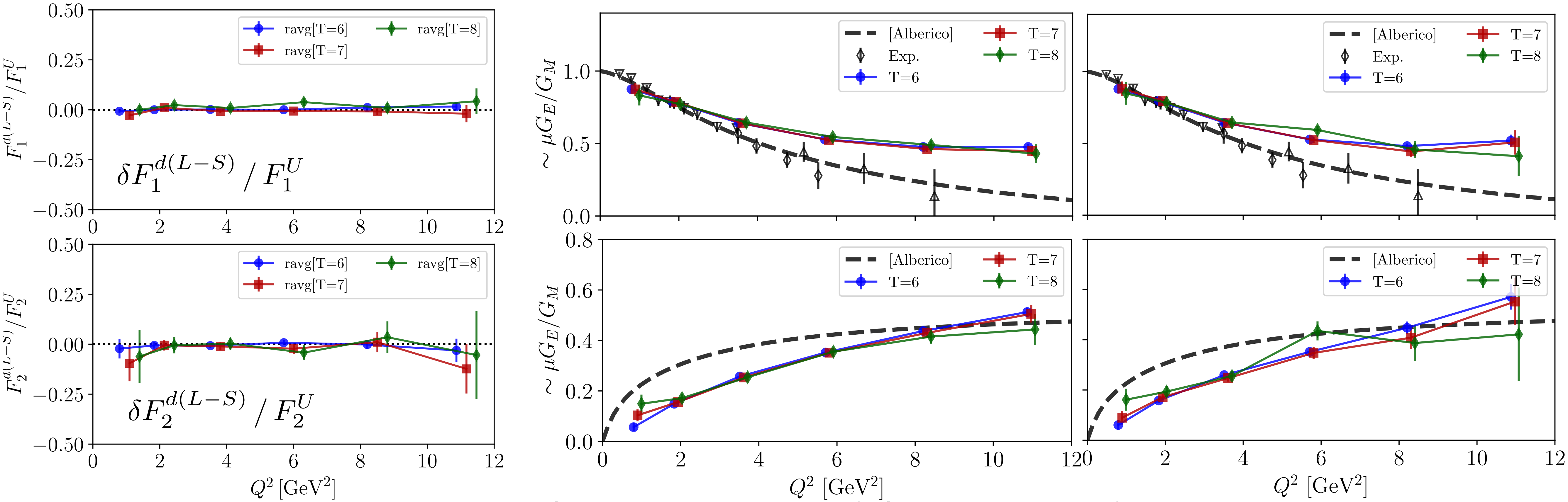
$$P = \frac{1}{3}\left[2U - D\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$
$$N = \frac{1}{3}\left[2D - U\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$

- limited effect on G_E/G_M *(within stat. noise)*
- larger effect on flavor-decomposition of $F_{1,2}$

disconnected (L–S), relative to conn.

G_E/G_M, conn. only

G_E/G_M, conn.+disc.



D5 ensemble (m_π=280 MeV, a=0.094 fm), t_{sep}= 0.50÷0.75 fm

Summary

- High-statistics lattice calculations of nucleon form factors at high-momentum
  - up to $Q^2 \lesssim 10\ \mathrm{GeV}^2$
  - two lattice spacings $a \gtrsim 0.07$ fm
  - two pion masses $m_\pi \gtrsim 170$ MeV

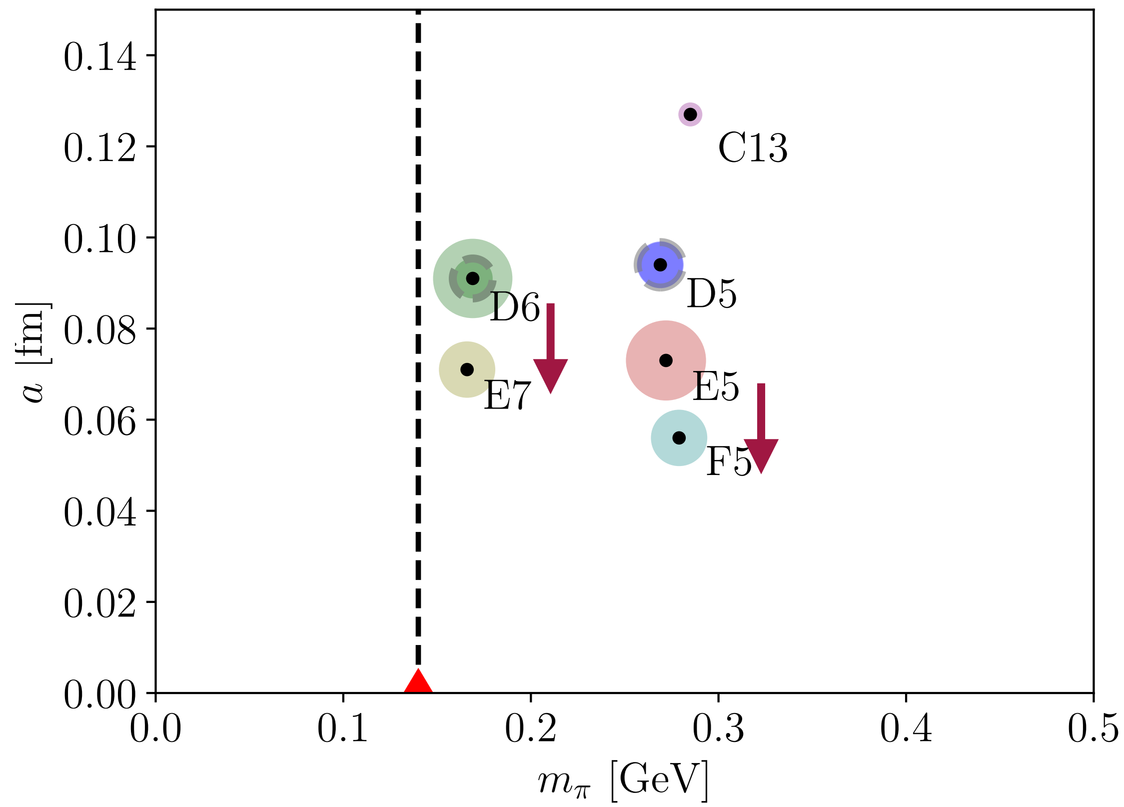
- Quark-disconnected contributions evaluated at $a \approx 0.09$ fm, m_π down to 170 MeV
  - little impact below $Q^2 \lesssim 6\ \mathrm{GeV}^2$
  - still major source of uncertainty (esp. above $Q^2 \gtrsim 8\ \mathrm{GeV}^2$)
  - large stoch. uncertainty, longer t_{sep} data needed

- Form factor results overshoot experimental data x(2 ... 2.5) ;
  G_E/G_M ratios in qualitative agreement
  - Non-physical quarks masses?
  - Discretization?
  - Excited states (most likely) — *novel lattice correlator analysis techniques may help*

Important cross-check with experiments,
relevant for calculations of relativistic nucleon matrix elements
as well as TMDs, PDFs, DAs ...

Outlook

- Upcoming next-gen computing facilities (e.g. Jupiter at Julich SC)
  - More statistics (x10) to control excited state systematics
  - Smaller lattice spacings (0.09 → 0.07 → 0.05 fm), physical and 2x physical pion mass

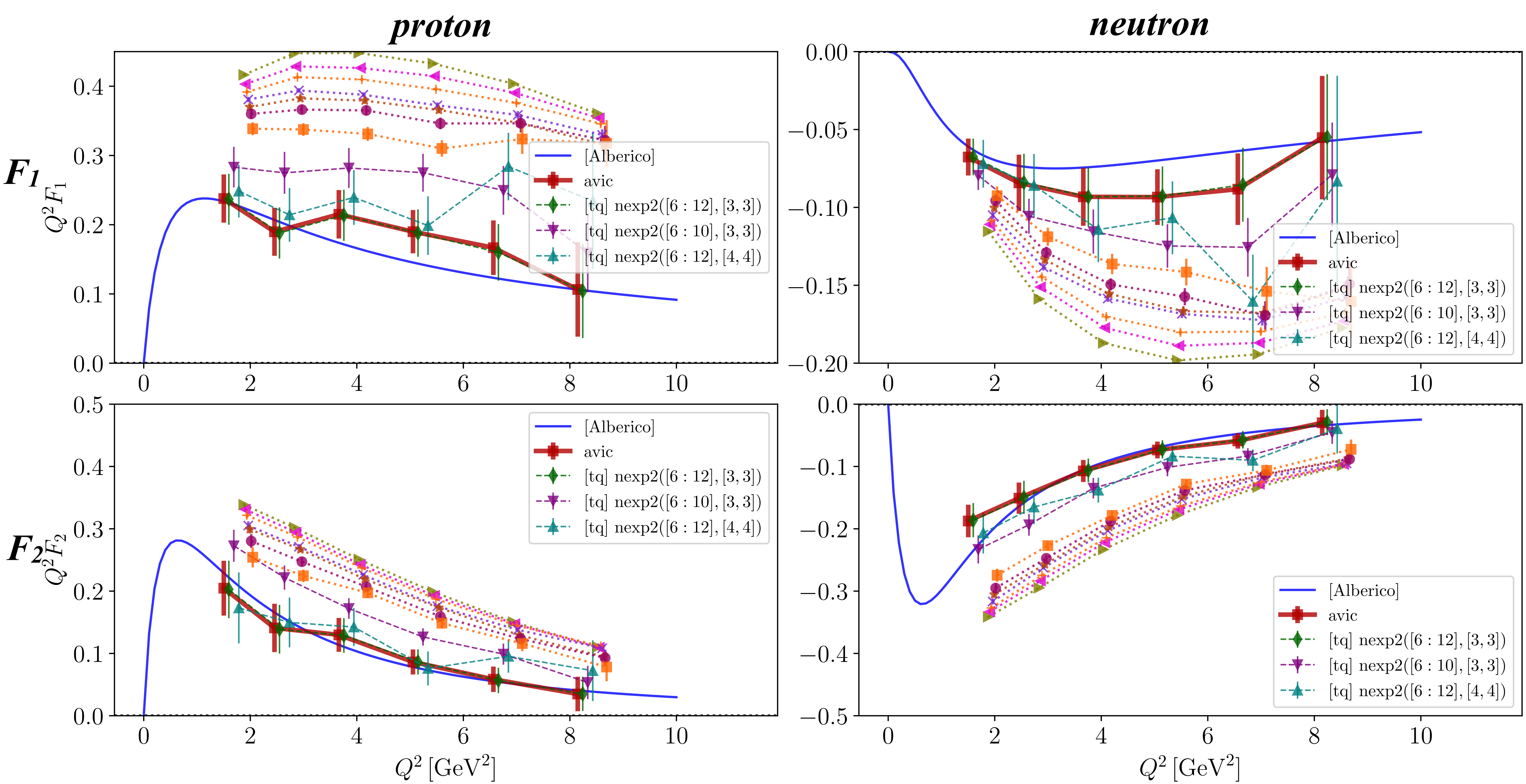


- Effective theory for large-momentum form factors of nucleon-like states?
- Explanation for potential G_E/G_M universality?

BACKUP

Form Factors: plateau, 2-state fits, AIC-average

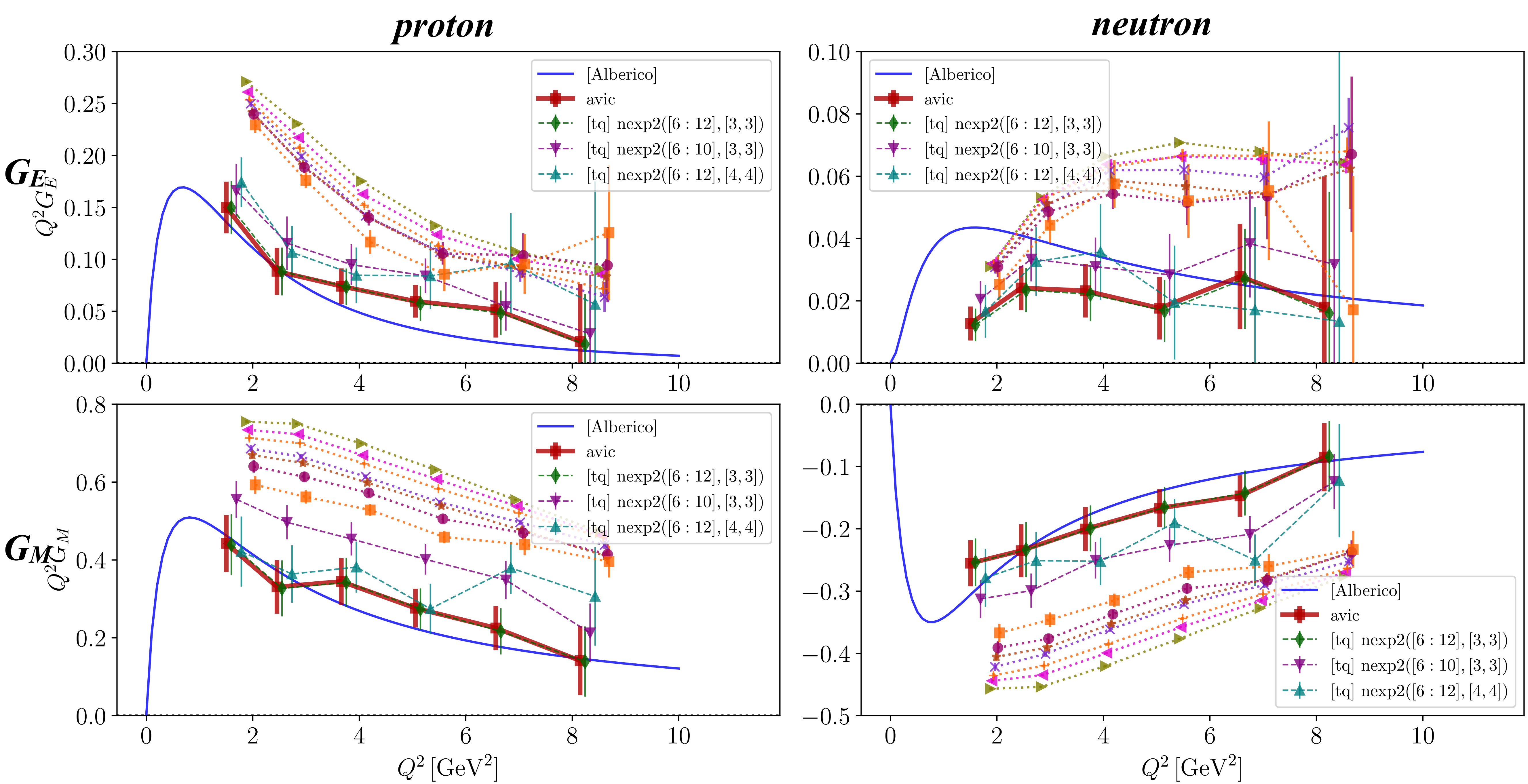
- D6 ensemble (260k samples) : fits with t_{sep}= *0.7÷1.1 fm* and t_{sep}=*0.5÷1.1 fm* range
- Dotted lines: comparison of 1-state approx. for various fixed t_{sep}
- *Phenomenology (dashed)* : [Alberico et al, PRC79:065204 (2008)]



- *No disconnected diagrams*

Form Factors: plateau, 2-state fits, AIC-average

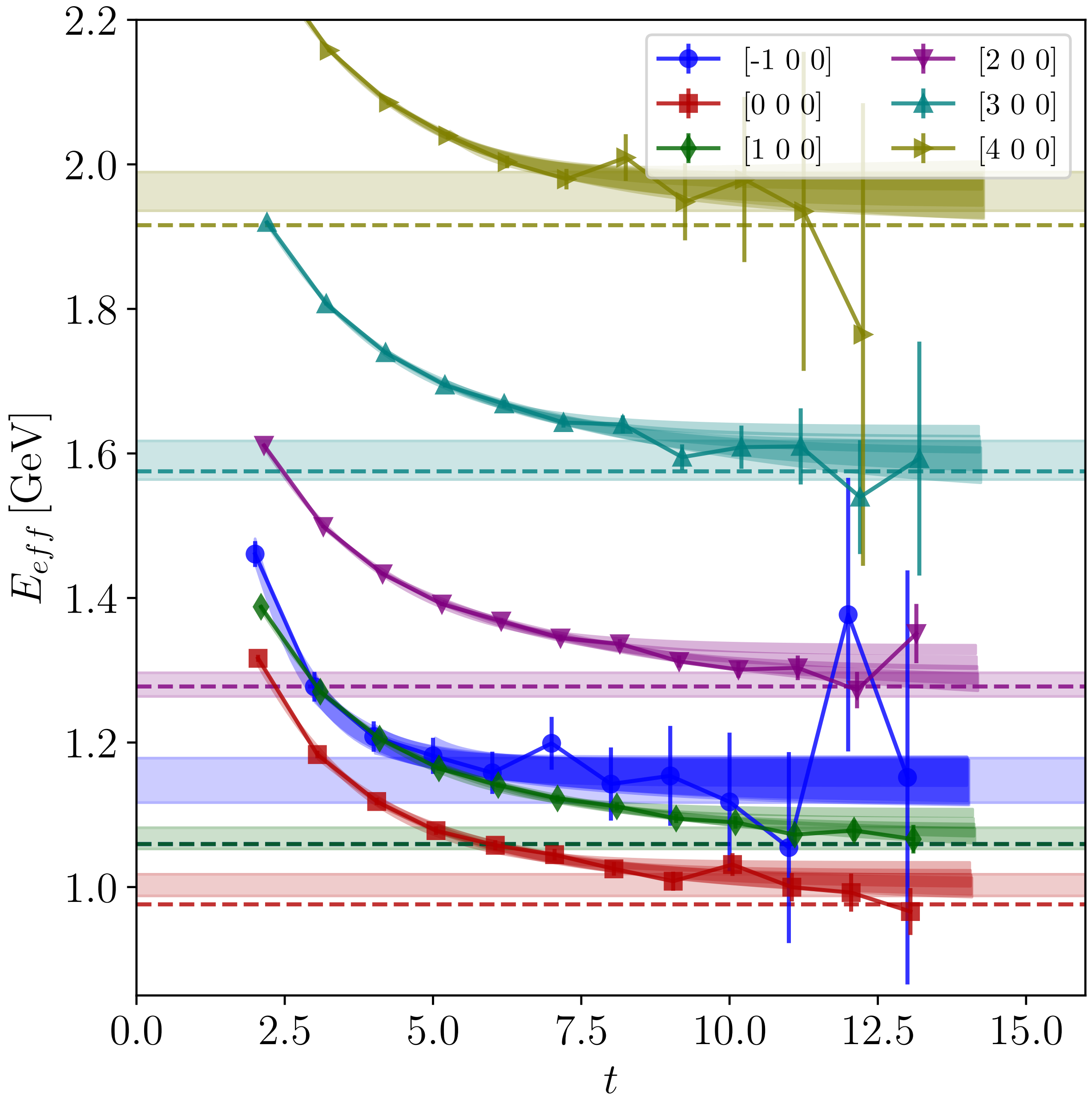
- D6 ensemble (260k samples) : fits with ***t_{sep}= 0.7÷1.1 fm*** and ***t_{sep}=0.5÷1.1 fm*** range
- Dotted lines: comparison of 1-state approx. for various fixed ***t_{sep}***
- *Phenomenology (dashed)* : [Alberico et al, PRC79:065204 (2008)]

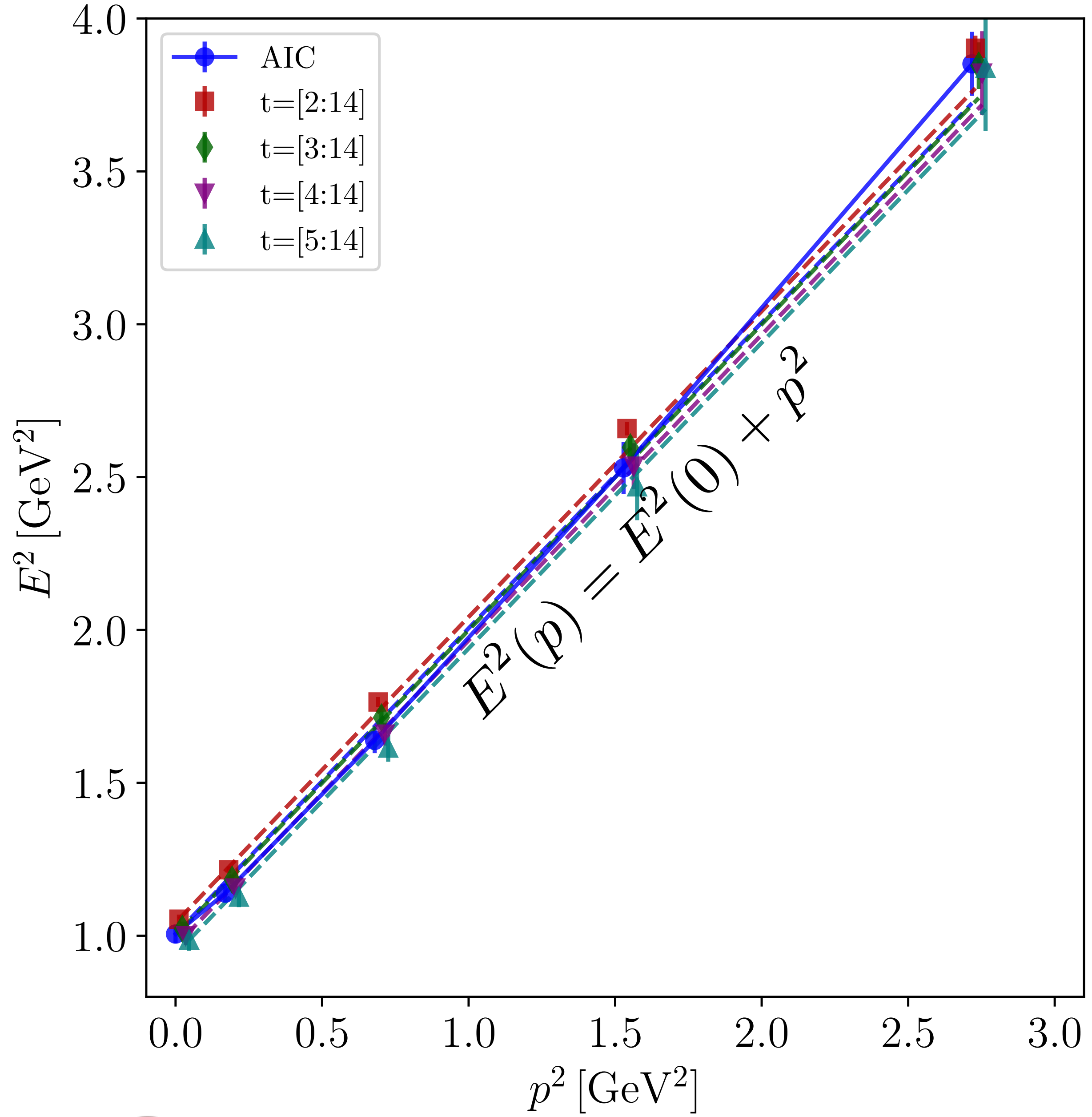


- *No disconnected diagrams*

Lattice Nucleon Energy & Dispersion Relation (D5)

- D5 : $m\pi$ = 280 MeV , spacing a = 0.094 fm , 86k MC samples





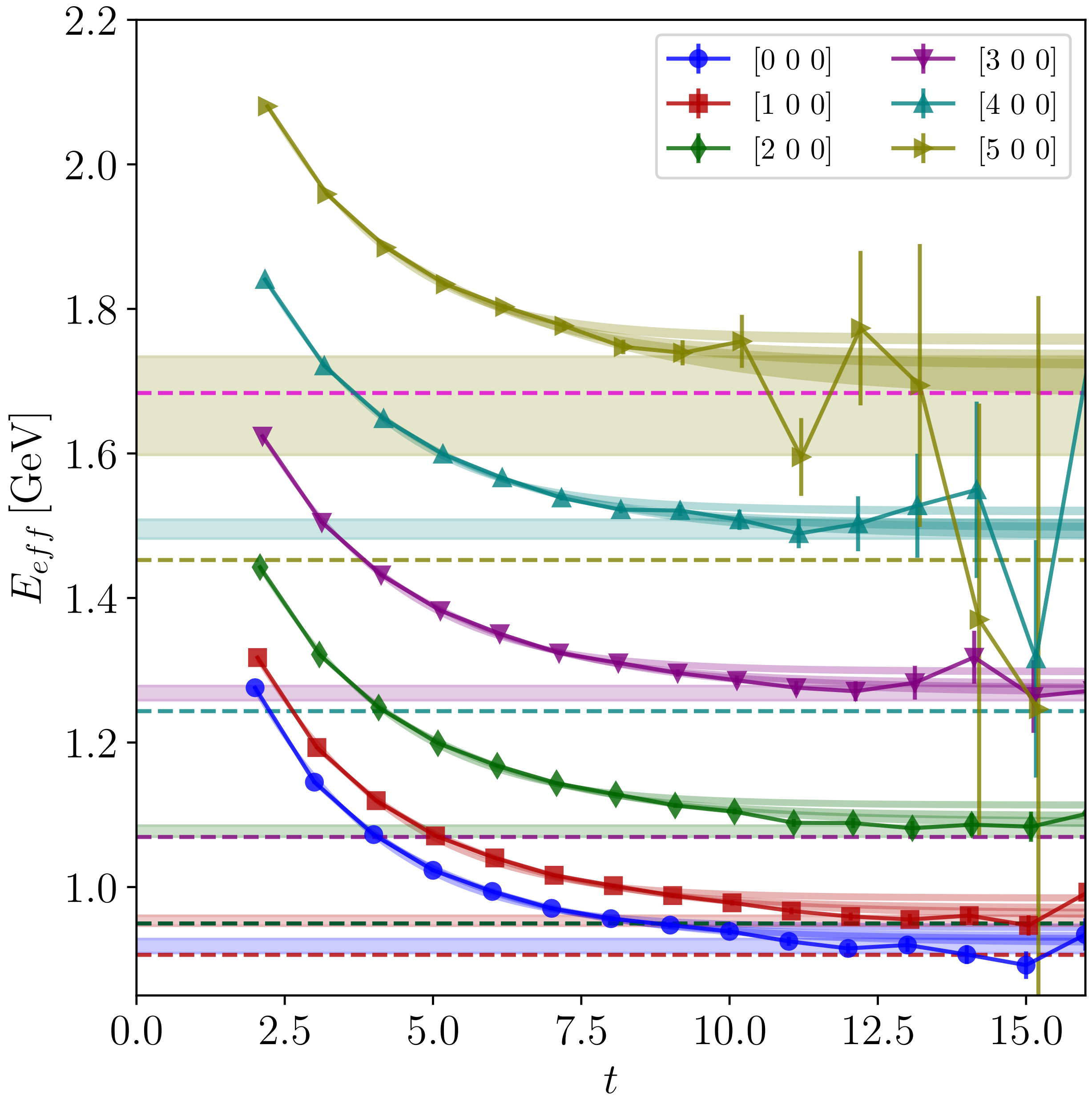
- 2-state fits compared to Effective energy

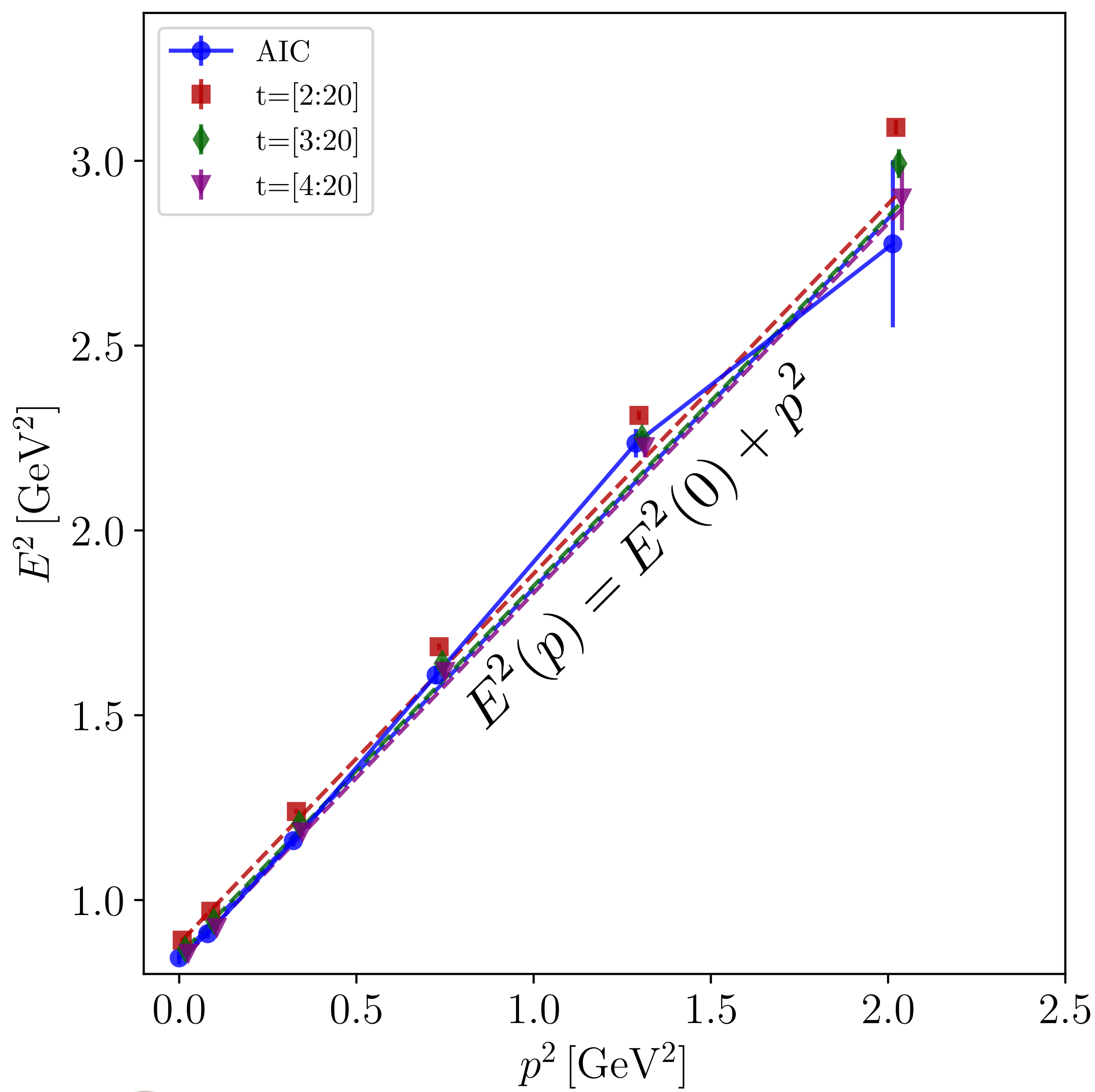
$$E_{eff} = \frac{1}{a} \log \frac{C_{N\bar{N}}(t)}{C_{N\bar{N}}(t+a)}$$

- Lattice energies vs. continuum dispersion relation (dashed)

Lattice Nucleon Energy & Dispersion Relation (D6)

- D6 : $m\pi =$ 170 MeV , spacing $a =$ 0.092 fm , 261k MC samples





- 2-state fits compared to Effective energy

$$E_{eff} = \frac{1}{a} \log \frac{C_{N\bar{N}}(t)}{C_{N\bar{N}}(t+a)}$$

- Lattice energies vs. continuum dispersion relation (dashed)

Disconnected-U,D & Strange vs. Connected (D6)

- *relative correction* $F_{1,2}^{disc}$ / $F_{1,2}^{conn}$ from plateau averages $\mathbf{t_{sep}}$**= 0.5÷0.74 fm,** $Q^2 \lesssim 8$ GeV^2
- **D6 ensemble (mπ=170 MeV, a=0.092 fm)**, 727 configs ⊗ 128 samples of ⟨N$\bar{\text{N}}$⟩
- partial noise cancellation between L=U/D and S in proton & neutron

$$P = \frac{1}{3}\left[2U - D\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$

$$N = \frac{1}{3}\left[2D - U\right]_{conn} + \frac{1}{3}\left[L - S\right]_{disc}$$

disconnected L=U or D ***disconnected S*** ***disconnected (L–S)***