

Compton amplitude and the nucleon structure functions from lattice QCD

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Granada, Lattice 2017

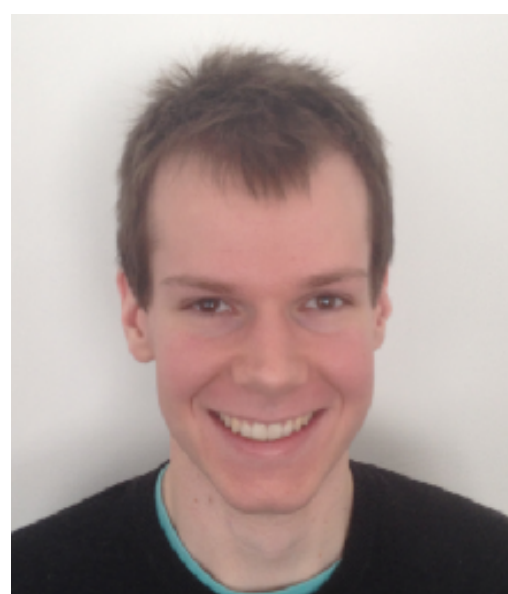
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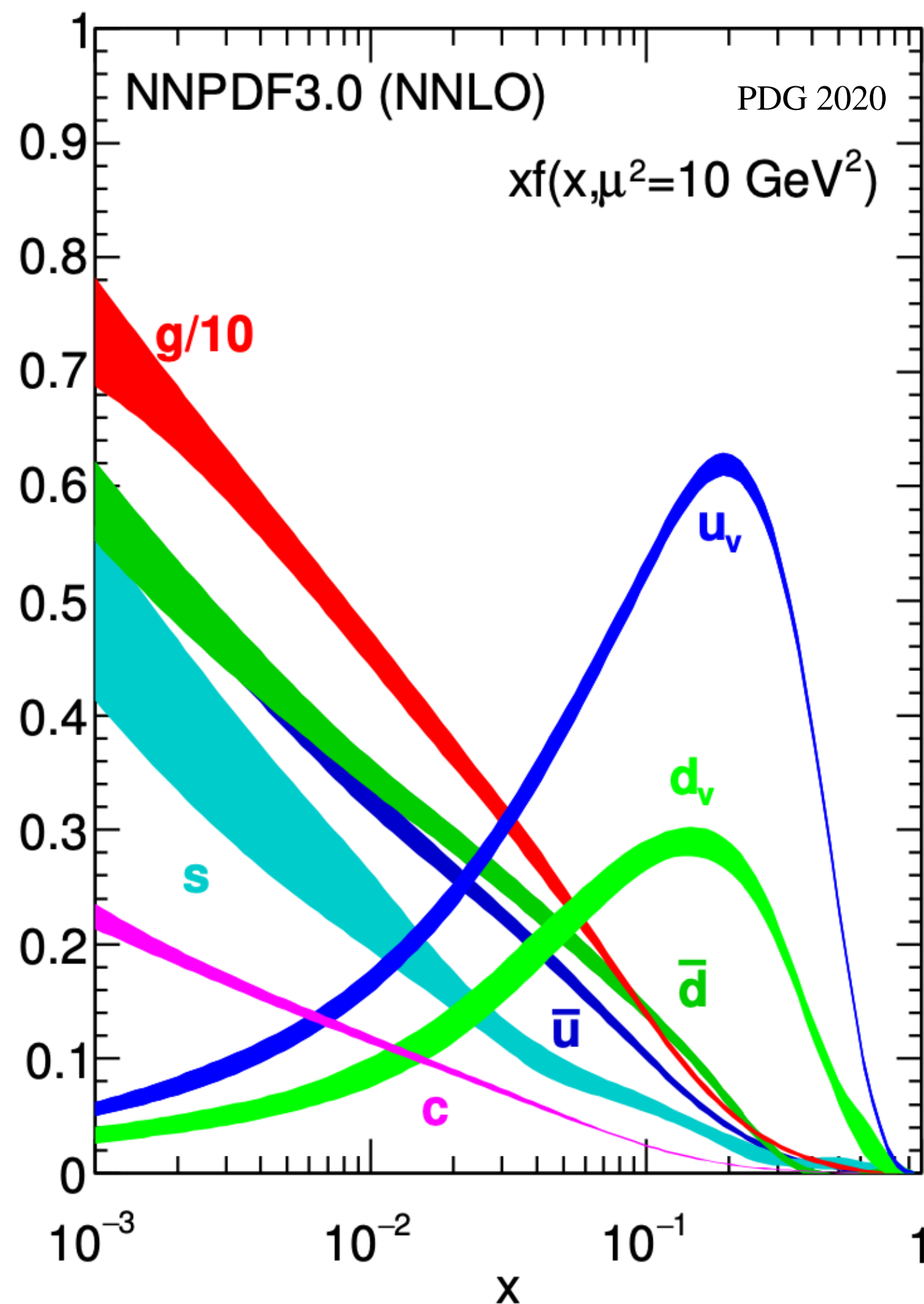


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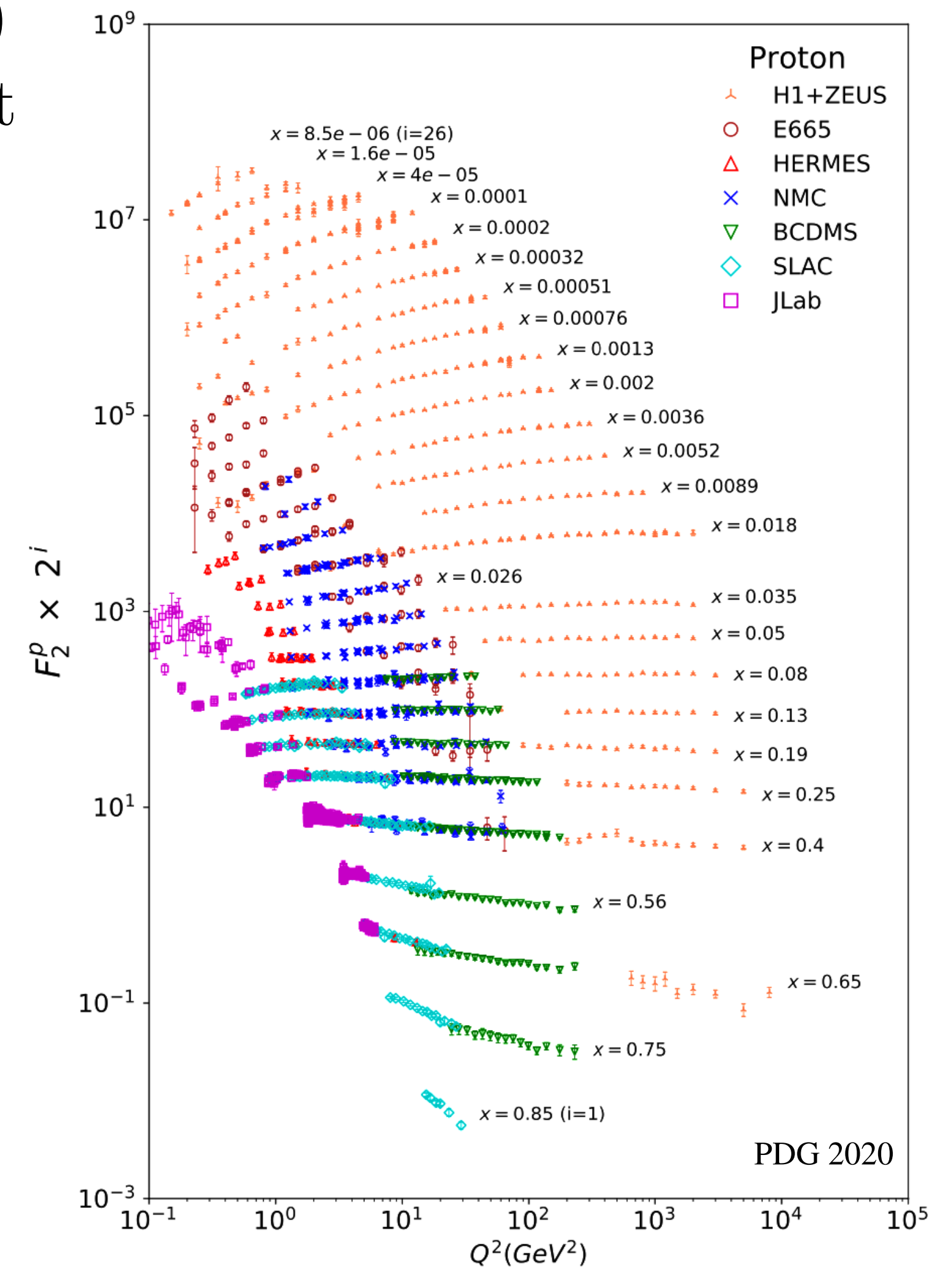


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Motivation



- Nucleon structure (leading twist)
- Parton distributions from first principles
- Constraining the high- and low- x regions better
- $F_2 \propto (q + \bar{q})$, $F_3^{\gamma Z} \propto (q - \bar{q})$
- Scaling and Power corrections
 - Q^2 cuts of global QCD analyses
 - Large- x , low- Q^2 :
 - Higher-twist contributions
 - Target mass corrections



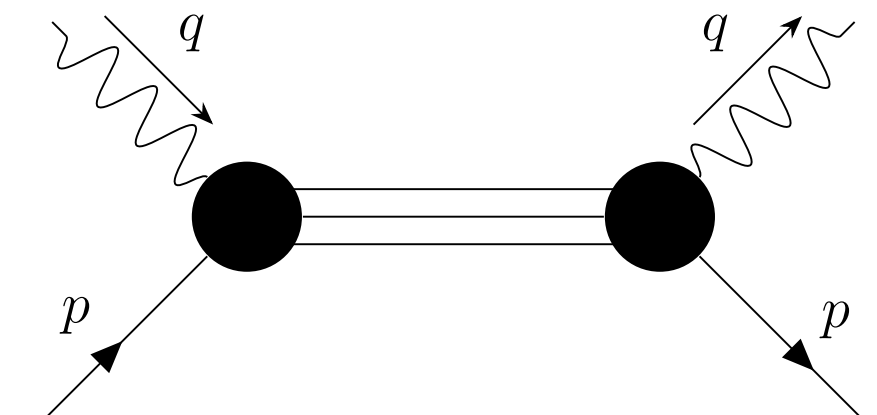
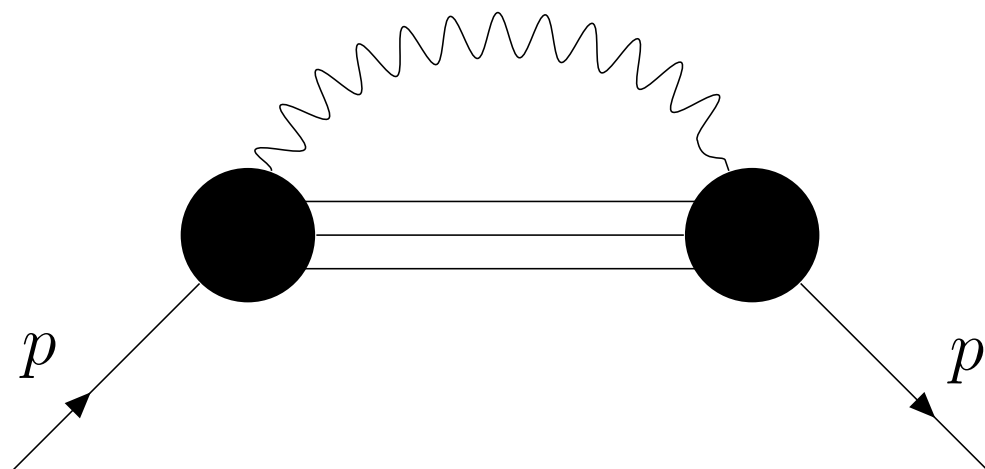
Motivation | Subtraction term

● Cottingham formula:

W.N. Cottingham, *Annals Phys.* 25, 424 (1963)
 J. C. Collins, *Nucl. Phys.*, B149:90–100, (1979)
 [Erratum: *Nucl. Phys.* B915,392(2017)]
 A. Walker-Loud, C. E. Carlson, G. A. Miller, *PRL* 108, 232301 (2012)
 A.W. Thomas, X.G. Wang, R.D. Young *PRC* 91 (2015) 1, 015209

$$\delta M^\gamma = \delta M^{\text{el}} + \delta M^{\text{inel}} + \delta M_{\text{sub}}^{\text{el}} + \delta M_{\text{sub}}^{\text{inel}} + \delta \tilde{M}^{\text{ct}}$$

$$\delta M^{\text{sub}} \sim -\frac{3\alpha_{em}}{16\pi M} \int^{\Lambda_0^2} dQ^2 T_1^{p-n}(0, Q^2)$$

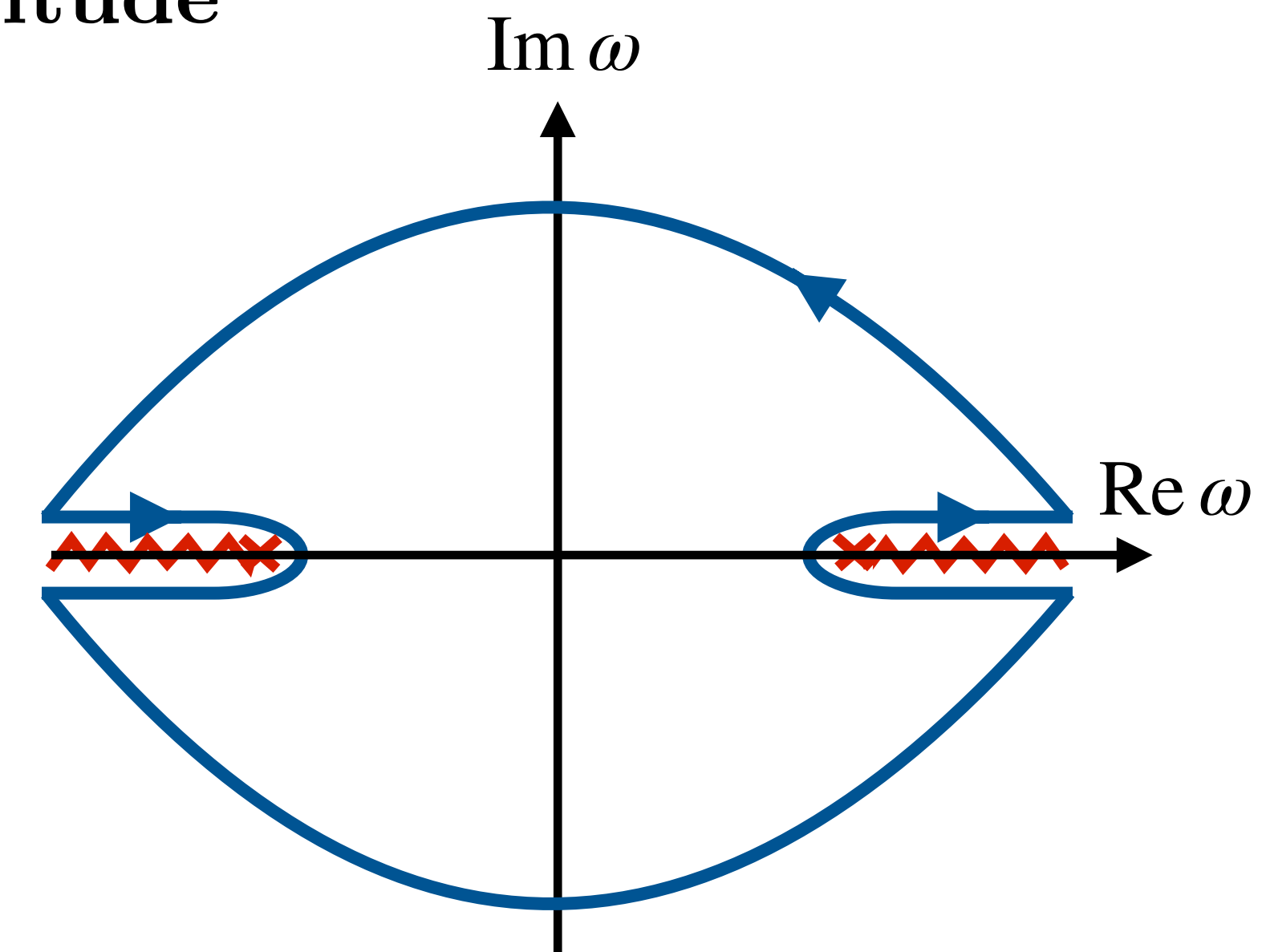


EM self energy is related to
the spin-avg. forward Compton amplitude

● Subtraction term $T_1(0, Q^2)$

$$\mathcal{F}_1(\omega, Q^2) - \mathcal{F}_1(\omega = 0, Q^2) = \frac{2\omega^2}{\pi} \int_1^\infty d\omega' \frac{\text{Im } \mathcal{F}_1(\omega', Q^2)}{\omega' (\omega'^2 - \omega^2 - i\epsilon)}$$

- dominant uncertainty
- not accessible via experiments



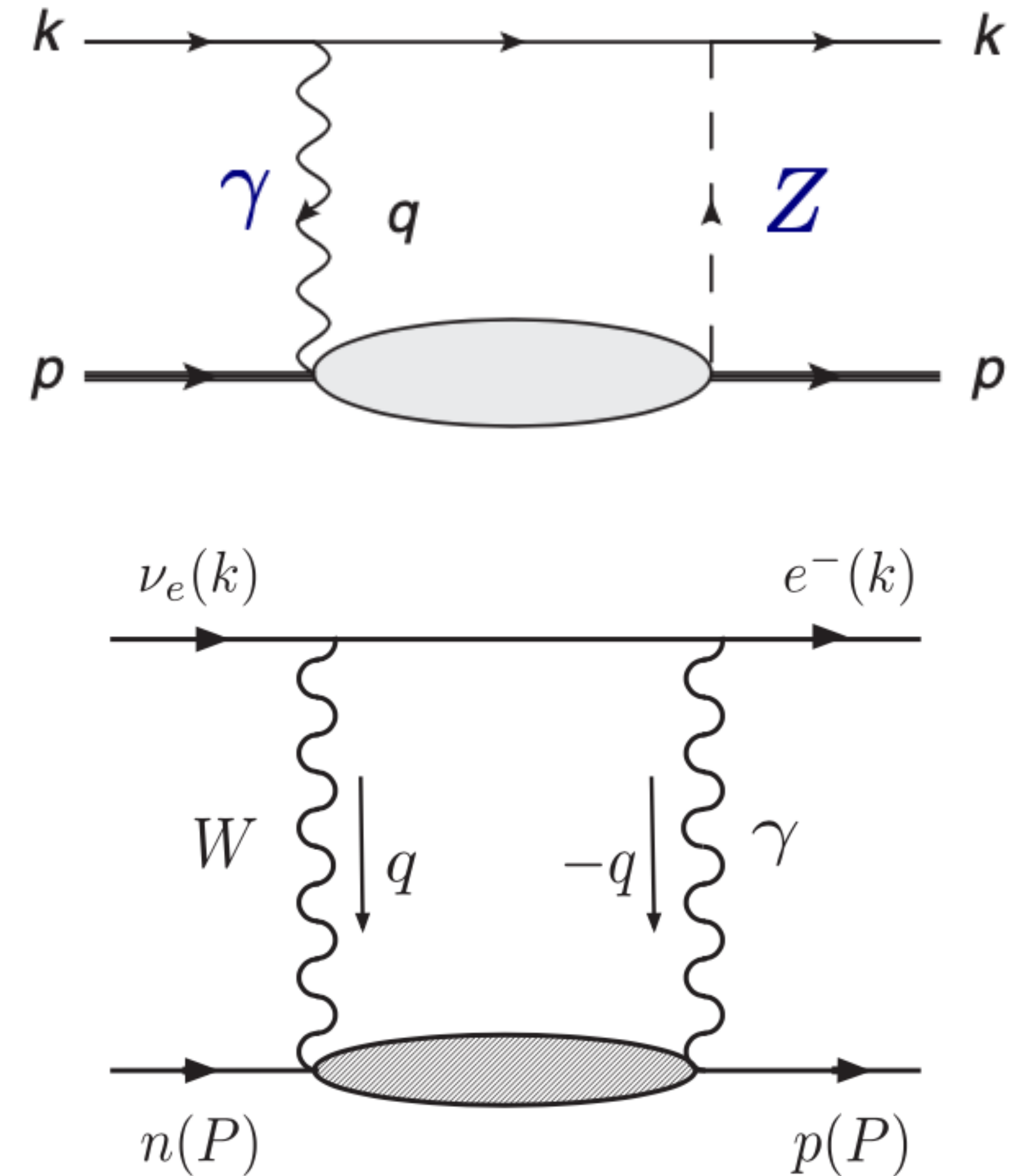
Motivation | EW Box

- Leading theoretical uncertainty in:
- Weak charge of the proton,

$$Q_W = (1 + \Delta_\rho + \Delta_e)(1 - 4 \sin^2 \theta_W(0) + \Delta'_e)$$

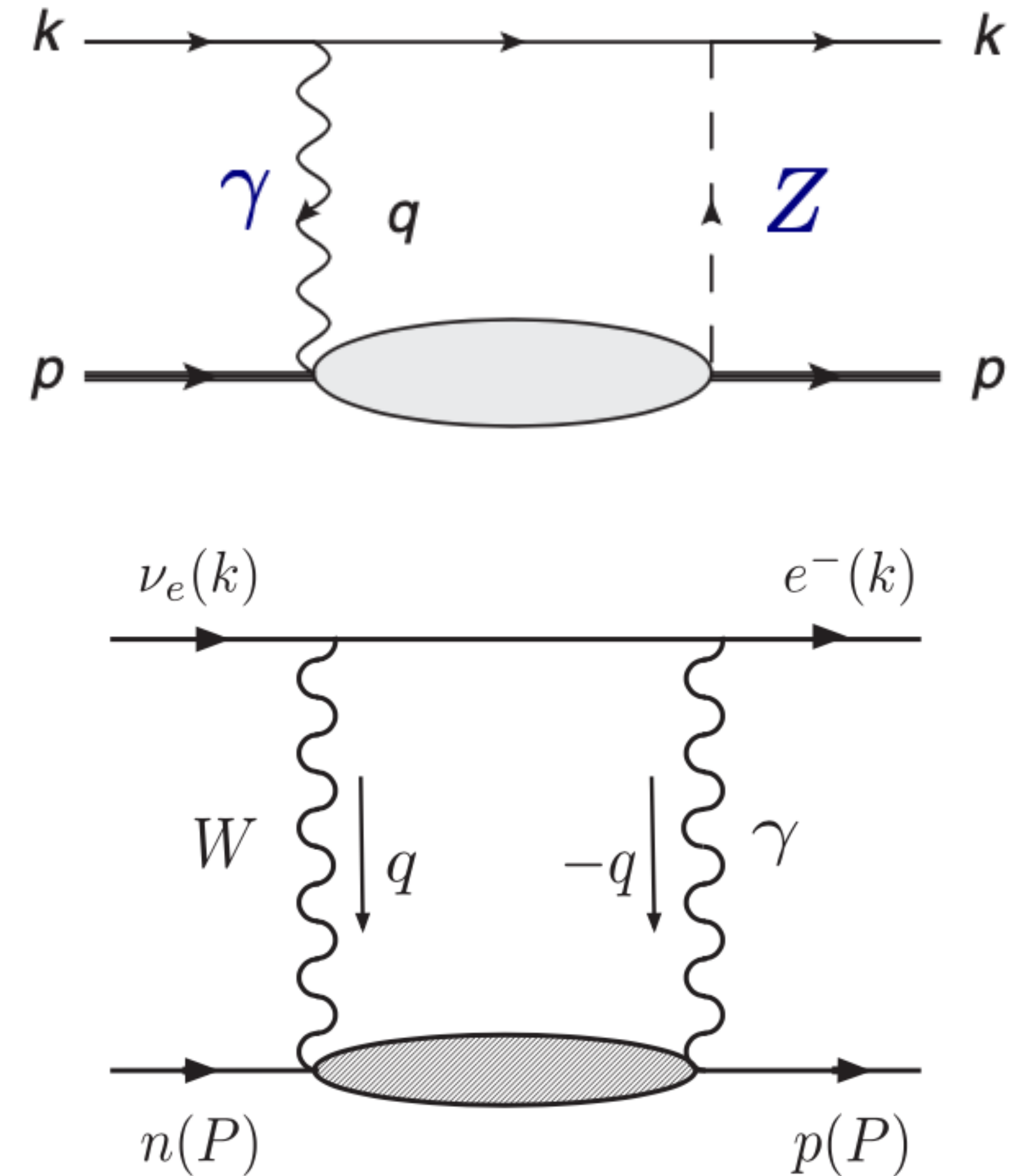
$$+ \square_{AA}^{WW} + \square_{AA}^{ZZ} + \square_{VA}^{\gamma Z}$$
- CKM matrix element extracted from superallowed β decays,

$$|V_{ud}|^2 = \frac{2984.432(3) \text{ s}}{\mathcal{F} t (1 + \Delta_R^V)} \rightarrow \propto \square_{VA}^{\gamma W}$$



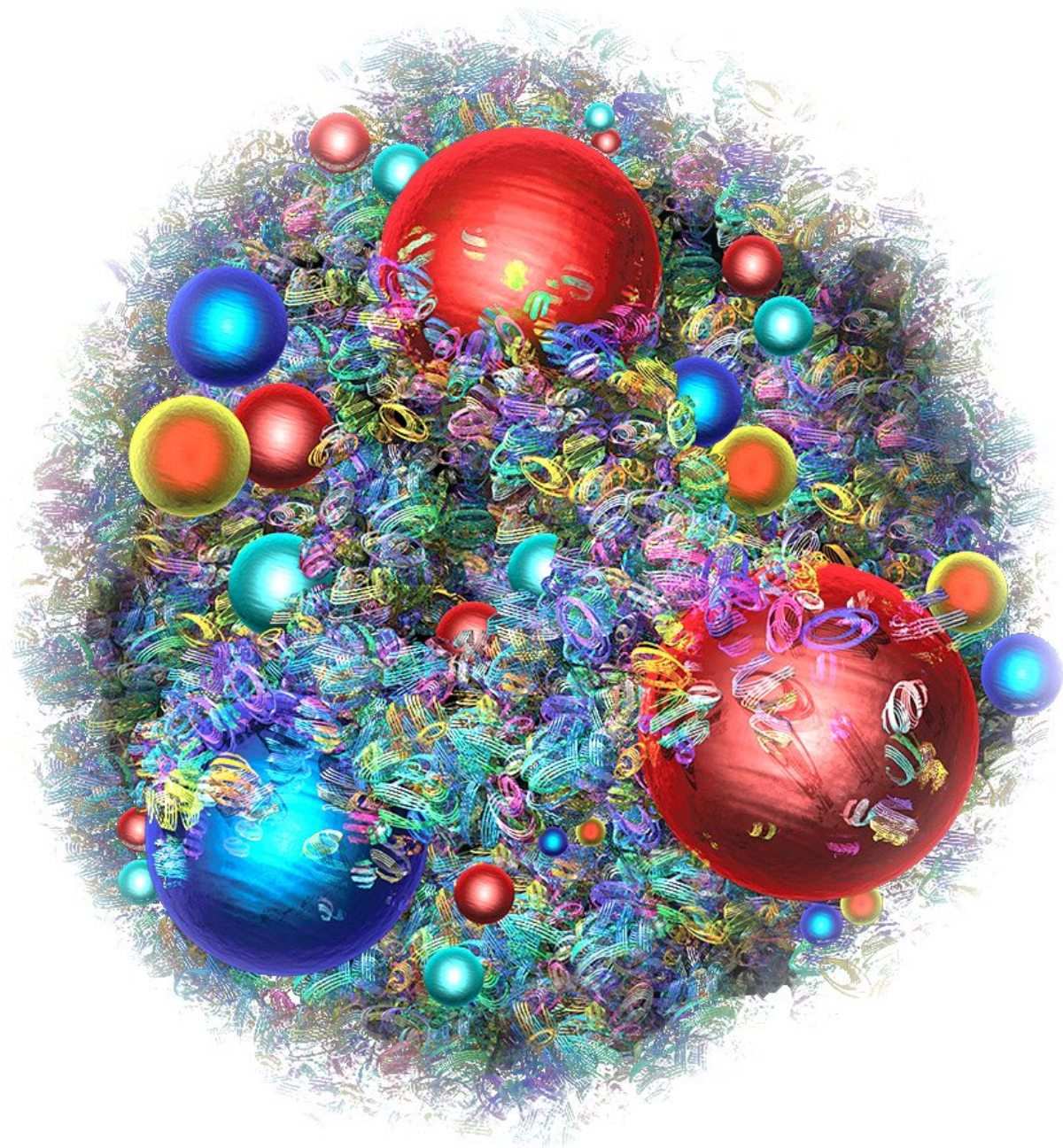
Motivation | EW Box

- Box diagrams proportional to an integral over the whole Q^2 range
$$\Box_A^{\gamma Z/W} \propto \int_0^\infty \frac{dQ^2}{Q^2} \mu_1^{(3)}(Q^2) (\dots)$$
- Low- Q^2 (non-perturbative) regime dominates the integral
- F_3 is experimentally poorly determined in low Q^2
- Lattice approach is ideal for a high-precision determination of moments



Outline

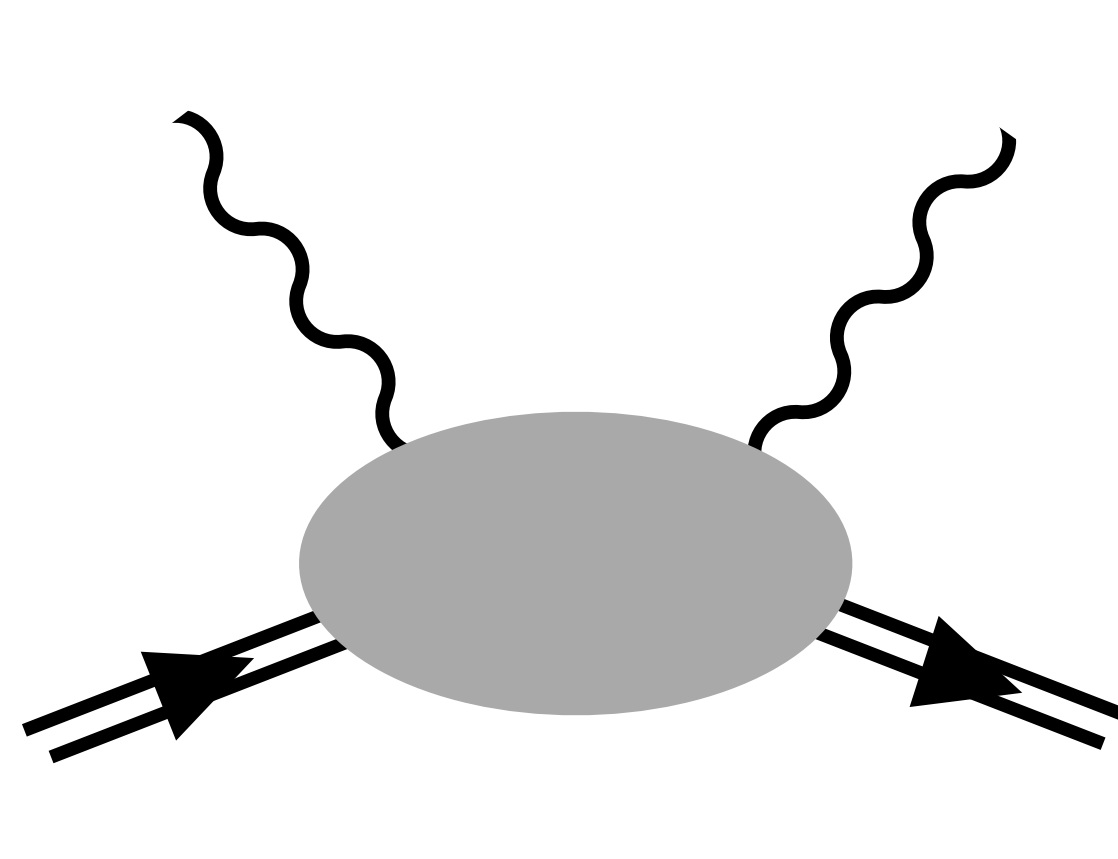
- Motivation



Credit: D Dominguez / CERN

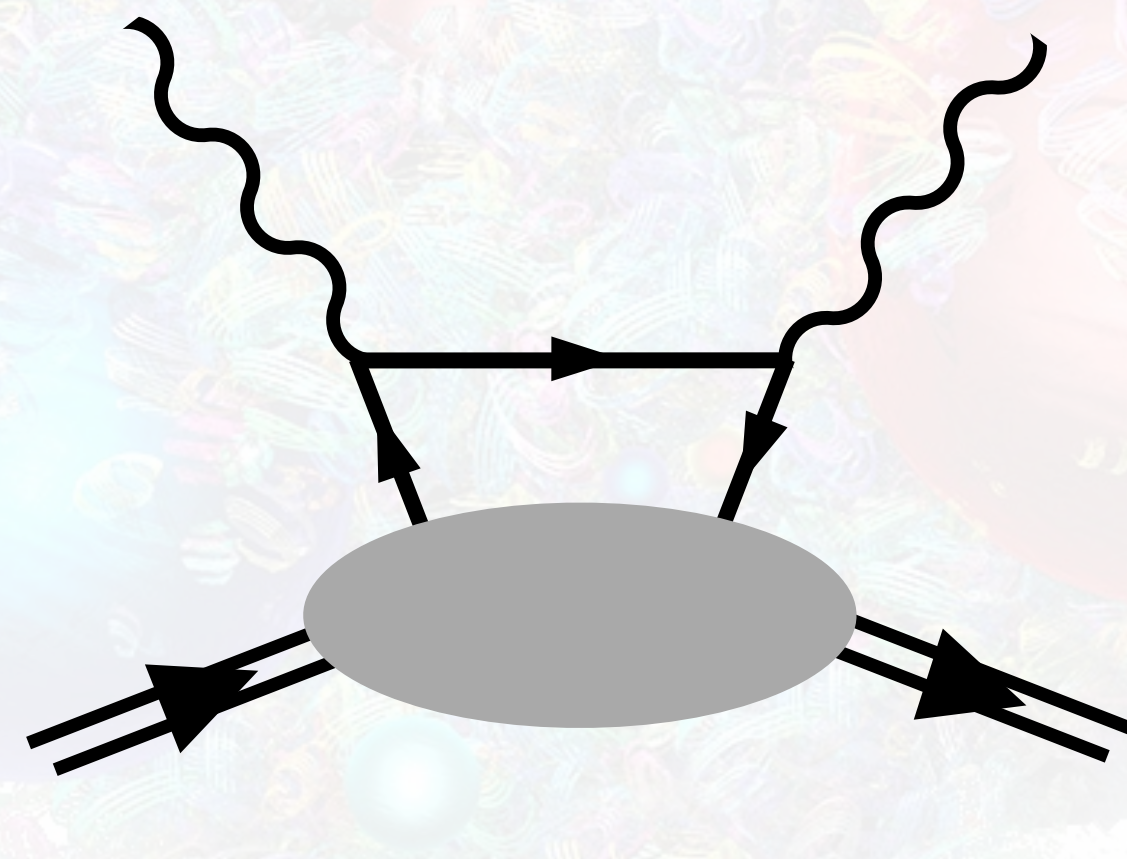
- Forward Compton Amplitude
- Feynman-Hellmann Theorem on the Lattice
- Nucleon Structure Functions
 - Unpolarised | F_1 & F_2
 - Parity-violating | F_3 & Gross-Llewellyn Smith sum rule
- Polarised | g_1 & Bjorken sum rule

Forward Compton Amplitude



A Feynman diagram representing forward Compton scattering. It features a central gray oval representing a nucleon. Two incoming fermion lines, each consisting of two parallel lines with arrows pointing right, enter the oval from the bottom. Two outgoing wavy lines, representing photons, exit the oval from the top.

=



A Feynman diagram representing a specific contribution to Compton scattering. It features a central gray oval representing a nucleon. Two incoming fermion lines, each consisting of two parallel lines with arrows pointing right, enter the oval from the bottom. Two outgoing wavy lines, representing photons, exit the oval from the top. Above the oval, a box diagram is shown with four fermion lines forming a square loop. The top and bottom lines of the box have arrows pointing right, while the left and right lines have arrows pointing left.

+

$$\mathcal{O}\left(\frac{M_N^2}{Q^2}, \frac{1}{Q^2}\right)$$

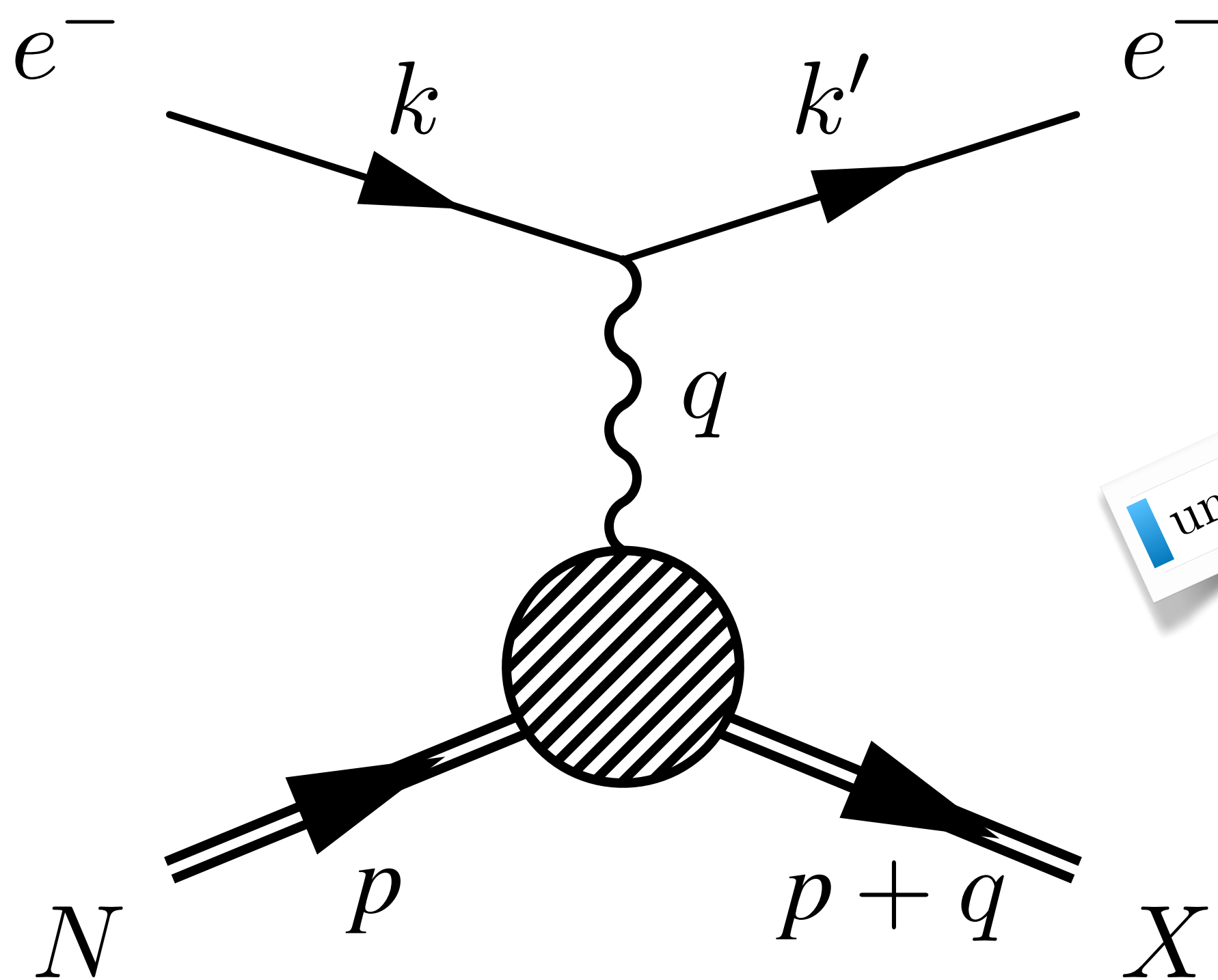
DIS and the Hadronic Tensor

Deep ($Q^2 \gg M^2$) inelastic ($W^2 \gg M^2$) scattering (DIS)

$$d\sigma \sim L_j^{\mu\nu} W_{\mu\nu}^j \quad j = \gamma, Z, \text{ and } \gamma Z \text{ (neutral) or } W \text{ (charged)}$$

leptonic tensor

hadronic tensor



unpolarised

$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4 z e^{iq \cdot z} \rho_{ss'} \langle p, s' | [J_\mu(z), J_\nu(0)] | p, s \rangle$$

$$\rho_{ss'} = \frac{1}{2} \delta_{ss'}$$

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1(x, Q^2) + \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) \frac{F_2(x, Q^2)}{p \cdot q}$$

Structure Functions

Forward Compton Amplitude

$$T_{\mu\nu}(p, q) = i \int d^4z e^{iq \cdot z} \rho_{ss'} \langle p, s' | \mathcal{T} \{ J_\mu(z) J_\nu(0) \} | p, s \rangle \quad , \text{ spin avg. } \rho_{ss'} = \frac{1}{2} \delta_{ss'} \quad \omega = \frac{2p \cdot q}{Q^2}$$

Same Lorentz decomposition as the Hadronic Tensor

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \mathcal{F}_1(\omega, Q^2) + \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) \frac{\mathcal{F}_2(\omega, Q^2)}{p \cdot q}$$

Compton Structure Functions (SF)

Optical theorem

$$\left| \begin{array}{c} \text{J}_\mu(q) \\ \text{N}(p) \end{array} \right|^2 \sim 2 \text{Im} \left(\begin{array}{c} \text{J}_\mu(q) \\ \text{N}(p) \end{array} \right)$$

$$W_{\mu\nu} \sim \int d^4x \langle p | [J_\mu(x), J_\nu(0)] | p \rangle$$

Structure Functions: $F_{1,2}(x, Q^2)$

$$T_{\mu\nu} \sim \int d^4x \langle p | T \{ J_\mu(x) J_\nu(0) \} | p \rangle$$

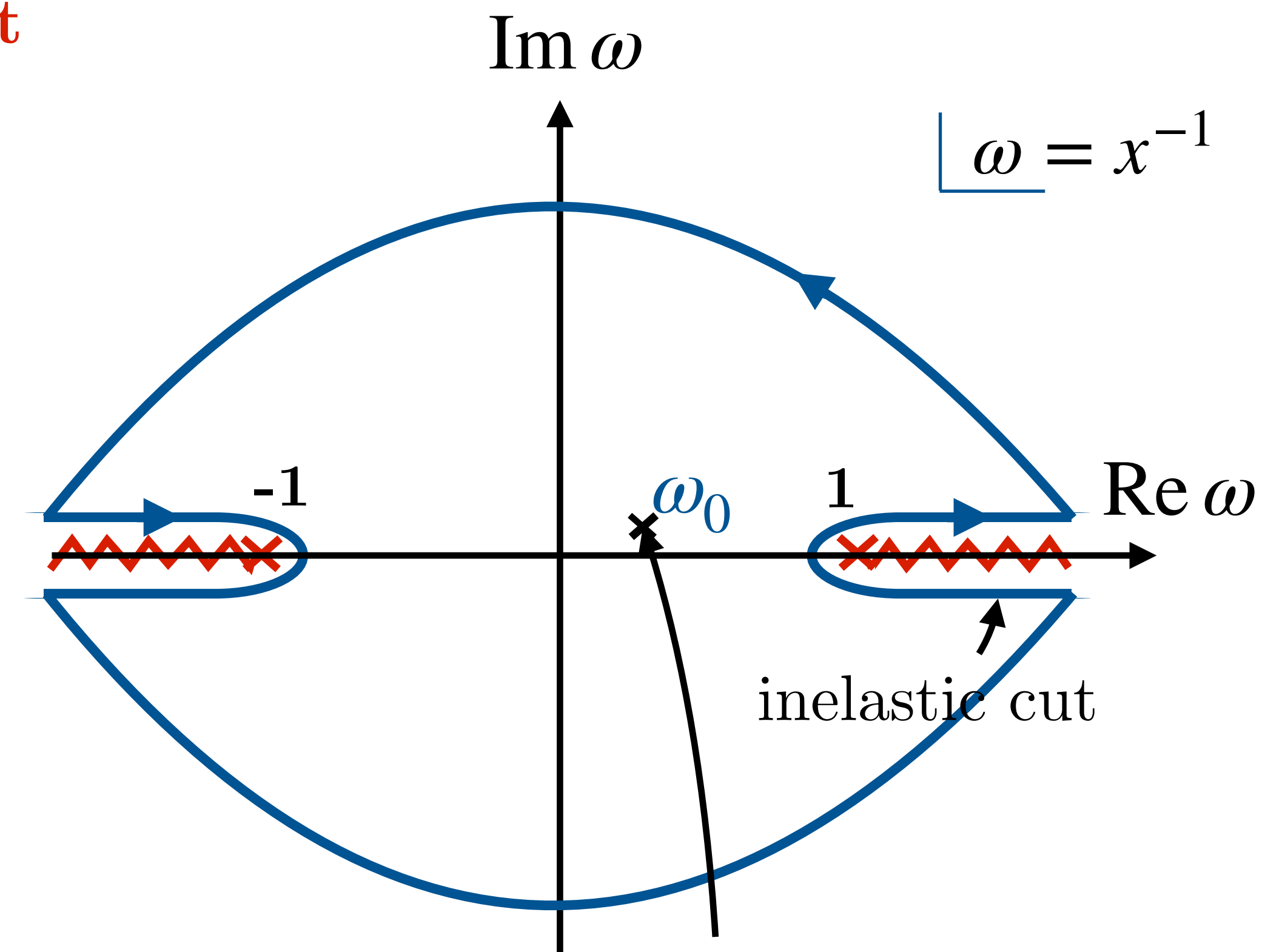
Compton Structure Functions: $\mathcal{F}_{1,2}(p \cdot q, Q^2)$

Nucleon Structure Functions

- we can write down dispersion relations and connect Compton SFs to DIS SFs:

$$\mathcal{F}_1(\omega, Q^2) = \mathcal{F}_1(0, Q^2) + 2\omega^2 \int_0^1 dx \frac{2x F_1(x, Q^2)}{1 - x^2\omega^2 - i\epsilon}$$

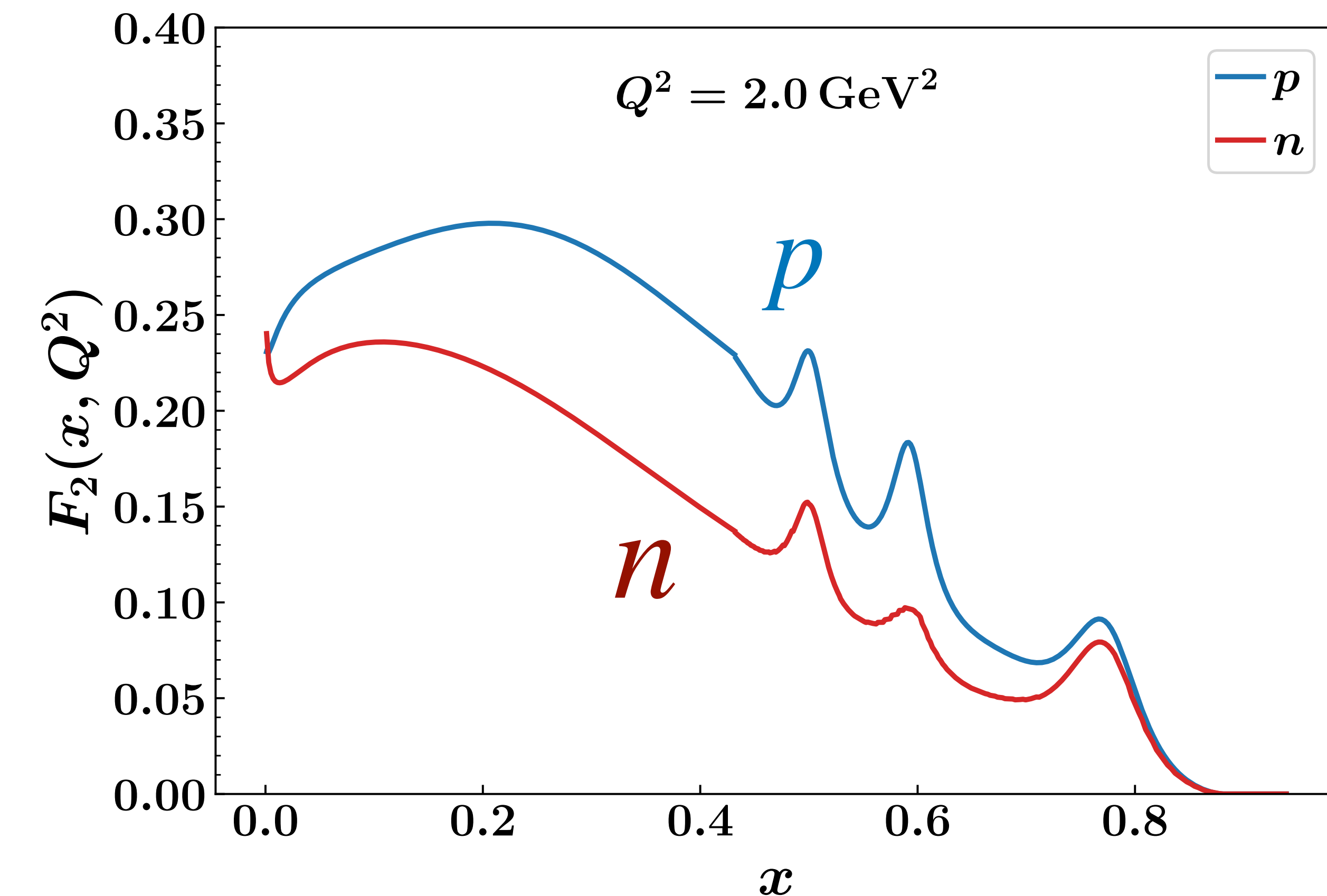
$$\mathcal{F}_2(\omega, Q^2) = 4\omega \int_0^1 dx \frac{F_2(x, Q^2)}{1 - x^2\omega^2 - i\epsilon}$$



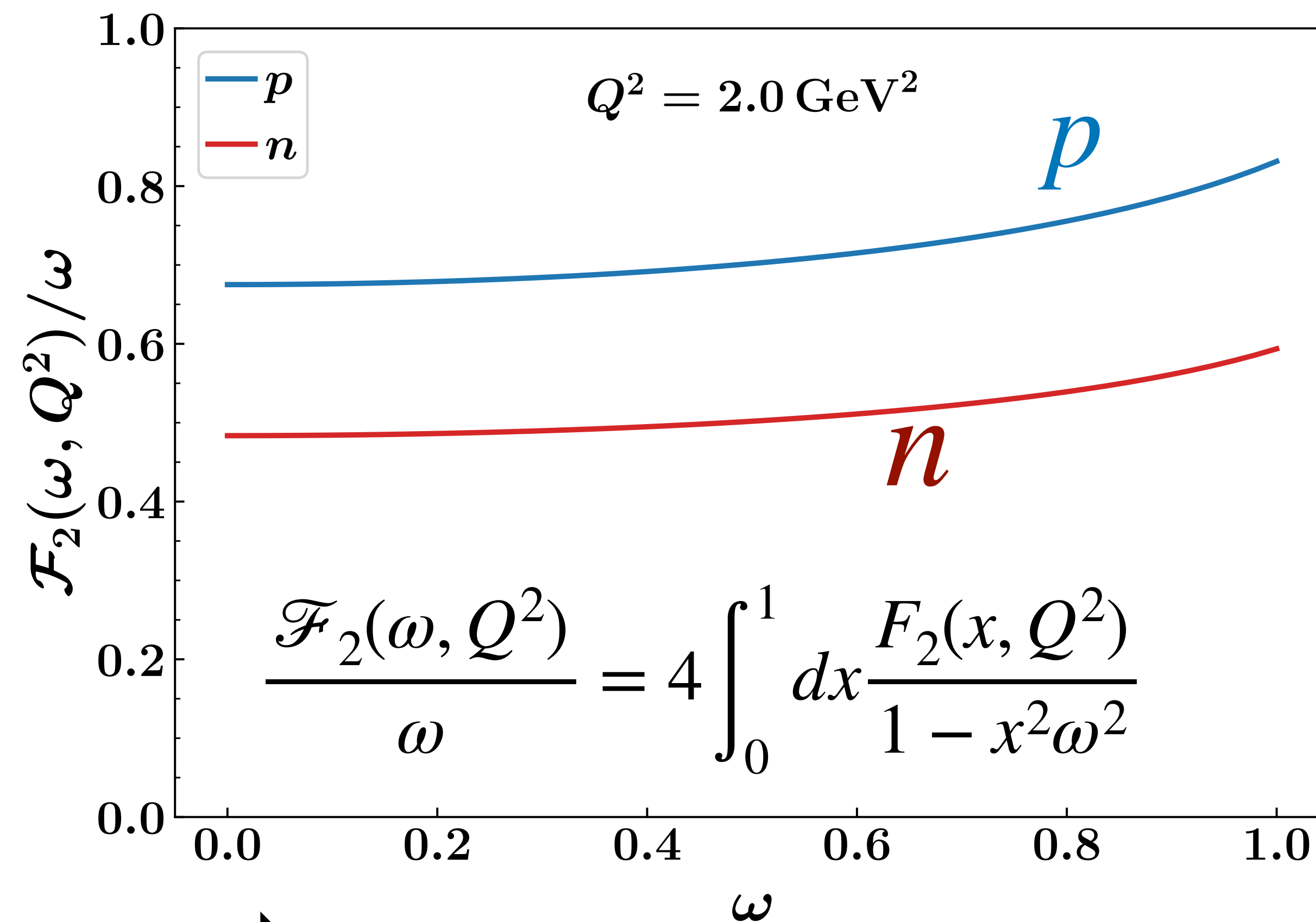
Compton Amplitude is an analytic function in the unphysical region $|\omega_0| < 1$

Shape of the Compton Amplitude

Structure functions



Compton Amplitudes

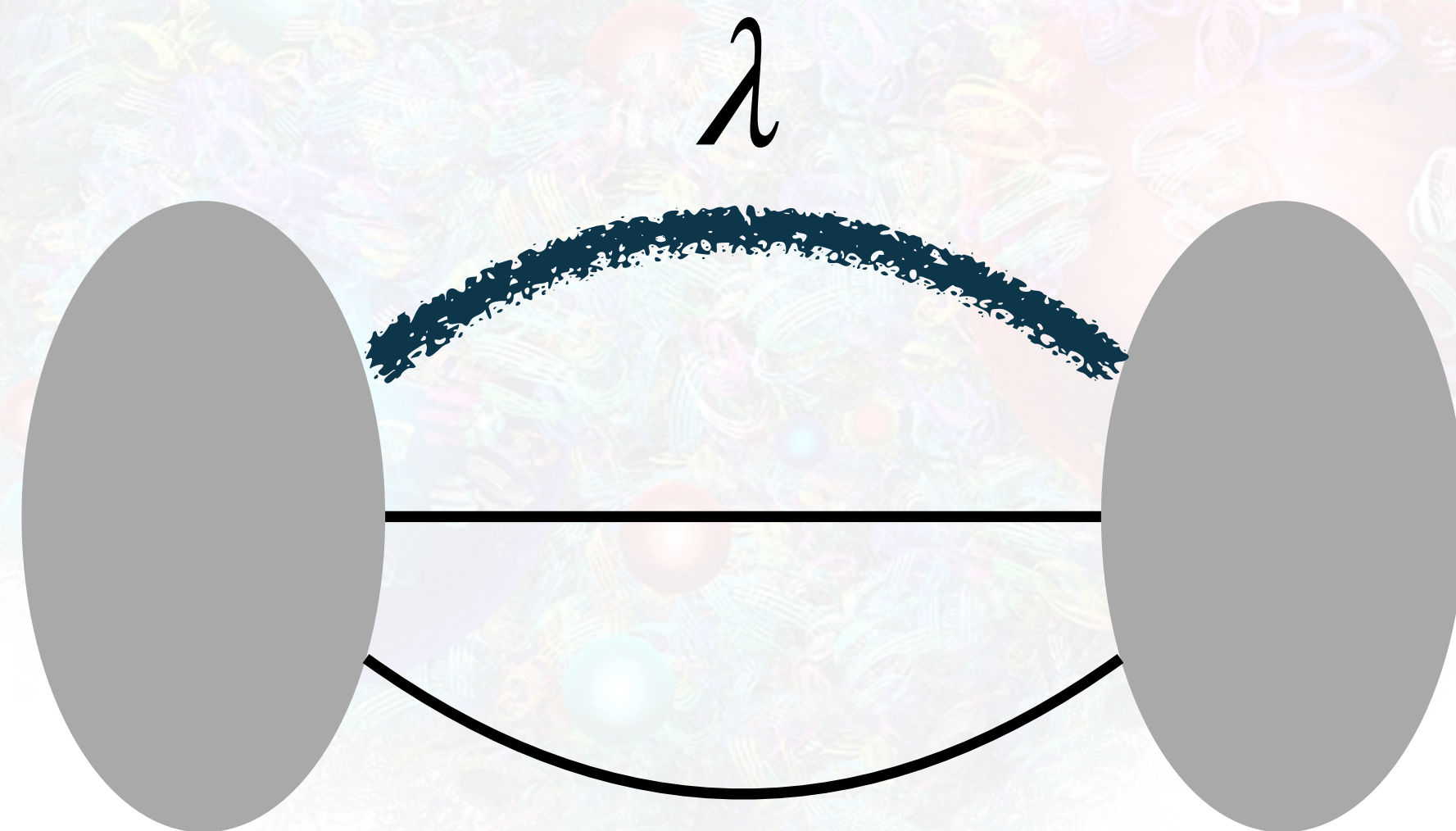


$$\frac{\mathcal{F}_2(\omega, Q^2)}{\omega} = 4 \int_0^1 dx \frac{F_2(x, Q^2)}{1 - x^2 \omega^2}$$

dispersion relation

High-W: M. Arneodo et al. [NMC],
PLB364, 107-115 (1995), [hep-ph/9509406]
Low-W: M.E. Christy and P.E. Bosted,
PRC81, 055213 (2010), [0712.3731]

Feynman-Hellmann Theorem on the Lattice



FH Theorem at 1st order

in Quantum Mechanics:

$$\frac{\partial E_\lambda}{\partial \lambda} = \langle \phi_\lambda | \frac{\partial H_\lambda}{\partial \lambda} | \phi_\lambda \rangle$$

H_λ : perturbed Hamiltonian of the system

E_λ : energy eigenvalue of the perturbed system

ϕ_λ : eigenfunction of the perturbed system

- expectation value of the perturbed system is related to the shift in the energy eigenvalue

in Lattice QCD: energy shifts in the presence of a weak external field

$$S \rightarrow S(\lambda) = S + \underset{\substack{\nearrow \\ \text{real parameter}}}{\lambda} \int d^4x \mathcal{O}(x) \quad \xrightarrow{\text{e.g. local bilinear operator}} \quad \bar{q}(x) \Gamma_\mu q(x) \quad , \Gamma_\mu \in \{\mathbf{1}, \gamma_\mu, \gamma_5 \gamma_\mu, \dots\}$$

@ 1st order

$$\frac{\partial E_\lambda}{\partial \lambda} = \frac{1}{2E_\lambda} \langle 0 | \mathcal{O} | 0 \rangle$$

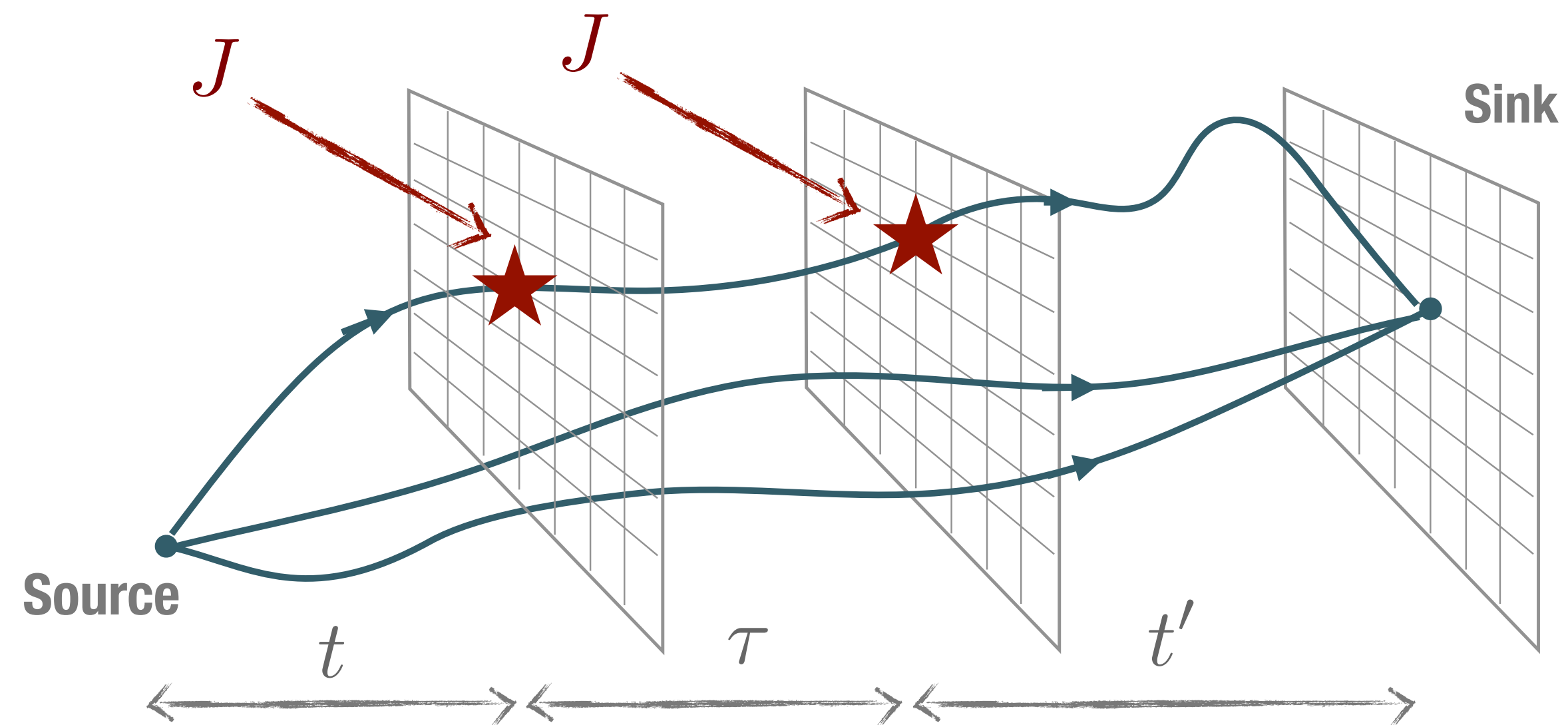
$E_\lambda \rightarrow$ spectroscopy, 2-pt function

$\langle 0 | \mathcal{O} | 0 \rangle \rightarrow$ determine 3-pt

Applications:

- σ - terms
- Form factors

Compton amplitude

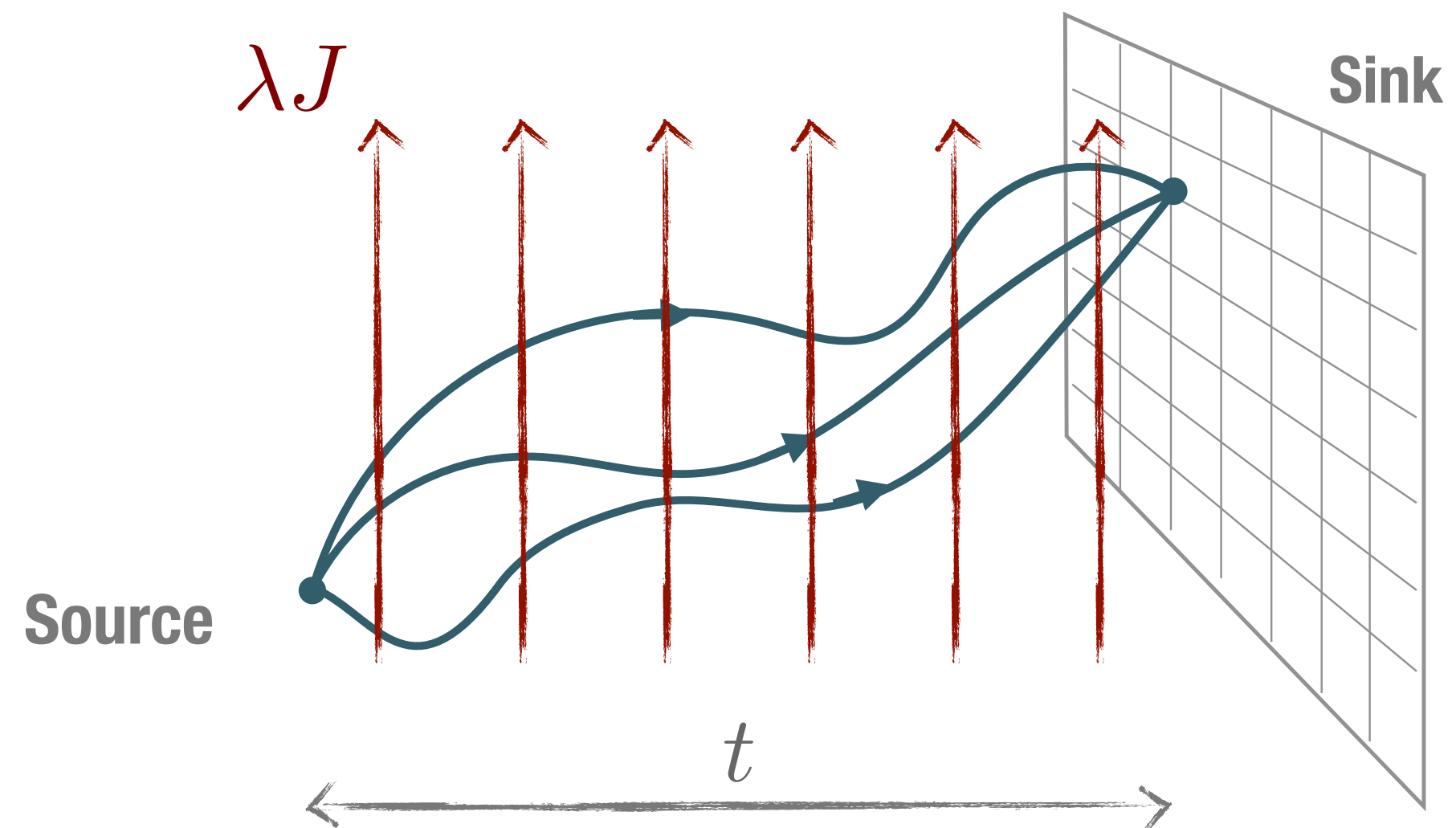


- **4-pt functions**

$$t, t' \gg \frac{1}{\Delta E}$$

$$\frac{\langle C_4(t, \tau, t') \rangle}{\langle C_2(t) \rangle \langle C_2(t') \rangle} \propto \langle N | J(\tau_E) J | N \rangle$$

$$\int_0^\infty d\tau_E \rightarrow \langle N | JJ | N \rangle$$



- **Feynman—Hellmann**

$$t \gg \frac{1}{\Delta E},$$

$$\left. \frac{\partial^2 E}{\partial \lambda^2} \right|_{\lambda \rightarrow 0} \propto \langle N | JJ | N \rangle$$

Nucleon Structure Functions

$\mathcal{F}_1$ subtraction term

PHYSICAL REVIEW D **111**, 094513 (2025)

Editors' Suggestion

Lattice QCD calculation of the Compton amplitude subtraction function

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Forward Compton Amplitude

$$T_{\mu\nu}(p, q) = i \int d^4z e^{iq \cdot z} \rho_{ss'} \langle p, s' | \mathcal{T} \{ J_\mu(z) J_\nu(0) \} | p, s \rangle \quad , \text{ spin avg. } \rho_{ss'} = \frac{1}{2} \delta_{ss'} \quad \omega = \frac{2p \cdot q}{Q^2}$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \mathcal{F}_1(\omega, Q^2) + \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) \frac{\mathcal{F}_2(\omega, Q^2)}{p \cdot q}$$

Simplest kinematics to directly isolate $\mathcal{F}_1$

$$J_3 J_3 \text{ and } q_3 = 0, \vec{p} = \vec{0}$$

$$T_{33}(\vec{0}, q) = \mathcal{F}_1(\omega = 0, Q^2) = T_1(0, Q^2) \equiv S_1(Q^2)$$

Strategy | Energy shifts

Isolate the 2nd-order energy shift

$$G_{\lambda}^{(2)}(\mathbf{p}; t) \sim A_{\lambda}(\mathbf{p}) e^{-E_{N_{\lambda}}(\mathbf{p})t}$$

$$E_{N_{\lambda}}(\mathbf{p}) = E_N(\mathbf{p}) + \lambda \left. \frac{\partial E_{N_{\lambda}}(\mathbf{p})}{\partial \lambda} \right|_{\lambda=0} + \frac{\lambda^2}{2!} \left. \frac{\partial^2 E_{N_{\lambda}}(\mathbf{p})}{\partial^2 \lambda} \right|_{\lambda=0} + \mathcal{O}(\lambda^3)$$

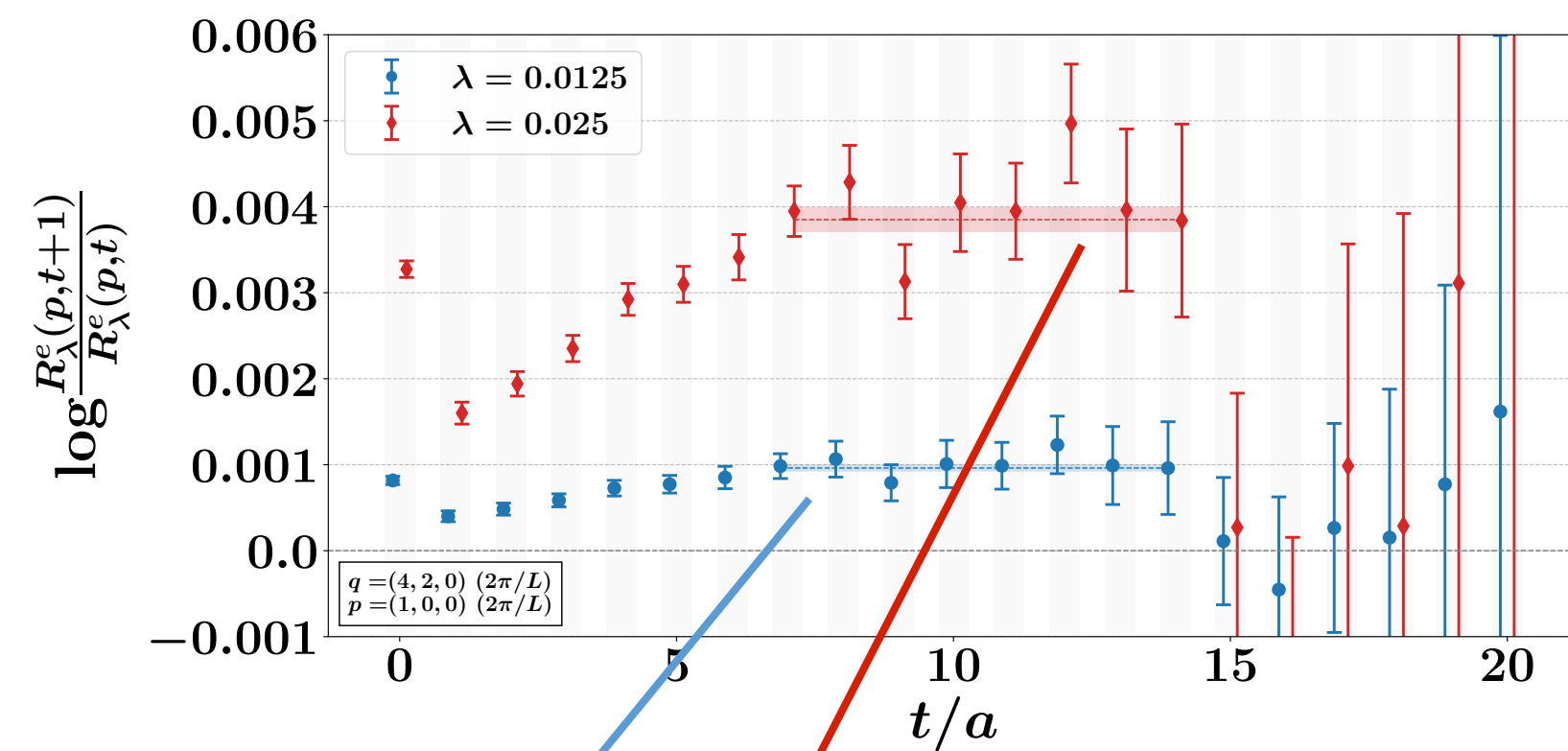
$$= E_N(\mathbf{p}) + \Delta E_N^o(\mathbf{p}) + \Delta E_N^e(\mathbf{p})$$

Construct a ratio of perturbed to unperturbed 2-pt functions

$$R_{\lambda}^e(\mathbf{p}, t) \equiv \frac{G_{+\lambda}^{(2)}(\mathbf{p}, t) G_{-\lambda}^{(2)}(\mathbf{p}, t)}{(G^{(2)}(\mathbf{p}, t))^2}$$

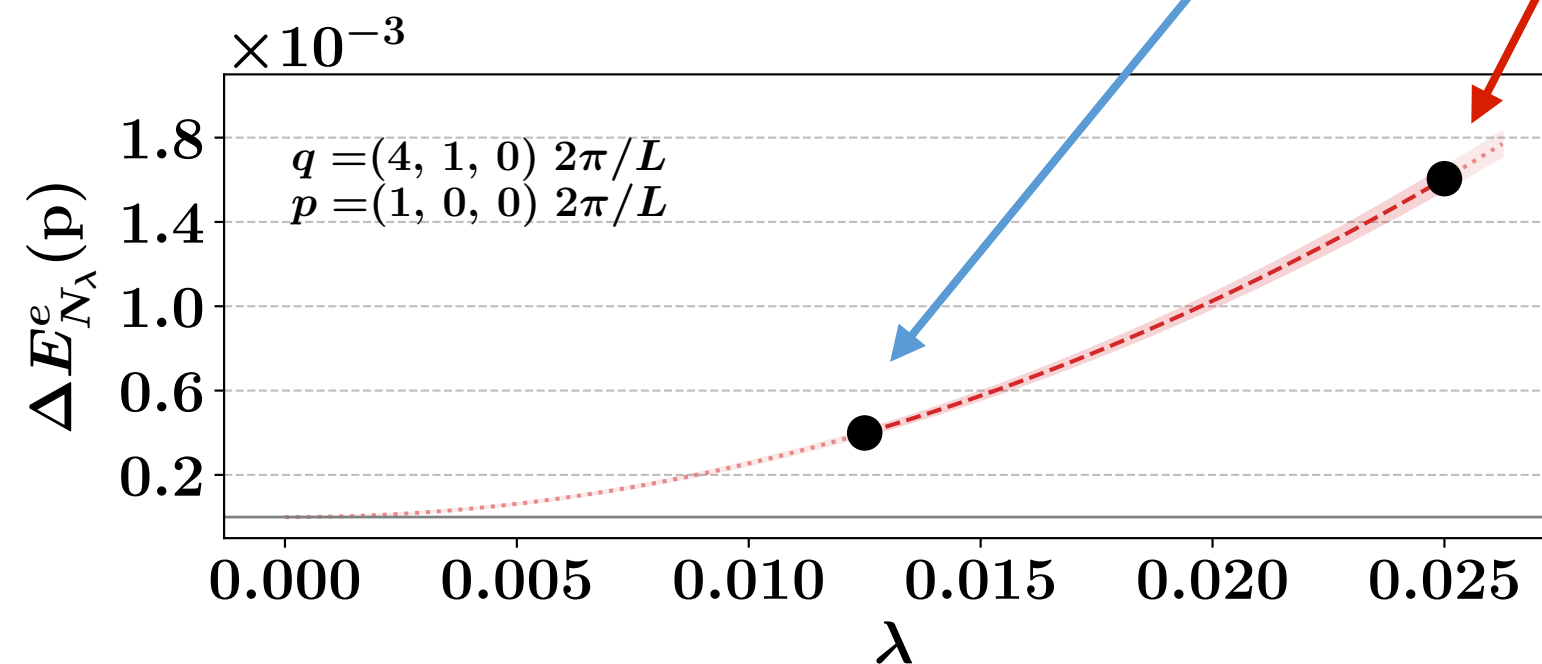
$$\xrightarrow{t \gg 0} A_{\lambda}(\mathbf{p}) e^{-2\Delta E_{N_{\lambda}}^e(\mathbf{p})t}$$

● Extract energy shifts for each $|\lambda|$



● Get the 2nd order shift $\left. \frac{\partial^2 E_N^{\lambda}(p)}{\partial \lambda^2} \right|_{\lambda=0}$

● Energy shift is related to the Amplitude



$$\left. \frac{\partial^2 E_N^{\lambda}(p)}{\partial \lambda^2} \right|_{\lambda=0} = - \frac{S_1(Q^2)}{E_N(p)}$$

Subtraction term S_1

● **Flavour decomposition**

● Good agreement with OPE:

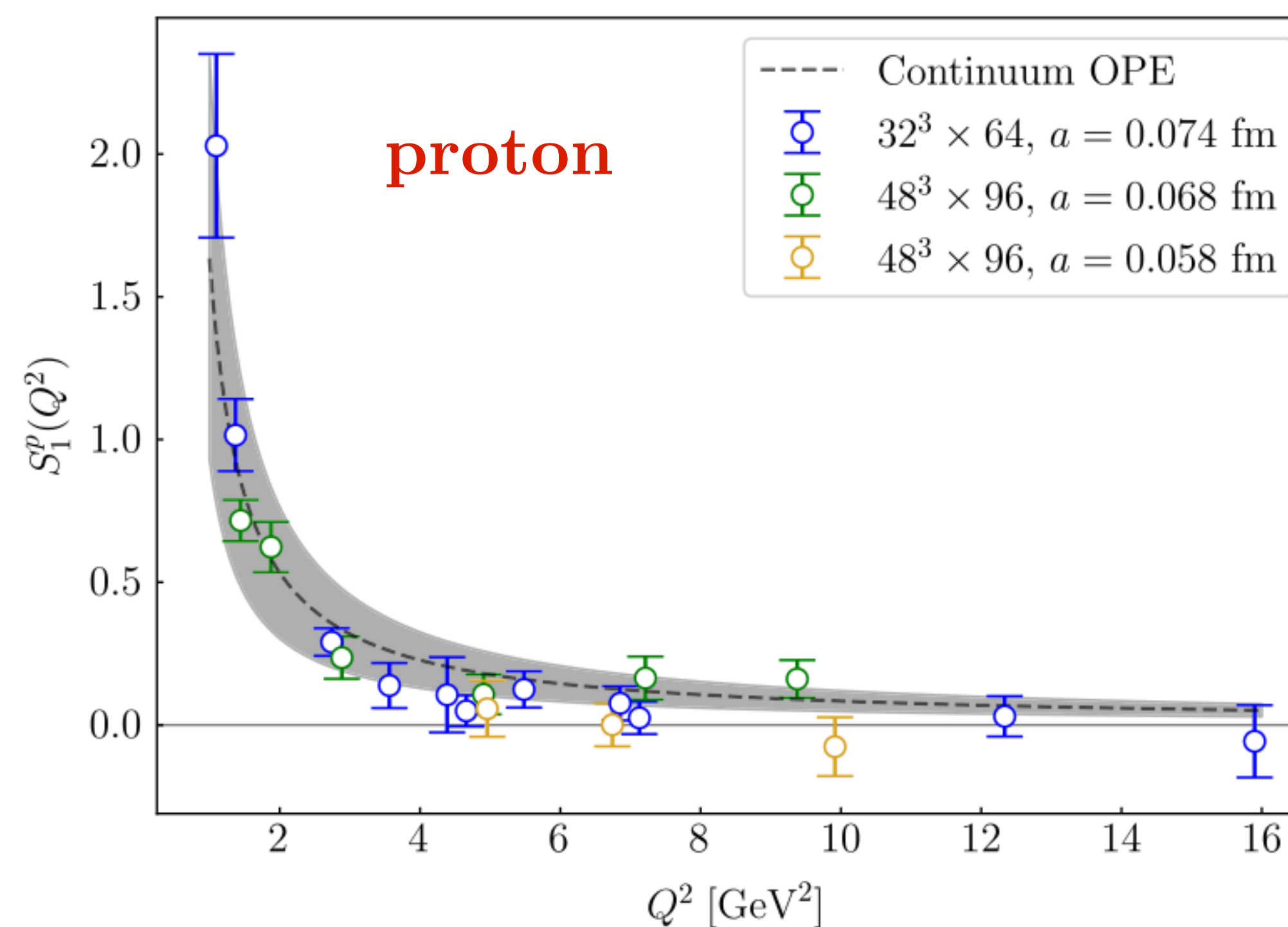
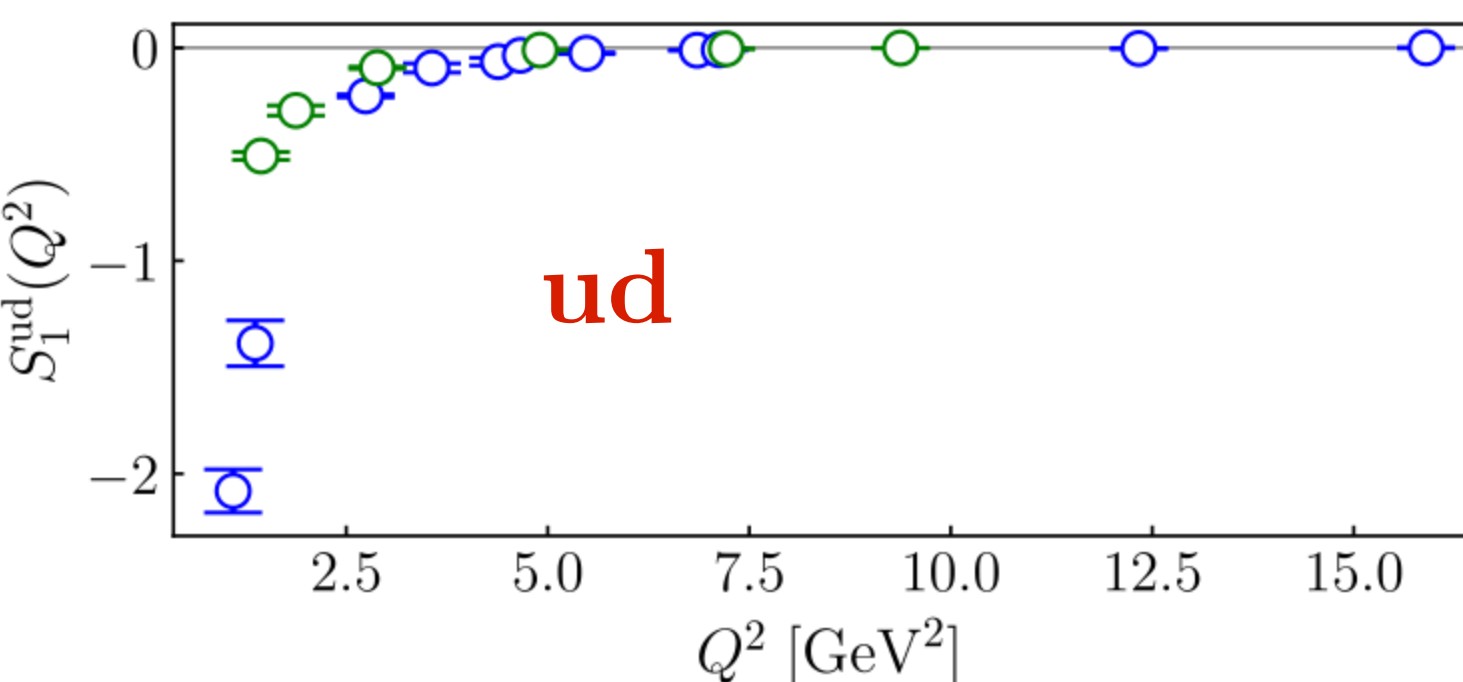
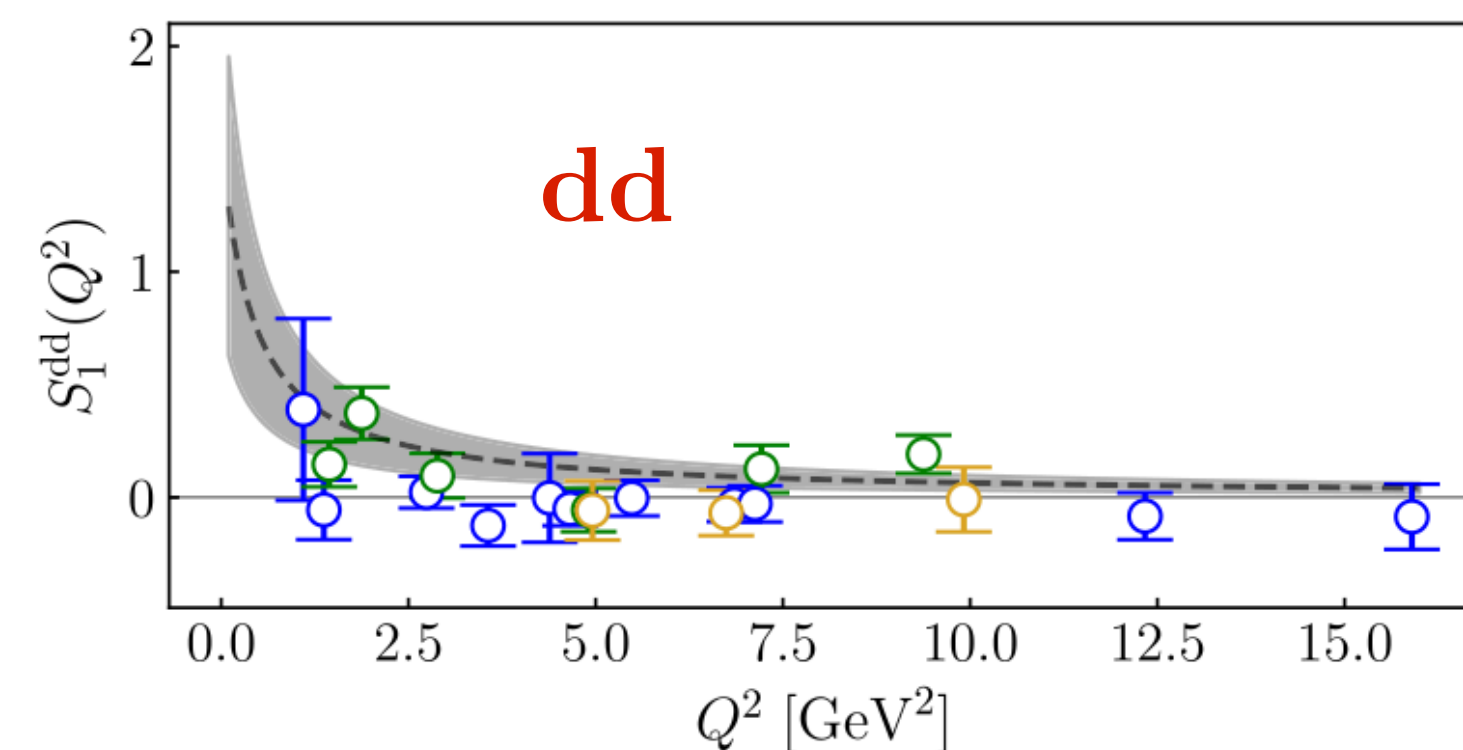
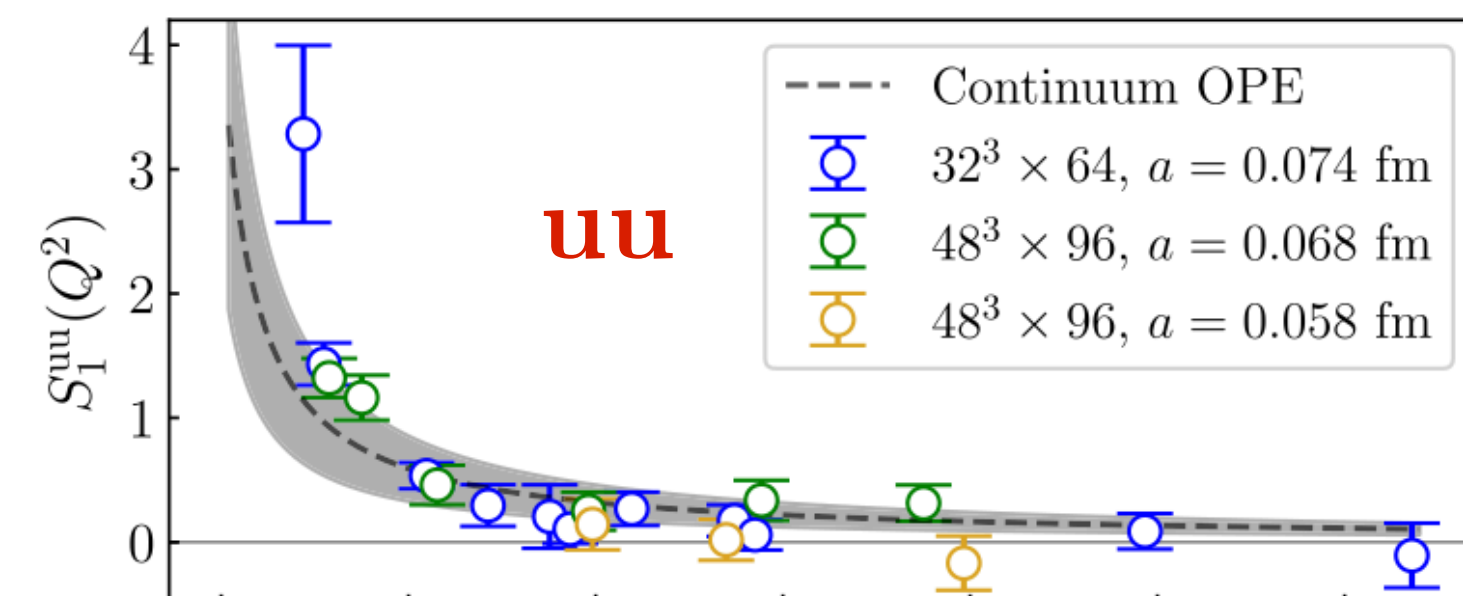


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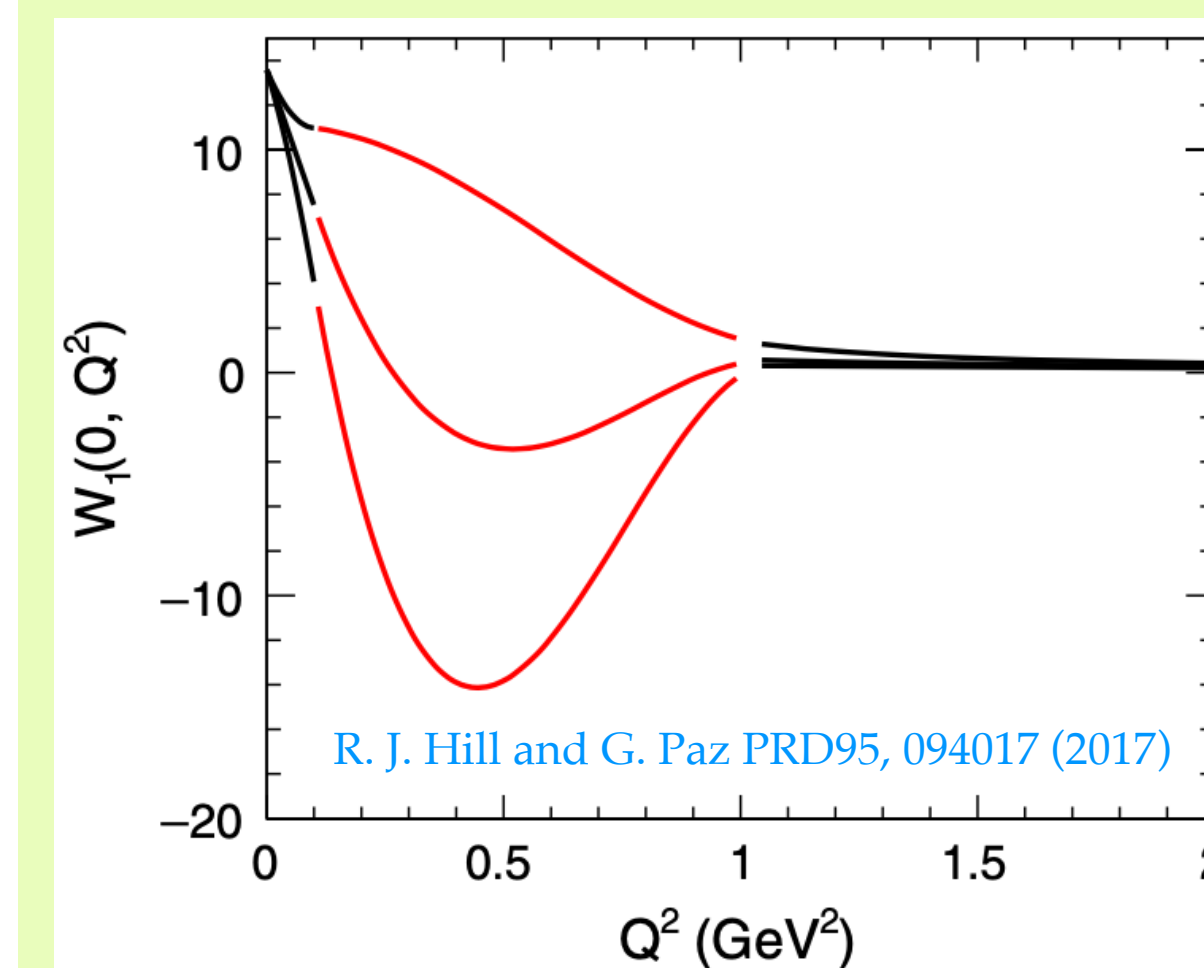


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$$S_1^{\text{OPE}}(Q^2) = \frac{4m_p^2}{Q^2} \sum_q e_q^2 \left(\underset{\substack{\uparrow \\ \text{momentum fraction}}}{a_2^q} - \frac{m_q}{m_p} \underset{\substack{\uparrow \\ \text{scalar charge}}}{g_S^q} \right)$$



- Low- and high- Q^2 regions are known
- Prospect of constraining the mid- Q^2 region using LQCD results



Nucleon Structure Functions

$\mathcal{F}_2$ & higher-twist

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Moments and power corrections of longitudinal and transverse proton structure functions from lattice QCD

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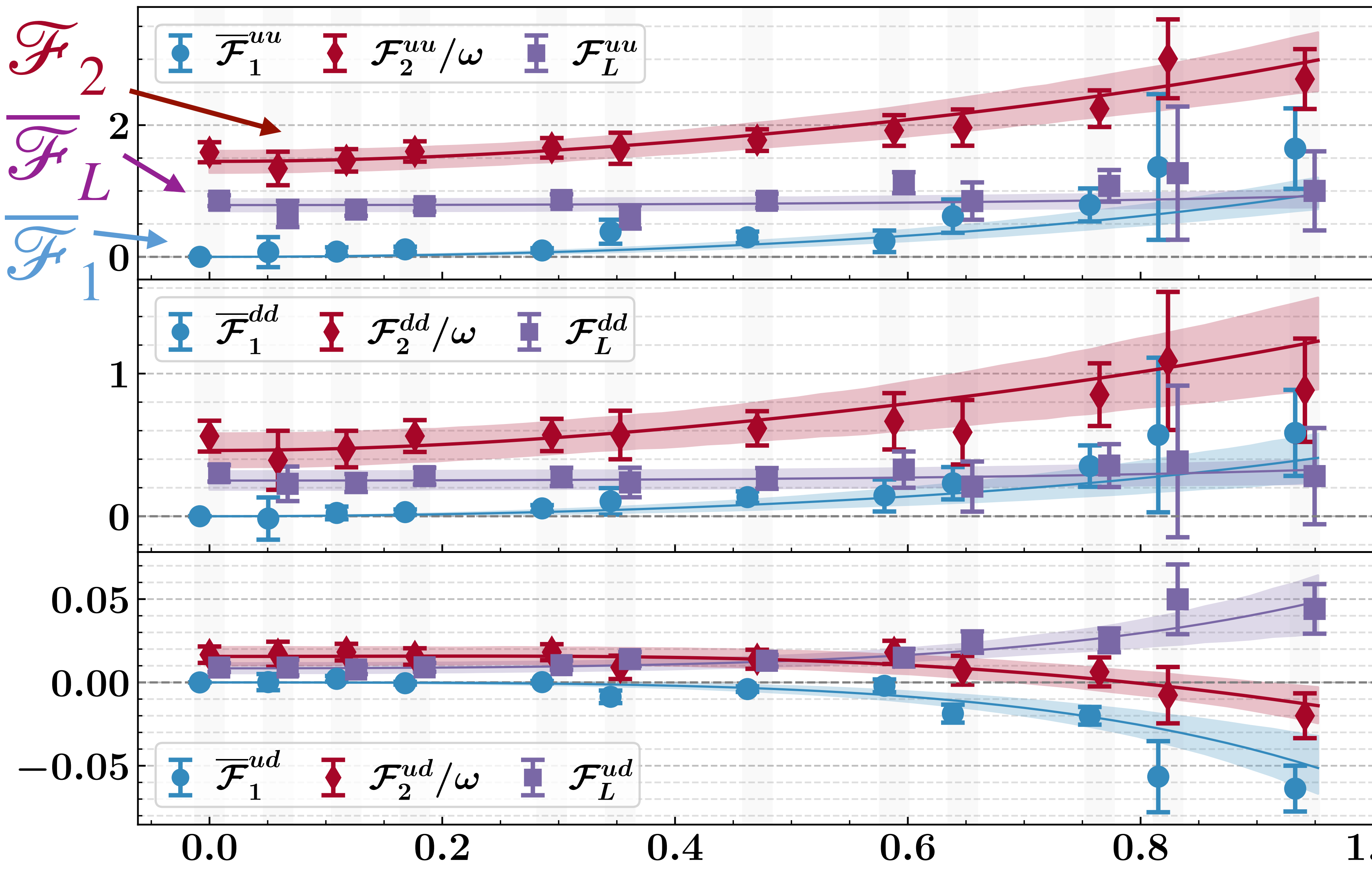
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$\mathcal{F}_{1,2,L}$ Compton Amplitudes

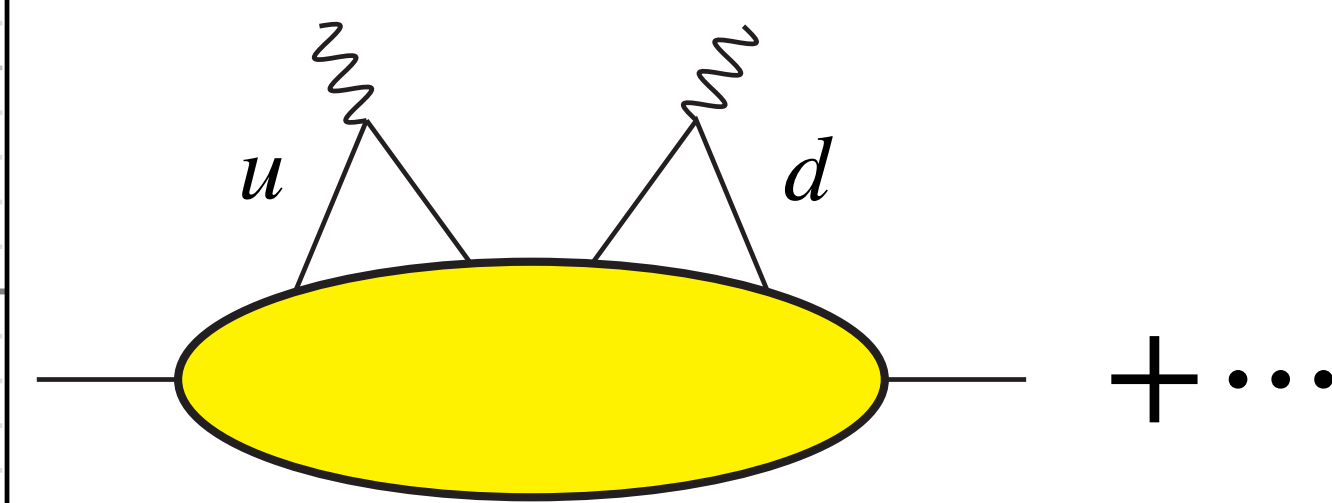
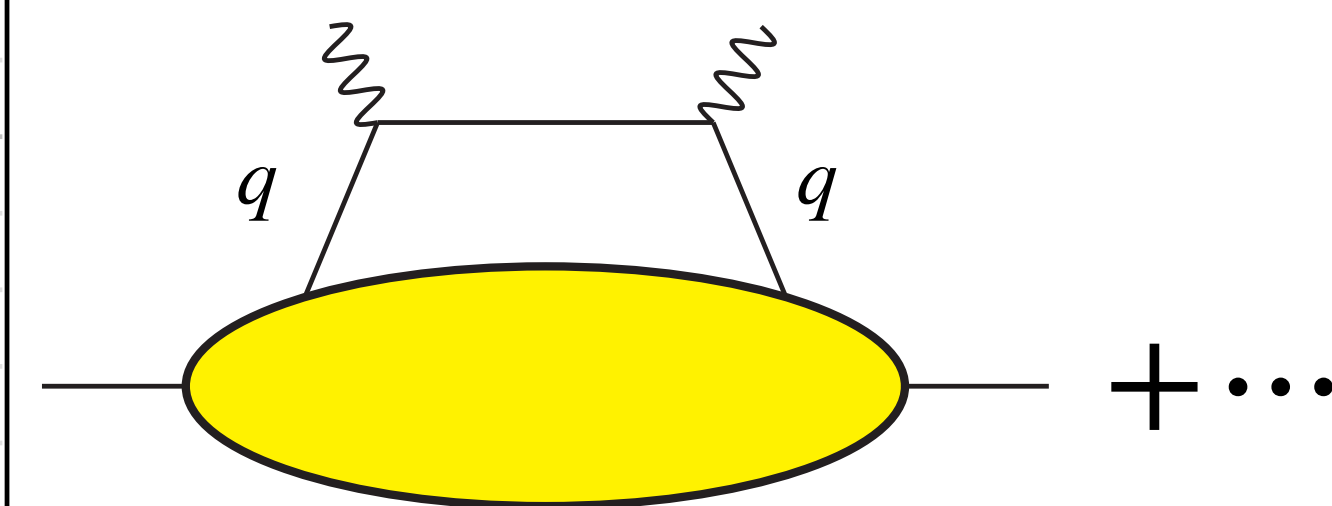


48³x96, 2+1 flavour

$a = 0.068$ fm

$m_\pi \sim 420$ MeV

$Q^2 \sim 4.9$ GeV²



$\mathcal{F}_2$ | Moments

- using the Taylor expansion, $\frac{1}{1 - (x\omega)^2} = \sum_{n=1}^{\infty} (x\omega)^{2n-2}$, to expand the dispersion relation

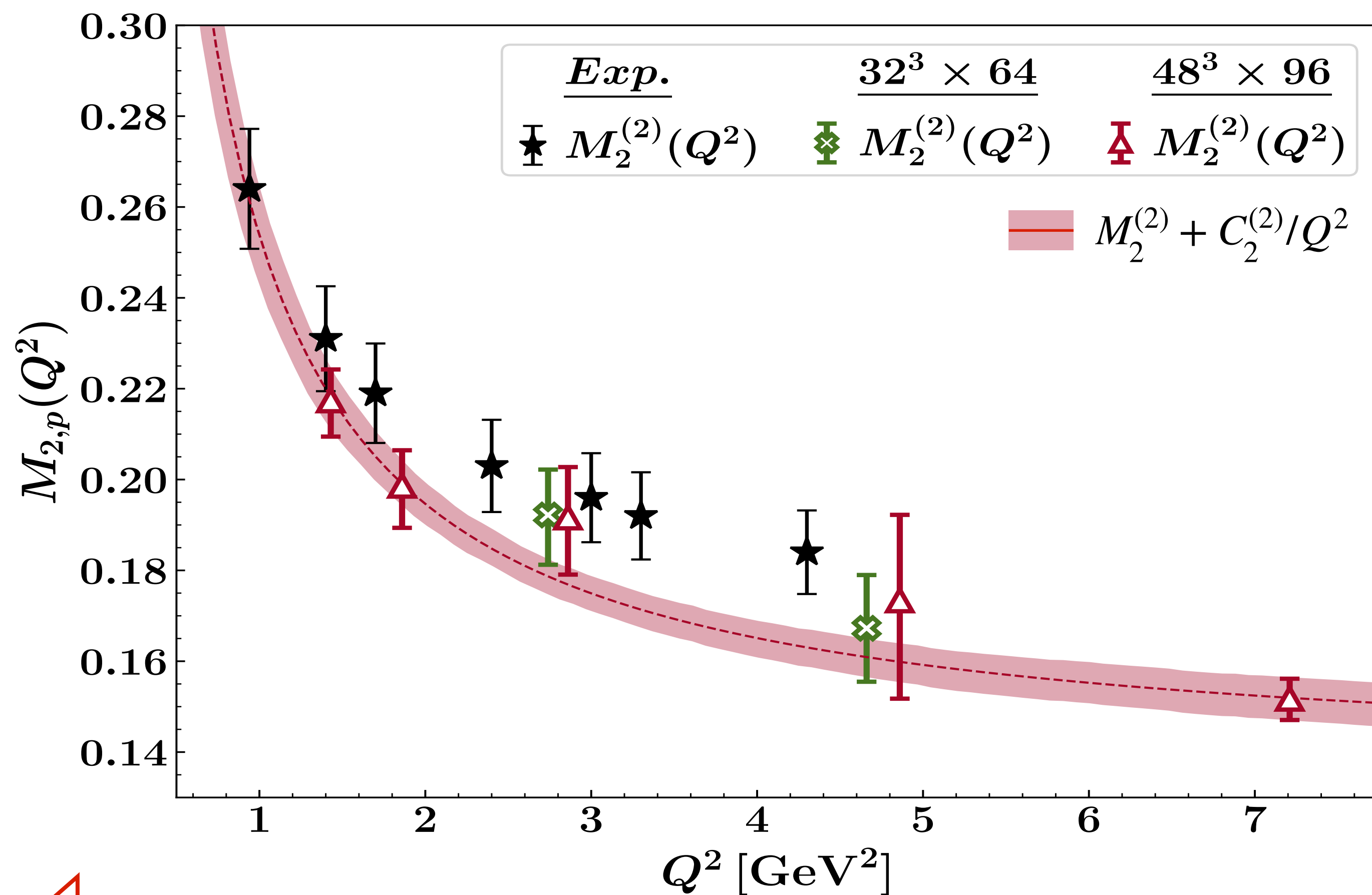
$$\begin{aligned}\mathcal{F}_2(\omega, Q^2) &= \sum_{n=1}^{\infty} 4\omega^{2n-1} \int_0^1 dx x^{2n-2} F_2(x, Q^2) \\ &= \sum_{n=1}^{\infty} 4\omega^{2n-1} M_{2n}^{(2)}(Q^2)\end{aligned}$$

$$\omega = \frac{2p \cdot q}{Q^2}$$

- Compton structure functions are given in terms of towers of physical Mellin moments

Moments | proton F_2

● Unique ability to study the Q^2 dependence of the moments!



- Global PDF-fit cuts $\sim 1 - 10 \text{ GeV}^2$
- Need $Q^2 > 10 \text{ GeV}^2$ data to directly access partonic moments

● Power corrections below $\sim 3 \text{ GeV}^2$?

● Naive modelling via

$$M_2^{(2)}(Q^2) = M_2^{(2)} + C_2^{(2)}/Q^2$$

Exp $M_2^{(2)}$: C. S. Armstrong, R. Ent, C. E. Keppel, S. Liuti, G. Niculescu, and I. Niculescu, [Phys. Rev. D 63, 094008](#) (2001), arXiv:hep-ph/0104055.

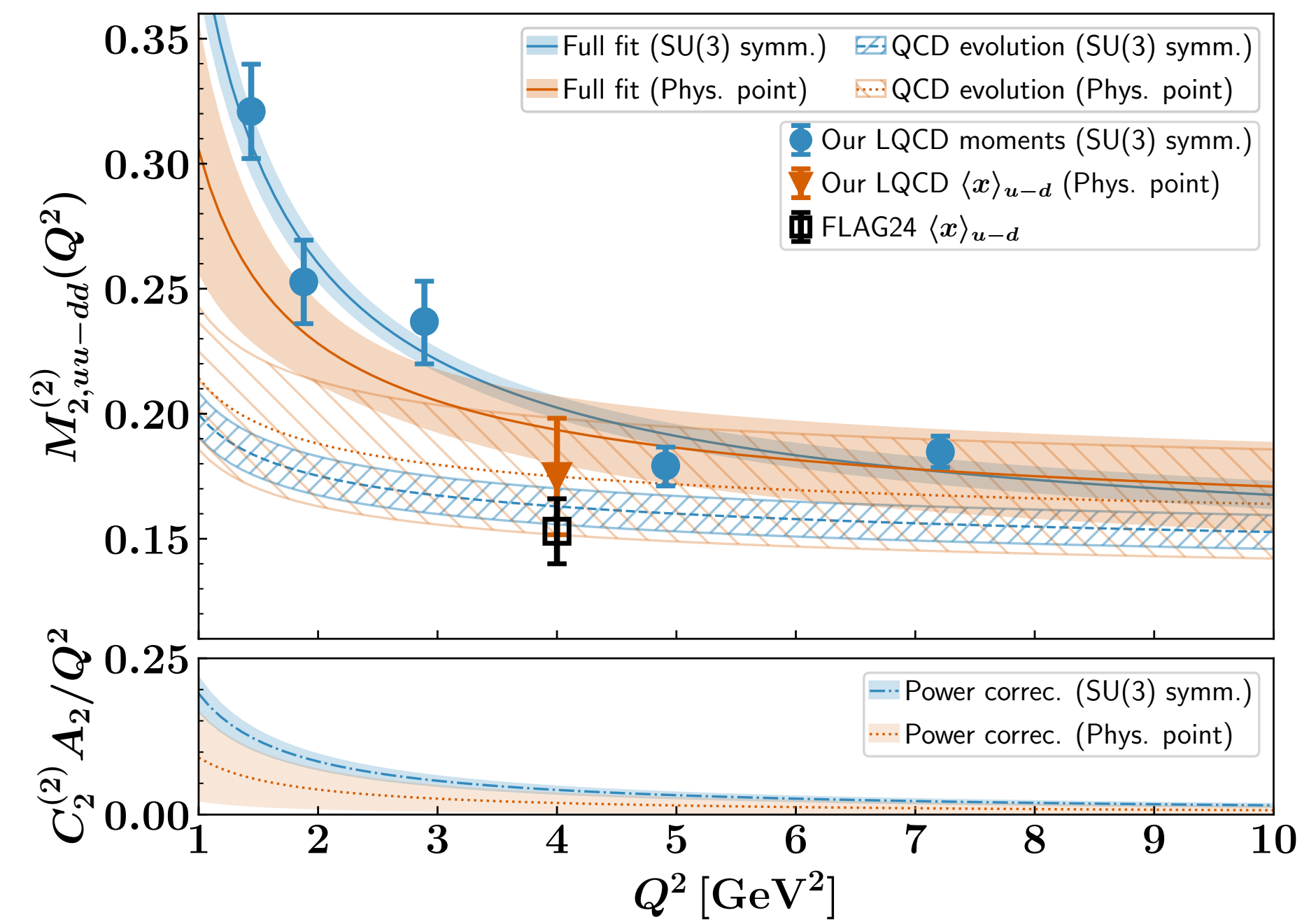
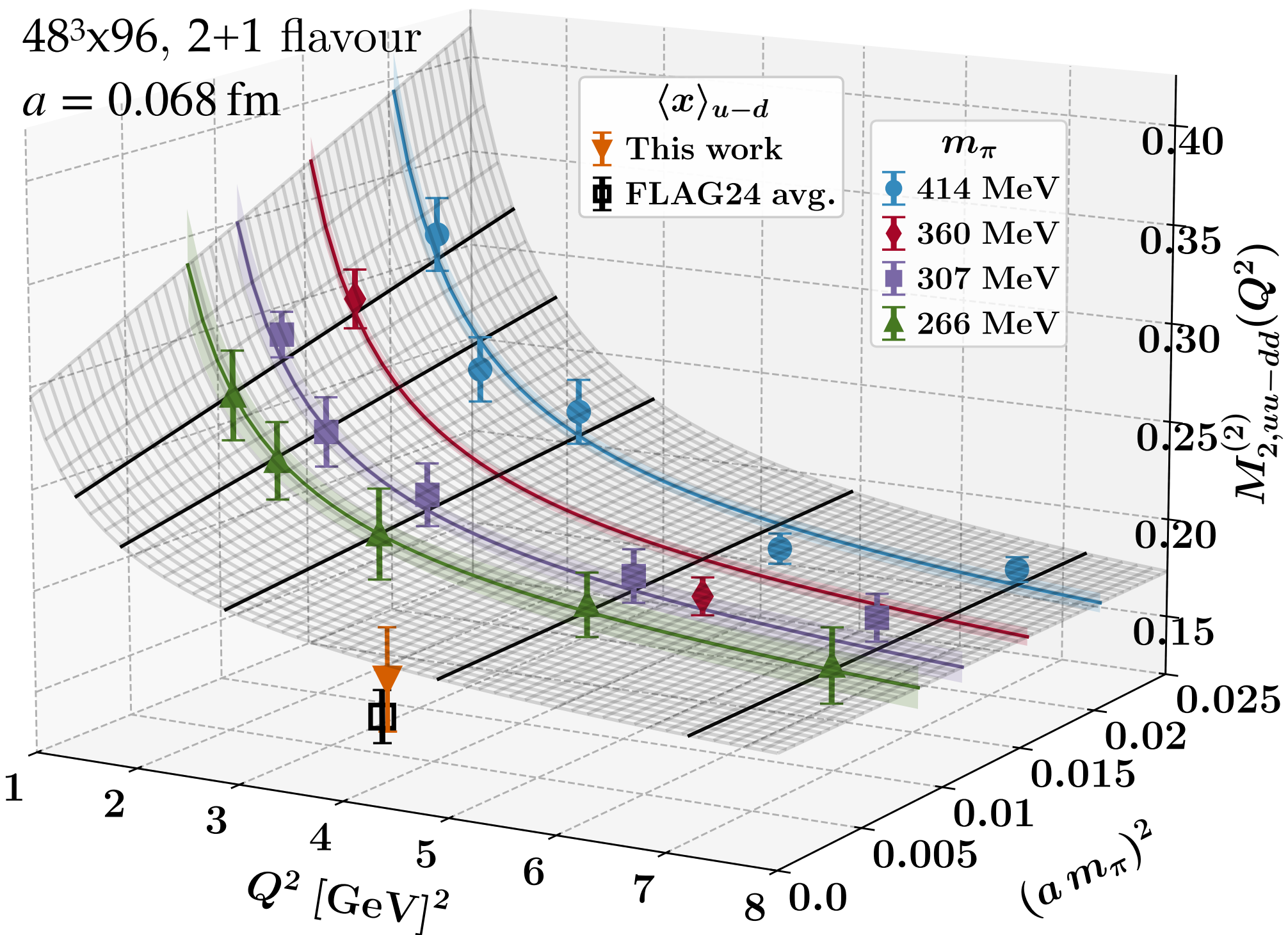
Power corrections

Scaling

Momentum fraction and Higher-twist

$$M_{2,uu-dd}^{(2)}(Q^2, am_\pi^2) = C_{2,uu-dd}^{(2)}\left(\frac{Q^2}{\mu^2}, g(\mu)\right) \left[\underbrace{\langle x \rangle_{u-d}(\mu, am_\pi^2)}_{\text{leading-twist PDF momentum fraction}} + \underbrace{\frac{A_2^{uu-dd}(\mu, am_\pi^2)}{Q^2}}_{\text{twist-4 contributions}} \right]$$

known (NNLO) leading-twist Wilson coeff



● Clear indication for quark mass dependence of higher-twist contributions

Parity-violating Structure Functions

F_3 & Gross-Llewellyn Smith sum rule

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Gross-Llewellyn Smith sum rule from lattice QCD

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R. D. Young¹ and J. M. Zanotti¹

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Nucleon Structure Functions | F_3

- for $\mu \neq \nu$ and $p_\mu = q_\mu = 0$, and $\beta \neq 0$, we isolate,

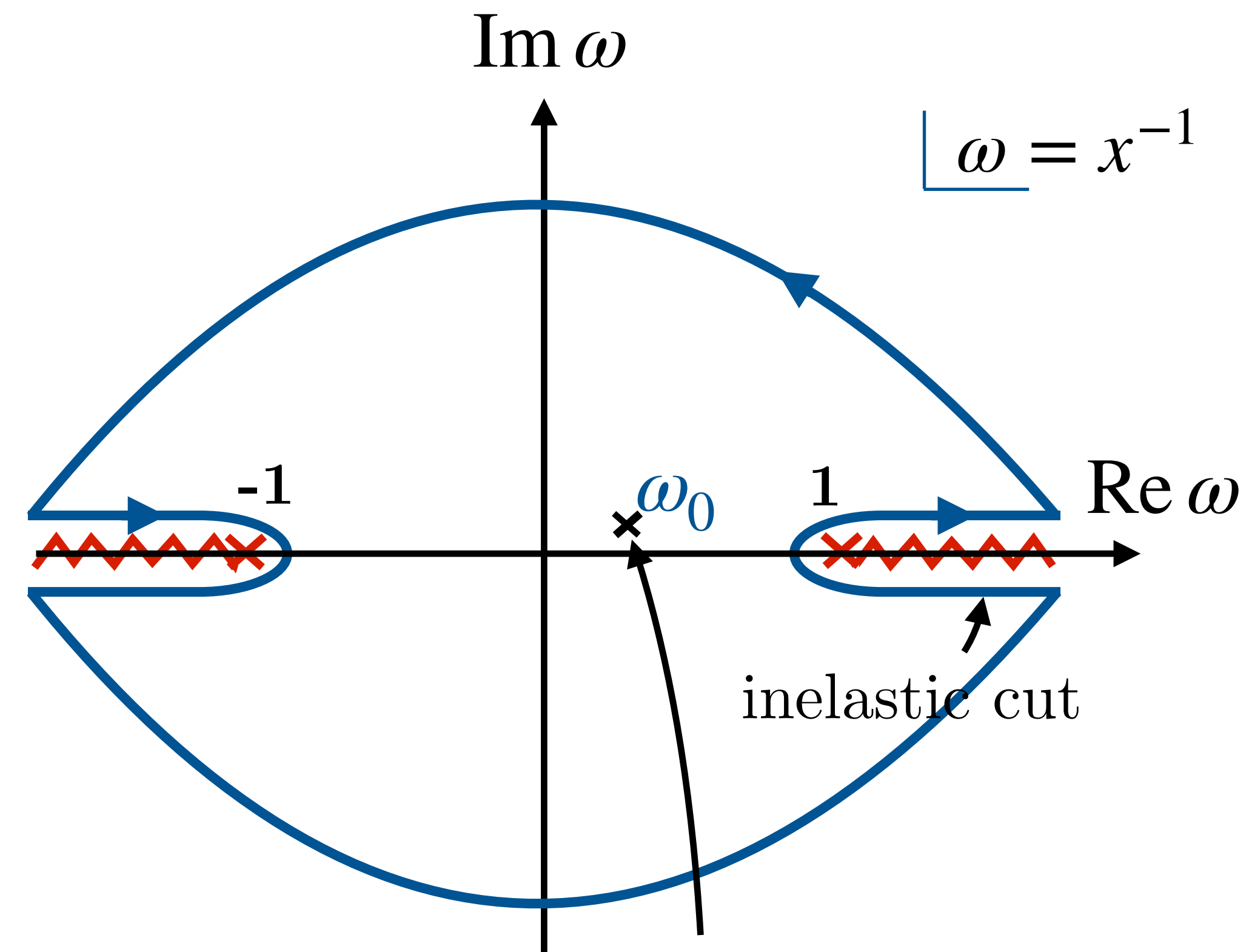
$$T_{\mu\nu}(p, q) = i \varepsilon^{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{2p \cdot q} \mathcal{F}_3(\omega, Q^2)$$

- we can write down dispersion relations and connect Compton SFs to DIS SFs:

$$\mathcal{F}_3(\omega, Q^2) = 4\omega \int dx \frac{F_3(x, Q^2)}{1 - x^2\omega^2}$$

- The 1st moment**

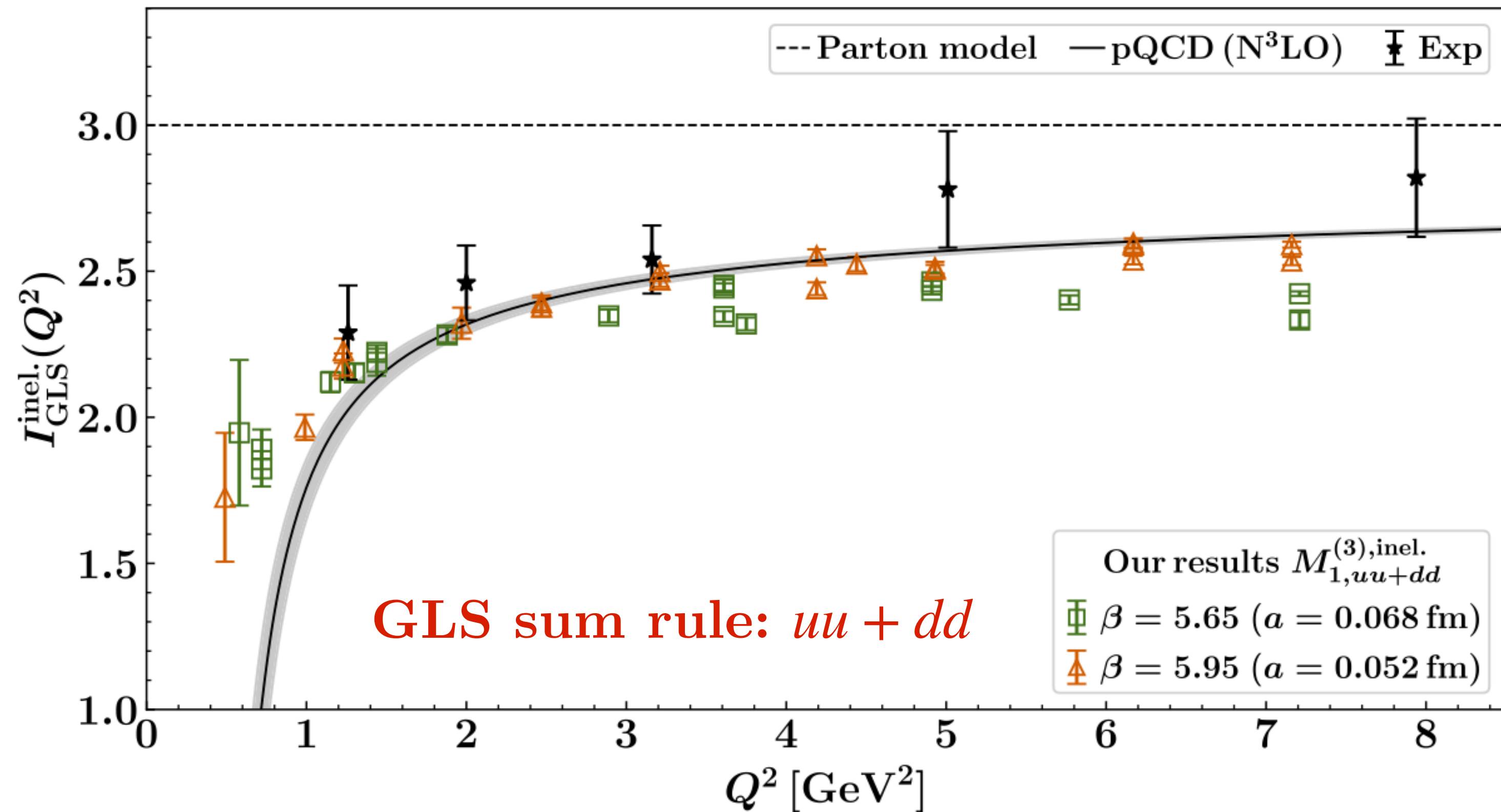
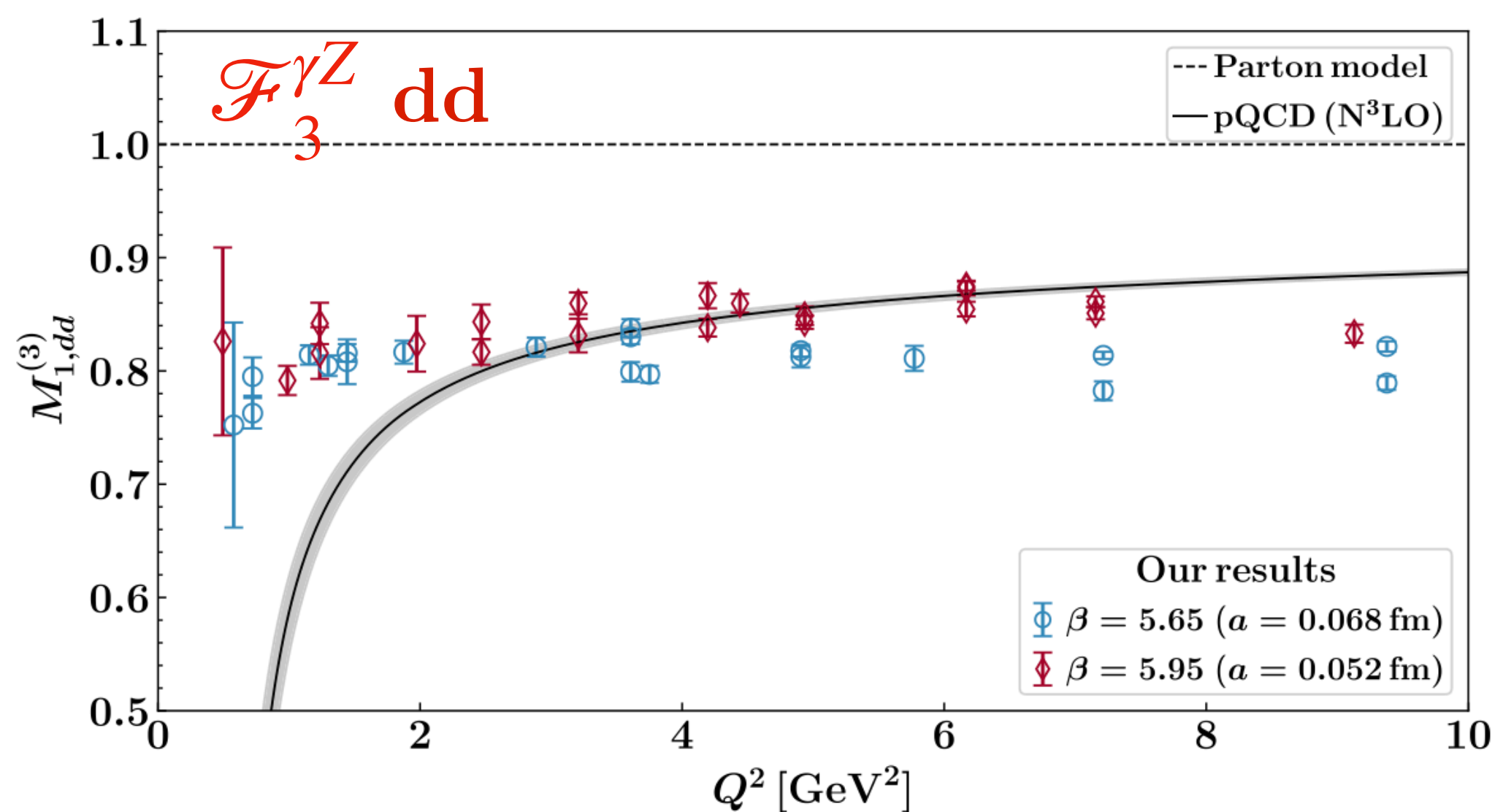
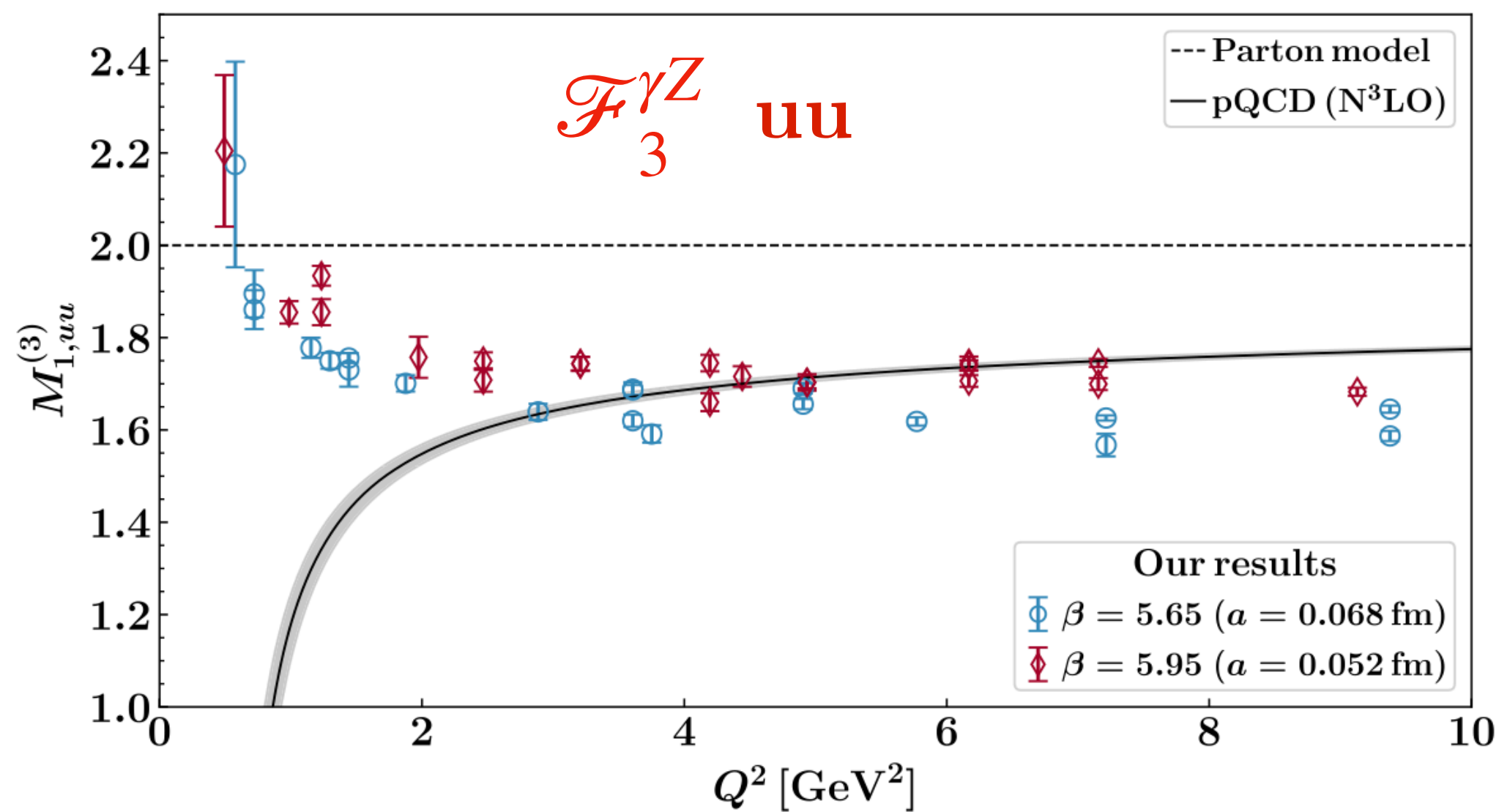
$$M_1^{(3)}(Q^2) = \int_0^1 dx F_3(x, Q^2) = \left. \frac{\mathcal{F}_3(\omega, Q^2)}{4\omega} \right|_{\omega=0}$$



Compton Amplitude is an analytic function in the unphysical region $|\omega_0| < 1$

$\mathcal{F}_3^{\gamma Z}$ and the GLS sum rule

$a = 0.068, 0.052$ fm
 $m_\pi \sim 410$ MeV
 $48^3 \times 96$, 2+1 flavour



$$I_{\text{GLS}}^{\text{inel.}}(Q^2) = 3 \left(1 + \sum_{i=1}^3 \left(\frac{\alpha_s(Q^2)}{\pi} \right)^i C_i(n_f) \right) + \frac{\Delta_{HT}}{Q^2}$$

known (N³LO) Wilson coeff. Higher-twist

Polarised Structure Functions

g_1
and the
Bjorken sum rule



THE UNIVERSITY
of ADELAIDE

FACULTY OF SCIENCES, ENGINEERING AND TECHNOLOGY

SCHOOL OF PHYSICS, CHEMISTRY AND EARTH SCIENCES

Round the Twist:
Exploring Higher Twist Phenomenology
with Lattice Quantum Chromodynamics

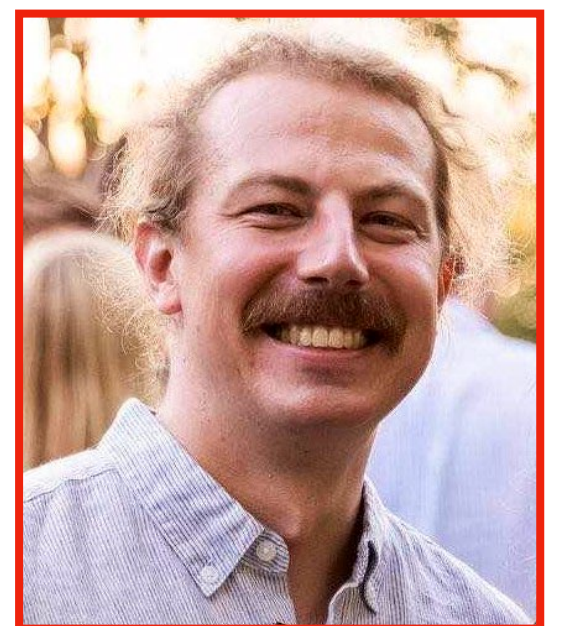
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July 2025

Polarised Structure Functions

$$T_{[\mu\nu]}(p, q, s) = i\varepsilon^{\mu\nu\alpha\beta} \frac{q_\alpha}{p \cdot q} \left[s_\beta \tilde{g}_1(\omega, Q^2) + \left(s_\beta - \frac{s \cdot q}{p \cdot q} p_\beta \right) \tilde{g}_2(\omega, Q^2) \right]$$

- Similar to the unpolarised case, we can extract $\tilde{g}_1$ and $\tilde{g}_2$
- via an OPE analysis: the first moment of $g_1(x)$ is related to axial current matrix elements

$$\Gamma_1(Q^2) = \int_0^1 g_1^{(u-d)}(x, Q^2) dx = \underbrace{(\Delta u - \Delta d)}_{\equiv g_A} C_1(\alpha_s(Q^2))$$

$$\text{where, } C_1(\alpha_s(Q^2)) = 1 - \frac{\alpha_s(Q^2)}{\pi} - \mathcal{O}(\alpha_s^2)$$

- $g_2(x)$ is twist-3, holds information on quark-gluon correlations
- Wandzura-Wilczek decomposition

$$g_2(x, Q^2) = -g_1(x, Q^2) + \int_x^1 g_1(y, Q^2) dy + \bar{g}_2(x, Q^2)$$
- The Buckhardt — Cottingham sum rule

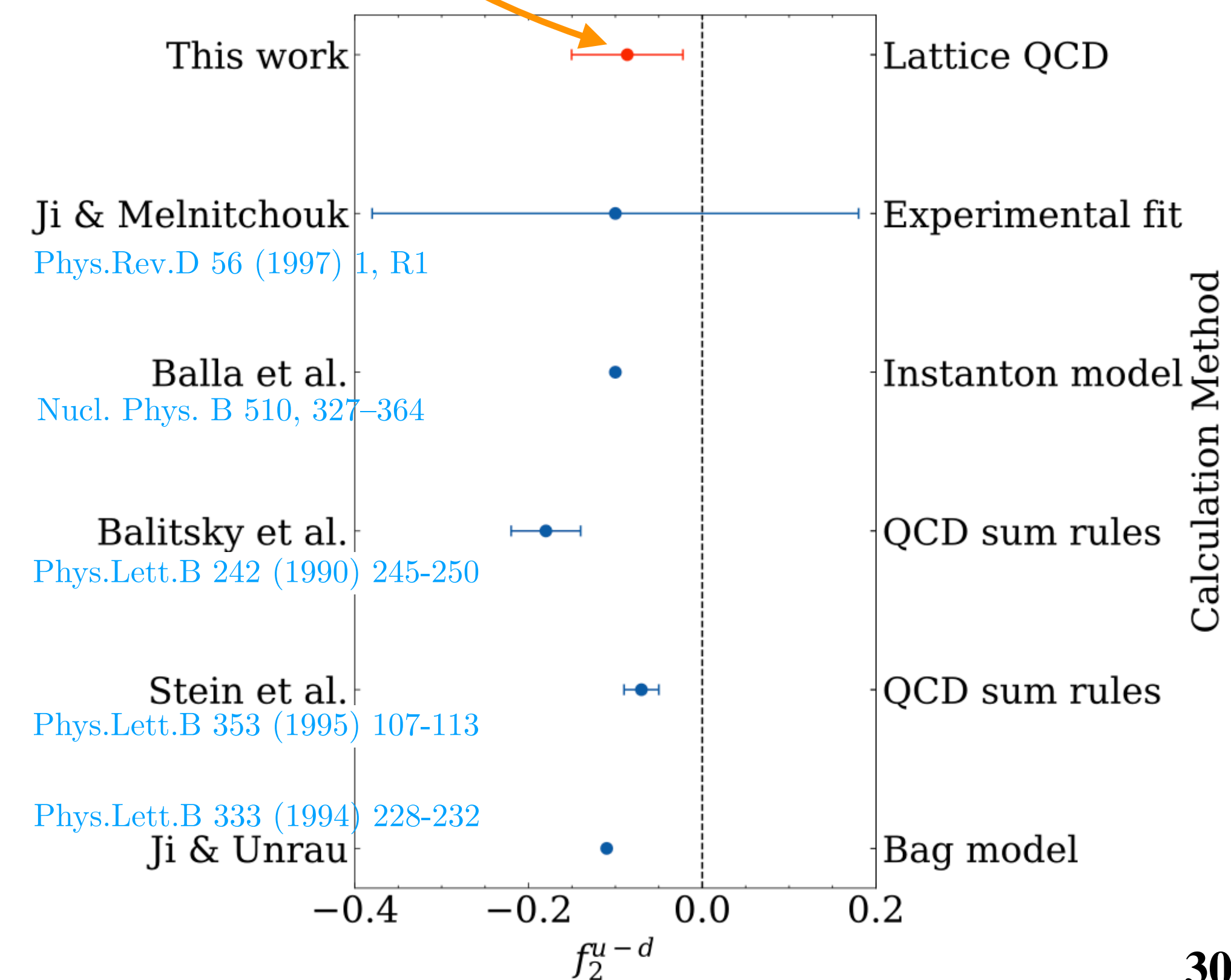
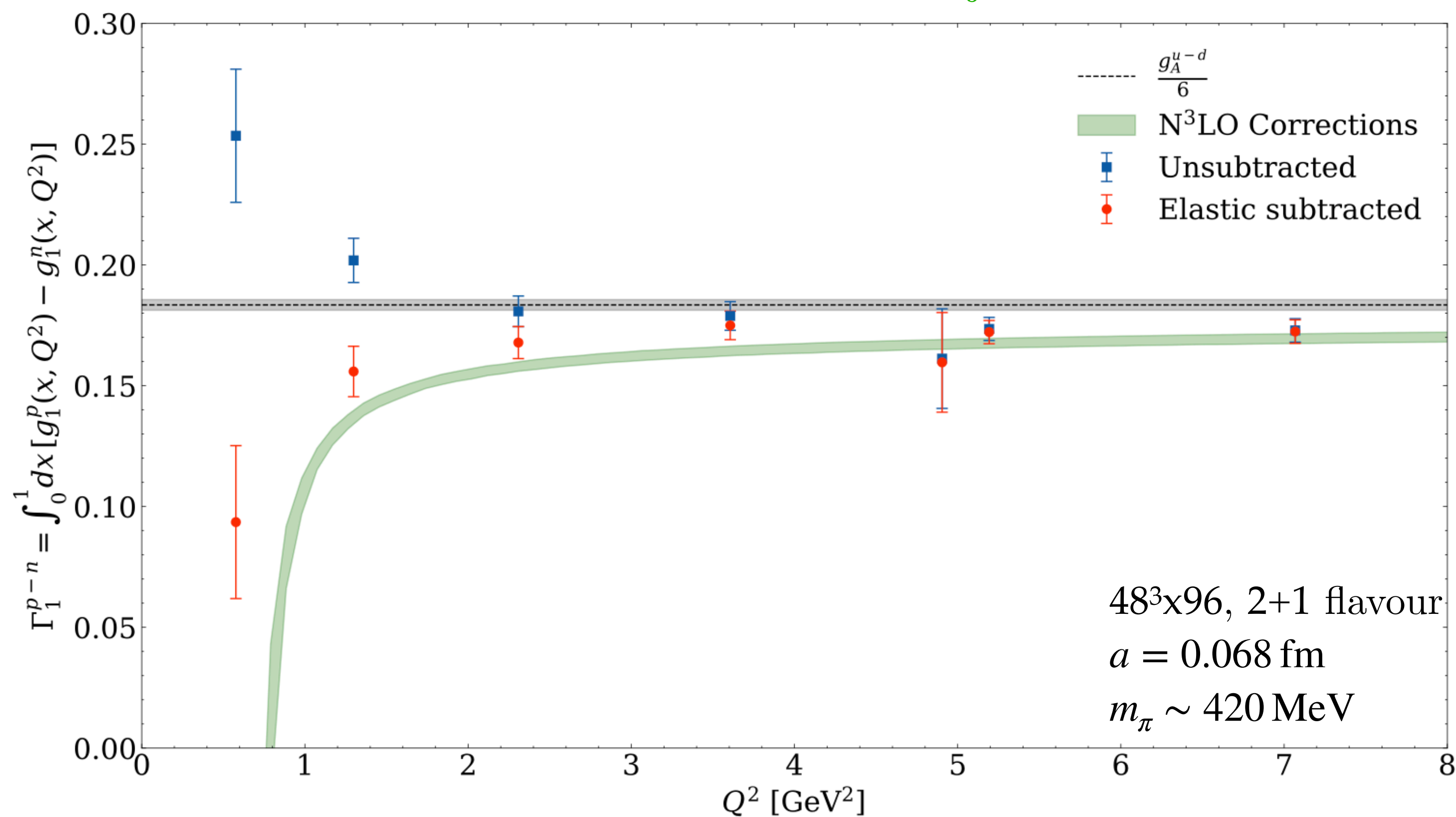
$$\int_0^1 g_2(x, Q^2) dx = 0$$

Bjorken sum rule and Higher twist

$$\int_0^1 dx g_1^{(f)}(x, Q^2) = \frac{1}{4} a_0^{(f)} + \frac{m_N^2}{9Q^2} \left(a_2^{(f)} + 4d_2^{(f)} - 4f_2^{(f)} \right) + \mathcal{O}\left(\frac{m_N^4}{Q^4}\right)$$

Our inelastic g_1 moments pQCD calculate from estimate

Bjr sum rule 3-pt matrix elements

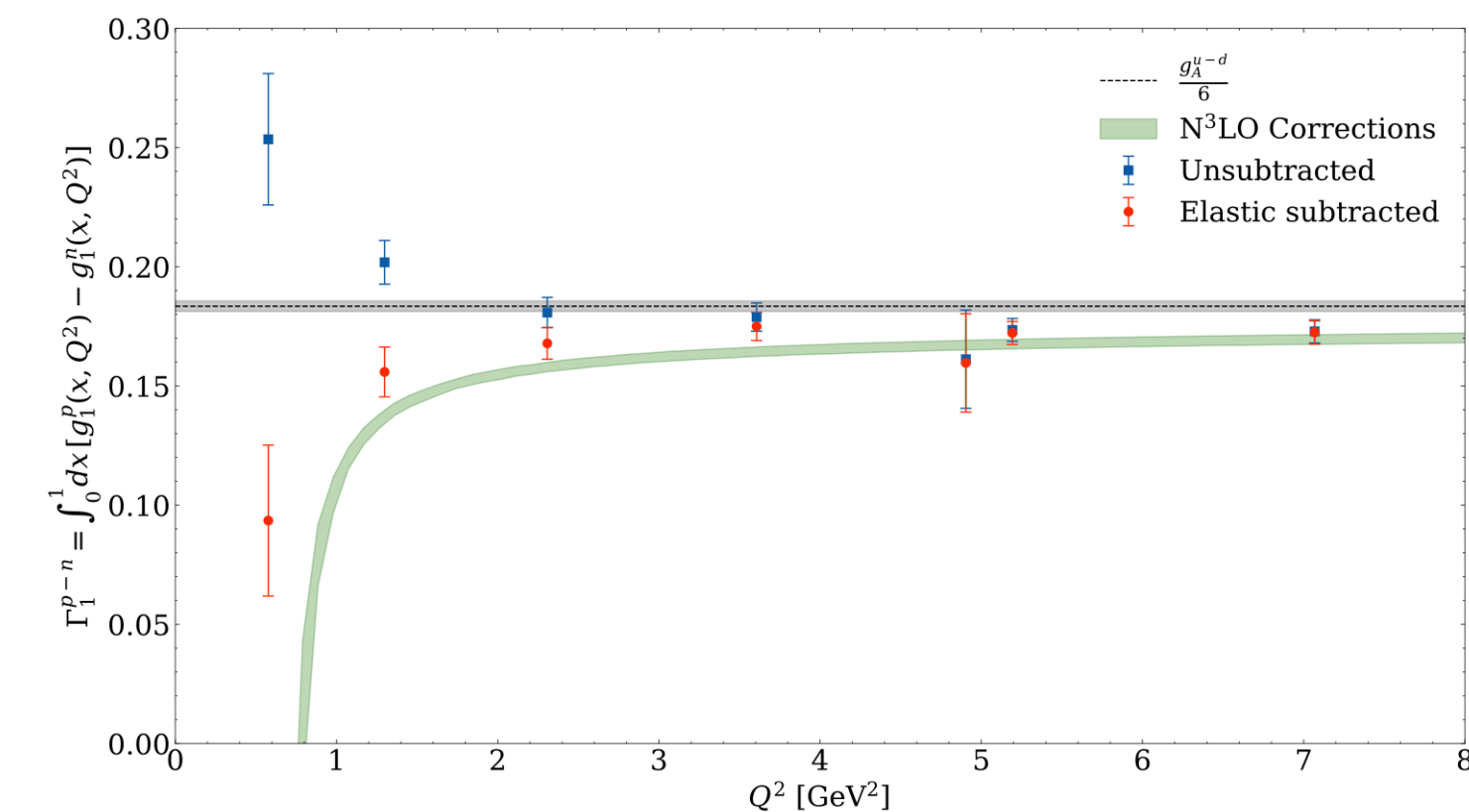
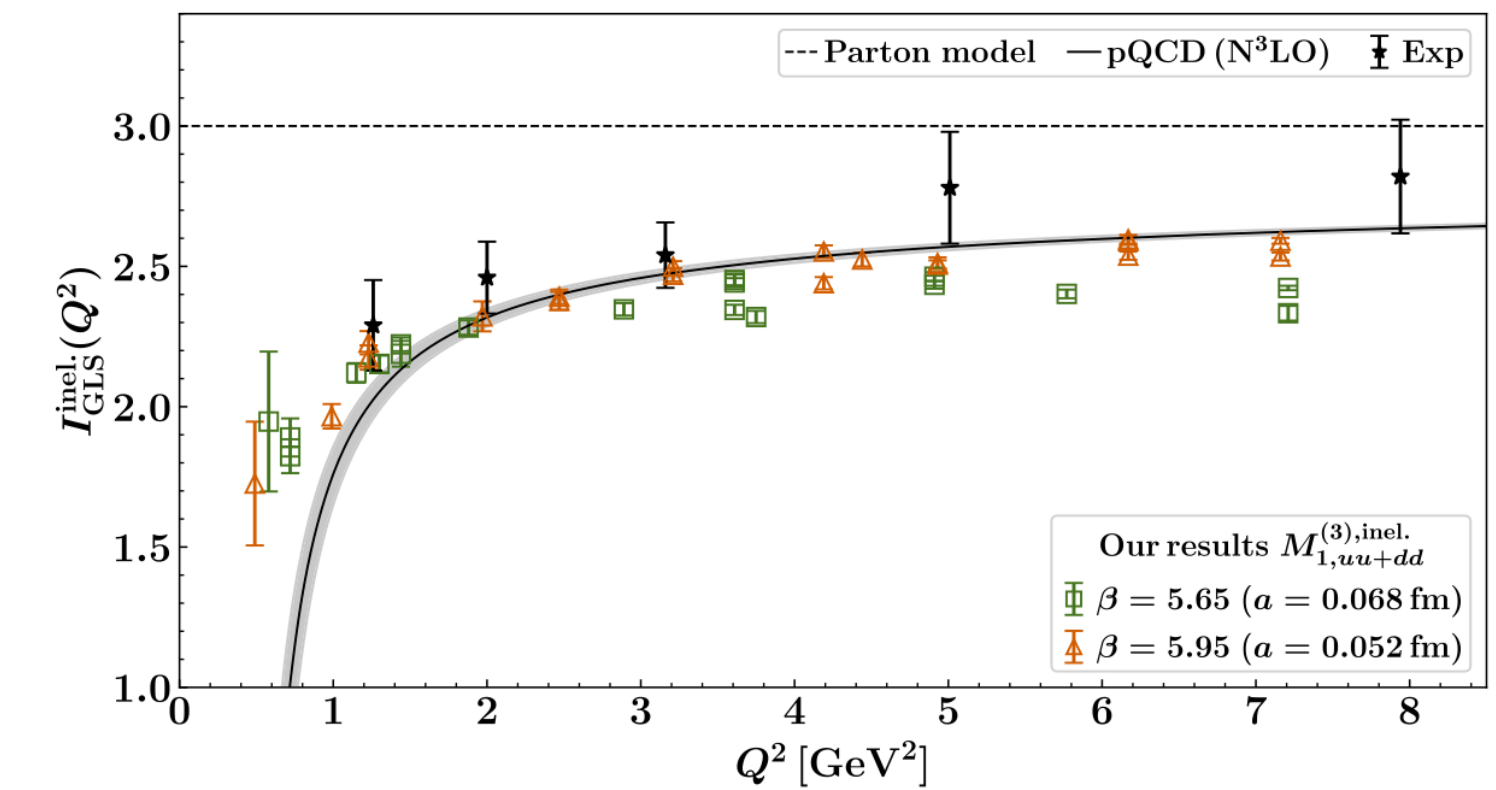
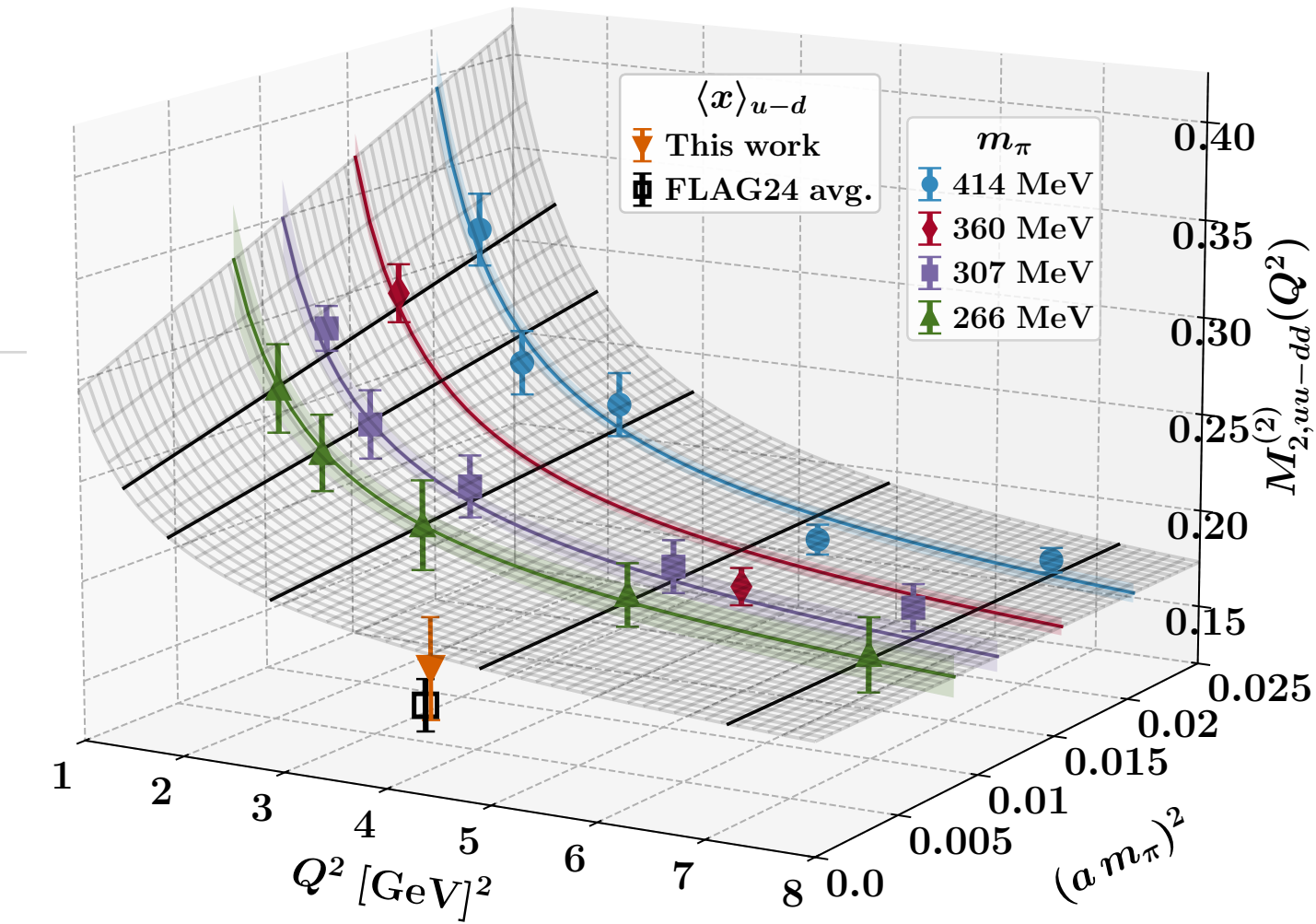


Summary & Outlook

- Physical Compton amplitude, can be matched to OPE
- ☑ Can extract moments of DIS structure functions,
- ☑ Systematic investigation of power corrections, higher-twist effects and scaling is within reach,
- ☑ A direct calculation of F_1 subtraction term,
- ☑ First look at the mass-dependence of higher-twist contributions from F_2 ,
- ☑ F_3 moments, testing GLS sum rule. Pathway to α_s and EW box diagrams, and higher-twist effects,
- ☑ g_1 moments, testing the Bjorken sum rule. Pathway to g_A , α_s , and estimating higher-twist matrix elements.

■ Our approach can be extended to:

- ☑ Deuteron: N. Humphrey et al. (QCDSF), PoS(QCHSC24)083 [arXiv:2505.03182]
- ☑ GPDs: A. Hannaford-Gunn et al. *Phys.Rev.D* **105**, 014502 [arXiv:2110.11532], and *Phys.Rev.D* **110** (2024) 1, 014509 [2405.06256]



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- KUC, RDY and JMZ are supported by the Australian Research Council grants DP190100297, DP220103098, and DP240102839.

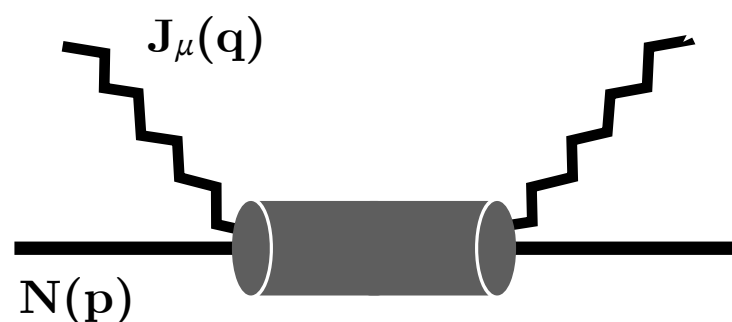


Backup

Compton amplitude via the FH relation at 2nd order

- **unpolarised Compton Amplitude**

$$T_{\mu\mu}(p, q) = \int d^4z e^{i\mathbf{q}\cdot\mathbf{z}} \langle N(p) | \mathcal{T} \{ J_\mu(z) J_\mu(0) \} | N(p) \rangle$$



- **Action modification**

local EM current

$$J_\mu(z) = \sum_q e_q \bar{q}(z) \gamma_\mu q(z)$$

$$S \rightarrow S(\lambda) = S + \lambda \int d^4z (e^{i\mathbf{q}\cdot\mathbf{z}} + e^{-i\mathbf{q}\cdot\mathbf{z}}) J_\mu(z)$$

- **2nd order derivatives of the 2-pt correlator, $G_\lambda^{(2)}(\mathbf{p}; t)$, in the presence of the external field**

$$\left. \frac{\partial^2 G_\lambda^{(2)}(\mathbf{p}; t)}{\partial \lambda^2} \right|_{\lambda=0} = \left(\frac{\partial^2 A_\lambda(\mathbf{p})}{\partial \lambda^2} - t A(\mathbf{p}) \frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial \lambda^2} \right) e^{-E_N(\mathbf{p})t} \quad \text{from spectral decomposition}$$

$$\left. \frac{\partial^2 G_\lambda^{(2)}(\mathbf{p}; t)}{\partial \lambda^2} \right|_{\lambda=0} = \frac{A(\mathbf{p})}{2E_N(\mathbf{p})} t e^{-E_N(\mathbf{p})t} \int d^4z (e^{iq\cdot z} + e^{-iq\cdot z}) \langle N(\mathbf{p}) | \mathcal{T} \{ \mathcal{J}(z) \mathcal{J}(0) \} | N(\mathbf{p}) \rangle$$

from path integral

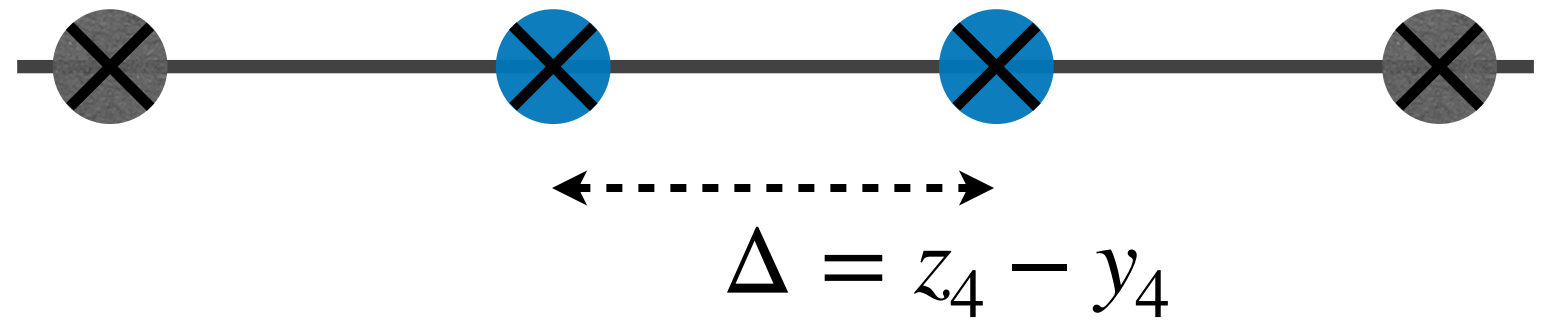
- **equate the time-enhanced terms:**

$$\left. \frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial \lambda^2} \right|_{\lambda=0} = - \frac{1}{2E_N(\mathbf{p})} \overbrace{\int d^4z (e^{iq\cdot z} + e^{-iq\cdot z}) \langle N(\mathbf{p}) | \mathcal{J}(z) \mathcal{J}(0) | N(\mathbf{p}) \rangle}^{T_{\mu\mu}(p, q)} + (q \rightarrow -q)$$

Compton amplitude is related to the second-order energy shift

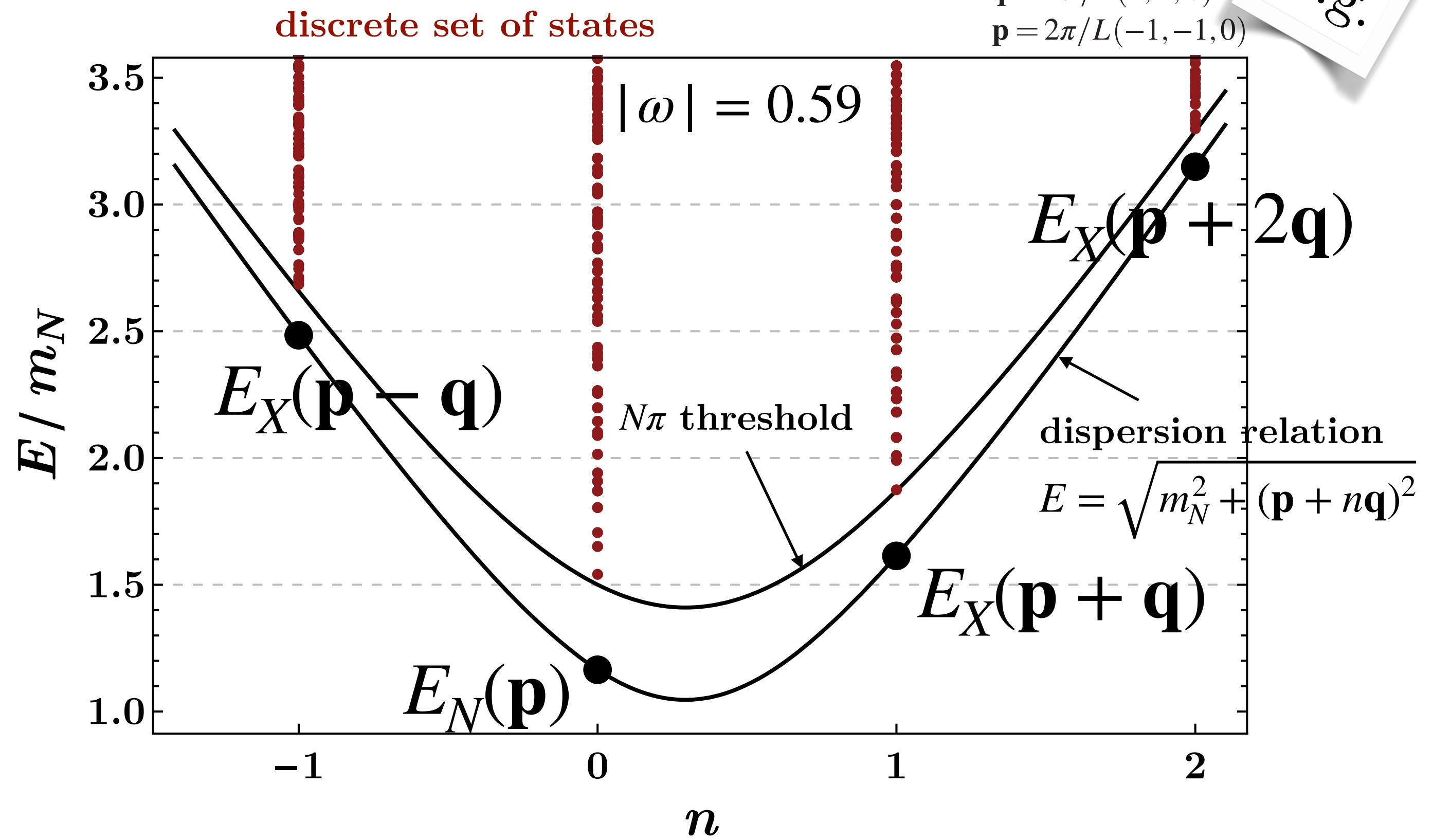
Compton amplitude via the FH relation at 2nd order

- relevant contribution comes from the ordering where the currents are sandwiched

$$\chi(t) \quad \mathcal{J}(z_4) \quad \mathcal{J}(y_4) \quad \bar{\chi}(0) \sim e^{-E_N(\mathbf{p})t} \int d\Delta e^{-(E_X(\mathbf{p} + \mathbf{q}) - E_N(\mathbf{p}))\Delta} (t - \Delta)$$


$\Delta = z_4 - y_4$

- under the condition $|\omega| < 1$,
 $E_X(\mathbf{p} + n\mathbf{q}) \gtrsim E_N(\mathbf{p})$,
 so the intermediate states cannot go on-shell
- ground state dominance is ensured in the large time limit



Calculation Details | S_1

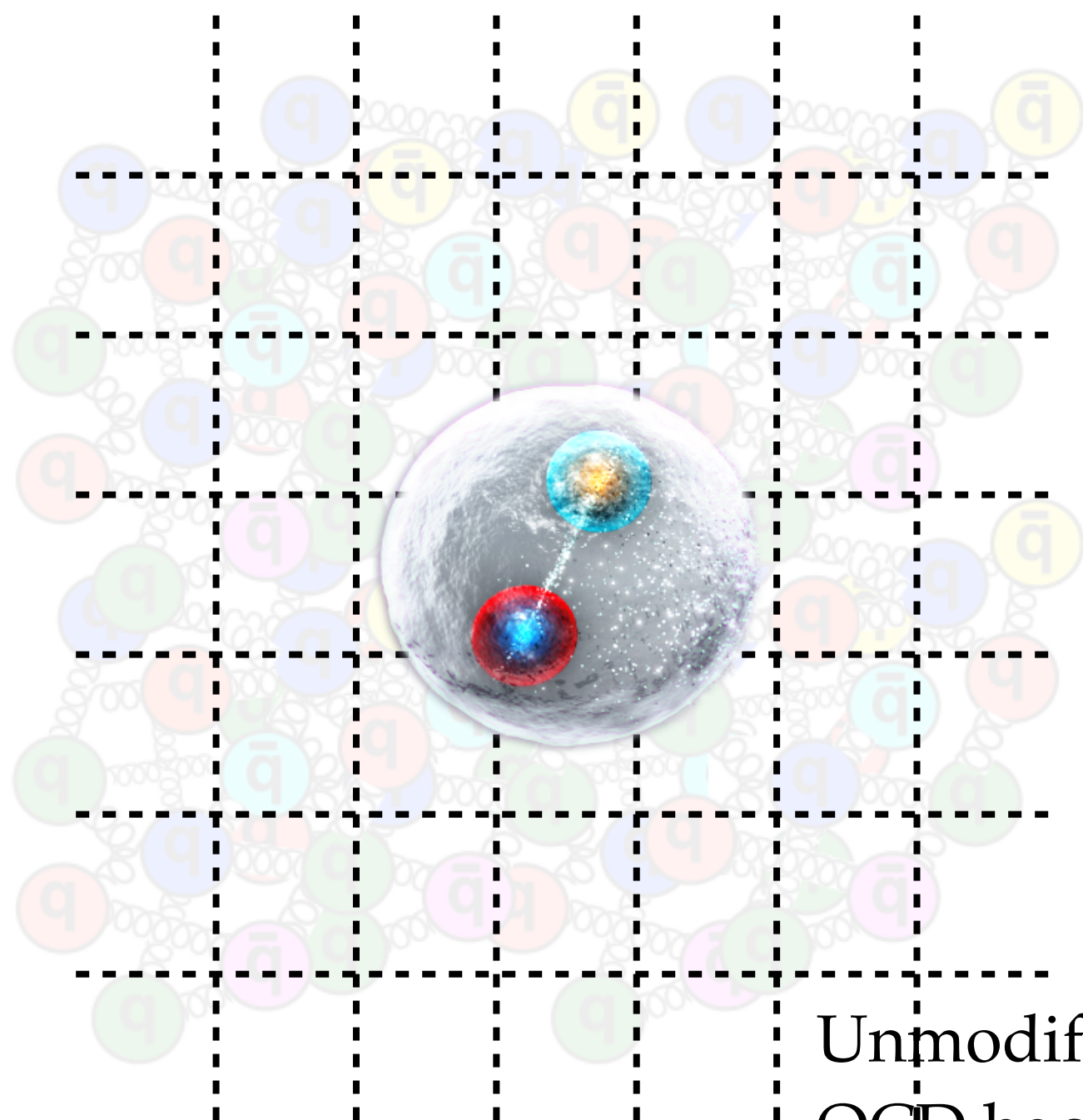
QCDSF/UKQCD configurations

2+1 flavour (u/d+s)

NP-improved Clover action

[PRD 79, 094507 \(2009\)](#), [arXiv:0901.3302 \[hep-lat\]](#)

N_f	$L^3 \times T$	β	κ	$a[\text{fm}]$	$m_\pi[\text{MeV}]$	Z_V
2 + 1	$32^3 \times 64$	5.50	0.120900	0.074	470	0.86
2 + 1	$48^3 \times 96$	5.65	0.122005	0.068	410	0.87
2 + 1	$48^3 \times 96$	5.80	0.122810	0.058	430	0.88



Unmodified
QCD background

- Local EM current insertion, $J_\mu(x) = Z_V \bar{q}(x) \gamma_\mu q(x)$ (valence only)
- 4 Distinct field strengths, $\lambda = [\pm 0.0125, \pm 0.025]$
- Up to $\mathcal{O}(10^4)$ measurements for each pair of Q^2 and λ
- Connected diagrams only

S_1 | lattice artefacts

- **tree-level mass-dependent lattice operator product expansion (LOPE)**

$$\mathcal{T}_{\mu\nu} = \bar{\psi}\gamma_\mu S_W(p+q)\gamma_\nu\psi + \bar{\psi}\gamma_\nu S_W(p-q)\gamma_\mu\psi,$$

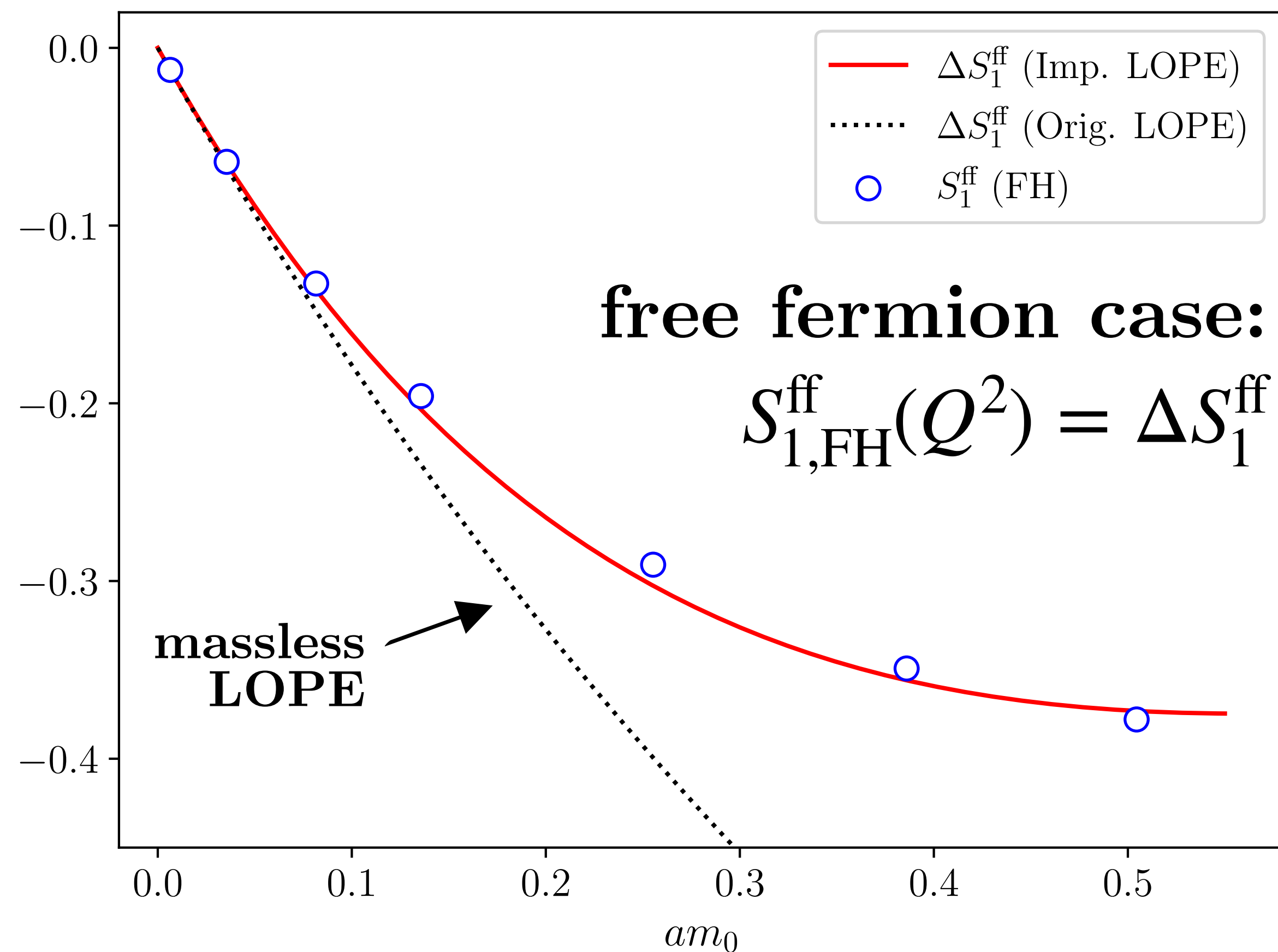
with the Wilson quark propagator,

$$S_W(k) = a \frac{M(k) - i\gamma_\mu \sin(ak_\mu)}{M(k)^2 + \sum_\mu \sin^2(ak_\mu)},$$

where $M(k) = am_0 + \sum_\rho [1 - \cos(ak_\rho)]$,

- **giving the correction**

$$\Delta S_1 = \frac{4m_p a Z_V^2 g_S^{\text{bare}} \sum_\rho [\cos(aq_\rho) - 1]}{\sum_\rho \sin^2(aq_\rho) + M^2(m_0, q)}$$



Calculation Details | F_3

QCDSF/UKQCD configurations

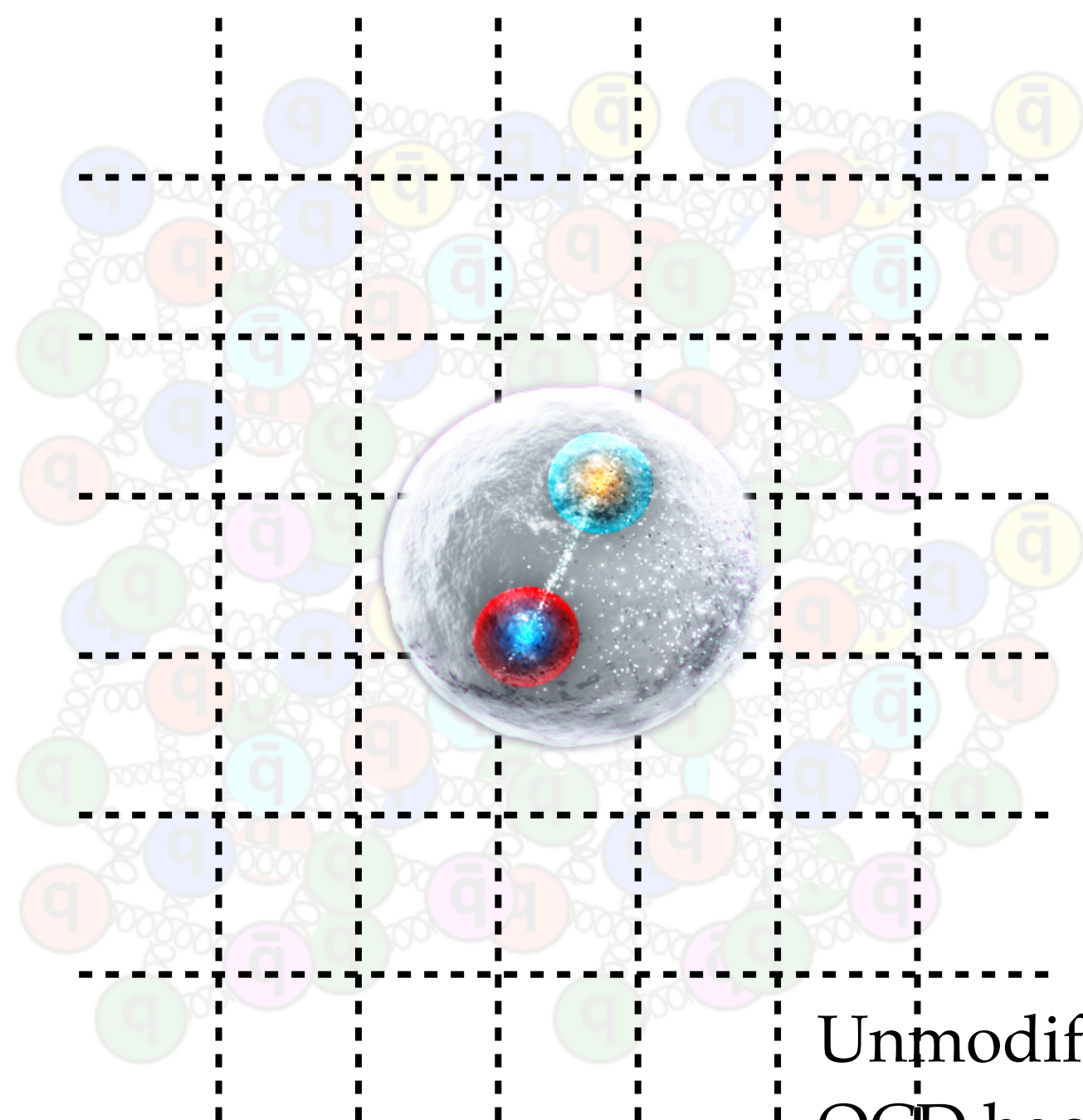
$48^3 \times 96$, 2+1 flavour (u/d+s)

$\beta = \begin{pmatrix} 5.65 \\ 5.95 \end{pmatrix}$, NP-improved Clover action

PRD 79, 094507 (2009), arXiv:0901.3302 [hep-lat]

$m_\pi \sim 410$ MeV, $\sim \text{SU}(3)$ sym.

$$m_\pi L \sim \begin{bmatrix} 6.9 \\ 5.3 \end{bmatrix} \quad a \sim \begin{bmatrix} 0.068 \\ 0.052 \end{bmatrix} \text{ fm}$$



Unmodified
QCD background

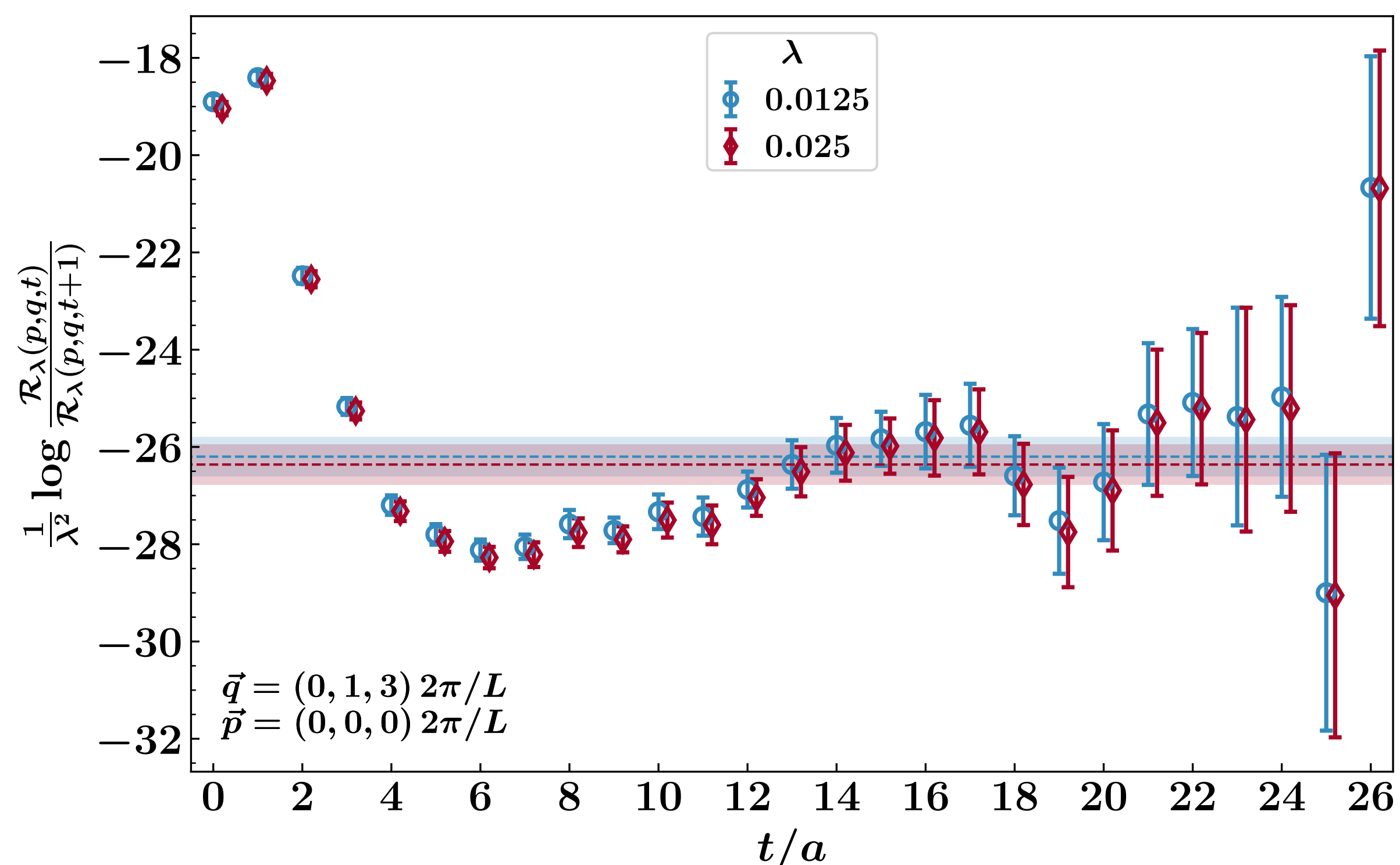
- Local EM and axial current insertion,
 $J_\mu^{V[A]}(x) = Z_{V[A]} \bar{q}(x) \gamma_\mu [\gamma_5] q(x)$
 (valence only)
- 4 Distinct field strengths, $\lambda = [\pm 0.0125, \pm 0.025]$
- Current momenta $0.1 \lesssim Q^2 \lesssim 10 \text{ GeV}^2$
- Roughly 500 measurements
- Nucleon at rest: $\vec{p} = (0,0,0)$ thus $\omega = 0$, varying $\vec{q}$
- Connected 2-pt only, no disconnected since F_3 is non-singlet

Energy shifts | F_3

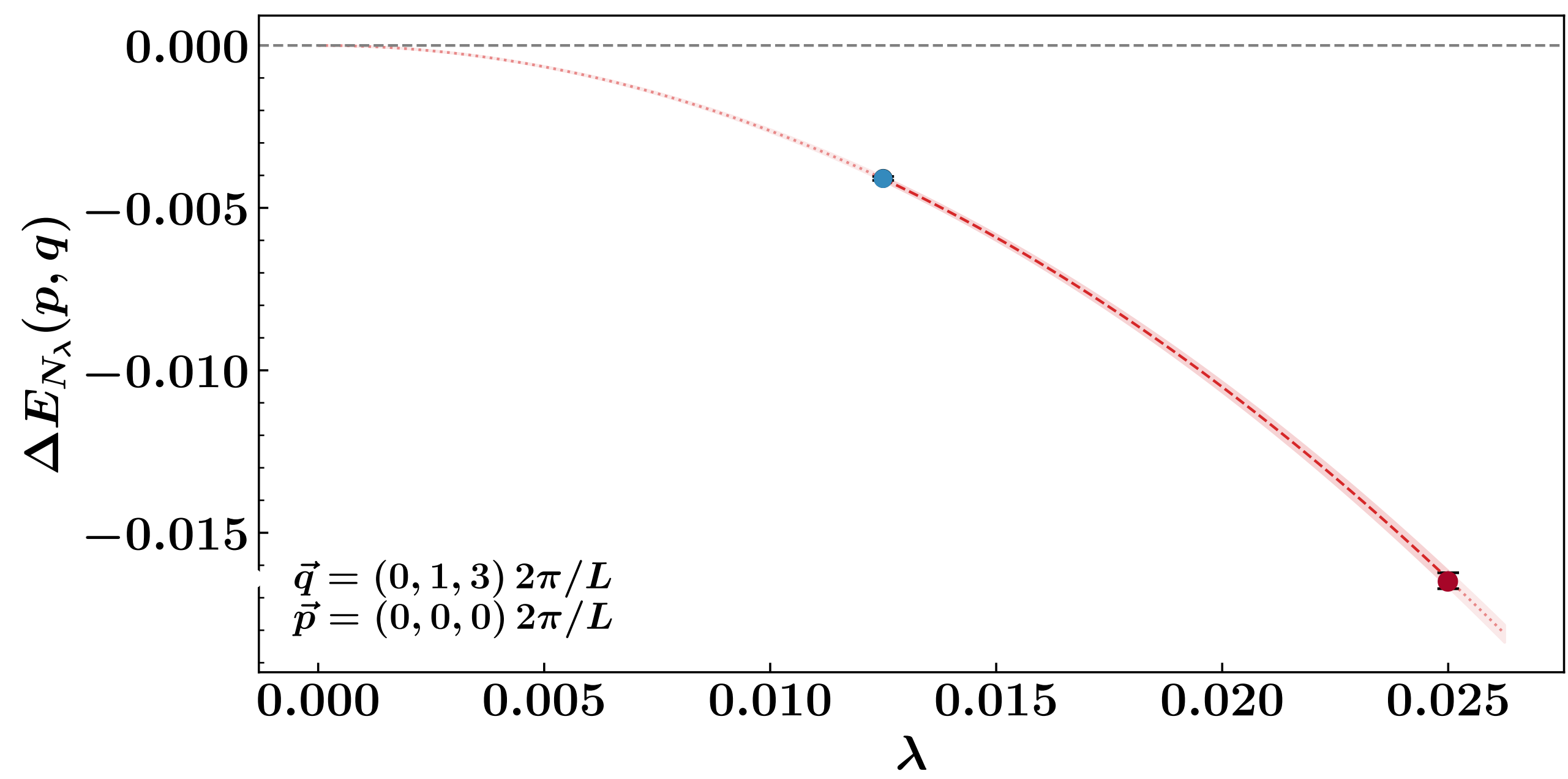
- Ratio of perturbed 2-pt functions

$$\mathcal{R}_\lambda^{qq}(p, t) \equiv \frac{G_{+\lambda_1^q, +\lambda_2^q}^{(2)}(p, t) G_{-\lambda_1^q, -\lambda_2^q}^{(2)}(p, t)}{G_{+\lambda_1^q, -\lambda_2^q}^{(2)}(p, t) G_{-\lambda_1^q, +\lambda_2^q}^{(2)}(p, t)} \rightarrow A_\lambda e^{-4\Delta E_{N_\lambda}(p) t}$$

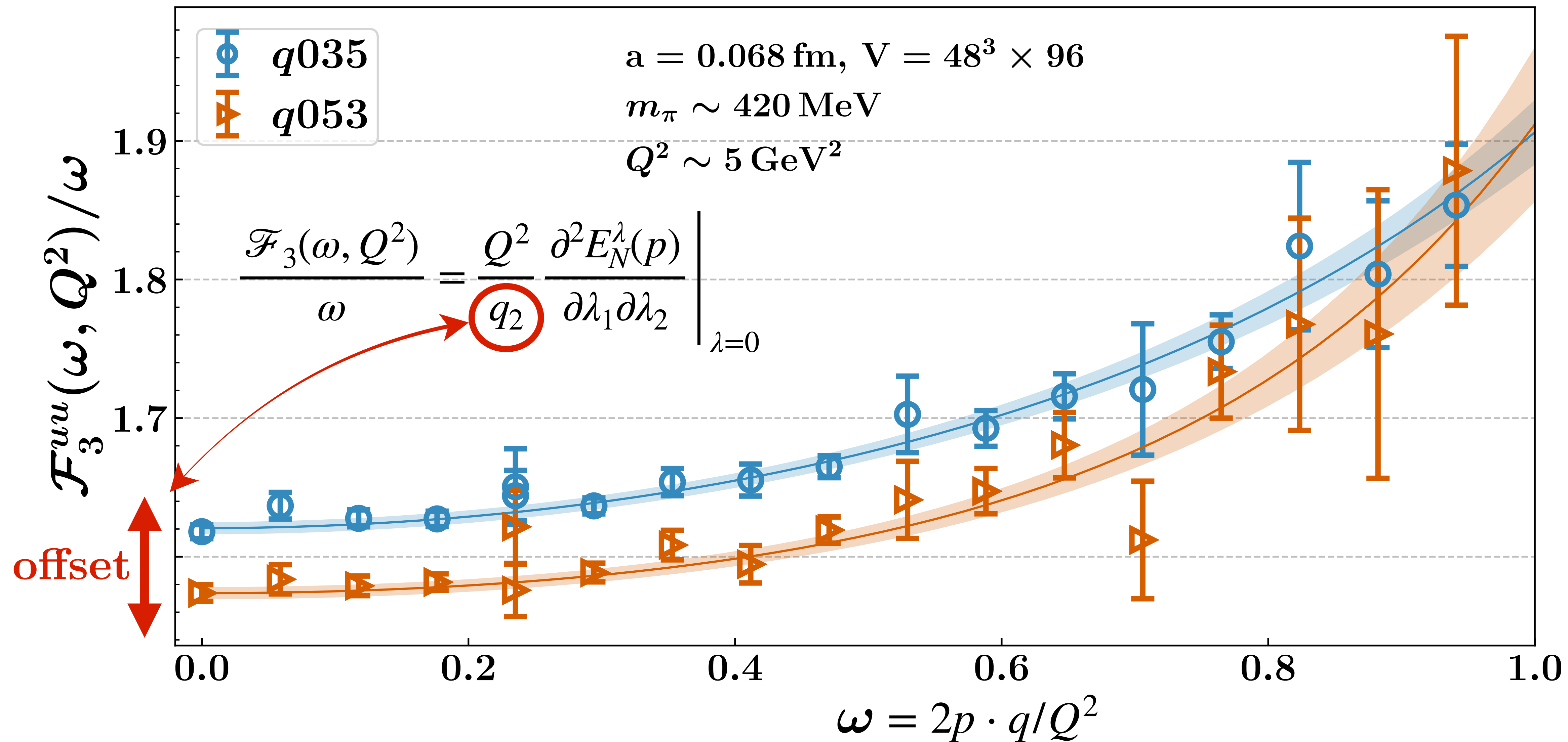
- Extract energy shifts for each $|\lambda|$



- Get the 2nd order derivative $\left. \frac{\partial^2 E_N^\lambda(p)}{\partial \lambda_1 \partial \lambda_2} \right|_{\lambda=0}$



$\mathcal{F}_3$ | unimproved



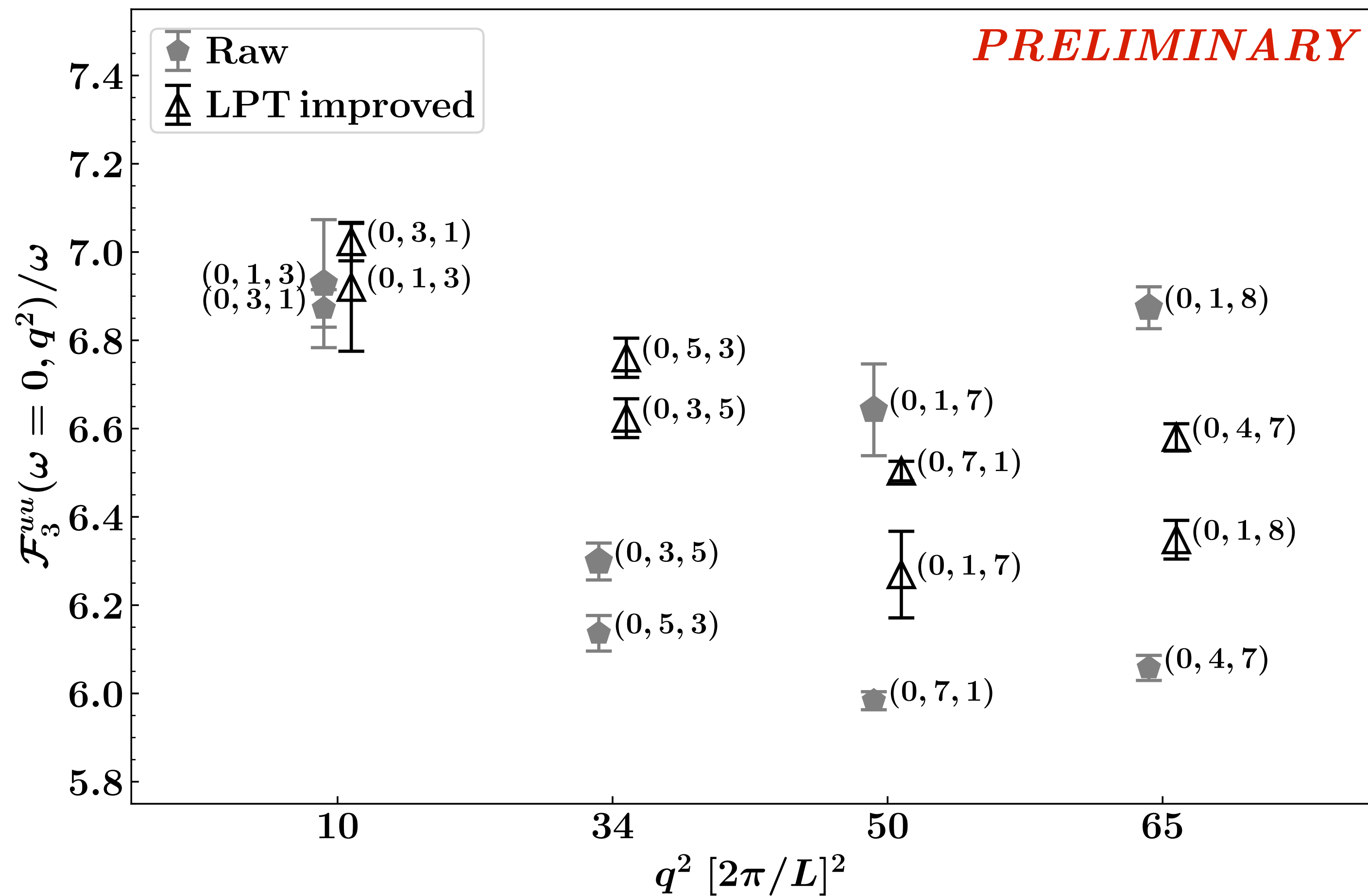
Syst. 1: LPT improvement

$$\frac{\mathcal{F}_3(\omega, Q^2)}{\omega} = \frac{Q^2}{\omega} \frac{\partial^2 E_N^\lambda(p)}{\partial \lambda_1 \partial \lambda_2} \bigg|_{\lambda=0}$$

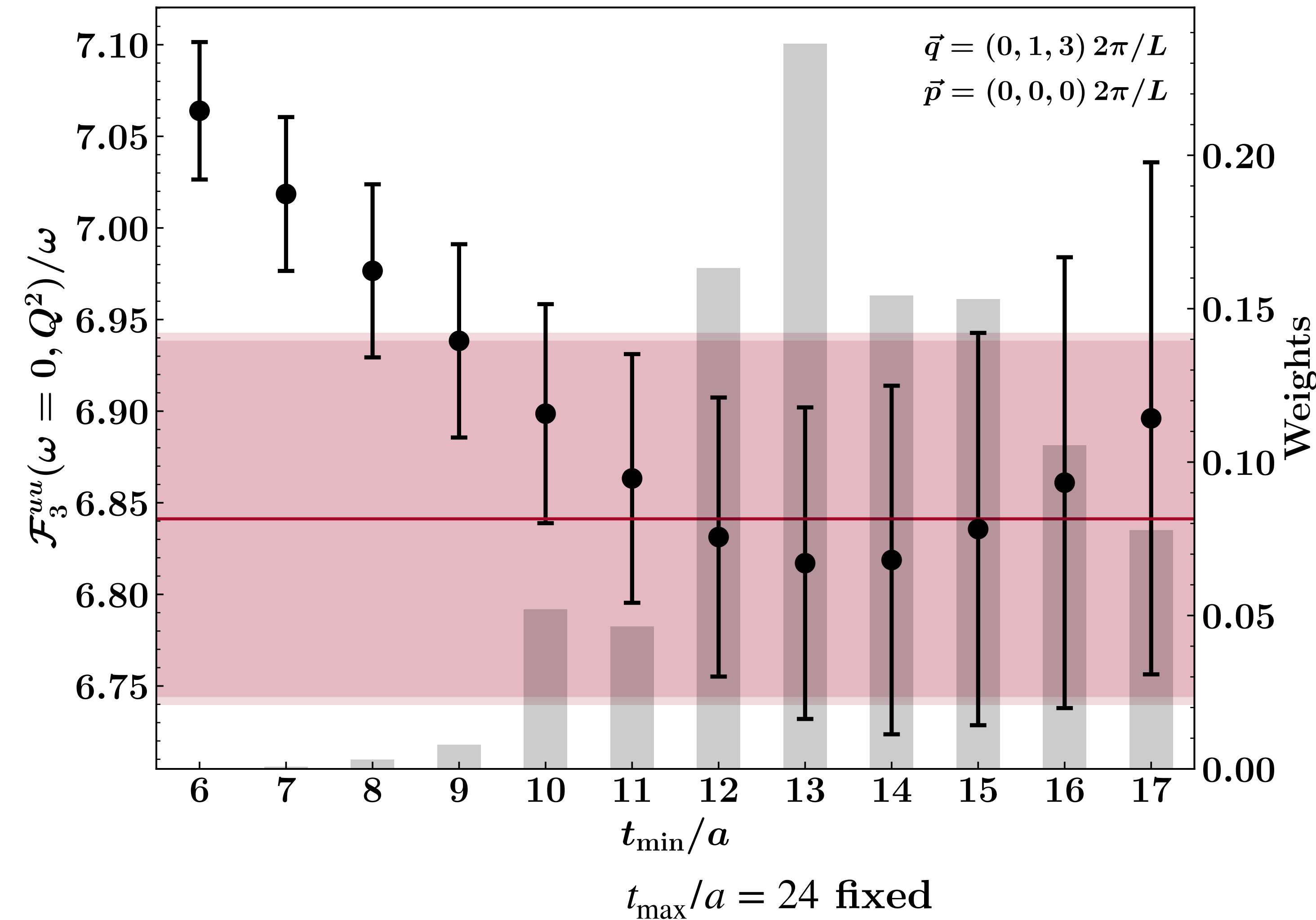
introduces discretisation error due to broken rotational symmetry

- Replace the kinematic factor by a lattice OPE motivated factor

$$\frac{Q^2}{q_2} \rightarrow \frac{\sum_i \sin^2 q_i + \left[\sum_i (1 - \cos q_i) \right]^2}{\sin q_2}$$



Syst. 2: Weighted averaging



● **Red line (mean):** $\bar{\mathcal{O}} = \sum_f w^f \mathcal{O}^f$

● **Red band (total uncertainty):**

$$\delta_{\text{stat}} \bar{\mathcal{O}}^2 = \sum_f w^f (\delta \bar{\mathcal{O}}^f)^2$$

$$\delta_{\text{sys}} \bar{\mathcal{O}}^2 = \sum_f w^f (\mathcal{O}^f - \bar{\mathcal{O}})^2$$

$$\delta \bar{\mathcal{O}} = \sqrt{\delta_{\text{stat}} \bar{\mathcal{O}}^2 + \delta_{\text{sys}} \bar{\mathcal{O}}^2}$$

● **Weights:** $w^f = \frac{p_f (\delta \mathcal{O}^f)^{-2}}{\sum_{f'} p_{f'} (\delta \mathcal{O}^{f'})^{-2}}$
 where p_f is the one sided p-value of the ratio fits