Hyperon Time-like Form Factors at BESIII

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EINN 2025

16th European Research Conference on Electromagnetic Interaction with Nucleons and Nuclei

28 October – 1 November 2025, Paphos, Cyprus

Agenda

Basic notions on form factors

Extraction of form factors from the data

Dispersion relations, advantages, drawbacks

Hyperons, polarization and phases

Λ: dispersive procedure, data and results

Σ^+: dispersive procedure, data and results

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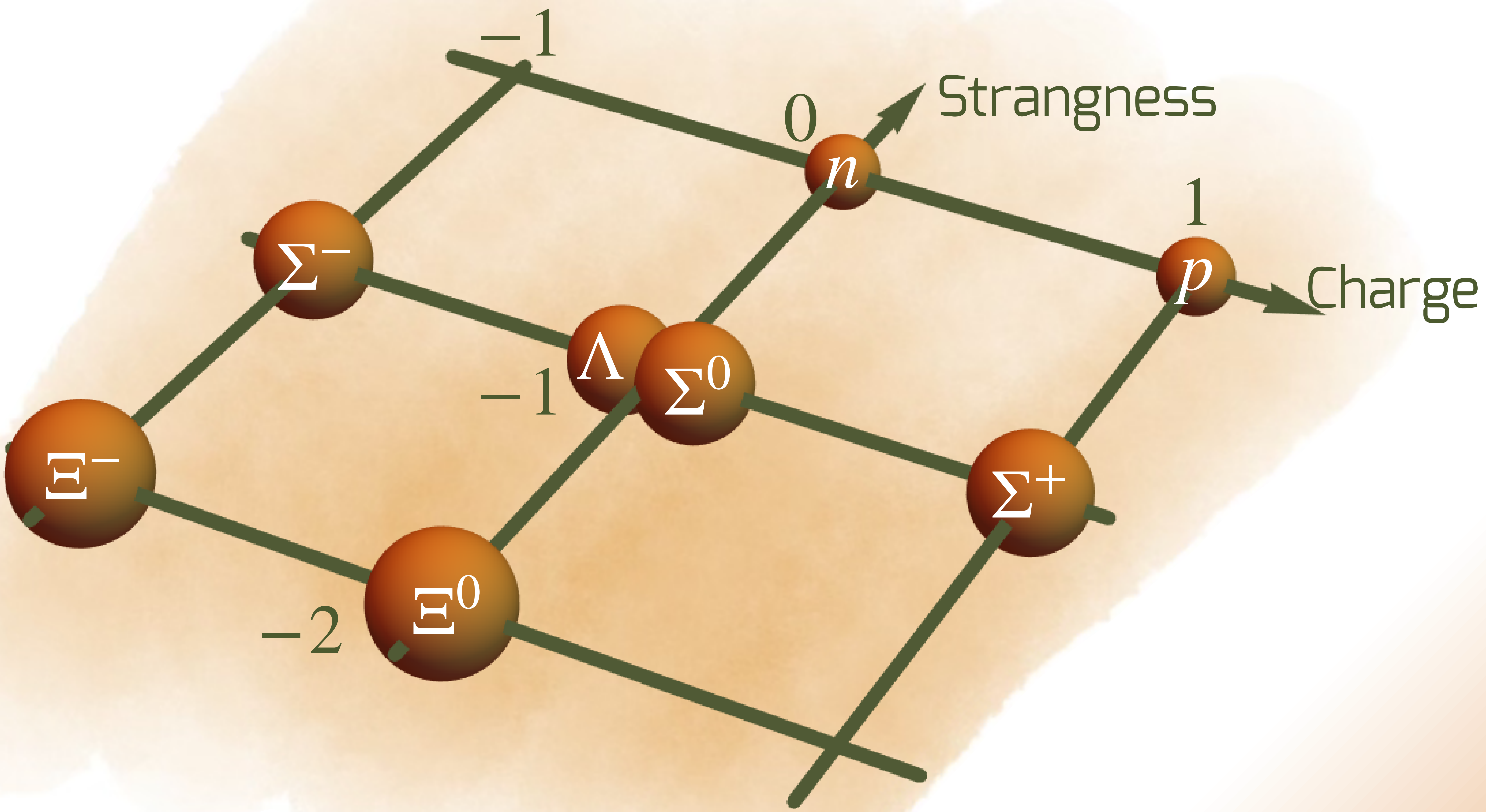
Strange baryons

A **hyperon** is a spin-1/2 baryon having only **light valence quarks and strangeness**

Hyperons are the **six** non nucleons of the spin 1/2 baryon octet



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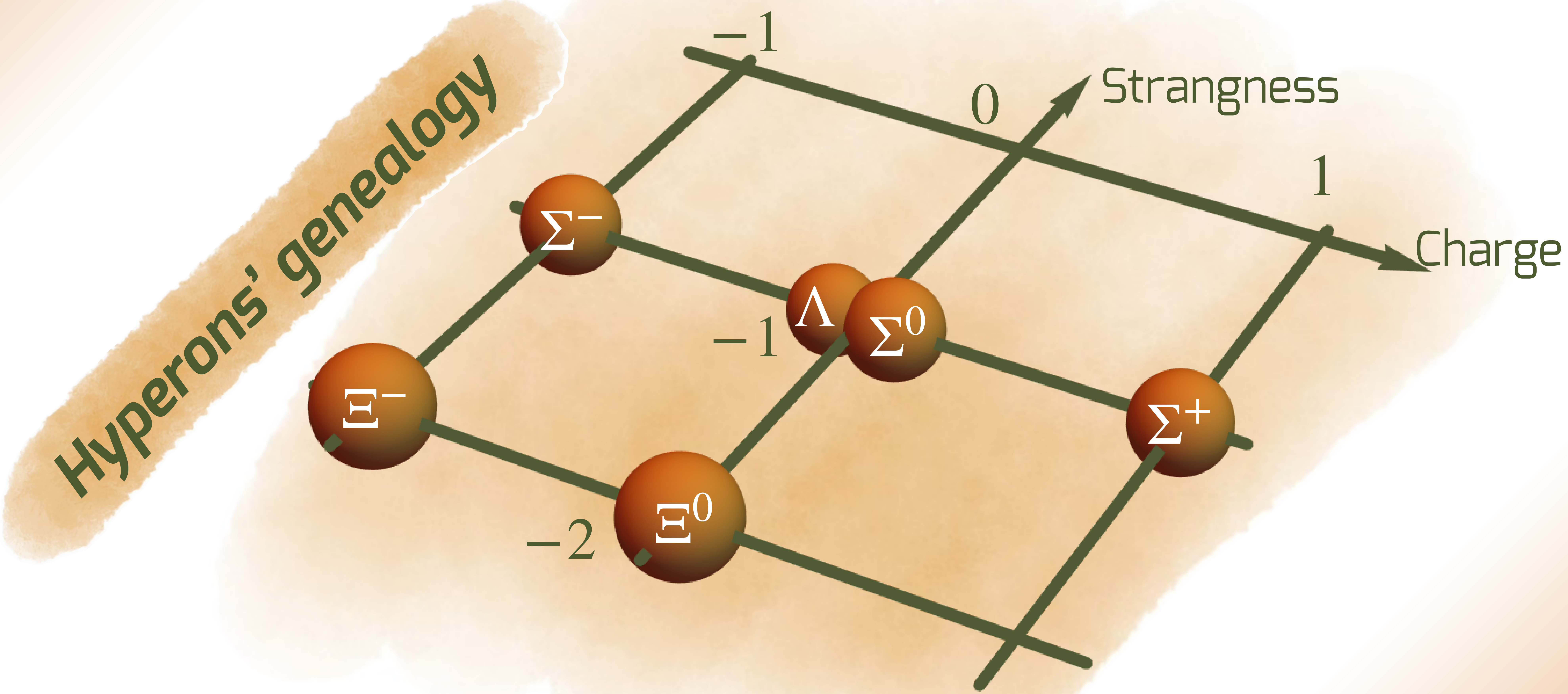

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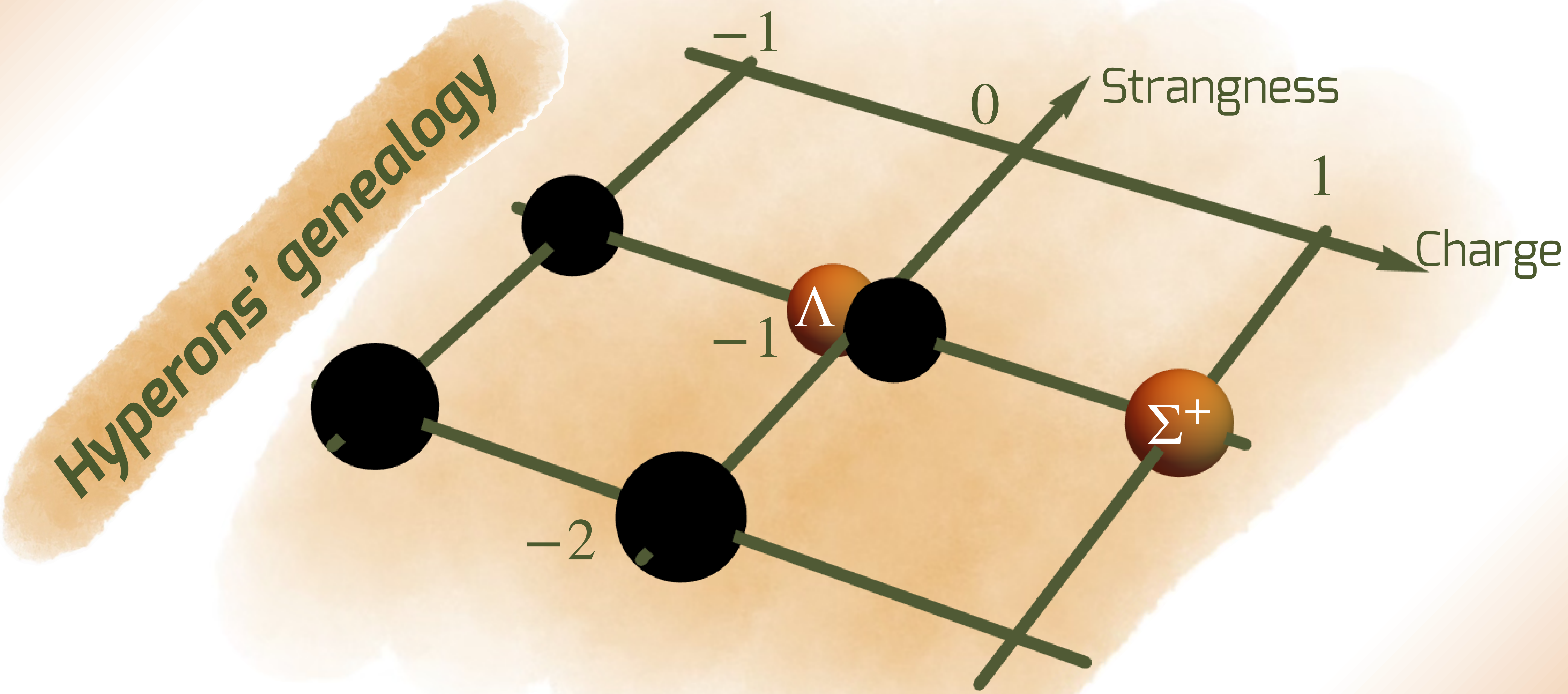

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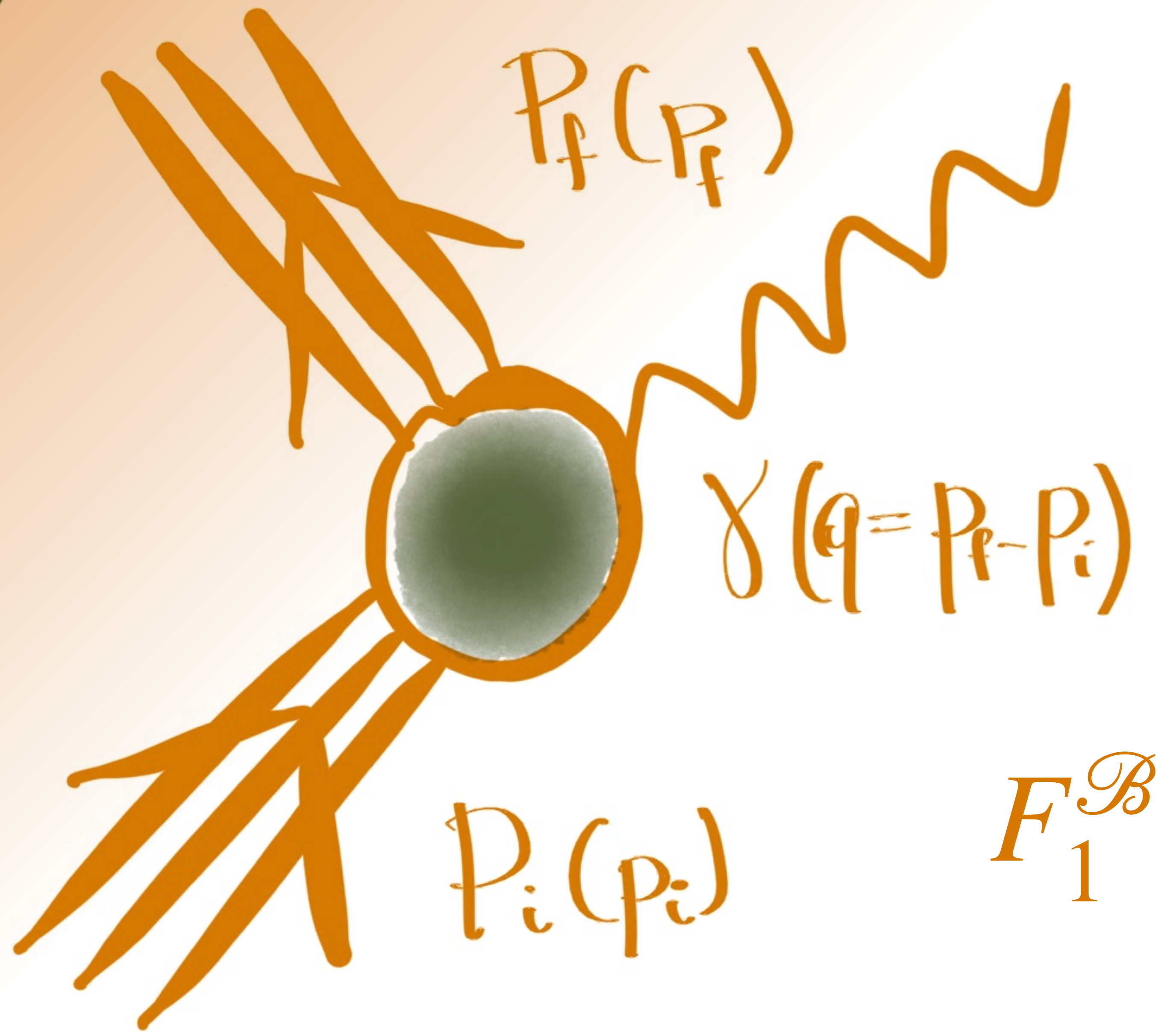
Baryon vertex



Electromagnetic four-current of the spin-1/2 baryon $\mathscr{B}$

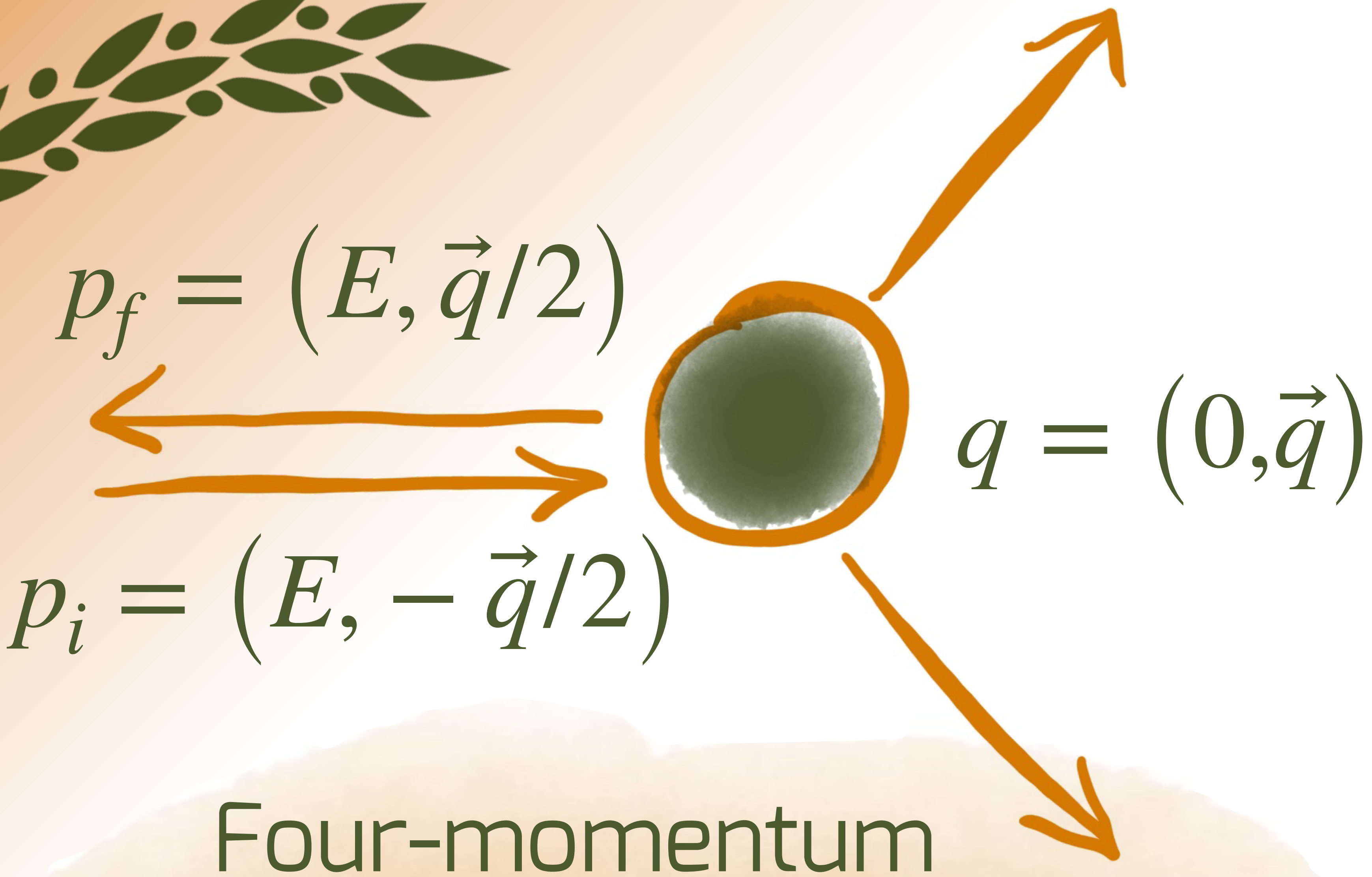
$$\left\langle P_f \middle| J^{\mu}_{\text{EM}}(0) \middle| P_i \right\rangle = e\bar{u}(p_f)\left[\gamma^{\mu}F_1^{\mathscr{B}}\left(q^2\right) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2M_{\mathscr{B}}}F_2^{\mathscr{B}}\left(q^2\right)\right]u(p_i)$$

$F_1^{\mathscr{B}}\left(q^2\right)$ and $F_2^{\mathscr{B}}\left(q^2\right)$ are Dirac and Pauli form factors

Normalizations at zero squared momentum transferred, $q^2 = 0$,

- $F_1^{\mathscr{B}}(0) = Q_{\mathscr{B}}$ — $Q_{\mathscr{B}}$ is the electric charge
- $F_2^{\mathscr{B}}(0) = \kappa_{\mathscr{B}}$ — $\kappa_{\mathscr{B}}$ is the anomalous magnetic moment

Breit frame



Four-momentum
coincides with three-momentum
$\Downarrow$
Sachs form factors
represent Fourier transforms

$$G_E^{\mathscr{B}}(q^2) = F_1^{\mathscr{B}}(q^2) + \frac{q^2}{4M_{\mathscr{B}}^2} F_2^{\mathscr{B}}(q^2)$$

$$G_M^{\mathscr{B}}(q^2) = F_1^{\mathscr{B}}(q^2) + F_2^{\mathscr{B}}(q^2)$$

In the Breit frame there is **no energy exchange**

$$\left\langle P_f \left| J^\mu_{\mathrm{EM}}(0) \right| P_i \right\rangle = J^\mu_{\mathrm{EM}} = \left(J^0_{\mathrm{EM}}, \vec{J}_{\mathrm{EM}} \right)$$

$$J^0_{\mathrm{EM}} = e \left(F_1^{\mathscr{B}}(q^2) + \frac{q^2}{4M_{\mathscr{B}}^2} F_2^{\mathscr{B}}(q^2) \right)$$

$$\vec{J}_{\mathrm{EM}} = e\, \bar{u}(p_f) \vec{\gamma} u(p_i) \left(F_1^{\mathscr{B}}(q^2) + F_2^{\mathscr{B}}(q^2) \right)$$

Normalizations at zero squared momentum transferred, $q^2 = 0$,

$$G_E^{\mathscr{B}}(0) = Q_{\mathscr{B}}$$

$$G_M^{\mathscr{B}}(0) = Q_{\mathscr{B}} + \kappa_{\mathscr{B}} = \mu_{\mathscr{B}}$$

$\mu_{\mathscr{B}}$ is the **total magnetic moment**

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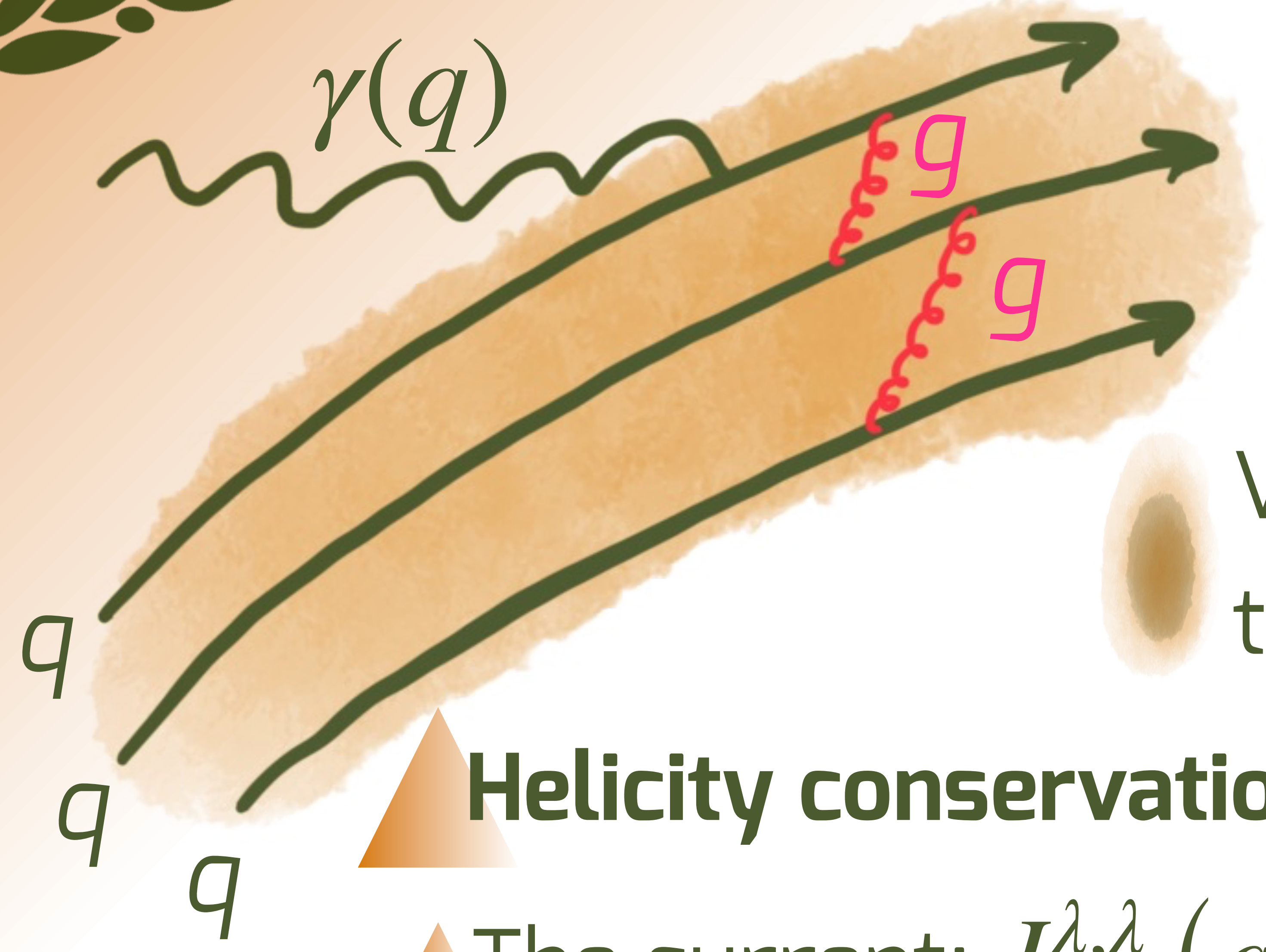
Asymptotics



- In **perturbative QCD** Dirac, Pauli and Sachs form factors as $q^2 \to -\infty$ follow power laws driven by **counting rules**
- Valence quarks exchange gluons to distribute the photon momentum transfer q

Helicity conservation

- The current: $J^{\lambda,\lambda}(q^2) \propto G_M^{\mathscr{B}}(q^2)$
- 2 gluon propagators
- $G_M^{\mathscr{B}}(q^2) \sim (q^2)^{-2}$

Helicity flip

- The current: $J^{\lambda,-\lambda}(q^2) \propto G_E^{\mathscr{B}}(q^2)/\sqrt{q^2}$
- [2 gluon propagators]/$\sqrt{q^2}$
- $G_E^{\mathscr{B}}(q^2) \sim (q^2)^{-2}$

Dirac and Pauli form factors

$$F_1^{\mathscr{B}}(q^2) \underset{q^2\to-\infty}{\sim} (q^2)^{-2}$$

$$F_2^{\mathscr{B}}(q^2) \underset{q^2\to-\infty}{\sim} (q^2)^{-3}$$

Ratio of Sachs form factors

$$\frac{G_E^{\mathscr{B}}(q^2)}{G_M^{\mathscr{B}}(q^2)} \underset{q^2\to-\infty}{\sim} \text{[constant]}$$

[V. A. Matveev, R. M. Muradian, A. N. Tavkhelidze, LNC 7 (1973) 719
S. J. Brodsky, G. R. Farrar, PRL 31 (1973) 1153
M. V. Galynsky, E. A. Kuraev JETPL 96 (2012) 6]

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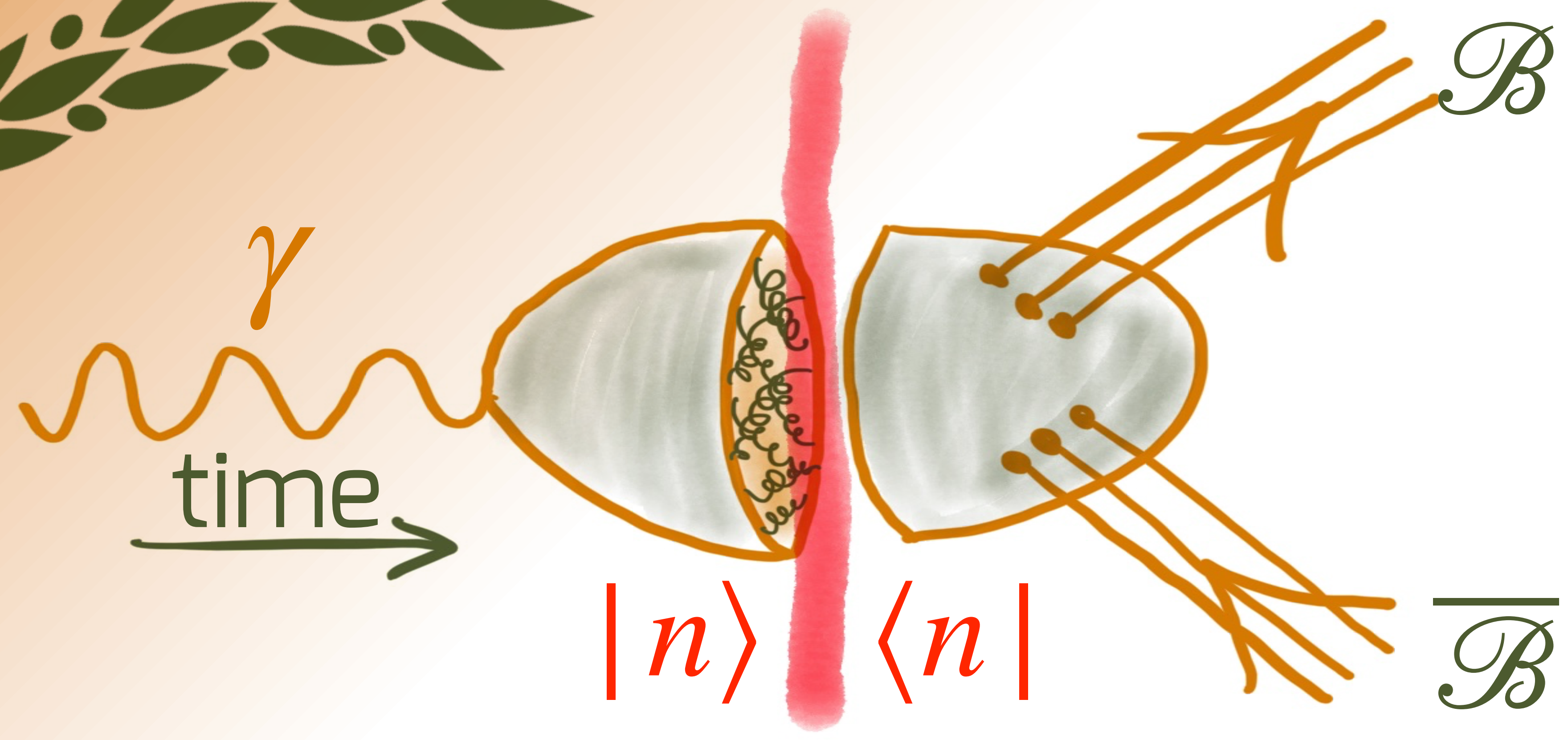
Time-like form factors



Crossing symmetry

$$\langle P(p')\,|\,j^\mu\,|\,P(p)\rangle \longrightarrow \langle \overline{P}(p')P(p)\,|\,j^\mu\,|\,0\rangle$$

Form factors are complex functions of q^2

Optical theorem

$$\mathrm{Im}\left(\langle \overline{P}(p')P(p)\,|\,j^\mu\,|\,0\rangle\right) \sim \sum_n \langle \overline{P}(p')P(p)\,|\,J_{\mathscr{B}}\,|\,n\rangle\langle n\,|\,j^\mu\,|\,0\rangle \Longrightarrow \begin{cases} \mathrm{Im}\left(F^{\mathscr{B}}_{1,2}\left(q^2\right)\right) \neq 0 \\ \text{for } q^2 > 4M_\pi^2 \end{cases}$$

$|\,n\rangle$ is an on-shell intermediate state, i.e., $|\,n\rangle = 2\pi, 3\pi, 4\pi, \ldots$

Phragmén-Lindelöf theorem

If $f(z) \to f_1$ as $z \to \infty$ along the straight line L_1, $f(z) \to f_2$ as $z \to \infty$ along the straight line L_2, and $f(z)$ is regular and bounded in the angle between, then $f_1 = f_2 \equiv f_{12}$ and $f(z) \to f_{12}$ **uniformly in the region between L_1 and L_2.**

Behavior in the time-like region

$$\underbrace{\lim_{q^2\to-\infty} G^{\mathscr{B}}_{E,M}(q^2)}_{\text{space-like } (L_1)} = \underbrace{\lim_{q^2\to+\infty} G^{\mathscr{B}}_{E,M}(q^2)}_{\text{time-like } (L_2)}$$

$$G^{\mathscr{B}}_{E,M}(q^2) \underset{q^2\to+\infty}{\sim} \left(q^2\right)^{-2} \in \mathbb{R}$$

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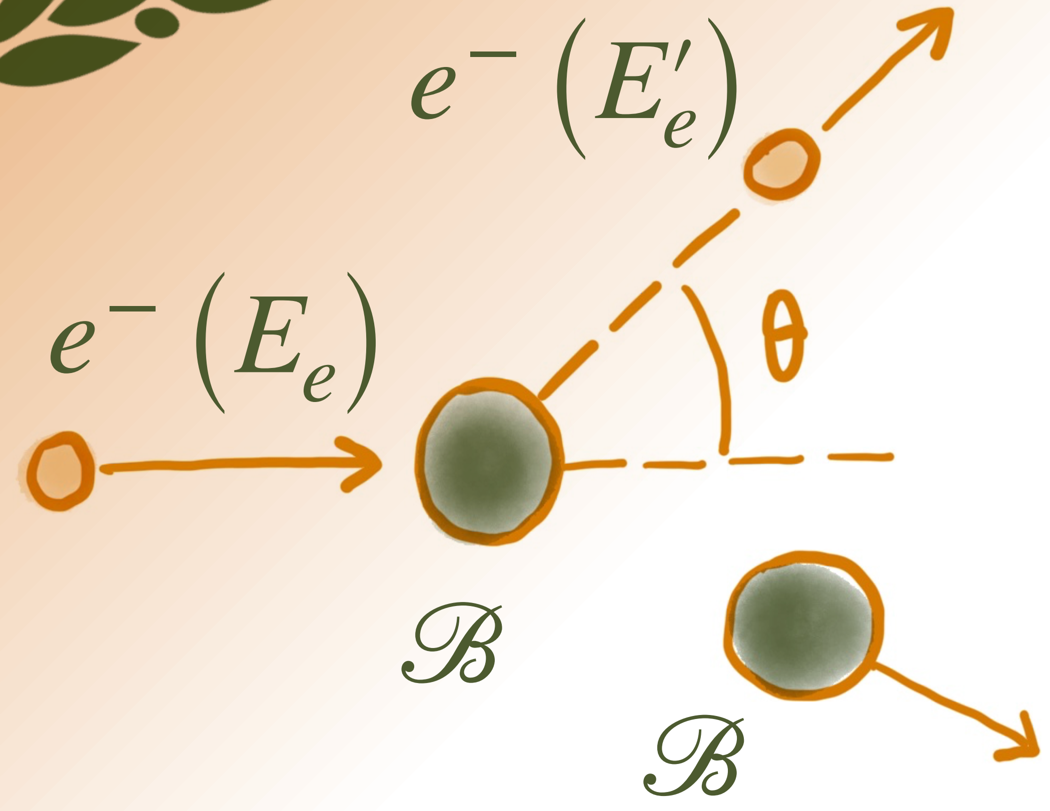
Cross sections



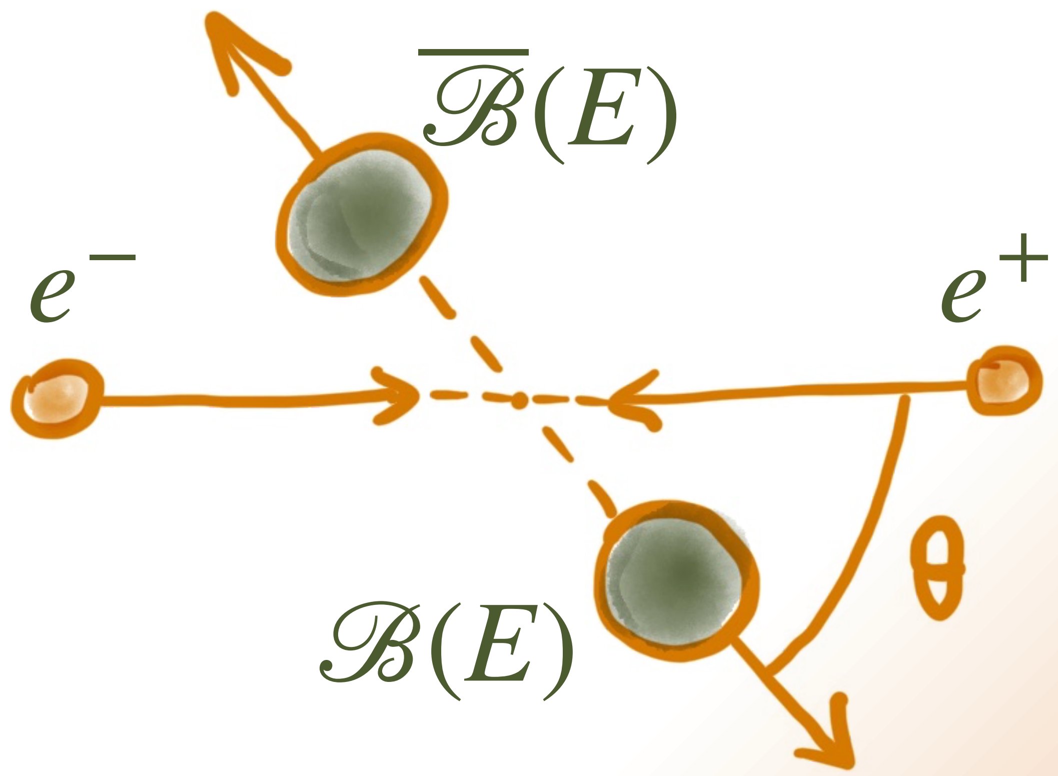
Elastic scattering differential cross section in Born approximation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E'_e \cos^2(\theta/2)}{4E_e^3 \sin^4(\theta/2)} \left[\left(G_E^{\mathscr{B}}\right)^2 - \tau \left(1 + 2(1-\tau)\tan^2(\theta/2)\right) \left(G_M^{\mathscr{B}}\right)^2 \right] \frac{1}{1-\tau}$$

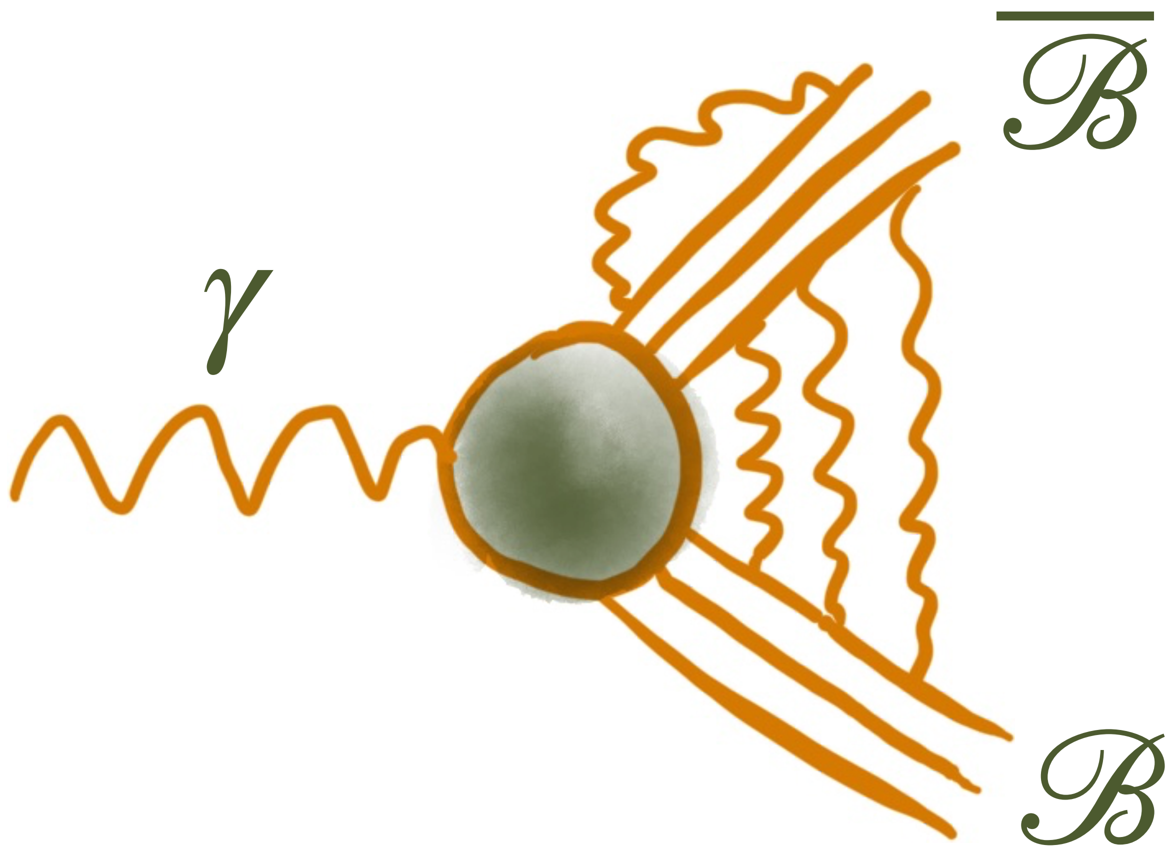
Annihilation differential cross section in Born approximation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta \mathscr{C}}{16E^2} \left[\left(1 + \cos^2(\theta)\right) \left|G_M^{\mathscr{B}}\right|^2 + \frac{1}{\tau} \sin^2(\theta) \left|G_E^{\mathscr{B}}\right|^2 \right]$$

$$\tau = E^2/M_{\mathscr{B}}^2$$
$$\beta = \sqrt{1 - 1/\tau}$$



Coulomb correction



Only S-wave
$\mathscr{B}\overline{\mathscr{B}}$ Coulomb
final state interaction

$$\mathscr{C} = \frac{\pi\alpha/\beta}{1 - e^{-\pi\alpha/\beta}}$$

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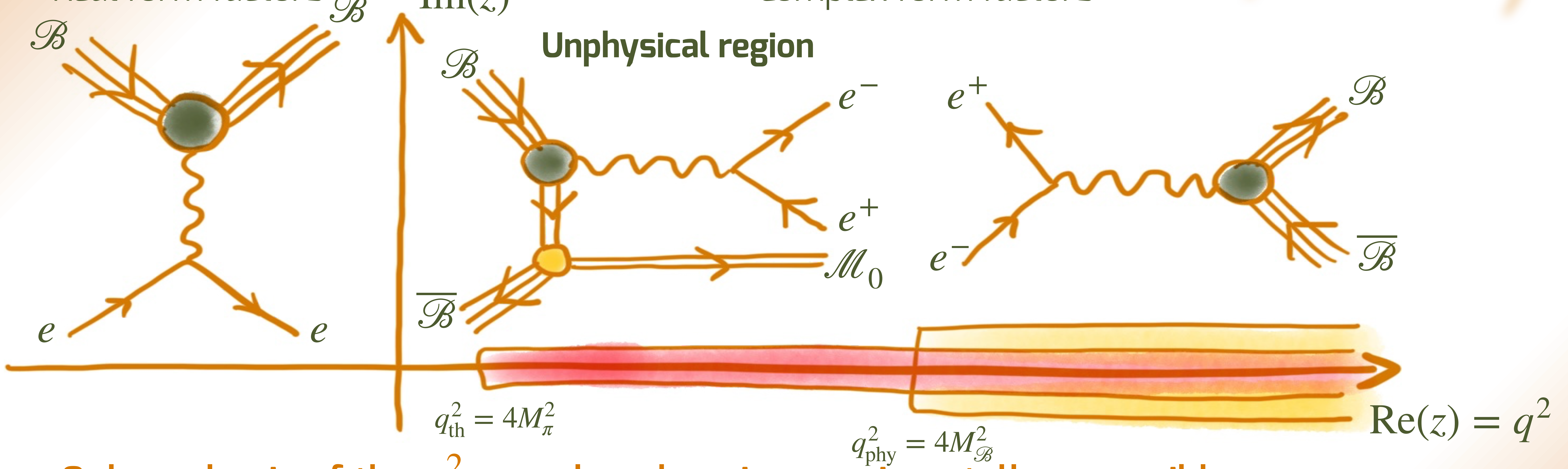
Analyticity

Space-like region
Real form factors

Time-like region
Complex form factors

q^2 complex plane



Only real axis of the q^2-complex plane is experimentally accessible

Space-like region	Time-like region*	Time-like region	
$q^2 < 0$	$q^2_{\rm th} < q^2 \leq q^2_{\rm phy}$	$q^2 > q^2_{\rm phy}$	
$e\mathscr{B} \to e\mathscr{B}$	$\mathscr{B}\overline{\mathscr{B}} \to e^+e^-\mathscr{M}_0$	$e^+e^- \leftrightarrow \mathscr{B}\overline{\mathscr{B}}$	$e^+e^- \leftrightarrow \mathscr{B}\overline{\mathscr{B}}$ (pol.)
$G_E^{\mathscr{B}}, G_M^{\mathscr{B}}$	$\left\|G_E^{\mathscr{B}}\right\|, \left\|G_M^{\mathscr{B}}\right\|$	$\left\|G_E^{\mathscr{B}}\right\|, \left\|G_M^{\mathscr{B}}\right\|$	$\left\|G_E^{\mathscr{B}}\right\|, \left\|G_M^{\mathscr{B}}\right\|, \arg\left(G_E^{\mathscr{B}}/G_M^{\mathscr{B}}\right)$

[*With $(\mathscr{B}, \mathscr{M}_0) = (p, \pi^0)$: C. Adamuscin, E. A. Kuraev, E. Tomasi-Gustafsson, F. Maas PRC 75, 045205;E. A. Kuraev et al., JETP 115, 93; G. I. Gakh, E. Tomasi-Gustafsson, A. Dbeyssi, A. G. Gakh PRC 86, 025204]

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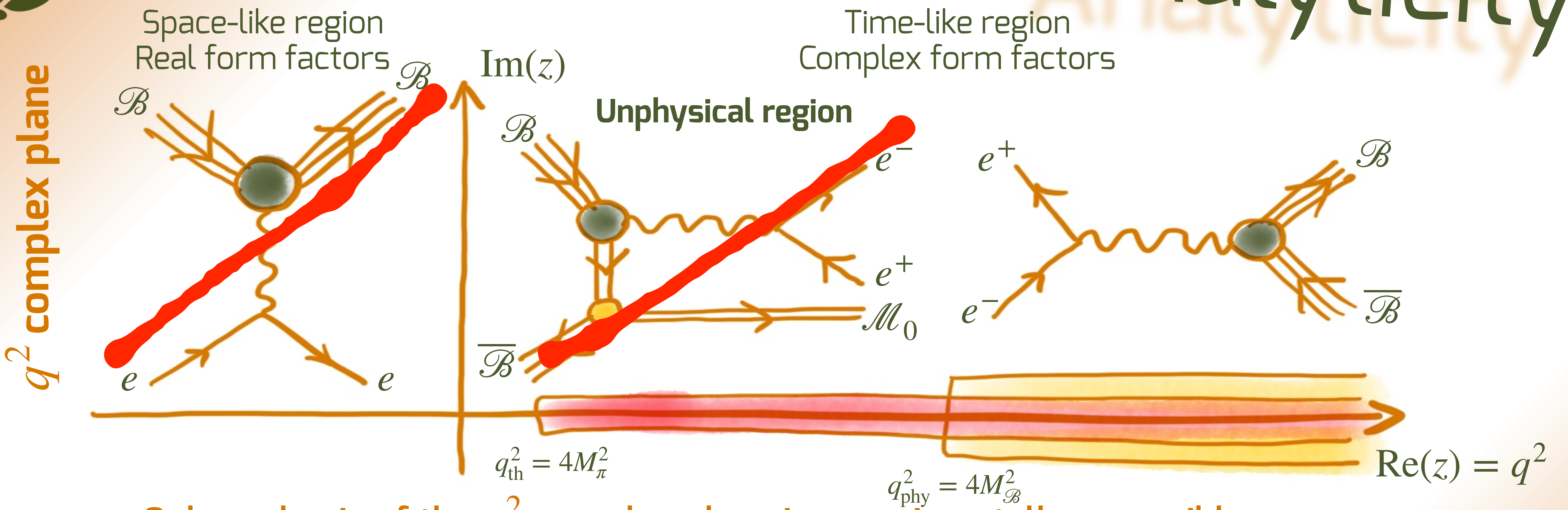
Analyticity



Only real axis of the q^2-complex plane is experimentally accessible

~~**Space-like region**~~	~~**Time-like region***~~	**Time-like region**	
$\cancel{q^2 < 0}$	$\cancel{q^2_{\mathrm{th}} < q^2 \leq q^2_{\mathrm{phy}}}$	$q^2 > q^2_{\mathrm{phy}}$	
$\cancel{e\mathscr{B} \to e\mathscr{B}}$	$\cancel{\mathscr{B}\overline{\mathscr{B}} \to e^+e^-\mathscr{M}_0}$	$e^+e^- \leftrightarrow \mathscr{B}\overline{\mathscr{B}}$	$e^+e^- \leftrightarrow \mathscr{B}\overline{\mathscr{B}}$ (pol.)
$\cancel{G_E^{\mathscr{B}},\ G_M^{\mathscr{B}}}$	$\cancel{\left\lvert G_E^{\mathscr{B}}\right\rvert,\ \left\lvert G_M^{\mathscr{B}}\right\rvert}$	$\left\lvert G_E^{\mathscr{B}}\right\rvert,\ \left\lvert G_M^{\mathscr{B}}\right\rvert$	$\left\lvert G_E^{\mathscr{B}}\right\rvert,\ \left\lvert G_M^{\mathscr{B}}\right\rvert,\ \arg\left(G_E^{\mathscr{B}}/G_M^{\mathscr{B}}\right)$

Hyperons

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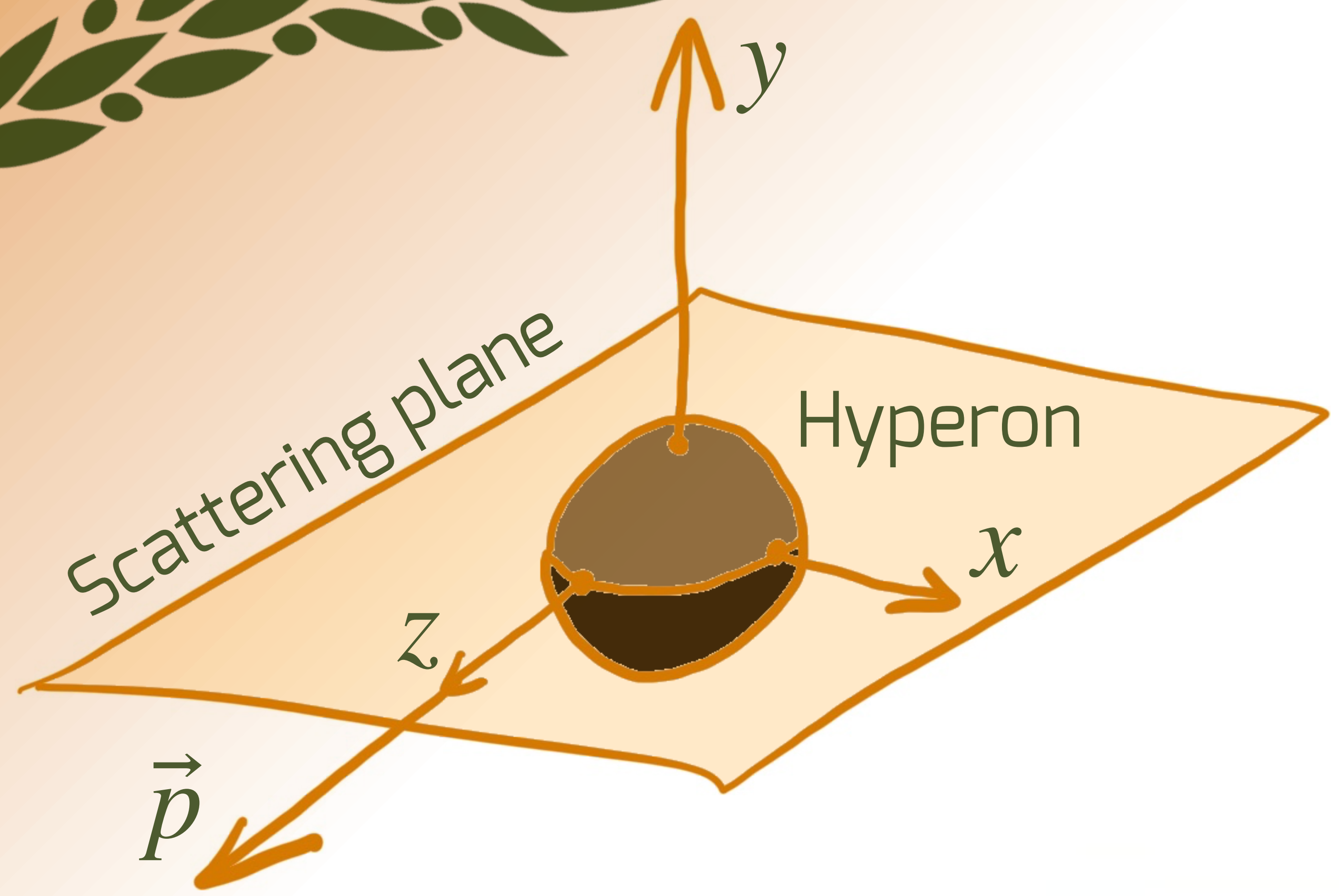
Polarization



The ratio $G_E^{\mathscr{B}}(q^2)/G_M^{\mathscr{B}}(q^2)$ is complex for $q^2 > q^2_{\rm th}$

$$\frac{G_E^{\mathscr{B}}(q^2)}{G_M^{\mathscr{B}}(q^2)} = \frac{\left|G_E^{\mathscr{B}}(q^2)\right|}{\left|G_M^{\mathscr{B}}(q^2)\right|} e^{i\rho_{\mathscr{B}}(q^2)}$$

The polarization depends on the relative phase $\rho_{\mathscr{B}}(q^2)$

[A. Z. Dubnickova, S. Dubnicka, M. P. Rekalo, NC A109 (1996) 241]

Y polarization vector

$\vec{\mathscr{P}} = \left(\mathscr{P}_x, \mathscr{P}_y, \mathscr{P}_z\right)$

$$\mathscr{P}_y = -\frac{\sin(2\theta)\sin(\rho_{\mathscr{B}})}{D\sqrt{\tau}}\frac{\left|G_E^{\mathscr{B}}\right|}{\left|G_M^{\mathscr{B}}\right|} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \equiv \mathscr{A}_y$$

Does not depend on P_e

$$\mathscr{P}_x = -P_e\frac{2\sin(2\theta)\cos(\rho_{\mathscr{B}})}{D\sqrt{\tau}}\frac{\left|G_E^{\mathscr{B}}\right|}{\left|G_M^{\mathscr{B}}\right|}$$

$$\mathscr{P}_z = P_e\frac{2\cos(\theta)}{D}$$

Does not depend on the relative phase $\rho_{\mathscr{B}}$

$$D = 1 + \cos^2(\theta) + \frac{\left|G_E^{\mathscr{B}}\right|^2}{\left|G_M^{\mathscr{B}}\right|^2}\frac{\sin^2(\theta)}{\tau} \qquad \tau = \frac{q^2}{4M_{\mathscr{B}}^2}$$

P_e: electron polarization

θ: scattering angle

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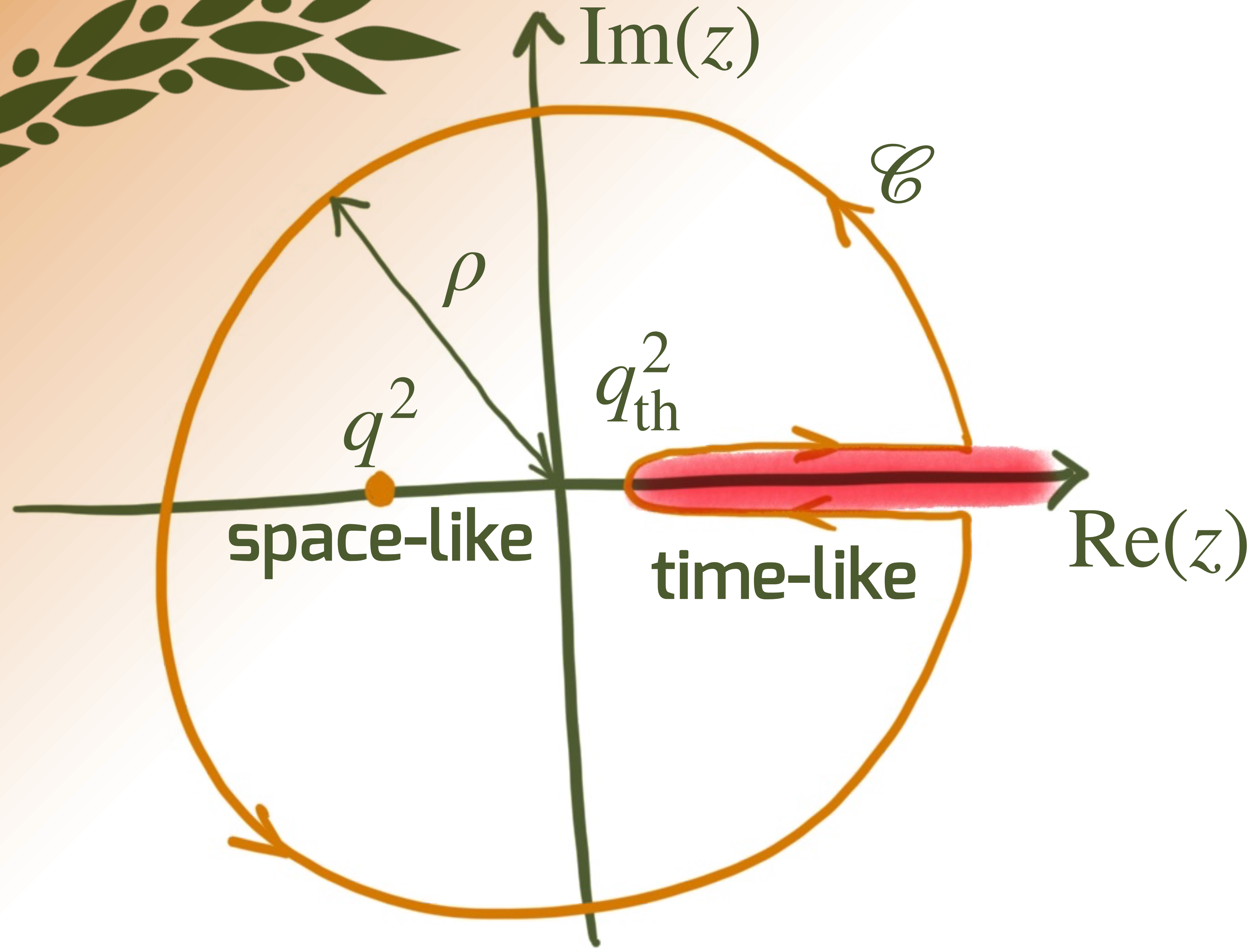
Dispersion relations



- The form factors are analytic on the q^2-complex plane with a multiple cut $\left(q^2_{\rm th}, \infty\right)$
- Dispersion relation for the imaginary part $\left(q^2 < 0\right)$

$$G(q^2) = \lim_{\rho\to\infty} \frac{1}{2i\pi}\oint_{\mathscr{C}} \frac{G(z)}{z-q^2}dz = \frac{1}{\pi}\int_{q^2_{\rm th}}^{\infty} \frac{\mathrm{Im}\left(G(s)\right)}{s-q^2}ds$$

Dispersion relation for the logarithm $\left(q^2 < 0\right)$
[B. V. Geshkenbein, Yad. Fiz. 9 (1969) 1232]

$$\ln\left(G(q^2)\right) = \frac{\sqrt{q^2_{\rm th}-q^2}}{\pi}\int_{q^2_{\rm th}}^{\infty} \frac{\ln\left|G(s)\right|}{(s-q^2)\sqrt{s-q^2_{\rm th}}}ds$$

Experimental inputs

Space-like data on real values of form factors from:
$e\mathscr{B} \to e\mathscr{B}$ and $e^{-\uparrow}\mathscr{B} \to e^{-}\mathscr{B}^{\uparrow}$, with polarization

Time-like data on moduli of form factors from:
$e^+e^- \leftrightarrow \mathscr{B}\overline{\mathscr{B}}$

Time-like data on the phase of $G_E^{\mathscr{B}}/G_M^{\mathscr{B}}$ from:
$e^+e^- \leftrightarrow \mathscr{B}^{\uparrow}\overline{\mathscr{B}}$, with polarization

Theoretical ingredients

Analyticity $\Longrightarrow$ convergence relations

Normalization and **threshold** values

Asymptotic behavior $\Longrightarrow$ super-convergence relations

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Advantages and drawbacks

(Phys. Rev. D 104 (2021) 116016]

Advantages

- Dispersion relations are based on unitarity and analyticity $\Longrightarrow$ **model-independent**
- Dispersion relations relate data from different precesses in different energy regions

$$\left[\begin{array}{c}\text{space-like}\\ \text{form factor}\\ e\mathscr{B} \to e\mathscr{B}\end{array}\right] = \int_{q^2_{\rm th}}^{\infty}\left[\begin{array}{c}\text{Im(form factor) or ln(|form factor|)}\\ \text{over the time-like cut } \left(q^2_{\rm th}, \infty\right)\\ e^+e^- \to \mathscr{B}\overline{\mathscr{B}} + \text{theory}\end{array}\right] dq^2$$

- Normalizations and theoretical constraints can be directly implemented
- Form factors can be computed in the whole q^2-complex plane
- Using ratios, poles cancel out

Drawbacks

- Very long-range integration

Remedy #1
pQCD power laws

Remedy #2
Subtracted dispersion relations

- No data in the unphysical region, crucial in dispersive analyses

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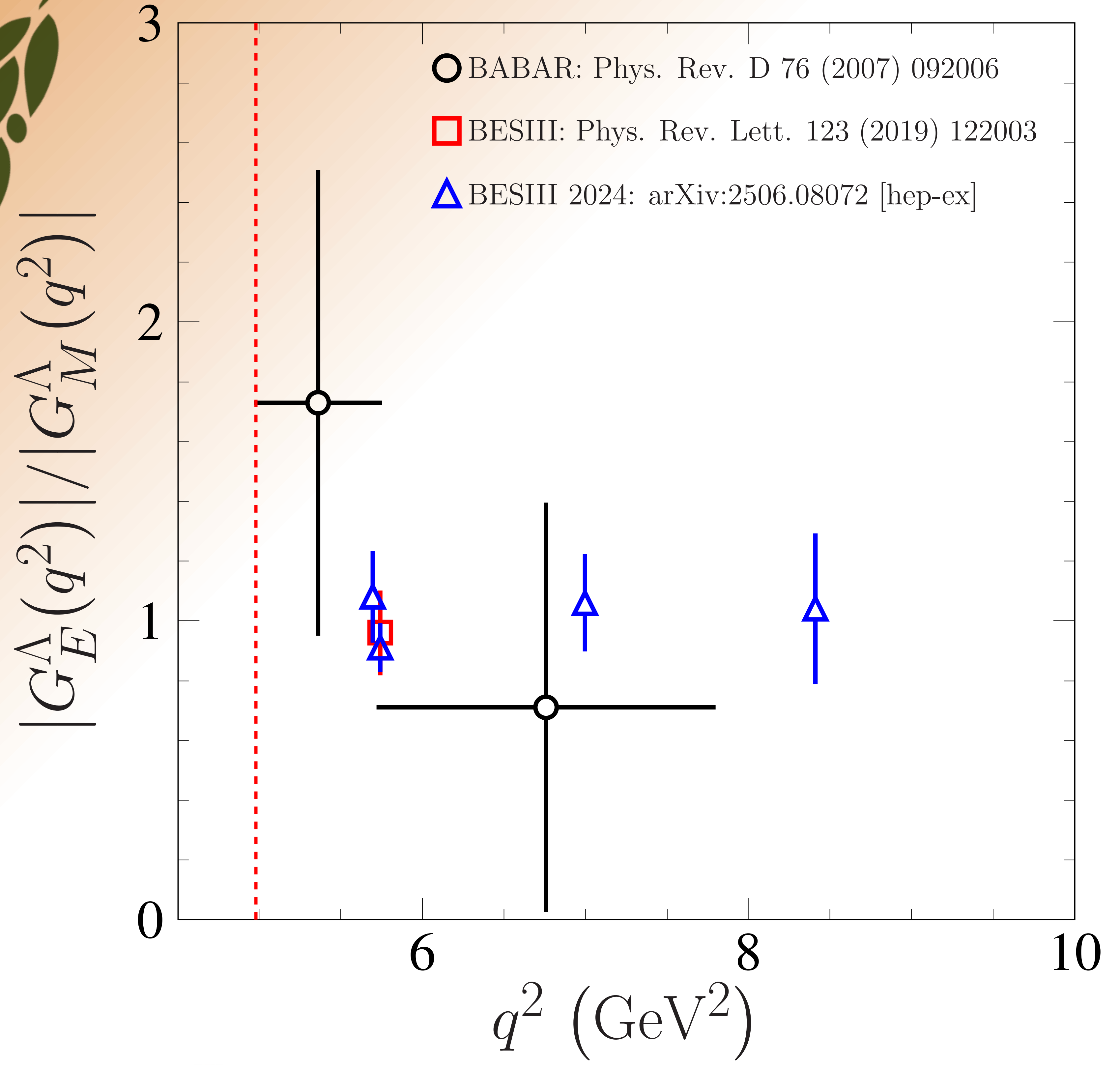
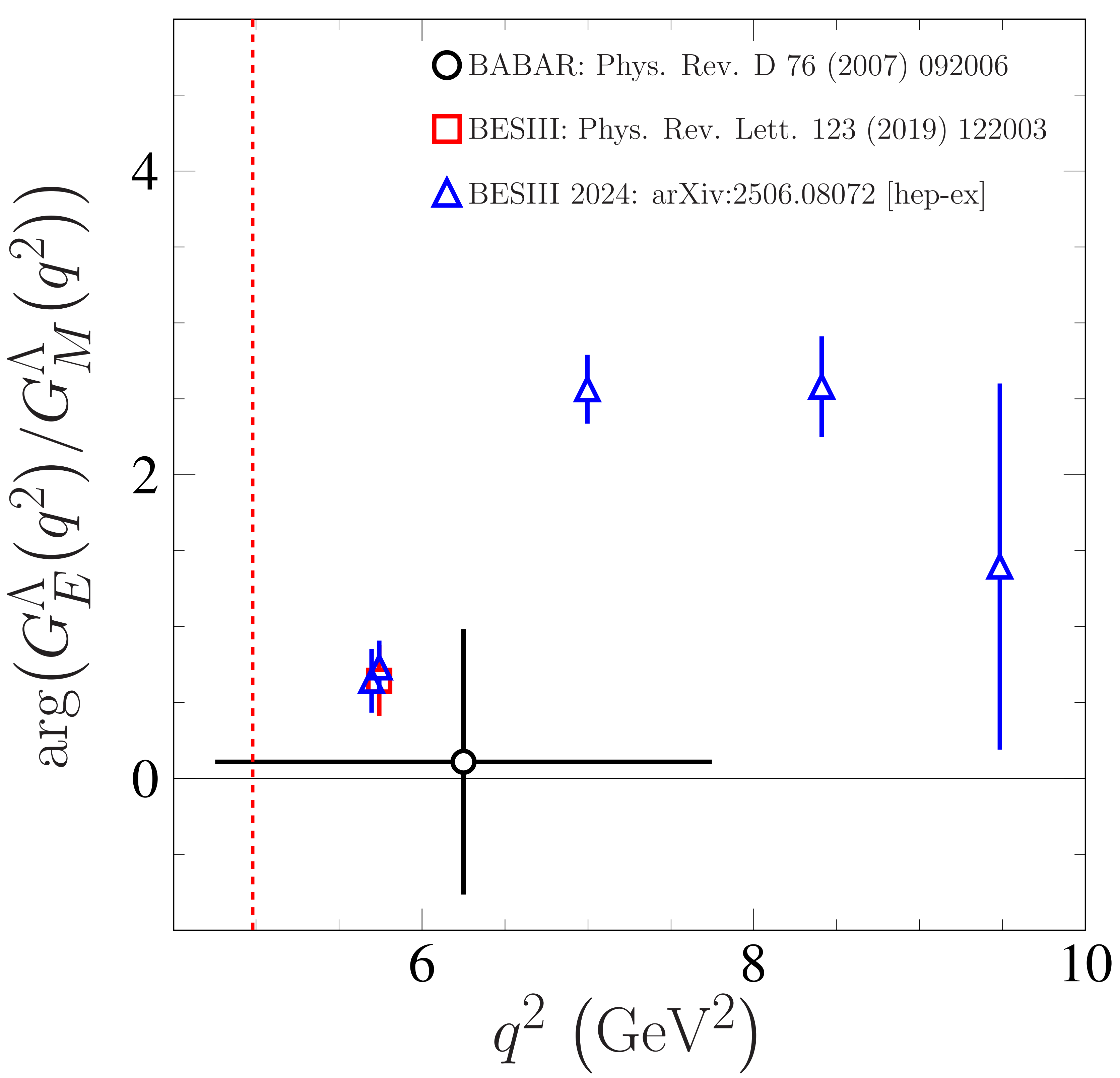
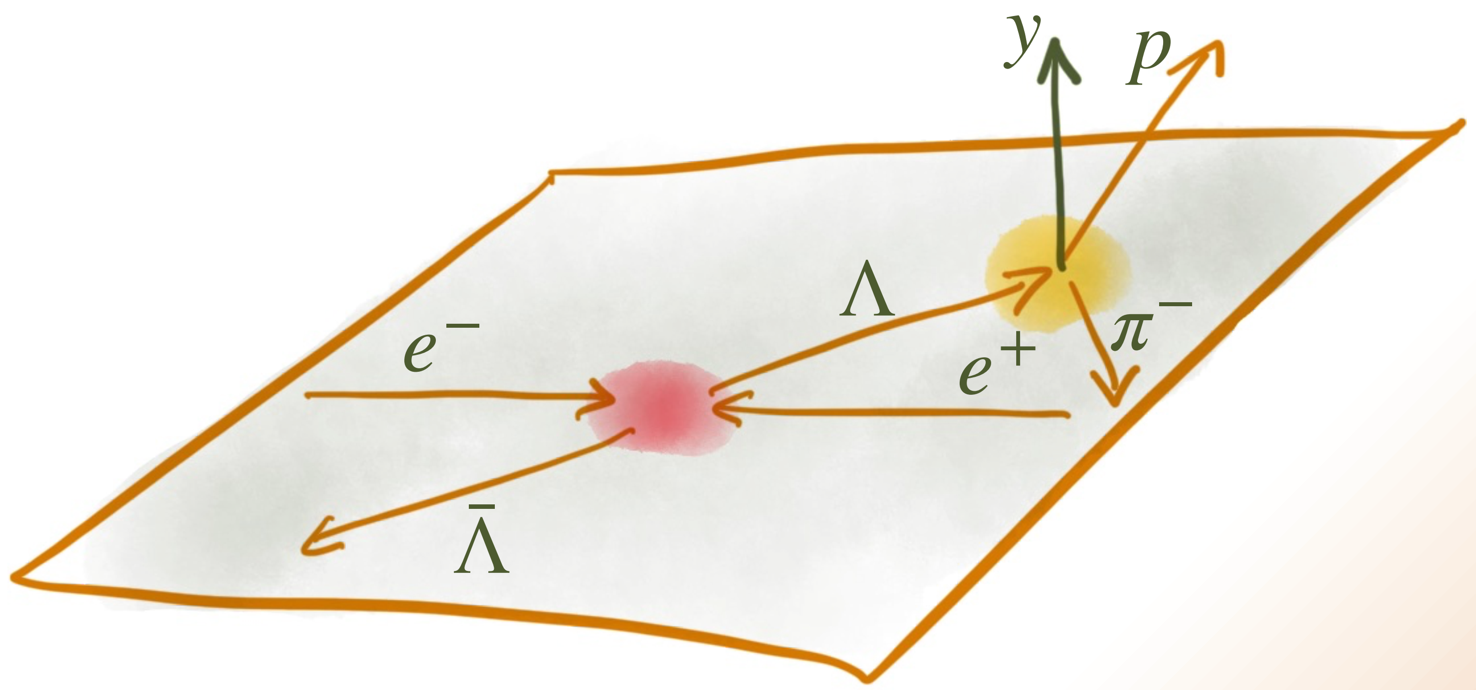
Polarization on Λ





"…the weak parity-violating decay gives straightforward access to the polarization."



Polarization ⟶ sine of the relative phase

Spin correlation ⟶ cosine of the relative phase

No indication on the determination of the relative phase

$$\mathscr{P}_y^\Lambda = -\frac{2M_\Lambda\sqrt{q^2}\sin(2\theta)\left|G_E^\Lambda/G_M^\Lambda\right|\sin\left(\arg\left(G_E^\Lambda/G_M^\Lambda\right)\right)}{q^2\left(1+\cos^2(\theta)\right)+4M_\Lambda^2\left|G_E^\Lambda/G_M^\Lambda\right|^2\sin^2(\theta)}$$

Is the determination of the phase meaningful?

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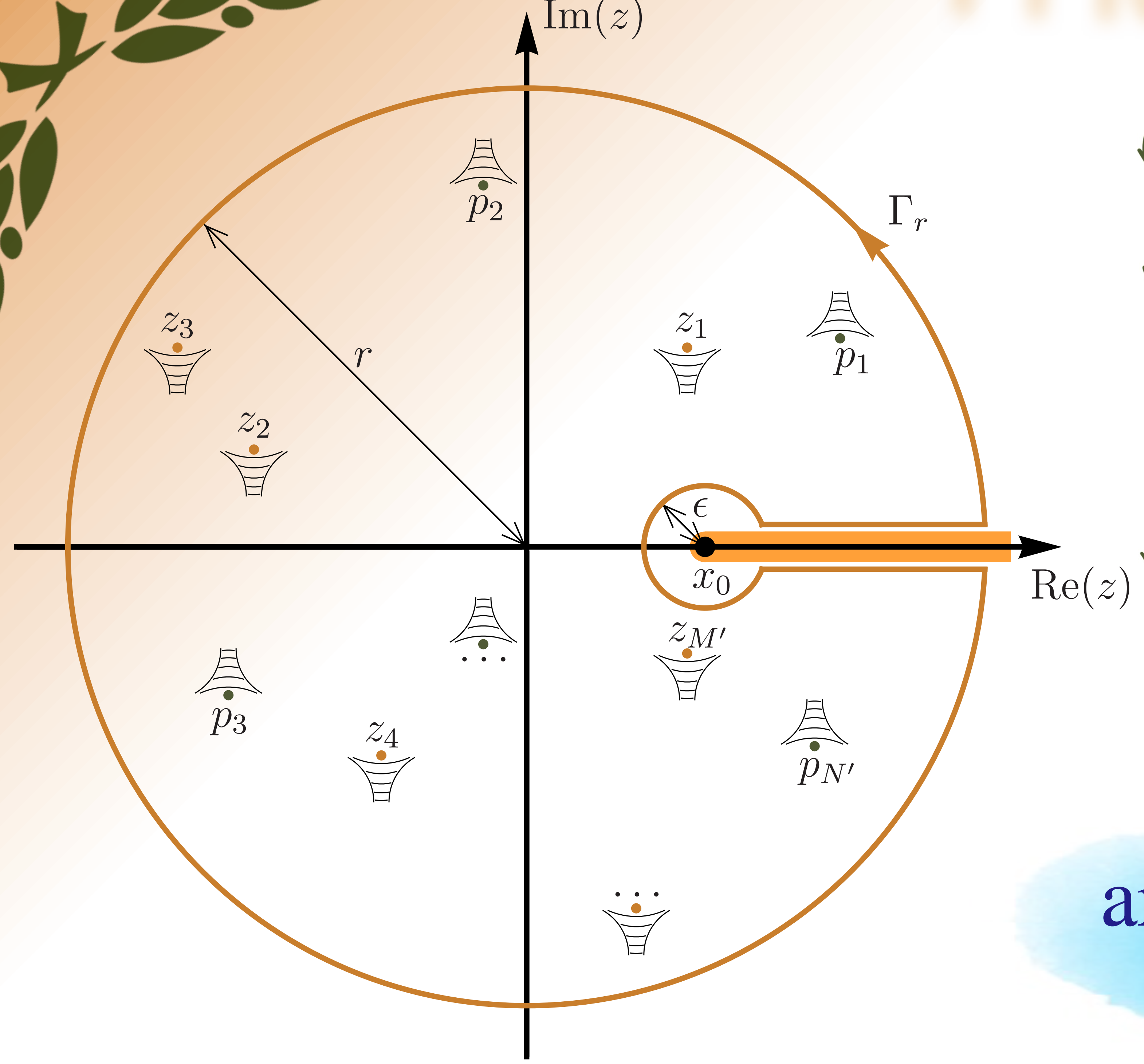
The meaning of determination



- Consider the function $R(z)$ with poles $\{p_j\}_{j=1}^N$, zeros $\{z_k\}_{k=1}^M$ and the real cut (x_0, ∞)
- The residue theorem over the Γ_r contour

$$\lim_{r\to\infty} \frac{1}{2i\pi} \oint_{\Gamma_r} \frac{d\ln(R(z))}{dz} dz = M - N.$$

- Considering single contributions

$$\lim_{r\to\infty} \frac{1}{2i\pi} \oint_{\Gamma_r} \frac{d\ln(R(z))}{dz} dz = \frac{\arg(R(\infty)) - \arg(R(x_0))}{\pi}.$$

$$\arg(R(\infty)) - \arg(R(x_0)) = \pi(M - N)$$

Levinson's theorem

- Form factors are analytic in the q^2 complex plane with the real positive cut $\left(q^2_{\mathrm{th},\Lambda}, \infty\right)$
- **Assuming no zeros for G_M^Λ**, the ratio G_E^Λ/G_M^Λ has the same analyticity domain
- Form factors and hence the ratio G_E^Λ/G_M^Λ are real for $q^2 \in \left(-\infty, q^2_{\mathrm{th},\Lambda}\right)$

$$\lim_{q^2\to q^{2-}_{\mathrm{th},\Lambda}} \arg\left(\frac{G_E^\Lambda(q^2)}{G_M^\Lambda(q^2)}\right) = \begin{cases} 0 & G_E^\Lambda(q^{2-}_{\mathrm{th},\Lambda})/G_M^\Lambda(q^{2-}_{\mathrm{th},\Lambda}) > 0 \\ \pm\pi & G_E^\Lambda(q^{2-}_{\mathrm{th},\Lambda})/G_M^\Lambda(q^{2-}_{\mathrm{th},\Lambda}) < 0 \end{cases}$$

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Dispersive procedure

The ratio $R_\Lambda(q^2) = \dfrac{G_E^\Lambda(q^2)}{G_M^\Lambda(q^2)} \Longrightarrow \left\{ \begin{array}{c} G_E^\Lambda(0) = 0 \\ G_E^\Lambda(q^2_{\mathrm{phy},\Lambda}) = G_M^\Lambda(q^2_{\mathrm{phy},\Lambda}) \end{array} \right\} \Longrightarrow \left\{ \begin{array}{r} R_\Lambda(0) = 0 \\ R_\Lambda(q^2_{\mathrm{phy},\Lambda}) = 1 \end{array} \right\}$

The asymptotic behavior

$$R_\Lambda(q^2) = \frac{G_E^\Lambda(q^2)}{G_M^\Lambda(q^2)} = \mathcal{O}(1) \text{ as } q^2 \to \pm\infty$$

Dispersion relations for the **imaginary** and **real** part with subtraction at $q^2 = 0$

$$R_\Lambda(q^2) = R_\Lambda(0) + \frac{q^2}{\pi}\int_{q^2_{\mathrm{th},\Lambda}}^{\infty} \frac{\mathrm{Im}(R_\Lambda(s))}{s(s-q^2)}ds = \frac{q^2}{\pi}\int_{q^2_{\mathrm{th},\Lambda}}^{\infty} \frac{\mathrm{Im}(R_\Lambda(s))}{s(s-q^2)}ds\,, \quad \forall\, q^2 \notin [q^2_{\mathrm{th},\Lambda}, \infty)$$

$$\mathrm{Re}(R_\Lambda(q^2)) = \frac{q^2}{\pi}\mathrm{Pr}\int_{q^2_{\mathrm{th},\Lambda}}^{\infty} \frac{\mathrm{Im}(R_\Lambda(s))}{s(s-q^2)}ds\,, \quad \forall\, q^2 \in [q^2_{\mathrm{th},\Lambda}, \infty)^+$$

The subtraction ensures the null normalization at $q^2 = 0$

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Parametrization for $R_\Lambda(q^2)$

The ratio $R_\Lambda(q^2)$ is parametrized through the set of Chebyshev polynomials $\{T_j(x)\}_{j=0}^{P}$

$$\mathrm{Im}\left(R_\Lambda(q^2)\right) \equiv Y_\Lambda\left(q^2; \vec{C}, q^2_{\rm asy}\right) = \begin{cases} \sum_{j=0}^{P} C_j T_j\left(x(q^2)\right) & q^2_{\rm th,\Lambda} < q^2 < q^2_{\rm asy} \\ 0 & q^2 \geq q^2_{\rm asy} \end{cases}$$

$$x(q^2) = 2\frac{q^2 - q^2_{\rm th,\Lambda}}{q^2_{\rm asy} - q^2_{\rm th,\Lambda}} - 1$$

$$q^2 \in \left[q^2_{\rm th,\Lambda}, q^2_{\rm asy}\right] \Rightarrow x \in [-1,1]$$

Theoretical constraints on $Y_\Lambda\left(q^2; \vec{C}, q^2_{\rm asy}\right)$

$R_\Lambda\left(q^2_{\rm th,\Lambda}\right)$ is real $\Longrightarrow Y_\Lambda\left(q^2_{\rm th,\Lambda}; \vec{C}, q^2_{\rm asy}\right) = 0$

$R_\Lambda\left(q^2_{\rm phy,\Lambda}\right)$ is real $\Longrightarrow Y_\Lambda\left(q^2_{\rm phy,\Lambda}; \vec{C}, q^2_{\rm asy}\right) = 0$

$R_\Lambda\left(q^2 \geq q^2_{\rm asy}\right)$ is real $\Longrightarrow Y_\Lambda\left(q^2 \geq q^2_{\rm asy}; \vec{C}, q^2_{\rm asy}\right) = 0$

Theoretical constraints on $\mathrm{Re}\left(R_\Lambda(q^2)\right)$

$$\mathrm{Re}\left(R_\Lambda\left(q^2_{\rm phy,\Lambda}\right)\right) = \frac{q^2_{\rm phy,\Lambda}}{\pi} \mathrm{Pr}\int_{q^2_{\rm th,\Lambda}}^{q^2_{\rm asy}} \frac{Y_\Lambda\left(s; \vec{C}; q^2_{\rm asy}\right)}{s\left(s - q^2_{\rm asy}\right)} ds = 1$$

$$\left|\mathrm{Re}\left(R_\Lambda\left(q^2_{\rm asy}\right)\right)\right| = \frac{q^2_{\rm asy}}{\pi} \left|\mathrm{Pr}\int_{q^2_{\rm th,\Lambda}}^{q^2_{\rm asy}} \frac{Y_\Lambda\left(s; \vec{C}; q^2_{\rm asy}\right)}{s\left(s - q^2_{\rm asy}\right)} ds\right| = 1$$

Experimental constraints in the time-like region $\left(q^2 > q^2_{\rm phy,\Lambda}\right)$

$\sin\left(\arg\left(R_\Lambda(q^2)\right)\right)$: data points from BABAR and from BESIII

$\left|R_\Lambda(q^2)\right|$: data points from BABAR and from BESIII

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Definition of χ^2

$$\chi^2(\vec{C}, q^2_{\rm asy}) = \chi^2_{|R|} + \chi^2_{\phi} + \tau_{\rm phy}\chi^2_{\rm phy} + \tau_{\rm asy}\chi^2_{\rm asy} + \tau_{\rm curv}\chi^2_{\rm curv}$$

Data: $\left\{q_j^2, |R_j|, \delta|R_j|\right\}_{j=1}^{8} \longrightarrow \chi^2_{|R|} = \sum_{j=1}^{8} \frac{\left(\sqrt{X_\Lambda^2(q_j^2) + Y_\Lambda^2(q_j^2)} - |R_j|\right)^2}{\left(\delta|R_j|\right)^2}$, with: $X_\Lambda(q^2) \equiv \mathrm{Re}\left(R_\Lambda(q^2)\right)$

Data: $\left\{q_k^2, \sin(\phi_k), \delta\sin(\phi_k)\right\}_{k=1}^{7} \longrightarrow \chi^2_{\phi} = \sum_{k=1}^{7} \frac{\left(\sin\left(\arctan\left(Y_\Lambda(q_k^2)/X_\Lambda(q_k^2)\right)\right) - \sin(\phi_k)\right)^2}{\left(\delta\sin(\phi_k)\right)^2}$

Constraint at $q^2 = q^2_{\rm phy,\Lambda} \longrightarrow \chi^2_{\rm phy} = \left(1 - X_\Lambda(q^2_{\rm phy,\Lambda})\right)^2$

Constraint at $q^2 = q^2_{\rm asy} \longrightarrow \chi^2_{\rm asy} = \left(1 - X_\Lambda(q^2_{\rm asy})^2\right)^2$

The values of multipliers $\tau_{\rm phy}$ and $\tau_{\rm asy}$ are chosen larger enough to nullify the corresponding χ^2's so that the conditions are exactly verified

Oscillation damping $\longrightarrow \chi^2_{\rm curv} = \int_{q^2_{\rm th,\Lambda}}^{q^2_{\rm asy}} \left(\frac{d^2 Y_\Lambda(s)}{ds^2}\right)^2 ds$

The integral equation obtained by the dispersion relations is an ill-posed problem whose solution has to be regularized

The value of the regularization parameter $\tau_{\rm curv}$ is selected in order to attenuate spurious oscillations

Too large values of $\tau_{\rm curv}$ would cancel physical information

Too small values $\tau_{\rm curv}$ would leave an unreliable level of noise

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Our parameterization

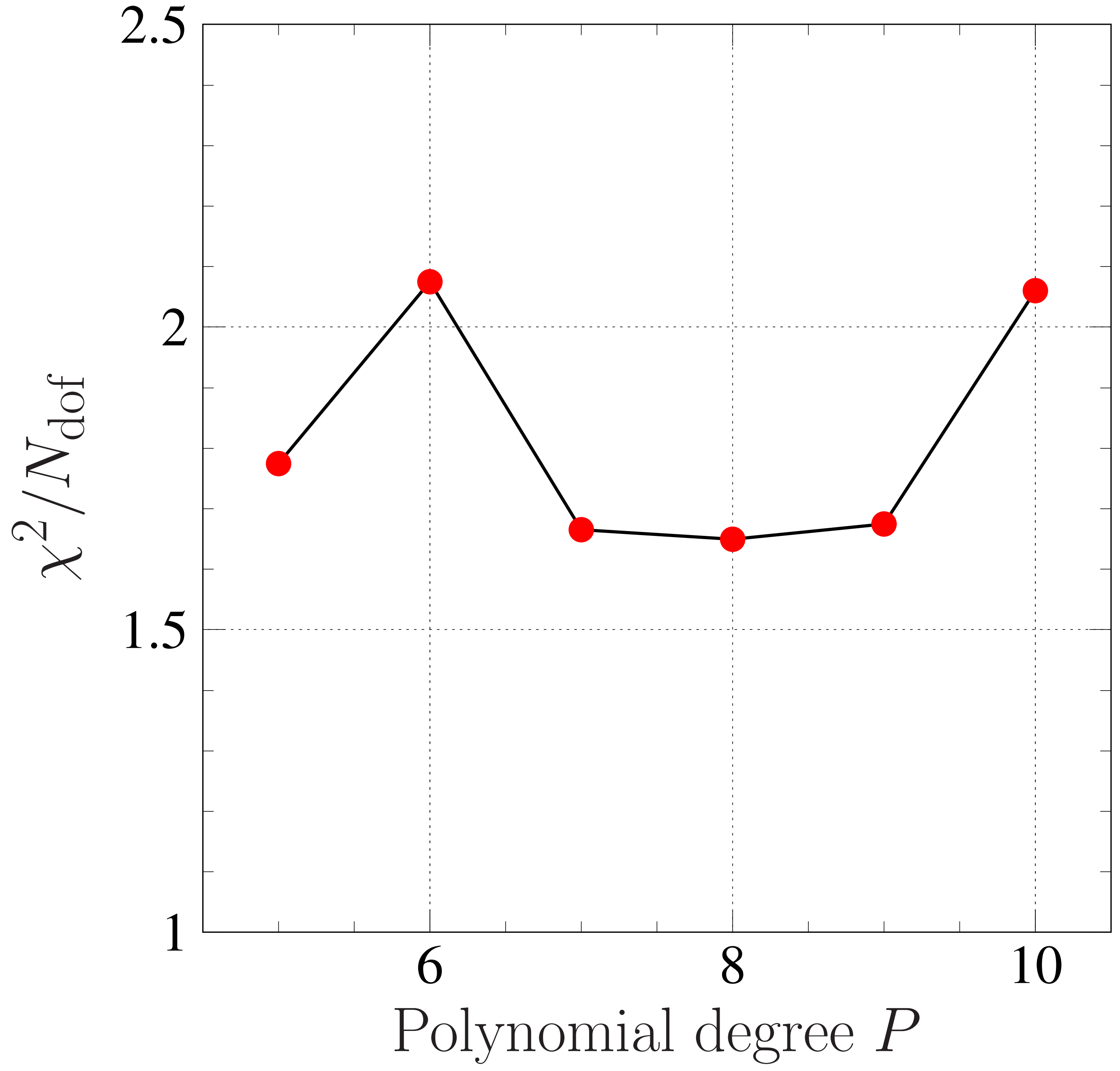
Theoretical constraints $Y_\Lambda\left(q^2_{\mathrm{th},\Lambda}; \vec{C}, q^2_{\mathrm{asy}}\right) = Y_\Lambda\left(q^2_{\mathrm{phy},\Lambda}; \vec{C}, q^2_{\mathrm{asy}}\right) = Y_\Lambda\left(q^2_{\mathrm{asy}}; \vec{C}, q^2_{\mathrm{asy}}\right) = 0$

determine the three coefficients: C_0, C_1, C_2

The asymptotic threshold q^2_{asy} is left as a free parameter

By considering $(P+1)$ **Chebyshev** polynomials there are $(P-2)$ **free coefficients**

There are **13 independent data points** $\Rightarrow N_{\mathrm{dof}} = 13 - (P-2) - 1 = 14 - P$



$\tau_{\mathrm{phy}} = 10^4$

The real part of $R_\Lambda\left(q^2\right)$ is forced to the unity at $q^2 = q^2_{\mathrm{phy},\Lambda}$

$\tau_{\mathrm{asy}} = 0$

No constraint for the real part of $R_\Lambda\left(q^2\right)$ at $q^2 = q^2_{\mathrm{asy}}$

$\tau_{\mathrm{curv}} = 0.05$

Low-degree polynomials do not need strong damping

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Results and considerations

At thresholds $q^2_{\mathrm{th},\Lambda}$ and q^2_{asy} the ratio is **real** hence the phase is an **integer multiple** of π radiants

$$N_{\mathrm{th(asy)}} = \frac{1}{\pi}\arg\left(\frac{G_E^\Lambda\left(q^2_{\mathrm{th},\Lambda(\mathrm{asy})}\right)}{G_M^\Lambda\left(q^2_{\mathrm{th},\Lambda(\mathrm{asy})}\right)}\right) \in \mathbb{Z}$$

The lack of data prevents obtaining unique pairs $\left(N_{\mathrm{th}}, N_{\mathrm{ays}}\right)$

The strong theoretical constraints reduce to **2** the number of possible pairs $\left(N_{\mathrm{th}}, N_{\mathrm{ays}}\right)$ compatible with the few data points

A **Monte Carlo** procedure, defined to make a statistical study of the results, gives the probability of occurrence of each pair $\left(N_{\mathrm{th}}, N_{\mathrm{ays}}\right)$

N_{th}	0	0	-1	1	-1
N_{asy}	3	4	1	3	2
$\%$	82.1	9.7	2.8	2.3	2.1

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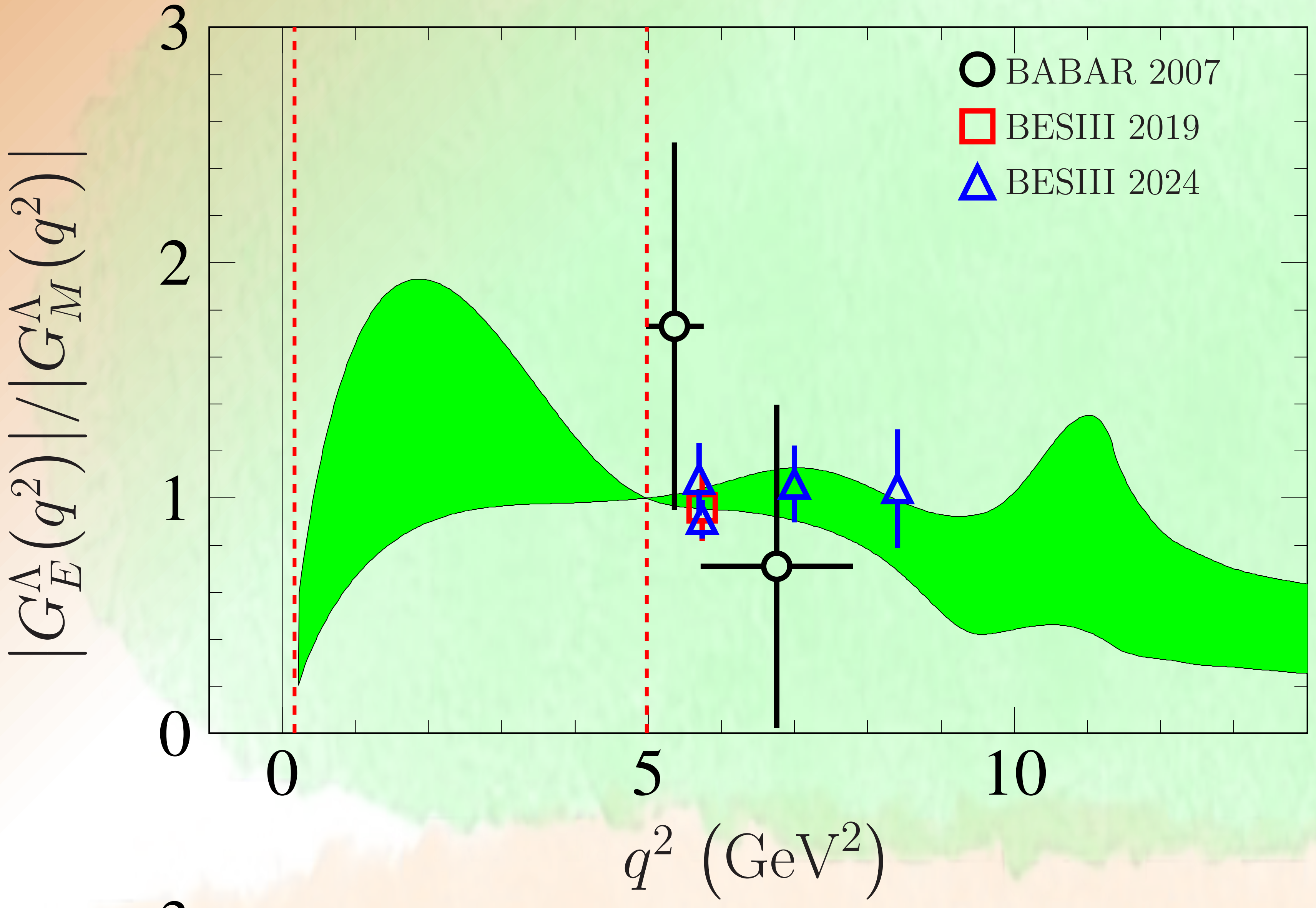
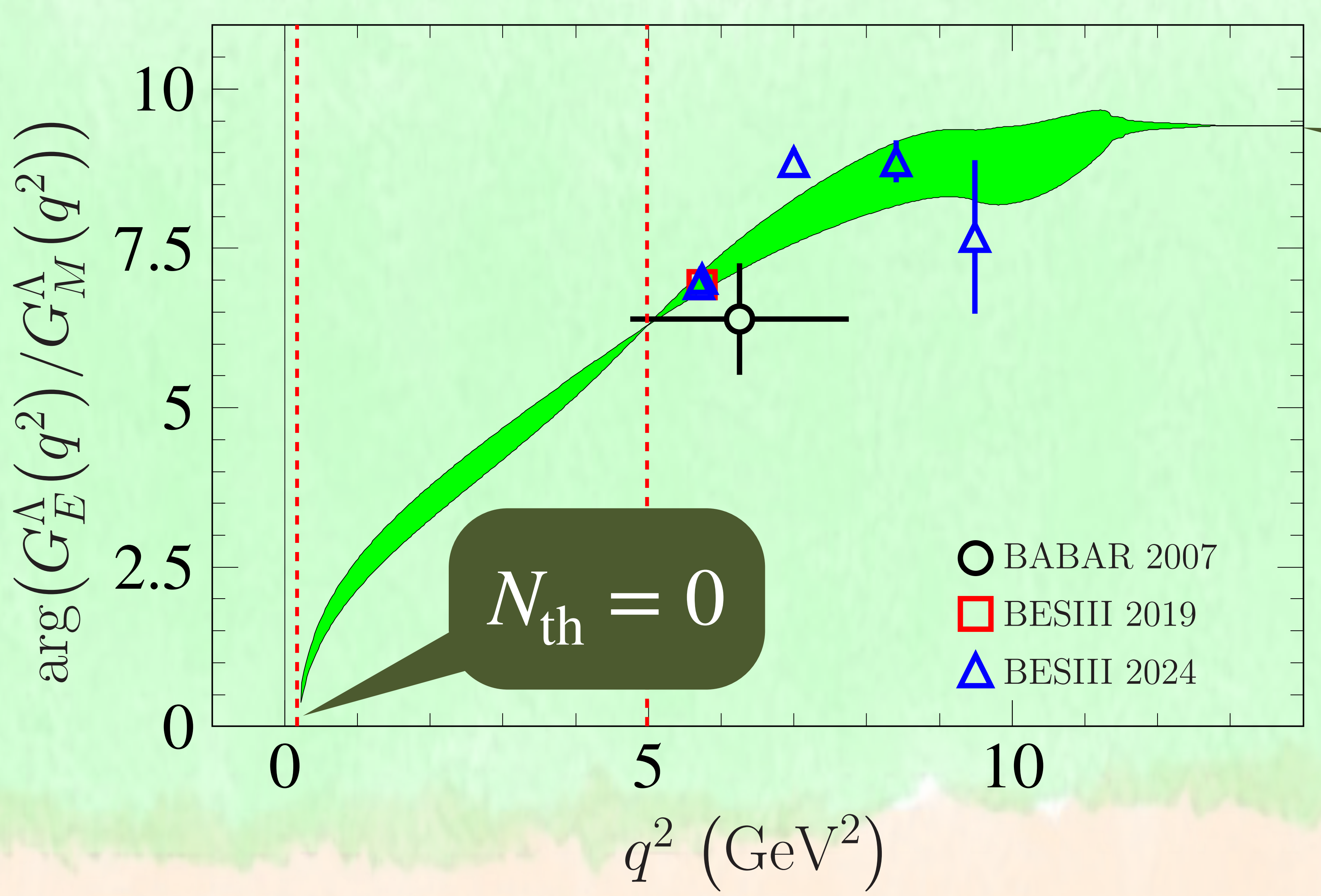
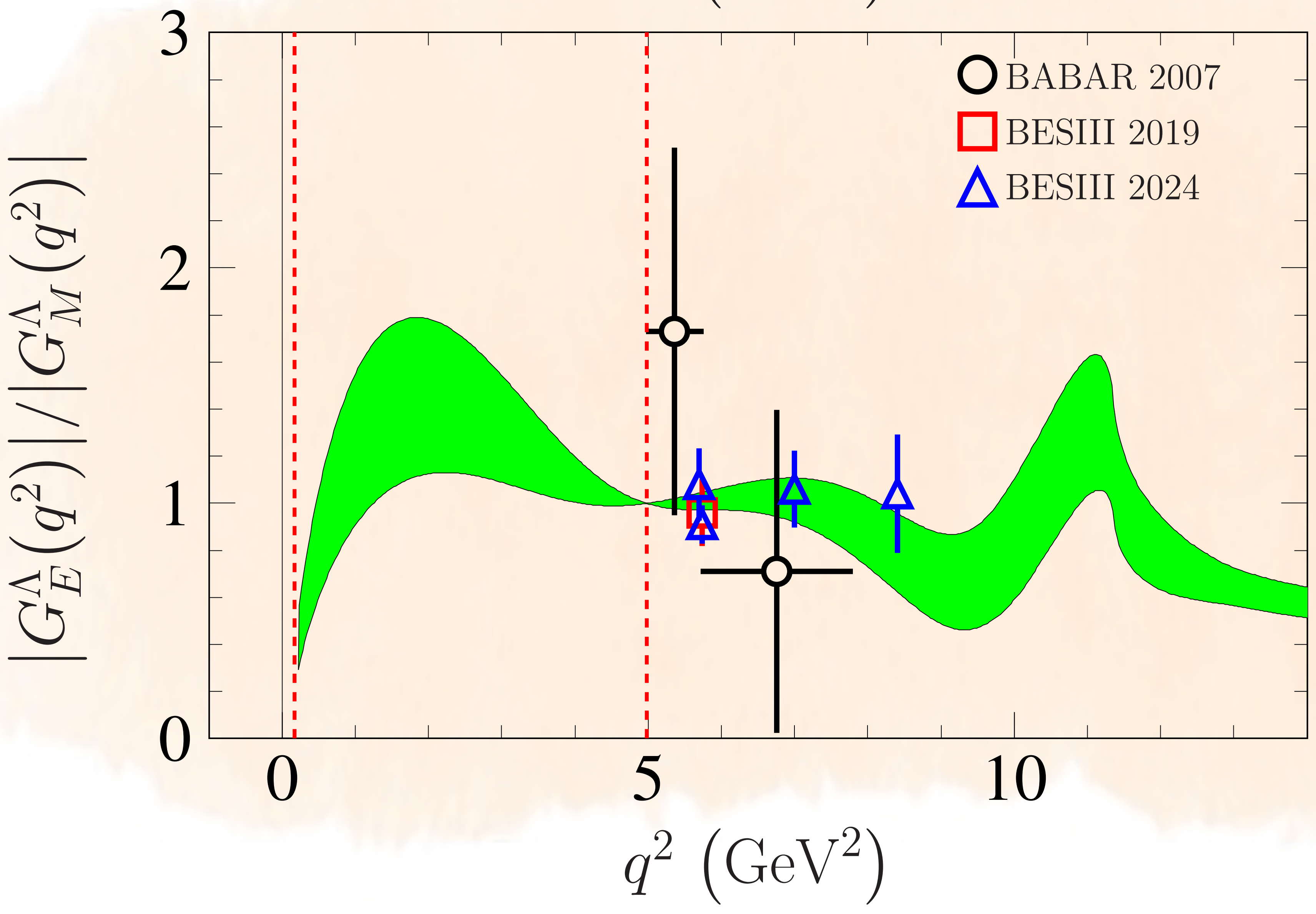
Λ results

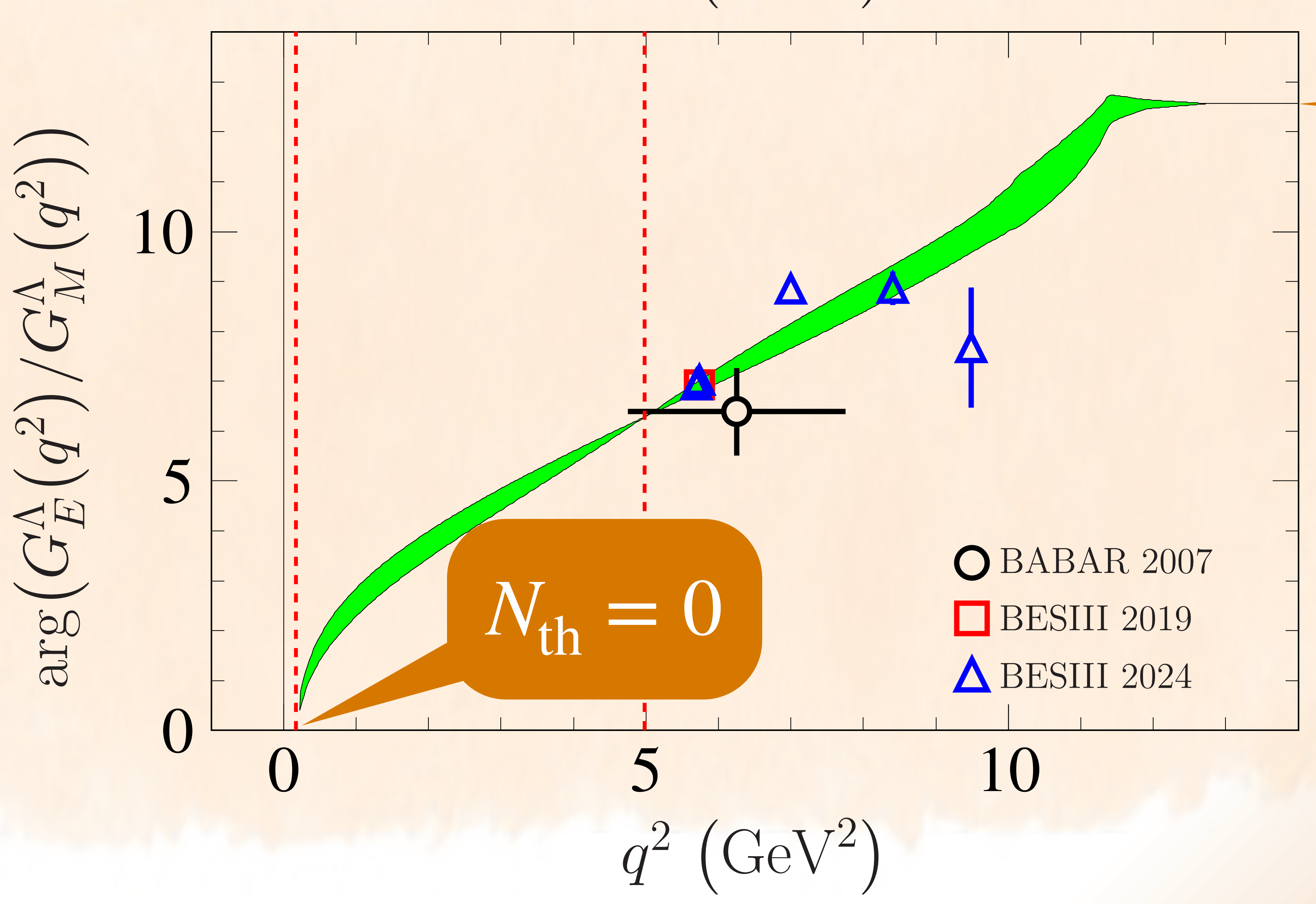




$N_{\rm asy} = 3$

Probability of 82.1 %





$N_{\rm asy} = 4$

Probability of 9.7 %

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Charge radius

Dynamical and static features of the baryon Λ can be inferred from the complete knowledge of its form factors as functions of q^2

The charge radius square of extended particles is given by the first derivative of the electric form factor at $q^2 = 0$

$$\langle r_E^2 \rangle = 6 \frac{dG_E(q^2)}{dq^2}\bigg|_{q^2=0}$$

In the Breit frame, where $q = (0, \vec{q})$ is purely space-like, the electric form factor is the Fourier transform of the spacial charge distribution

For a **neutral baryon**, like the Λ, the Sachs form factors at $q^2 = 0$ are normalized as: $G_E(0) = 0$ and $G_M(0) = \mu \neq 0$, then the charge radius squared is also proportional to the first derivative at $q^2 = 0$ of the ratio $R(q^2) = G_E(q^2)/G_M(q^2)$

$$\frac{dR(q^2)}{dq^2}\bigg|_{q^2=0} = \frac{1}{G_M(q^2)}\left(\frac{dG_E(q^2)}{dq^2} - \frac{\overbrace{G_E(q^2)}^{0 \text{ at } q^2=0}}{G_M(q^2)}\frac{dG_M(q^2)}{dq^2}\right)\bigg|_{q^2=0} = \frac{1}{G_M(q^2)}\frac{dG_E(q^2)}{dq^2}\bigg|_{q^2=0} = \frac{1}{\mu}\frac{\langle r_E^2 \rangle}{6}$$

The first derivative at $q^2 = 0$ of the ratio $R(q^2)$ is computed by means of the dispersione relation for the imaginary part

$$\langle r_E^2 \rangle = 6\mu \frac{dR(q^2)}{dq^2}\bigg|_{q^2=0} = \frac{6\mu}{\pi}\int_{q_{\rm th}^2}^{\infty} \frac{\mathrm{Im}(R(s))}{s^2} ds = \frac{6\mu}{\pi \Delta q^2} \sum_{j=0}^{P} C_j \int_{-1}^{1} \frac{T_j(x)dx}{\left(x + 1 + q_{\rm th}^2/\Delta q^2\right)^2}$$

with $\Delta q^2 = (q_{\rm asy}^2 - q_{\rm th}^2)/2$

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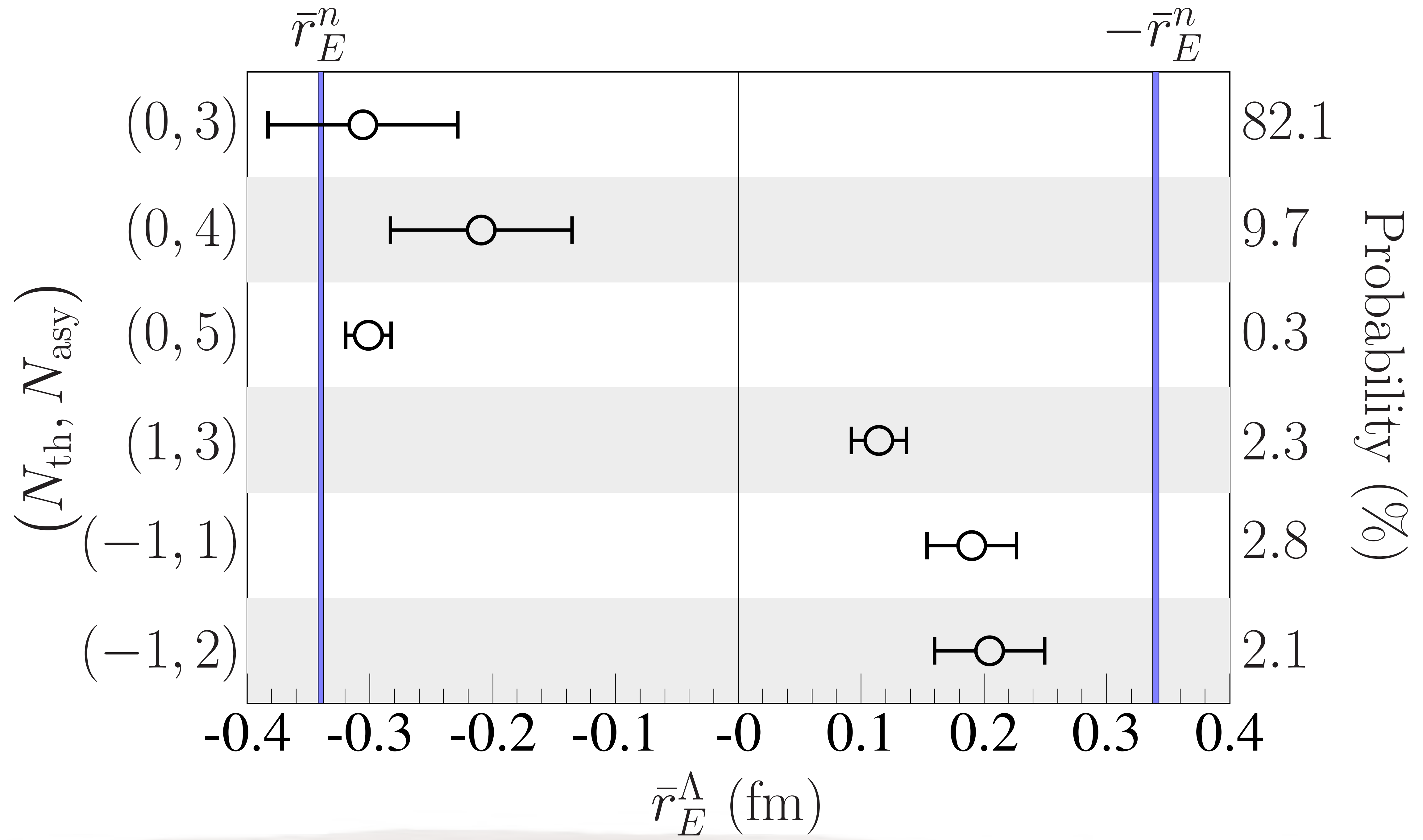
Λ's charge radii

The neutron has a negative squared charge radius: $\langle r^2_{n,E} \rangle = -\,0.1155 \pm 0.0017\,\mathrm{fm}^2$

To have a better understanding of the linear extension of the baryon we define

$$\bar{r}_E \equiv \mathrm{Sign}\left(\langle r_E^2 \rangle\right)\sqrt{\left|\langle r_E^2 \rangle\right|}$$



The fact that the most probable values of $\bar{r}^\Lambda_E$ are compatible with $\bar{r}^n_E$ can be interpreted in terms of the amount of **time** that the valence quarks of the same charge spend at a certain **distance from the center** of the baryon

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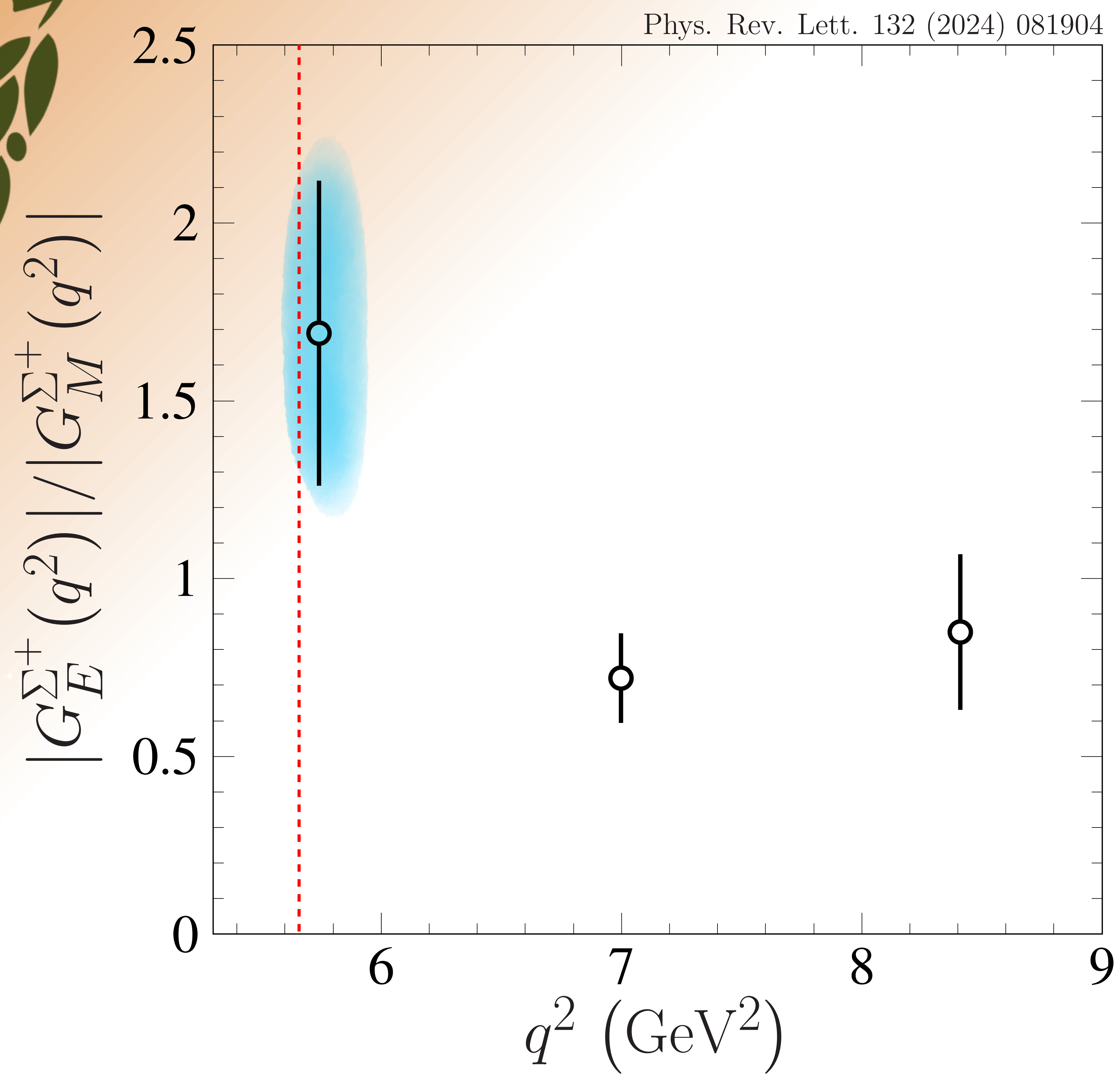
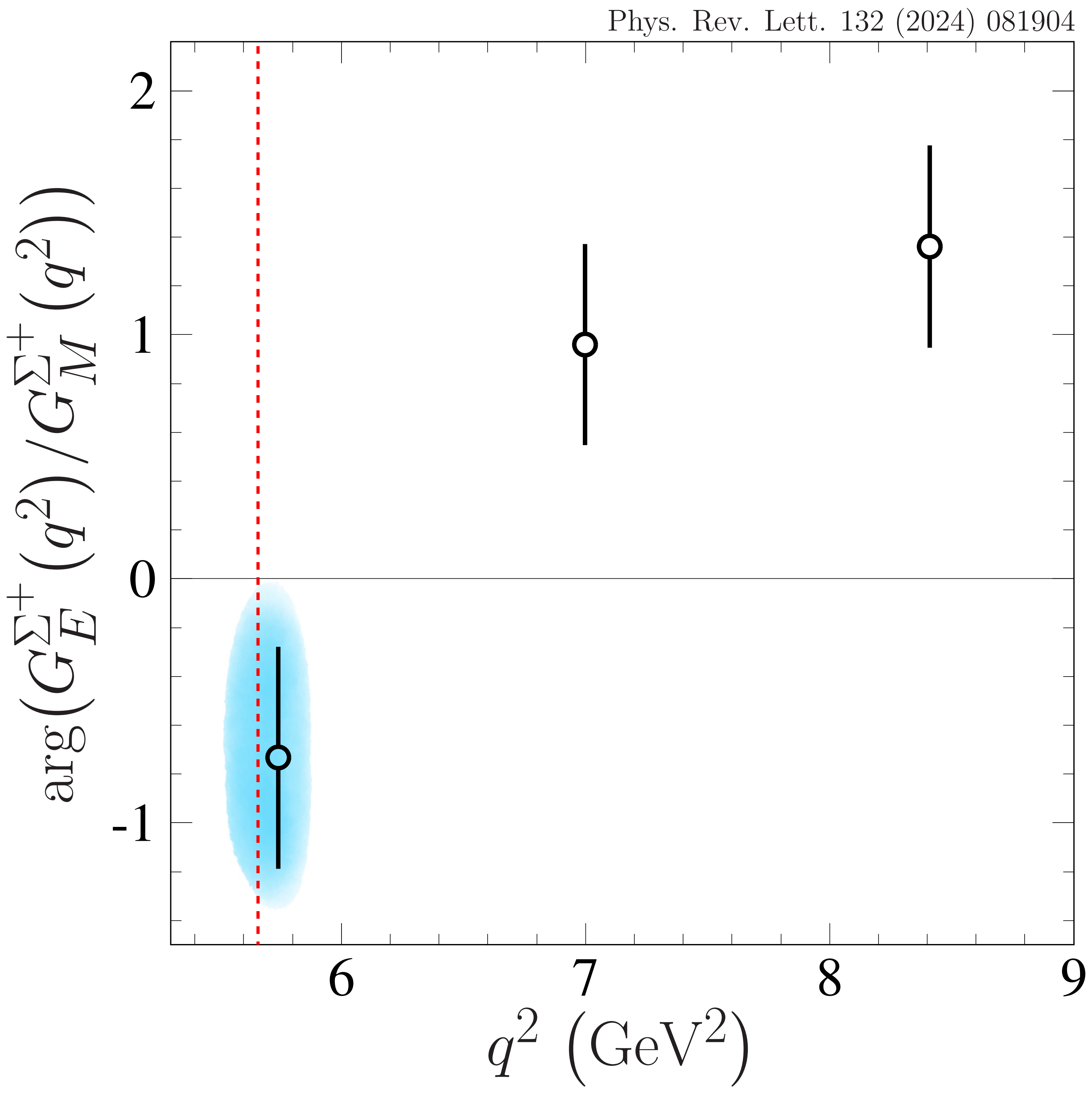
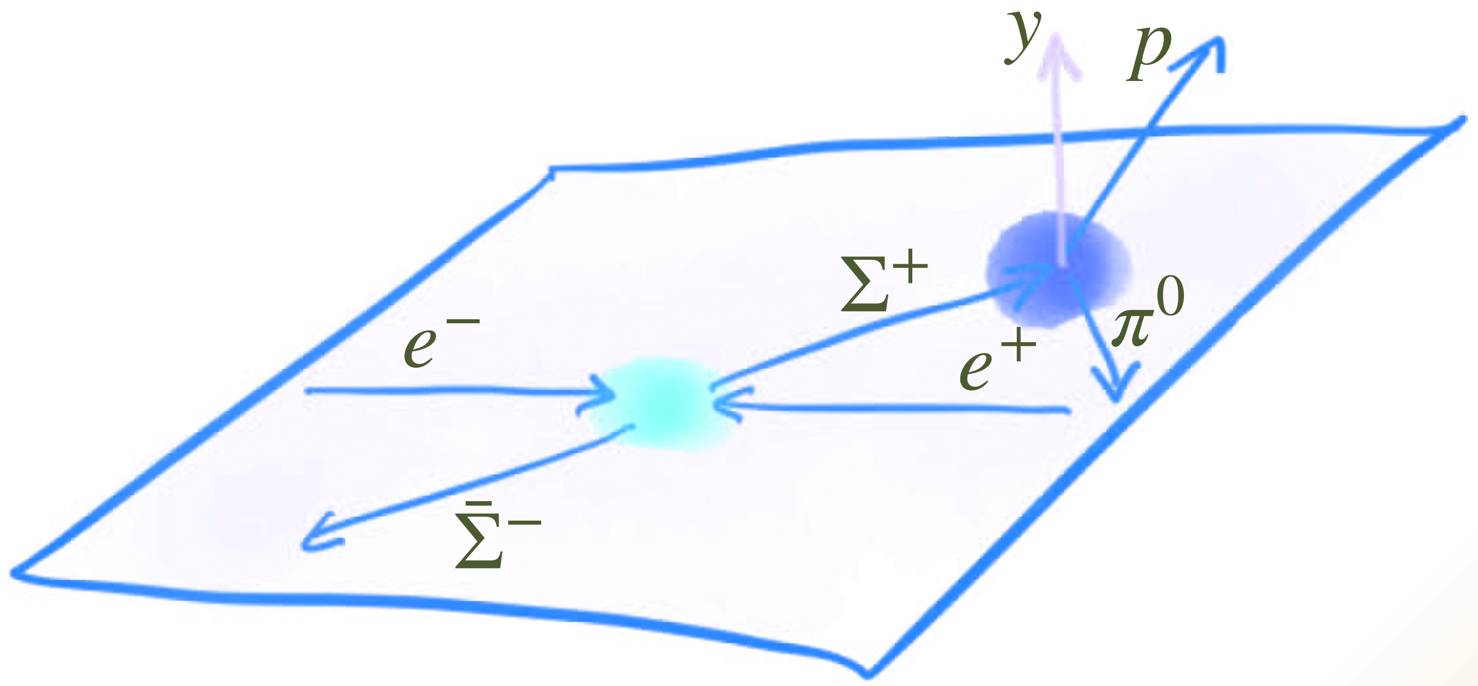
Polarization on Σ^+







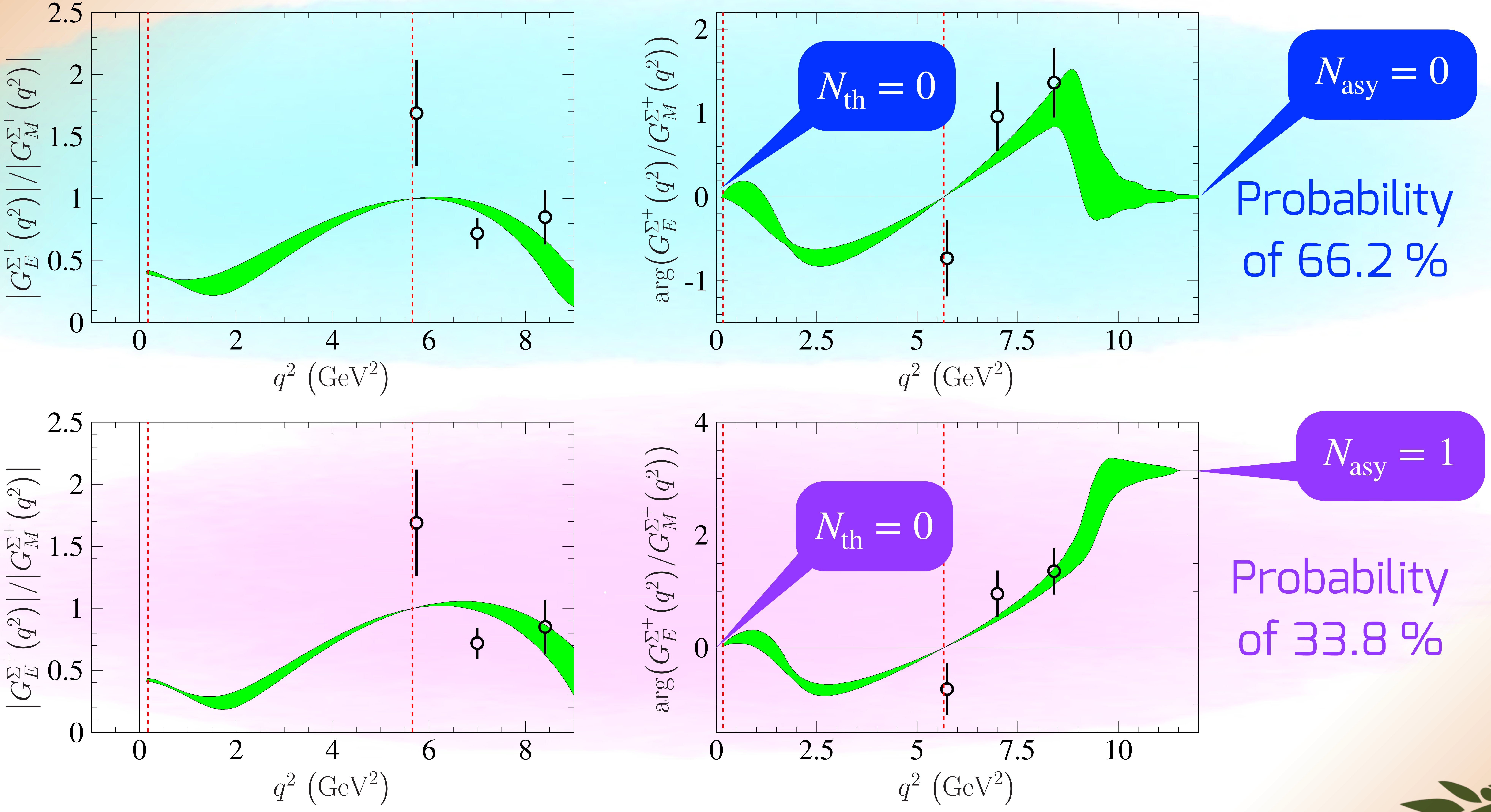
Anomalous threshold value of

$R_{\Sigma^+}(q^2) = G_E^{\Sigma^+}(q^2)/G_M^{\Sigma^+}(q^2)$?

$$\mathscr{P}_y^{\Sigma^+} = -\frac{2M_{\Sigma^+}\sqrt{q^2}\,\sin(2\theta)\left|G_E^{\Sigma^+}/G_M^{\Sigma^+}\right|\sin\left(\arg\left(G_E^{\Sigma^+}/G_M^{\Sigma^+}\right)\right)}{q^2\left(1+\cos^2(\theta)\right)+4M_{\Sigma^+}^2\left|G_E^{\Sigma^+}/G_M^{\Sigma^+}\right|^2\sin^2(\theta)}$$

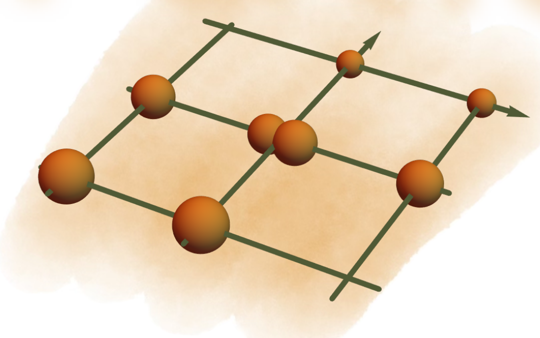
Σ^+ results

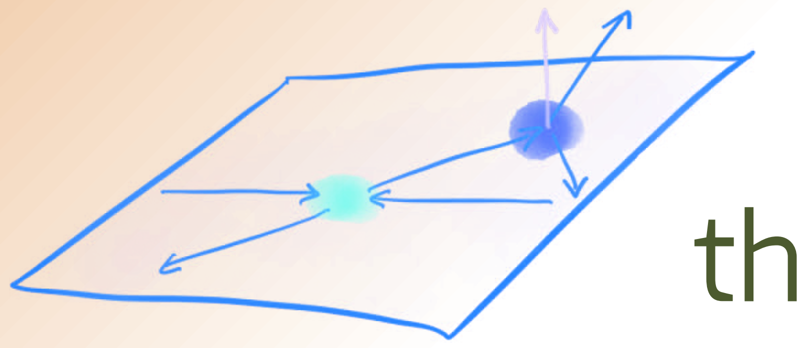
- Still **unstable** because of lack of data and unwanted threshold value
- Best for now: Chebyshev polynomial with $P = 4$ and $\tau_{\text{curv}} \simeq 10^{-1}$



Considerations

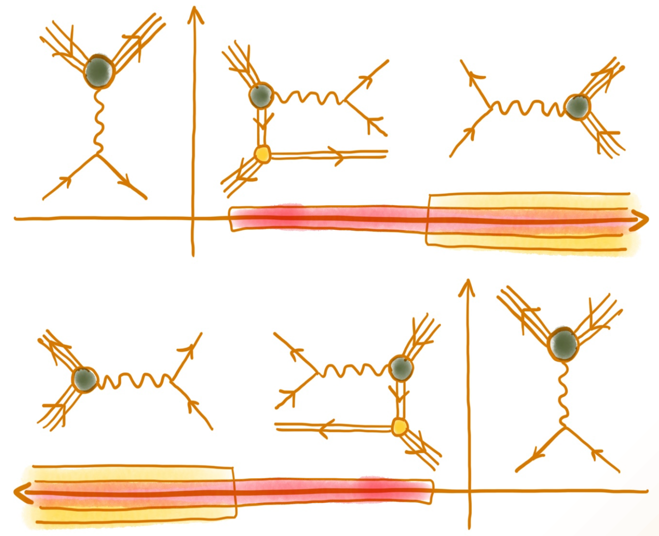
BESIII worked and is working hard to measure **hyperons' strong**, **electromagnetic** and **strong-electromagnetic** dynamics

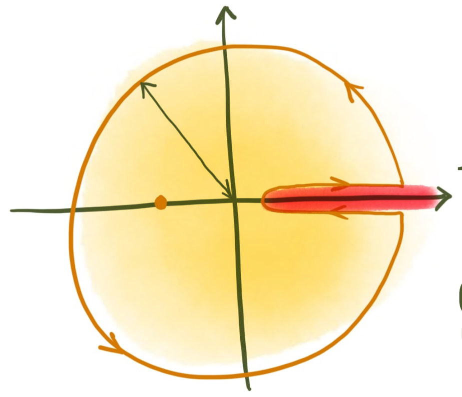




Hyperons' weak decays allow to access **polarization observables** and through them the **complex structure** of their electromagnetic dynamics

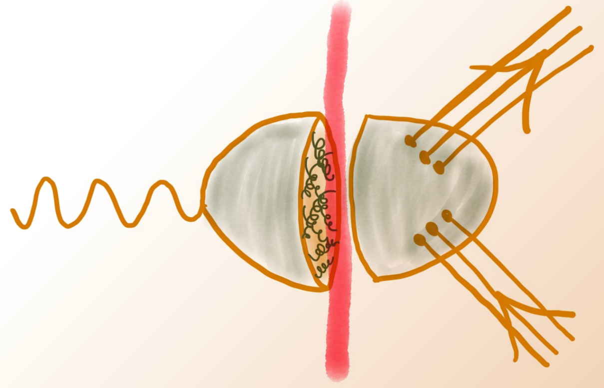
A **dispersive procedure** based on first principles allows to extract information from data on **moduli** and **phases** of form factors' ratios





Λ: stable information on the presence of **two** additional non trivial **space-like zeros** are obtained together with clues in its charge radius

Σ^+: first results are promising and could help in understanding the **unexpected threshold behavior.** New data will be crucial



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The end
of the beginning...

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