TMD FACTORIZATION AT NLP

Ignazio Scimemi

In collaboration with M. Jaarsma, O. Del Rio, W. Waalewijn, hep-ph/2507.03072 and A. Vladimirov, V. Moos, *JHEP* 01 (2022) 110

MINISTERIO DE CIENCIA E INNOVACIÓN
AGENCIA ESTATAL DE INVESTIGACIÓN
PID2022-136510NB-C31,
Comunidad de Madrid
2020-T1/TIC-20204, 2022-T1/TIC-24024
Comisión Europea
Marie Skłodowska-Curie Actions – Staff Exchanges, HORIZON-MSCA-2023-SE-01-101182937-HeI

OUTLINE

- FACTORIZATION OF CURRENTS
- FACTORIZATION OF HADRONIC TENSOR
- STARTING PHENOMENOLOGY
- FACTORIZATION:SCET vs BFM vs..
- ..

FACTORIZATION OF THE CROSS SECTION

$$W^{\mu\nu}_{\mathrm{DY}} = \int \frac{d^4y}{(2\pi)^4} e^{-i(yq)} \sum_X \langle p_1, p_2 \,|\, J^{\mu\dagger}(y) \,|\, X \rangle \langle X \,|\, J^{\nu}(0) \,|\, p_1, p_2 \rangle$$

$$W^{\mu\nu}_{\mathrm{SIDIS}} = \int \frac{d^4y}{(2\pi)^4} e^{-i(yq)} \sum_X \langle p_1 \,|\, J^{\mu\dagger}(y) \,|\, p_2, X \rangle \langle p_2, X \,|\, J^{\nu}(0) \,|\, p_1 \rangle$$

In order to arrive to some factorization we need some kinematical limits

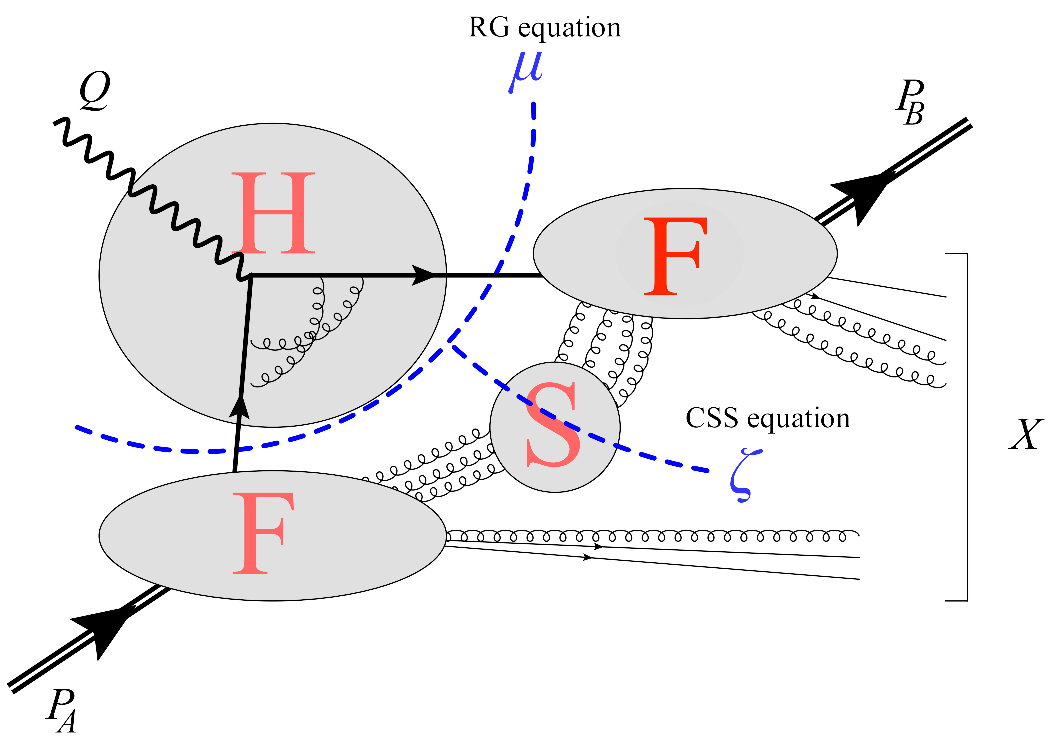
$$Q^2 \gg \Lambda^2, \qquad Q^2 \gg \mathbf{q_T}^2 = \text{fixed}$$

$$Q^2 = q^2 = 2q^+q^- - \mathbf{q_T}^2 \quad q_\mu W^{\mu\nu} = 0$$

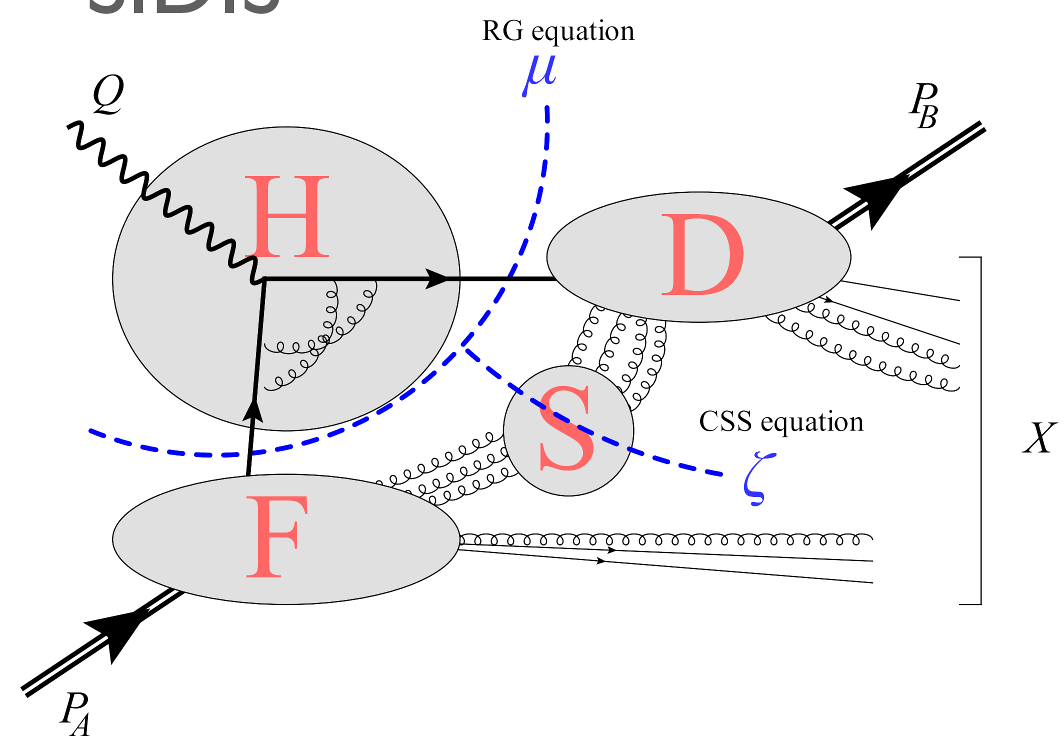
We would also like to be consistent with transversality of the hadronic tensor

BIG PICTURE I

DY



SIDIS



The μ-scale allows to separate hard interactions from the rest.
We need another scale to divide soft interactions

There are several ways to understand this separation:

Diagrammatic (Collins), SCET, Background field method

BASICS: MODES AND EXPANSIONS

$\lambda \sim q_T/Q$ Expansion parameter

We have several modes, fixed by momenta

$$n\text{-collinear} \sim (\lambda^2, 1, \lambda)\,, \quad \bar{n}\text{-collinear} \sim (1, \lambda^2, \lambda)\,, \quad \text{soft} \sim (\lambda, \lambda, \lambda)$$

$$\partial^\mu \phi \sim (\lambda^2, 1, \lambda)\phi\,, \quad \partial^\mu \phi \sim (1, \lambda^2, \lambda)\phi\,, \quad \partial^\mu \phi \sim (\lambda, \lambda, \lambda)\phi$$

$$A^\nu \sim (\lambda^2, 1, \lambda)\,, \quad A^\nu \sim (1, \lambda^2, \lambda)\,, \quad A^\nu \sim (\lambda, \lambda, \lambda)$$

The explicit Lagrangian formulation differs slightly among authors in the literature:

- *SCET (Becher, Neubert; Echevarria, I.S., Idilbi; Chiu et al…, Ebert et al., 2011-2022),*
- *Background method without soft modes (Vladimirov, Moos, I.S., 2022),*
- ***Background method with soft modes (Del Rio, Jaarsma, I.S. ,Waalewijn, arXiv:2507.03072).***

BACKGROUND FIELD METHOD

BFM has been recently re-discovered to prove TMD factorization at NLP (Vladimirov, Scimemi, Moos, JHEP 01 (2022) 110). BFM is an extension of QCD which is recovered when dynamical fields vanish

Dynamical Fields

$$|\Psi\rangle = \Psi[\phi;\phi;\phi]\,|0\rangle$$

This statement however must be refined:

- *Naive calculations of collinear modes overlap. We need to understand **zero-bin subtraction** to define TMD*
- *The **zero-bin subtraction** is rapidity-regulator dependent*
- *At LP we partially include soft radiation into TMD definitions to set it free of rapidity divergences*
- *At NLP we find also special rapidity or endpoint divergences: Their cancellation is achieved at the level of hadronic tensor*

SCET

SCET proves that different modes do not interact at the Lagrangian level, based on the "method of regions" (Beneke, Smirnov). The basic concept is that every perturbative integral can be reconstructed as a sum of integrals that contain only fields of a single mode.

$$|\Psi\rangle = \Psi[\phi;\phi;\phi]|0\rangle = |\Psi_c\rangle|\Psi_s\rangle|\Psi_{\bar{c}}\rangle$$

This statement however must be refined:

- *Soft modes are explicit*
- ***The zero-bin subtraction is rapidity-regulator dependent and related to soft fields***
- ***At NLP we find also special rapidity or endpoint divergences: Their cancellation is achieved at the level of hadronic tensor***
- *The NLP is currently achieved using label formalism in momentum space (Ebert, Gao, Stewart, JHEP 06 (2022) 007)*

BACKGROUND FIELD METHOD WITH SOFT MODES

The only way to check the agreement is to introduce soft modes in BFM

Dynamical Fields

$$|\Psi\rangle = \Psi[\phi;\phi;\phi;\phi]\,|0\rangle$$

What we discovered:

- *zero-bin subtraction can be associated with soft modes explicitly (like in SCET)*
- *TMD definitions at LP agrees with literature*
- *At NLP we find also special rapidity or endpoint divergences: **The cancellation of these divergences is achieved at the level of matrix elements** (I.e. without evoking a cancellation between collinear and anticollinear sectors): we have a proper definition of factorized higher twists*
- ***The agreement between BFM and SCET at the moment is valid only at one loop.** It should be checked at higher orders*
- ***Pheno: jets in final states***

BFM+S

All fields are re-written is its components

$$\psi = \psi + \psi + \psi + \varphi\,, \qquad A^\mu = A^\mu + A^\nu + A^\mu + B^\mu\,.$$

This produced a sum of terms for the QCD action with replicas for each mode

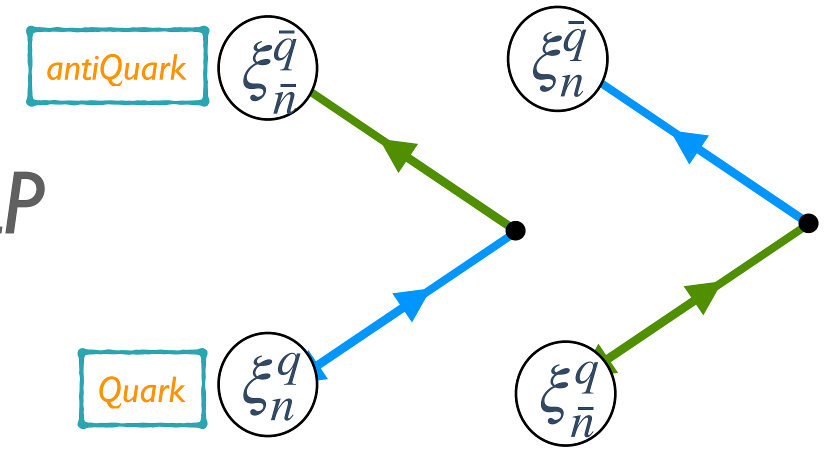
$$S_{\rm QCD}[\bar\psi,\psi,A] = S_{\rm QCD}[\bar\psi,\psi,A] + S_{\rm QCD}[\bar\psi,\psi,A] + S_{\rm QCD}[\bar\psi,\psi,A] + S_{\rm QCD}[\bar\varphi,\varphi,B] + S_{\rm int}[\bar\varphi,\varphi,B;\bar\psi,\psi,A;\bar\psi,\psi,A;\bar\psi,\psi,A]$$

All connected graphs are deduced from

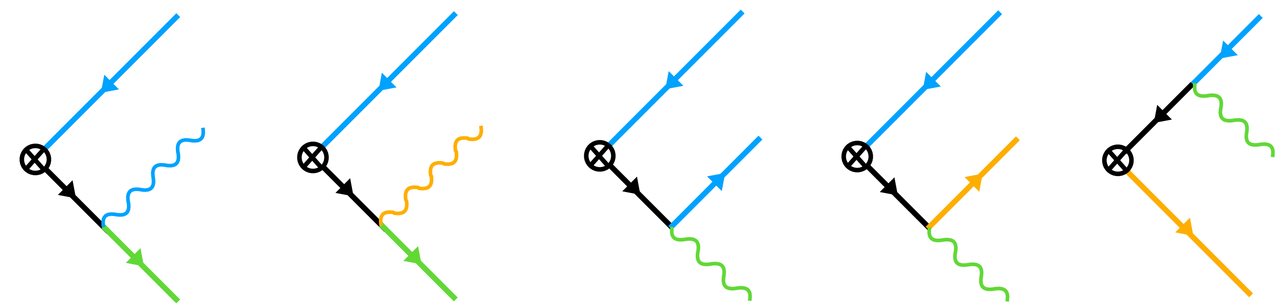
$$e^{{\rm i}S_{\rm eff}[\bar\psi,\psi,A;\bar\psi,\psi,A;\bar\psi,\psi,A]} = e^{{\rm i}S_{\rm QCD}[\bar\psi,\psi,A]}e^{{\rm i}S_{\rm QCD}[\bar\psi,\psi,A]}e^{{\rm i}S_{\rm QCD}[\bar\psi,\psi,A]} \times \int \mathcal{D}\bar\varphi\,\mathcal{D}\varphi\,\mathcal{D}B\, e^{{\rm i}S_{\rm int}[\bar\varphi,\varphi,B;\bar\psi,\psi,A;\bar\psi,\psi,A;\bar\psi,\psi,A]}e^{{\rm i}S_{\rm QCD}[\bar\varphi,\varphi,B]}\,.$$

BFM+S CURRENT OPERATOR

LP



NLP



Step1: current power expansion $J^\mu_{\rm eff}(0) = \left[J^\mu_{\rm eff}(0)\right]^{(2)} + \left[J^\mu_{\rm eff}(0)\right]^{(2.5)} + \left[J^\mu_{\rm eff}(0)\right]^{(3)} + \mathcal{O}(\lambda^{7/2})$

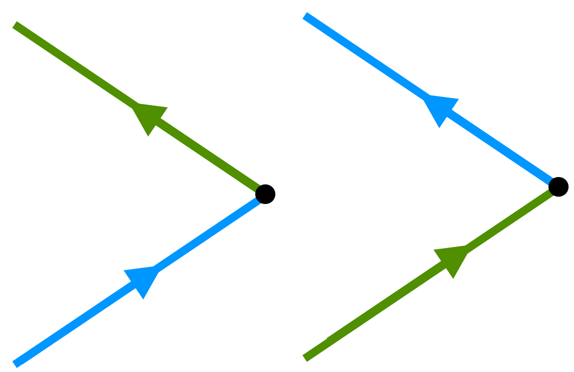
$\left[J^\mu_{\rm eff}(0)\right]^{(k)} \sim \lambda^k$.

$$
\begin{aligned}
J^\mu_{\rm eff}(0) = {} & \bar\psi\gamma^\mu\left[1 - g\frac{1}{\mathrm{i}\partial^-}A^- + g\frac{1}{\mathrm{i}\partial^+}A^+ - g\frac{1}{\mathrm{i}\partial^-}A^- + g\frac{1}{\mathrm{i}\partial^+}A^+\right]\psi + (n\leftrightarrow\bar n) \\
& + g\Bigg\{-n^\mu\bar\psi\left(\not{A}_T - \frac{\mathrm{i}\not{\partial}_T}{\mathrm{i}\partial^-}A^-\right)\frac{1}{\mathrm{i}\partial^+}\psi - n^\mu\bar\psi\left(\not{A}_T - \frac{\mathrm{i}\not{\partial}_T}{\mathrm{i}\partial^-}A^-\right)\frac{1}{\mathrm{i}\partial^+}\psi \\
& \quad - \bar\psi\gamma^\mu_T\left(\frac{1}{\mathrm{i}\partial^-}A^\rho_T - \frac{\mathrm{i}\partial^\rho_T}{(\mathrm{i}\partial^-)^2}A^-\right)\frac{\mathrm{i}\partial^\rho_T}{\mathrm{i}\partial^+}\psi - \frac{1}{2}\bar\psi\gamma^\mu_T\left(\frac{\mathrm{i}\not{\partial}_T}{\mathrm{i}\partial^-}\not{A}_T - \frac{(\mathrm{i}\not{\partial}_T)^2}{(\mathrm{i}\partial^-)^2}A^-\right)\frac{1}{\mathrm{i}\partial^+}\psi \\
& \quad - \frac{1}{2}\bar\psi\gamma^\mu_T\gamma^-\left(\not{A}_T - \frac{\mathrm{i}\not{\partial}_T}{\mathrm{i}\partial^+}A^+\right)\frac{1}{\mathrm{i}\partial^-}\psi - \frac{1}{2}\bar\psi\gamma^\mu_T\gamma^-\left(\not{A}_T - \frac{\mathrm{i}\not{\partial}_T}{\mathrm{i}\partial^+}A^+\right)\frac{1}{\mathrm{i}\partial^-}\psi \\
& \quad + (n\leftrightarrow\bar n) + \mathrm{h.c.}\Bigg\} + \mathcal{O}(g^2,\lambda^{7/2})\,.
\end{aligned}
$$

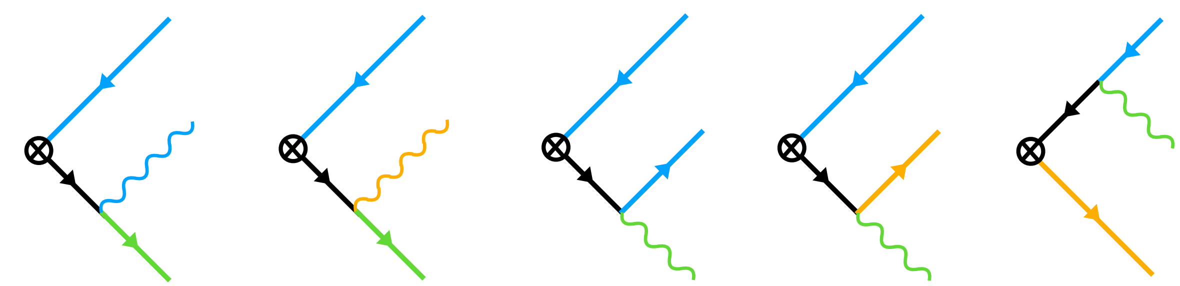
$$
\begin{aligned}
\frac{1}{(\partial^-)^k}f(y) &= \frac{(-1)^{k-1}}{\Gamma(k)}\int_{\bar L}^0 \mathrm{d}w^+\,(w^+)^{k-1}\,f(y+w^+\bar n)\,,\\
\frac{1}{(\partial^+)^k}g(y) &= \frac{(-1)^{k-1}}{\Gamma(k)}\int_{L}^0 \mathrm{d}w^-\,(w^-)^{k-1}\,f(y+w^- n)\,.
\end{aligned}
$$

BFM+S CURRENT OPERATOR

LP



NLP



Step II: Wilson lines insertion (we use δ-regulator)

$$\chi = \frac{\gamma^+\gamma^-}{2} W^\dagger \psi\,, \qquad \mathcal{A}_T^\rho = W^\dagger \left[\mathrm{i}D_T^\rho, W\right], \qquad \Psi_n = S_n^\dagger \psi\,, \qquad \mathcal{A}_{T,n}^\rho = S_n^\dagger \left[\mathrm{i}D_T^\rho, S_n\right],$$

$$\chi = \frac{\gamma^-\gamma^+}{2} W^\dagger \psi\,, \qquad \mathcal{A}_T^\rho = W^\dagger \left[\mathrm{i}D_T^\rho, W\right]. \qquad \Psi_{\bar{n}} = S_{\bar{n}}^\dagger \psi\,, \qquad \mathcal{A}_{T,\bar{n}}^\rho = S_{\bar{n}}^\dagger \left[\mathrm{i}D_T^\rho, S_{\bar{n}}\right].$$

Step III: hard coefficients. C-,P-,T-symmetries, RPI, current conservation,

$$\mathrm{i}\partial_\mu J^\mu_{\mathrm{eff}} = \mathcal{O}(\lambda^{\frac{7}{2}})\,.$$

impose relations among the hard coefficients..

BFM+S CURRENT OPERATOR

LP

Hard coefficient — Soft WL

$$\left[J^{\mu}_{\text{eff}}(0)\right]^{(2)} = (\gamma^{\mu}_{T})_{\alpha\beta}\int \mathrm{d}y^{+}\,\mathrm{d}y^{-}\,C_1(y^{+},y^{-})\,\bar{\chi}_{\alpha}(y^{+}\bar{n})\,S^{\dagger}_{n}S_{\bar{n}}\,\chi_{\beta}(y^{-}n)\ +\mathrm{h.c.}$$

Fermion+WL — Fermion+WL

NLP

$$\begin{aligned}
\left[J^{\mu}_{\text{eff}}(0)\right]^{(3)} = &-n^{\mu}\int \mathrm{d}y^{+}\,\mathrm{d}y^{-}\,C_1(y^{+},y^{-})\,\bar{\chi}(y^{+}\bar{n})\,S^{\dagger}_{n}S_{\bar{n}}\,\frac{\mathrm{i}\not{\partial}_{T}}{\mathrm{i}\partial^{+}}\chi(y^{-}n)\\
&-\int \mathrm{d}y^{+}\,\mathrm{d}y^{-}\,C_1(y^{+},y^{-})\,\bar{\chi}(y^{+}\bar{n})\,S^{\dagger}_{n}S_{\bar{n}}\mathcal{A}_{T,\bar{n}}\,\frac{n^{\mu}}{\mathrm{i}\partial^{+}}\chi(y^{-}n)\\
&-(\gamma^{\mu}_{T})_{\alpha\beta}\int \mathrm{d}y^{+}\,\mathrm{d}y^{-}\,C_1(y^{+},y^{-})\,\bar{\chi}_{\alpha}(y^{+}\bar{n})\,S^{\dagger}_{n}S_{\bar{n}}\,\frac{1}{\mathrm{i}\partial^{-}}\mathcal{A}^{\rho}_{T,\bar{n}}\,\frac{\mathrm{i}\partial^{\rho}_{T}}{\mathrm{i}\partial^{+}}\chi_{\beta}(y^{-}n)\\
&+\int \mathrm{d}y_1^{+}\,\mathrm{d}y_2^{+}\,\mathrm{d}y^{-}\,C_2(\{y_1^{+},y_2^{+}\},y^{-})\\
&\quad\times\left(\frac{\bar{n}^{\mu}}{\mathrm{i}\partial^{-}}-\frac{n^{\mu}}{\mathrm{i}\partial^{+}}\right)\bar{\chi}(y_1^{+}\bar{n})\mathcal{A}_{T}(y_2^{+}\bar{n})\,S^{\dagger}_{n}S_{\bar{n}}\,\chi(y^{-}n)\\
&-\frac{1}{2}(\gamma^{\mu}_{T}\gamma^{\rho}_{T}\gamma^{\sigma}_{T})_{\alpha\beta}\int \mathrm{d}y^{+}\,\mathrm{d}y^{-}\,\mathrm{d}z^{+}\,C_3(y^{+},z^{+},y^{-})\\
&\quad\times\bar{\chi}_{\alpha}(y^{+}\bar{n})\,S^{\dagger}_{n}S_{\bar{n}}\,\frac{\mathrm{i}\partial^{\rho}_{T}}{\mathrm{i}\partial^{-}}\mathcal{A}^{\sigma}_{T,\bar{n}}(z^{+}\bar{n})\,\frac{1}{\mathrm{i}\partial^{+}}\chi_{\beta}(y^{-}n)\\
&-\frac{1}{2}(\gamma^{\mu}_{T}\gamma^{-}\gamma^{\rho}_{T})_{\alpha\beta}\int \mathrm{d}y_1^{+}\,\mathrm{d}y_2^{+}\,\mathrm{d}y^{-}\,C_5(\{y_1^{+},y_2^{+}\},y^{-})\\
&\quad\times\bar{\chi}_{\alpha}(y_1^{+}\bar{n})\,S^{\dagger}_{n}S_{\bar{n}}\,\mathcal{A}^{\rho}_{T}(y^{-})\,S^{\dagger}_{\bar{n}}S_{n}\,\frac{1}{\mathrm{i}\partial^{-}}\chi_{\beta}(y_2^{+}\bar{n})\\
&+(n\leftrightarrow\bar{n})+\mathrm{h.c.}
\end{aligned}$$

Many of these contributions are zero when evaluated on states

BFM+S: HADRONIC TENSOR (UNSUBTRACTED)

NLP:

Kinematic PC

$$[\mathcal{W}^{\mu\nu}]^{(3,2)}_{Cn} = \left(-\frac{\bar{n}^\mu}{q^-}\right)\int \frac{\mathrm{d}^2 b}{(2\pi)^2}\, e^{+\mathrm{i}q_T\cdot b_T}\int \frac{\mathrm{d}b^+}{2\pi}\, e^{+\mathrm{i}q^- b^+}\int \frac{\mathrm{d}b^-}{2\pi}\, e^{+\mathrm{i}q^+ b^-}$$
$$\times (\gamma_T^\rho)_{\alpha\beta}(\gamma_T^\nu)_{\gamma\delta}\int \mathrm{d}y^+\,\mathrm{d}y^-\, C_1(y^+,y^-)\int \mathrm{d}z^+\,\mathrm{d}z^-\, C_1^*(z^+,z^-)$$
$$\times \langle P|\,\bar{\chi}_{i,\alpha}(b_T + b^- n + y^- n)\,|X\rangle\langle X|\,\chi_{l,\delta}(z^- n)\,|P\rangle$$
$$\times \langle 0|\,\mathrm{i}\partial_T^\rho \chi_{j,\beta}(b_T + b^+\bar{n} + y^+\bar{n})\,|p,X\rangle\langle p,X|\,\bar{\chi}_{k,\gamma}(z^+\bar{n})\,|0\rangle$$
$$\times \langle 0|\,\left[S_{\bar{n}}^\dagger S_n(b_T)\right]_{ij}|X\rangle\langle X|\left[S_n^\dagger S_{\bar{n}}(0)\right]_{kl}|0\rangle\,.$$

NLP:

Soft gluon PC

$$[\mathcal{W}^{\mu\nu}]^{(3,2)}_{En} = \left(-\frac{\bar{n}^\mu}{q^-}\right)\int \frac{\mathrm{d}^2 b}{(2\pi)^2}\, e^{+\mathrm{i}q_T\cdot b_T}\int \frac{\mathrm{d}b^+}{2\pi}\, e^{+\mathrm{i}q^- b^+}\int \frac{\mathrm{d}b^-}{2\pi}\, e^{+\mathrm{i}q^+ b^-}$$
$$\times (\gamma_T^\rho)_{\alpha\beta}(\gamma_T^\nu)_{\gamma\delta}\int \mathrm{d}y^+\,\mathrm{d}y^-\, C_1(y^+,y^-)\int \mathrm{d}z^+\,\mathrm{d}z^-\, C_1^*(z^+,z^-)$$
$$\times \langle P|\,\bar{\chi}_{i,\alpha}(b_T + b^- n + y^- n)\,|X\rangle\langle X|\,\chi_{l,\delta}(z^- n)\,|P\rangle$$
$$\times \langle 0|\,\chi_{j,\beta}(b_T + b^+\bar{n} + y^+\bar{n})\,|p,X\rangle\langle p,X|\,\bar{\chi}_{k,\gamma}(z^+\bar{n})\,|0\rangle$$
$$\times \langle 0|\,\left[S_{\bar{n}}^\dagger S_n \mathcal{A}^\rho_{T,n}(b_T)\right]_{ij}|X\rangle\langle X|\left[S_n^\dagger S_{\bar{n}}(0)\right]_{kl}|0\rangle\,,$$

NEW!

NLP:

Genuine PC

$$[\mathcal{W}^{\mu\nu}]^{(3,2)}_{Dn} = \left(\frac{n^\mu}{q^+}-\frac{\bar{n}^\mu}{q^-}\right)\int \frac{\mathrm{d}^2 b}{(2\pi)^2}\, e^{+\mathrm{i}q_T\cdot b_T}\int \frac{\mathrm{d}b^+}{2\pi}\, e^{+\mathrm{i}q^- b^+}\int \frac{\mathrm{d}b^-}{2\pi}\, e^{+\mathrm{i}q^+ b^-}$$
$$\times (\gamma_T^\rho)_{\alpha\beta}(\gamma_T^\nu)_{\gamma\delta}\int \mathrm{d}y^-\,\mathrm{d}y_1^+\mathrm{d}y_2^+\, C_2(\{y_1^+,y_2^+\},y^-)\int \mathrm{d}z^-\,\mathrm{d}z^+\, C_1^*(z^-,z^+)$$
$$\times \langle P|\,\bar{\chi}_{i,\alpha}(b_T + b^- n + y^- n)\,|X\rangle\langle X|\,\chi_{l,\delta}(z^- n)\,|P\rangle$$
$$\times \langle 0|\,\left[\mathcal{A}_T^\rho(b_T + b^+\bar{n} + y_2^+\bar{n})\chi(b_T + b^+\bar{n} + y_1^+\bar{n})\right]_{j,\beta}|p,X\rangle\langle p,X|\,\bar{\chi}_{k,\gamma}(z^+\bar{n})\,|0\rangle$$
$$\times \langle 0|\,\left[S_{\bar{n}}^\dagger S_n(b_T)\right]_{ij}|X\rangle\langle X|\left[S_n^\dagger S_{\bar{n}}(0)\right]_{kl}|0\rangle\,,$$

SUBTRACTION METHOD STEPS

- SPLIT THE COLLINEAR BACKGROUND FIELDS UP INTO A PURE COLLINEAR FIELD AND A BACKGROUND FIELD THAT CONTAINS THE OVERLAP WITH THE SOFT REGION.

$$S_{\mathrm{QCD}}[\phi + \phi] = \int \mathrm{d}^d x \left[\bar{\psi}(x) \left(i\not{\partial} + g\not{A}(x) + g\not{A}(x) \right) \psi(x) + \bar{\psi}(x) g\not{A}(x) \psi(x) + \bar{\psi}(x) g\not{A}(x) \psi(x) \right| + \text{field strength} + S_{\mathrm{QCD}}[\phi]$$

- REDEFINE THE PURE COLLINEAR FIELDS SUCH THAT THE LEADING-POWER INTERACTIONS WITH THE OVERLAP BACKGROUND FIELD ARE REMOVED.

$$\psi \to S_n \psi + \psi \qquad\qquad A_T^\mu \to S_n A_T{}^\mu S_n^\dagger + A_T{}^\mu$$

- SEPARATE THE COLLINEAR MATRIX ELEMENTS INTO PURE COLLINEAR AND OVERLAP PARTS

- INVERT THE RELATIONS FROM PREVIOUS STEP TO OBTAIN THE PURE-COLLINEAR MATRIX ELEMENTS.

FACTORIZATION AT LP

TMD definitions free of rapidity divergences

$$\left[\mathcal{F}_{q,11}^{\text{bare}}(x, b_T, \zeta)\right]_{\delta\alpha} = \int \frac{\mathrm{d}b^-}{2\pi}\, e^{+\mathrm{i}b^- q^+}\, \frac{\langle P|\, \bar{\chi}_{i,\alpha}(b_T + b^- n)\, |X\rangle\, \langle X|\, \chi_{i,\delta}(0)\, |P\rangle}{\sqrt{S\left(b_T, \zeta(\delta^+/q^+)^2\right)}},$$

$$\left[\mathcal{D}_{q,11}^{\text{bare}}(z, b_T, \bar{\zeta})\right]_{\beta\gamma} = \int \frac{\mathrm{d}b^+}{2\pi}\, e^{+\mathrm{i}b^+ q^-}\, \frac{\langle 0|\, \chi_{i,\beta}(b_T + b^+ \bar{n})\, |p, X\rangle\, \langle p, X|\, \bar{\chi}_{i,\gamma}(0)\, |0\rangle}{\sqrt{S\left(b_T, \bar{\zeta}(\delta^-/q^-)^2\right)}},$$

TMD factorized hadronic tensor

$$W_{\text{LP}}^{\mu\nu}(q) = \left[\mathcal{W}_{\text{sub.}}^{\mu\nu}\right]_A^{(2,2)} = \frac{1}{N_c}(\gamma_T^\mu)_{\alpha\beta}(\gamma_T^\nu)_{\gamma\delta}\, H_1(q^2) \int \frac{\mathrm{d}^2 b_T}{(2\pi)^2}\, e^{\mathrm{i}b_T\cdot q_T} \times \sum_q \left\{ \left[\mathcal{F}_{q,11}\right]_{\delta\alpha} \left[\mathcal{D}_{q,11}\right]_{\beta\gamma} + \left[\mathcal{F}_{\bar{q},11}\right]_{\beta\gamma} \left[\mathcal{D}_{\bar{q},11}\right]_{\delta\alpha} \right\}.$$

SUBTRACTION METHOD AT NLP: KINEMATIC PC

Useful identities: soft factor vs CS kernel (K) and combining terms..

$$S^{\text{bare}}(b_T, 2\delta^+\delta^-) = \exp\Big[K^{\text{bare}}(b_T)\log(2\delta^+\delta^-) + s^{\text{bare}}(b_T)\Big]$$

$$\frac{\partial_T^\rho S^{\text{bare}}(b_T,\Delta_1^2)}{S^{\text{bare}}(b_T,\Delta_1^2)} - \frac{\partial_T^\rho S^{\text{bare}}(b_T,\Delta_2^2)}{S^{\text{bare}}(b_T,\Delta_2^2)} = \partial_T^\rho K^{\text{bare}}(b_T)\ln\left(\frac{\Delta_1^2}{\Delta_2^2}\right).$$

…one obtains the kinematic power corrections. Here only derivatives of LP functions appear

$$\begin{aligned}
W^{\mu\nu}_{\text{kNLP}}(q) &= [\mathcal{W}^{\mu\nu}_{\text{sub.}}]^{(3,2)}_{C+E} + [\mathcal{W}^{\mu\nu}_{\text{sub.}}]^{(2,3)}_{C+E} \\
&= -\frac{\mathrm{i}}{N_c}\frac{1}{q^+}\left[n^\mu(\gamma_T^\rho)_{\alpha\beta}(\gamma_T^\nu)_{\gamma\delta} + n^\nu(\gamma_T^\mu)_{\alpha\beta}(\gamma_T^\rho)_{\gamma\delta}\right] H_1(q^2)\int\frac{\mathrm{d}^2b_T}{(2\pi)^2}\,e^{\mathrm{i}b_T\cdot q_T} \\
&\quad\times\sum_q\Big\{[\partial_\rho\mathcal{F}_{q,11}]_{\delta\alpha}[\mathcal{D}_{q,11}]_{\beta\gamma} + \frac{1}{2}[\partial_\rho K]\ln\left(\frac{\zeta}{\bar\zeta}\right)[\mathcal{F}_{q,11}]_{\delta\alpha}[\mathcal{D}_{q,11}]_{\beta\gamma} \\
&\qquad + [\partial_\rho\mathcal{F}_{\bar q,11}]_{\beta\gamma}[\mathcal{D}_{\bar q,11}]_{\delta\alpha} + \frac{1}{2}[\partial_\rho K]\ln\left(\frac{\zeta}{\bar\zeta}\right)[\mathcal{F}_{\bar q,11}]_{\beta\gamma}[\mathcal{D}_{\bar q,11}]_{\delta\alpha}\Big\} \\
&\quad - \frac{\mathrm{i}}{N_c}\frac{1}{q^-}\left[\bar n^\mu(\gamma_T^\rho)_{\alpha\beta}(\gamma_T^\nu)_{\gamma\delta} + \bar n^\nu(\gamma_T^\mu)_{\alpha\beta}(\gamma_T^\rho)_{\gamma\delta}\right] H_1(q^2)\int\frac{\mathrm{d}^2b_T}{(2\pi)^2}\,e^{\mathrm{i}b_T\cdot q_T} \\
&\quad\times\sum_q\Big\{[\mathcal{F}_{q,11}]_{\delta\alpha}[\partial_\rho\mathcal{D}_{q,11}]_{\beta\gamma} + \frac{1}{2}[\partial_\rho K]\ln\left(\frac{\bar\zeta}{\zeta}\right)[\mathcal{F}_{q,11}]_{\delta\alpha}[\mathcal{D}_{q,11}]_{\beta\gamma} \\
&\qquad + [\mathcal{F}_{\bar q,11}]_{\beta\gamma}[\partial_\rho\mathcal{D}_{\bar q,11}]_{\delta\alpha} + \frac{1}{2}[\partial_\rho K]\ln\left(\frac{\bar\zeta}{\zeta}\right)[\mathcal{F}_{\bar q,11}]_{\beta\gamma}[\mathcal{D}_{\bar q,11}]_{\delta\alpha}\Big\}.
\end{aligned}$$

SUBTRACTION METHOD AT NLP: ENDPOINT DIVERGENCES

$$\left[F_{21}^{\text{naive}}(x,\xi,b_T)\right]_{\delta\alpha} = \int \frac{\mathrm{d}b_1^-}{2\pi}\frac{\mathrm{d}b_2^-}{2\pi}\, e^{-\mathrm{i}\bar{\xi}xb_1^- P^+}\, e^{-\mathrm{i}\xi x b_2^- P^+} \times \langle P|\left[\bar{\chi}(b_T+b_1^- n)\mathcal{A}_T^\rho(b_T+b_2^- n)\right]_{i,\alpha}|X\rangle\langle X|\chi_{i,\delta}(0)|P\rangle\,.$$

Higher twist operators have also ED

ED appear already at LO

$$\left[F_{21}^{\text{naive}}(x,\xi,b_T)\right]_{\delta\alpha}^{\text{LO}} = a_s \frac{b_T^\sigma}{b_T^2}\theta(-\xi x)\left[\gamma_T^\sigma\gamma_T^\rho\gamma^- x - 2(1+\xi x)\frac{g_T^{\rho\sigma}}{\xi - \mathrm{i}\delta^+/xP^+}\gamma^-\right]_{\delta\alpha}$$

ED are due to soft gluon limit of tw3 distributions. They overlap with

$$\left[F_{21}^{\text{o.s.}}(x,\xi,b_T)\right]_{\delta\alpha} = \mathrm{i}xP^+\int \frac{\mathrm{d}b_1^-}{2\pi}\frac{\mathrm{d}b_2^-}{2\pi}\, e^{-\mathrm{i}\bar{\xi}xb_1^- P^+}\, e^{-\mathrm{i}\xi x b_2^- P^+}\langle P|\bar{\chi}_\alpha(b_T+b_1^- n)|X\rangle\langle X|\chi_\delta(0)|P\rangle \times \langle 0|S_{\bar{n}}^\dagger S_n(b_T)\,\mathcal{A}_{T,n}^\rho(b_T+b_2^- n)|X\rangle\langle X|S_n^\dagger S_{\bar{n}}(0)|0\rangle\,.$$

ED cancellation check LO $\quad \lim_{\xi\to 0}\xi\left(\left[F_{21}^{\text{naive}}(x,\xi,b_T)\right]_{\delta\alpha}^{\text{LO}} - \left[F_{21}^{\text{o.s.}}(x,\xi,b_T)\right]_{\delta\alpha}^{\text{LO}}\right) = 0\,,$

The final contribution to the hadronic tensor is

$$\int \mathrm{d}\xi\, H_2(\xi,Q^2)\left(F_{21}^{\text{naive}}(x,\xi,b_T) - F_{21}^{\text{o.s.}}(x,\xi,b_T)\right)D_{11}(z,b_T) = \text{finite}\,.$$

SUBTRACTION METHOD AT NLP: GENUINE PC

$$\left[\mathcal{F}^{\text{bare}}_{q,21}(x,\xi,b_T,\zeta)\right]_{\delta\alpha}$$
$$= \mathrm{i}q^+ \int \frac{\mathrm{d}b_1^-}{2\pi}\frac{\mathrm{d}b_2^-}{2\pi} e^{-\mathrm{i}\bar{\xi}b_1^- q^+} e^{-\mathrm{i}\xi b_2^- q^+} \left\{ \frac{\langle P|\left[\bar{\chi}(b_T+b_1^- n)\mathcal{A}_T(b_T+b_2^- n)\right]_{i,\alpha}|X\rangle\langle X|\chi_{i,\delta}(0)|P\rangle}{\sqrt{S(b_T,\zeta(\delta^+/q^+)^2)}} \right.$$
$$- \frac{\langle P|\left[\bar{\chi}(b_T+b_1^- n)\,\gamma_T^\rho\right]_{i,\alpha}|X\rangle\langle X|\chi_{i,\delta}(0)|P\rangle}{\sqrt{S(b_T,\zeta(\delta^+/q^+)^2)}}$$
$$\left. \times \frac{1}{N_c}\mathrm{tr}\left[\langle 0|S_{\bar{n}}^\dagger S_n(b_T)\,\mathcal{A}^\rho_{T,n}(b_T+b_2^- n)|X\rangle\langle X|S_n^\dagger S_{\bar{n}}(0)|0\rangle\right]\right\}$$

Tw3 contributions subtractions happen in 2 steps

Tw3 contributions to the hadronic tensor

$$W^{\mu\nu}_{\text{gNLP}}(q) = \left[\mathcal{W}^{\mu\nu}_{\text{sub.}}\right]^{(3,2)}_D + \left[\mathcal{W}^{\mu\nu}_{\text{sub.}}\right]^{(2,3)}_D$$
$$= \frac{\mathrm{i}}{N_c}\left[\frac{\bar{n}^\mu}{q^-} - \frac{n^\mu}{q^+}\right](\mathbb{1})_{\alpha\beta}(\gamma_T^\nu)_{\gamma\delta}\int \mathrm{d}\xi\, H_2(\xi,q^2)\int\frac{\mathrm{d}^2 b_T}{(2\pi)^2}e^{\mathrm{i}b_T\cdot q_T}$$
$$\times \sum_q \left\{ \left[\mathcal{F}_{q,21}\right]_{\delta\alpha}\left[\mathcal{D}_{q,11}\right]_{\beta\gamma} - \left[\mathcal{F}_{\bar{q},21}\right]_{\beta\gamma}\left[\mathcal{D}_{\bar{q},11}\right]_{\delta\alpha}\right.$$
$$\left. + \left[\mathcal{F}_{q,11}\right]_{\delta\alpha}\left[\mathcal{D}_{q,21}\right]_{\beta\gamma} - \left[\mathcal{F}_{\bar{q},11}\right]_{\beta\gamma}\left[\mathcal{D}_{\bar{q},21}\right]_{\delta\alpha}\right\}$$
$$+ \frac{\mathrm{i}}{N_c}\left[\frac{\bar{n}^\nu}{q^-} - \frac{n^\nu}{q^+}\right](\gamma_T^\mu)_{\alpha\beta}(\mathbb{1})_{\gamma\delta}\int \mathrm{d}\xi\, H_2^*(\xi,q^2)\int\frac{\mathrm{d}^2 b_T}{(2\pi)^2}e^{\mathrm{i}b_T\cdot q_T}$$
$$\times \sum_q \left\{ \left[\mathcal{F}_{q,12}\right]_{\delta\alpha}\left[\mathcal{D}_{q,11}\right]_{\beta\gamma} - \left[\mathcal{F}_{\bar{q},12,}\right]_{\beta\gamma}\left[\mathcal{D}_{\bar{q},11}\right]_{\delta\alpha}\right.$$
$$\left. + \left[\mathcal{F}_{q,11}\right]_{\delta\alpha}\left[\mathcal{D}_{q,12}\right]_{\beta\gamma} - \left[\mathcal{F}_{\bar{q},11}\right]_{\beta\gamma}\left[\mathcal{D}_{\bar{q},12}\right]_{\delta\alpha}\right\},$$

COMPARISON WITH LITERATURE

$$
\begin{aligned}
&[\mathcal{F}^{\text{bare}}_{q,21}(x,\xi,b_T,\zeta)]_{\delta\alpha} \\
&= \mathrm{i}q^+ \int \frac{\mathrm{d}b_1^-}{2\pi}\frac{\mathrm{d}b_2^-}{2\pi} e^{-\mathrm{i}\bar{\xi}b_1^- q^+} e^{-\mathrm{i}\xi b_2^- q^+} \Bigg\{ \frac{\langle P| \left[\bar{\chi}(b_T+b_1^- n)\mathcal{A}_T(b_T+b_2^- n)\right]_{i,\alpha} |X\rangle \langle X| \chi_{i,\delta}(0) |P\rangle}{\sqrt{S(b_T,\zeta(\delta^+/q^+)^2)}} \\
&\qquad - \frac{\langle P| \left[\bar{\chi}(b_T+b_1^- n)\,\gamma_T^\rho\right]_{i,\alpha} |X\rangle \langle X| \chi_{i,\delta}(0) |P\rangle}{\sqrt{S(b_T,\zeta(\delta^+/q^+)^2)}} \\
&\qquad \times \frac{1}{N_c}\mathrm{tr}\left[\langle 0| S_{\bar{n}}^\dagger S_n(b_T)\,\mathcal{A}^\rho_{T,n}(b_T+b_2^- n) |X\rangle \langle X| S_n^\dagger S_{\bar{n}}(0) |0\rangle\right] \Bigg\}
\end{aligned}
$$

Tw3 contributions subtractions happen in 2 steps

Rodini, Vladimirov *Phys.Rev.D* 110 (2024) 3, 034009

$$
\begin{aligned}
\mathbf{\Phi}^{[\Gamma]}_{\mu,12}(x_1,x_2,x_3,b) &= \Phi^{[\Gamma]}_{\mu,12}(x_1,x_2,x_3,b) - [\mathcal{R}_{12}\otimes\Phi_{11}]^{[\Gamma]}_\mu(x_1,x_2,x_3,b), \\
\mathbf{\Phi}^{[\Gamma]}_{\mu,21}(x_1,x_2,x_3,b) &= \Phi^{[\Gamma]}_{\mu,21}(x_1,x_2,x_3,b) - [\mathcal{R}_{21}\otimes\Phi_{11}]^{[\Gamma]}_\mu(x_1,x_2,x_3,b),
\end{aligned}
$$

where $\mathcal{R}$ are kernels that could be computed perturbatively, and $\otimes$ is an integral convolution.

The subtraction term is not associated with any matrix element, it is observed that the kernel is related to derivative of the CS kernel and deduced from their previous work at one loop JHEP 08 (2022) 031. In our approach all subtraction are defined using matrix elements and not evoking special a cancellation among sectors

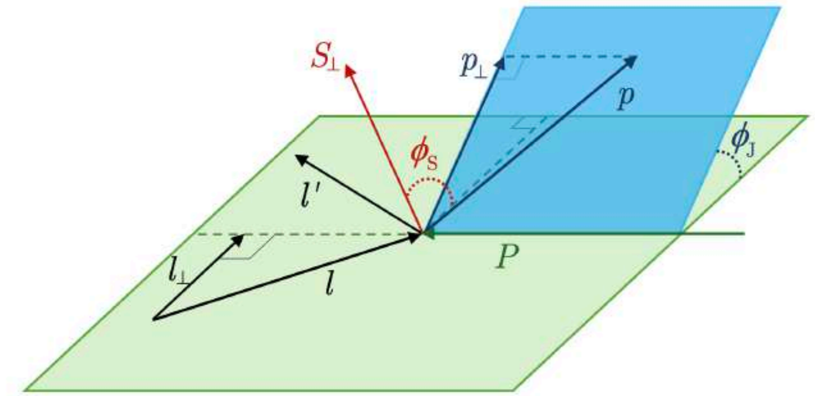
COMPARISONS WITH LITERATURE

SCET (Ebert, Gao Stewart JHEP 06 (2022) 007) uses label formalism and a series of intermediate matching (hard, hard-collinear, hard-anti-collinear..) so that some operators in the current are different (i.e. inverse derivatives do not exist in label formalism). Most of these operators are related to soft modes and vanish at the level of hadronic tensor, however the Wilson coefficient of soft interactions agree with us only if hard coefficients and CS kernel $(\mathscr{K})$ are related by

$$H_2(\xi, Q^2) = H_1(Q^2) + \left[H_2 \otimes \mathscr{K}\right](\xi, Q^2) + \mathcal{O}(\xi)$$

This relationship holds at one-loop level.

PHENOMENOLOGY



$$e(\ell) + h(P) \to e(\ell') + \mathrm{Jet(p)} + \mathrm{X}\,.$$

Jets are mostly perturbative objects (within some percent), they have also a calculable power expansion with tw3 operators (del Castillo, Jaarsma, Scimemi, Waalewijn, *JHEP* 02 (2024) 074)

✗ ***Leading power.***

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi_J\,\mathrm{d}\phi_S\,\mathrm{d}\boldsymbol{q}^2} = \frac{\alpha_{\mathrm{em}}^2 y}{8Q^2}\Bigg\{ & \frac{2-2y+y^2}{y^2} F_{UU,T} + \frac{2(2-y)\sqrt{1-y}}{y^2}\cos\phi_J F_{UU}^{\cos\phi_J} \\ & + \lambda_e \frac{2\sqrt{1-y}}{y}\sin\phi_J F_{LU}^{\sin\phi_J} + S_\parallel \frac{2(2-y)\sqrt{1-y}}{y^2}\sin\phi_J F_{UL}^{\sin\phi_J} \\ & + \lambda_e S_\parallel \left[\frac{2-y}{y} F_{LL} + \frac{2\sqrt{1-y}}{y}\cos\phi_J F_{LL}^{\cos\phi_J})\right] \\ & + |S_\perp| \Bigg[\frac{2-2y+y^2}{y^2}\sin(\phi_J-\phi_S) F_{UT,T}^{\sin(\phi_J-\phi_S)} \\ & + \frac{2(2-y)\sqrt{1-y}}{y^2}\left(\sin(\phi_S) F_{UT}^{\sin(\phi_S)} + \sin(2\phi_J-\phi_S) F_{UT}^{\sin(2\phi_J-\phi_S)}\right)\Bigg] \\ & + \lambda_e |S_\perp| \Bigg[\frac{2-y}{y}\cos(\phi_J-\phi_S) F_{LT}^{\cos(\phi_J-\phi_S)} \\ & + \frac{2\sqrt{1-y}}{y}\left(\cos(\phi_S) F_{LT}^{\cos(\phi_S)} + \cos(2\phi_J-\phi_S) F_{LT}^{\cos(2\phi_J-\phi_S)}\right)\Bigg]\Bigg\},\end{aligned}$$

✗ Studied in the paper..

PHENOMENOLOGY

Each form factor is a product of a TMD and a jet observable I.e.

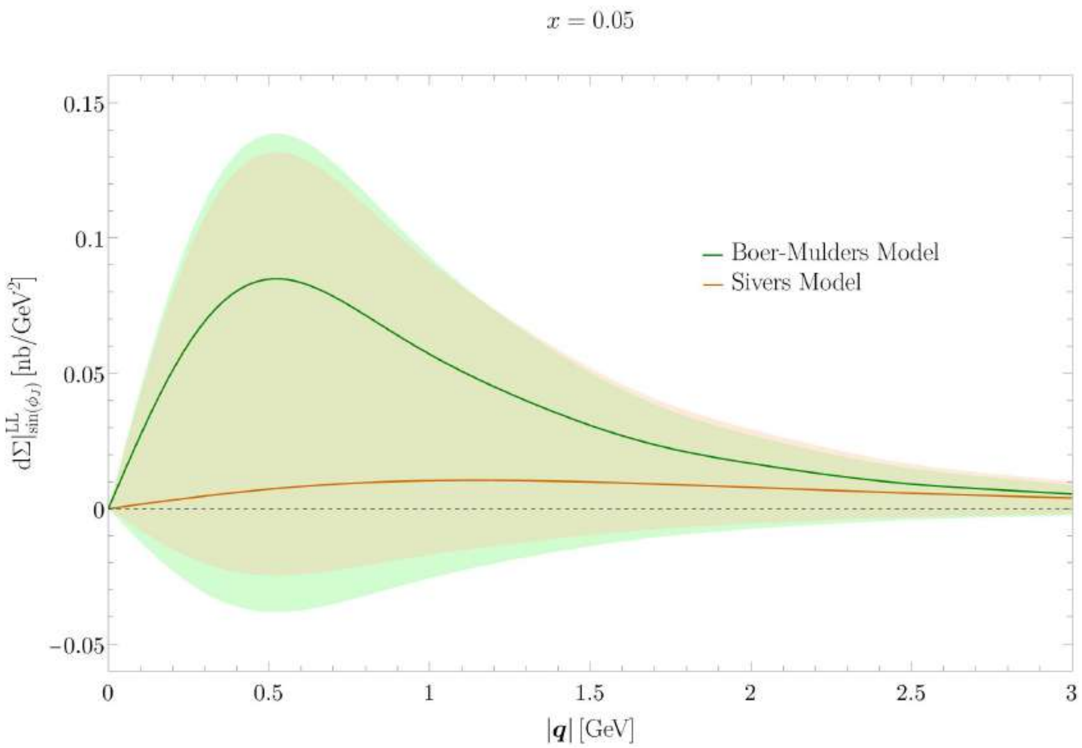
$$
\begin{aligned}
F_{UU,T} &= \mathcal{J}_{0,0}\left[f_1 J_{11}\right], \\
F_{UU}^{\cos\phi_J} &= -\frac{2|\boldsymbol{q}|}{Q}\Big\{\mathcal{J}_{0,0}\left[f_1 J_{11}\right] + \mathcal{J}_{1,1}\left[f_1 J'_{11}\right] \\
&\qquad + \mathrm{Re}\left(\mathcal{J}^{(2)}_{1,1}\left[f_1 J_{21}\right]\right) + \mathrm{Im}\left(\mathcal{J}^{(2)}_{1,1}\left[f_2^{\perp} J_{11}\right]\right)\Big\}, \\
F_{LU}^{\sin\phi_J} &= \frac{2|\boldsymbol{q}|}{Q}\Big\{\mathrm{Im}\left(\mathcal{J}^{(2)}_{1,1}\left[f_1 J_{21}\right]\right) - \mathrm{Re}\left(\left[f_2^{\perp} J_{11}\right]\right)\Big\}, \\
F_{UL}^{\sin\phi_J} &= -\frac{2|\boldsymbol{q}|}{Q}\mathrm{Im}\left(\mathcal{J}^{(2)}_{1,1}\left[g_1 J_{21} + g_{2L}^{\perp} J_{11}\right]\right),
\end{aligned}
$$

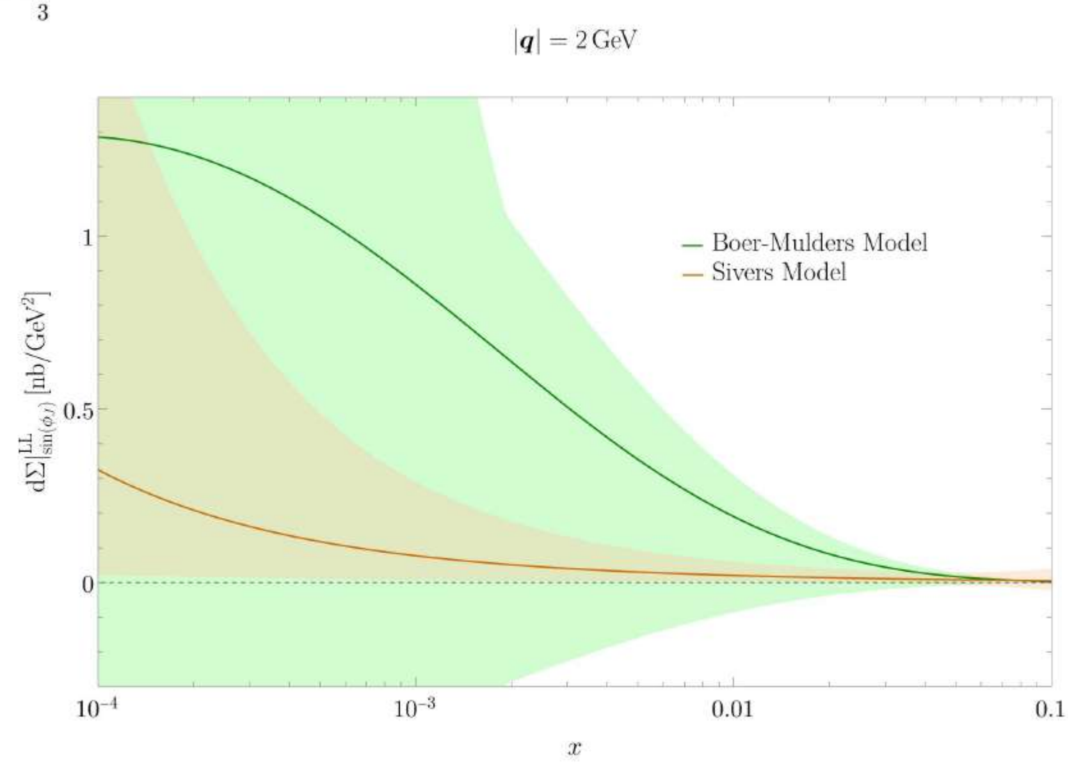
.....

Leading power.

Studied in the paper..

PHENOMENOLOGY





CONCLUSIONS

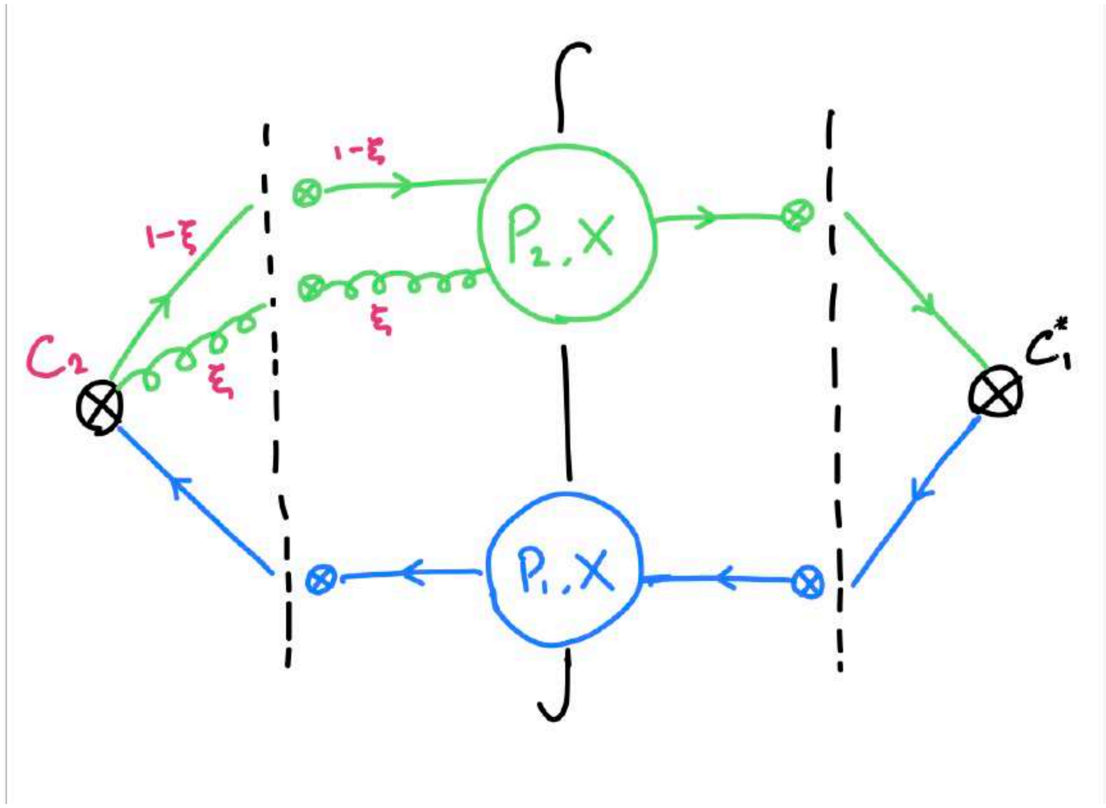
- THE STUDY OF NLP CORRECTIONS TO TMD FACTORABLE CROSS SECTION HAS JUST STARTED AND IT IS OUR PRESENT FRONTIER. EIC AND EXPERIMENTS OF SIMILAR OR LOWER ENERGY MAY BE STRONGLY AFFECTED BY THEM
- WE HAVE PROVIDED AN OPERATORIAL AND OPERATIVE DEFINITION OF CURRENTS AND HADRONIC TENSOR IN SIDIS. EXTENSION TO DY AND E+E- IS STRAIGHT IN BFM+S.
- THE ZERO-BIN SUBTRACTIONS OF NLP TERMS CAN BE EXPRESSED IN TERMS OF SOFT MATRIX ELEMENTS. THIS FACILITATES THE NONPERTURBATIVE ESTIMATES OF ALL MATRIX ELEMENTS AND EVOLUTION COMPARED TO PREVIOUS RESULTS
- HIGHER ORDER CALCULATIONS CAN BE NECESSARY
- PHENOMENOLOGY FOR HADRONS AND JETS IS JUST STARTING

Thanks!


QCD
evolution
2026
MAY, 11-15
S. Lorenzo de El Escorial
https://indico.fis.ucm.es/event/37/

BACK UP

DESARROLLOS RECIENTES: NUEVAS OBSERVABLES A NLP



Jets de higher twist

Producción de di-jet in e^+e^-:
Asymmetrias a NLP

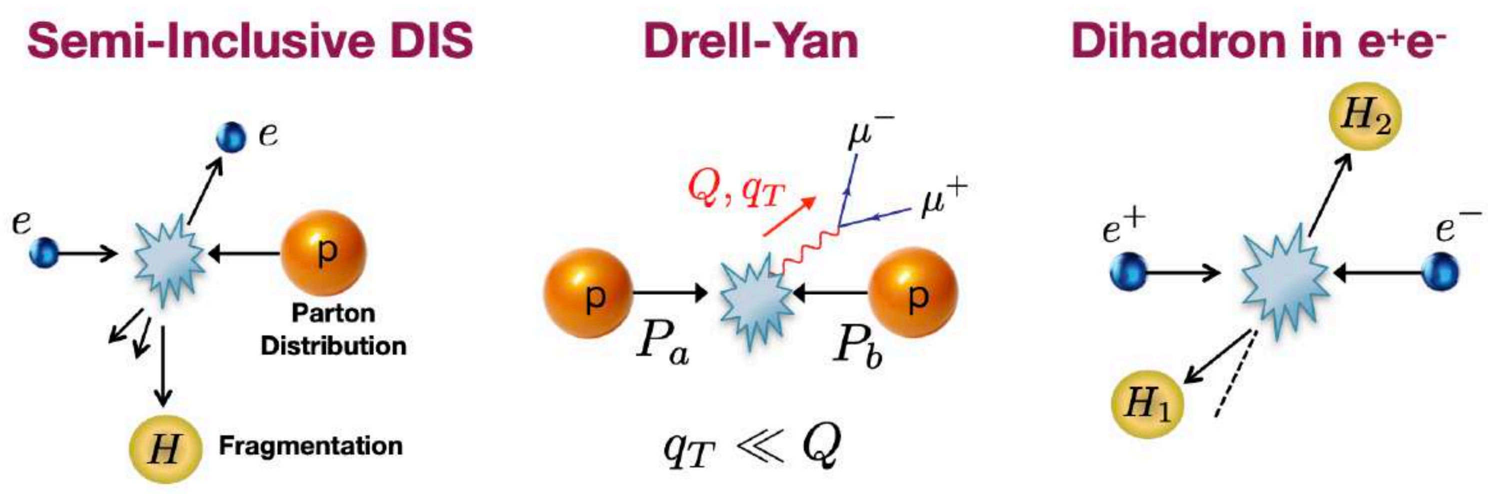
$$
\begin{aligned}
W^{\mu\nu} = & -N_c\, g_T^{\mu\nu}\, H_1(Q^2) \int_0^\infty \mathrm{d}\mathbf{b}^2\, \frac{J_0(|\mathbf{b}||\mathbf{q}|)}{2\pi}\, J_{11}(\mathbf{b}^2)\, J_{11}(\mathbf{b}^2) \\
& + N_c \left[\frac{n^\mu q_T^\nu}{q^+} + \frac{n^\nu q_T^\mu}{q^+}\right] H_1(Q^2) \int_0^\infty \mathrm{d}\mathbf{b}^2\, \frac{J_1(|\mathbf{b}||\mathbf{q}|)}{2\pi|\mathbf{b}||\mathbf{q}|}\, J'_{11}(\mathbf{b}^2)\, J_{11}(\mathbf{b}^2) \\
& + N_c \left[\frac{\bar{n}^\mu q_T^\nu}{q^-} - \frac{n^\mu q_T^\nu}{q^+}\right] \int_0^1 \mathrm{d}\xi\, H_2(\xi, Q^2) \int_0^\infty \mathrm{d}\mathbf{b}^2\, \frac{J_1(|\mathbf{b}||\mathbf{q}|)}{2\pi|\mathbf{b}||\mathbf{q}|} \\
& \times \left\{ J_{21}(\xi, \mathbf{b}^2)\, J_{11}(\mathbf{b}^2) - J_{11}(\mathbf{b}^2)\, J_{21}(\xi, \mathbf{b}^2) \right\}
\end{aligned}
$$

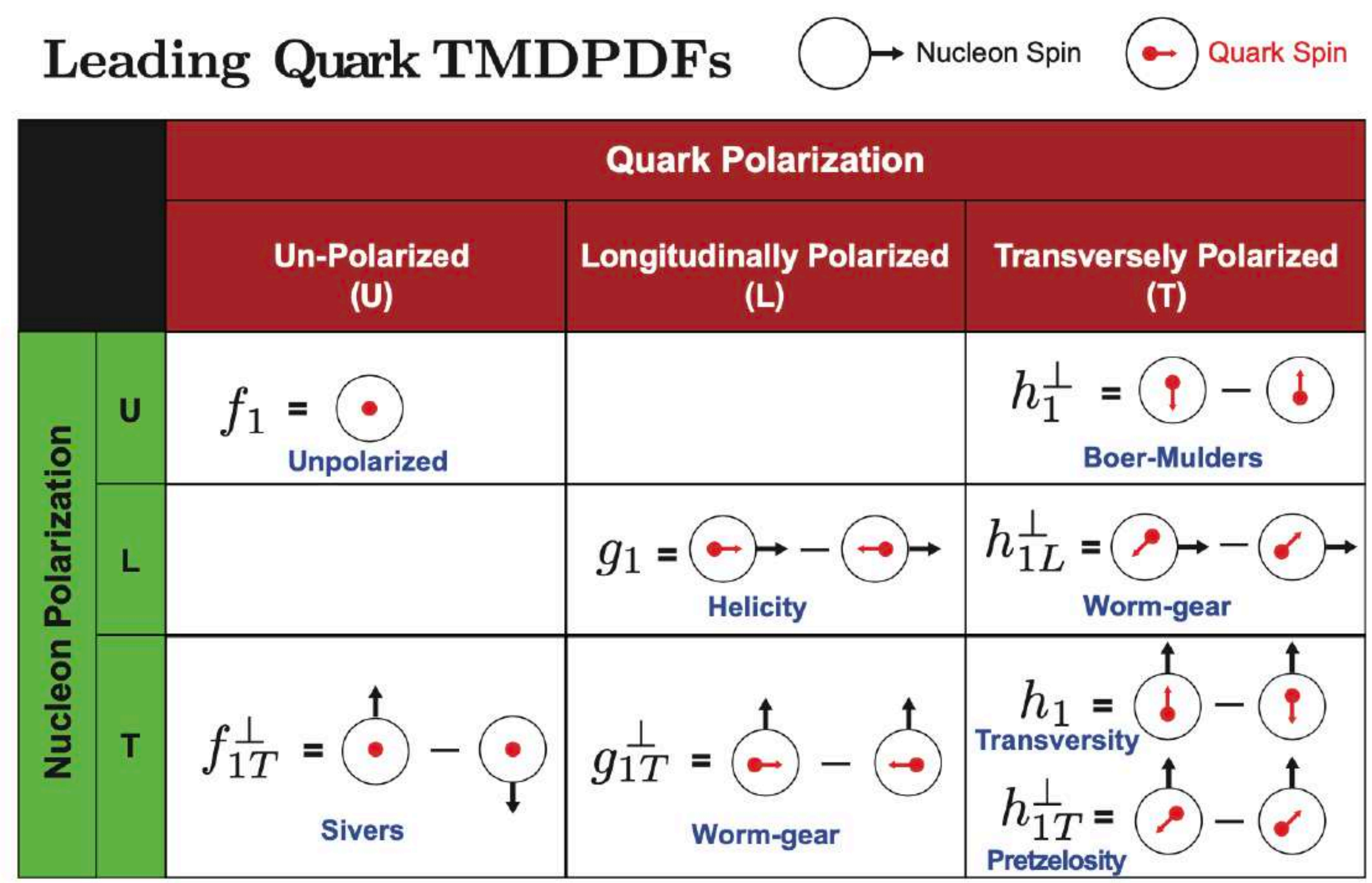
Pictures from M. Jaarsma talk at SCET2024,

R. F. Del Castillo, *M. Jaarsma, I. Scimemi., W. Waalewijn, JHEP* 02 (2024) 074

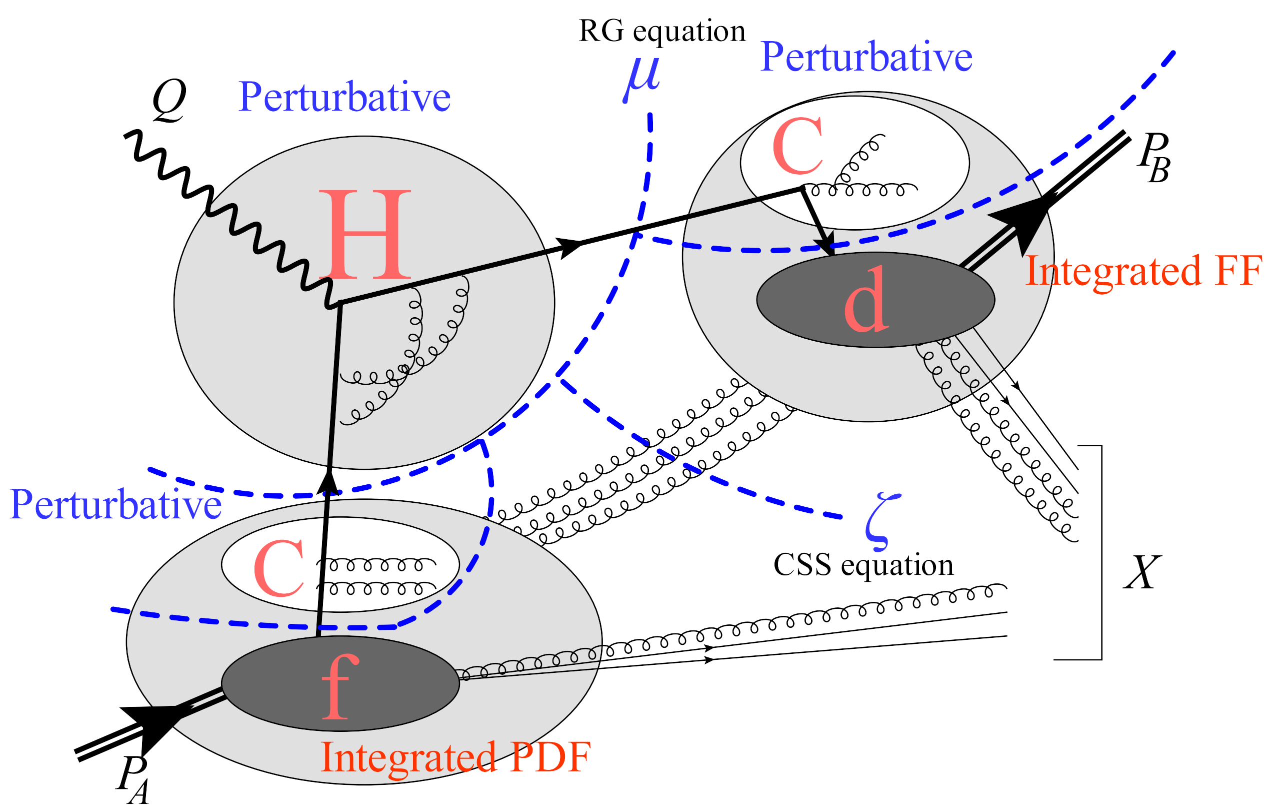
FACTORIZATION

A physical process is typically characterized by several energy scales. By expanding the cross sections into powers of ratios of small scales to large ones, fundamental QCD distributions/correlations are discovered.





BIG PICTURE II



We can also do a small-b asymptotic limit and recover collinear distributions+Sudakov double log resummation. This can be useful for phenomenology in some cases.

BFM+S: HADRONIC TENSOR (UNSUBTRACTED)

Each current enters in the hadronic tensor..

$$W^{\mu\nu} = \int \frac{\mathrm{d}^4 b}{(2\pi)^4}\, e^{+\mathrm{i}q\cdot b}\, \langle P|\, J^{\mu}_{\text{eff}}(b)\, |p, X\rangle \langle p, X|\, J^{\nu}_{\text{eff}}(0)\, |P\rangle$$

..and we have a power expansion of the hadronic tensor

$$\left[\mathcal{W}^{\mu\nu}\right]^{(\kappa,\eta)} = \int \frac{\mathrm{d}^4 b}{(2\pi)^4}\, e^{+\mathrm{i}q\cdot b}\, \langle P|\, \left[J^{\mu}_{\text{eff}}(b)\right]^{(\kappa)} |p, X\rangle \langle p, X|\, \left[J^{\nu}_{\text{eff}}(0)\right]^{(\eta)} |P\rangle$$

LP

$$\begin{aligned}
\left[\mathcal{W}^{\mu\nu}\right]^{(2,2)}_{A} = &\int \frac{\mathrm{d}^2 b}{(2\pi)^2}\, e^{+\mathrm{i}q_T\cdot b_T} \int \frac{\mathrm{d}b^+}{2\pi}\, e^{+\mathrm{i}q^- b^+} \int \frac{\mathrm{d}b^-}{2\pi}\, e^{+\mathrm{i}q^+ b^-} \\
&\times (\gamma^{\mu}_{T})_{\alpha\beta}(\gamma^{\nu}_{T})_{\gamma\delta} \int \mathrm{d}y^+\, \mathrm{d}y^-\, C_1(y^+, y^-) \int \mathrm{d}z^+\, \mathrm{d}z^-\, C_1^*(z^+, z^-) \\
&\times \langle P|\, \bar{\chi}_{i,\alpha}(b_T + b^- n + y^- n)\, |X\rangle\langle X|\, \chi_{l,\delta}(z^- n)\, |P\rangle \\
&\times \langle 0|\, \chi_{j,\beta}(b_T + b^+ \bar{n} + y^+ \bar{n})\, |p, X\rangle\langle p, X|\, \bar{\chi}_{k,\gamma}(z^+ \bar{n})\, |0\rangle \\
&\times \langle 0|\, \left[S^{\dagger}_{\bar{n}} S_n(b_T)\right]_{ij} |X\rangle\langle X|\, \left[S^{\dagger}_{n} S_{\bar{n}}(0)\right]_{kl} |0\rangle\,.
\end{aligned}$$

Plus terms that disappear with zero bien subtraction (see later)

BFM+S

In our case we are interested in the hadronic tensor of the cross section

$$W^{\mu\nu} = \int \frac{\mathrm{d}^4\mathrm{b}}{(2\pi)^4}\, e^{+\mathrm{i}\mathrm{b}\cdot\mathrm{q}} \langle P \,|\, J^{\mu}(b) \,|\, p, X \rangle \langle p, X \,|\, J^{\nu}(0) \,|\, P \rangle$$

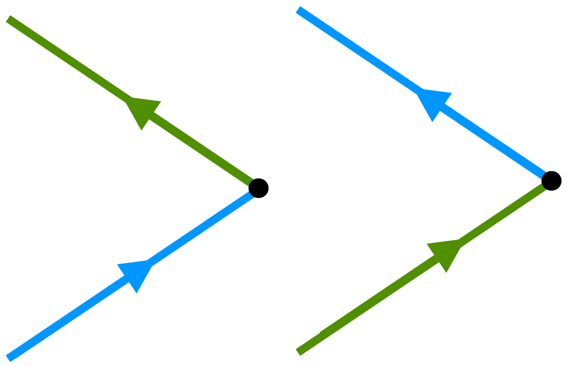
Each e.m. current is analyzed separately BFM+S

$$J^{\mu}_{\mathrm{eff}} = \int \mathcal{D}\bar{\varphi}\,\mathcal{D}\varphi\,\mathcal{D}B\,(\bar{\varphi} + \bar{\psi} + \bar{\psi} + \bar{\psi})\gamma^{\mu}(\varphi + \psi + \psi + \psi)\; e^{\mathrm{i}S_{\mathrm{int}}[\bar{\varphi},\varphi,B;\bar{\psi},\psi,A;\bar{\psi},\psi,A;\bar{\psi},\psi,A]} e^{\mathrm{i}S_{\mathrm{QCD}}[\bar{\varphi},\varphi,B]}$$

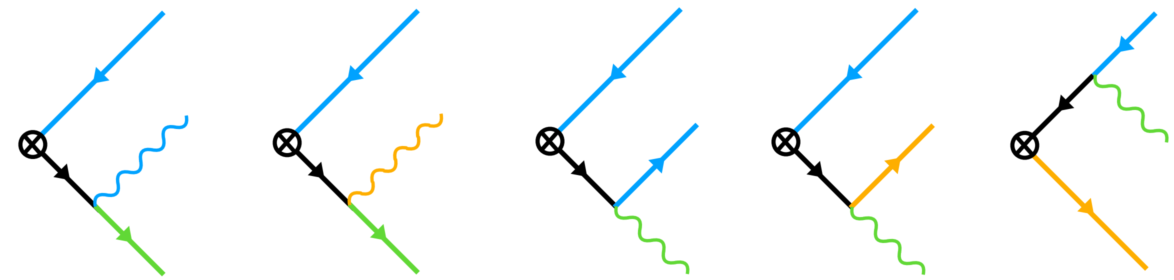
- Integrate out hard modes
- Match on gauge invariant building blocks (in SCET)
- Generalize to all orders in perturbation theory

BFM+S CURRENT OPERATOR

LP



NLP



Step II: Wilson lines insertion

$$\chi = \frac{\gamma^+\gamma^-}{2} W^\dagger \psi \,, \qquad \mathcal{A}^\rho_T = W^\dagger \left[\mathrm{i} D^\rho_T, W \right] \,, \qquad \Psi_n = S^\dagger_n \psi \,, \qquad \mathcal{A}^\rho_{T,n} = S^\dagger_n \left[\mathrm{i} D^\rho_T, S_n \right] \,,$$

$$\chi = \frac{\gamma^-\gamma^+}{2} W^\dagger \psi \,, \qquad \mathcal{A}^\rho_T = W^\dagger \left[\mathrm{i} D^\rho_T, W \right] . \qquad \Psi_{\bar{n}} = S^\dagger_{\bar{n}} \psi \,, \qquad \mathcal{A}^\rho_{T,\bar{n}} = S^\dagger_{\bar{n}} \left[\mathrm{i} D^\rho_T, S_{\bar{n}} \right] .$$

Wilson lines with δ-regulator

$$W(x) = \lim_{\delta^+ \to 0} \mathcal{P} \left\{ \exp \left[ig \int_L^0 \mathrm{d}y^+ \, e^{-s\delta^+ |y^-|} A^+(x + y^- n) \right] \right\} ,$$

$$W(x) = \lim_{\delta^- \to 0} \mathcal{P} \left\{ \exp \left[ig \int_{\bar{L}}^0 \mathrm{d}y^+ \, e^{-\bar{s}\delta^- |y^+|} A^-(x + y^+ \bar{n}) \right] \right\} ,$$

$$S_n(x) = \lim_{\delta^+ \to 0} \mathcal{P} \left\{ \exp \left[ig \int_L^0 \mathrm{d}y^+ \, e^{-s\delta^+ |y^-|} A^+(x + y^- n) \right] \right\} ,$$

$$S_{\bar{n}}(x) = \lim_{\delta^- \to 0} \mathcal{P} \left\{ \exp \left[ig \int_{\bar{L}}^0 \mathrm{d}y^+ \, e^{-\bar{s}\delta^- |y^+|} A^-(x + y^+ \bar{n}) \right] \right\} ,$$

SUBTRACTION METHOD AT LP

LP $\chi \to S_{\bar{n}}^{\dagger} S_n \chi + \Psi_{\bar{n}}$ $\mathscr{A}^{\mu} \to S_{\bar{n}}^{\dagger} S_n \mathscr{A}^{\mu} S_n^{\dagger} S_{\bar{n}} + \mathscr{A}_{\bar{n}}^{\mu}$

A. Idilbi, T. Mehen, Phys. Rev. D75 (2007) 114017

$$\langle P | \bar{\chi}_{\alpha}(b_{\perp} + xp^{+}\bar{n}) | X \rangle \langle X | \chi_{\beta}(0) | P \rangle^{\text{sub, bare}} = \frac{\langle P | \bar{\chi}_{\alpha}(b_{\perp} + xp^{+}\bar{n}) | X \rangle \langle X | \chi_{\beta}(0) | P \rangle^{\text{uns, bare}}}{\langle 0 | S_n^{\dagger} S_{\bar{n}}(b_{\perp}) S_{\bar{n}}^{\dagger} S_n(0) | 0 \rangle} = \frac{O_{\alpha\beta}^{\text{uns, bare}}(xp^{+}, b_{\perp})}{S(b_{\perp})}$$

All this reshapes the soft decor of the hadronic tensor

$$\begin{aligned}
[\mathcal{W}^{\mu\nu}]_A^{(2,2)} = &\int \frac{\mathrm{d}^2 b}{(2\pi)^2}\, e^{+\mathrm{i}q_T\cdot b_T} \int \frac{\mathrm{d}b^+}{2\pi}\, e^{+\mathrm{i}q^- b^+} \int \frac{\mathrm{d}b^-}{2\pi}\, e^{+\mathrm{i}q^+ b^-} \\
&\times (\gamma_T^{\mu})_{\alpha\beta}(\gamma_T^{\nu})_{\gamma\delta} \int \mathrm{d}y^+\, \mathrm{d}y^-\, C_1(y^+, y^-) \int \mathrm{d}z^+\, \mathrm{d}z^-\, C_1^*(z^+, z^-) \\
&\times \langle P | \bar{\chi}_{i,\alpha}(b_T + b^- n + y^- n) | X \rangle \langle X | \chi_{l,\delta}(z^- n) | P \rangle \\
&\times \langle 0 | \chi_{j,\beta}(b_T + b^+ \bar{n} + y^+ \bar{n}) | p, X \rangle \langle p, X | \bar{\chi}_{k,\gamma}(z^+ \bar{n}) | 0 \rangle \\
&\times \langle 0 | \left[S_{\bar{n}}^{\dagger} S_n(b_T)\right]_{ij} | X \rangle \langle X | \left[S_n^{\dagger} S_{\bar{n}}(0)\right]_{kl} | 0 \rangle .
\end{aligned}$$

$$\begin{aligned}
= &\int \frac{\mathrm{d}^2 b}{(2\pi)^2}\, e^{+\mathrm{i}q_T\cdot b_T} \int \frac{\mathrm{d}b^+}{2\pi}\, e^{+\mathrm{i}q^- b^+} \int \frac{\mathrm{d}b^-}{2\pi}\, e^{+\mathrm{i}q^+ b^-} \\
&\times (\gamma_T^{\mu})_{\alpha\beta}(\gamma_T^{\nu})_{\gamma\delta} \int \mathrm{d}y^+\, \mathrm{d}y^-\, C_1(y^+, y^-) \int \mathrm{d}z^+\, \mathrm{d}z^-\, C_1^*(z^+, z^-) \\
&\times \langle P | \bar{\chi}_{\alpha}(b_T + b^- n + y^- n) | X \rangle \langle X | \chi_{\delta}(z^- n) | P \rangle \\
&\times \langle 0 | \chi_{\beta}(b_T + b^+ \bar{n} + y^+ \bar{n}) | p, X \rangle \langle p, X | \bar{\chi}_{\gamma}(z^+ \bar{n}) | 0 \rangle \\
&\div \langle 0 | \left[S_{\bar{n}}^{\dagger} S_n(b_T)\right] | X \rangle \langle X | \left[S_n^{\dagger} S_{\bar{n}}(0)\right] | 0 \rangle .
\end{aligned}$$

SUBTRACTION METHOD AT NLP: KINEMATIC PC

Overlap Subtraction

$$\langle 0|\, \mathrm{i}\partial_T^\rho \chi_\beta(b_T + b^+\bar{n})\, |p, X\rangle\, \langle p, X|\, \bar{\chi}_\gamma(0)\, |0\rangle$$
$$\to \langle 0|\, \mathrm{i}\partial_T^\rho \chi_\beta(b_T + b^+\bar{n})|p, X\rangle\langle p, X|\bar{\chi}_\gamma(0)|0\rangle\langle 0|\left[S_{\bar{n}}^\dagger S_n(b_T)\right]|X\rangle\langle X|\left[S_n^\dagger S_{\bar{n}}(0)\right]|0\rangle$$
$$+ \langle 0|\chi_\beta(b_T + b^+\bar{n})|p, X\rangle\langle p, X|\bar{\chi}_\gamma(0)|0\rangle\langle 0|\mathrm{i}\partial_T^\rho\left[S_{\bar{n}}^\dagger S_n(b_T)\right]|X\rangle\langle X|\left[S_n^\dagger S_{\bar{n}}(0)\right]|0\rangle$$

Useful identity

$$S_{\bar{n}}^\dagger S_n \mathcal{A}_{T,n}^\rho = \frac{1}{2}\mathrm{i}\partial_T^\rho\left[S_{\bar{n}}^\dagger S_n\right] + \frac{1}{2}\left[\mathcal{A}_{T,\bar{n}}^\rho S_{\bar{n}}^\dagger S_n + S_{\bar{n}}^\dagger S_n \mathcal{A}_{T,n}^\rho\right]$$

Subtracted hadronic tensor

$$\begin{aligned}
&[\mathcal{W}_{\text{sub.}}^{\mu\nu}]_{Cn}^{(3,2)} + [\mathcal{W}_{\text{sub.}}^{\mu\nu}]_{En}^{(3,2)} \\
&= \left(-\frac{\bar{n}^\mu}{q^-}\right)\int \frac{\mathrm{d}^2 b}{(2\pi)^2}\, e^{+\mathrm{i}q_T\cdot b_T}\int \frac{\mathrm{d}b^+}{2\pi}\, e^{+\mathrm{i}q^- b^+}\int \frac{\mathrm{d}b^-}{2\pi}\, e^{+\mathrm{i}q^+ b^-} \\
&\quad\times (\gamma_T^\rho)_{\alpha\beta}(\gamma_T^\nu)_{\gamma\delta}\int \mathrm{d}y^+\,\mathrm{d}y^-\, C_1(y^+, y^-)\int \mathrm{d}z^+\,\mathrm{d}z^-\, C_1^*(z^+, z^-) \\
&\quad\times \langle P|\, \mathrm{i}\partial_T^\rho \bar{\chi}_\alpha(b_T + b^- n + y^- n)\, |X\rangle\langle X|\, \chi_\delta(z^- n)\, |P\rangle \\
&\quad\times \langle 0|\, \chi_\beta(b_T + b^+\bar{n} + y^+\bar{n})\, |p, X\rangle\langle p, X|\, \bar{\chi}_\gamma(z^+\bar{n})\, |0\rangle \\
&\quad\div \langle 0|\, S_{\bar{n}}^\dagger S_n(b_T)\, |X\rangle\langle X|\, S_n^\dagger S_{\bar{n}}(0)\, |0\rangle \\
&- \frac{1}{2}\left(-\frac{\bar{n}^\mu}{q^-}\right)\int \frac{\mathrm{d}^2 b}{(2\pi)^2}\, e^{+\mathrm{i}q_T\cdot b_T}\int \frac{\mathrm{d}b^+}{2\pi}\, e^{+\mathrm{i}q^- b^+}\int \frac{\mathrm{d}b^-}{2\pi}\, e^{+\mathrm{i}q^+ b^-} \\
&\quad\times (\gamma_T^\rho)_{\alpha\beta}(\gamma_T^\nu)_{\gamma\delta}\int \mathrm{d}y^+\,\mathrm{d}y^-\, C_1(y^+, y^-)\int \mathrm{d}z^+\,\mathrm{d}z^-\, C_1^*(z^+, z^-) \\
&\quad\times \langle P|\, \bar{\chi}_\alpha(b_T + b^- n + y^- n)\, |X\rangle\langle X|\, \chi_\delta(z^- n)\, |P\rangle \\
&\quad\times \langle 0|\, \chi_\beta(b_T + b^+\bar{n} + y^+\bar{n})\, |p, X\rangle\langle p, X|\, \bar{\chi}_\gamma(z^+\bar{n})\, |0\rangle \\
&\quad\times \langle 0|\, \mathrm{i}\partial_T^\rho\left[S_{\bar{n}}^\dagger S_n(b_T)\right]|X\rangle\langle 0|\, S_n^\dagger S_{\bar{n}}(0)\, |0\rangle \\
&\quad\div \langle 0|\, S_{\bar{n}}^\dagger S_n(b_T)\, |X\rangle\langle X|\, S_n^\dagger S_{\bar{n}}(0)\, |0\rangle\,.
\end{aligned}$$