

# Theoretical Perspectives on **Electromagnetic Hadronic Physics**



**Marco Radici** 





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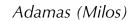
Adamas (Milos)



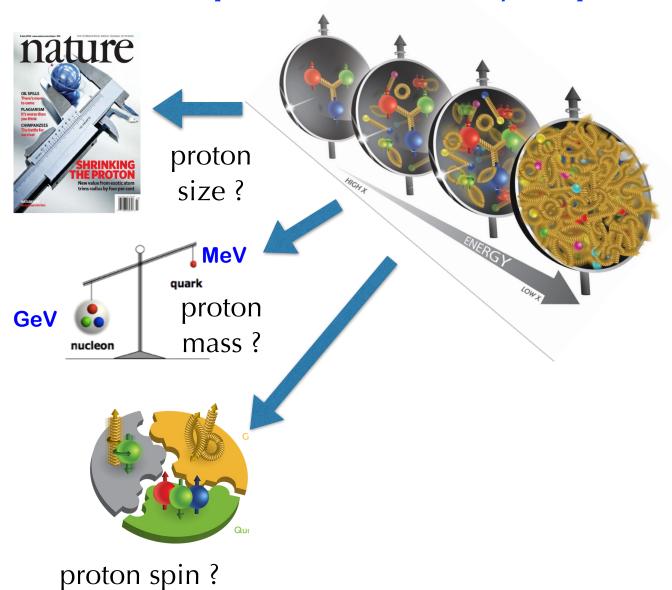


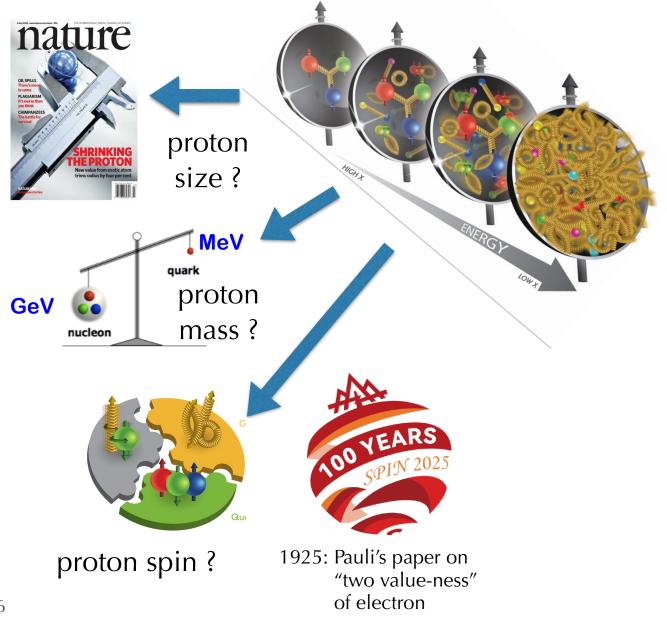


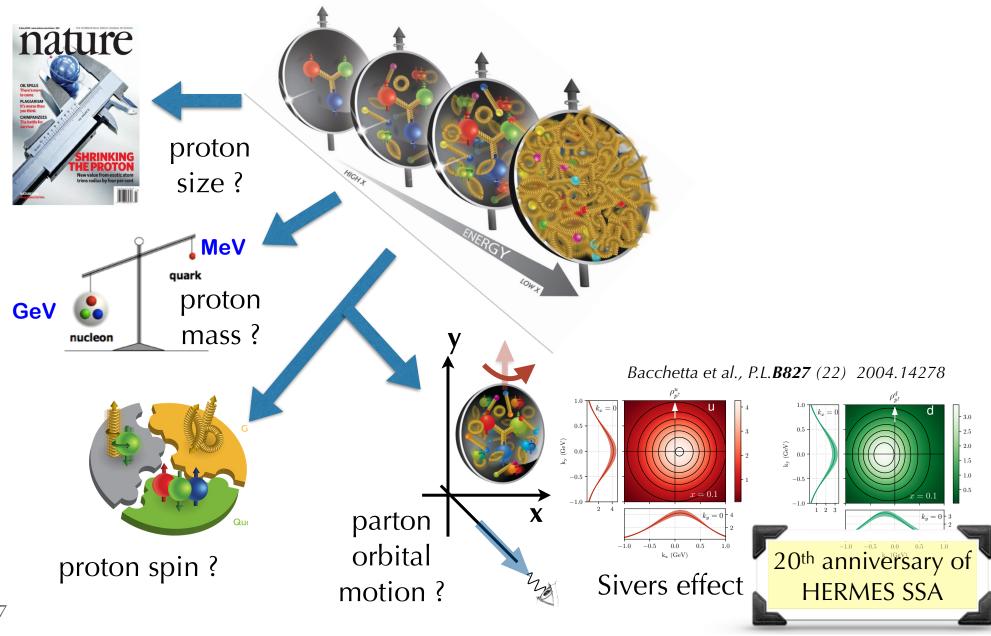
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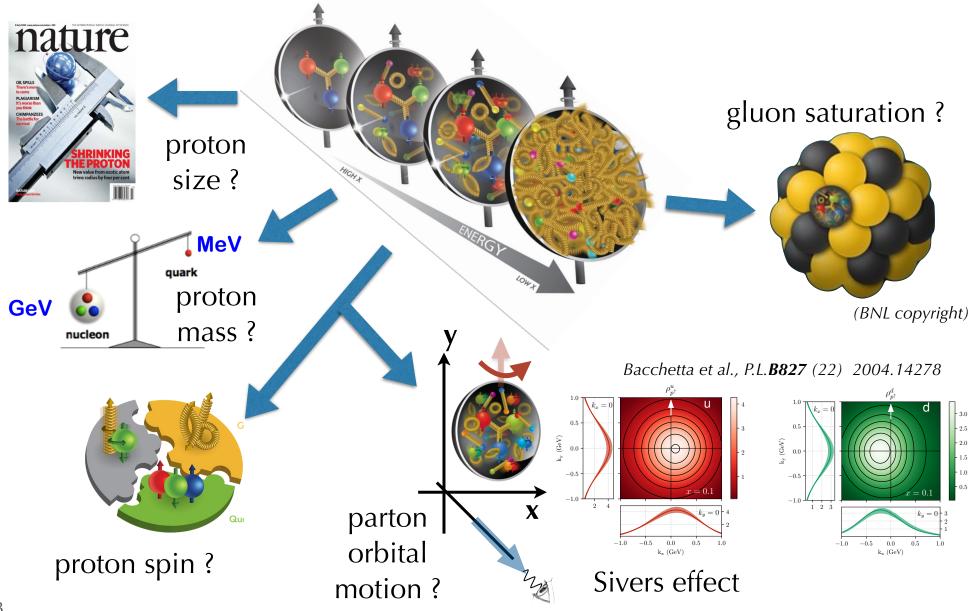






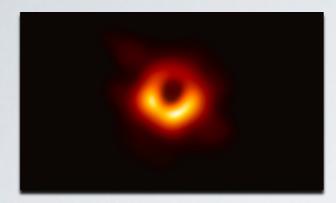




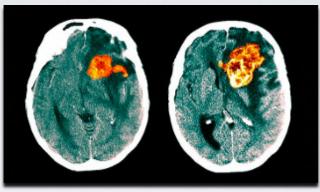


# Seeing is believing - the power of imaging

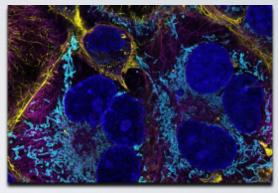
courtesy Kong Tu (BNL)



black hole (Event Horizon telescope)



CT scan sequence of brain with glioblastoma



3D image of myelin (coating of nerve fibres)

 $10^{12} \,\mathrm{m}$   $10^{-9} \,\mathrm{m}$ 

Imaging: a powerful scientific method to understand Nature from astronomical .... to .... microscopic scale

Muldi-dim. imaging: fundamental to enhance this power!



#### (incomplete) list of EINN25 topics

muon g-2 dispersion relations

chiral EFT and nuclear structure

spectroscopy: hybrids, glueballs, charmonium-like states..

e.m. and weak N form factors low-energy dark sector proton (charge) radius

TMD GPD BSM

Cold & Hot QCD

N e.m. polarizabilities



EIC (and ePIC)

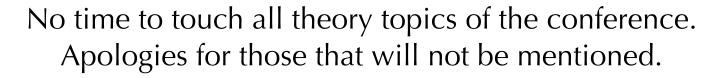
strange sector and EoS of neutron stars

2N- & 3N-correlations

#### (incomplete) list of EINN25 topics

muon g-2

dispersion relations



In this talk, main focus on tools for imaging: "(femto-)tomography" of the Nucleon (with specific reference to very recent results, typically > 2024..)

low-

**TMD** 

**GPD** 

EIC (and ePIC)

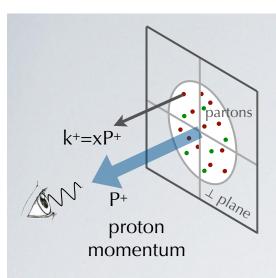
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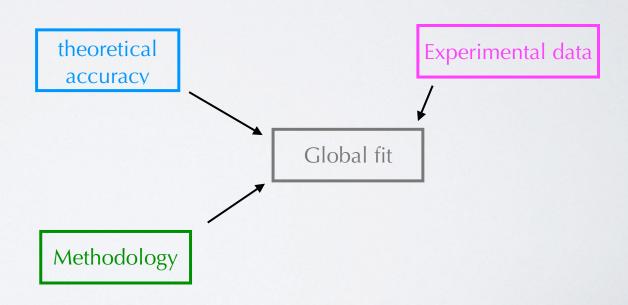
#### **Nucleon "femtography"** Multi-dim. maps of Nucleon structure Lorcé et al., JHEP **05** (11) Burkardt & Pasquini, EPJ A52 (16) F.T. $\mathbf{b}_{\perp} \leftrightarrow \Delta_{\perp}$ at $\xi = 0$ Boussarie et al. (TMD Handbook). 2304.03302 $W(x, \mathbf{k}_{\perp}, \mathbf{b}_{\perp})$ $F(x, \mathbf{k}_{\perp}, \boldsymbol{\xi}, \Delta^2)$ Wigner **GTMD** ∫dk⊥ ∫d**b**⊥ ∫dk⊥ $\rho(x,b_{\perp})$ F.T. $b_{\perp} \leftrightarrow \Delta_{\perp}$ at $\xi=0$ $q(x, \mathbf{k}_{\perp})$ $H(x, \xi, \Delta^2)$ **GPD TMD** impact PDF $\int dx x^{n-1}$ ∫dk⊥ $\sum_{n=0}^{\infty} A_{nk}(t = \Delta^2) (2\xi)^k$ $F_n(t)$ q(x)generalised form factors **PDF** form factors



# PDF: Parton Distribution Functions (1D mapping)

Ongoing major effort in **increasing precision** and **estimate uncertainties**, demanded by reproducing new data (primarily, LHC)

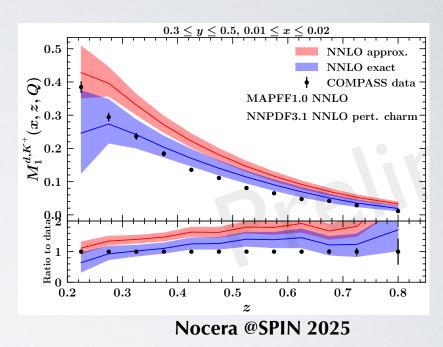
reducing the uncertainty depends on the accuracy of all ingredients entering a global fit



Exact NNLO corrections for unpolarized SIDIS long. polarized SIDIS NC-, CC-unpolarized SIDIS

**Significant impact on NNLO FFs** 

Goyal et al., 2312.17711; Bonino et al., 2401.16281 Bonino et al., 2404.08597; Goyal et al., 2404.09959 Bonino et al., 2506.19926



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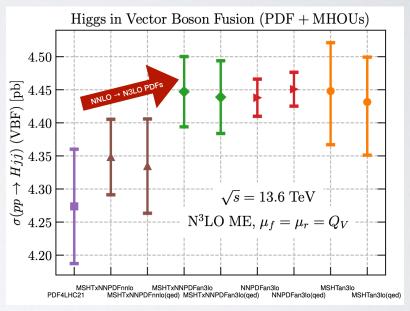
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#### **Significant impact on NNLO FFs**

approximate N<sup>3</sup>LO (aN<sup>3</sup>LO) PDF sets for unpolarized PDFs in inclusive processes

MSHT 2207.04739 NNPDF 2402.18635

NNLO → aN³LO unexpected significant shift



Cridge et al., 2411.05373

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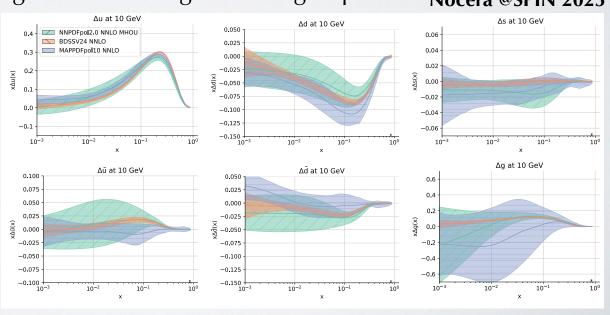
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Polarized PDFs at NNLO: good agreement among different groups

#### Nocera @SPIN 2025



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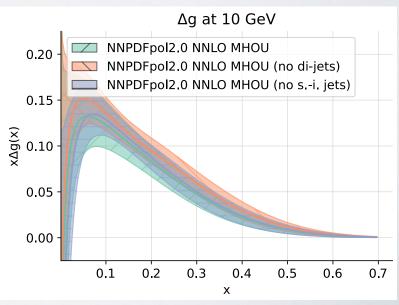
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See also recent extraction at NLO from polarized DIS+SIDIS+pp

Cocuzza et al. (JAM), 2506.13616

#### Methodology

Parametrization: reduce th. bias by replacing standard Mellin-transformable expressions with **Neural Networks** (NN)

NN can have O(~100) parameters  $\rightarrow$  need M.L.-inspired techniques to find minimum  $\chi^2$ 

NN flexibility → risk of overfitting → **cross-validation tests** 

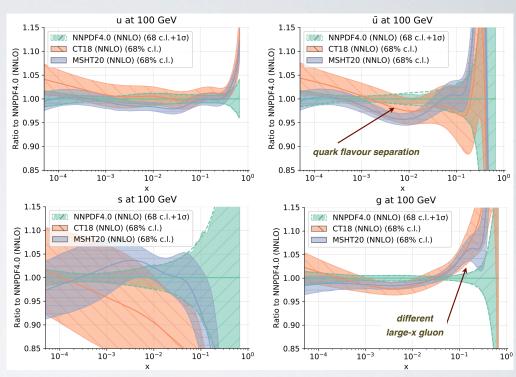
**Closure tests**: statistical validation of methodology and PDF faithfulness (generate pseudodata from known model and try to fit it)

Generalization tests: how well do PDF sets describe new data not included in the fit?

need exact NNLO predictions (PineAPPLE, NNLOJET,...) need to include in  $\chi^2$  all exp. and th. uncertainties: bin-by-bin correlations, MHO,  $\alpha_s$ , PDF, QED corrections,...

Uncertainties at large x in unpolarized PDFs (for strangeness, over all x's..)

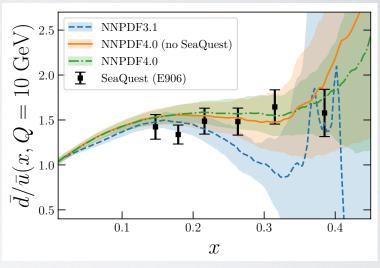
impacts BSM searches at LHC  $x \sim \frac{M_X}{\sqrt{s}}$ 



Uncertainties at large x in unpolarized PDFs (for strangeness, over all x's..)

impacts BSM searches at LHC  $x \sim \frac{M_X}{\sqrt{s}}$ 

impacts antimatter asymmetry explorations (SeaQuest)  $\bar{d}(x)/\bar{u}(x) \stackrel{x \to 1}{\to} ?$ 

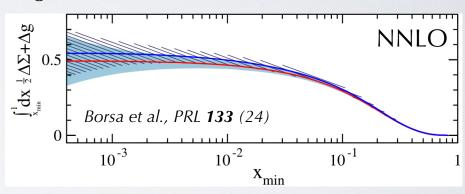


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polarized PDFs: small-x singlet helicity  $\Delta\Sigma$  and  $\Delta g$ , is there room for OAM?



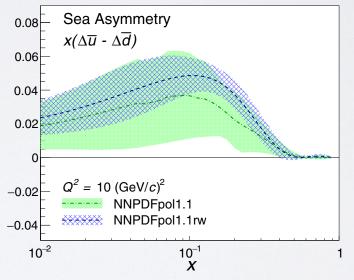
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antimatter helicity asymmetry?



Nocera, 1702.05077

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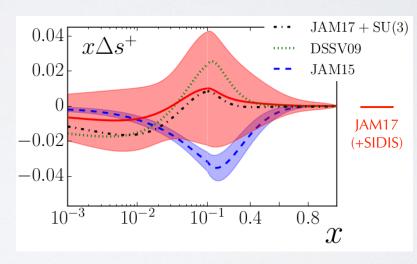
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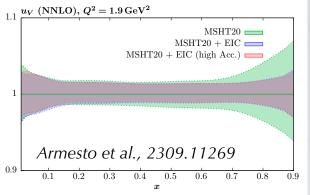
JAM, 1705.05889

#### **Expected impact of the EIC**

Uncertainties at large x in unpolarized PDFs (for strangen

impacts BSM searches at LHC  $x \sim \frac{M_X}{\sqrt{s}}$ 

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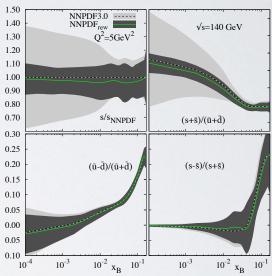


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large impact on strange and sea-quark ratio  $R_{\rm s}$ 



EIC Yellow Report, 2103.05419

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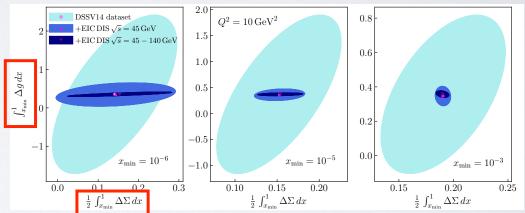
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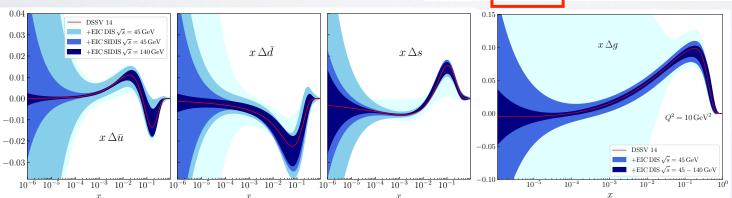
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polarized PDFs: small-x singlet helicity  $\Delta\Sigma$  and  $\Delta g$ , is there room for OAM?

impact on helicity contribution to spin sum rule





EIC Yellow Report, 2103.05419

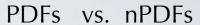
Borsa et al., 2007.08300

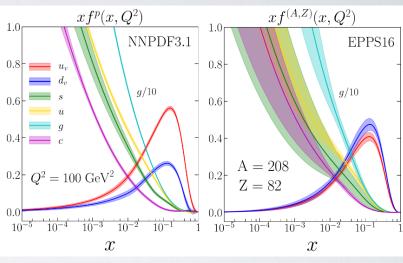
#### **Nuclear PDFs (nPDFs)**

data sets: LHC (Z, W, HQs, dijets, ALICE), RHIC, v and NC DIS, pA and  $\pi A$  Drell-Yan

nPDF sets: KSAG20 NLO+NNLO PRD 104 (21)
TUJU21 " " PRD 105 (22)
EPPS21 NLO EPJ C82 (22)
nNNPDF3.0 " EPJ C82 (22)
nCTEQ15HQ " PRD 105 (22)

differ in datasets and fitting methodology





Nocera @DIS 2025

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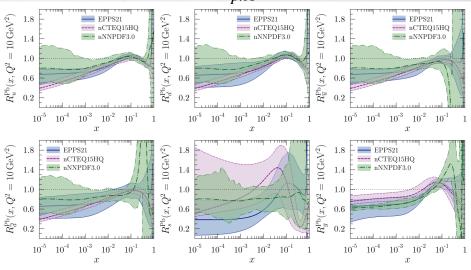
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differ in datasets and fitting methodology

$$R_q^A(x, Q^2) = f_{p/A}^q(x, Q^2) / f_p^q(x, Q^2)$$

evidence of (anti-)shadowing in all partons good agreement except strange EPPS21 impact of dijet data, particularly on gluon potential impact of target mass corrections at large x and small Q  $_{Prog.Part.Nucl.Phys.\ 136\ (24)}$  impact of NNLO on gluon at x > 0.03



ongoing work on integrated QCD fit of PDFs and nPDFs

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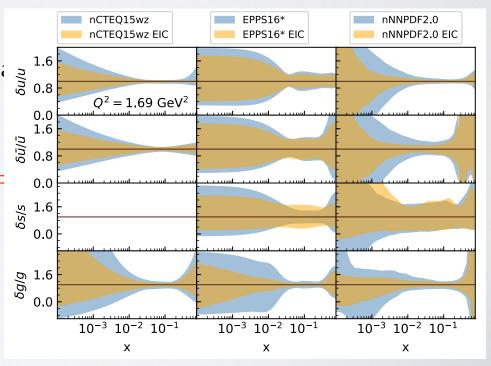
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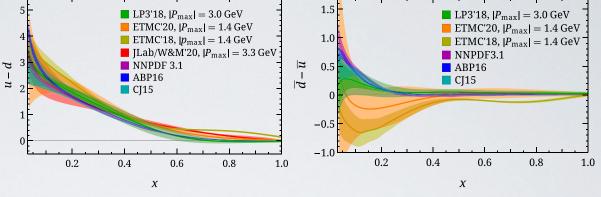
expected EIC impact

EIC Yellow Report, 2103.05419

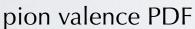


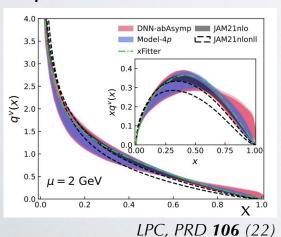
#### **Comparison with lattice**

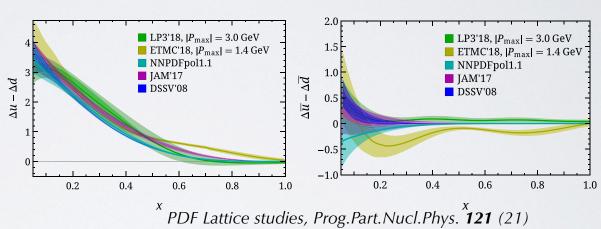
isovector unpolarized PDFs



isovector helicity PDFs

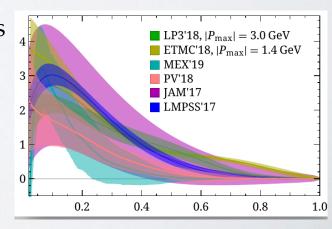




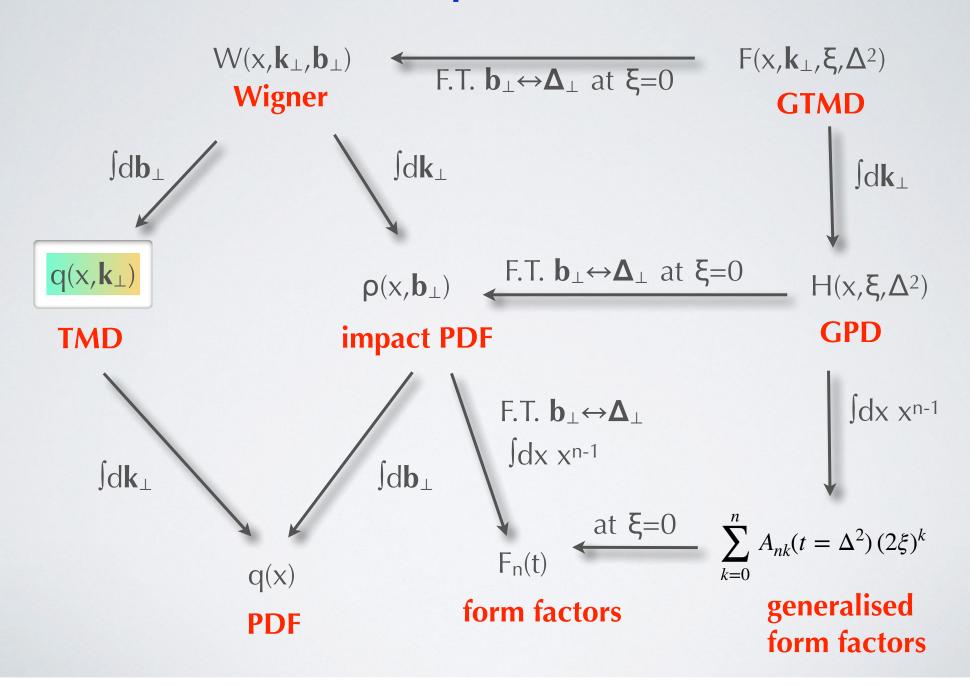


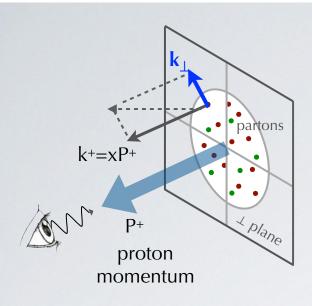
isovector transversity PDFs

talk by Delmar, Wednesday (Workshop 1)



#### Nucleon "femtography" Multi-dim. maps of Nucleon structure





# TMDs: Transverse Momentum Dependent PDFs (and FFs) from 1D to 3D mapping

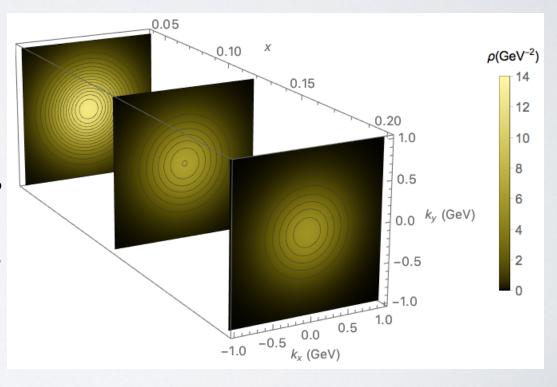
How do partons move and orbit inside hadrons?

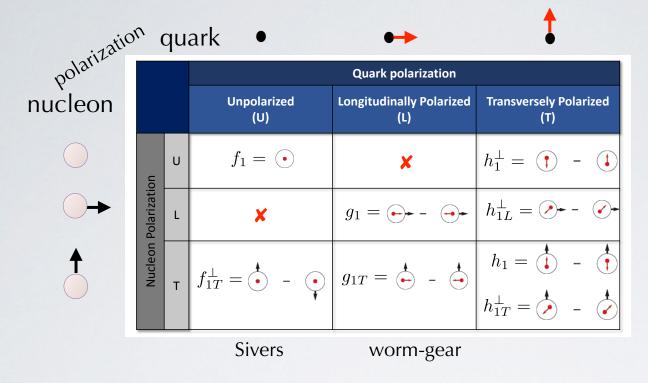
How wide is the distribution?

How does it change with x?

How does it change with flavor?

What if the hadron is polarized?





Mulders & Tangerman, N.P. B461 (96)Boer & Mulders, P.R. D57 (98)

**Boer-Mulders** 

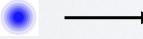
Kotzinian-Mulders

transversity

Each entry: - has a probabilistic interpretation;  $\times$  = forbidden by parity invariance

- is connected to deformations induced by spin-momentum correlations







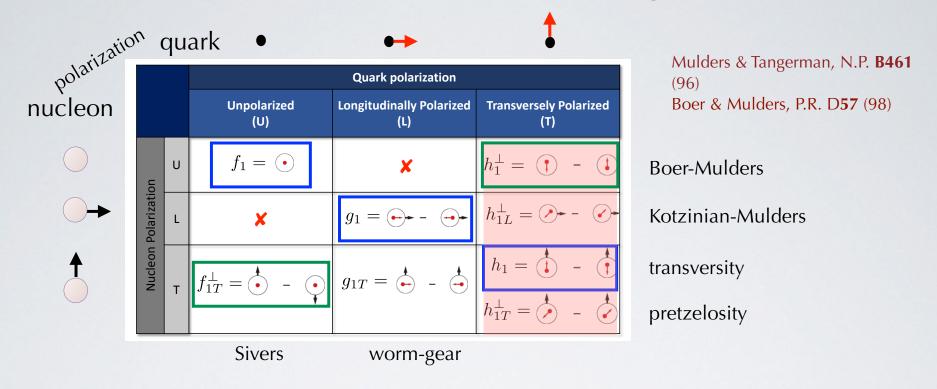






- can be extracted from a specific measurable spin asymmetry

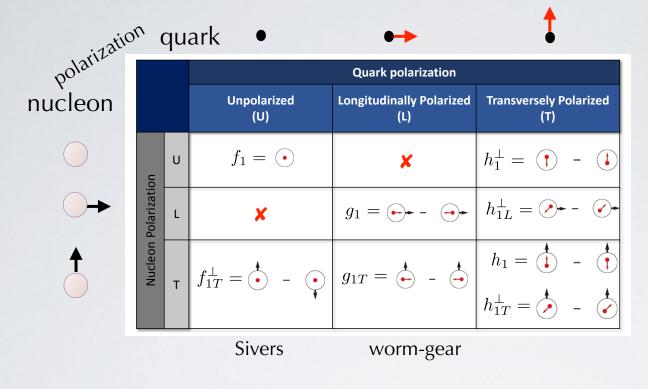
-  $f_1$  enters the denominator of each asymmetry  $\rightarrow$  influences all others



collinear PDFs surviving integration in k<sub>T</sub>

chiral-odd: need a chiral-odd partner in the cross section

naïve T-odd: vanishing without residual color final-state interactions
→ non universal, but in a predictable way..



Mulders & Tangerman, N.P. **B461** (96) Boer & Mulders, P.R. **D57** (98)

**Boer-Mulders** 

Kotzinian-Mulders

transversity

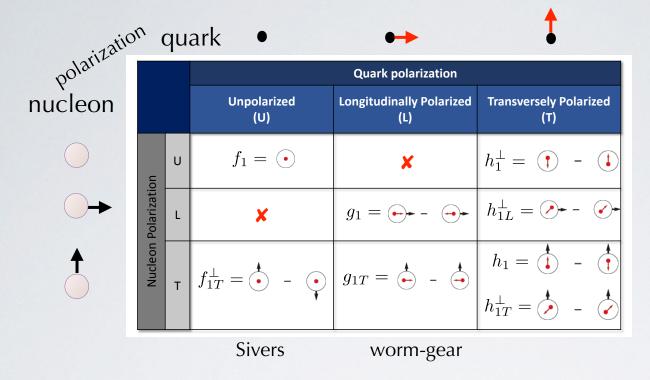
pretzelosity

Good knowledge of the x-dependence of f<sub>1</sub> and g<sub>1</sub>

Good knowledge of the k<sub>T</sub>-dependence of f<sub>1</sub> (also for pions)

Fair knowledge of  $f_{1T}^{\perp}$  and  $h_1$  (mainly, x-dependence)

Some hints about the others



Mulders & Tangerman, N.P. **B461** (96) Boer & Mulders, P.R. **D57** (98)

**Boer-Mulders** 

Kotzinian-Mulders

transversity

pretzelosity

similar table for TMD FFs; and also for gluon TMD PDFs, but with some differences

Gluons carry "two color charges" → intricate T-oddness: WW-type and dipole-type

Different color structures → different mechanisms → different processes to extract them

WW-type: heavy quarkonia, di-jets dipole-type: photon-jet factorization under investigation

Complementary role of colliders: RHIC, LHC, EIC, NICA

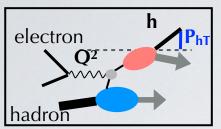
similar classification for subleading-twist TMD PDFs

talk by Scimemi, Wednesday

## Where TMDs?

Fourier Transf. to b<sub>T</sub> space: from convolution to simple product

#### SIDIS



factorization conditions

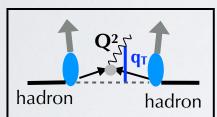
$$M^2 \ll Q^2 \qquad q_T^2 = \frac{P_{hT}^2}{z^2} \ll Q^2$$

$$\frac{d\sigma}{dxdzdq_TdQ} \sim \mathcal{H}^{\text{SIDIS}}(Q^2) \frac{1}{2\pi} \int_0^\infty db_T b_T J_0(b_T, q_T) \ \tilde{f}_1^q(x, b_T^2; Q^2) \ \tilde{D}_1^{q \to h}(z, b_T^2; Q^2) \ + \mathcal{O}(q_T^2/Q^2, M^2/Q^2)$$
hard part
$$\boxed{\text{TMDPDF}} \ \boxed{\text{TMDFF}}$$

Ji, Ma, Yuan, P.R. D**71** (05); Rogers & Aybat, P.R. D**83** (11)

Collins & Metz, P.R.L. 93 (04) 252001

#### Drell-Yan



factorization conditions

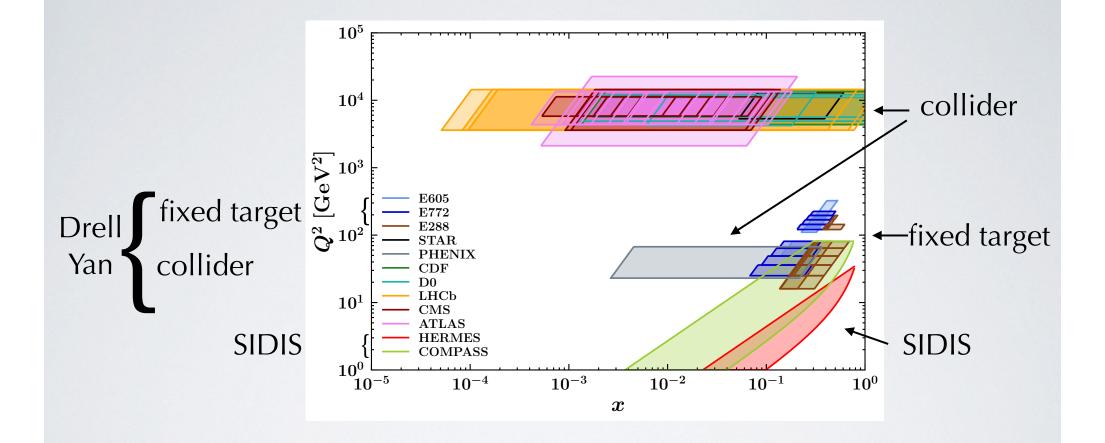
$$M^2 \ll Q^2 \qquad q_T^2 \ll Q^2$$

$$\frac{d\sigma}{dq_T dy dQ} \sim \begin{array}{c} \mathcal{H}^{\mathrm{DY}}(Q^2) \ \frac{1}{2\pi} \int_0^\infty db_T \ b_T \ J_0(b_T, q_T) \ \ \tilde{f}_1^{\bar{q}}(x_A, b_T^2; Q^2) \ \ \tilde{f}_1^q(x_B, b_T^2; Q^2) + \mathcal{O}(q_T^2/Q^2, M^2/Q^2) \\ \text{hard part} \end{array}$$

Collins, Soper, Sterman, N.P. B**250** (85) 199

Echevarria, Idilbi, Scimemi, JHEP 07 (12)

## Kinematic coverage of exp. data

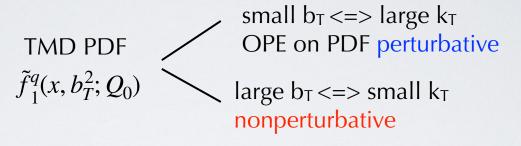


Complementarity between SIDIS and Drell-Yan For unpolarized TMDs  $O(\lesssim 2000)$  points, smaller but comparable with unpolarized PDFs [O(5000)]

## **Evolution of TMDs**

very different from PDFs: no cross talk between different x; no quark  $\leftrightarrow$  gluon transition two scales,  $\mu$  (virtuality cutoff) and  $\zeta$  (rapidity cutoff), usually taken  $\mu^2 = \zeta = Q^2$ 

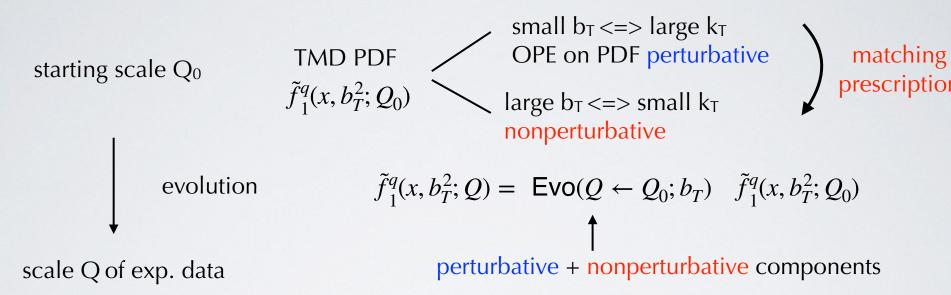
starting scale Q<sub>0</sub>



matching

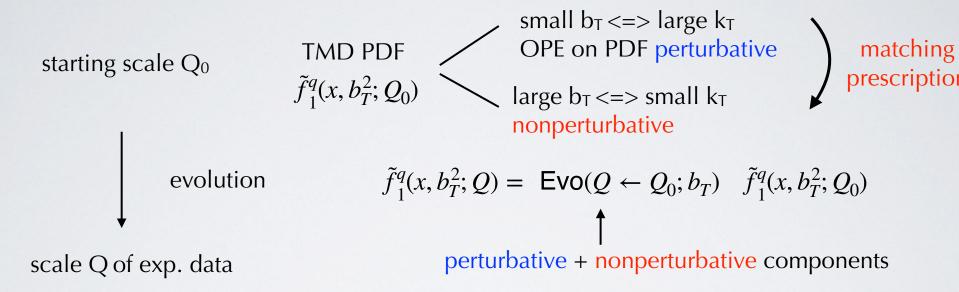
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+ hard part  ${\mathcal H}$  perturbative

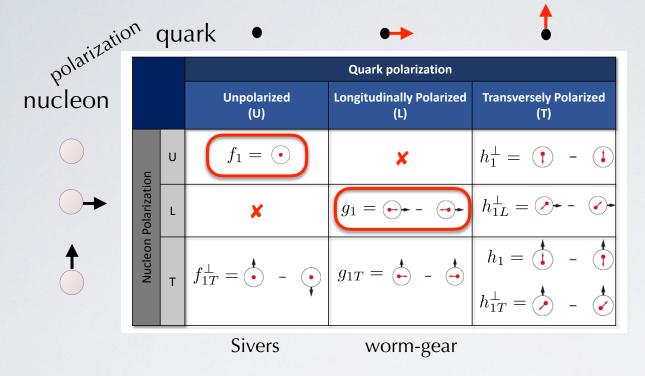
similarly for TMD FF  $ilde{D}_1^{q o h}$ 

every component in red is arbitrary => depends on parameters to be fitted to data

## Quality of TMD extraction:

- perturbative accuracy - size of data set and best  $\chi^2$ 

## quark TMD PDFs at leading twist



Mulders & Tangerman, N.P. **B461** (96) Boer & Mulders, P.R. D**57** (98)

**Boer-Mulders** 

Kotzinian-Mulders

transversity

pretzelosity

Just quick flash about very recent (~2024-25) results on unpolarized TMD PDF (  $f_1$  ) and helicity TMD PDF (  $g_1$  )

overview talk by Pitonyak, Thursday

## Most recent extractions of Nucleon TMD f<sub>1</sub>

	Accuracy	SIDIS	Drell-Yan	N of points	$\chi^2/N_{points}$	Flavor dep.
PV 2017 arXiv:1703.10157	NLL	>	V	8059	1.5	×
SV 2017 arXiv:1706.01473	N <sup>3</sup> LL	×	✓ (LHC)	309	1.23	X
BSV 2019 arXiv:1902.08474	N <sup>3</sup> LL	×	✓ (LHC)	45 <i>7</i>	1.17	X
SV 2019 arXiv:1912.06532	N <sup>3</sup> LL(-)	>	✓ (LHC)	1039	1.06	X
PV 2019 arXiv:1912.07550	N <sup>3</sup> LL	×	✓ (LHC)	353	1.07	X
MAPTMD 2022 arXiv:2206.07598	N <sup>3</sup> LL(-)	>	✓ (LHC)	2031	1.06	X
ART23 arXiv:2305.07473	N <sup>4</sup> LL(-)	×	✓ (LHC)	627	0.96	V
MAPTMD 2024 arXiv:2405.13833	N <sup>3</sup> LL	>	✓ (LHC)	2031	1.08	V
MAPNN 2025 arXiv:2502.04166	N <sup>3</sup> LL	×	✓ (LHC)	482	0.97	X
ART25 arXiv:2503.11201	N <sup>4</sup> LL(-)	>	✓ (LHC)	1209	1.05	V



# Most recent extractions of Nucleon TMD f<sub>1</sub>

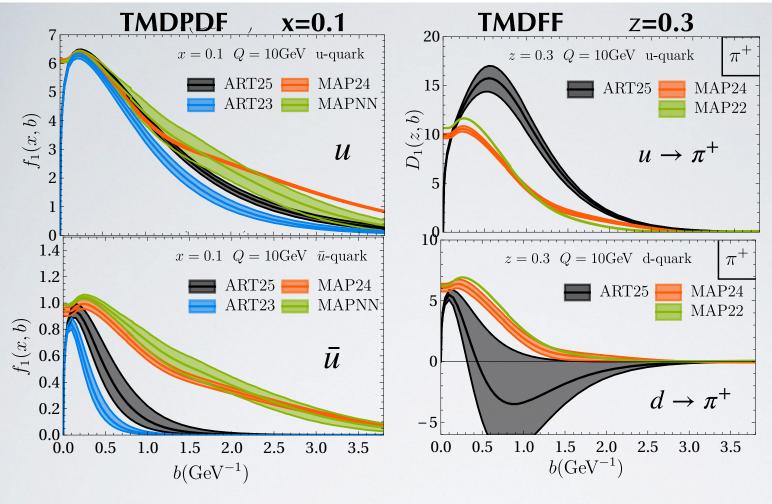
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PV 2017 arXiv:1703.10157	NLL	V	V	8059	1.5	×	
Global fits now able to discriminate between k <sub>T</sub> distributions of different flavor							
BSV 2019 arXiv:1902.08474	N <sup>3</sup> LL	×	✓ (LHC)	457	1.17	×	
SV 2019 arXiv:1912.06532	N <sup>3</sup> LL(-)	V	✓ (LHC)	1039	1.06	×	
PV 2019 arXiv:1912.07550	N <sup>3</sup> LL	×	✓ (LHC)	353	1.07	×	
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First extraction of TMDs using Neural Networks



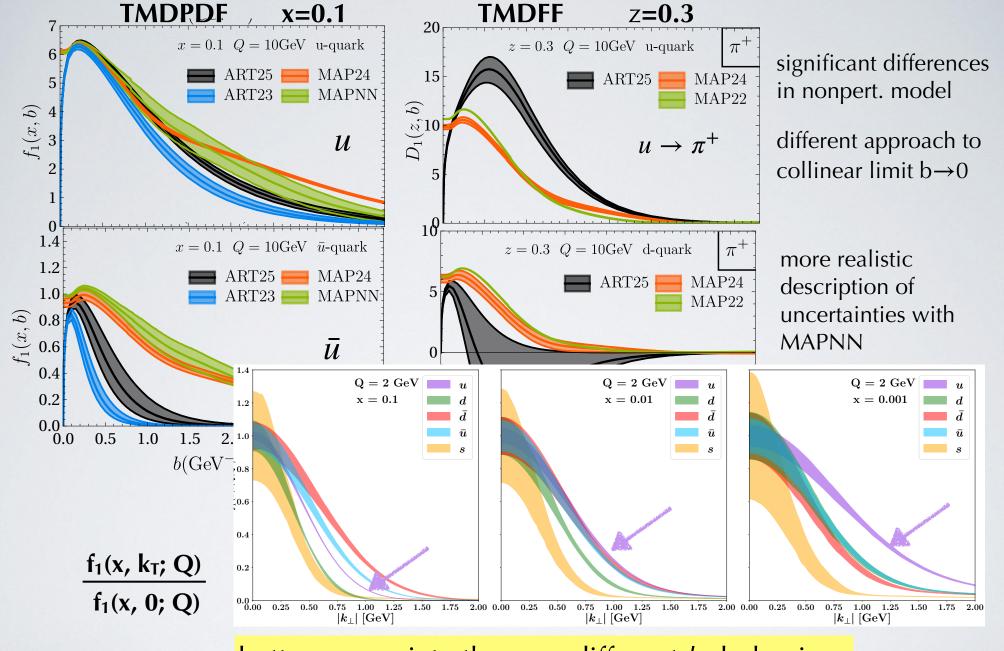


significant differences in nonpert. model

different approach to collinear limit  $b\rightarrow 0$ 

more realistic description of uncertainties with MAPNN

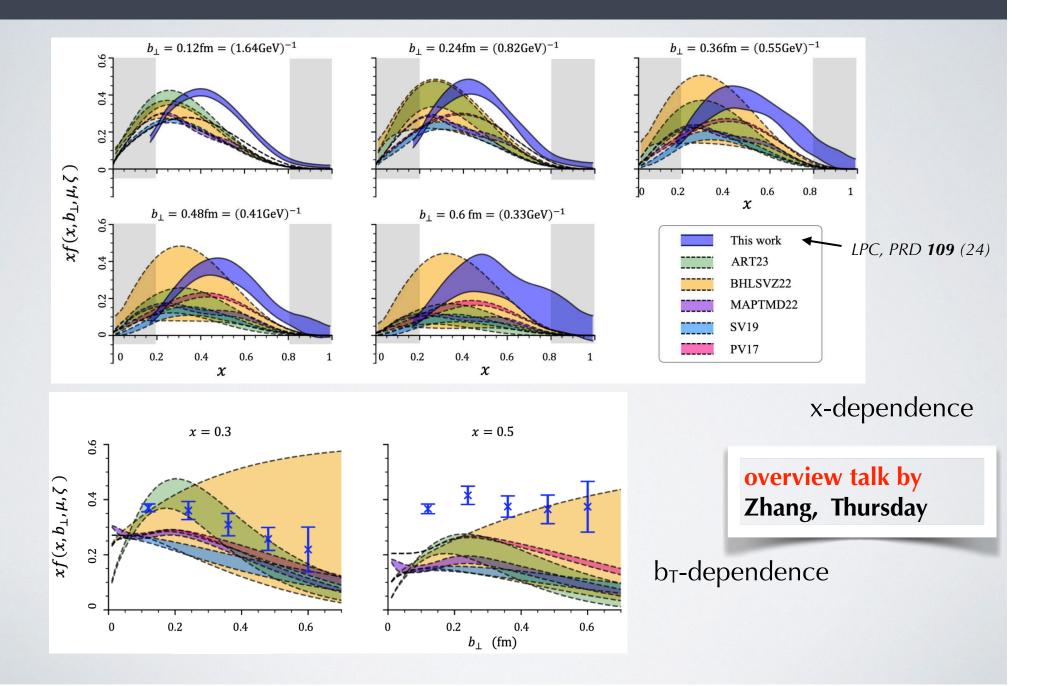
**overview talk by Pitonyak, Thursday** 



better appreciate the very different  $k_T$  behavior that changes with x

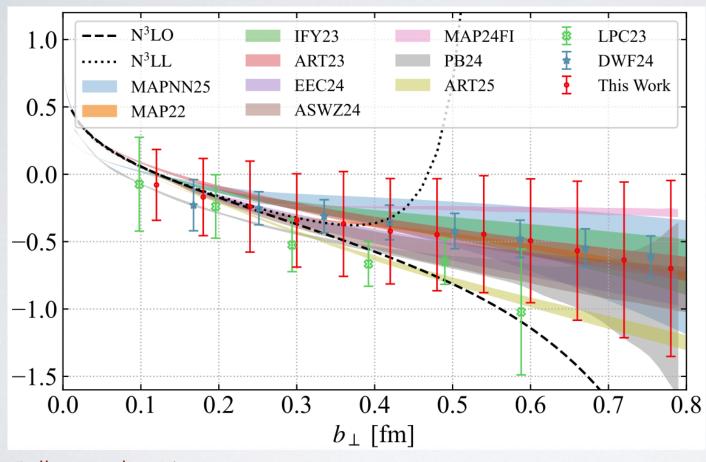
Bacchetta et al. (MAP), JHEP **08** (24), 2405.13833

# Comparison with lattice: TMDs in b<sub>T</sub> space



# Comparison with lattice: Collins-Soper kernel

universal flavor-independent drives evolution in rapidity  $\zeta$  perturbative + nonperturbative



Bollweg et al., arXiv:2504.04625

remarkable agreement with pheno ↔ lattice

#### Lattice

DWF24 Bollweg et al., arXiv:2403.00664

LPC23 Chu et al. (LPC), arXiv:2306.06488

ASWZ24 Avkhadiev et al., arXiv:2402.06725

#### **Pheno**

IFY23 (ResBos)
Isaacson et al.,
arXiv:2311.09916
EEC24
Kang et al.,
arXiv:2410.21435

PB24 Martinez et al., arXiv:2412.21116

+ MAPTMD-22, -24, -NN

+ ART-23, -25

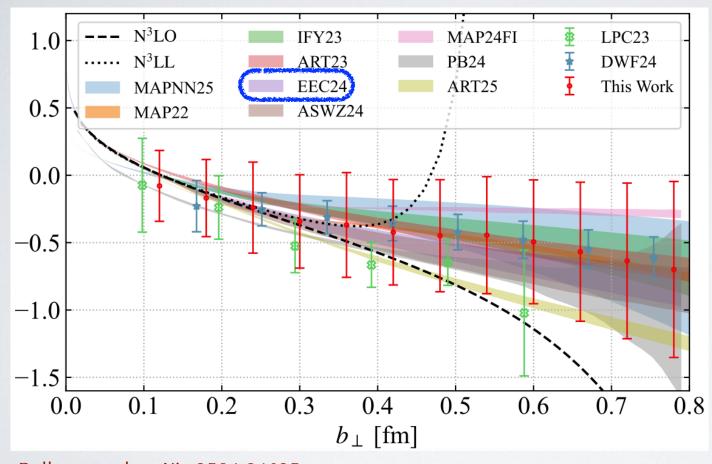
#### pQCD

N<sup>3</sup>LL Vladimirov, arXiv:1610.05791

N<sup>3</sup>LO Li&Zhu, arXiv:1604.01404

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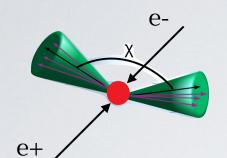
+ ART-23, -25

#### pQCD

N<sup>3</sup>LL Vladimirov, arXiv:1610.05791

N<sup>3</sup>LO Li&Zhu, arXiv:1604.01404

# Additional info: Energy-Energy Correlator in e+e-



$$EEC(\zeta) = \frac{d\sigma^{e+e-}}{d\zeta} = \sum_{ij} \int d\sigma_{e+e-\to ij+X} \frac{E_i E_j}{Q^2} \, \delta\left(\zeta - \frac{1 - \cos\chi_{ij}}{2}\right)$$

energy-weighted e+e- cross section measures correlation of two energy flows along specific directions

for 
$$\zeta \rightarrow 1 \ (\chi \rightarrow \pi)$$
 factorized formula

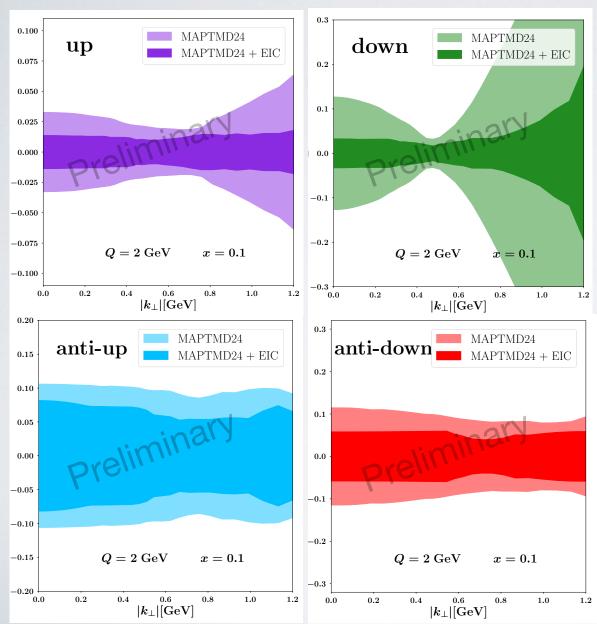
for 
$$\zeta \to 1$$
 ( $\chi \to \pi$ )
factorized formula:
$$\frac{d\sigma}{d\zeta} = \frac{\sigma_0}{8} \sum_{q\bar{q}} H_{q\bar{q}} \int db_T J_0(b_T Q \sqrt{1-\zeta}) \text{ Jet}^q \text{ Jet}^{\bar{q}}$$

$$\operatorname{Jet}^{q}(b_{T}) = \sum_{\text{hadrons}} \int_{0}^{1} dz \, z^{3} \, \operatorname{TMDFF}^{q \to h}(z, b_{T}) = \operatorname{Jet}_{\text{pert}}^{q} + \operatorname{Jet}_{NP}^{q} \longleftarrow \text{ fitted to data}$$

sensitivity only to Collins-Soper kernel

use OPAL, SLD, TOPAZ, TASSO, DELPHI,... data to constrain CS kernel

# The EIC impact at x=0.1, Q=2 GeV



 MAPTMD24
 2031

 EIC
 # pts.
 lumi [fb-1]

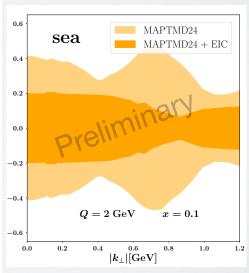
 5x41
 1273
 2.85

 10x100
 1611
 51.3

18x275 1648 10

(simulation campaign May '24, **only π+ production**)

L. Rossi, Ph.D. Thesis and in preparation

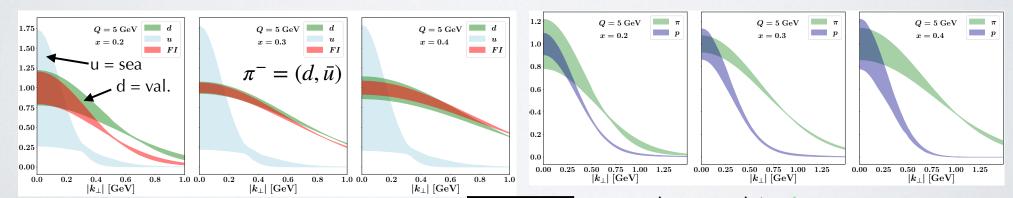


## Extractions of Pion TMD f<sub>1</sub>

	Accuracy	Drell- Yan	N of points	$\chi^2/N_{points}$	Flavor dep.
Wang et al., 2017 JHEP <b>08</b> (17) 137	NLL	>	96	1.61	×
Vladimirov 2019 JHEP <b>10</b> (19) 090	NNLL'	>	80	1.44	×
MAPTMDPion22 PRD <b>107</b> (23)	N <sup>3</sup> LL(-)	>	138	1.55	×
JAM23 PRD <b>108</b> (23)	NNLL	>	93	1.37	×
MAPTMDPion25 2509.25098	N <sup>3</sup> LL	>	101	1.22	<b>~</b>

simultaneous extraction ofpion PDF & TMDproton TMD

**MAPTMDPion25** First fit sensitive to flavor dep. of  $k_T$  distributions: valence - sea, 6 parameters E615, E537  $\pi^-$ -N Drell-Yan, N TMDs from MAPTMD24 with same CS kernel



 $f_1(x, k_T; Q)$ 

 $f_1(x, 0; Q)$ 

valence and sea have different k<sub>T</sub> distributions

FI = flavor-independent ~intermediate between valence & sea down quark in **pion** vs. **proton** in  $k_T$  space, proton is narrower than pion

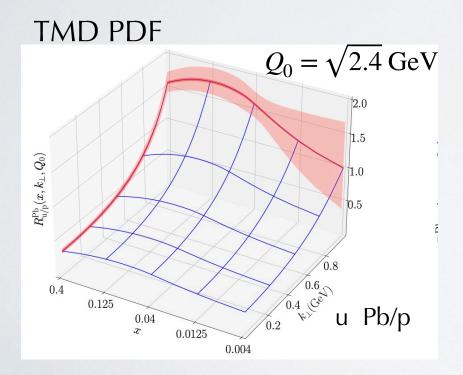
same finding in JAM23, lattice, and chiral QCD

## Nuclear TMD f<sub>1</sub><sup>A</sup>

#### Global fit using TMD factorization

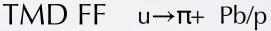
Alrashed et al., PRL 129 (22)

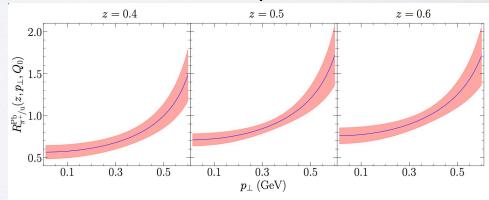
Alrashed et al., arXiv:2312.09226



Collaboration	Process	Baseline	seline Nuclei		$\chi^2$
JLAB <u>[49]</u>	$\mathrm{SIDIS}(\pi)$	D C, Fe, Pb		36	41.7
HERMES <u>[40]</u>	$\mathrm{SIDIS}(\pi)$	D	Ne, Kr, Xe	18	10.2
RHIC <u>[43]</u>	DY	p	$\mathbf{A}\mathbf{u}$	4	1.3
E772 [41]	DY	D	C, Fe, W	16	40.2
E866 [42]	DY	Be	Fe, W	28	20.6
CMS [63]	$\gamma^*/Z$	N/A	Pb	8	10.4
ATLAS <u>[83]</u>	$\gamma^*/Z$	N/A	Pb	7	13.3
Total				117	137.8

 $\chi^2/N_{data} = 1.275$ 

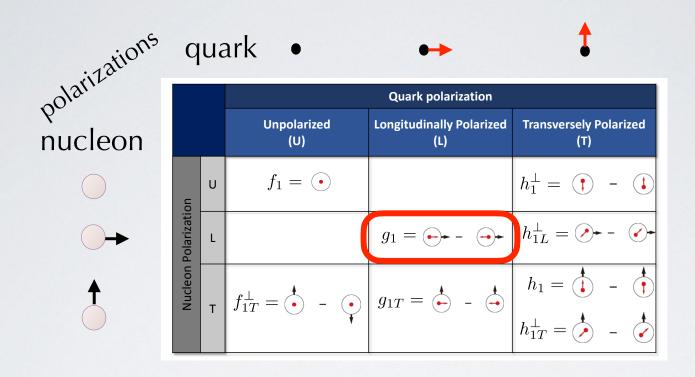




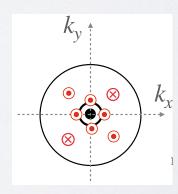
(anti-)shadowing visible

**k**<sup>⊤</sup> broadening

# **Helicity TMD PDF**



- How does polarization of quarks distort their k<sub>T</sub>?
- Do quarks with spin parallel to proton spin have larger / smaller k<sub>T</sub> than those with spin antiparallel?

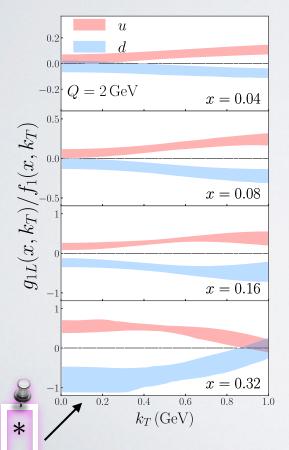


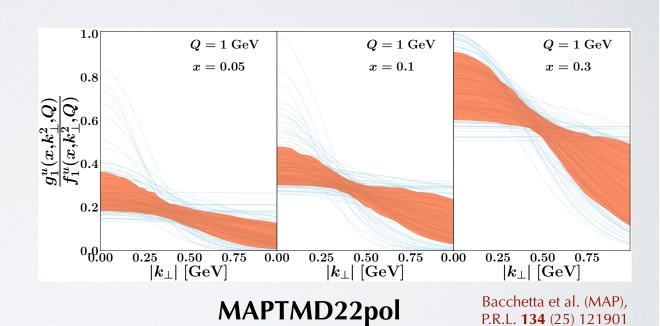
	Accuracy	SIDIS	Drell-Yan	N of points	<b>X</b> <sup>2</sup> /N	Flavor dep.
MAPTMD22pol Bacchetta et al. (MAP) P.R.L. 134 (25) 121901	NNLL	/	×	291	1.09	×
<b>YLSZM</b> Yang et al. (TNT) P.R.L. <b>134</b> (25) 121902	NNLL	>	X	253	0.74	×

 $\leftarrow$  enforced  $|g_1| \le f_1$ 

accuracy limited by gluon Wilson coeffs.

data from SIDIS DSA A<sub>1</sub>





**YLSZM** 

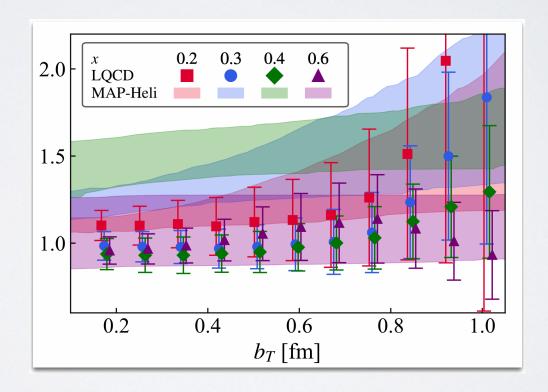
Yang et al., P.R.L. **134** (25) 121902

# Comparison with lattice: isovector g<sub>1</sub>/f<sub>1</sub> ratio

input from lattice

$$\frac{g_{1L}^{\Delta u_{+} - \Delta d_{+}}(x, b_{T}; \mu, \zeta)}{g_{A} f_{1}^{u_{v} - d_{v}}(x, b_{T}; \mu, \zeta)} \qquad \mu = \sqrt{\zeta} = 1.62 \text{ GeV}$$

Bollweg et al., arXiv:2505.18430



# Comparison with lattice: isovector g<sub>1</sub>/f<sub>1</sub> ratio

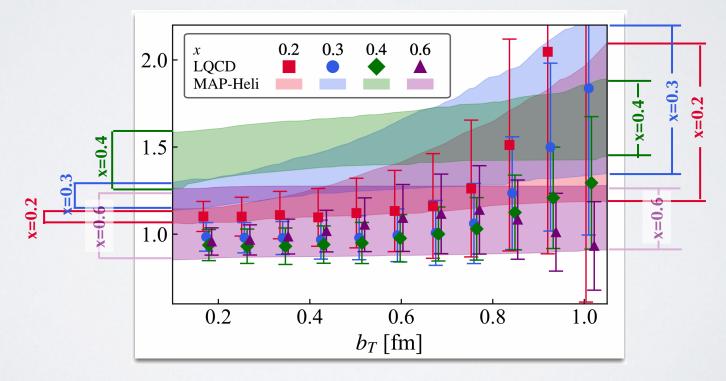
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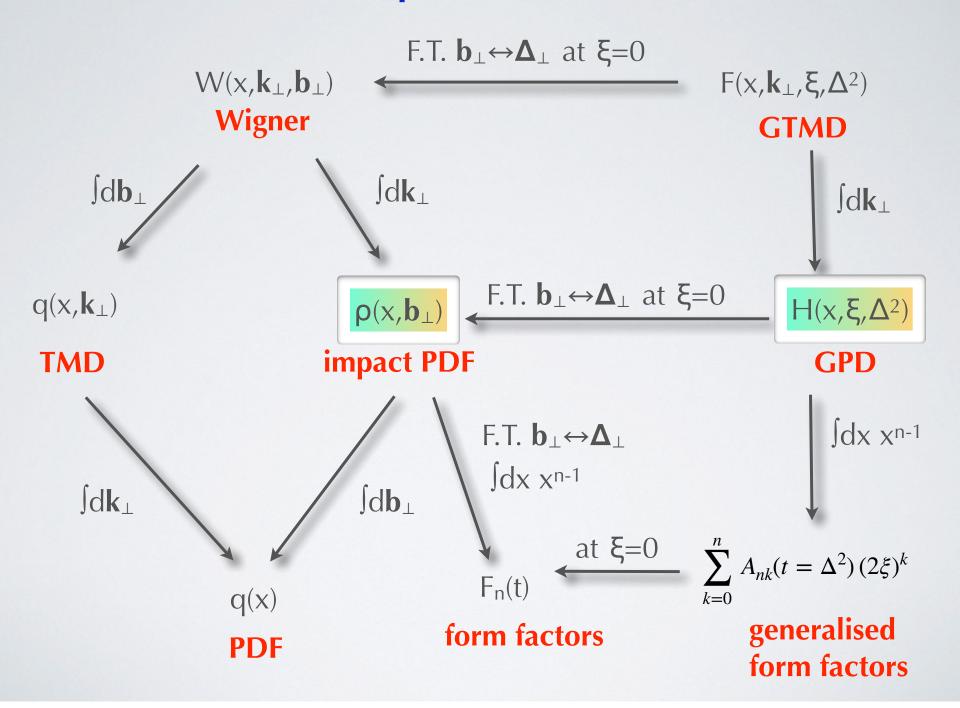
$$\mu = \sqrt{\zeta} = 1.62 \text{ GeV}$$

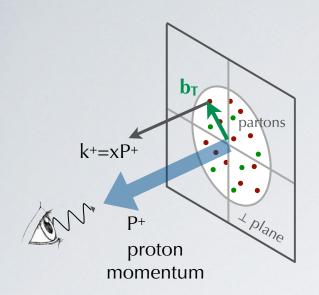
compatibility for x=0.2, 0.6 partial " for x=0.3, 0.4

Bollweg et al., arXiv:2505.18430



## Multi-dim. maps of Nucleon structure



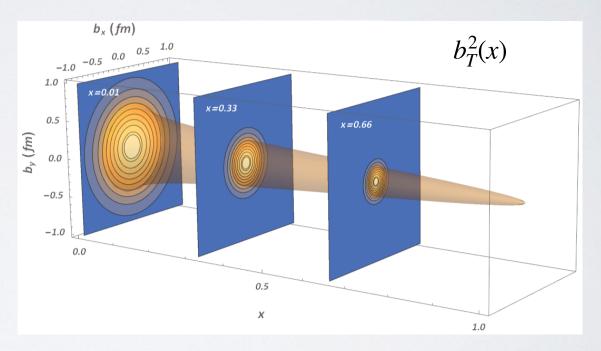


# **GPDs:** access to 1D (momentum) x 2D spatial distribution of partons inside hadrons

Where are partons at a given momentum x?

2D spatial density at given momentum x

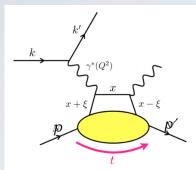
Hadron tomography



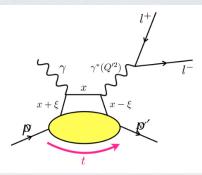
for  $x\rightarrow 1$ , active parton carries the whole momentum and represents the center of momentum; low-x sea quarks populate the periphery

Lorcé et al. (Encyclopedia Part. Phys.), 2507.12664 Dupré et al., PRD **95** (17)

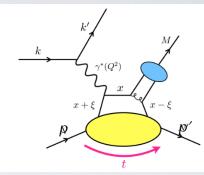
## Where GPDs?



Deeply Virtual **Compton Scattering** (DVCS)



Time-like **Compton Scattering** (TCS)



Deeply Virtual Meson Production (DVMP)

Th overview by Sznajder, Thursday exp. talks by Niccolai, Wednesday Van Hulse, Thursday

Factorization for  $|t| \ll Q^2$ , s

Universality of GPDs in different processes

exclusive processes  $\rightarrow$  GPDs are scattering amplitudes, depending on GPD(x,  $\xi$ , t)

$$x = \frac{(k+k')^+}{(p+p')^+} = \frac{\bar{k}^+}{P^+}$$

average longitudinal momentum of partons

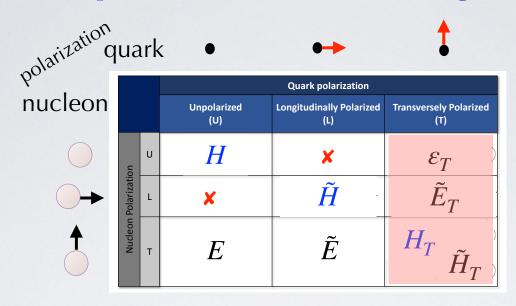
$$x = \frac{(k+k')^{+}}{(p+p')^{+}} = \frac{\bar{k}^{+}}{P^{+}} \qquad \qquad \xi = \frac{(p-p')^{+}}{(p+p')^{+}} = -\frac{\Delta^{+}}{2P^{+}}$$

skewness: fraction of longitudinal momentum transfer

$$t = (p - p')^2 = \Delta^2$$

momentum transfer squared (t-channel)

## quark GPDs at leading twist



similar table for gluons

GPDs in blue in the collinear limit reduce to PDFs:  $H^q(x,0,0) \to f_1^q(x)$ ,  $\tilde{H}^q \to g_1^q$ ,  $H_T^q \to h_1^q$ Chiral-odd GPDs not accessible in DVCS

At  $\xi=0$ , Fourier Transf.  $\Delta_{\perp}\leftrightarrow b_{\perp}$  gives a probability density: the impact parameter PDF

no data in this limit, extrapolation needed

→ model dependence

$$\rho(x, \mathbf{b}_{\perp}) = \int \frac{d\mathbf{\Delta}_{\perp}}{(2\pi)^2} e^{-i\mathbf{b}_{\perp} \cdot \mathbf{\Delta}_{\perp}} H^q(x, 0, \Delta^2)$$

Burkardt, PRD **62** (00)

$$\langle b_T^2(x) \rangle \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}^{0.5} \\ \begin{bmatrix} \frac{3}{2}$$

# Polynomiality of GPDs and spin sum rule

$$\int_{-1}^{1} dx \, x^{m} \, H(x, \xi, t) = \sum_{j=0}^{\left[\frac{m}{2}\right]} \xi^{2j} \, A_{2j}(t) + \operatorname{mod}(m, 2) \, \xi^{m+1} \, C_{m+1}(t) \qquad \text{very general principle (Lorentz covariance)}$$

$$\int_{-1}^{1} dx \, x^{m} \, E(x, \xi, t) = \sum_{j=0}^{\left[\frac{m}{2}\right]} \xi^{2j} \, B_{2j}(t) - \operatorname{mod}(m, 2) \, \xi^{m+1} \, C_{m+1}(t) \qquad \qquad \text{Radyushkin, P.L. B449 (99)}$$
81

m=0 → connection to elastic form factors (only for chiral-even GPDs)

$$\int_{-1}^{1} dx \, H^q(x,\xi,t) = F_1^q(t) \qquad \int_{-1}^{1} dx \, E^q(x,\xi,t) = F_2^q(t)$$
 similarly,  $\tilde{H} \leftrightarrow g_A(t)$ ,  $\tilde{E} \leftrightarrow g_P(t)$ 

# Polynomiality of GPDs and spin sum rule

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81

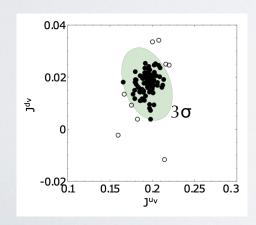
 $m=0 \rightarrow connection to elastic form factors$ (only for chiral-even GPDs)

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similarly,  $\tilde{H} \leftrightarrow g_{A}(t)$ ,  $\tilde{E} \leftrightarrow g_{P}(t)$ 

$$m=1 \rightarrow Ji \text{ spin sum rule}$$

m=1 
$$\rightarrow$$
 Ji spin sum rule  $J^q = \frac{1}{2} \int_{-1}^{1} dx \, x \, \left[ H^q(x, \xi, 0) + E^q(x, \xi, 0) \right] = A^q(0) + B^q(0)$ 

Ji, P.R.L. 78 (97) 610



recent result fitting elastic FF and lattice data compatible with

Diehl-Kroll GPD model (fitted to elastic data)

Bacchetta & M.R. (fitting SIDIS data + Burkardt's lensing function Sivers ↔ GPD E)

$$J^{u_v} = 0.195 \pm 0.010$$
 Cichy et al., PRD **110**  $J^{d_v} = 0.0173 \pm 0.0046$  (24)

$$J^{u_v} = 0.230^{+0.009}_{-0.024}$$
 EPJ **C73** (13)  $J^{d_v} = -0.004^{+0.010}_{-0.016}$ 

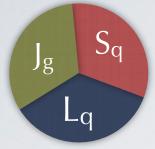
$$J^{u} = 0.229 \pm 0.002^{+0.008}_{-0.012}, J^{\bar{u}} = 0.015 \pm 0.003^{+0.001}_{-0.000}$$

$$J^{d} = -0.007 \pm 0.003^{+0.020}_{-0.005}, J^{\bar{d}} = 0.022 \pm 0.005^{+0.001}_{-0.000}$$

Bacchetta & M.R., PRL 107 (11)

## pizza "Tre Stagioni"

pizza "Quattro Stagioni"



© M. Burkardt

PRL 78 (97) 610

Ji sum rule

$$1/2 = \underbrace{L_q + S_q + J_g}_{J_q}$$

- manifestly gauge invariant
- "ready" for lattice (Jq, Jg from moments of GPD)
- \* Lq does not satisfy canonical relations

intuitive L<sub>q</sub> - L<sub>q</sub> difference:

color Lorentz force acting on active quark and

producing a torque along its trajectory

Burkardt, PRD 88 (13)



NPB 337 (90) 509

Jaffe-Manohar

$$1/2 = L_q + S_q + L_g + S_g$$

- $L_q$  and  $L_g$  satisfy canonical relations
- \* each term gauge dependent (but measurable)
- $L_q$  and  $L_g$  can be computed from Wigner distributions, or from twist-3 GPDs

Lorcé & Pasquini, PRD 84 (11)

also, exploratory calculations on lattice

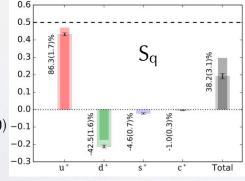
Engelhardt, PRD 95 (17); 102 (20)

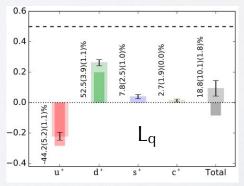
dark bars: connected diagrams

Lattice calculations

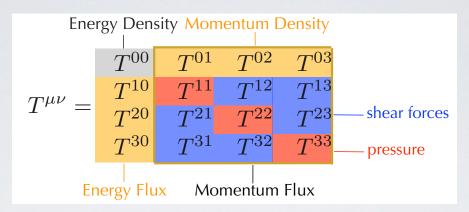
of  $S_q$  and  $L_q = (J_q - S_q)$ 

Alexandrou et al., PRD **101** (20) -0.1





## **QCD Energy-Momentum Tensor (EMT)**



$$\langle P' | T_{\mu\nu}^{q,g} | P \rangle = \bar{u}(P') \left[ \frac{A^{q,g}(t)}{M_N} \frac{P_{\mu}P_{\nu}}{M_N} + \frac{J^{q,g}(t)}{M_N} \frac{i(P_{\mu}\sigma_{\nu\rho} + P_{\nu}\sigma_{\mu\rho}) \Delta^{\rho}}{2M_N} + \frac{D^{q,g}(t)}{5M_N} \frac{\Delta_{\mu}\Delta_{\nu} - g_{\mu\nu} \Delta^2}{5M_N} + \bar{c}_{q,g}(t) g_{\mu\nu} \right] u(P)$$

#### Charges

Aq.g (0) = mass and momentum of partons Jq.g (0) = angular momentum of partons Dq.g (0) = "D-term" related to internal stability  $\bar{c}^{q,g}$  = "anomalous" contribution to mass (trace  $T^{\mu}_{\mu}$ ) Connection to GPDs

$$\int dx \, x \, H(x, \xi, t) = \frac{A(t) + \frac{4}{5} \, D(t) \, \xi^2}{\int dx \, x \, E(x, \xi, t) = \frac{2J(t) - A(t) - \frac{4}{5} \, D(t) \, \xi^2}$$

## Mechanical properties of the Nucleon

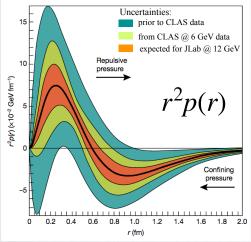
Fourier Transf. of EMT in Breit frame

$$T_{ij}(\mathbf{r}) = s(r) \left( \frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + p(r) \delta_{ij}$$
  
shear force isotopic pressure

Lorcé et al, P.L. **B776** (18) 38 Lorcé et al., E.P.J. **C79** (19) 1 Polyakov, P.L. **B555** (03) 57

$$s(r) = -\frac{1}{2M}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}$$
 [ F.T. of D(t) ]

$$p(r) = \frac{1}{6M} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr}$$
 [ F.T. of D(t) ]



Girod et al., Nature **557** (18) 7705

## **Mechanical properties of the Nucleon**

Fourier Transf. of EMT in Breit frame

$$T_{ij}(\mathbf{r}) = s(r) \left( \frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + p(r) \delta_{ij}$$
  
shear force isotopic pressure

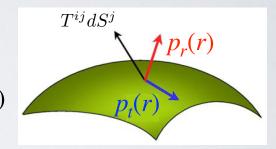
Lorcé et al., P.L. **B776** (18) 38 Lorcé et al., E.P.J. **C79** (19) 1 Polyakov, P.L. **B555** (03) 57

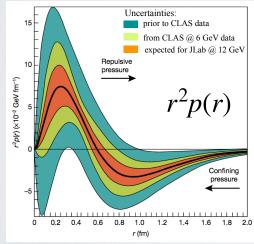
$$s(r) = -\frac{1}{2M}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}$$
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$$p(r) = \frac{1}{6M} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr}$$
 [ F.T. of D(t) ]

$$p_r(r) = p(r) + \frac{2}{3}s(r)$$

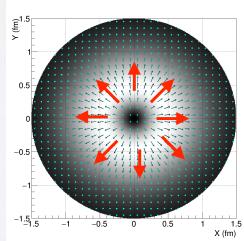
$$p_t(r) = p(r) - \frac{1}{3}s(r)$$



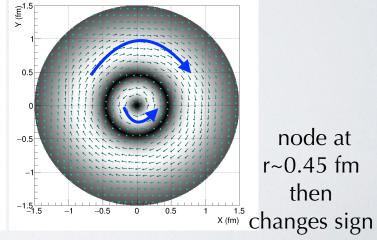


Girod et al., Nature **557** (18) 7705

#### radial pressure p<sub>r</sub>(r)



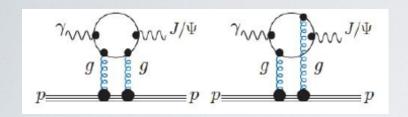
#### tangential pressure p<sub>t</sub>(r)



always positive, fades away with r

Burkardt et al, Prog.Part.Nucl.Phys. **131** (23)

## **Gluonic EMT form factors**



$$\int \int V DVMP \text{ suggested to address the gluon EMT} 
g g g g g 
p 
$$\frac{d\sigma}{dt} \sim |\mathcal{M}_{\gamma p \to J/\psi}(t)|^2 \sim |\langle P'|T_{\mu}^{\mu g}|P\rangle|^2 \xrightarrow{t \to 0} \begin{cases} A^g(0) \\ D^g(0) \\ \bar{c}^g(0) \end{cases}$$$$

$$D^g(0) \rightarrow \text{gluon contribution to} \qquad \text{mechanical radius} \qquad r_p = \sqrt{\frac{\int d\mathbf{r} r^2 p_r(r)}{\int d\mathbf{r} p_r(r)}}$$

$$\text{mass radius} \qquad r_m = \sqrt{\frac{\int d\mathbf{r} r^2 T^{00}(r)}{\int d\mathbf{r} T^{00}(r)}}$$

lattice / pheno / model analyses → mechanical and mass radii < charge radius gluon contribution extends proton size w.r.t. quark

Hackett et al, PRL 132 (24)

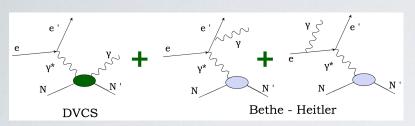
 $\bar{c}^g(0) \rightarrow \text{gluon contribution to proton mass from trace anomaly}$ 

talk by Castelli, Wednesday

**Problems:** - factorization th. holds only for  $V = J/\psi$ , Y production at threshold

- lowest-order approximation in  $M_N$  /  $M_V$  and  $\alpha_s$ 

## **Deconvolution problem**



GPDs from interference amplitude DVCS - Bethe-Heitler

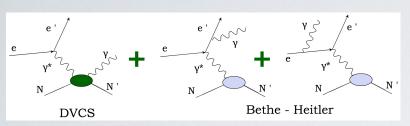
interference cross section contains convolutions of perturbative kernels and Compton Form Factors (CFF)

$$CFF(\xi, t) = PV \int_{-1}^{1} dx \frac{GPD(x, \xi, t)}{x - \xi} - i\pi GPD(x = \pm \xi, \xi, t) + o\left(\frac{1}{Q^{2}}\right)$$
Re(CFF)

How to deconvolute GPDs from CFFs ?

problem of shadow GPDs Bertone et al, PRD 103 (21) Cichy et al, PRD 110 (24)

## **Deconvolution problem**



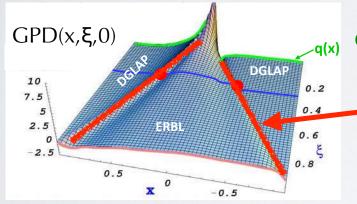
GPDs from interference amplitude DVCS - Bethe-Heitler

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$$CFF(\xi, t) = PV \int_{-1}^{1} dx \frac{GPD(x, \xi, t)}{x - \xi} - i\pi GPD(x = \pm \xi, \xi, t) + o\left(\frac{1}{Q^2}\right)$$

$$Re(CFF)$$
How to dec

How to deconvolute GPDs from CFFs?



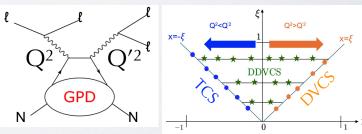
 $\mathsf{GPD}(x,0,0) = \mathsf{PDF}(x)$ 

Access only to the  $(x = \pm \xi, \xi, t)$  dependence

problem of shadow GPDs Bertone et al, PRD 103 (21) Cichy et al, PRD 110 (24)

**Alternatives**: - access  $x \neq \pm \xi$  dependence with **DDVCS** (proposal approved (C1) by JLab PAC53)

- use lattice



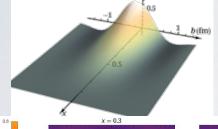
Deja et al, PRD **107** (23) Alvarado et al., PRC **111** (25)

### **Lattice calculations of GPDs**

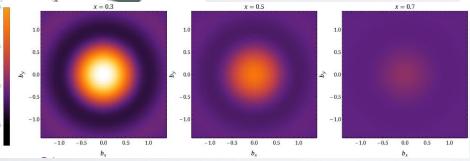
Access x-dependence of GPDs using LaMET theory, matching parton correlators in Euclidean space to Light-cone ones with perturbative coefficients and for large

hadron momenta  $P_z \gg M$ 

$$q(x, \mathbf{b}) = \int d\mathbf{\Delta}_{\perp} e^{-i\mathbf{b}\cdot\mathbf{\Delta}_{\perp}} H^{q}(x, 0, t = -\mathbf{\Delta}_{\perp}^{2})$$



Lin, PRL 127 (21)



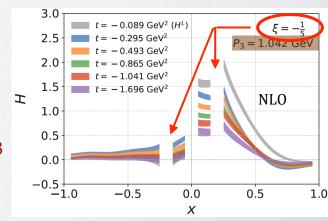
 $E(x, \xi = 0, Q^2)$   $E(x, \xi = 0, Q^2)$ 0.5  $Q^2(\text{GeV}^2)$ 0.5  $Q^2(\text{GeV}^2)$ 1.0 0.5  $Q^2(\text{GeV}^2)$ 

Also for  $\tilde{H}$  and for pion H

Lin (MSULatt), PLB 824 (22); 846 (23)

talks by Lin, Miller, Wednesday Cichy, Zhang overview, Thursday new: at  $\xi \neq 0$  using asymmetric frame

ETMC, arXiv:2508.17998



### **Conclusions**

Significant progresses in many areas

Complementarity between experiments, phenomenology, theory, and lattice

New theory developments: precision (higher-order correction), novel processes..

New robust methodologies and data science tools for reliable phenomenological information

Significant contributions from lattice, guiding both theory and experiments in "intricate" regions

Future more precise data, particularly from EIC, can turn the field to a higher level

Enjoy the rest of the workshop!

