RUB

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Chiral symmetry and nuclear interactions: New developments

„The purpose of this work is not to improve our detailed picture of nuclear forces, […] but rather to take a first step toward identifying those **aspects of nuclear forces** and pion-nucleon interactions **that can be derived from the symmetry properties of quantum chromodynamics**.“

Steven Weinberg, NPB 363 (91)

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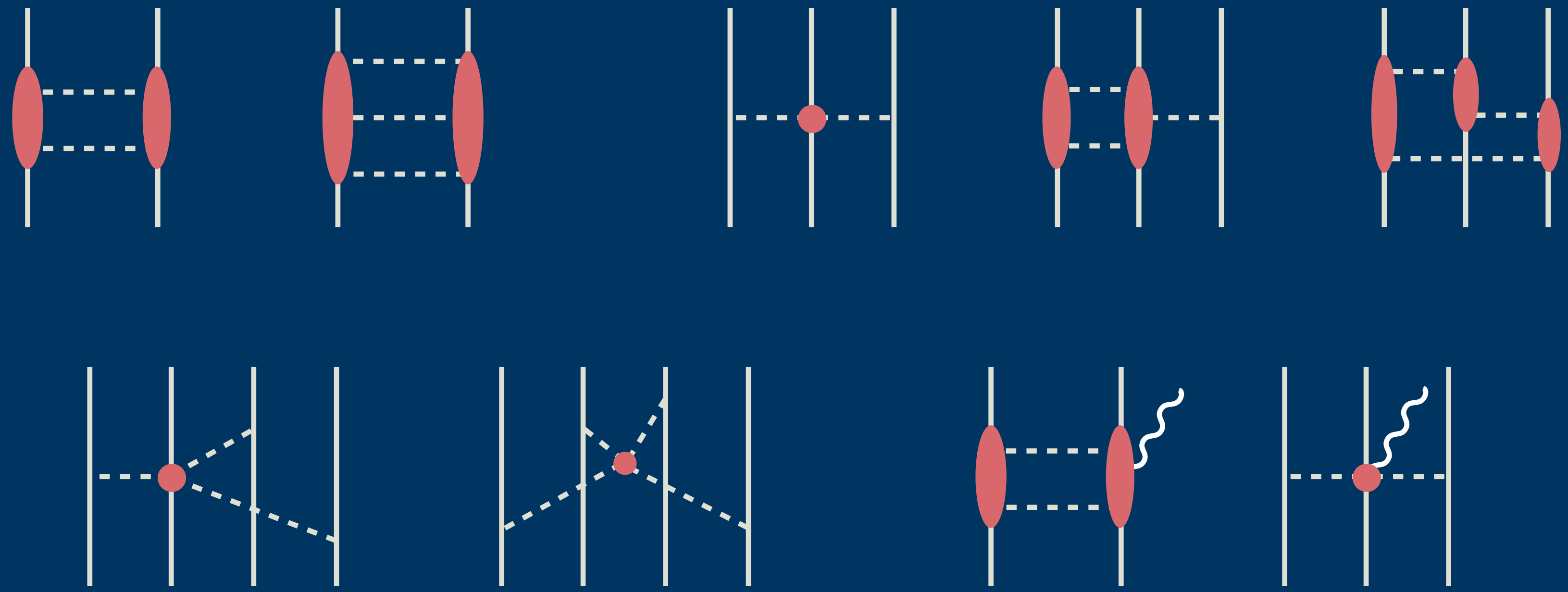
+ chiral symmetry (predictive power, connection to QCD)



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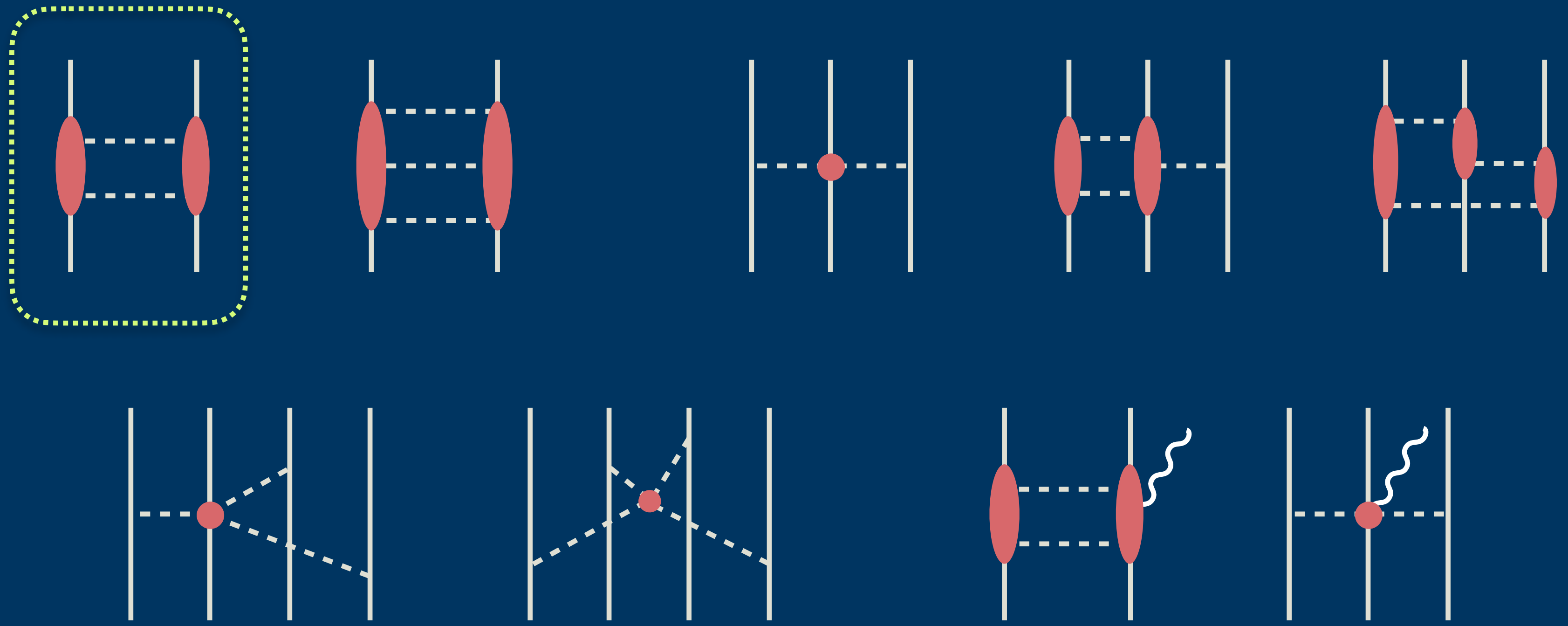

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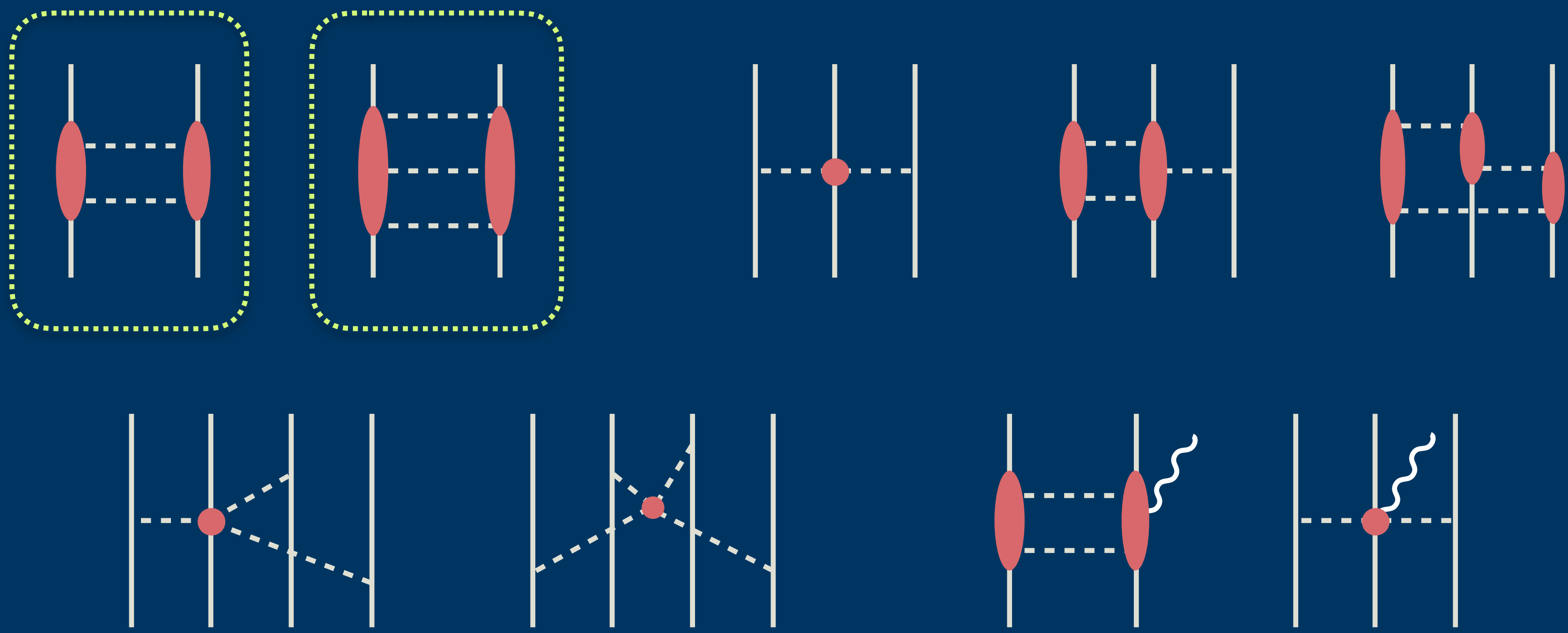

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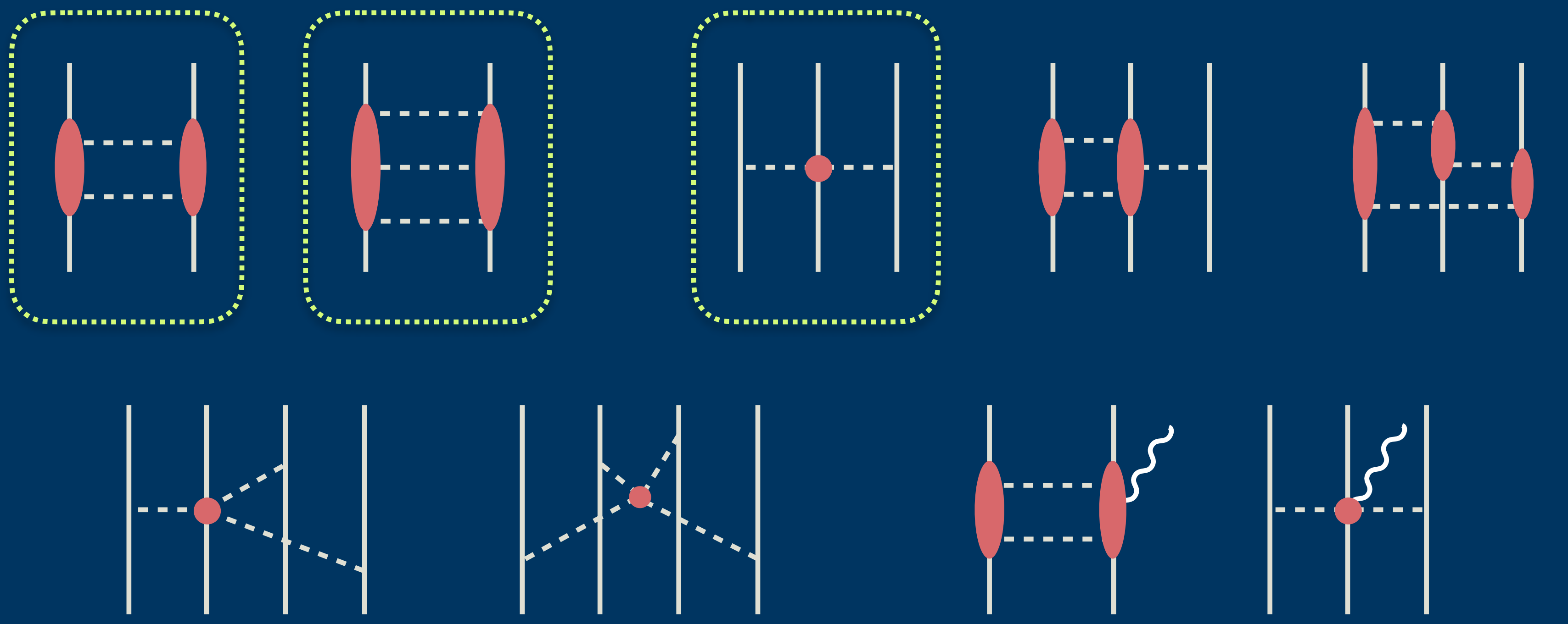

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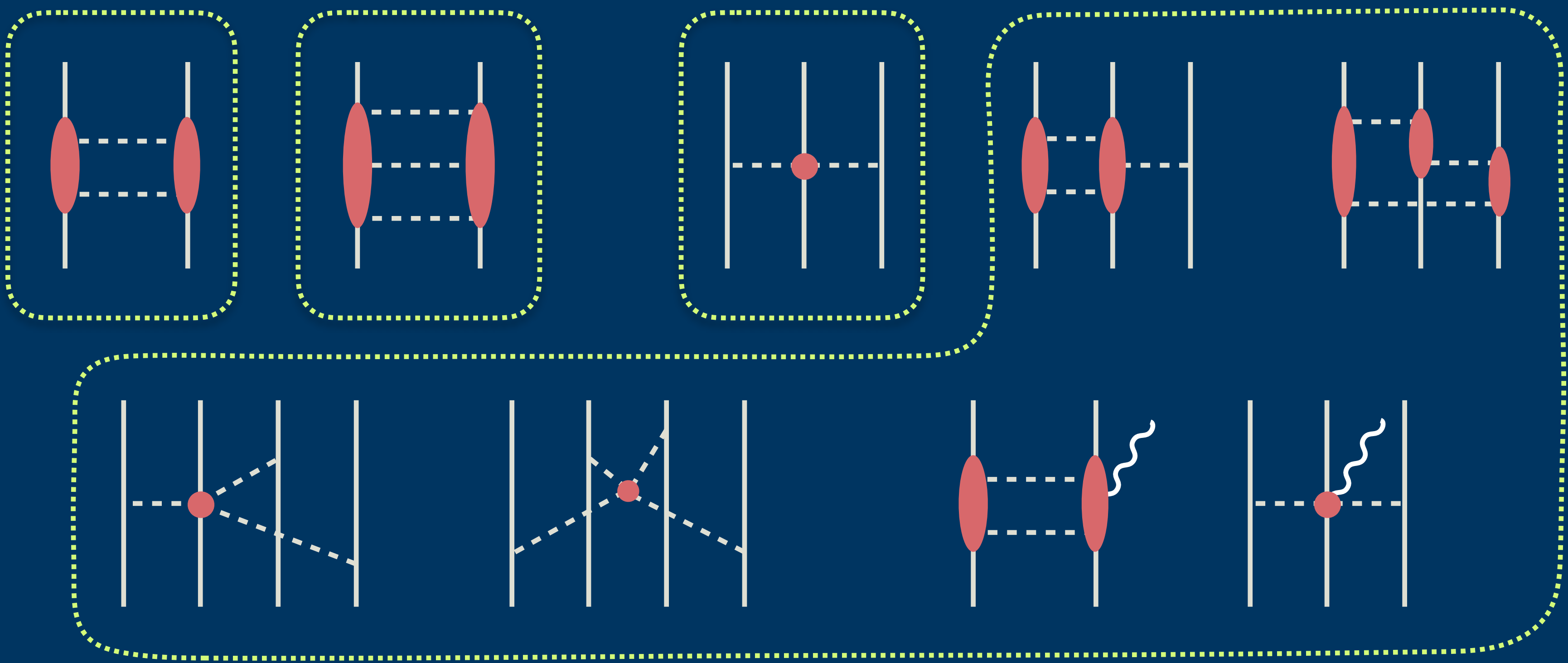

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— Symmetry-preserving regularization [H. Krebs, EE, PRC 110 (24) 044003; PRC 110 (24) 044004]

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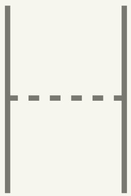

$$\mathscr{L}_{pv} = -\frac{g}{2m_N}\bar{N}\gamma^5\gamma^\mu\tau N\cdot\partial_\mu\pi$$

$$\cancel{\mathscr{L}_{ps} = -ig\bar{N}\gamma^5\tau N\cdot\pi}$$

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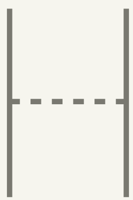

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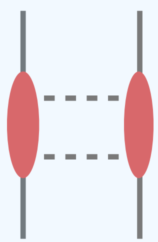


$\longleftarrow$ strongly constrained by χ symmetry:

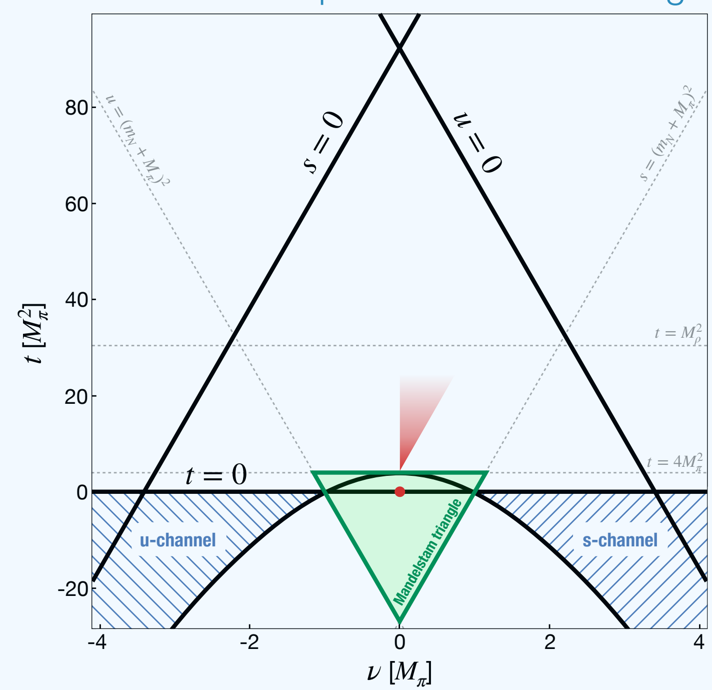
$\mathscr{L}_{ps}$ vs. $\mathscr{L}_{pv}$ matters, also $\pi\pi$, $\pi\pi N$, etc. interactions play a role…

Dispersive representation:

$$V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty}\mu d\mu\,\frac{\rho(\mu)}{q^2+\mu^2} + \ \ldots$$

$\rho(\mu)$ can be extracted from (analytically continued) $T_{\pi N}(s,t)$ obtained in ChPT (need data to fix LECs)

Mandelstam plane for πN scattering



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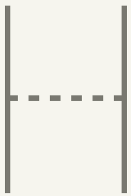

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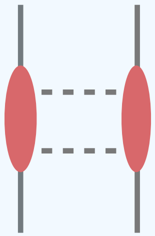

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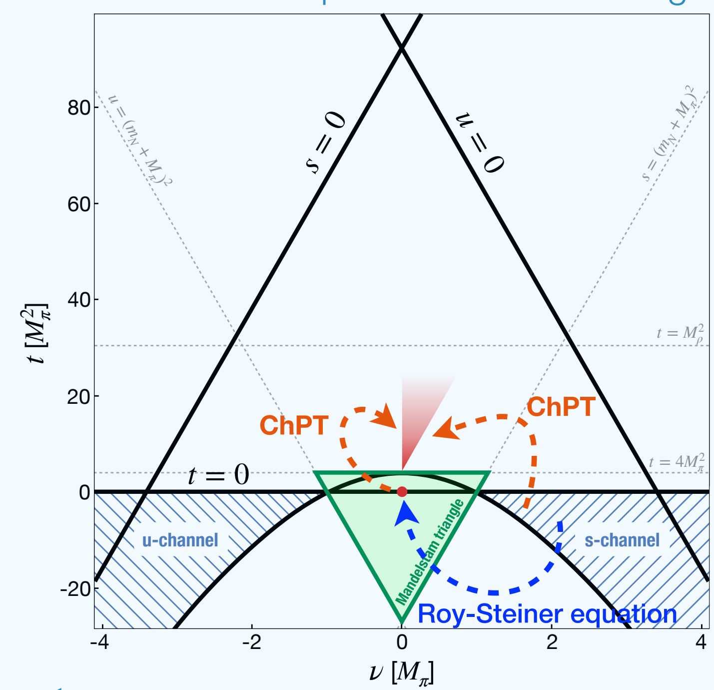
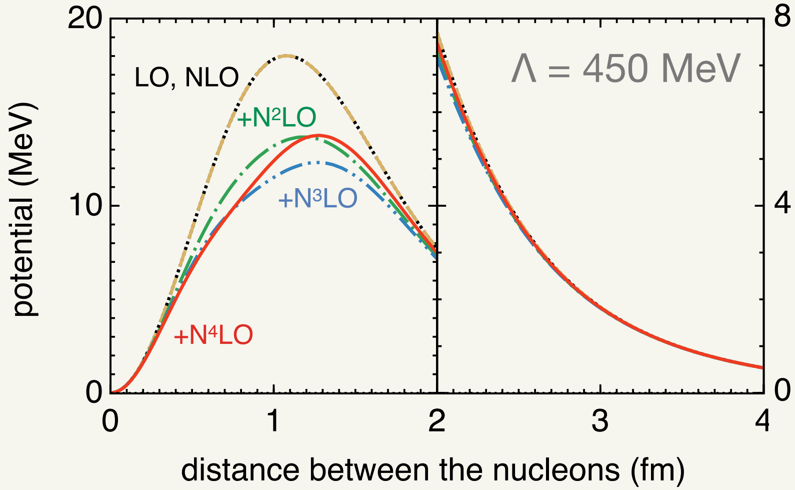


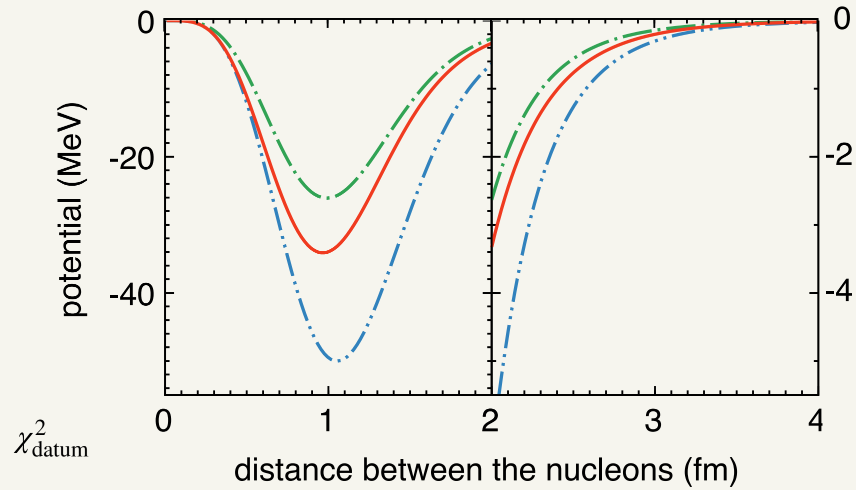
$\Rightarrow$ parameter-free predictions for $V_{2\pi}(r)$ at $r \gtrsim M_\pi^{-1}$

Chiral expansion of the long-range NN force

Isovector tensor potential



Isoscalar central potential

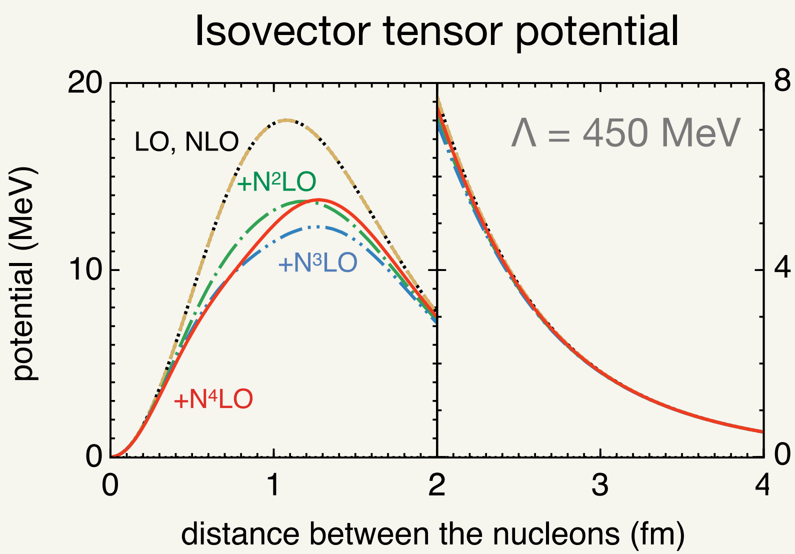


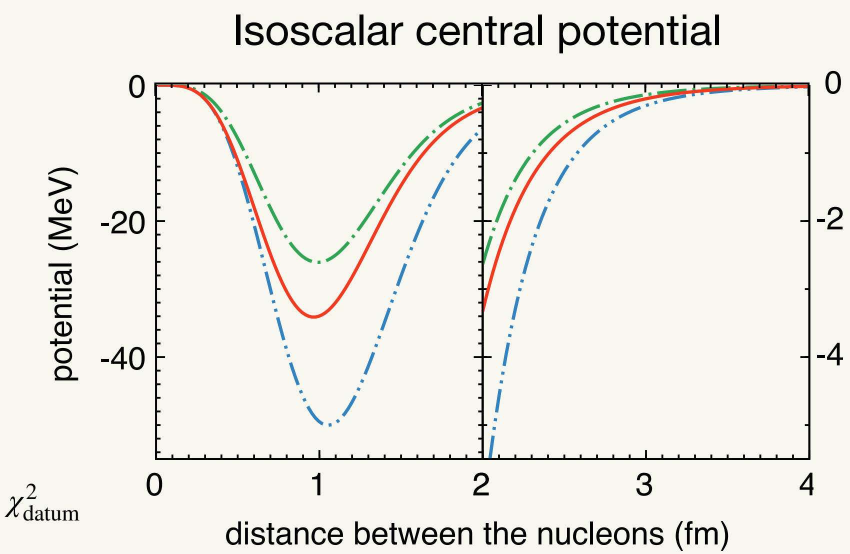
SMS NN potentials Reinert, Krebs, EE, EPJA 54 (2018) 86; PRL 126 (2021) 092501

$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^{\,2} + M_\pi^2} \; e^{-\frac{\vec{q}^{\,2} + M_\pi^2}{\Lambda^2}} + \text{subtraction}, \qquad V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\,\mu \frac{\rho(\mu)}{\vec{q}^{\,2} + \mu^2} \; e^{-\frac{\vec{q}^{\,2} + \mu^2}{2\Lambda^2}} + \text{subtractions}$$

\+ nonlocal (Gaussian) cutoff for contacts

Chiral expansion of the long-range NN force





Can we test these predictions in NN scattering?

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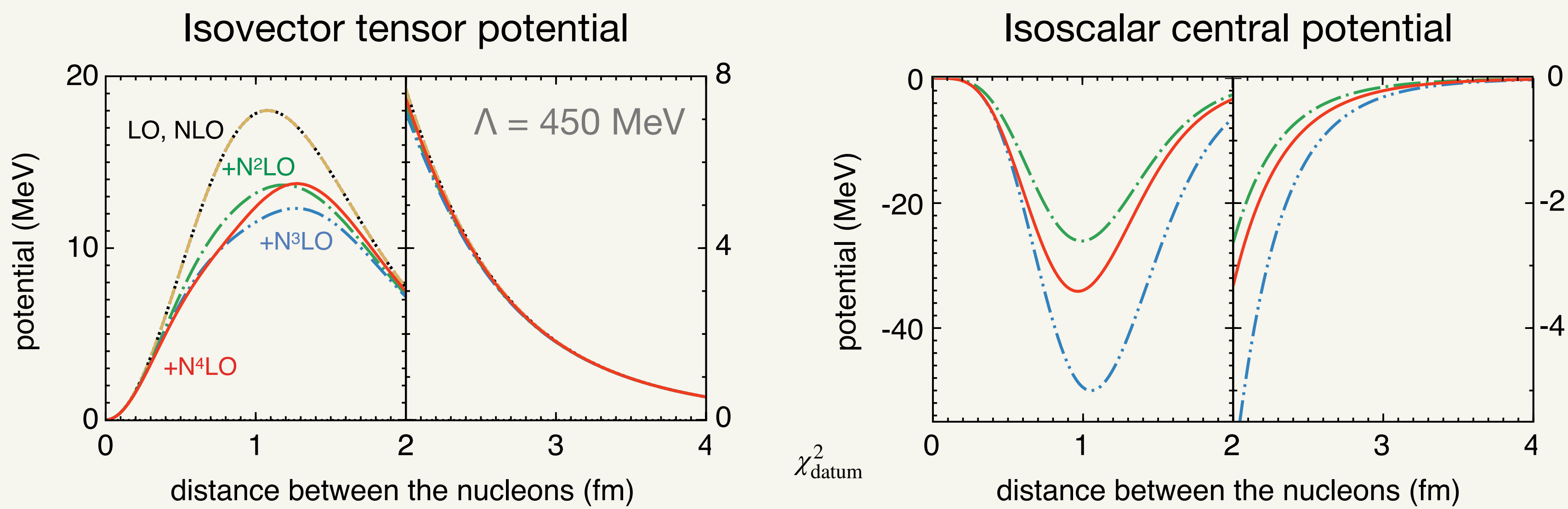

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$\chi^2_{\rm datum}$ for the description of neutron-proton and proton-proton scattering data

E_{lab} bin	CD Bonn	Nijm I	Nijm II	Reid 93	Bochum N^{4}LO$^+$
0-300 MeV	1.042	1.061	1.070	1.078	1.013

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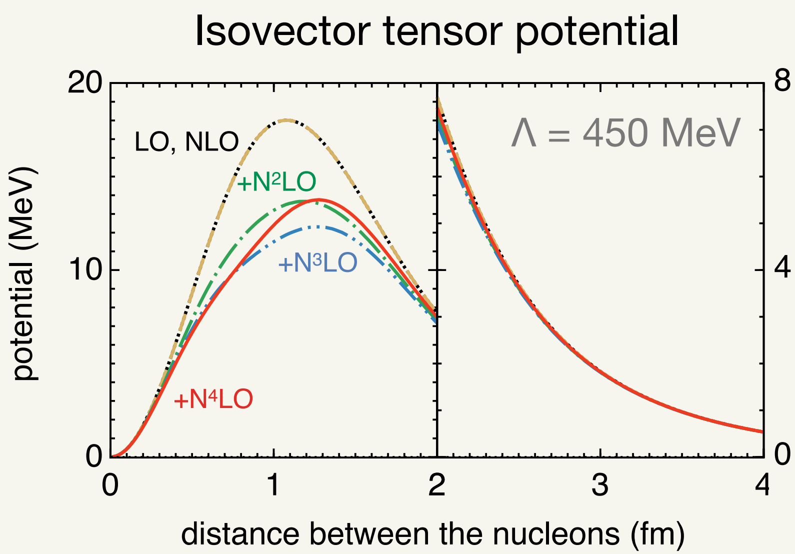

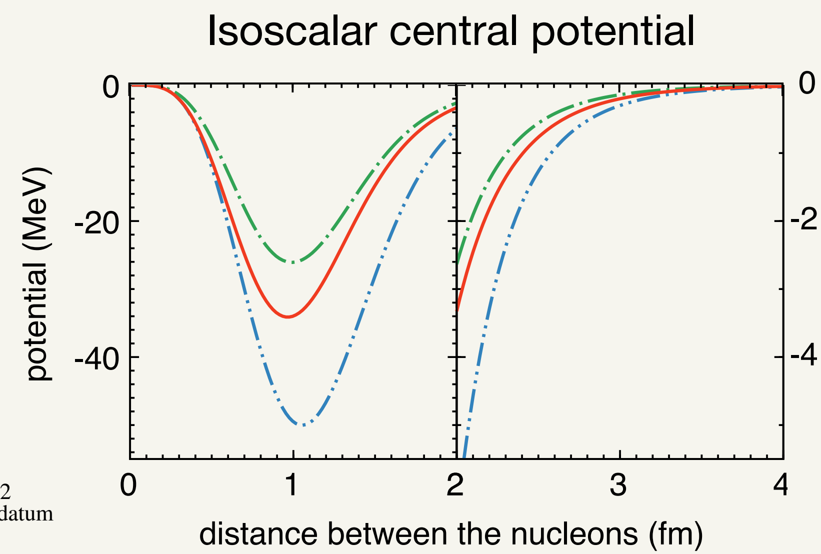

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Switching off the 2π-exchange leads to $\chi^2_{\rm datum} \sim 1.99$ (i.e., 50σ away!), consistent fits only for $E_{\rm lab} \leq 135$ MeV…

Three-pion exchange

What about the 3π-exchange?

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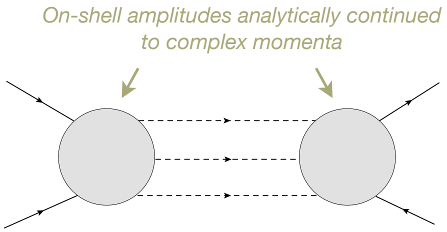
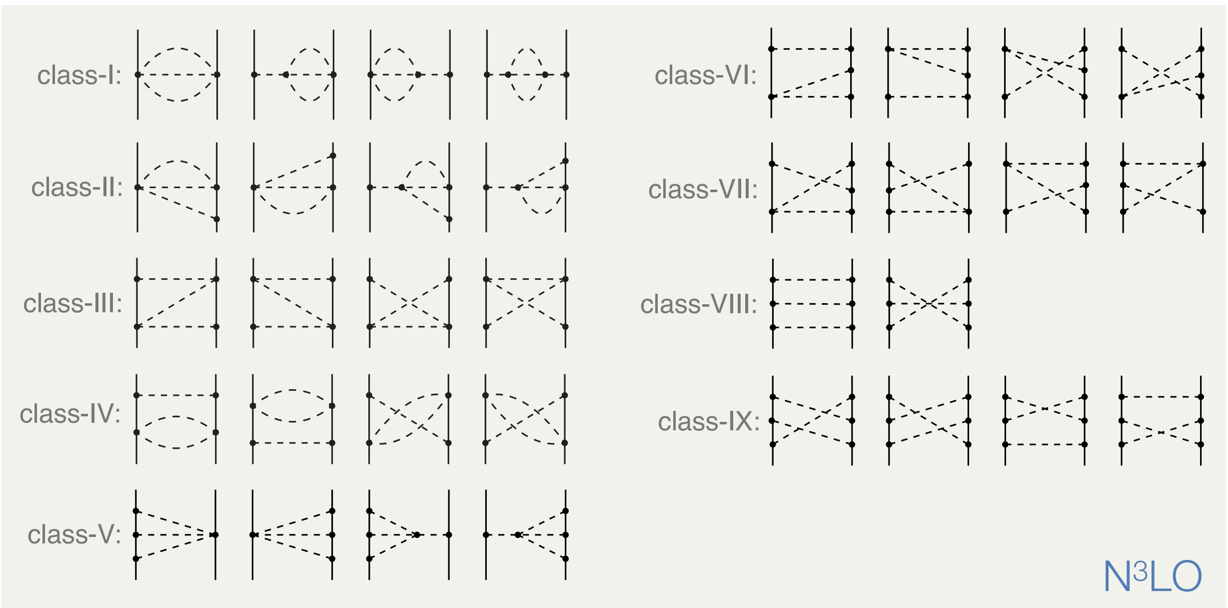
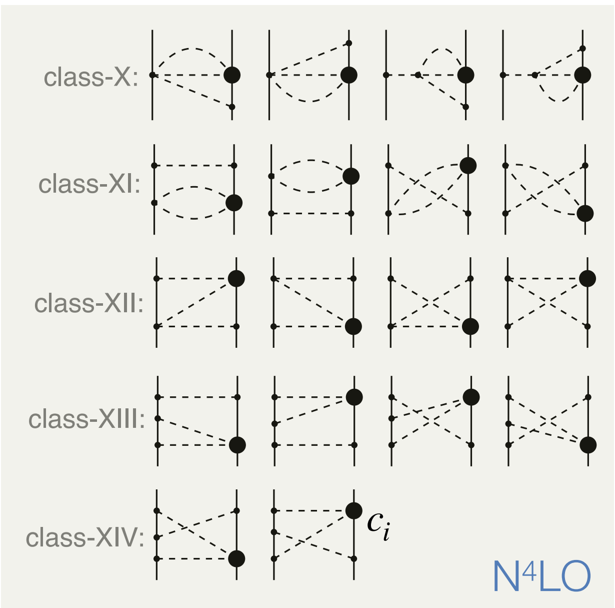
What about the 3π-exchange? Tour-de-force calculation by N. Kaiser using the Cutkosky cutting rules N. Kaiser, PRC61 (2000), PRC62 (2000), PRC63 (2001)



$$\mathrm{Im}\Big[V\big(q_\mu q^\mu = \mu^2 > 9M_\pi^2\big)\Big] \;=\; \int d\Gamma_3 \;\mathrm{Ampl}_1 \times \mathrm{Ampl}_2$$





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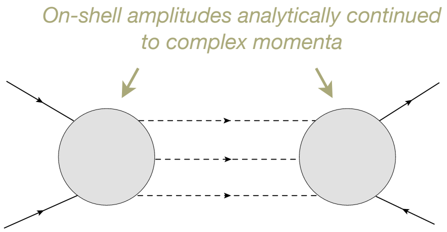

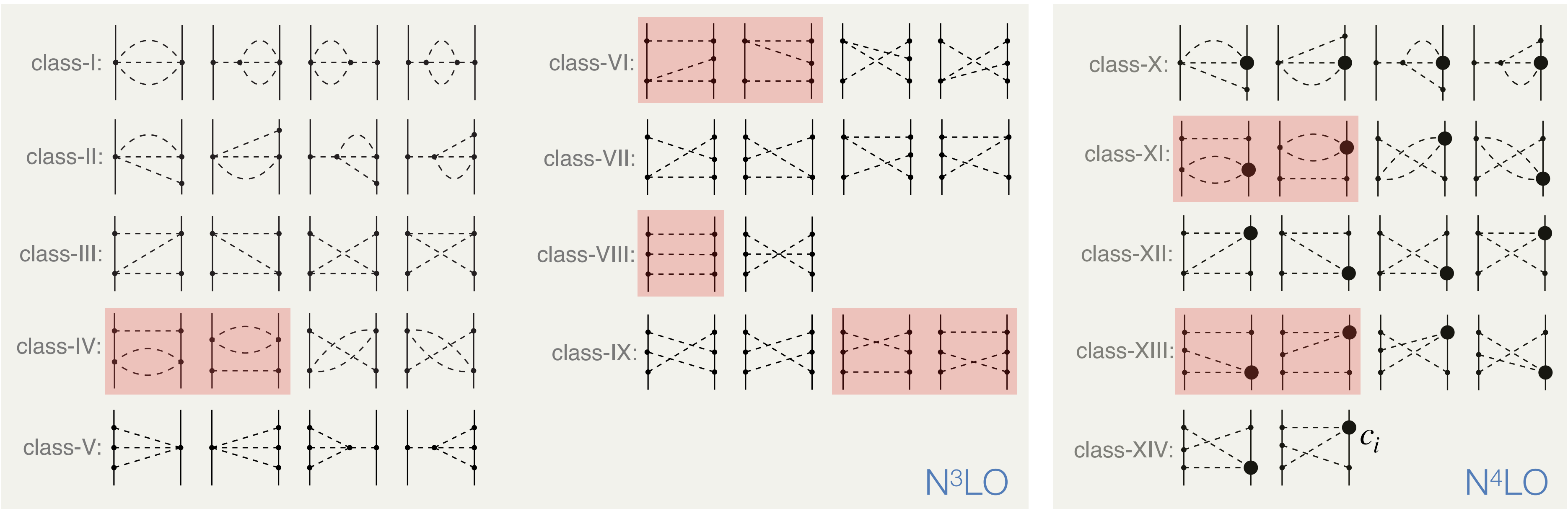

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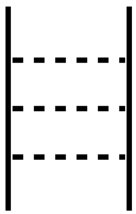
- Main concern: Potentials from reducible-like diagrams are scheme-dependent. Are the results of Norbert consistent with our potentials obtained using the Method of Unitary Transformation?

3π-exchange using the Method of UT

Victor Springer, Hermann Krebs, EE, PRC 112 (2025) 034004

The 3π-exchange potential has been re-derived using the Method of UT [PhD thesis of Victor Springer]

- First, calculated the corresponding energy denominators, e.g.:



$$V_{3\pi} = \int d^3l_1\, d^3l_2\, \hat{O}(\sigma_i, \tau_i, l_i) \bigg[\frac{\omega_1^4 + \omega_2\omega_1^3 + \omega_2^2\omega_1^2 + \omega_2^3\omega_1 + \omega_2^4}{4\omega_1^5\omega_2^5\,(\omega_1+\omega_2)\,\omega_3} - \frac{1}{4\omega_1^5\omega_2^2\,(\omega_1+\omega_3)} - \frac{1}{4\omega_1^4\omega_2^2\,(\omega_1+\omega_3)^2}$$
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where $\omega_i = \sqrt{\vec{l}_i^{\,2} + M_\pi^2}$

- Found the corresponding 4-dim expressions by matching and used the same method as Norbert (Cutkosky rules) to calculate the spectral functions.

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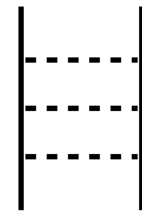

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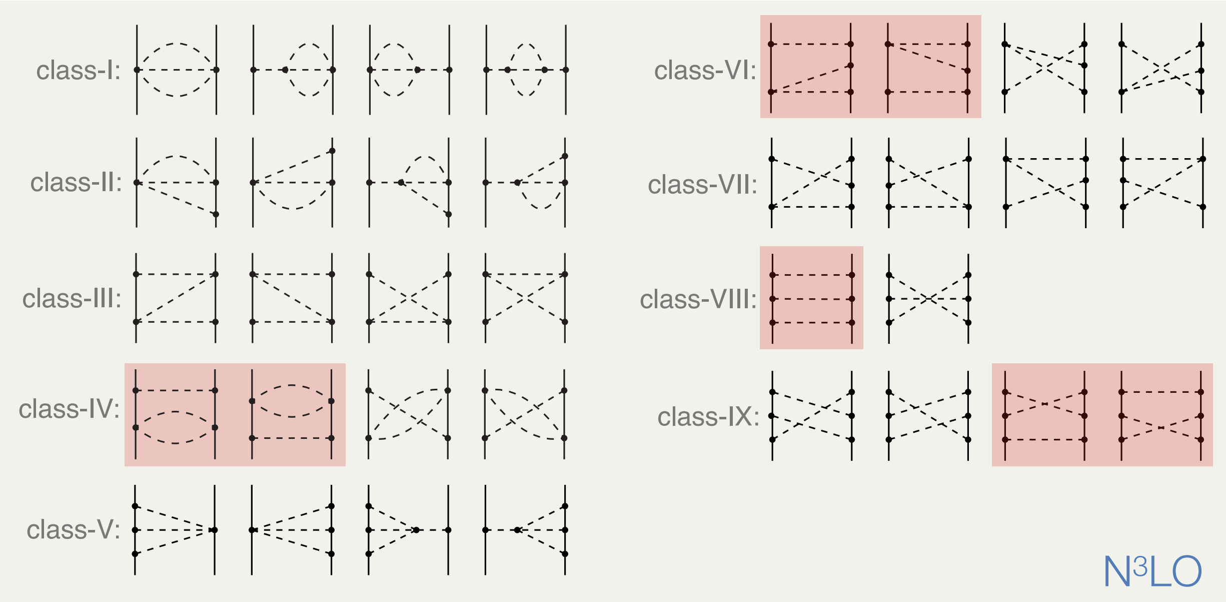

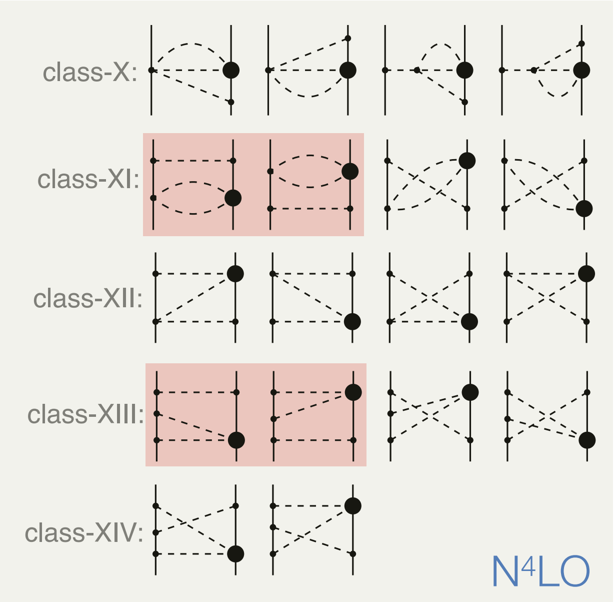

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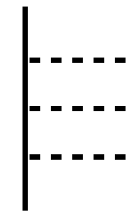

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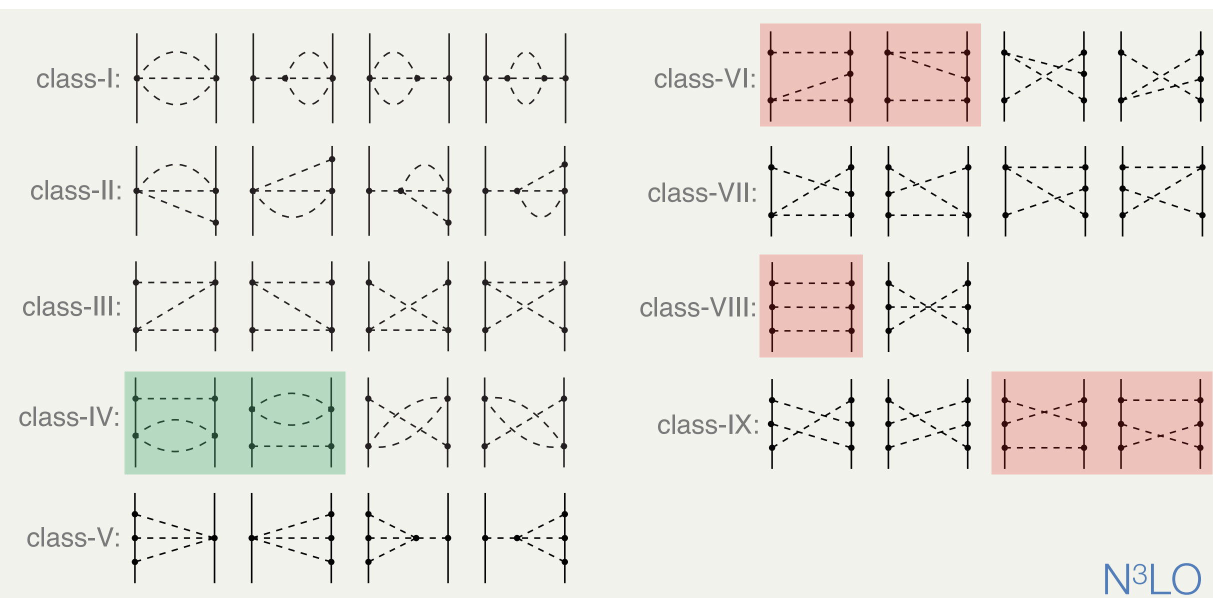
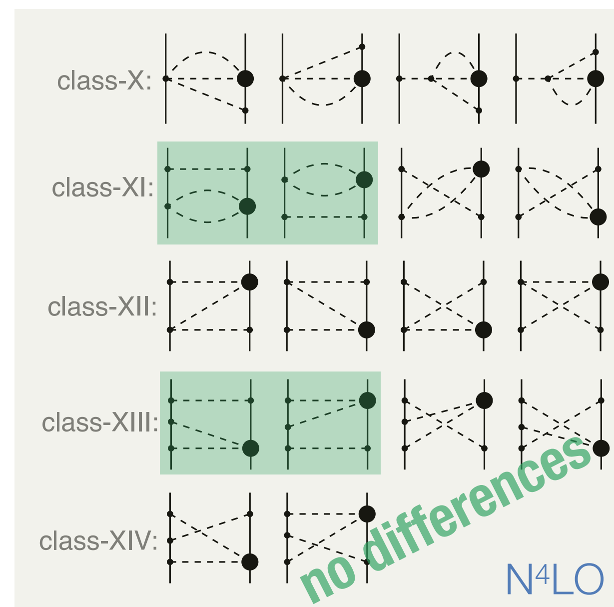
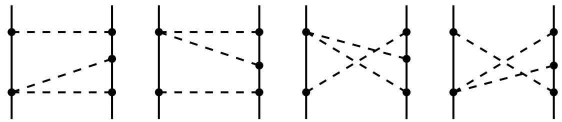




3π-exchange using the Method of UT

Victor Springer, Hermann Krebs, EE, PRC 112 (2025) 034004

For example, for the class-VI: 

— Norbert finds the only non-vanishing contributions:

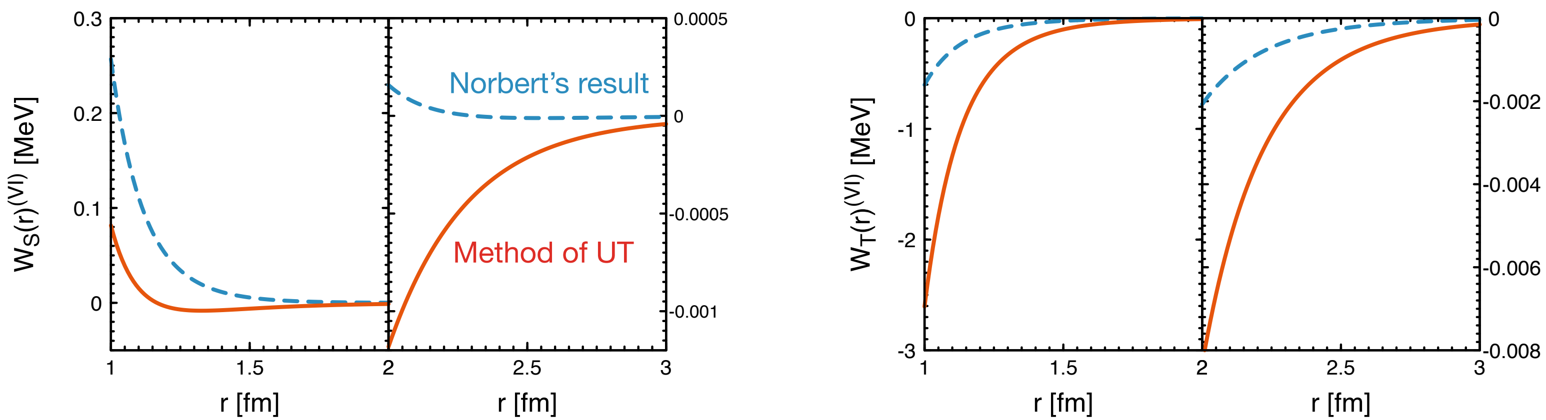
$$\mathrm{Im}\, W_S(i\mu) = \frac{2g_A^4}{(8\pi F_\pi^2)^3} \iint_{z^2 \leq 1} d\omega_1 d\omega_2 \left\{ -k_1^2 - \frac{5}{3}\mu\omega_1 + (\mu\omega_1 - M_\pi^2)\left(z + \frac{k_2}{k_1}\right) \frac{\arccos(-z)}{\sqrt{1-z^2}} \right\}, \qquad \mathrm{Im}\, W_T(i\mu) = \ldots$$

where $k_{1,2} = \sqrt{\omega_{1,2}^2 - M_\pi^2}, \qquad z k_1 k_2 = \omega_1\omega_2 - \mu(\omega_1 + \omega_2) + \frac{1}{2}(\mu^2 + M_\pi^2)$

— Method of UT:

$$\delta W_S(r) = \frac{2}{3} V_S(r) = -\frac{g_A^4}{3\,(8\pi F^2)^3} \frac{e^{-3M_\pi r}}{r^5} M_\pi^2 \,(1 + M_\pi r)^2$$

$$\delta W_T(r) = \frac{2}{3} V_T(r) = -\frac{g_A^4}{3\,(8\pi F^2)^3} \frac{e^{-3M_\pi r}}{r^7} (1 + M_\pi r)^2 \left(3 + 3M_\pi r + M_\pi^2 r^2\right)$$



phenomenological implications still to be explored…

Probing χ symmetry in peripheral neutron-α scattering

Yilong Yang, EE, Jie Meng, Lu Meng, Pengwei Zhao, PRL 135 (25) 172502

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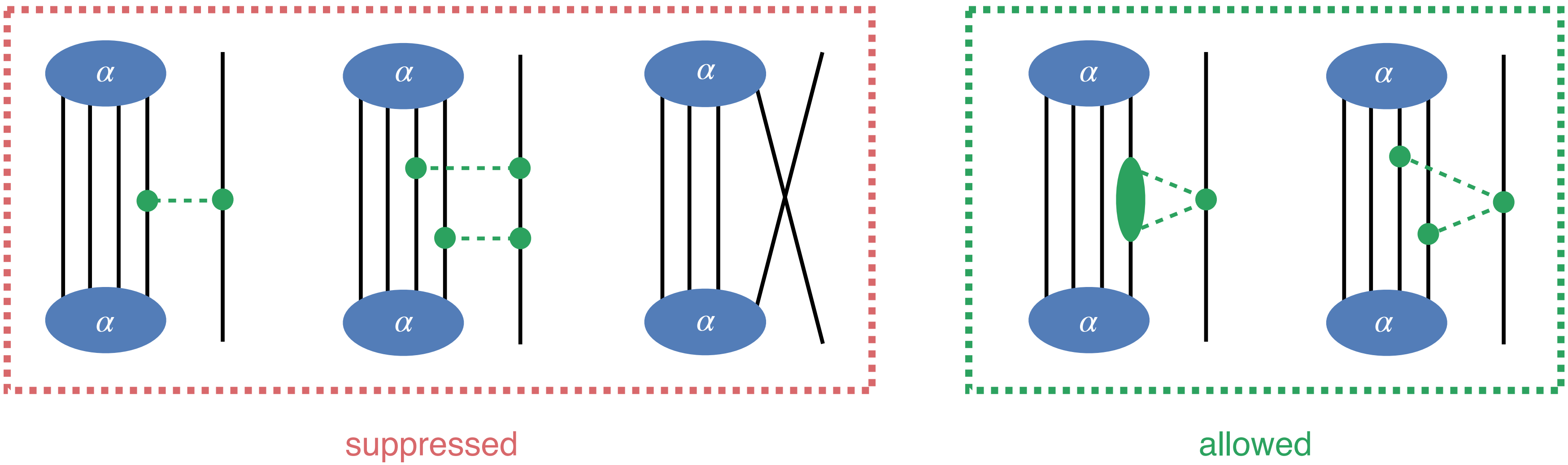


Suggested long ago by Manoel Robilotta,

but so far only S-, P-waves considered Carlson, Nollett, Lynn, Lazauskas, Kravvaris, Shirokov, Zhang, Mercenne, Bagnarol, …

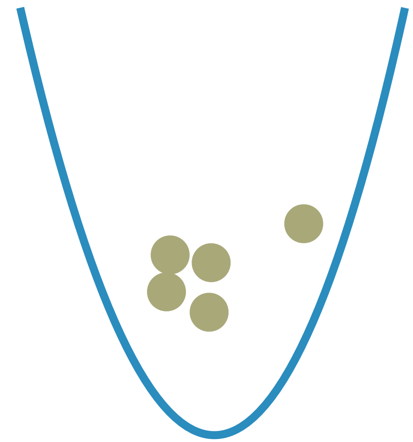
Probing χ symmetry in peripheral n-α scattering

Yilong Yang, EE, Jie Meng, Lu Meng, Pengwei Zhao, PRL 135 (25) 172502

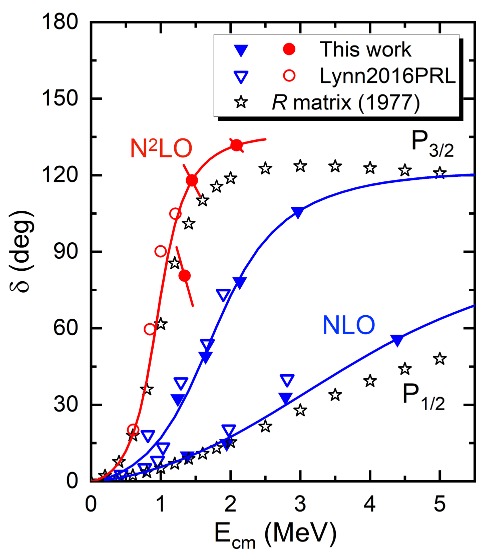
- Calculate the $^5\mathrm{He}_l$ GS energy ϵ_l in a harmonic trap (for $l = 2$) and use the Busch formula:

$$k^{2l+1}\cot\delta_l(\epsilon_l) = (-1)^{l+1}(4\mu\omega)^{l+1/2}\frac{\Gamma\left[(3+2l)/4-\epsilon_l/(2\omega)\right]}{\Gamma\left[(1-2l)/4-\epsilon_l/(2\omega)\right]}$$

Busch et al., Found. Physical. 28 (2008) 549;
Suzuki et al., PRA 80 (2009) 033601



Benchmarks for P-waves

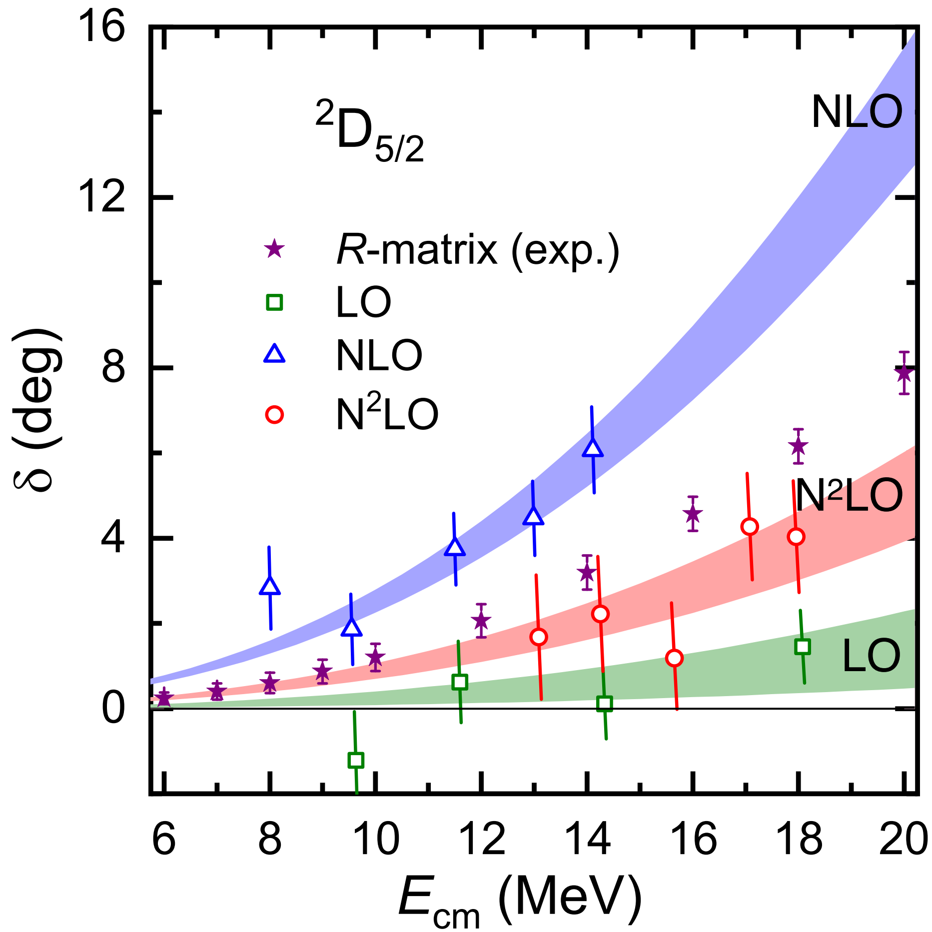


- Use VMC with a neural network [Y. L. Yang, P. W. Zhao, 2404.04203] to prepare an accurate ^{5}He trial state to be used in Diffusion MC calculation.

- Use local NN interaction up to N^{2}LO [Gezerlis et al. PRC90 (2014)] and locally regularized 3NF [Lynn et al. PRC96 (2017)] with the softest r-space cutoff $R = 1.2$ fm (sign problem…)

Probing χ symmetry in peripheral n-α scattering

Yilong Yang, EE, Jie Meng, Lu Meng, Pengwei Zhao, PRL 135 (25) 172502



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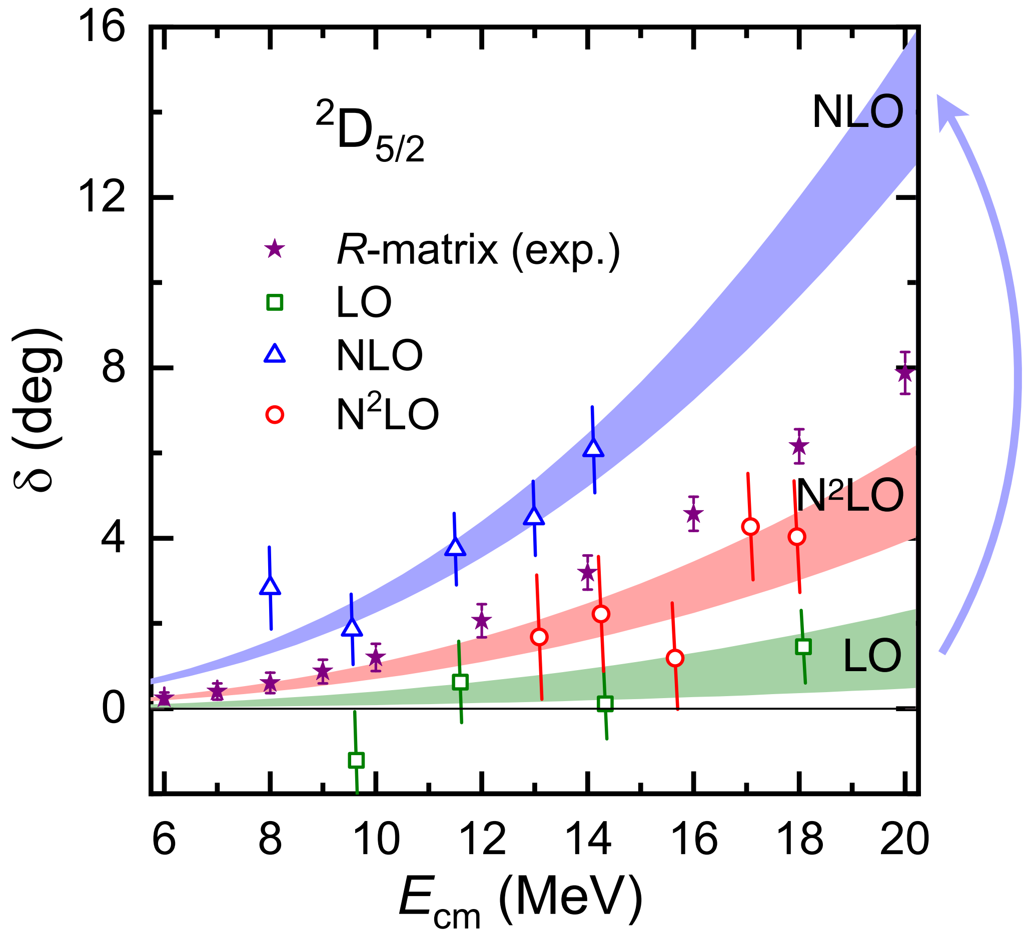

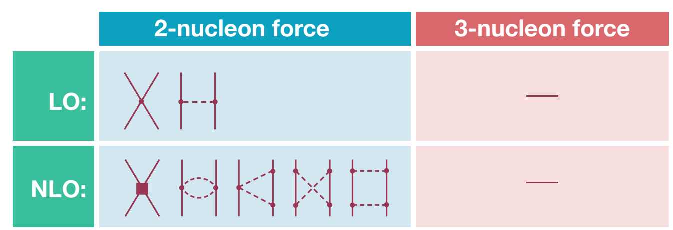

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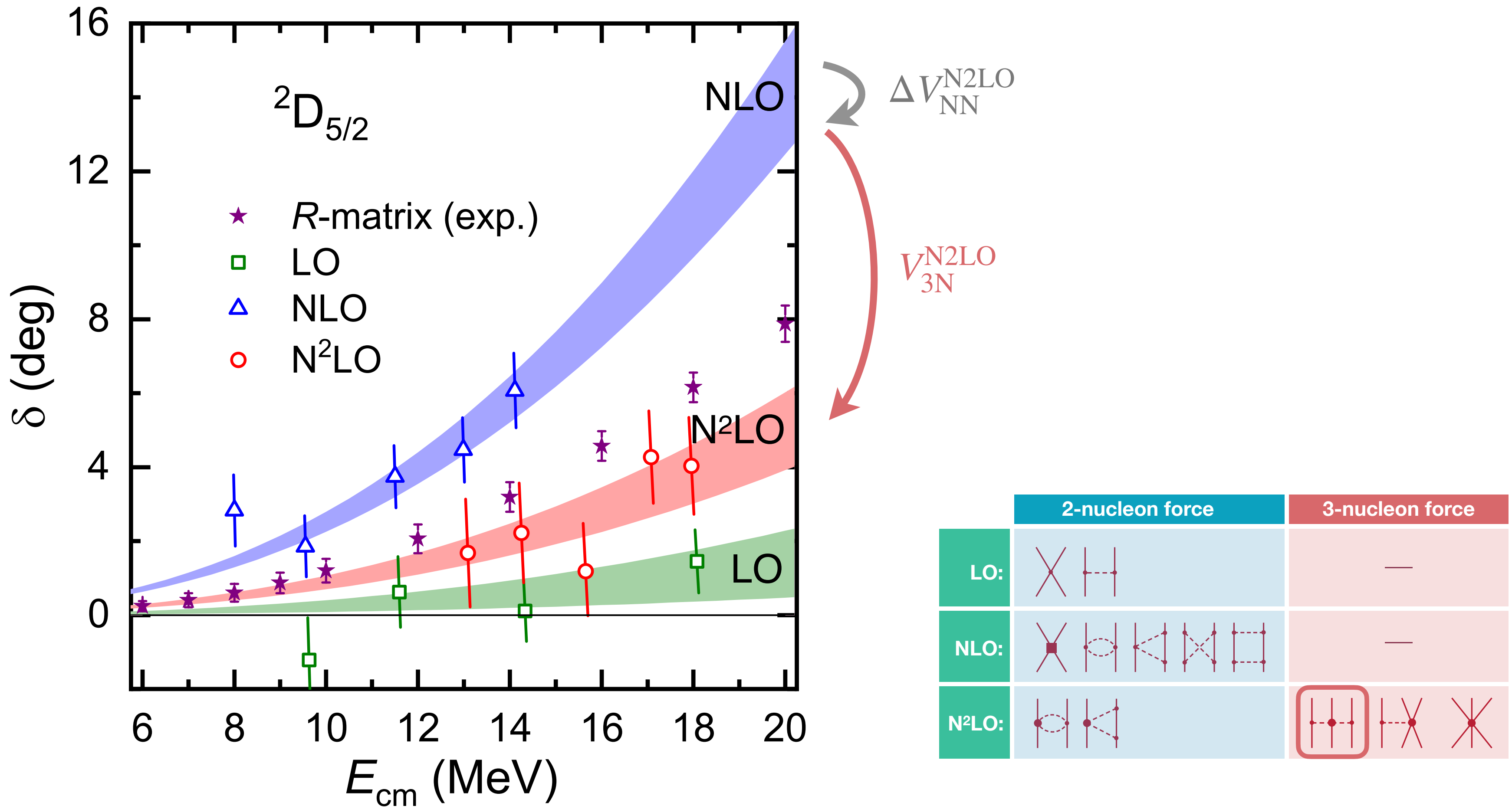
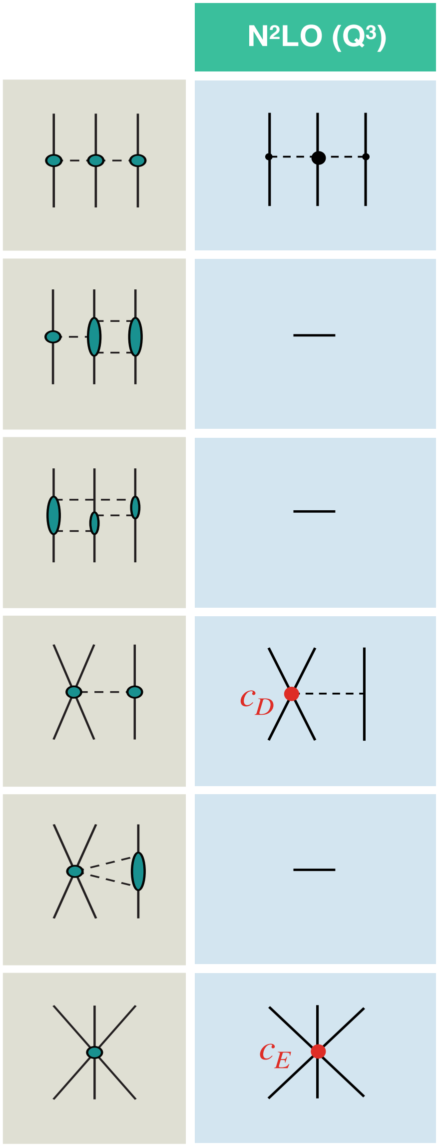


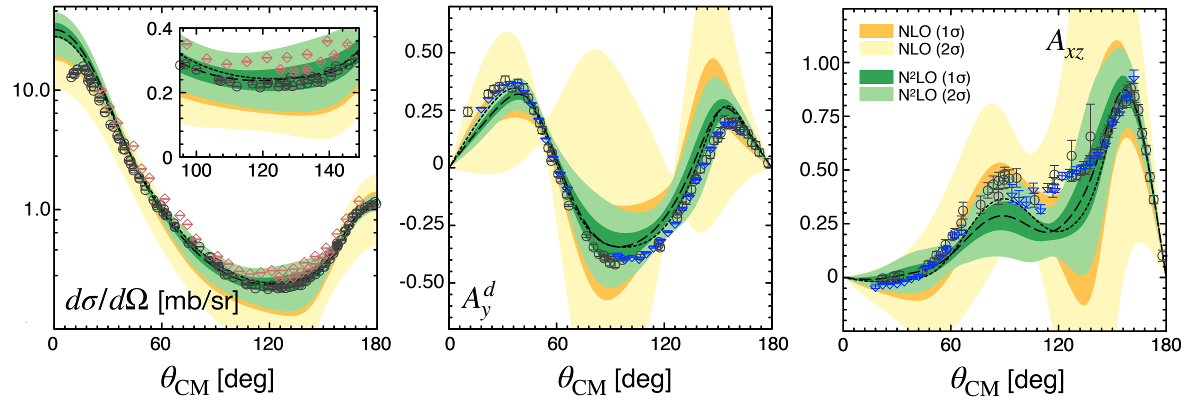
Peripheral $n\alpha$-scattering provides a sensitive probe of the long-range 3NF (governed by the chiral symmetry)

More results using the leading 3NF

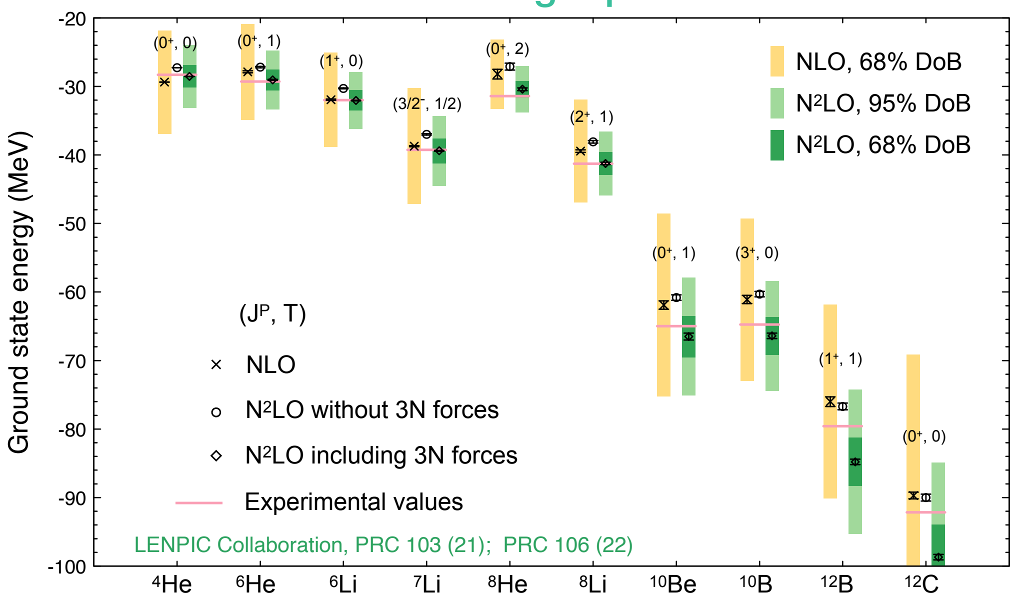
LENPIC Collaboration, PRC 103 (2021), PRC 106 (2022), see also Endo, EE, Naidon, Nishida, Sekiguchi, Takahashi, EPJA 61 (2025) 9



Elastic Nd scattering at 135 MeV



Predictions for light p-shell nuclei



More results using the leading 3NF

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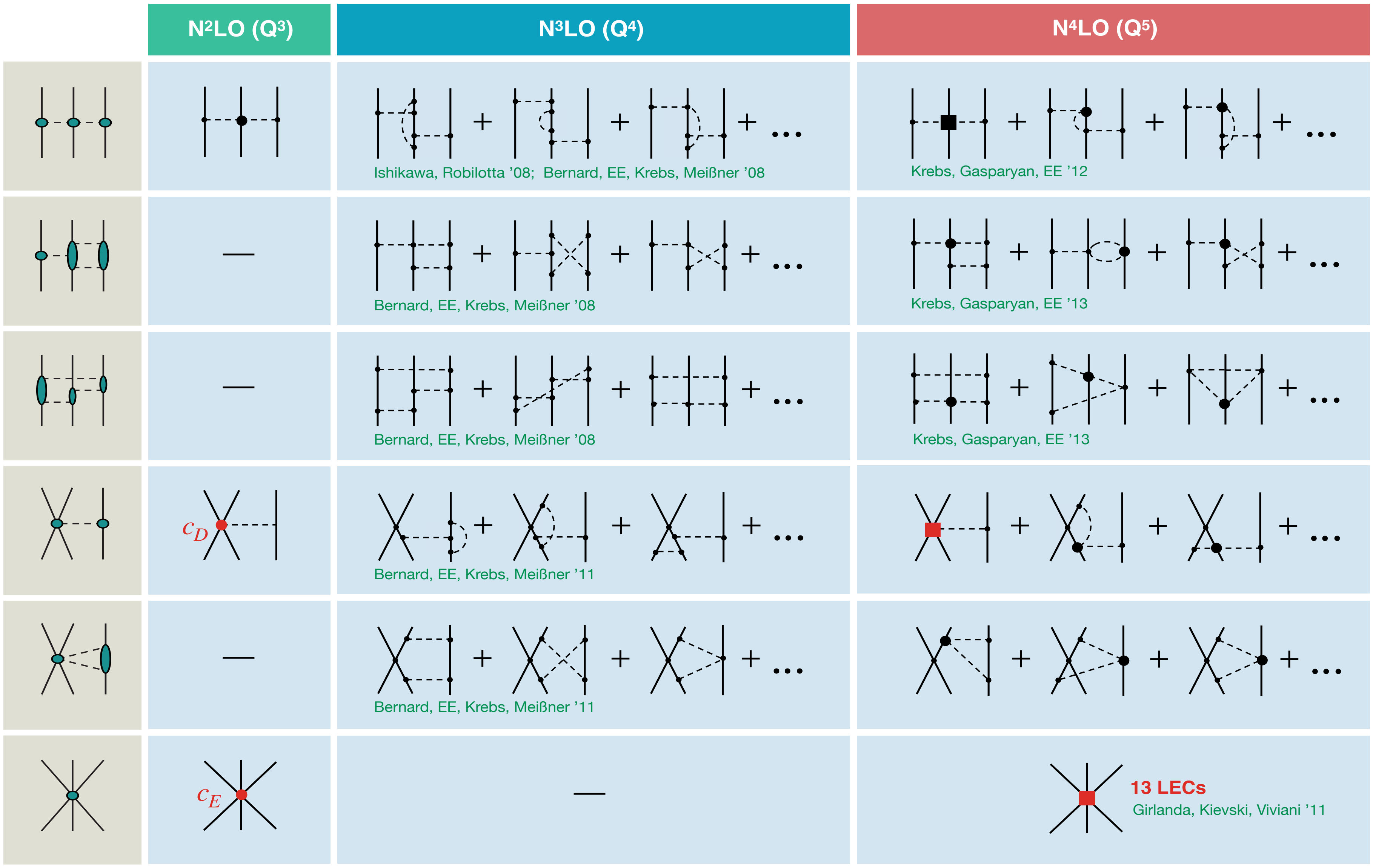

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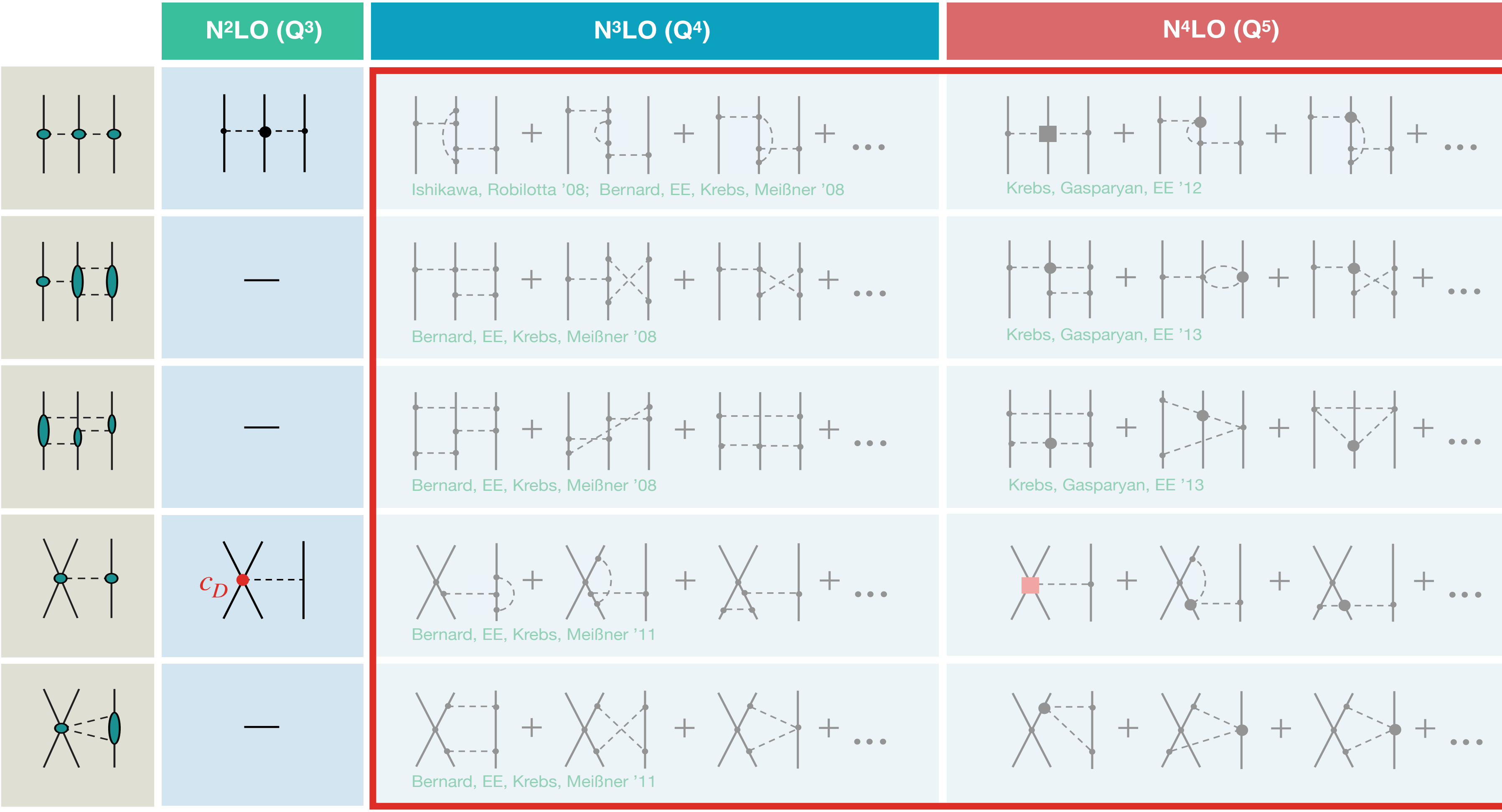
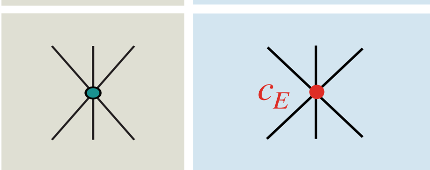




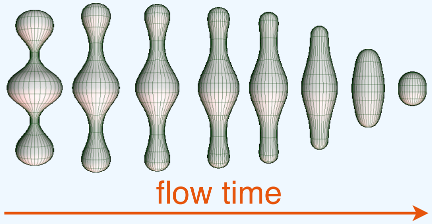
Mixing DimReg and CutoffReg violates chiral symmetry EE, Krebs, Reinert ’19

DANGER: momentum cutoff for pions breaks chiral symmetry!

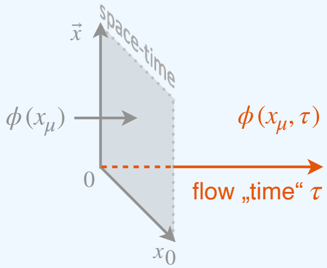
Chiral EFT using gradient flow

Krebs, EE, PRC 110 (2024) 044004; PRC 110 (2024) 04405

Gradient flows: methods for smoothing manifolds
(e.g., Ricci flow used in the proof of the Poincaré conjecture)



Gradient flow as a regulator in field theory



Flow equation: $\frac{\partial}{\partial \tau}\phi(x,\tau) = -\frac{\delta S[\phi]}{\delta \phi(x)}\Big|_{\phi(x)\to\phi(x,\tau)}$

subject to the boundary condition $\phi(x,0) = \phi(x)$

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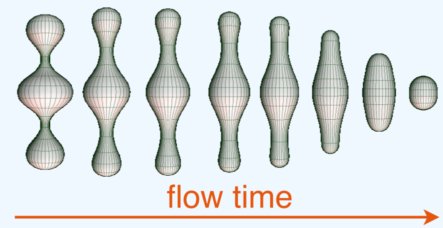
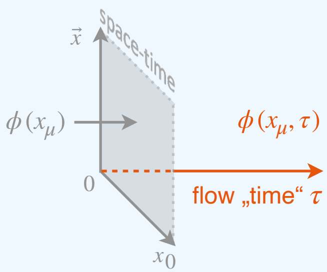


Gradient flow as a regulator in field theory



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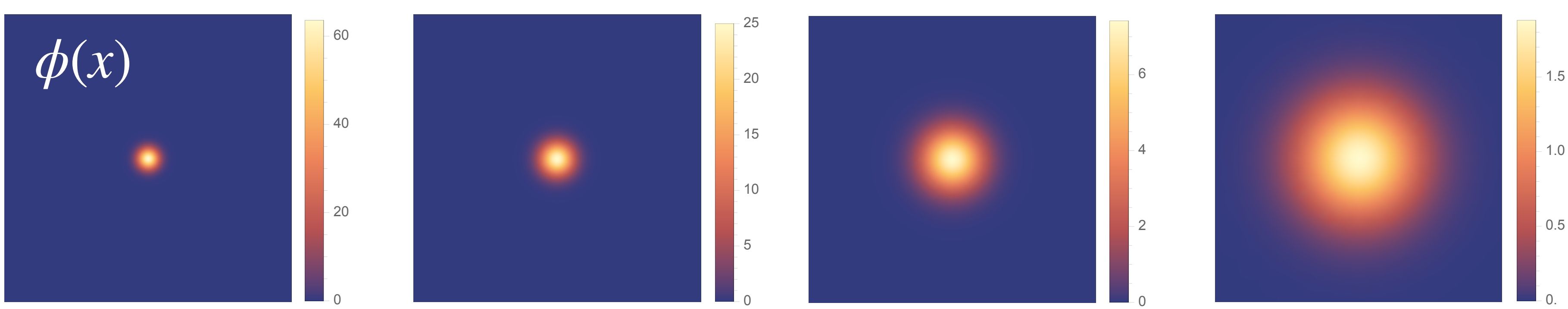
Free scalar field:

$$\left[\partial_\tau - (\partial^x_\mu \partial^x_\mu - M^2)\right]\phi(x,\tau) = 0 \quad \Rightarrow \quad \phi(x,\tau) = \int d^4y\, \overbrace{\underbrace{G(x-y,\tau)}_{\text{heat kernel}}}^{G(x,\tau) = \frac{\theta(\tau)}{16\pi^2\tau^2} e^{-\frac{x^2+4M^2\tau^2}{4\tau}}} \phi(y) \quad \Rightarrow \quad \tilde{\phi}(q,\tau) = e^{-\tau(q^2+M^2)}\tilde{\phi}(q)$$



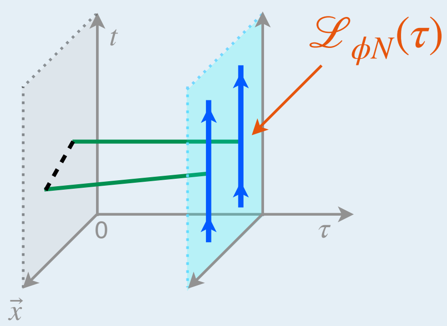
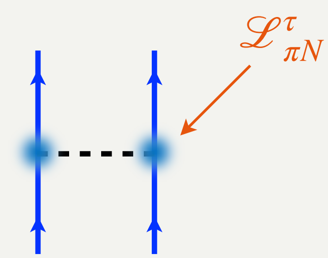
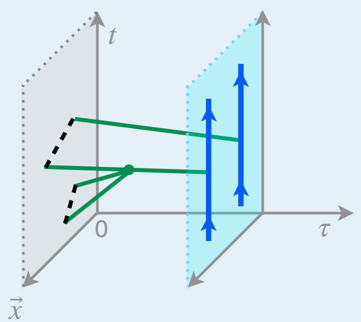
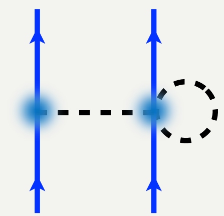
$\tau = 0$ $\tau = 1$ $\tau = 2$ $\tau = 4$

Chiral EFT using gradient flow

Krebs, EE, PRC 110 (2024) 044004; PRC 110 (2024) 04405

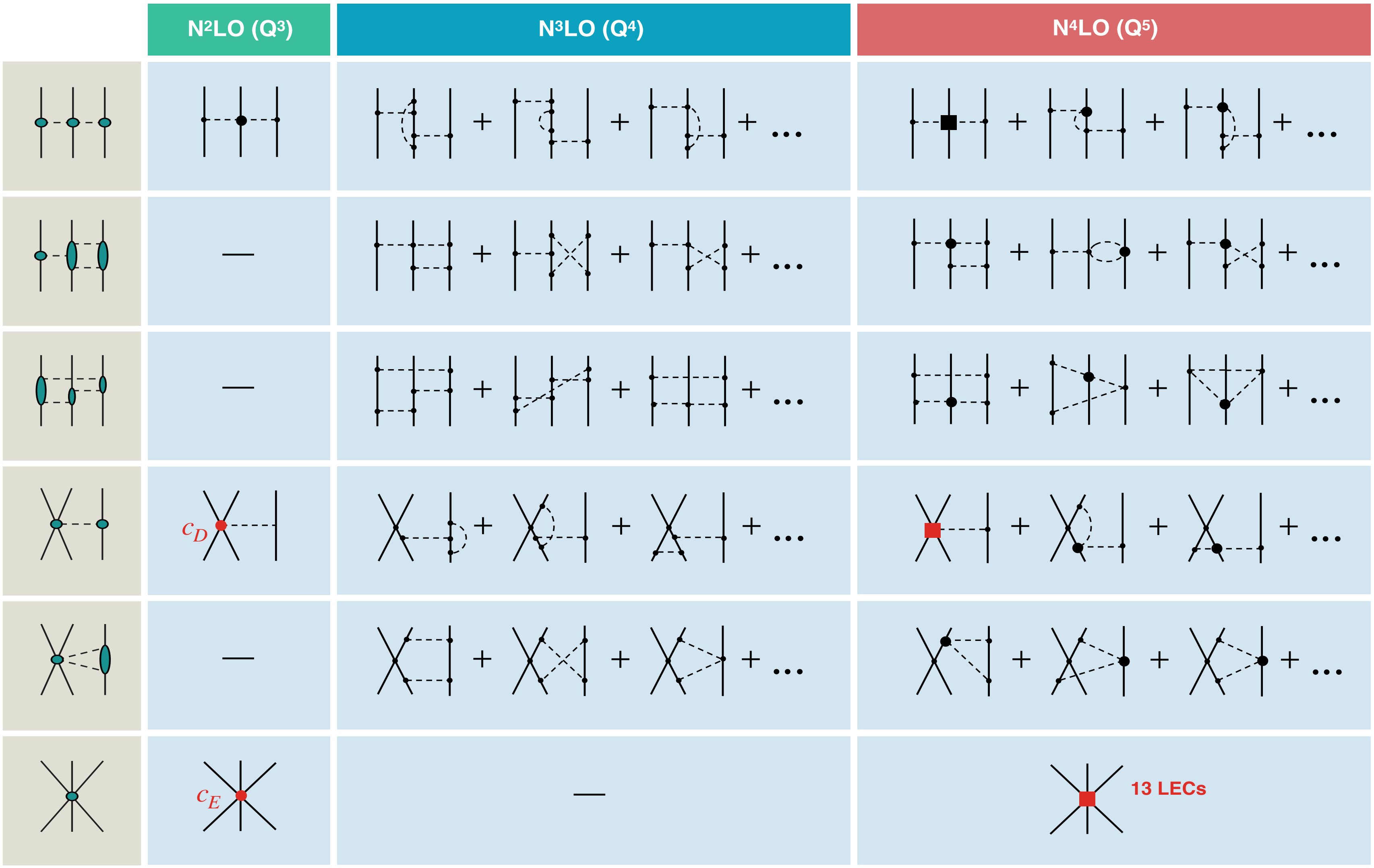
Introduce generalized pion fields $\phi(\tau)$, $\phi(0) = \pi$, that fulfills the *covariant* gradient flow equation.

Regularization is achieved by requiring N to „live“ at a fixed τ: $\mathscr{L}_{\pi N} \rightarrow \mathscr{L}_{\phi N}(\tau)$

Local field theory in 5d	**Smeared (non-local) theory in 4d**
	
	

Chiral EFT using gradient flow

Krebs, EE, PRC 110 (2024) 044004; PRC 110 (2024) 04405



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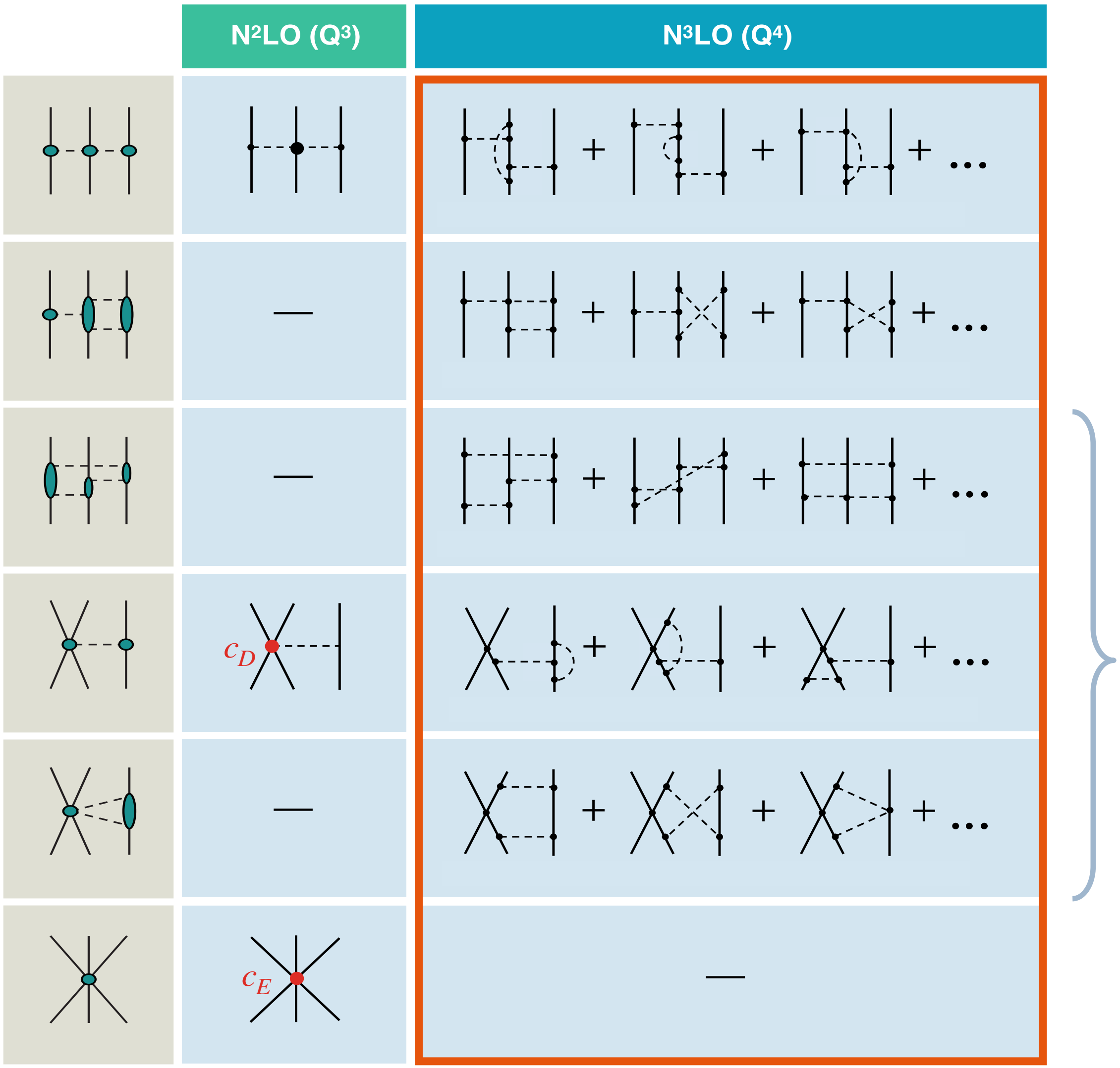

Partial-wave decomposition in progress

Kai Hebeler, Andreas Nogga, Kacper Topolnicki, …

Has been re-derived using Gradient Flow regularization

Chiral EFT using gradient flow

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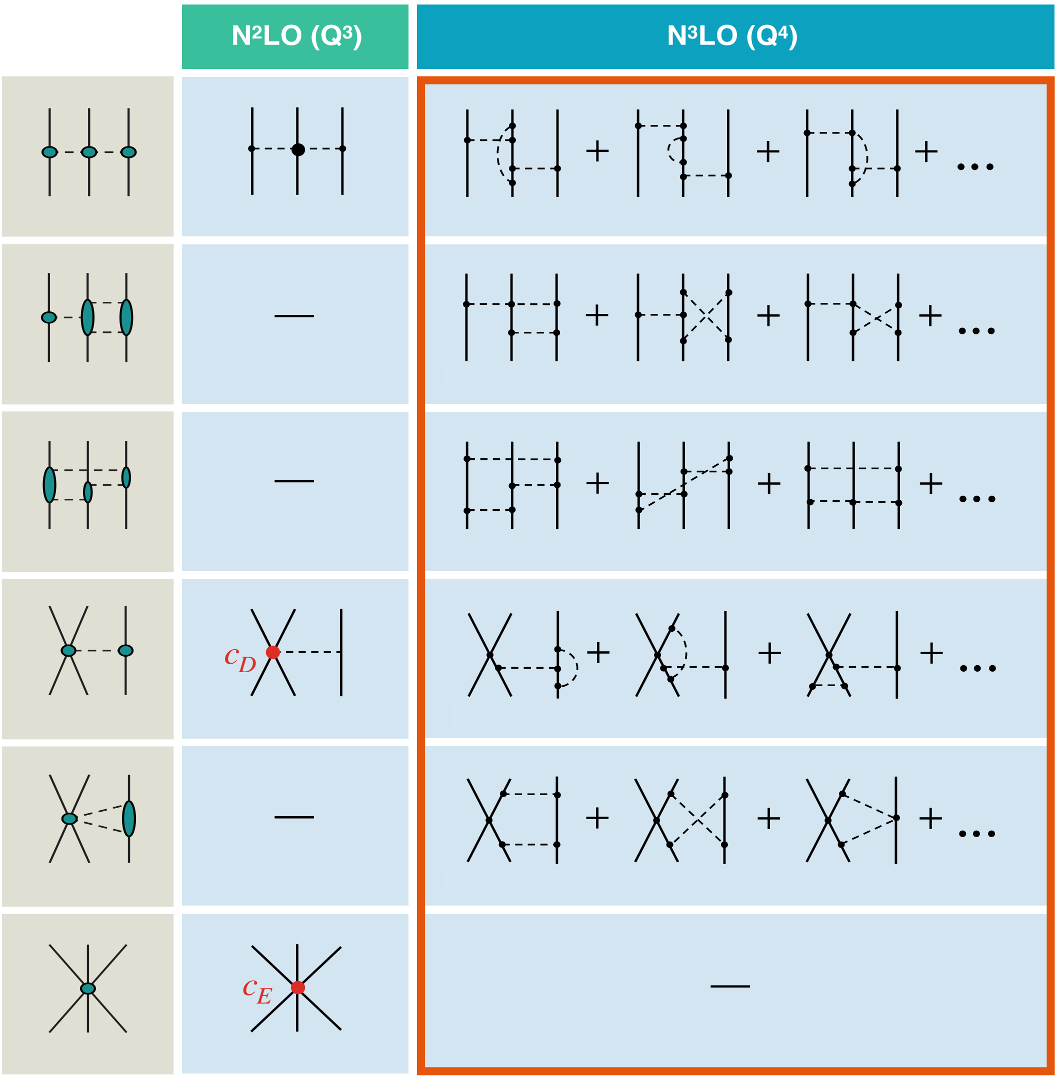

First (preliminary) results for ^{3}H using SMS 2NF@N^{4}LO$^+$, $\Lambda = 450$ MeV:

$\delta B_{3\mathrm{H}} = -\,315$ keV (repulsive)

$\delta B_{3\mathrm{H}} = 308$ keV (attractive)

Partial-wave decomposition in progress

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LENPIC

Summary and outlook

- **Chiral symmetry** and its breaking pattern play the **key role** for understanding low-energy nuclear physics

- Chiral EFT has already become a **precision tool in the NN sector**

- Symmetry-preserving **Gradient Flow regularization** puts chiral EFT on a firm basis and opens the avenue for precision calculations beyond the 2-body system

Thank you for your attention