Dalitz-plot decomposition of $e^+e^- \to \pi^+\pi^- J/\psi$ process from 4.13 to 4.36 GeV employing dispersive analysis

Viktoriia Ermolina

Institut für Kernphysik, Johannes Gutenberg-Universität Mainz

26.10 – 01.11 EINN, Workshop 1, Paphos, Cyprus

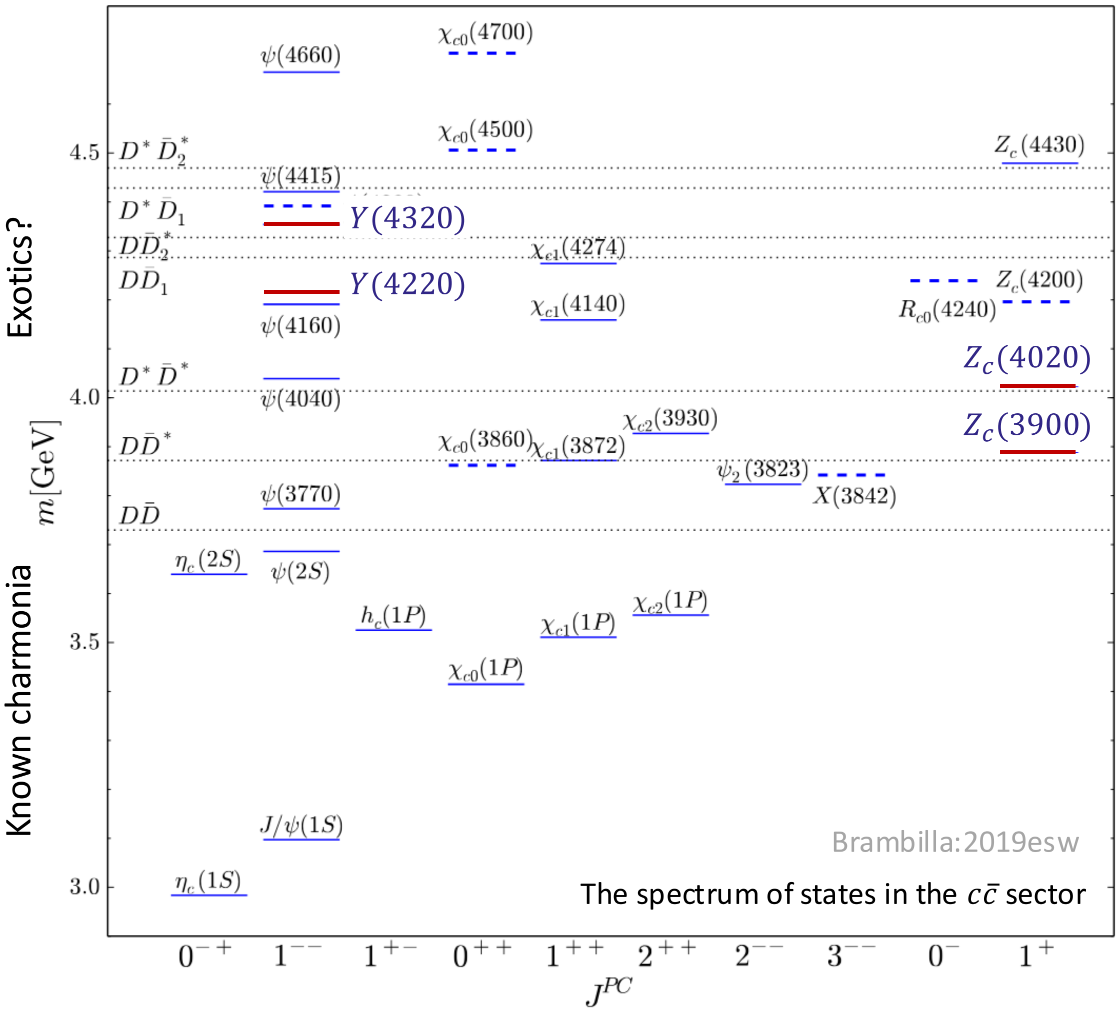
In collaboration with I. Danilkin, M. Vanderhaeghen

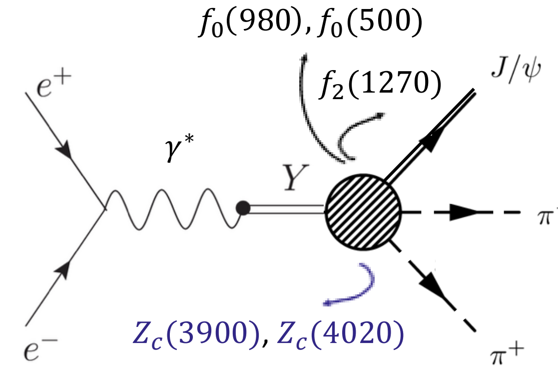
Plan

1. Introduction – field of study
2. Motivation – what we want to achieve
3. Formalism – how to do it
4. Results & Conclusion

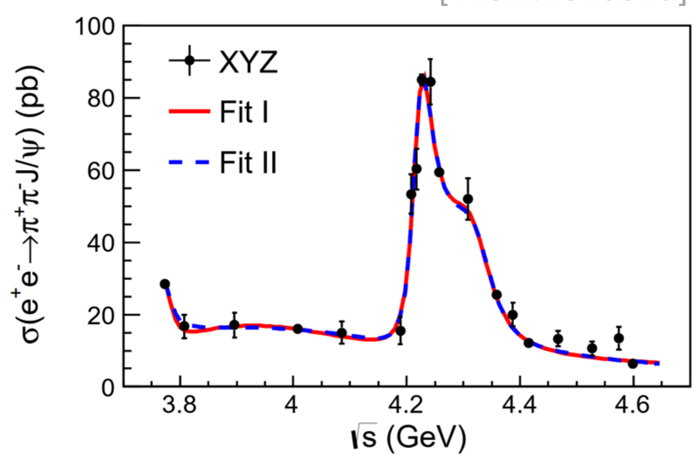
Introduction – observable charmonium-like states

XYZ-states: Last two decade's phenomena lying beyond conventional quark model expectations





[BESIII:2016bnd]



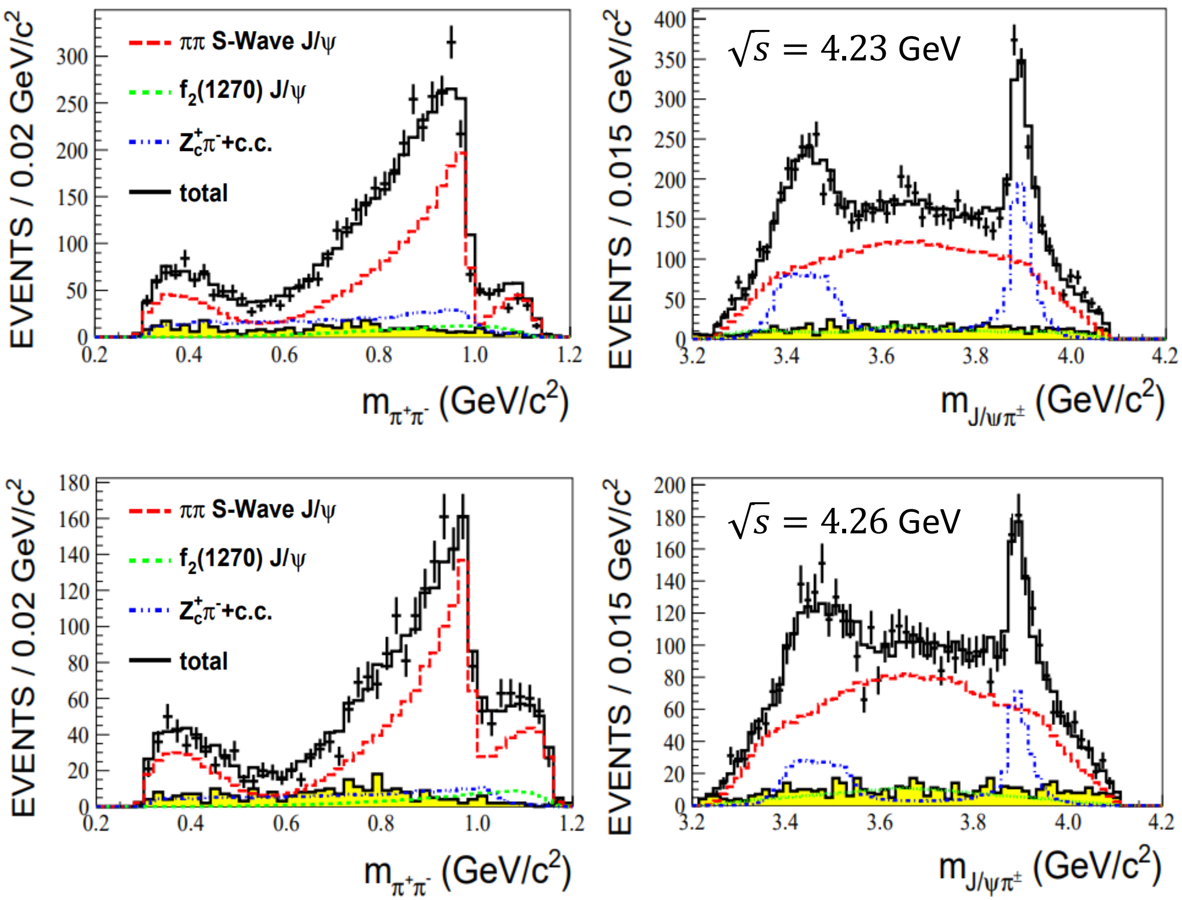
$\sqrt{s} - e^+e^-$ c.m. energies

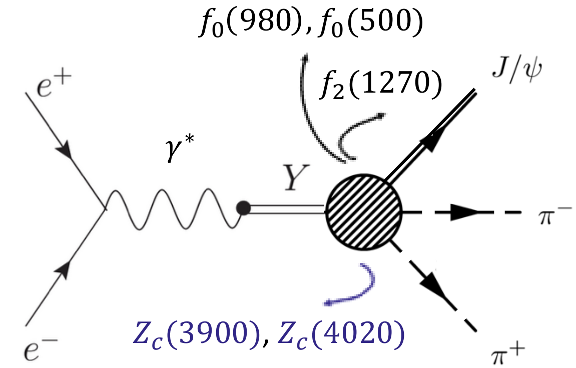
Y-states: 2005: BaBar reports $Y(4260)$ in $e^+e^- \to \gamma_{ISR}\pi^+\pi^-J/\psi$ 2017: BESIII resolves $Y(4220)$ and $Y(4320)$

[BaBar:2005hhc, Belle:2007dxy]

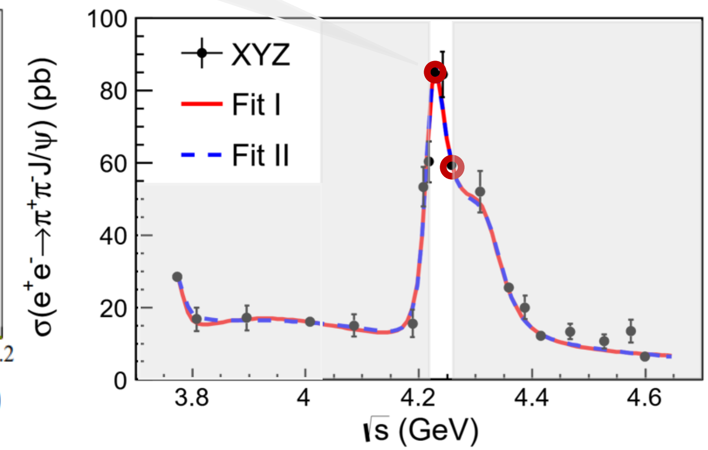
Introduction – observable charmonium-like states

Zc-states: Two points of the cross-section contains more information [BESIII:2017bua]





[BESIII:2016bnd]

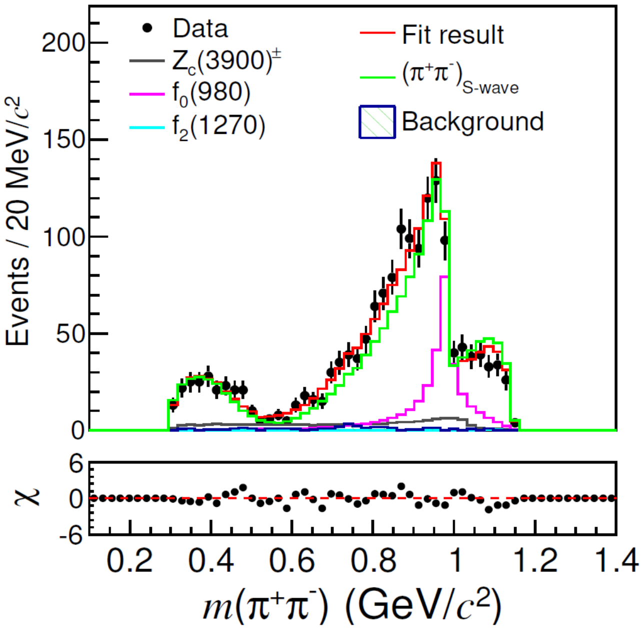


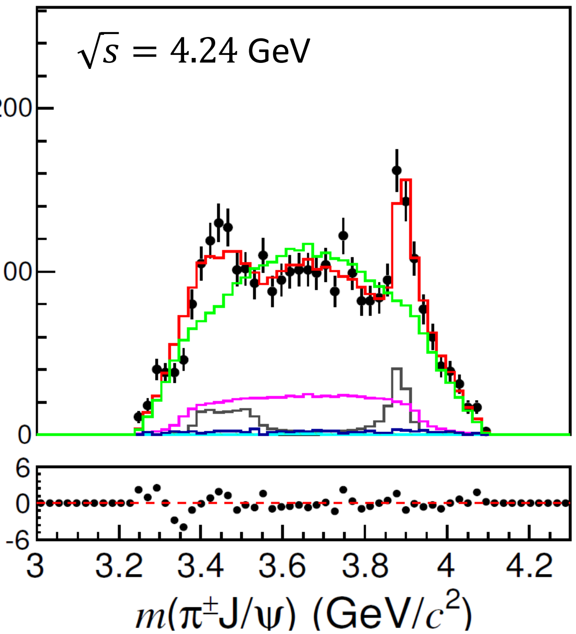
$\sqrt{s}$ – e^+e^- c.m. energies

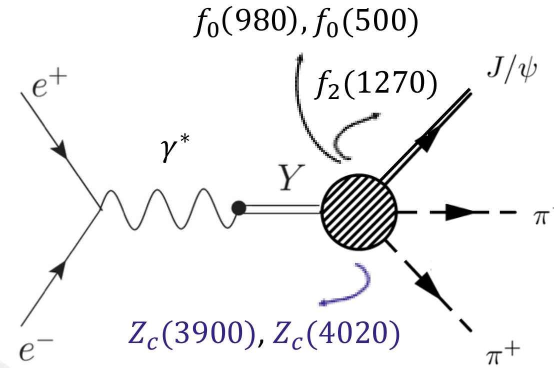
- Strong correlation of **Y**'s to the Z_c states and resonances of the $\pi^+\pi^-$ system
- Invariant mass distributions have been thoroughly analysed [Chen:2019dbo, Danilkin:2020kce, Du:2022jjv, Chen:2023def, Ermolina:2024uln]

Zc-states: Each point of the cross-section contains more information [BESIII:2025qkn]

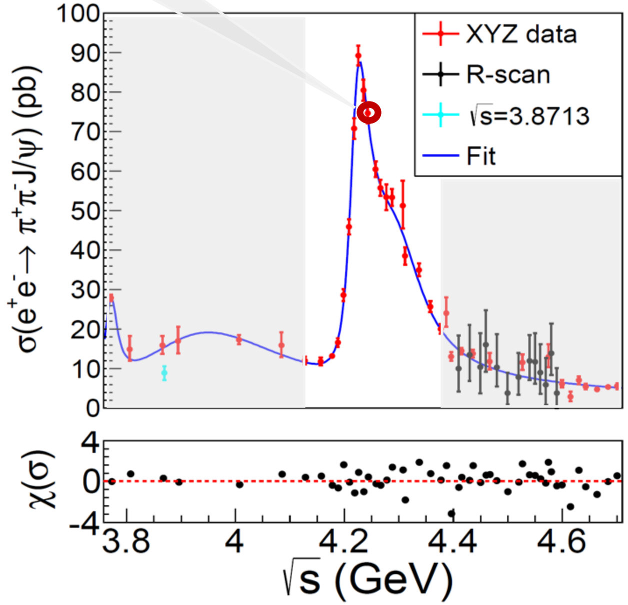
- New invariant mass distributions for 21 energy points from 4.13 – 4.36 GeV







[BESIII:2022qal]



Aims:

❶ A large set of data of different energy points enables to **extract $\sqrt{s}$ dependence**

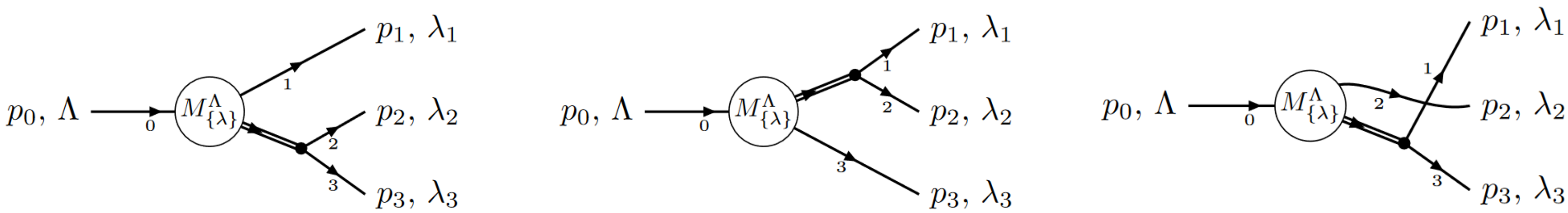
❷ It would allow to **predict both** cross-section **and** invariant mass distributions at other energies

❸ Potentially **constrain** properties of **Y states** and also **Zc states**

Formalism – Dalitz-plot decomposition

Dalitz-plot decomposition (DPD): [JPAC:2019ufm] – extension of helicity formalism [Chung:2007nn]

- A decay is represented as a cascade of sequential decays into particles with helicities $\lambda_{1,2,3}$: $(J,\Lambda) \to \{\lambda\}$



$$M^{\Lambda}_{\{\lambda\}} = \sum_{\nu} \underbrace{D^{J*}_{\Lambda,\nu}(\Omega)}_{\text{decay-plane orientation}} \times \underbrace{O^{\nu}_{\{\lambda\}}(m^2_{12}, m^2_{23})}_{\text{Dalitz-plot function}}$$

- Model-independent factorization: Wigner D-function of Euler angles $(\Omega) = (\varphi_1, \theta_1, \varphi_{23})$ × Mandelstam variables m^2_{ij} function
- Rotation of the system as a rigid body

$$O^{\nu}_{\{\lambda\}} = \sum_{(ij)k} \sum_{s}^{(ij)\to i,j} \sum_{\tau} \sum_{\{\lambda'\}} n_J n_s\, d^{J}_{\nu,\tau-\lambda'_k}(\hat{\theta}_{k(1)}) H^{0\to(ij),k}_{\tau,\lambda'_k} X_s(\sigma_k) d^{s}_{\tau,\lambda'_i-\lambda'_j}(\theta_{ij}) H^{(ij)\to i,j}_{\lambda'_i,\lambda'_j} d^{j_1}_{\lambda'_1,\lambda_1}(\zeta^1_{k(0)}) d^{j_2}_{\lambda'_2,\lambda_2}(\zeta^2_{k(0)}) d^{j_3}_{\lambda'_3,\lambda_3}(\zeta^3_{k(0)})$$

$\hookrightarrow \propto$ parametrization angles (m^2_{ij}), all $1 \to 2$ helicity couplings $H(m^2_{ij})$ and resonant contributions $X_R(m^2_{ij})$

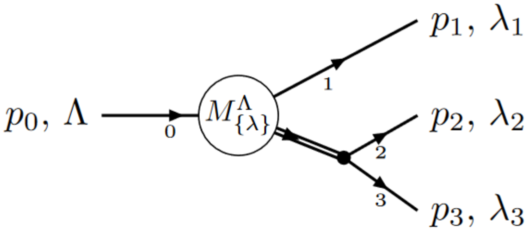
We validate this parametrization taking $H(m^2_{ij})$ from Lagrangian calculation [Ermolina:2024uln]

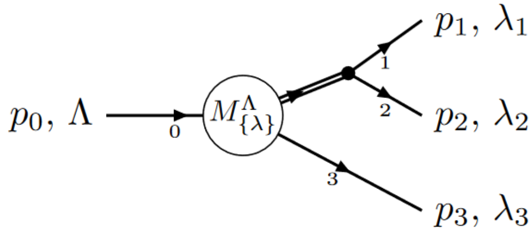
- $Z_c^+ \to \pi^+ J/\psi,\ Z_c^- \to \pi^- J/\psi$ symmetry allows to relate their helicity couplings $\to$ reduces degrees of freedom

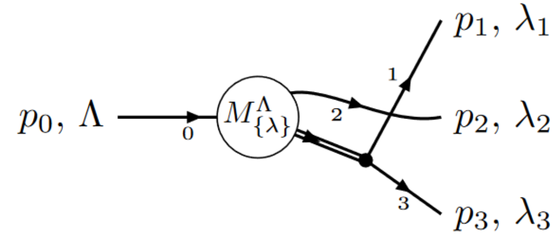
Formalism – Dalitz-plot decomposition

Dalitz-plot decomposition (DPD): [JPAC:2019ufm] – extension of helicity formalism [Chung:2007nn]

- A decay is represented as a cascade of sequential decays into particles with helicities $\lambda_{1,2,3}$: $\quad (J,\Lambda) \to \{\lambda\}$







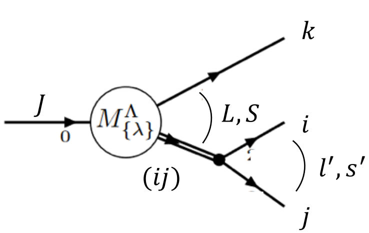
$$M^{\Lambda}_{\{\lambda\}} = \sum_{\nu} \underbrace{D^{J*}_{\Lambda,\nu}(\Omega)}_{\text{decay-plane orientation}} \times \underbrace{O^{\nu}_{\{\lambda\}}(m^2_{12}, m^2_{23})}_{\text{Dalitz-plot function}}$$

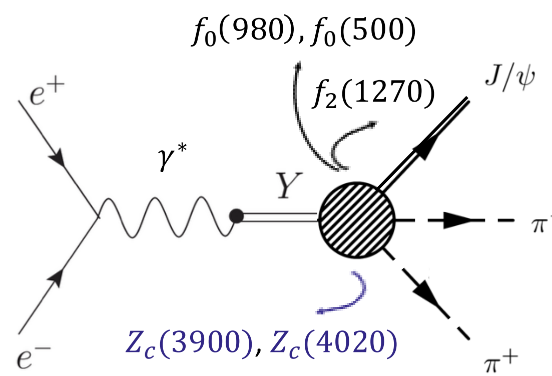
- Model-independent factorization: Wigner D-function of Euler angles $(\Omega) = (\varphi_1, \theta_1, \varphi_{23})$ × Mandelstam variables m^2_{ij} function
- Rotation of the system as a rigid body

In e^+e^- c.m. the contributions of photon helicity $\Lambda = 0$ is supressed by the factor of $2m_e^2/s$

Cross-section: $$\mathrm{d}\sigma = \frac{e^2}{64(2\pi)^4 s} \sum_{\{\lambda\}} \left| M^{\Lambda=+1}_{\{\lambda\}} \right|^2 \, \mathrm{d}m^2_{12}\, \mathrm{d}m^2_{23}\, \mathrm{d}\Omega$$

- Helicity couplings $H(m^2_{ij})$ can be transformed to a different basis according to LS of $1 \to 2$ vertex



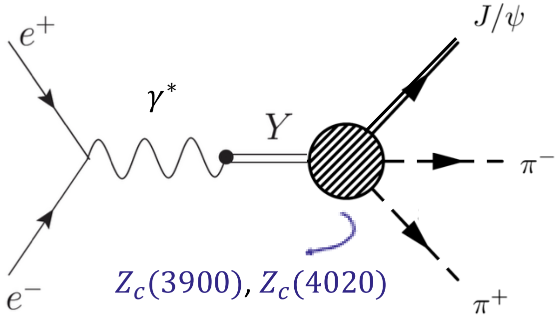


Functions $X_R(m^2_{ij})$ are the only model-dependent elements – resonant contributions

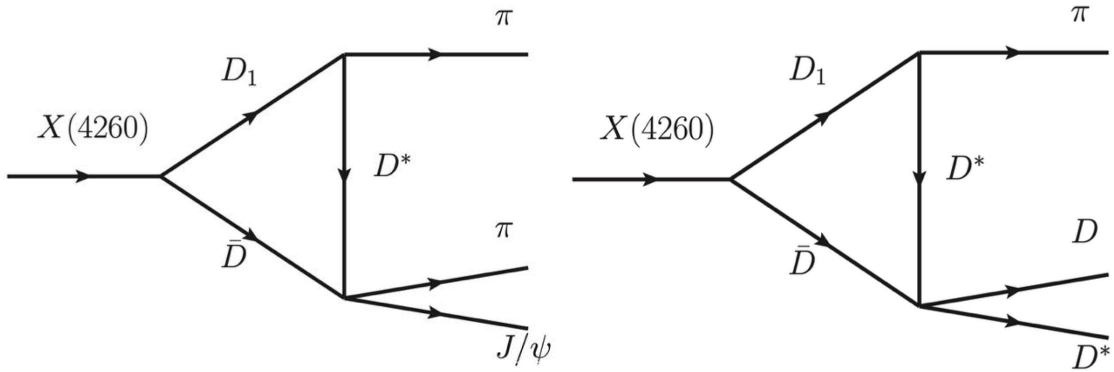
Functions $X_R(m_{ij}^2)$ are the only model-dependent elements → Breit-Wigner parametrization: narrow resonances

- $Z_c(3900)$ was first observed in the same process at $\sqrt{s} = 4.26\ GeV$ [BESIII:2013ris]
- Quantum number established to be $Z_c(3900) = 1^+$ [BESIII:2017bua]
- Nature – investigated

Tetraquark	Hybrid
Molecular state	Virtual state



Not a kinematic effect coming from anomalous singularities



Triangle diagram cannot explain the experimental results [Gong:2016jzb]

Pole in unphysical Riemann sheet

$$X_{Z_c}(m_{\pi J/\psi}^2) \to \frac{1}{m_{\pi J/\psi}^2 - M_{Z_c}^2 + iM_{Z_c}\Gamma_{Z_c}}$$

Room for improvement: K-matrix

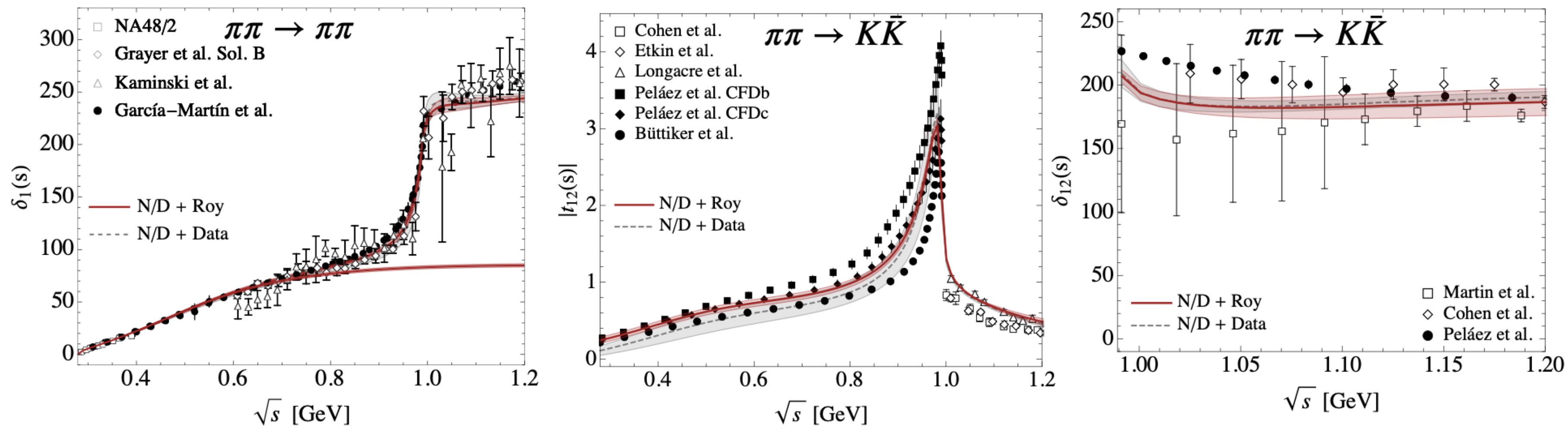
Formalism – $\pi\pi$ contribution

Functions $X_R(m_{ij}^2)$ are the only model-dependent elements

- **B**reit-Wigner parametrization: narrow resonances
- **D**ispersive treatment

Omnes function $\Omega^{(J)}$ by definition fulfils unitarity relation on a right-hand cut

$$\Omega^{(0)} = \begin{pmatrix} \Omega_{\pi\pi\to\pi\pi} & \Omega_{\pi\pi\to K\bar{K}} \\ \Omega_{K\bar{K}\to\pi\pi} & \Omega_{K\bar{K}\to K\bar{K}} \end{pmatrix}$$



	Our results		Roy-like analyses	
	pole position, MeV	couplings, GeV	pole position, MeV	couplings, GeV
$\sigma/f_0(500)$	$458(10)^{+7}_{-15} - i\,256(9)^{+5}_{-8}$	$\pi\pi : 3.33(8)^{+0.12}_{-0.20}$ $K\bar{K} : 2.11(17)^{+0.27}_{-0.11}$	$449^{+22}_{-16} - i\,275(15)$	$\pi\pi : 3.45^{+0.25}_{-0.29}$ $K\bar{K} : -$
$f_0(980)$	$993(2)^{+2}_{-1} - i\,21(3)^{+2}_{-4}$	$\pi\pi : 1.93(15)^{+0.07}_{-0.12}$ $K\bar{K} : 5.31(24)^{+0.04}_{-0.24}$	$996^{+7}_{-14} - i\,25^{+11}_{-6}$	$\pi\pi : 2.3(2)$ $K\bar{K} : -$

Approach: data-driven N/D ansatz [Danilkin:2020pak]

❶ Incorporates final-state interaction $\pi\pi/K\bar{K}$ ❷ Accurately reflects phase shifts ❸ No unitarity violation

❹ Contribution from left-hand cuts can be absorbed in subtraction polynomial [Danilkin:2020kce]

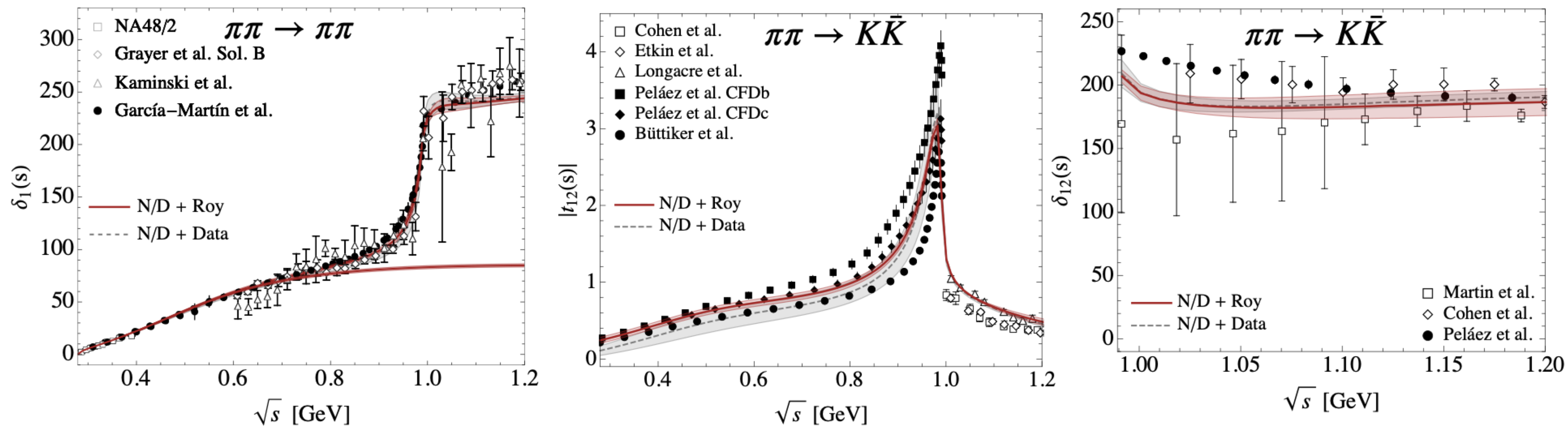
Formalism – $\pi\pi$ contribution

Functions $X_R(m_{ij}^2)$ are the only model-dependent elements

- Breit-Wigner parametrization: narrow resonances
- Dispersive treatment

Omnes function $\Omega^{(J)}$ by definition fulfils unitarity relation on a right-hand cut

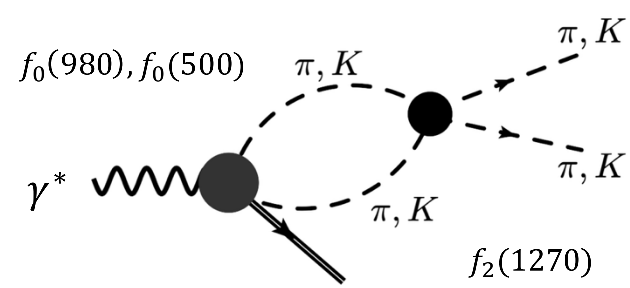
$$\Omega^{(0)} = \begin{pmatrix} \Omega_{\pi\pi\to\pi\pi} & \Omega_{\pi\pi\to K\bar{K}} \\ \Omega_{K\bar{K}\to\pi\pi} & \Omega_{K\bar{K}\to K\bar{K}} \end{pmatrix}$$



The same set of subtraction constants to describe $\pi\pi$ or $K\bar{K}$ final state

$$X_{\pi\pi}(m_{\pi\pi}^2) \to (a + b\, m_{\pi\pi}^2)\Omega_{\pi\pi\to\pi\pi}(m_{\pi\pi}^2) + (c + d\, m_{\pi\pi}^2)\Omega_{\pi\pi\to K\bar{K}}(m_{\pi\pi}^2)$$

$$X_{\pi\pi}(m_{\pi\pi}^2) \to (a + b\, m_{\pi\pi}^2)\Omega_{K\bar{K}\to\pi\pi}(m_{\pi\pi}^2) + (c + d\, m_{\pi\pi}^2)\Omega_{K\bar{K}\to K\bar{K}}(m_{\pi\pi}^2)$$



The Omnes function for $f_2(1270)$ is constructed directly from the phase shift

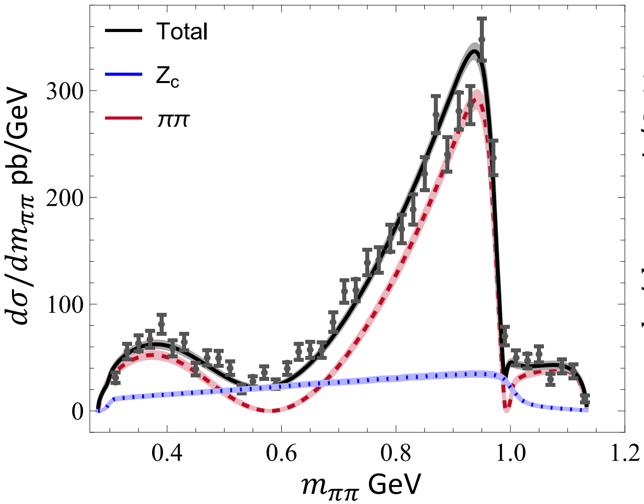
$$X_{f_2(1270)}(m_{\pi\pi}^2) \to e\Omega^{(2)}(m_{\pi\pi}^2)$$

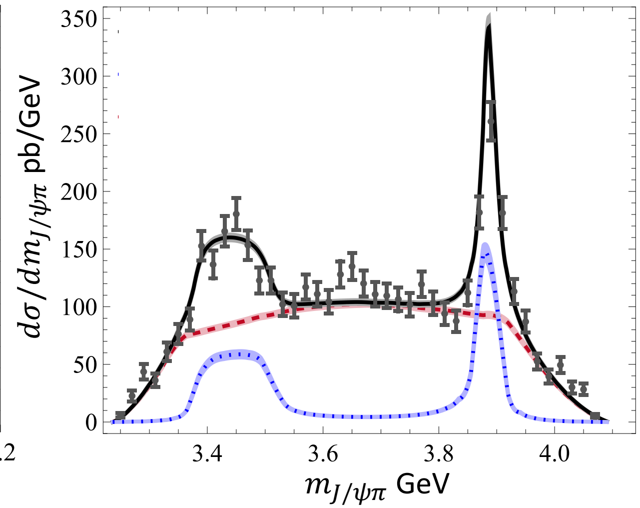
a, b, c, d, e – fit parameters

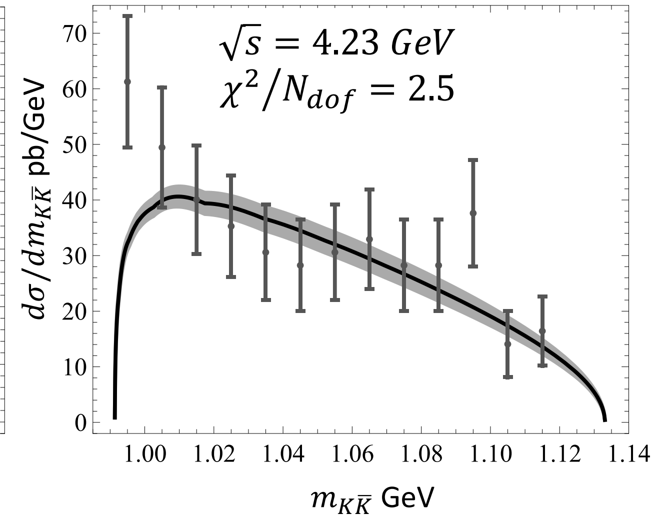
$$\Omega^{(2)}(m_{\pi\pi}^2) = \exp\left(\frac{m_{\pi\pi}^2}{\pi}\int_{x_{th}}^{\infty}\frac{\mathrm{d}x}{x}\frac{\delta(x)}{x - m_{\pi\pi}^2}\right)$$

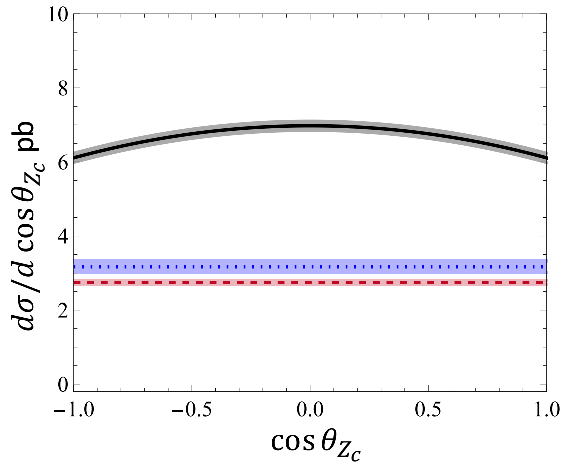
Results – Simultaneous fit to $e^+e^- \to \pi\pi(K\bar{K})J/\psi$

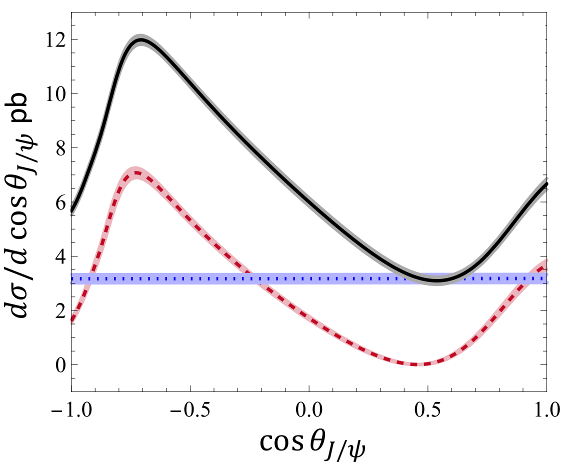
- Fixed energy fit to $\sqrt{s} = 4.23$, 4.26 GeV to [BESIII:2017bua] and $e^+e^- \to K\bar{K}J/\psi$ cross-section [BESIII:2022joj]











1. Rescattering strongly affects the shape
2. Polar angle $\cos\theta_{Z_c}$ – symmetric
   Helicity angle $\cos\theta_{J/\psi}$ – asymmetric

[Ermolina:2024uln]

$Z_c(3900) = 1^+$

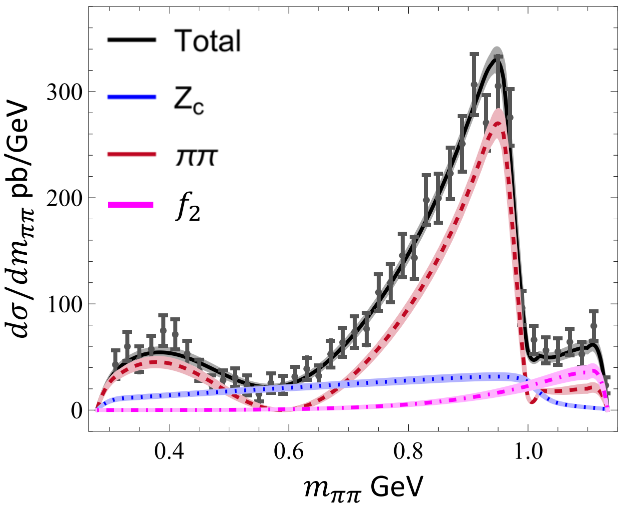
LS combinations	
$\gamma^* \to Z_c^\pm \pi^\mp$	$(0,1), (2,1)$
$Z_c^\pm \to J/\psi\pi^\mp$	$(0,1), (2,1)$
$\gamma^* \to f_0\, J/\psi$	$(0,1), (2,1)$
$f_0 \to \pi^+\pi^-$	$(0,0)$

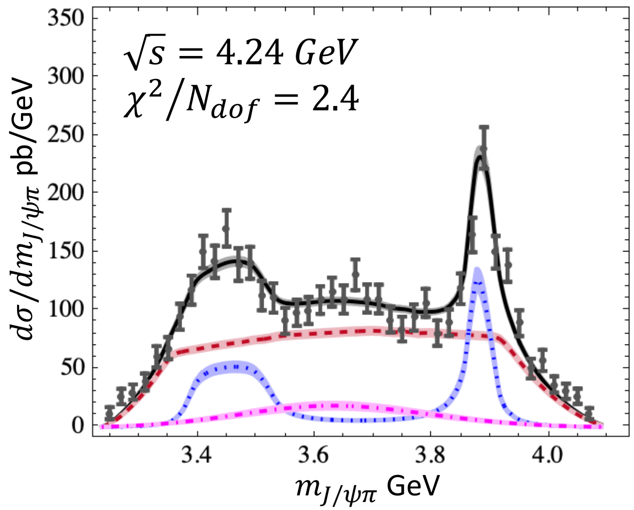
- Fixed mass and width of $Z_c(3900)$ to [BESIII:2017bua]

  5 parameters: subtraction constants + product of LS-couplings entering vertices with Z_c

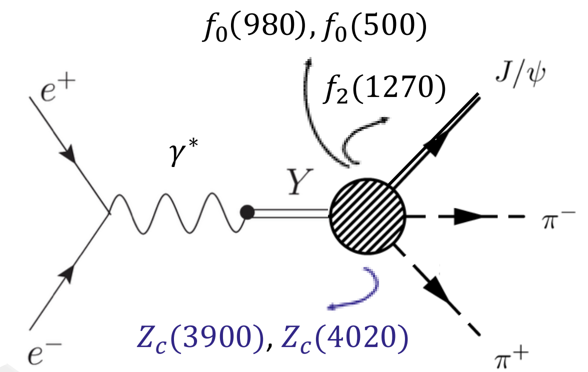
Results – Different energies of $e^+e^- \to \pi^+\pi^- J/\psi$

- Fixed energy fit to the new data [BESIII:2025qkn]

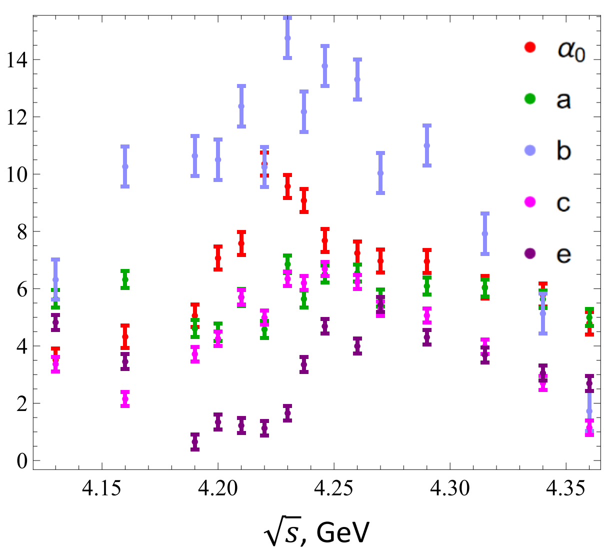




- Parameter space: LS-coupling α_0, subtractions a, b, c, e



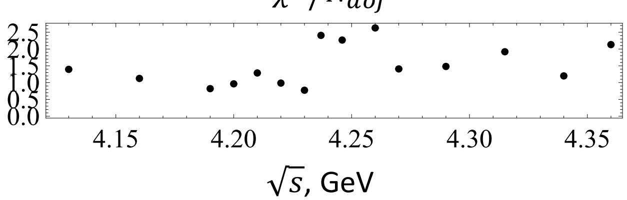
Parameter value



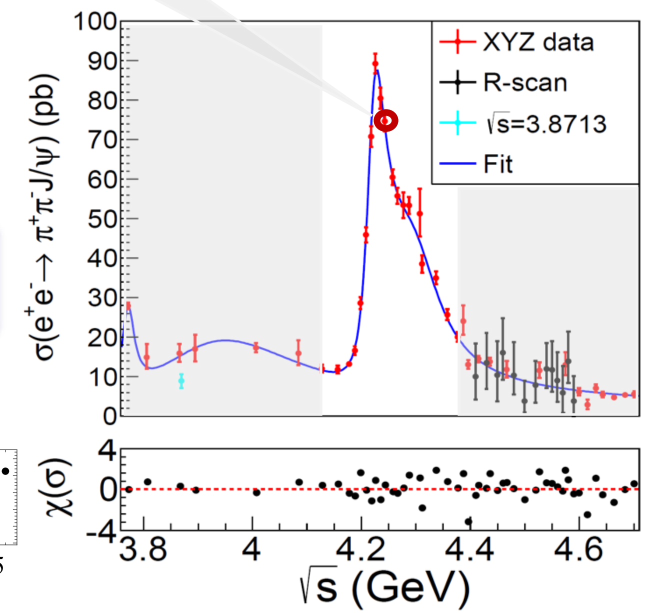
- Fixed mass and width of $Z_c(3900)$ from the data

Parameters of fixed $\sqrt{s}$ description follows cross-section pattern

χ^2/N_{dof}



[BESIII:2022qal]



Results – Global fit to $e^+e^- \to \pi^+\pi^- J/\psi$

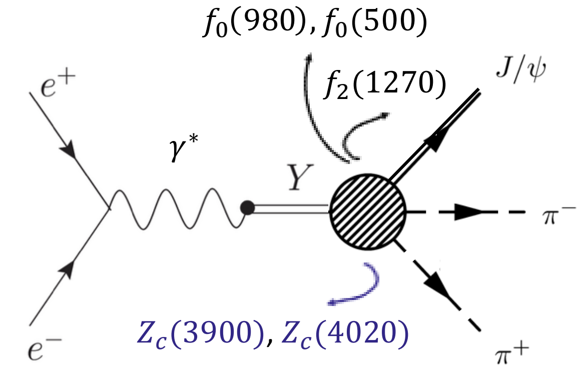
How to catch $\sqrt{s}$ dependence?

Describe all the data for all the energies available

- Introduce energy-dependent subtractions [Hoferichter:2014vra, Hoferichter:2016duk]

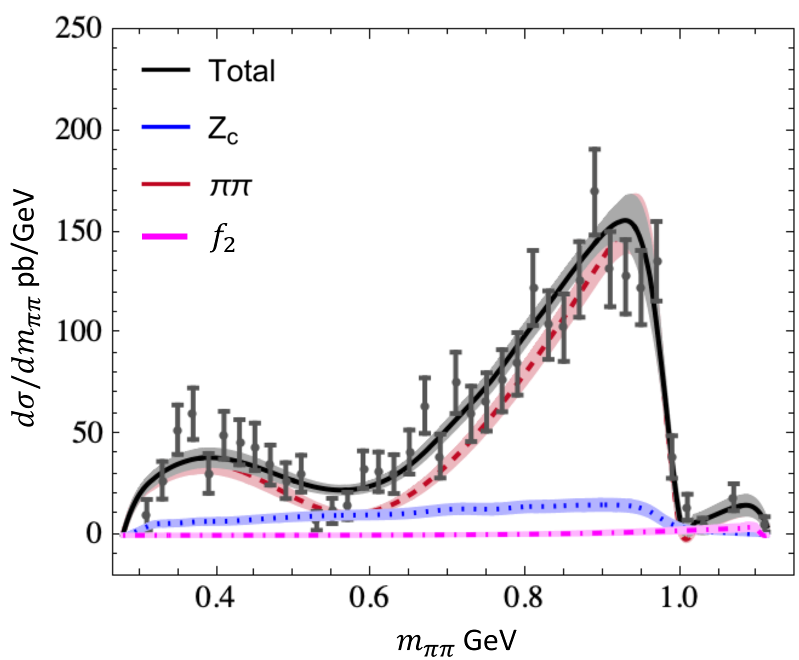
Each parameters turns into 3: $c_{Y_1}, c_{Y_2}, \varphi$

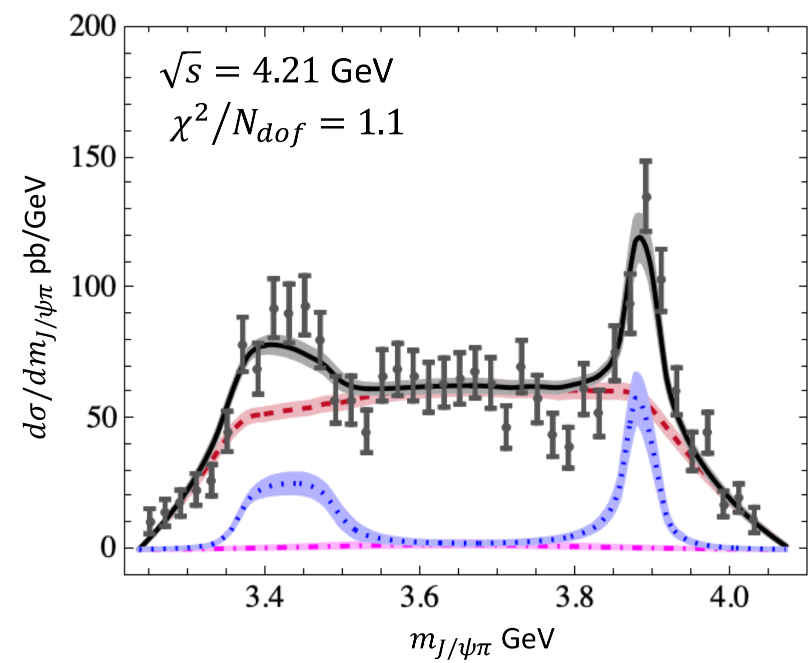
Coherent sum of both Y's BW: $$\frac{c_{Y_1}}{s - M_{Y_1}^2 + iM_{Y_1}\Gamma_{Y_1}} + \frac{c_{Y_2} e^{i\varphi}}{s - M_{Y_2}^2 + iM_{Y_2}\Gamma_{Y_2}}$$



Global fit to the data, with parameters $c_{Y_1}, c_{Y_2}, \varphi$ being truly constants.

Y's mass and width – fixed to [BESIII:2022qal]



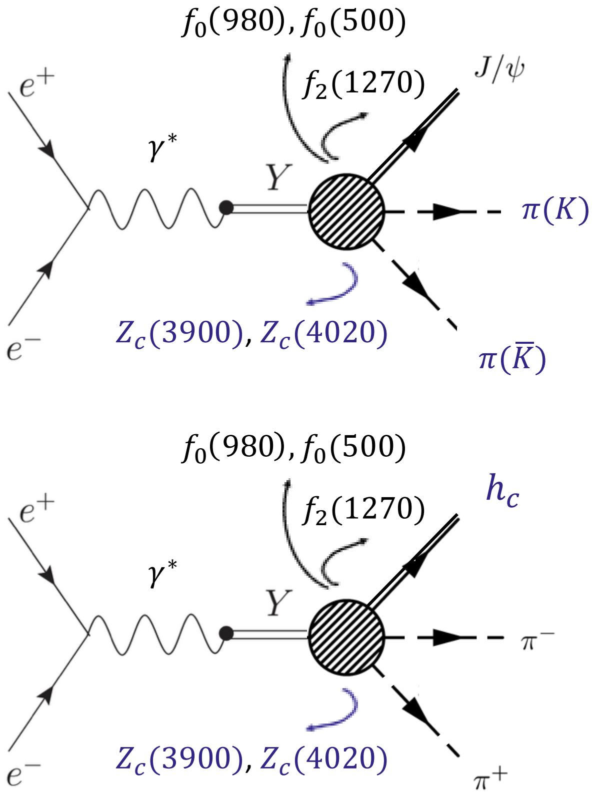


Conclusion – Results and further prospects

Results:

1. Validated the Dalitz-plot decomposition formalism via Lagrangian cross-checks
2. Described $e^+e^- \to \pi\pi(K\bar{K})J/\psi$ data and studied angular dependence
3. Described the $e^+e^- \to \pi\pi J/\psi$ data for all available energies

   Introduced and tested simultaneous global description of data
4. Currently studying the $e^+e^- \to \pi\pi h_c$ data to determine quantum numbers of $Z_c(4020)$
5. Stablished the correspondence of conventional helicity formalism, DPD and covariant tensor formalism



Prospects:

- Study precise relations between parameters to reduce the number of them
- Consider different $\sqrt{s}$ dependence models (e.g. introduce constant background term) [Hoferichter:2014vra, BESIII:2022qal, BESIII:2025qkn]
- **Constrain** the properties of **Y** and **Zc** states

Thank you!