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TWO-PHOTON EXCHANGE EFFECTS IN MUONIC HYDROGEN

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CONTENTS

WHY THIS MATTERS

- Lamb shift of muonic hydrogen:

$$\Delta E_{\mathrm{LS}}^{\mathrm{th}}(\mu\mathrm{H}) = \left[206.0344(3) - 5.2259\, r_p^2/\mathrm{fm}^2 + 0.0289(25)\right] \mathrm{meV}$$

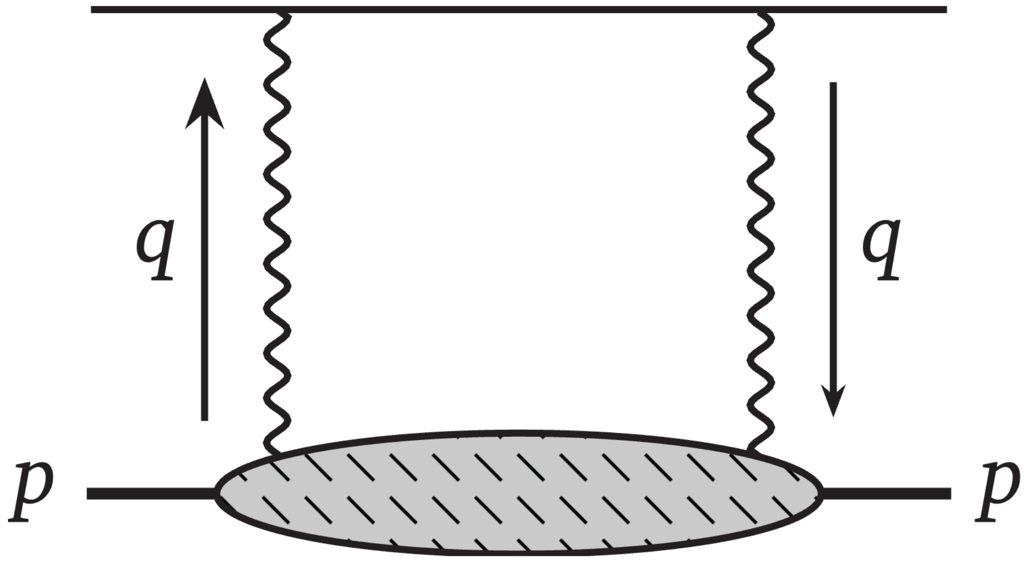
$$\Delta E_{\mathrm{LS}}^{\mathrm{exp}}(\mu\mathrm{H}) = 202.3706(23)\, \mathrm{meV}$$

where r_p is the proton charge radius

- Large (2γ exchange) hadronic uncertainties
- Catch up to experiments (proposed 5-times improvement!)
- Update parametrization of structure functions with the latest data
- Analytic parametrization **over the whole energy region**

THE THEORY BEHIND

Forward two-photon exchange (TPE)

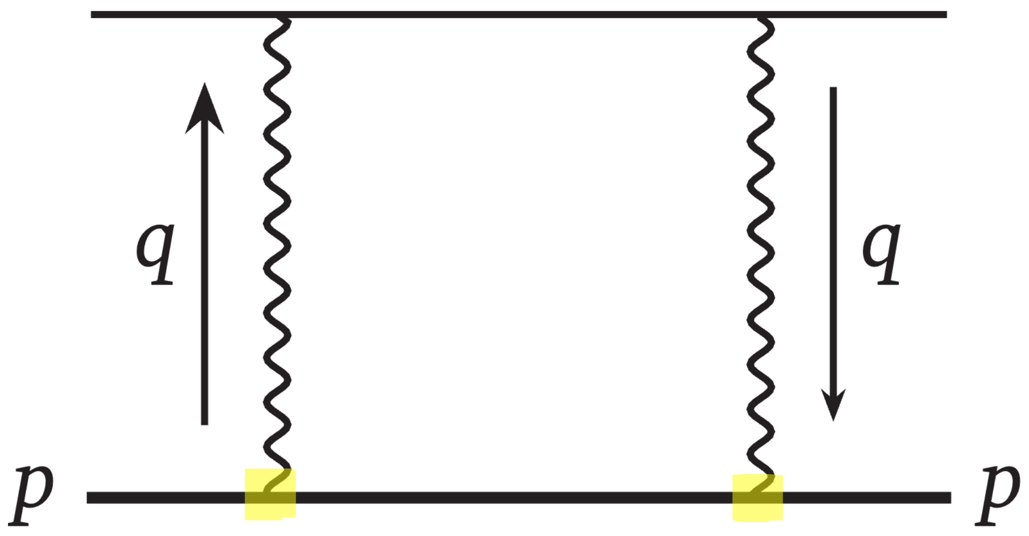


How to parametrize the proton?

- Unpolarized case

THE THEORY BEHIND

Forward two-photon exchange (TPE)



+ crossed channel

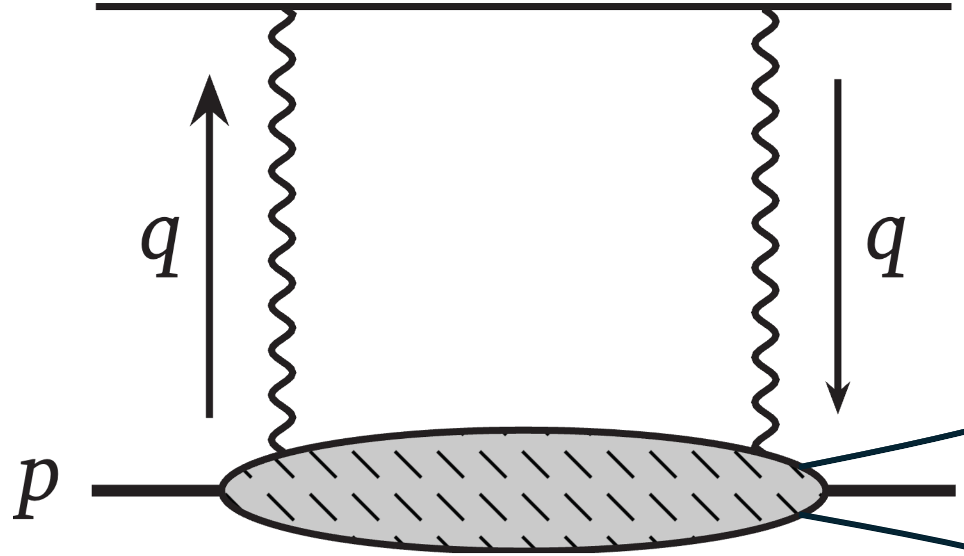
How to parametrize the proton?

- Unpolarized case
- Born contributions (well-known)

$$\Gamma^\mu = F_D(q^2)\gamma^\mu + F_P(q^2)i\sigma^{\mu\nu}\frac{q_\nu}{2M_N},$$

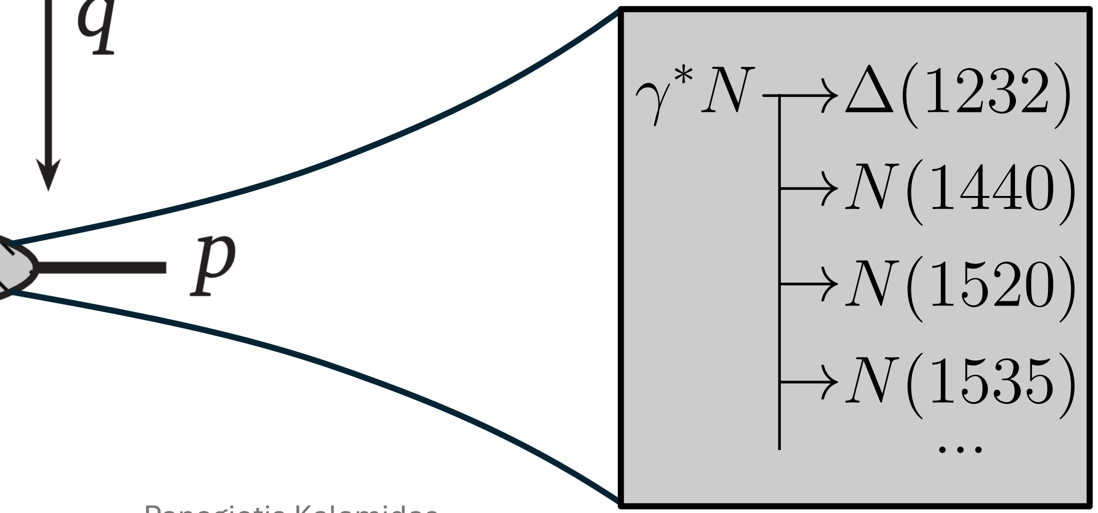
THE THEORY BEHIND

Forward two-photon exchange (TPE)



How to parametrize the proton?

- Unpolarized case
- Born contributions (well-known)
- Non-Born contributions



THE THEORY BEHIND

Unpolarized Compton Tensor

$$T^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2) + \frac{1}{M_N^2}\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2} q^\nu\right) T_2(\nu, Q^2)$$

Connect to structure functions

$$\operatorname{Im} T_1(\nu, Q^2) = \frac{e^2}{4M_N} F_1(x, Q^2)$$

$$\operatorname{Im} T_2(\nu, Q^2) = \frac{e^2}{4\nu} F_2(x, Q^2)$$

Parametrize structure functions and fit to data

THE THEORY BEHIND

Dispersion relations (DR)

$$\mathrm{Re}\,T_1(\nu,Q^2) = \mathrm{Re}\,T_1^{\mathrm{Born}}(\nu,Q^2) + T_1^{\mathrm{subt}}(0,Q^2) + \frac{2\nu^2}{\pi}\mathcal{P}\int_{\nu_{\mathrm{th}}}^{\infty} \mathrm{d}\nu' \, \frac{\mathrm{Im}\,T_1(\nu',Q^2)}{\nu'(\nu'^2-\nu^2)}$$

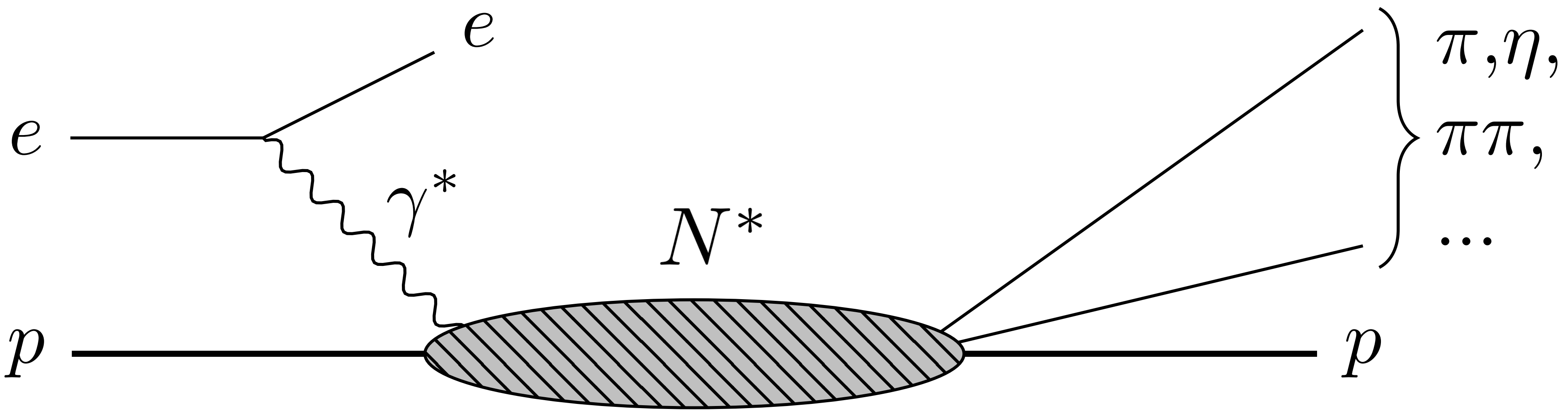
$$\mathrm{Re}\,T_2(\nu,Q^2) = \mathrm{Re}\,T_2^{\mathrm{Born}}(\nu,Q^2) + \frac{2}{\pi}\mathcal{P}\int_{\nu_{\mathrm{th}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu'}{\nu'^2-\nu^2}\mathrm{Im}\,T_2(\nu',Q^2),$$

- $T_2(\nu,Q^2)$ obeys unsubtracted DR
- $T_1(\nu,Q^2)$ needs once subtracted DR $\Rightarrow$ Assumptions are needed
- Low energy expansion (LEX): $T_1^{\mathrm{subt}}(0,Q^2) \to \beta_{M1}Q^2 + \mathcal{O}(Q^4)$

where β_{M1} is the magnetic polarizability

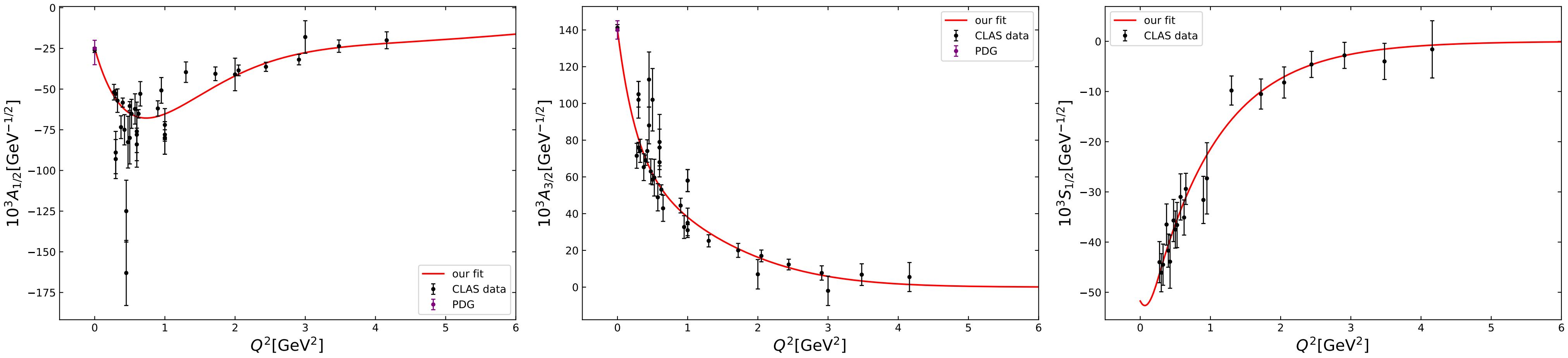
THE RESONANCE REGION

- Incorporate current 4* resonances (the ones we have data for!)
- Use **exclusive data** from CLAS experiment
- Fit $\gamma^* N \to N^*$ helicity amplitudes ($A_{1/2}$, $A_{3/2}$, $S_{1/2}$)



THE RESONANCE REGION

- N(1520) example



- Low-Q^2 behaviour covered!
- For $S_{1/2}(Q^2)$, consider the Siegert limit: $S_{1/2}(Q^2) \propto |\mathbf{q}|$ (Ramalho, 2019)
- Predict resonance contribution to inclusive cross section

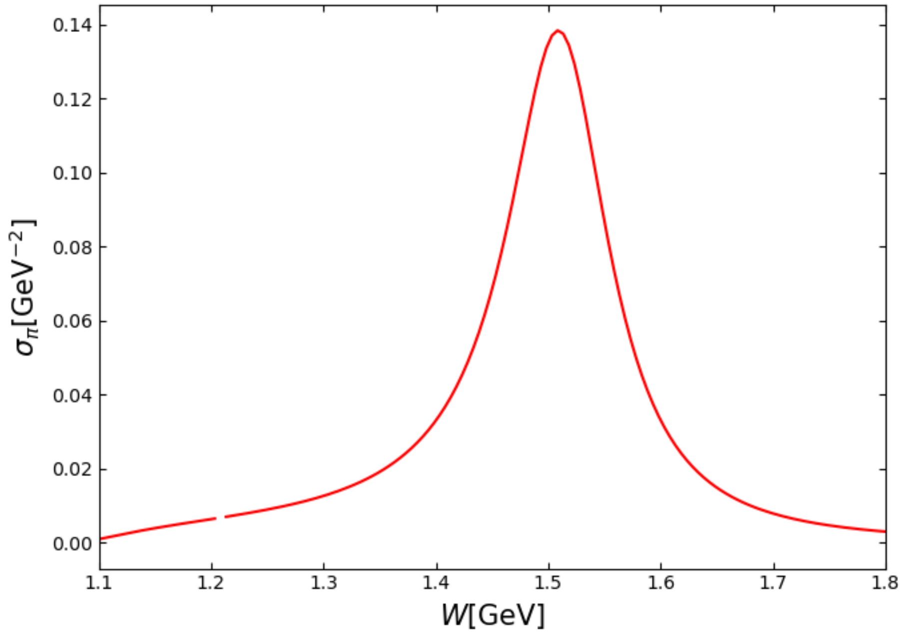
THE RESONANCE REGION

- Use Breit-Wigner parametrization for cross-sections
- Three channels considered:

$$\pi N,\ \eta N,\ \pi\pi N$$

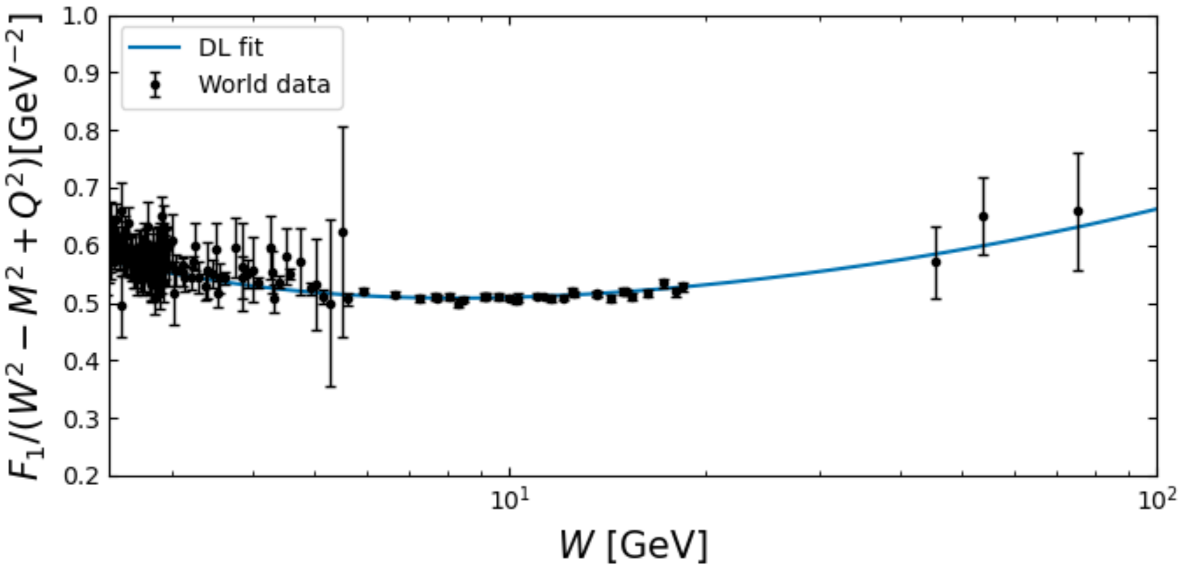
- N(1520) example $\longrightarrow$
- Resonance near threshold

$\Longrightarrow$Flatté parametrization needed



THE HIGH ENERGY REGIME

- Known high energy behavior
- Fit to world DIS data (Donnachie and Landshoff, 2004)



- Goal: Connect it smoothly to our parametrization

THE HIGH ENERGY REGIME

- Parametrize only the non-resonant part

$$F_1(\nu, Q^2) = F_1^{\text{Res}}(\nu, Q^2) + F_1^{\text{NRes}}(\nu, Q^2)$$

with,

$$F_1^{\text{NRes}}(\nu, Q^2) = \sum_{i=\text{p}_s,\text{p}_h} f_i(Q^2)(2M_N\nu)^{\alpha_i}\left(1-\frac{\nu_0}{\nu}\right)^{a_i+\alpha_i}\left(1+\frac{\nu_0}{\nu}\right)^{a_i+\alpha_i} + \sum_{i=\text{R},\pi} f_i(Q^2)(2M_N\nu)^{\alpha_i}\left(1-\frac{\nu_0}{\nu}\right)^{a_i+\alpha_i}\left(1+\frac{\nu_0}{\nu}\right)^{b_i}$$

- 4 contributions: 2 pomerons, a ϱ, ω trajectory and a π loop term
- Only part **fitted to inclusive data**

THE HIGH ENERGY REGIME

- $T_1(\nu, Q^2) - T_1^{\mathrm{NRes}}(\nu, Q^2)$ satisfies an unsubtracted dispersion relation

  $\Longrightarrow T_1^{\mathrm{NRes}}(\nu, Q^2)$ parametrization fully determines subtraction function:

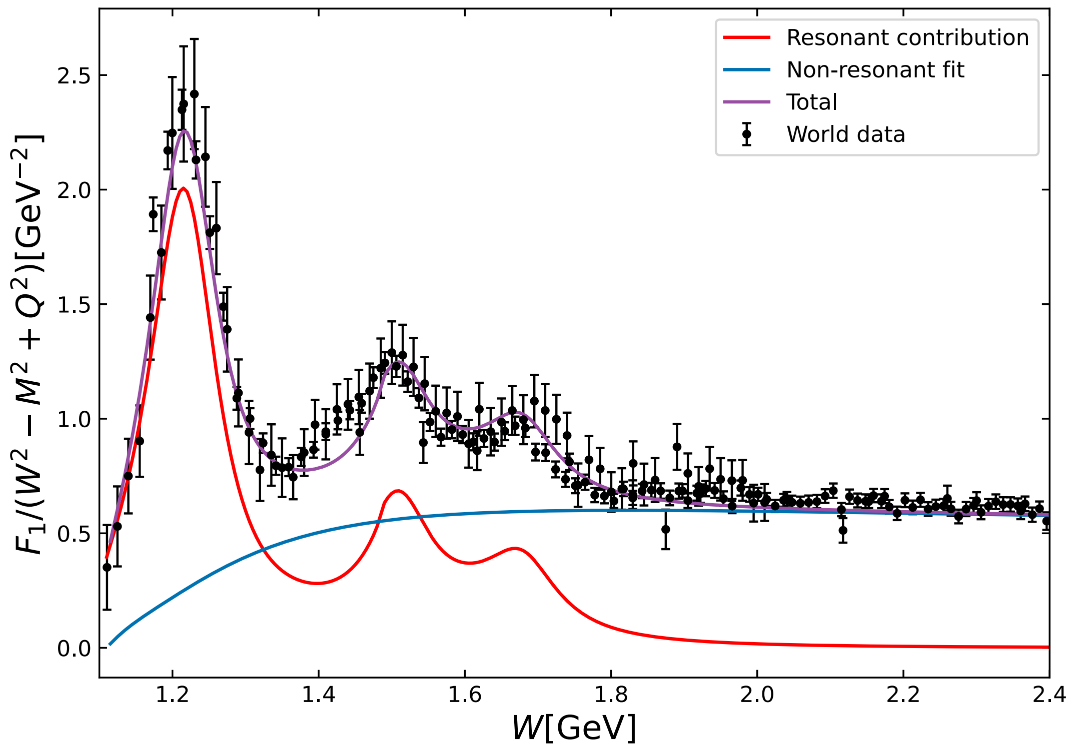
$$T_1^{\mathrm{subt}}(0, Q^2) = \frac{1}{4\pi M_N} F_D^2(Q^2) + T_1^{\mathrm{NRes}}(0, Q^2) + \frac{e^2}{2\pi M_N} \int_{\nu_{\mathrm{th}}}^{\infty} \frac{\mathrm{d}\nu'}{\nu'} F_1^{\mathrm{Res}}(\nu', Q^2)$$

- Non-resonant contribution

$$T_1^{\mathrm{NRes}}(0, Q^2) = \frac{2\alpha}{M_N} \Bigg\{ \sum_{i=\mathrm{p}_s, \mathrm{p}_h} f_i(Q^2)\, (4M_N\nu_0)^{\alpha_i} \sqrt{\pi} \frac{\Gamma(-\alpha_i)\Gamma(a_i + \alpha_i + 1)}{\Gamma(a_i + \alpha_i/2 + 1)\Gamma((1 - \alpha_i)/2)}$$
$$+ \sum_{i=\mathrm{R},\pi} f_i(Q^2)\, (2M_N\nu_0)^{\alpha_i} \frac{\Gamma(-\alpha_i)\Gamma(a_i + \alpha_i + 1)}{\Gamma(a_i + 1)} {}_2F_1(-b_i, -\alpha_i, a_i + 1, -1) \Bigg\}$$

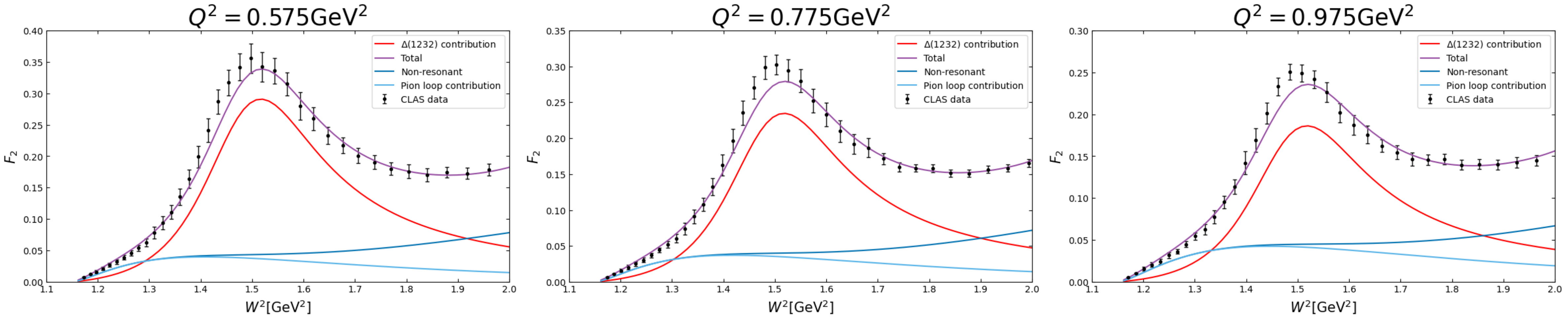
RESULTS… SO FAR

- Photoproduction $(Q^2 = 0)$
- Smooth connection to high energy fit
- General agreement with the data
- Potential improvement in second resonance region



RESULTS ... SO FAR

- $Q^2 \neq 0$ for the $\Delta(1232)$ resonance region



- Pion loop provides correct near-threshold behavior in first resonance region

OUTLOOK

- We use exclusive data for the resonance contribution and fit the non-resonant part
- Significantly different to previous inclusive fits (e.g. Bosted and Christy, ,2007)
- New fit will include latest Jefferson Lab data lacking from previous ones
- Application for new dispersive estimate of the two-photon exchange contribution to the Lamb shift in muonic H
- Planned extension to update hyperfine splitting prediction

Thank you for your attention!

Questions? Ideas? Suggestions?
Contact me at pakalami@uni-mainz.de