Nucleon electromagnetic and axial form factors: Analysis of LQCD data with ChPT and dispersive methods

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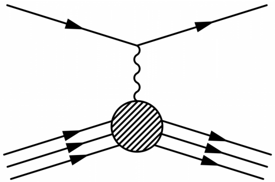
- Structure of protons and neutrons is encoded in form factors
  - EM current, $V^\mu = \bar{q}Q\gamma^\mu q$,
    $\langle N|\, V^\mu \,|N\rangle = \bar{u}' \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_N} F_2(q^2)\right] u$
    - $F_{1,2}$ are the Dirac and Pauli ffs
    - $G_E = F_1 - \frac{Q^2}{4m_N^2} F_2, \quad G_M = F_1 + F_2$
    - slope=charge radius, $\langle r_E^2 \rangle = 6 \left.\frac{\mathrm{d}G_E}{\mathrm{d}q^2}\right|_0$
  - Axial current, $A^\mu = \bar{q}\gamma^\mu\gamma_5 q$,
    $\langle N|\, A^\mu \,|N\rangle = \bar{u}' \left\{\gamma_\mu F_A(q^2) + \frac{q_\mu}{2m_N}\tilde{G}_P(q^2)\right\}\gamma_5 u$
  - F_A is the axial ff
    - $F_A(0) \to$ related to spin distribution
  - Weak interaction is $V - A$

    $\Rightarrow F_A$ is exrtracted from different ν scattering exp. with significant uncertainty

    - F_A is a key input to ν oscillation experiments



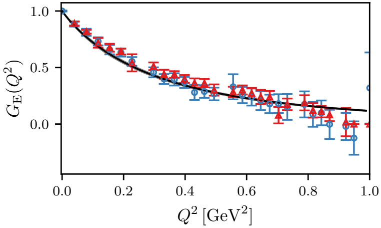


Figure 1: Djukanovic et al. [2102.07460]

Electromagnetic form factor at low q^2

- EM form factors of the nucleon
- Advent of the proton radius puzzle

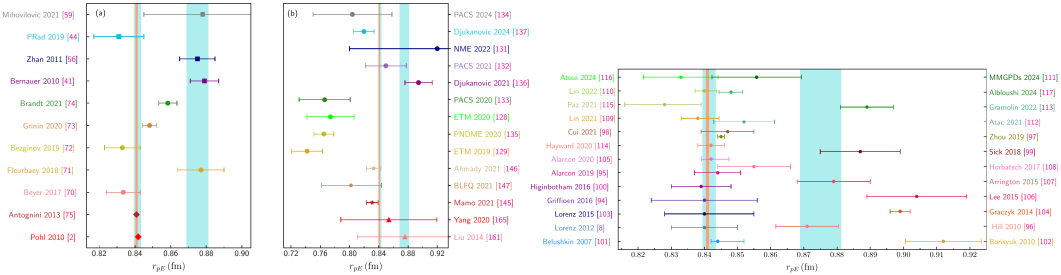
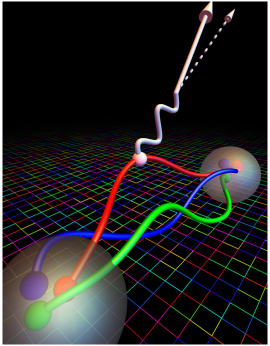


Figure 2: Fig. from [Goharipour et al., Nucl.Phys.B 1017 (2025)].

- Active QCD subject
  - Lattice: see Fig. (b)
  - Dispersive methods, e.g. [Lin et al. PRL 128 (2022) 052002, Alarcon et al. PRC 99 (2019) 044303]

Electromagnetic form factor at low q^2

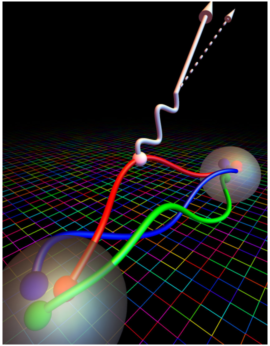


- **Lattice QCD**
  - Nucleon f.f. is a benchmark for LQCD
  - Uncertainties reduced for unphysical large M_π
  - Technical difficulties $\to$ recent progress
  - Experimental and lattice q^2 parametrisation:

$$\left.\begin{array}{l}\text{- dipole ansatz}\\ \text{- z-expansion of different orders}\\ \text{- ...}\end{array}\right\} \Longrightarrow \text{different } \langle r_1^2 \rangle\text{, and } F_{\mathrm{EM}} \text{ in general}$$

- Theoretical input needed

Electromagnetic form factor at low q^2



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    - dipole ansatz
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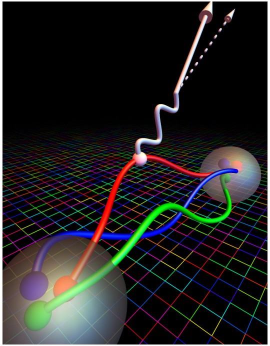
    - ...

    $\Longrightarrow$ different $\langle r_1^2 \rangle$, and F_{EM} in general

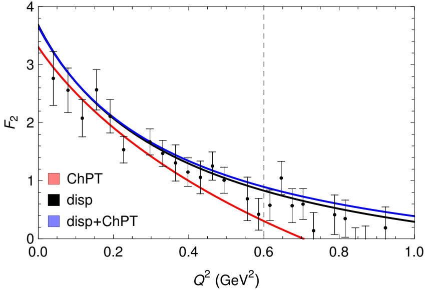
- Theoretical input needed
  - **Chiral Perturbation Theory** (χPT) $\rightarrow$ parametrise M_π and q^2 dep.
  - **Dispersion theory** $\rightarrow$ enlarge q^2 range
  - **Goal**: Disp+χPT = good q^2 and M_π description

Electromagnetic form factor at low q^2



- **Lattice QCD** parametrisation question
- **Chiral Perturbation Theory** (ChPT)
  - QCD based parametrisation of q^2 and M_π dependencies $\Longrightarrow$ extrapolate lattice results to the phys. point and extract $\langle r_i^2 \rangle$ and κ from the lattice simulations
- **Modern Baryon ChPT**
  - Relativistic EOMS:
    terms that break power counting are absorbed in LECs,
    e.g. κ [Geng et al PRL 101 (2008)], πN [Alarcon et al Annals Phys. 336 (2013)]
  - Relativistic+Dynamical Δ is necessary,
    e.g. πN [Yao et al JHEP 5 (2016)], ffs.[Hiller Blin PRD 96 (2017), Thürmann et al. PRC 103 (2021)]

EM Form Factors



(FA, Di An, Luis Alvarez-Ruso & Stefan Leupold)

- **ChPT** $\rightarrow$ parameterise M_π and q^2 dep.
- **Dispersion theory** $\rightarrow$ enlarge q^2 range (ρ resonance)
- **Goal**: Disp+χPT
  - $=$ good q^2 and M_π description of the LQCD EM form factors
  - accurate extrapolation to M_π^{phys}

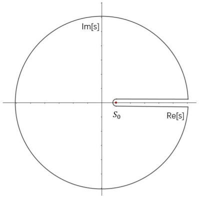
- Enlarge the q^2 range of χPT (ρ dynamics)
  1. Analiticity $\Rightarrow$ Disp. rel. (Cauchy)

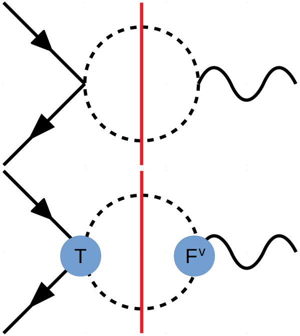
$$F(q^2) = \int_{4M_\pi^2}^{\infty} \frac{ds}{\pi} \frac{\Im F(s)}{s - q^2 - i\epsilon}$$

  2. Unitarity$\Rightarrow$ $\Im F = \frac{1}{2} \sum_n T_{\gamma^* n} T^\dagger_{n\bar{N}N}$, $n = \pi^+\pi^-, \ldots$
     - $\ell = 1$, $\pi\pi$ must be iso-vector (ρ channel)
       $\Rightarrow F_i = \frac{1}{2} F_i^{(s)} + F_i^{(v)} \frac{\sigma^3}{2}$, $F_i^{(v)} = F_i^p - F_i^n$
  3. Using full $N\bar{N}\pi\pi$ and $\gamma^*\pi\pi$ vertices with M_π dep.

$$F(q^2) = \frac{1}{12\pi} \int_{4M_\pi^2}^{\infty} \frac{ds}{\pi} \frac{T\, p_{\rm cm}^3\, F_\pi^{V*}}{s^{1/2}(s - q^2 - i\epsilon)} ,$$

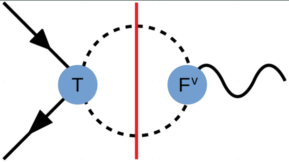
[Granados et al, EPJ A 53 (2017)]

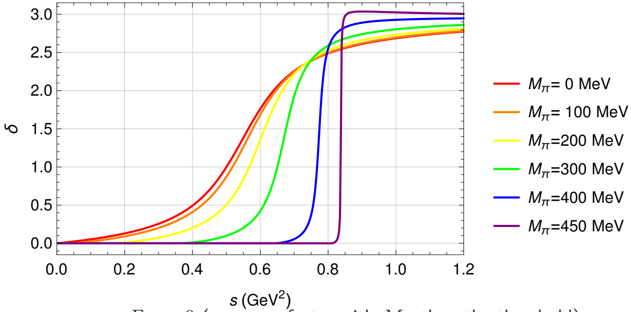




Dispersion theory

- $\pi\pi$ scattering included in F_π^V & T via Omnès with IAM δ (nonperturbative)
- M_π dep. taken into account
  - $\Omega(s) = \exp\left\{ s \int_{4M_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\delta(s')}{s'(s'-s-i\epsilon)} \right\}$
  - We fit NLO IAM δ to physical results [Garcia-Martin PRD 83(2011)]
  - We check that the M_π dependence is realistic
  - Our prediction of $m_\rho(M_\pi)$ agrees well with LQCD [Andersen et al, 1808.05007]





$\rho \nrightarrow \pi\pi$, $\Gamma_\rho \to 0$ (m_ρ runs faster with M_π than the threshold)

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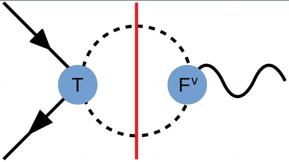
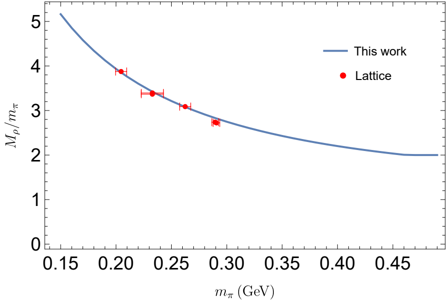




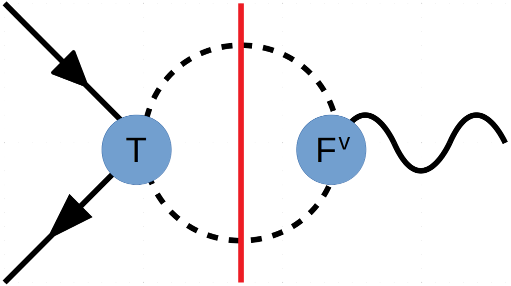
$\rho \not\to \pi\pi$, $\Gamma_\rho \to 0$ (m_ρ runs faster with M_π than the threshold)

Dispersion theory

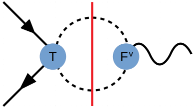
- Our two vertices, T and F_π^V, account for M_π dep.
  - $\boxed{F_\pi^V(s) = [1+\alpha_v s]\ \Omega(s)}$
  - $\mathrm{Im}F_{2\pi}(s) = F_v^*(s)\,\frac{p_{\mathrm{cm}}^3}{12\pi\sqrt{s}}\,T(s) \in \mathbb{R}$
  - $\boxed{T_i(s) = K_i(s) + \Omega(s)\,P_i + I_i(s)}$
    - P and K determined from p-wave projected πN in ChPT

$$\Rightarrow \boxed{F(q^2) = \frac{1}{12\pi}\int_{4M_\pi^2}^{\infty}\frac{ds}{\pi}\,\frac{T\ p_{\mathrm{cm}}^3\ F_\pi^{V*}}{s^{1/2}(s-q^2-i\epsilon)}}$$

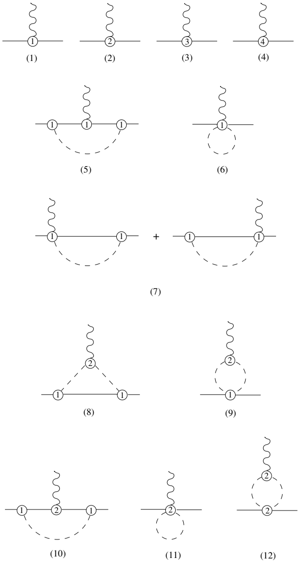
more in backup slides



- $\mathbf{F}_{\mathrm{EM}} = \mathbf{F}_{\mathrm{EM}}^{\mathrm{disp}} +$ **diagrams w/o 2π cut from χPT**
  - $F(q^2) = \int_{4M_\pi^2} \frac{\mathrm{d}s}{\pi} \frac{\mathrm{Im}F_{2\pi}(s)}{s-q^2} + F_{\chi\text{PT no } 2\pi}$
  - $F_{\mathrm{EM}}^{\mathrm{disp}}$



  - $+\chi$PT diagrams



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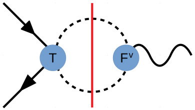
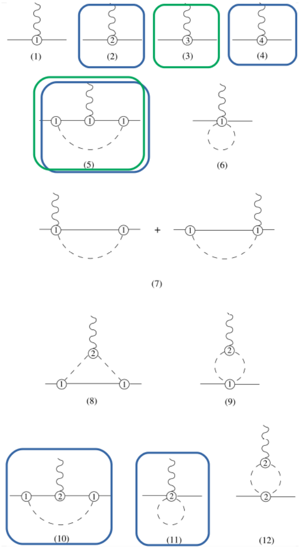


  - $+\chi$PT diagrams
    - Relativistic and with explicit $\Delta(1232)$ [Bauer et al., PRC 86 (2012)]
    - green: $F_1 - F_1(0)$
    - blue: F_2



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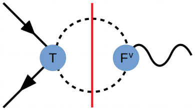

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    - green: $F_1 - F_1(0)$
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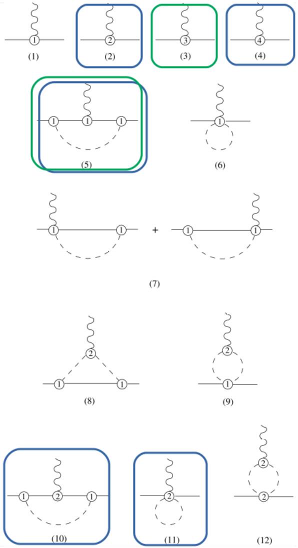
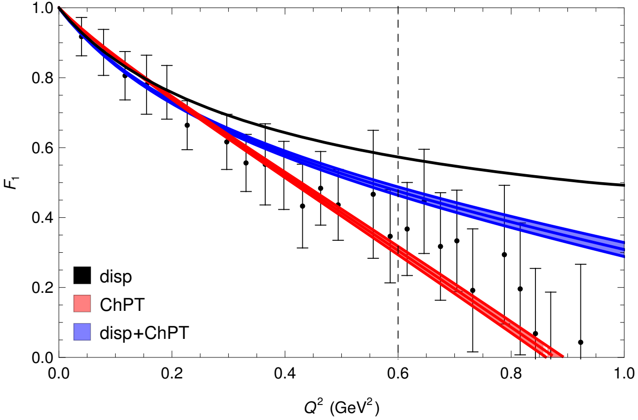
  - Disp and χPT renormalizations
    - At $\mathcal{O}(p^3)$ disp and χPT agree on the M_π nonanalyticities
    - $F_1^{\mathrm{point}} \sim F_1^{(9)\mathrm{ChPT}} \sim q^2 \log M_\pi$
    - differences in the renorm. are absorbed in LECs



- **Dirac f.f., F_1** FA, An, Alvarez-Ruso & Leupold PRD 108 (2023)
  - $\langle N| V^\mu |N\rangle = \bar{u}' \left[\gamma^\mu F_1 + \frac{i\sigma^{\mu\nu}q_\nu}{2m_N} F_2\right] u$
  - $F_1 = 1 + \frac{q^2}{6}\left[-12 d_6 + \langle r_1^{2(\log M_\pi)}\rangle \log M_\pi\right] + \mathcal{O}(p^4)$
  - Comparison with LQCD:
    10 CLS ensembles [Djukanovic et al. PRD 103 (2021)] $\longleftarrow$ controlled FV and discret. effects
  - In the χPT and disp+χPT F_1, d_6 is fitted to LQCD
  - Disp $\longrightarrow q^2$ curvature



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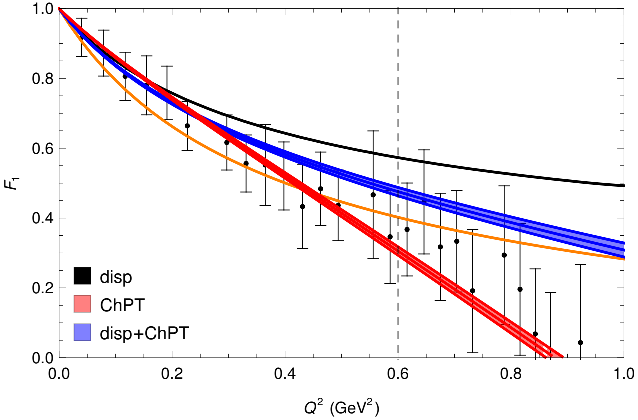
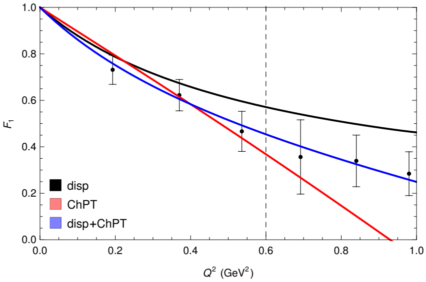
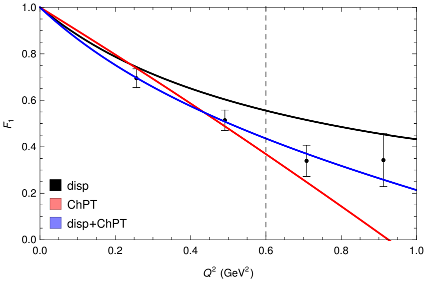




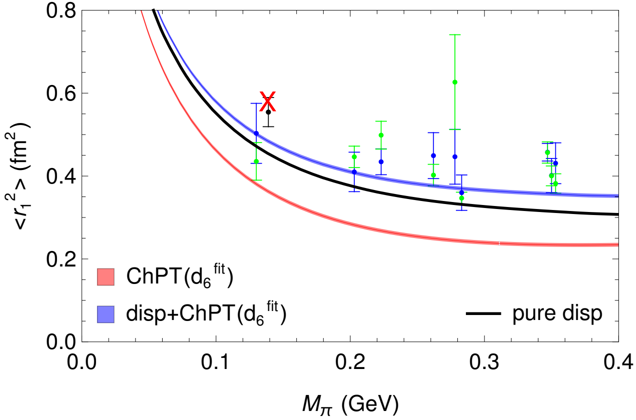
(h) H105 $M_\pi = 0.278$ GeV



(i) N302 $M_\pi = 0.353$ GeV

- **Disp+χPT describes well the M_π dep.**
  - Only one parameter fit, d_6
  - Describes data at Q^2 as high as $Q^2 = 0.6$ GeV^2 ($\sim m_\rho^2$) and all M_π ($M_\pi \lesssim 350$ MeV)
  - outperforms the pure dispersive and plain χPT descriptions

	Disp (prediction)	χPT	**Disp+χPT**
$d_6(\mu = m_\rho)$ (GeV^{-2})	-	0.074 ± 0.010	0.416 ± 0.010
$d_6(\mu = m_N)$ (GeV^{-2})	-	-0.422 ± 0.010	0.155 ± 0.010
χ^2/dof	$108.9/47 = 2.32$	$73.9/(47-1) = 1.61$	$24.6/(47-1) = 0.53$
$\langle r_1^2 \rangle_{\mathrm{phys}}$ (fm^2)	0.4541	0.3626 ± 0.0047	$\mathbf{0.4838} \pm \mathbf{0.0047}$



- $F_1^{(v)} = 1 + \frac{1}{6}\langle r_1^2\rangle^{(v)} q^2 + \mathcal{O}(q^4)$
- disp. has the $\log M_\pi$ predicted by chiral symmetry breaking
- $\langle r_1^2\rangle^{\text{PDG}} = 0.577$ fm^2, heavy baryon fit to LQCD from [Djukanovic PRD 103(2021)]: $\langle r_1^2\rangle^{\text{HB}} = 0.554 \pm 0.035$ fm^2

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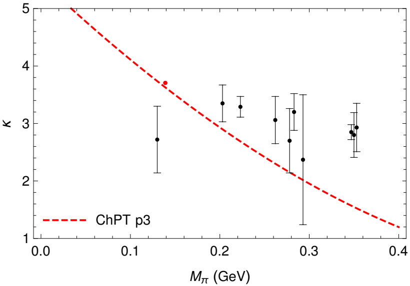
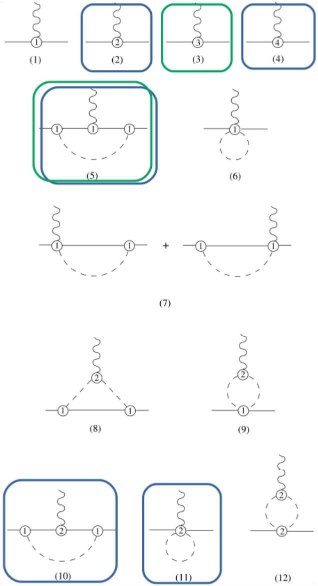
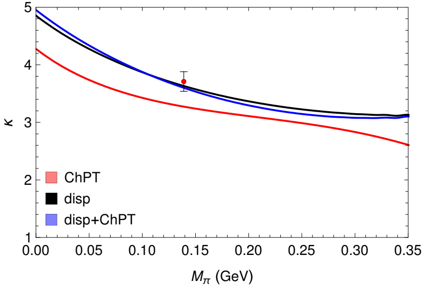
    $F_2 = \frac{F_M - F_E}{(1+Q^2/(4m^2))}$
  - $F_2^{(v)} = \kappa^{(v)} \left[1 + \frac{1}{6} \langle r_2^2 \rangle^{(v)} q^2 + \mathcal{O}(q^4) \right]$
  - $\mathcal{O}(p^3)$ χPT is not enough$F_2(0)^{o(3)} = c_6 - \left(\frac{\pi m_N g_A^2}{4\pi^2 F^2} \right) M_\pi$

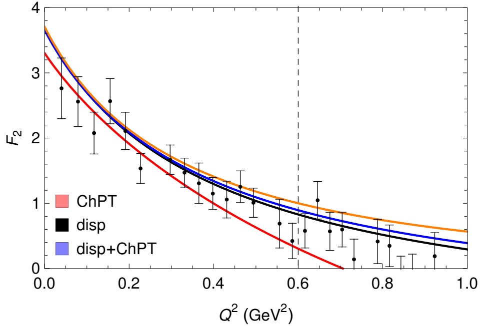


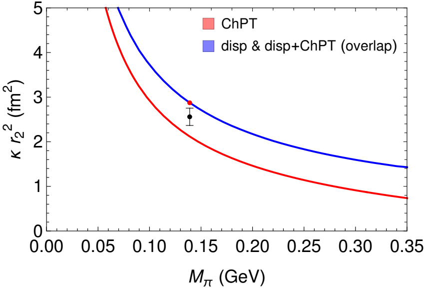
  - We implement the necessary $\mathcal{O}(p^4)$ contributions ($\not\Delta$)



- **Pauli f.f., F_2, fit to LQCD**
  - Constraints from πN scattering simplify the calculation
  - We are able to describe the F_2 and extract κ and $\langle r_2^2 \rangle$ (close to the PDG values)
  - When there are several orders and LECs involved it is interesting to perform an analysis of the weight of each order, i.e. a specific study of the convergence of the chiral series and the systematic error (including the systematic error of the LECs themsleves)
    - for the moment we have performed such a study of the chiral convergence only for F_A







Nucleon axial form factor at low q^2

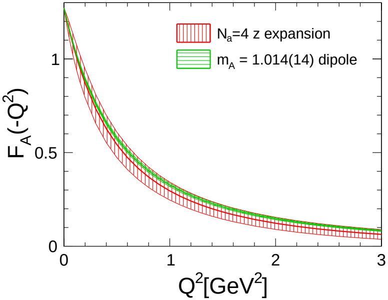
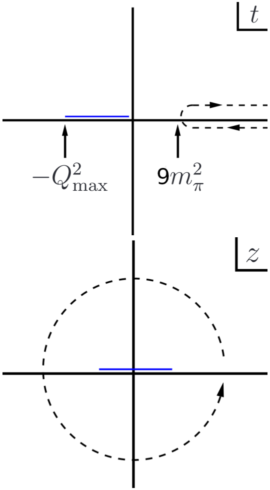


Figure 4: νD extract. [Meyer et al. 1603.03048]

- F_A **Experimental determinations**
  - $F_A(q^2)$ main source of uncertainty for QE νN scatt.
    - key input in ν oscillation experiments
  - old bubble chamber νD (ANL, BNL, FNL) data
  - Recent remarcable measurement:
    $\bar{\nu} p \to \mu n$ in MINERvA
    subtracting carbon from plastic [Cai et al Nature 614 (2023)]
- $\langle r_A^2 \rangle$ is a benchmark quantity for exp. and th.
  - interesting extraction from weak muon capture in muonic hydrogen (μH): $\mu^- p \to \nu_\mu n$
    chiral Ward id.: $\bar{g}_P = \bar{g}_P(\langle r_A^2 \rangle) + \mathcal{O}(t^2)$, no z-exp.
    (MuCap)[Andreev PRC 91(2015)] [Hill, Rept. Prog. Phys. 81 (2018)]
  - also related to pion-electroproduction in ChPT

Axial form factor at low q^2



- Experimental and lattice q^2 parametrisation is delicate:

  - dipole ansatz
  - z-expansion
  - ...

  $\Longrightarrow$ different $\langle r_A^2 \rangle$, and F_A in general

- At low q^2, **ChPT** can be used, also for M_π dependence
  - estimate uncertainty
- Our work on F_A
  - Goal: extract F_A & $\langle r_A^2 \rangle$ from a combined set of recent LQCD data using ChPT to parametrise the ff and estimate the systematic uncertainty

Axial form factor

- **Nucleon axial isovector form factor**
  - $F_A(q^2) = g_A \left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \mathcal{O}(q^4)\right]$
- **NNLO $\mathcal{O}(p^4)$ F_A in relativistic χPT** with explicit Δ
  - $F_A = \mathring{g}_A + 4d_{16}M_\pi^2 + d_{22}q^2 + \mathrm{loops}(M_\pi, q^2)$

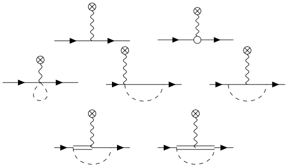


Figure 5: $\mathcal{O}(p)$ and $\mathcal{O}(p^3)$ (w. f. renormalisation not shown)

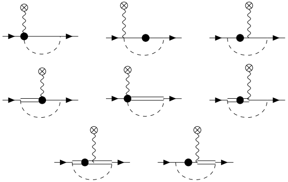
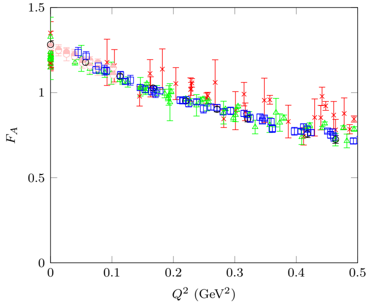


Figure 6: $\mathcal{O}(p^4)$

Axial form factor

- We perform a **meta-analysis of large set of recent LQCD results**
  - Many recent works $\Rightarrow$ substantial improvements
  - RQCD[1] + PNDME[2] + "Mainz"[3] + PACS[4] + ETMC[5]
  - More works will be included in an update [Djukanovic PRD 106 (2022) 074503, Jang PRD 109 (2024) 014503, Alexandrou PRD 109 (2024) 034503, Tsuji PRD 109 (2024) 094505]



- Active topic in ChPT, e.g. [Bernard et al. Phys.Lett.B 868 (2025), Hermsen et al Phys.Rev.D 109 (2024) 114029, FA & Alvarez-Ruso PRD 105 (2021)]

[1] Bali et al. JHEP 05 (2020)
[2] Park et al. 2103.05599
[3] Meyer et al. Modern Phys. A 34 (2019)
[4] Shintani et al. PRD 102 (2020)
[5] Alexandrou et al. PRD 103 (2021)

$g_A \equiv F_A(q^2 = 0)$ analysis

$g_A(M_\pi)$ study **FA & Alvarez-Ruso PRD 105 (2021)**

Motivations:

1. **test** the LECs extracted from πN **scattering**
2. **determine** d_{16} from fit to lattice:
   - source of uncertainty in m_q dependence of nuclear properties (ground-state and binding energies)
3. **study** more deeply the **convergence of g_A** in χPT

$g_A \equiv F_A(q^2 = 0)$ analysis

- $\left.\begin{array}{l}\pi N \to \pi\pi N \Rightarrow d_{16}, c_i \\ \pi N \to \pi N \Rightarrow c_i\end{array}\right\} \to$ combined fit Refs. [*]
- $g_A^{\text{phys}} = g_A(M_\pi^{\text{phys}}) \Rightarrow \mathring{g}_A$

$\Rightarrow$ Bad Δ-less $g_A(M_\pi)$ prediction

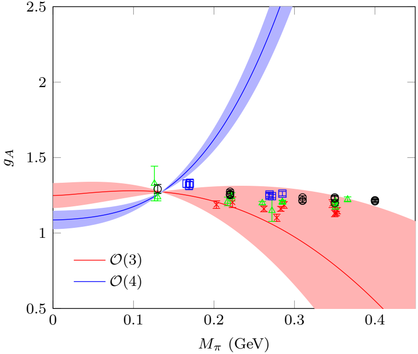


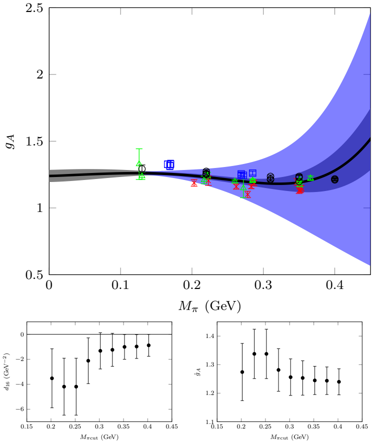
Table 1: $\mathcal{O}(p^4)$: steep rise (OK with [**])

	$\mathcal{O}(p^3)$	$\mathcal{O}(p^4)$
d_{16}	-2.2 ± 1.1	-1.86 ± 0.80
c_1	-	-0.89 ± 0.06
c_2	-	3.38 ± 0.15
c_3	-	-4.59 ± 0.09
c_4	-	3.31 ± 0.13

- $\Rightarrow$ **We include explicit $\Delta(1232)$ and fit d_{16} to lattice**
- $\Rightarrow$ **This reconciles g_A^{LQCD} with the πN LECs, c_i**
  (with the caveat of a slow convergence)
- we will discuss the whole $F_A(q^2)$ directly

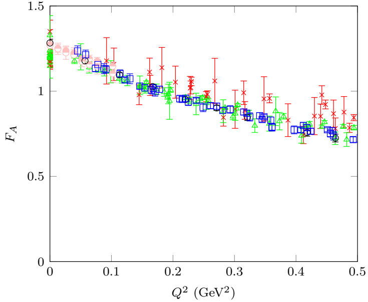
[*] Siemens et al. PRC 94 (2016), Siemens et al. PRC 96 (2017), (value converted to standard EOMS)
[**] Bernard et al PLD 639 (2006)

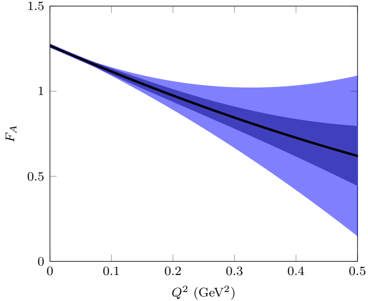
F_A fit



- $\mathbf{F_A}$ **fit procedure:**
  - **Difference between orders $\simeq$ theoretical uncertainty** [Epelbaum EPJA 53 (2015)]
    $$\Delta g_{A\chi}^{(4)} = \max\left\{ \left(\frac{M_\pi}{\Lambda}\right)^4 |\mathring{g}_A|, \left(\frac{M_\pi}{\Lambda}\right)^2 \left|g_A^{(3)}\right|, \frac{M_\pi}{\Lambda} |g_A^{(4)}| \right\}$$
  - $\Delta F_{A\chi}$ is added to LQCD errors in the χ^2
  - LECs have naturalness priors
  - fit range: χ^2 plateau $\Longrightarrow M_\pi^{\text{cut}} \simeq 400$ MeV, $Q^2_{\text{cut}} = 0.36$ GeV^2

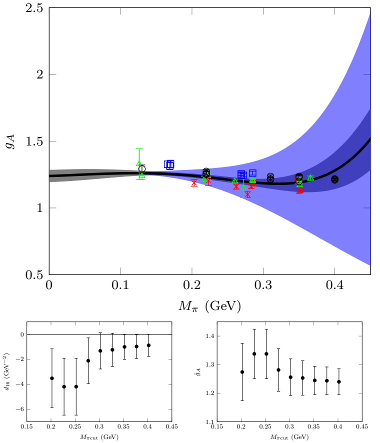
F_A fit





- **$\mathbf{F_A}$ fit procedure:**
  - **Difference between orders $\simeq$ theoretical uncertainty** [Epelbaum EPJA 53 (2015)]
    $$\Delta g_{A\chi}^{(4)} = \max\left\{ \left(\frac{M_\pi}{\Lambda}\right)^4 |\mathring{g}_A|, \left(\frac{M_\pi}{\Lambda}\right)^2 \left|g_A^{(3)}\right|, \frac{M_\pi}{\Lambda} |g_A^{(4)}| \right\}$$
  - $\Delta F_{A\chi}$ is added to LQCD errors in the χ^2
  - LECs have naturalness priors
  - fit range: χ^2 plateau $\Longrightarrow M_\pi^{\text{cut}} \simeq 400$ MeV, $Q^2_{\text{cut}} = 0.36$ GeV^2
  - lattice spacing corrections are implemented
  - only large vols., $M_\pi L \geq 3.5$
- **Fit results: good description**
  - accurate description at the physical point
  - Δ baryon is a necessary d.o.f.
  - $\mathcal{O}(p^5)$ still needed for full convergence

$F_A(q^2 = 0) = \boxed{g_A}$



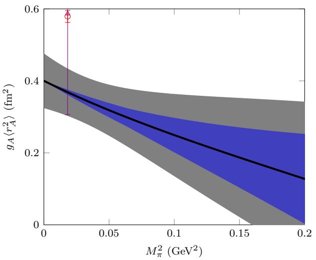
- **Axial charge results** from $F_A(q^2)$ fit
  - $g_A(M_{\pi\mathrm{phys}}) = 1.273 \pm 0.014$ vs $g_A^{\mathrm{exp}} = 1.2754(13)_{\mathrm{exp}}(2)_{\mathrm{RC}}$
    $\Rightarrow$ excellent agreement with exp.
    vs $g_A^{\mathrm{FLAG}} = 1.246 \pm 0.028$

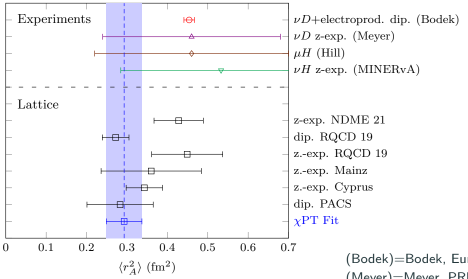
- $g_A(M_\pi) = \mathring{g}_A + 4d_{16}M_\pi^2 + \mathrm{loop}(M_\pi)$
- $\boxed{d_{16} = -1.46 \pm 1.00\ \mathrm{GeV}^{-2}}$

$\longrightarrow$ M_π dependence of long range nuclear forces

- Can not be extracted from πN elastic scattering
- In line with $d_{16} = -1.0 \pm 1.0\ \mathrm{GeV}^{-2}$ from $\pi N \to \pi\pi N$
  [Siemens et al. PRC 96 (2017)] (EOMS corrected)

$F_A = g_A\left(1+\frac{1}{6}\boxed{\langle r_A^2\rangle}\,q^2\right)$ axial radius





(Bodek)=Bodek, Eur. Phys. J. C 53, 349 (2008)
(Meyer)=Meyer, PRD 93, 113015 (2016)
(Hill)=Hill, Rept. Prog. Phys. 81 (2018)
(MINERvA)=Cai, Nature 614 (2023)

- **Our $\mathcal{O}(p^4)$ χPT extraction:**
  - M_π slope driven by loops with Δ
  - $d_{22} = 0.29 \pm 1.69$ GeV^{-2} (no assumptions on $\Delta\Delta\pi$ coupling enlarges error)
    - d_{22} compatible with $\mathcal{O}(p^3)$ π electroprod. Guerrero et al. PRD, 102 (2020)
  - $\boxed{\langle r_A^2\rangle(M_{\text{phys}}) = 0.293 \pm 0.044\ \text{fm}^2}$
  - Empirical determinations (model dependent) are in tension with ours and with most of LQCD extractions
  - Tipically the extracted $\langle r_A^2\rangle^{\text{phys}}$ value varies depending on the parametrisation
  - **Our QCD-based parametrisation leads to a value in line with most individual LQCD extractions**

Axial form factor

- Our ChPT analysis will be updated with new LQCD data
  - [Djukanovic PRD 106 (2022) 074503, Jang PRD 109 (2024) 014503, Alexandrou PRD 109 (2024) 034503, Tsuji PRD 109 (2024) 094505]

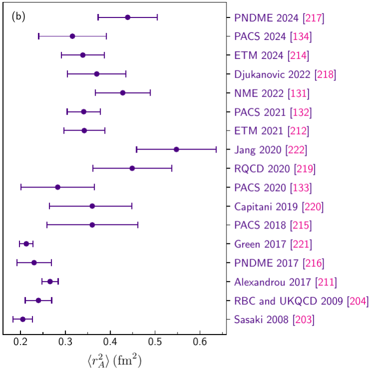


Figure 7: Fig. from [Goharipour et al., Nucl.Phys.B 1017 (2025)].

Conclusions and Outlook

- Remarcable work of the LQCD community in the recent years
- Isovector Dirac ff, F_1
  - Dispersive 2π cut necessary to describe curvature
  - The dispersive improvement of ChPT outperforms the separate descriptions
  - single LEC fits well the LQCD F_1 for $Q^2 \lesssim 0.6$ GeV^2 and $M_\pi \lesssim 350$ MeV
- F_A
  - Succesful description of LQCD $F_A(q^2)$ using $\mathcal{O}(p^4)$ relativistic χPT
    - Explicit $\Delta(1232)$ necessary
    - Fit describes data in $M_\pi^{\rm cut} \simeq 400$ MeV, $Q^2_{\rm cut} = 0.36$ GeV^2
  - In general there is tension between the experimental and lattice extractions of $\langle r_A^2 \rangle$
  - $\langle r_A^2 \rangle^{\rm phys} = 0.291 \pm 0.052 \ {\rm fm}^2$ extracted with systematic chiral error
- Relativisitc ChPT (and EFTs in general) are beneficial to control systematics
  - Moreover useful LECs extracted from these calculations: connection with other processes

Backup: F_{EM}

- **Summary of dispersive nucleon F_{EM}**

1. $F(q^2) \approx \int_{4M_\pi^2}^{\Lambda^2} \frac{\mathrm{d}s}{\pi} \frac{\mathrm{Im}F_{2\pi}(s)}{s-q^2-i\epsilon} + F_{\text{ChPT no }2\pi}$
2. $\mathrm{Im}F_{2\pi}(s) = F_\pi^*(s) \frac{p_{\mathrm{cm}}(s)^3}{12\pi\sqrt{s}} T(s)$, need $F_\pi(s)$ and $T(s) \hookrightarrow$ given by $\pi\pi$ scattering
3. We use t_{IAM} NLO with a Blatt-Weiskopf ff on t_2 to go to π smoothly: $\tilde{t}_2 = t_2/(1+r^2 p_{\mathrm{cm}}^2)$.
   - We fit $l_2 - 2l_1$ and r to $\delta(s)$ of [García-Martín] at M_π^{phys}
   - Describe well the $m_\rho(M_\pi)$ from LQCD
4. $F_\pi^V(s, M_\pi) = (1+\alpha_V(M_\pi))\Omega(s, M_\pi)$, α_V determined from $\mathrm{d}F_\pi^V(s)/\mathrm{d}s$ in ChPT and $\Omega \to$ improves F_π^V ρ peak
5. T has to respect Watson th. ($\Im F_{2\pi} \in \mathbb{R}$), therefore Omnes solution
   $$T(s) = K(s) + \Omega(s)\, P + \Omega(s) \int_{4M_\pi^2}^{\Lambda^2} \frac{ds'}{\pi} \frac{\sin\delta_1(s')\, K(s')}{|\Omega(s')|\,(s'-s-i\epsilon)}$$
   - P and K determined from ChPT πN p-wave projected
6. Subtrctions on $T_{1,2}$ and $F_{1,2}$ if M_π-indep
   - T_1 subtracted, $P_1 = P_1^{\mathrm{LO}}$, $T_1 = T_1^{\mathrm{LO}}$ enough for leading 1-loop F_1. (F_1 also trivially subtracted)
   - T_2 and F_2 unsubtracted due to M_π dependences (but still the Λ dep. mild)

Backup

- EM f.f.
  - $V^\mu = \bar{q} Q \gamma^\mu q$,
    $\langle N(p')| V^\mu(0) |N(p)\rangle = \bar{u}' \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2(q^2)\right] u$
  - $G_E^N = F_1^N - \frac{Q^2}{4m_N^2} F_2^N, \quad G_M^N = F_1^N + F_2^N$
  - $F_1 = 1 \times \left[1 + \frac{1}{6}\langle r_1^2\rangle q^2 + \mathcal{O}(q^4)\right]$
    $F_1(0) = F_E(0) = 1$ electric charge
  - $F_2 = \kappa \left[1 + \frac{1}{6}\langle r_2^2\rangle q^2 + \mathcal{O}(q^4)\right]$
    $\kappa = F_M(0) = \mu - 1$ anom. magn. mom.
  - $\langle r_E^2\rangle = \langle r_1^2\rangle + \frac{3}{2m_N^2}\kappa$ proton radius puzzle
  - $F_i = \frac{1}{2} F_i^{(s)} + F_i^{(v)} \frac{\sigma^3}{2}$
  - $F_1^{(v)} = 1 \times \left[1 + \frac{1}{6}\langle r_1^2\rangle^{(v)} q^2 + \mathcal{O}(q^4)\right]$
    $F_1^{(v)}(0) = F_E^{(v)}(0) = 1$ electric charge
  - $F_2^{(v)} = \kappa^{(v)} \left[1 + \frac{1}{6}\langle r_2^2\rangle^{(v)} q^2 + \mathcal{O}(q^4)\right]$
    $\kappa^{(v)} = F_M^{(v)}(0) = \mu^{(v)} - 1$ magnetic moment
  - $\langle r_E^2\rangle^{(v)} = \langle r_1^2\rangle^{(v)} + \frac{3}{2m_N^2}\kappa^{(v)}$ proton radius puzzle

$$G_P(t) = \frac{4mg_{\pi N}F_\pi}{M_\pi^2 - t} - \frac{2}{3}\, g_A\, m^2\, r_A^2 \, + \mathcal{O}(t, M_\pi^2)$$

[Bernard et al PRD 50 (1994)]

Extra

	$\mathcal{O}(p^3)$ $\not\Delta$	$\mathcal{O}(p^4)$ $\not\Delta$	$\mathcal{O}(p^3)$ Δ	$\mathcal{O}(p^4)$ Δ
$\mathring{g}_A$ (free)	1.1782 ± 0.0073		1.2041 ± 0.0074	1.274 ± 0.041
d_{16} (GeV^{-2}) (free)	-1.021 ± 0.048		0.983 ± 0.062	-1.46 ± 1.00
d_{22} (GeV^{-2}) (free)	1.275 ± 0.086		3.77 ± 1.96	0.29 ± 1.69 (free g_1)
h_A	-	-	1.35	1.35
g_1 (free)	-	-	-0.69 ± 0.69	0.66 ± 0.56
c_1 (GeV^{-1})	-	-0.89 ± 0.06	-	-1.15 ± 0.05
c_2 (GeV^{-1})	-	3.38 ± 0.15	-	1.57 ± 0.10
c_3 (GeV^{-1})	-	-4.59 ± 0.09	-	-2.54 ± 0.05
c_4 (GeV^{-1})	-	3.31 ± 0.13	-	2.61 ± 0.10
a_1 (GeV^{-1})	-	-	-	0.90
b_1 (GeV^{-2}) (free)	-	-	-	-0.27 ± 4.96
b_2 (GeV^{-2}) (free)	-	-	-	2.27 ± 2.28
$\tilde{b}_4$ (GeV^{-2}) (free)	-	-	-	-12.48 ± 1.28
x_1 (fm^{-2}) (free)	-8.4 ± 5.8	-	-5.6 ± 5.9	-0.25 ± 16.5 (consistent)
x_2 (fm^{-2}) (free)	-8.6 ± 2.6	-	-7.1 ± 2.6	-6.36 ± 4.20
x_3 (fm^{-1}) (free)	-0.25 ± 0.21	-	-0.08 ± 0.22	0.36 ± 0.47
y_1 (fm^{-2} GeV^{-2}) (free)	-100 ± 40	-	-76 ± 44	-64 ± 121
y_2 (fm^{-2} GeV^{-2}) (free)	-31 ± 21	-	-21 ± 22	-15 ± 46
y_3 (fm^{-1} GeV^{-2}) (free)	-0.63 ± 1.49	-	0.36 ± 1.63	2.54 ± 3.98
$\mathring{m}$ (GeV)	0.874	0.874	0.855	0.855
$\mathring{m}_\Delta$ (GeV)	-	-	1.166	1.166

Extra 2

	$\mathcal{O}(p^3)$ $\not\Delta$	$\mathcal{O}(p^4)$ $\not\Delta$	$\mathcal{O}(p^3)$ Δ	$\mathcal{O}(p^4)$ Δ
$\mathring{g}_A$ (free)	1.1782 ± 0.0073		1.2041 ± 0.0074	1.274 ± 0.041
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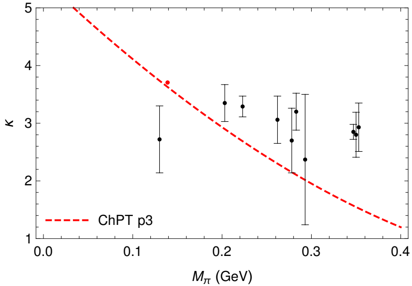
d_{22} (GeV^{-2}) (free)	1.275 ± 0.086		3.77 ± 1.96	0.29 ± 1.69 (free g_1)
h_A	-	-	1.35	1.35
g_1 (free)	-	-	-0.69 ± 0.69	0.66 ± 0.56
c_1 (GeV^{-1})	-	-0.89 ± 0.06	-	-1.15 ± 0.05
c_2 (GeV^{-1})	-	3.38 ± 0.15	-	1.57 ± 0.10
c_3 (GeV^{-1})	-	-4.59 ± 0.09	-	-2.54 ± 0.05
c_4 (GeV^{-1})	-	3.31 ± 0.13	-	2.61 ± 0.10
a_1 (GeV^{-1})	-	-	-	0.90
b_1 (GeV^{-2}) (free)	-	-	-	-0.27 ± 4.96
b_2 (GeV^{-2}) (free)	-	-	-	2.27 ± 2.28
$\tilde{b}_4$ (GeV^{-2}) (free)	-	-	-	-12.48 ± 1.28
x_1 (fm^{-2}) (free)	-8.4 ± 5.8	-	-5.6 ± 5.9	-0.25 ± 16.5 (consistent)
x_2 (fm^{-2}) (free)	-8.6 ± 2.6	-	-7.1 ± 2.6	-6.36 ± 4.20
x_3 (fm^{-1}) (free)	-0.25 ± 0.21	-	-0.08 ± 0.22	0.36 ± 0.47
y_1 (fm^{-2} GeV^{-2}) (free)	-100 ± 40	-	-76 ± 44	-64 ± 121
y_2 (fm^{-2} GeV^{-2}) (free)	-31 ± 21	-	-21 ± 22	-15 ± 46
y_3 (fm^{-1} GeV^{-2}) (free)	-0.63 ± 1.49	-	0.36 ± 1.63	2.54 ± 3.98
$\mathring{m}$ (GeV)	0.874	0.874	0.855	0.855
$\mathring{m}_\Delta$ (GeV)	-	-	1.166	1.166
χ^2/dof	$46.13/(127-9) = 0.391$		$39.17/(127-10) = 0.326$	$14.64/(127-13) = 0.129$ Δ trunc. overestim.
χ_0^2/dof	$857.31/(127-9) = 7.27$		$533.87/(127-10) = 4.45$	$196.58/(127-13) = 1.724$

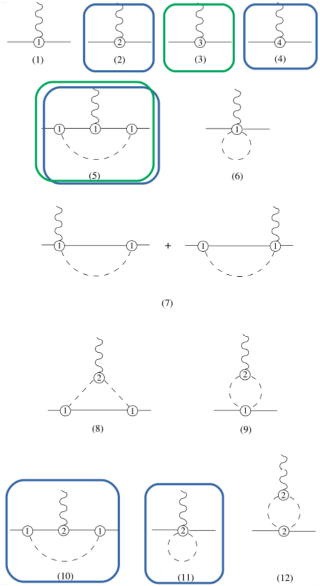
g_A Backup

	$\mathcal{O}(p^4)$ Δ
$\mathring{g}_A$ (free par.)	1.240 ± 0.046
d_{16} (free par.)	-0.88 ± 0.88
h_A	$h_A^{N_c} = 1.35$
g_1	$-\lvert g_1^{N_c}\rvert = -2.29$
c_1	-1.15 ± 0.05
c_2	1.57 ± 0.10
c_3	-2.54 ± 0.05
c_4	2.61 ± 0.10
a_1	0.90
$\tilde{b}_4$ (free par.)	-12.3 ± 1.0
$\mathring{m}$	0.855
$\mathring{m}_\Delta$	1.166
χ^2/dof	$11.14/(43-7) = 0.31$

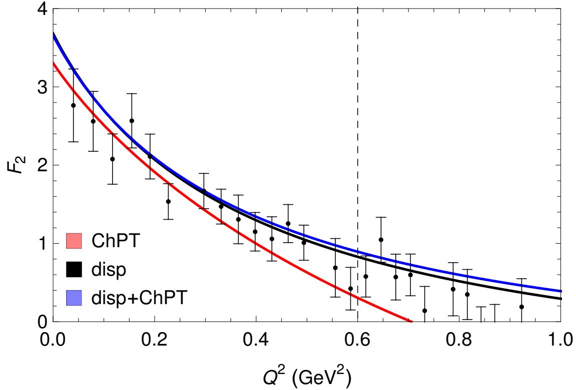
- **Pauli f.f., F_2**
  - $\langle N | V^\mu | N\rangle = \bar{u}' \left[\gamma^\mu F_1 + \frac{i\sigma^{\mu\nu}q_\nu}{2m_N} F_2\right] u,$
    $F_2 = \frac{F_M - F_E}{(1+Q^2/(4m^2))}$
  - $F_2^{(v)} = \kappa^{(v)} \left[1 + \frac{1}{6}\langle r_2^2\rangle^{(v)} q^2 + \mathcal{O}(q^4)\right]$
  - $\mathcal{O}(p^3)$ χPT is not enough $F_2(0)^{o(3)} = c_6 - \left(\frac{\pi m_N g_A^2}{4\pi^2 F^2}\right) M_\pi$

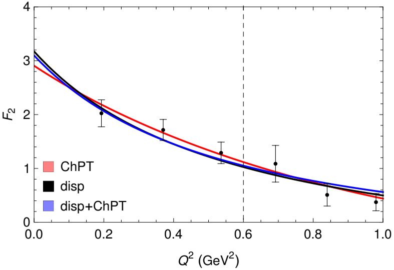


  - $\Longrightarrow$ include Δ $\mathcal{O}(p^4)$
  - disp+χPT $\mathcal{O}(p^4)$: $F_2 = F_2^{\rm disp} + F_2^{\rm tree} + F_2^{\chi \rm PTloop}$,
    $F_2^{\rm tree} = c_6 - 16 e_{106} m_N M_\pi^2 + 2q^2(d_6 + 2e_{74} m_N)$

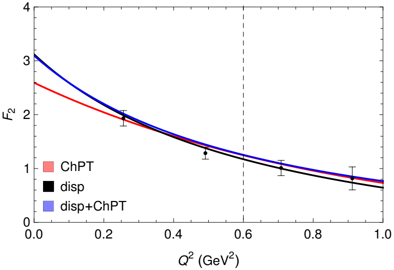


- F_2 **fit to LQCD**
  - In F_2^{disp} free c_6
  - In $F_2^{\chi\text{PT}}$ and $F_2^{disp+\chi\text{PT}}$, free c_6, e_{106}, e_{74}
  - χPT $\mathcal{O}(p^4)$ and disp separately are good enough to describe the data



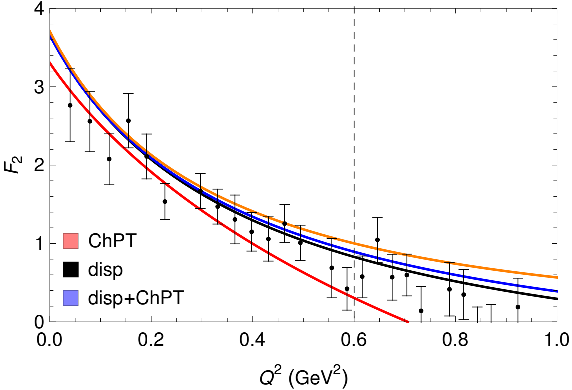


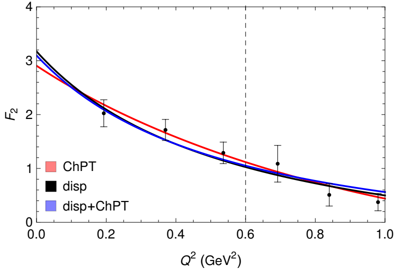
(a) H105 $M_\pi = 0.278$ GeV



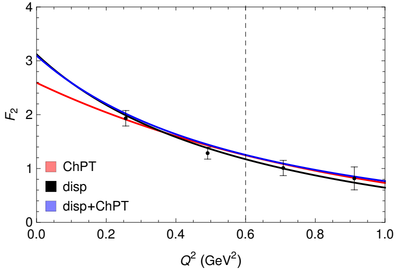
(b) N302 $m_\pi = 0.353$ GeV

- F_2 **fit to LQCD**
  - In $F_2^{\rm disp}$ free c_6
  - In $F_2^{\chi\rm PT}$ and $F_2^{disp+\chi\rm PT}$, free c_6, e_{106}, e_{74}
  - χPT $\mathcal{O}(p^4)$ and disp separately are good enough to describe the data

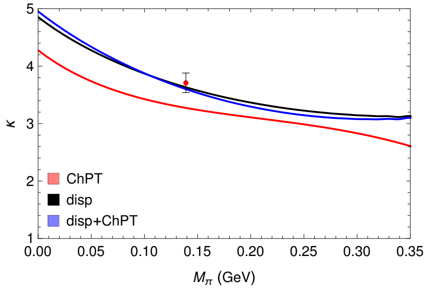


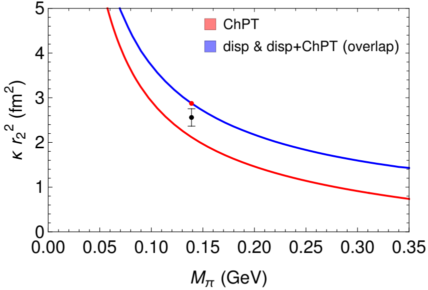


(a) H105 $M_\pi = 0.278$ GeV



(b) N302 $m_\pi = 0.353$ GeV





- $F_2^{(v)} = \kappa^{(v)} \left[1 + \frac{1}{6}\langle r_2^2\rangle^{(v)} q^2 + \mathcal{O}(q^4)\right]$

	disp+c_6	ChPT	disp+ChPT	HB	exp. (PDG)
χ^2/dof	$\frac{49.95}{47-2} = 1.110$	$\frac{44.18}{47-4} = 1.027$	$\frac{56.08}{47-4} = 1.304$		
χ_0^2/dof	1.09	1.027	1.283		
κ_{phys}	3.632 ± 0.037	3.423 ± 0.059	$\mathbf{3.605 \pm 0.067}$	3.71 ± 0.17	3.706
$\langle r_2^2\rangle_{\text{phys}}$ (fm^2)	0.792 ± 0.011	0.61885 ± 0.0069	$\mathbf{0.788 \pm 0.015}$	0.690 ± 0.042	0.7754 ± 0.0080

- In general **good $F_{1,2}$ description at different M_π values**
- **Extracted $\langle r_{1,2}^2\rangle$ and κ**