

Proton Generalized Parton Distributions

From the Lattice: Tensor Case

Joshua Miller

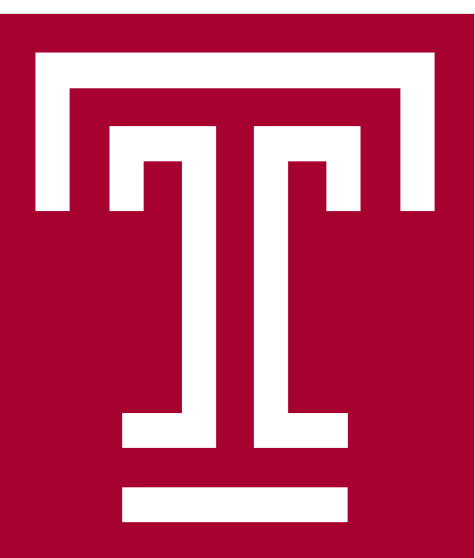
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Collaborators

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EINN2025

EINN 2025
Paphos, Cyprus
10/29/2025



Outline

❖ Work extends the approach for the unpolarized and axial cases [\[Bhattacharya, S. et al., Phys. Rev. D 106, 11, 114512 \(2022\)\]](#)

❖ Theoretical Formulation

❖ Lattice Formulation (focus of this talk)

❖ Background

❖ Lattice Methodology

❖ Results: Twist-2

- Matrix Elements
- Lorentz invariant amplitudes
- Quasi-GPDs
- Light-Cone GPDs

❖ Results: Twist-3

PHYSICAL REVIEW D **106**, 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya^{1,*} Krzysztof Cichy,² Martha Constantinou^{3,†} Jack Dodson,³ Xiang Gao,⁴ Andreas Metz,³ Swagato Mukherjee¹ Aurora Scapellato,³ Fernanda Steffens,⁵ and Yong Zhao⁴

[\[Bhattacharya, S. et al., Phys. Rev. D 109, 3, 034508 \(2024\)\]](#)

PHYSICAL REVIEW D **109**, 034508 (2024)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Axial-vector case

Shohini Bhattacharya^{1,*} Krzysztof Cichy,² Martha Constantinou^{3,†} Jack Dodson,² Xiang Gao,³ Andreas Metz^{1,2} Joshua Miller,^{2,‡} Swagato Mukherjee^{1,4} Peter Petreczky^{1,4} Fernanda Steffens,⁵ and Yong Zhao³

[\[Bhattacharya, S. et al., arXiv:2505.11288 \(2025\)\]](#)

Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Tensor case

Shohini Bhattacharya,^{1,*} Krzysztof Cichy,² Martha Constantinou,^{3,†} Andreas Metz,³ Joshua Miller,^{3,‡} Peter Petreczky,⁴ and Fernanda Steffens⁵

Generalized Parton Distributions

❖ GPDs are rich in information:

- Reflect spatial distribution of partons in transverse plane
- Hadron mechanical properties are stored in GPDs
- Information on spin

❖ ... but not well studied:

- extracted from off-forward kinematic (unlike PDFs)
- Multi-variable quantities; dependence upon x , t and ξ (unlike PDFs)
- Inferred from Compton form factors from experimental data (e.g., DVCS)
- Other processes proposed (SDHEP [J. Qiu et al, arXiv:2205.07846]) still require theoretical developments

❖ Transversity proton GPDs:

- Four GPDs: H_T , E_T , $\widetilde{H}_T$, $\widetilde{E}_T$

$$F_{\lambda,\lambda'}^{[i\sigma^{j+}\gamma_5]}(z, \Delta, P) = -i\epsilon^{-+ij}\bar{u}(p', \lambda') \left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta_\perp^i - \Delta^+\gamma_\perp^i}{2M}E_T + \frac{P^+\Delta_\perp^i - P_\perp^i\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P_\perp^i - P^+\gamma_\perp^i}{M}\widetilde{E}_T \right] u(p, \lambda)$$

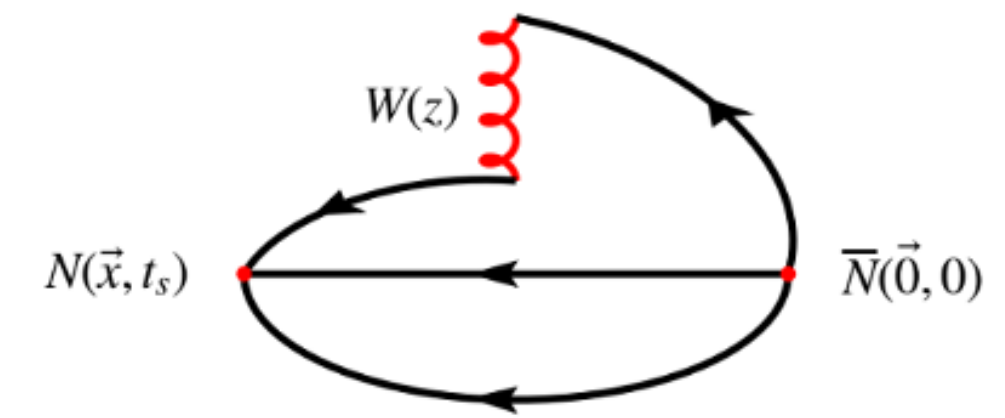
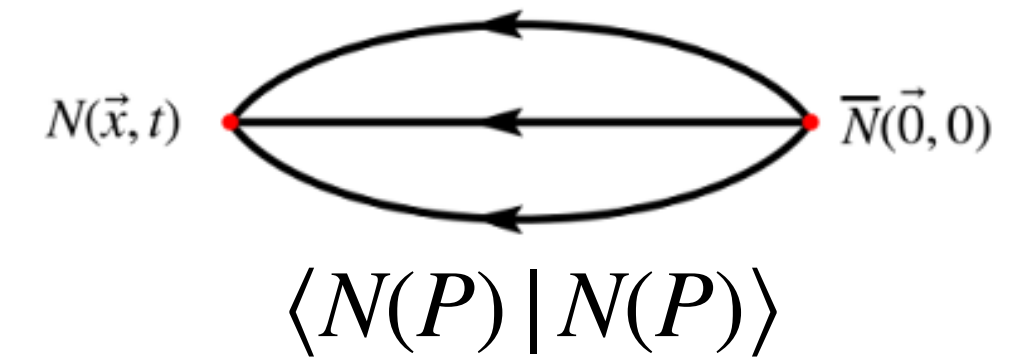
Methodology on the Lattice

- ❖ Choice of frame:
- **Symmetric:** $\vec{p}_i = P_3 \hat{z} - \vec{\Delta}/2, \quad \vec{p}_f = P_3 \hat{z} + \vec{\Delta}/2$
 - **Asymmetric:** $\vec{p}_i = P_3 \hat{z} - \vec{\Delta}, \quad \vec{p}_f = P_3 \hat{z}$

Methodology on the Lattice

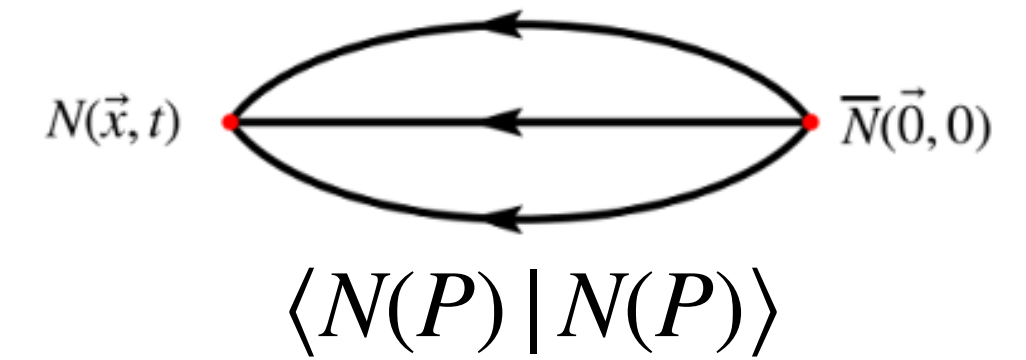
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- ❖ Calculate the 2-, 3-point correlation functions in the frame chosen

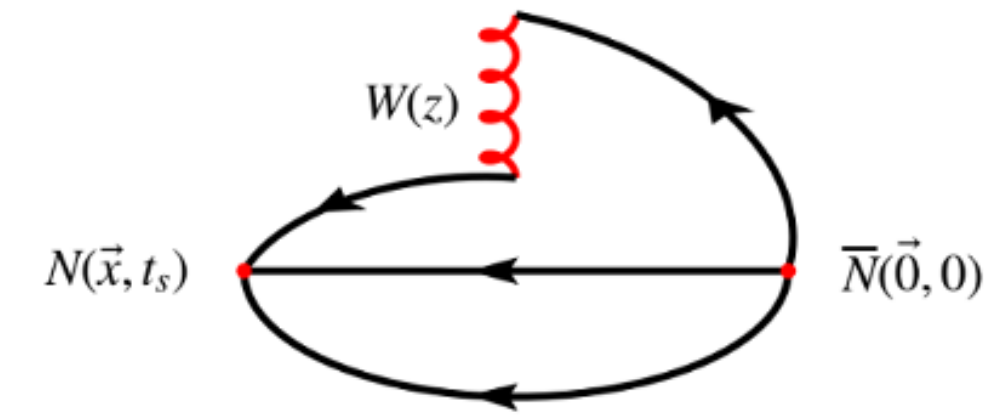


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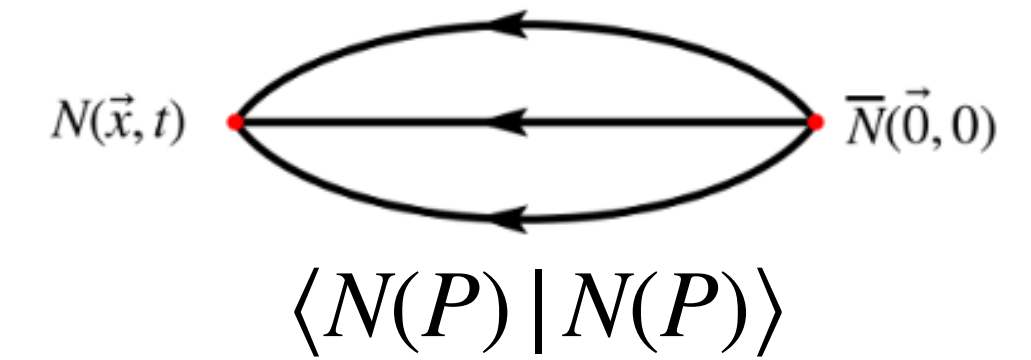


- ❖ Take an appropriate ratio of the 2- and 3-point correlation functions and isolate the ground state (plateau fit)

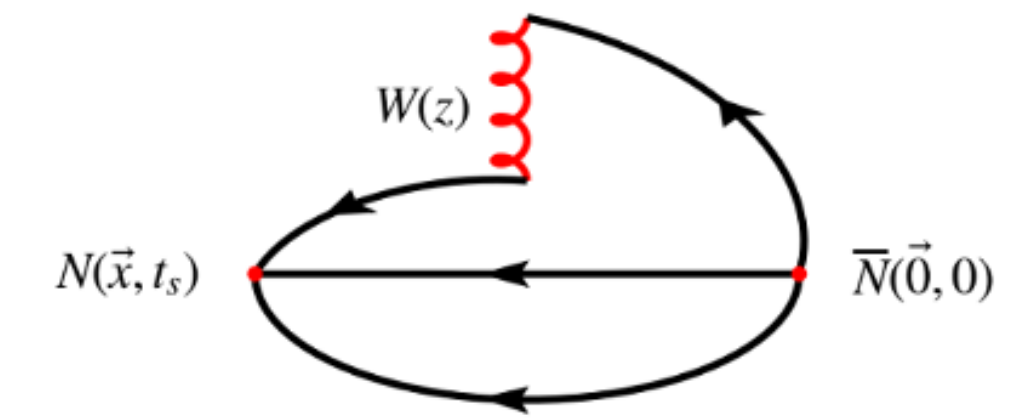
$$R_\mu(\Gamma_\kappa, z, p_f, p_i; t_s, \tau) = \frac{C_\mu^{3\text{pt}}(\Gamma_\kappa, z, p_f, p_i; t_s, \tau)}{C^{2\text{pt}}(\Gamma_0, p_f; t_s)} \sqrt{\frac{C^{2\text{pt}}(\Gamma_0, p_i, t_s - \tau) C^{2\text{pt}}(\Gamma_0, p_f, \tau) C^{2\text{pt}}(\Gamma_0, p_f, t_s)}{C^{2\text{pt}}(\Gamma_0, p_f, t_s - \tau) C^{2\text{pt}}(\Gamma_0, p_i, \tau) C^{2\text{pt}}(\Gamma_0, p_i, t_s)}} \xrightarrow[\tau \gg a]{t_s - \tau \gg a} \Pi_\mu(\Gamma_\kappa, z, p_f, p_i; t_s)$$

Methodology on the Lattice

- ❖ Choice of frame:
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- ❖ Take an appropriate ratio of the 2- and 3-point correlation functions and isolate the ground state (plateau fit)

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- ❖ Parameterize the matrix elements in terms of (Lorentz invariant) amplitudes

$$F_{\lambda, \lambda'}^{[i\sigma^{\mu\nu}\gamma_5]} = P^{[\mu} z^{\nu]} A_1 + \frac{P^{[\mu} \Delta^{\nu]}}{M^2} \gamma_5 A_2 + z^{[\mu} \Delta^{\nu]} \gamma_5 A_3 + \gamma^{[\mu} \left(\frac{P^{\nu]} }{M} A_4 + M z^{\nu]} A_5 + \frac{\Delta^{\nu]} }{M} A_6 \right) \gamma_5 + M \gamma_\alpha z^\alpha \left(P^{[\mu} z^{\nu]} A_7 + \frac{P^{[\mu} \Delta^{\nu]}}{M^2} A_8 + z^{[\mu} \Delta^{\nu]} A_9 \right) + i\sigma^{\mu\nu} \gamma_5 A_{10} + i\epsilon^{\mu\nu P z} A_{11} + i\epsilon^{\mu\nu z \Delta} A_{12}$$

12 linearly independent Lorentz invariant amplitudes

Methodology on the Lattice

SF: Symmetric Frame

AF: Asymmetric Frame

- ❖ Equate and relate the amplitude and quasi-GPD decomposition

$$F_{\lambda,\lambda'}^{[i\sigma^{j+}\gamma_5]}(z, \Delta, P) = -i\epsilon^{-+ij}\bar{u}(p', \lambda') \left[i\sigma^+ H_T + \frac{\gamma^+ \Delta_\perp^i - \Delta^+ \gamma_\perp^i}{2M} E_T + \frac{P^+ \Delta_\perp^i - P_\perp^i \Delta^+}{M^2} \widetilde{H}_T + \frac{\gamma^+ P_\perp^i - P^+ \gamma_\perp^i}{M} \widetilde{E}_T \right] u(p, \lambda)$$

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GPDs defined in SF with A_i in SF

$$\longrightarrow H_T = -2A_2 \left(1 - \frac{E^2 - P_3^2}{m^2} \right) + A_4 + A_{10}$$

$$\longrightarrow E_T = 2A_2 - A_4$$

$$\longrightarrow \widetilde{H}_T = -A_2 - \frac{m^2}{P_3}zA_{12}$$

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GPDs defined in SF with A_i in AF

$$\longrightarrow H_T = -2A_2 \left(1 - \frac{(E_i + E_f)^2 - \vec{\Delta}^2 - 4P_3^2}{4m^2} \right) + A_4 - \frac{(E_f + E_i)(E_f - E_i)}{2P_3}zA_8 + A_{10}$$

$$\longrightarrow E_T = 2A_2 - A_4 + \frac{(E_f + E_i)(E_f - E_i)}{2P_3}zA_8$$

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❖ Renormalization functions: RI-MOM

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- ❖ Renormalization functions: RI-MOM

- ❖ Transform from position to momentum space (Backus-Gilbert)

[Backus & Gilbert, Geophysical Journal International 16, 169 (1968)]

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- ❖ Renormalization functions: RI-MOM

- ❖ Transform from position to momentum space (Backus-Gilbert)

[Backus & Gilbert, Geophysical Journal International 16, 169 (1968)]

- ❖ Extract light cone-GPDs using one-loop matching formalism

[Liu, et al., Phys. Rev. D 100, 034006 (2019)]

Decomposition

Symmetric frame ($\xi = 0$)

$$\Pi_{01}^s(\Gamma_0) = iK \left(-\frac{\Delta_1 P_3^2}{4m^3} A_{T4} + \frac{\Delta_1 P_3}{4m} z A_{T5} + \frac{(E+m)\Delta_1}{4m^2} A_{T10} \right)$$

$$\Pi_{03}^s(\Gamma_0) = iK \left(\frac{P_3(\Delta_1^2 + \Delta_2^2)}{4m^3} A_{T6} - \frac{E(E(E+m) - P_3^2)}{2m^2} z A_{T11} \right)$$

Asymmetric frame ($\xi = 0$)

$$\Pi_{01}^a(\Gamma_0) = iK \left(\frac{(m - E_f)(m + E_f)\Delta_1}{4m^3} A_{T4} + \frac{P_3 \Delta_1}{4m} z A_{T5} + \frac{(E_f + m)\Delta_1}{4m^2} A_{T10} \right)$$

$$\Pi_{03}^a(\Gamma_0) = iK \left(\frac{(E_f - E_i)(E_f + E_i)P_3}{8m^3} A_{T4} + \frac{(E_i^2 - E_f^2)P_3}{4m^3} A_{T6} + \frac{(E_i - E_f)P_3}{4m^2} A_{T10} + \frac{(E_f + E_i)(E_f - E_i - 2m)(E_f + m)}{8m^2} z A_{T11} - \frac{(E_f - E_i)(E_f - E_i - 2m)(E_f + m)}{4m^2} z A_{T12} \right)$$

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Asymmetric frame ($\xi \neq 0$)

$$\Pi_{01}^a(\Gamma_0) = iK \left(\frac{(m - E_f)(m + E_f)\Delta_1}{4m^3} A_{T4} + \frac{P_3 \Delta_1}{4m} z A_{T5} + \frac{(E_f + m)\Delta_1}{4m^2} A_{T10} \right)$$

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$$\Pi_i^s \neq \Pi_i^a$$

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Coefficients become more complicated

Amplitudes frame independent

Asymmetric frame ($\xi \neq 0$)

$$\Pi_{01}^a(\Gamma_0) = iK \left(\frac{(m - E_f)(m + E_f)\Delta_1}{4m^3} A_{T4} + \frac{P_3 \Delta_1}{4m} z A_{T5} + \frac{(E_f + m)\Delta_1}{4m^2} A_{T10} \right)$$

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$$\Pi_i^s \neq \Pi_i^a \quad A_{Ti}^s = A_{Ti}^a$$

Decomposition

Symmetric frame ($\xi = 0$)

$$\Pi_{01}^s(\Gamma_0) = iK \left(-\frac{\Delta_1 P_3^2}{4m^3} A_{T4} + \frac{\Delta_1 P_3}{4m} z A_{T5} + \frac{(E+m)\Delta_1}{4m^2} A_{T10} \right)$$

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Coefficients become more complicated

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$$\Pi_i^s \neq \Pi_i^a \quad A_{Ti}^s = A_{Ti}^a$$



Frame dependence of matrix elements due to kinematic coefficients of A_{Ti}



Lattice Setup



❖ $N_f = 2 + 1 + 1$ Twisted mass fermions with a clover term

Parameters							
Ensemble	β	a [fm]	volume $L^3 \times T$	N_f	m_π [MeV]	Lm_π	L [fm]
cA211.32	1.726	0.093	$32^3 \times 64$	u, d, s, c	260	4	3.0

Lattice Setup



❖ $N_f = 2 + 1 + 1$ Twisted mass fermions with a clover term

Parameters							
Ensemble	β	a [fm]	volume $L^3 \times T$	N_f	m_π [MeV]	Lm_π	L [fm]
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❖ Calculation of symmetric and asymmetric frame

- Symmetric frame:
Each $\vec{\Delta}$ requires new calculation
- Asymmetric frame:
Several $\vec{\Delta}$ values grouped in the same production run
(e.g. $\{\vec{\Delta} = (100), (200), (300), \dots\}$)

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N/A	± 1.25	(0,0,0)	0	0	2	329	16	10528
symm	± 0.83	($\pm 2, 0, 0$), ($0, \pm 2, 0$)	0.69	0	8	67	8	4288
symm	± 1.25	($\pm 2, 0, 0$), ($0, \pm 2, 0$)	0.69	0	8	249	8	15936
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Lattice Setup



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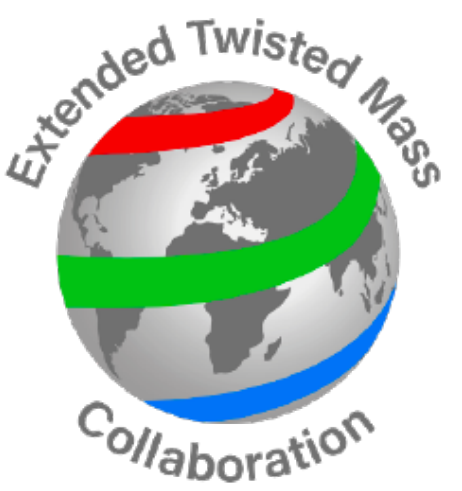


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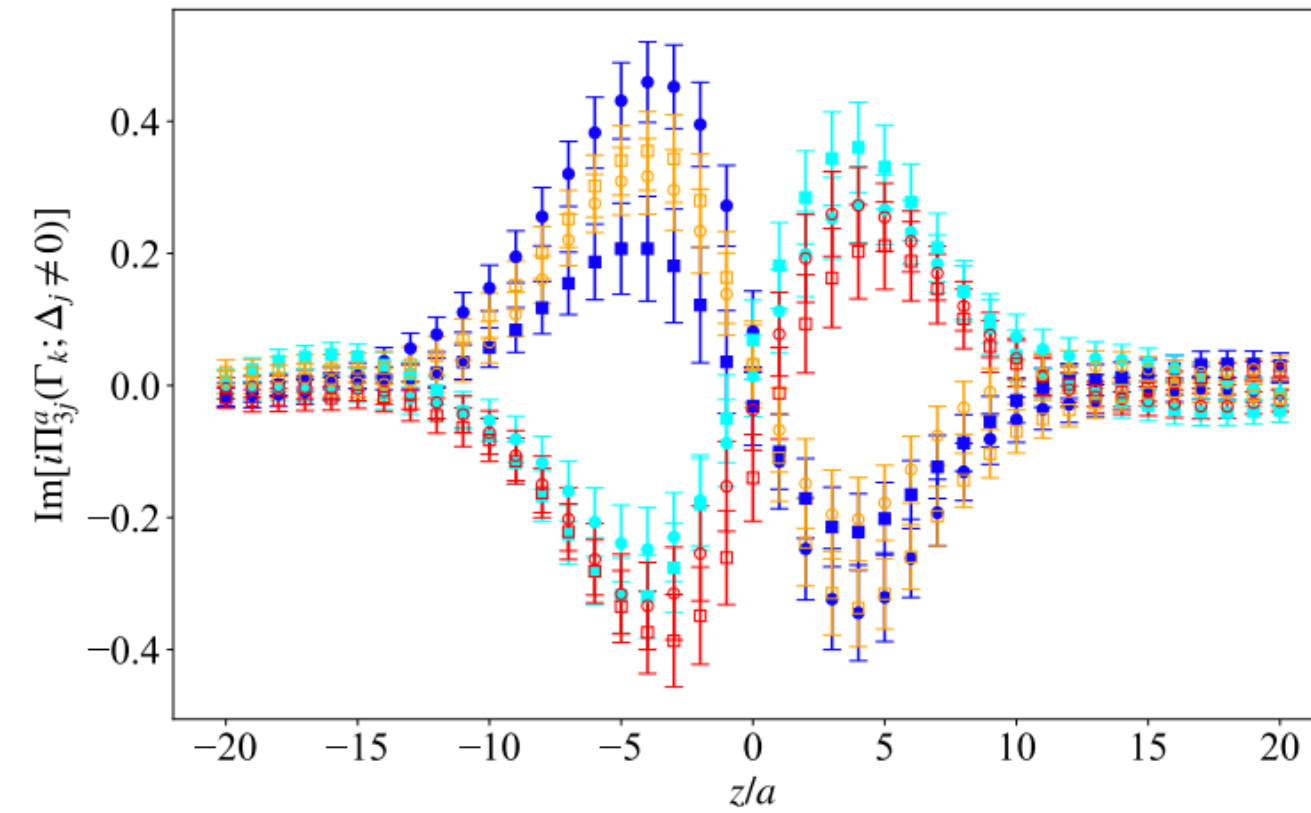
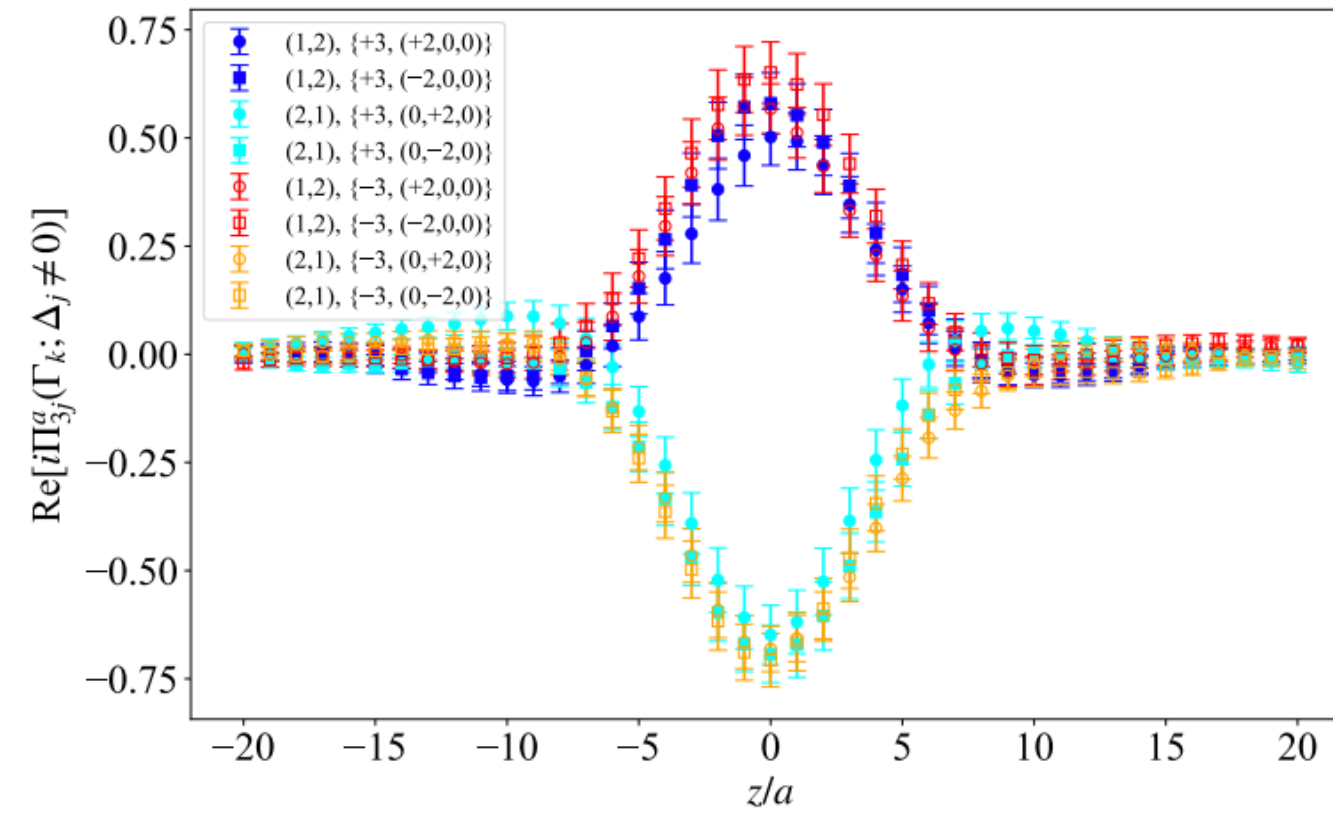
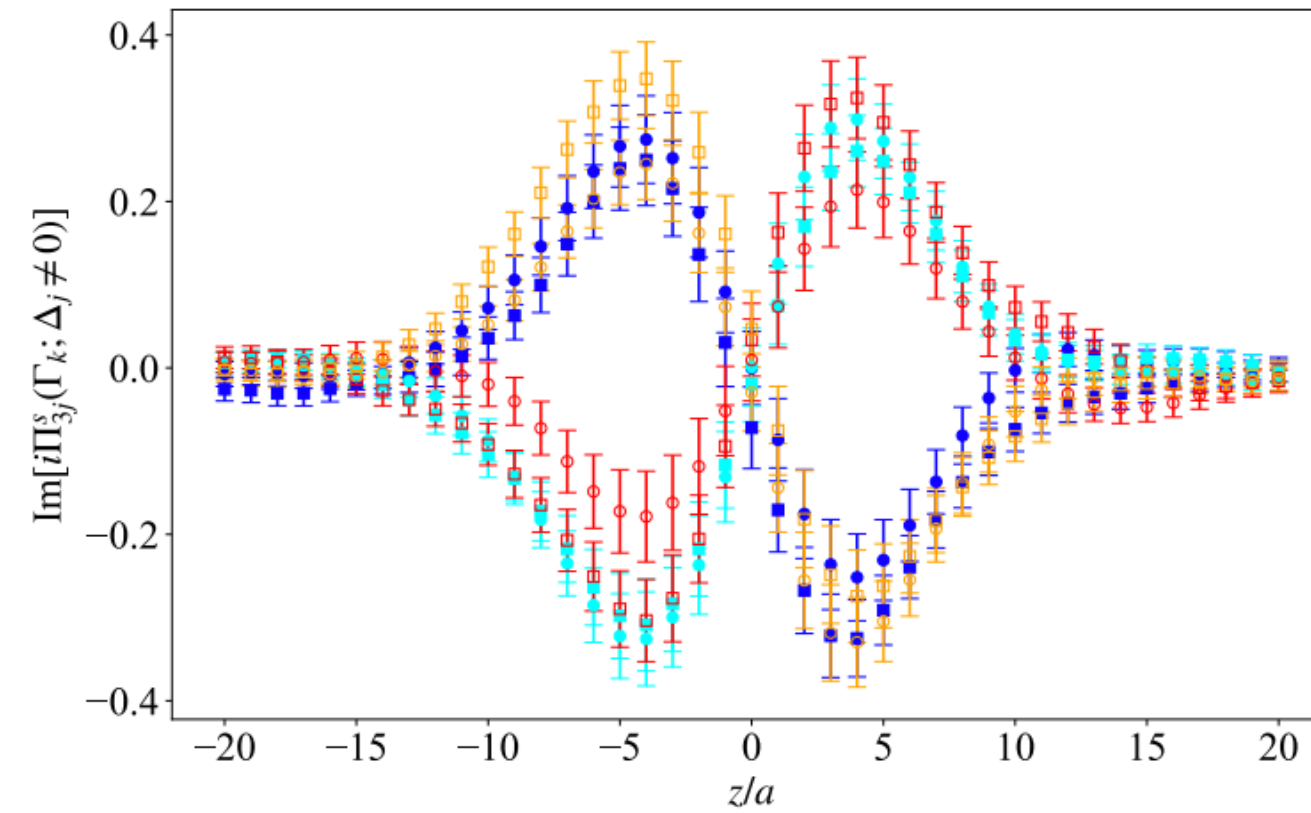
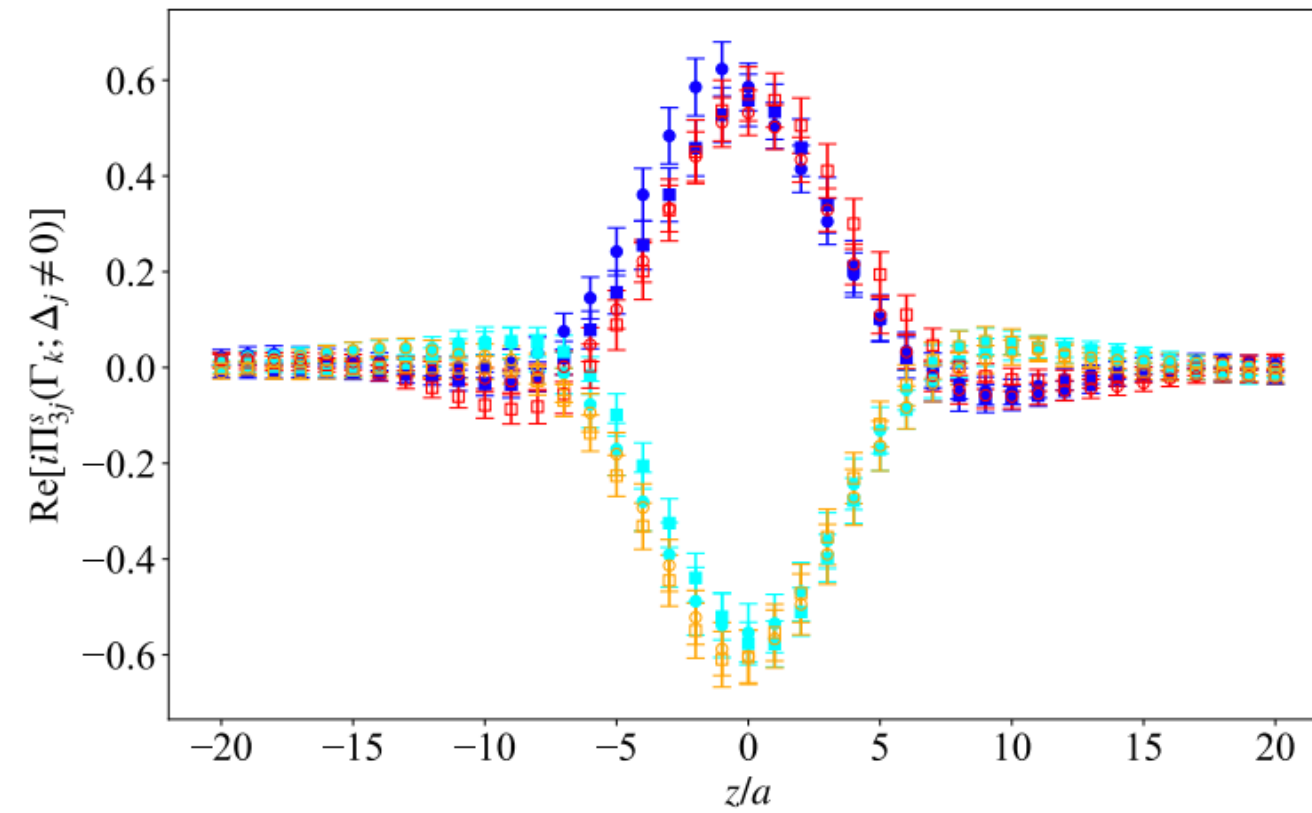
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❖ Exploitation of $\tilde{A}_i$ symmetry properties with respect to $(\pm P_3, \pm \vec{\Delta}, \pm z)$

Matrix Elements: $\Pi_{3j}^{s/a}(\Gamma_k, \Delta_j \neq 0)$

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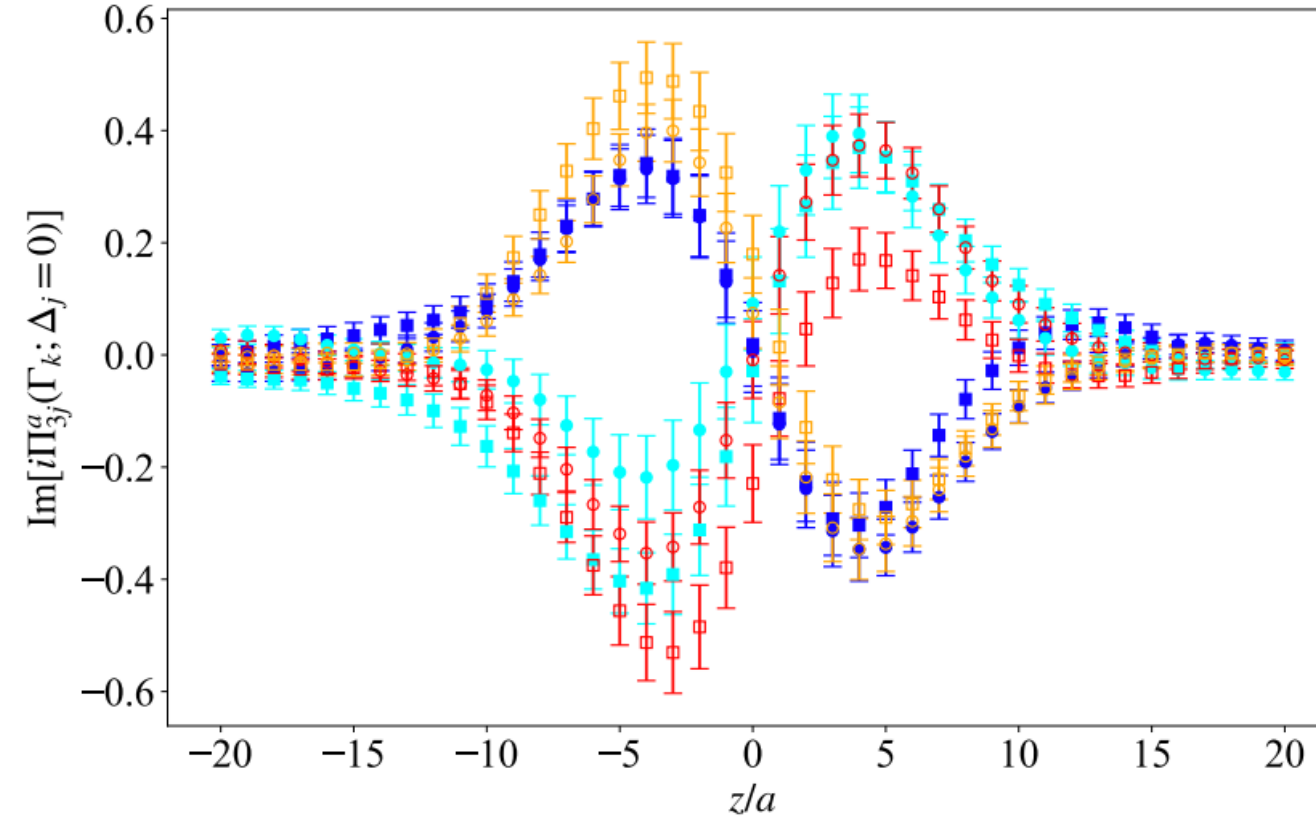
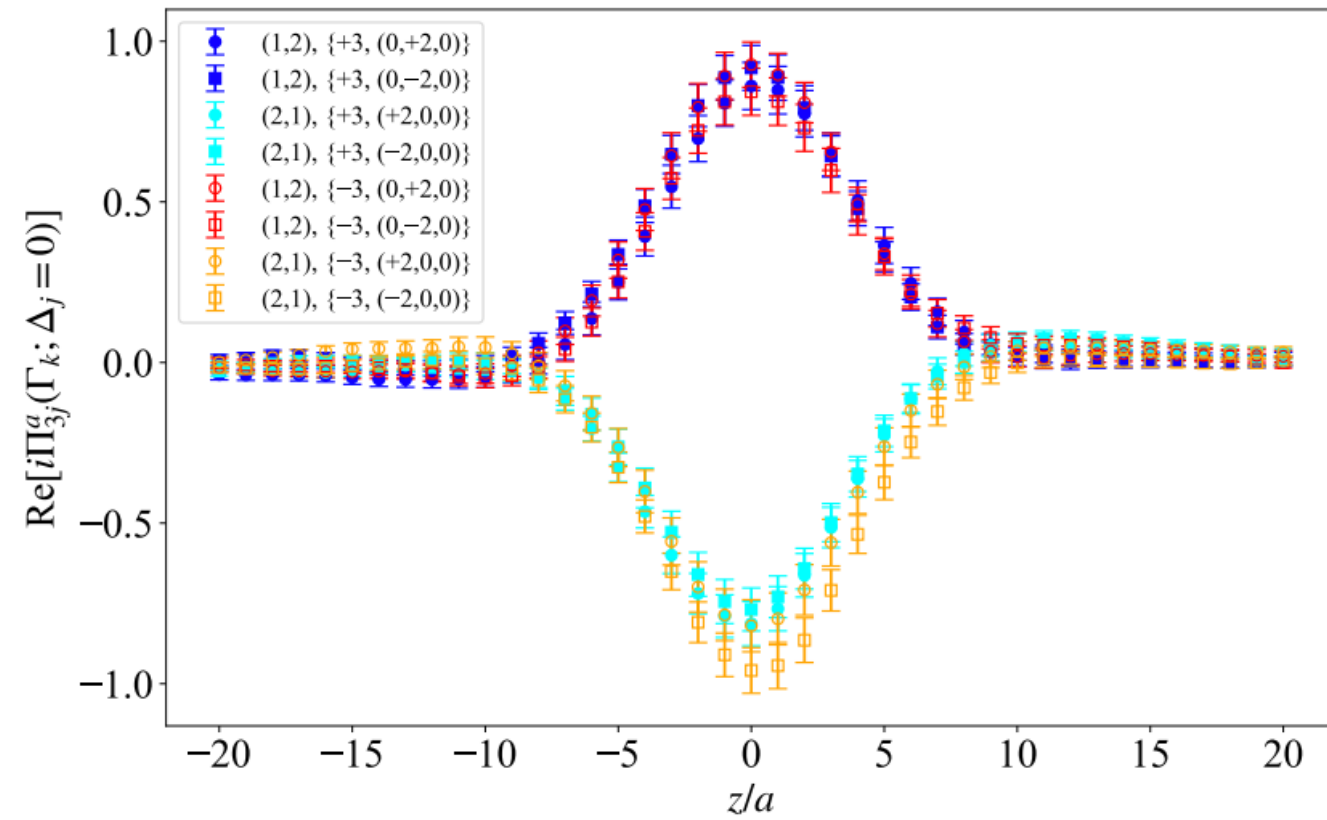
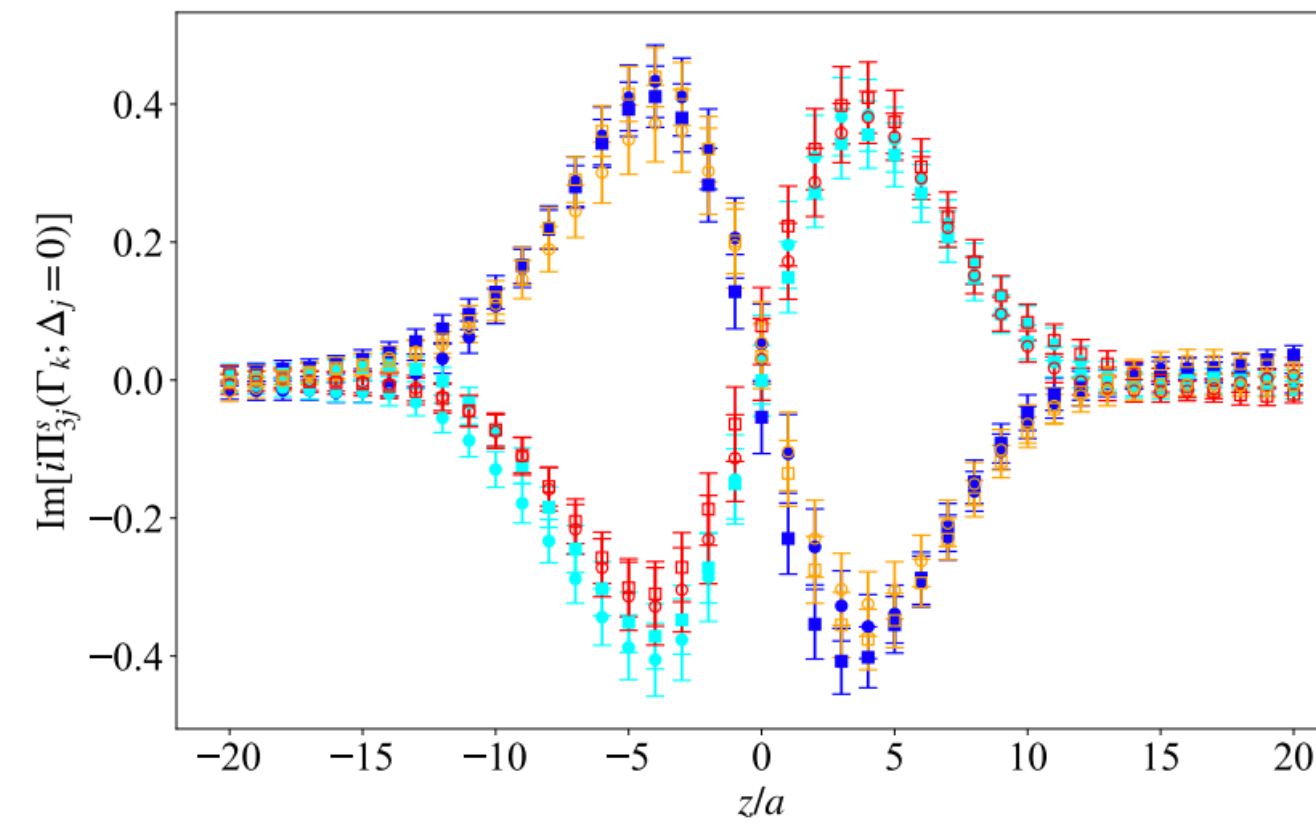
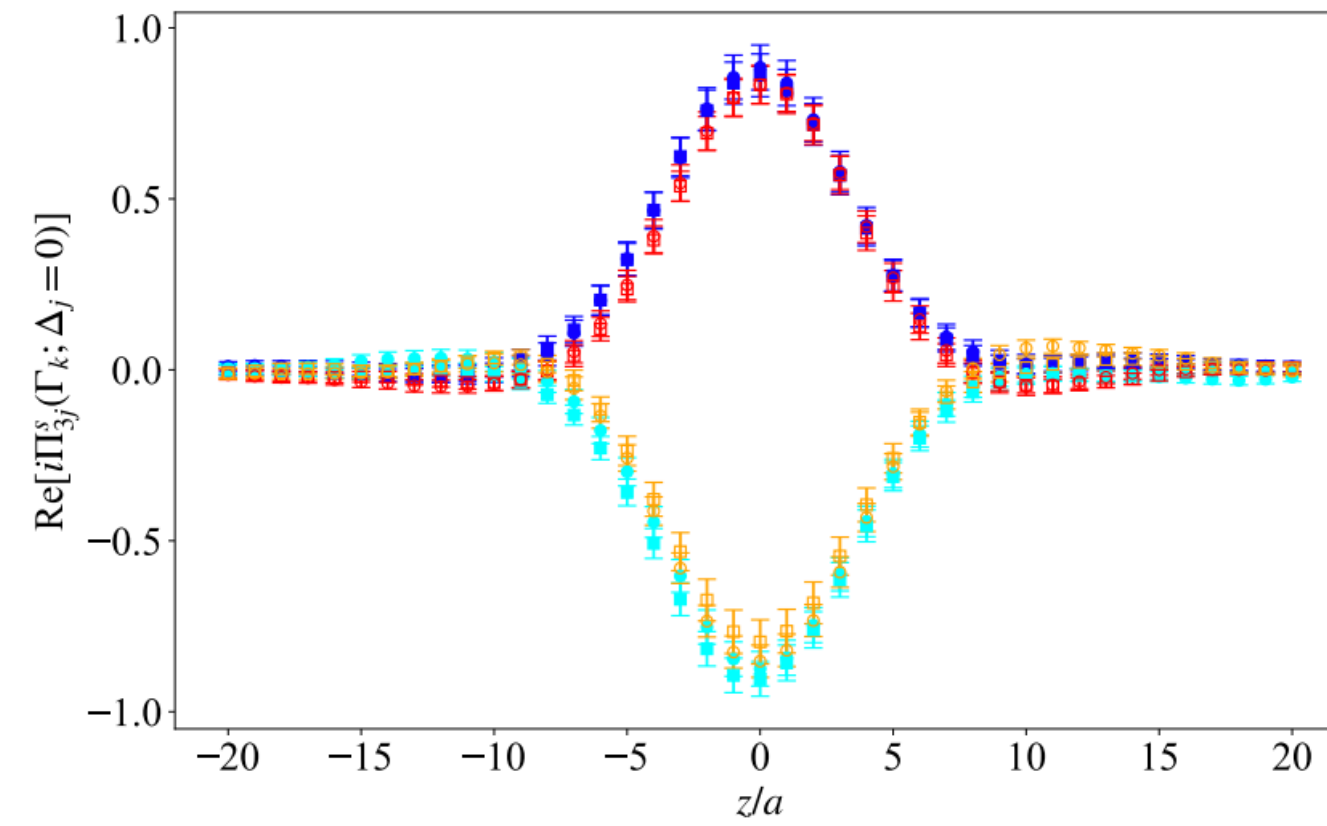
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Example: $\vec{\Delta} = (\Delta, 0, 0)$

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$$zA_{T1} \propto \Pi_{12}^s(\Gamma_1)$$

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$$zA_{T1} \propto \Pi_{02}^a(\Gamma_1), \Pi_{02}^a(\Gamma_3), \Pi_{12}^a(\Gamma_1), \Pi_{12}^a(\Gamma_3)$$

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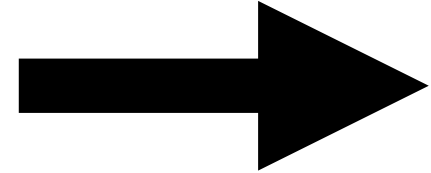
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- ❖ Disentangling amplitudes gets more difficult in (any) non-symmetric frame compared to the symmetric one
- ❖ Amplitudes are frame independent by construction
- ❖ Coefficients are frame dependent

Transversity Amplitudes

Ratio

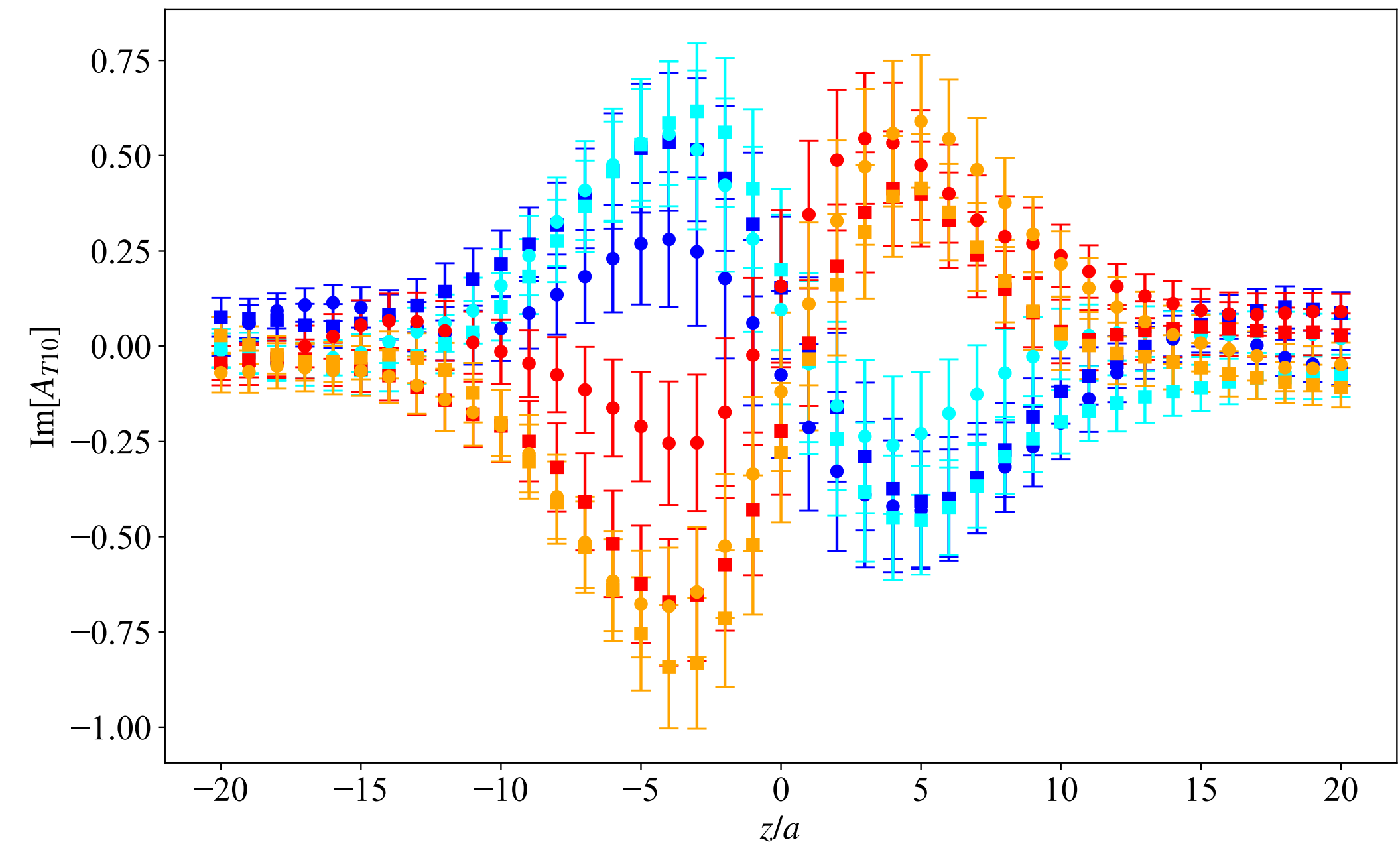
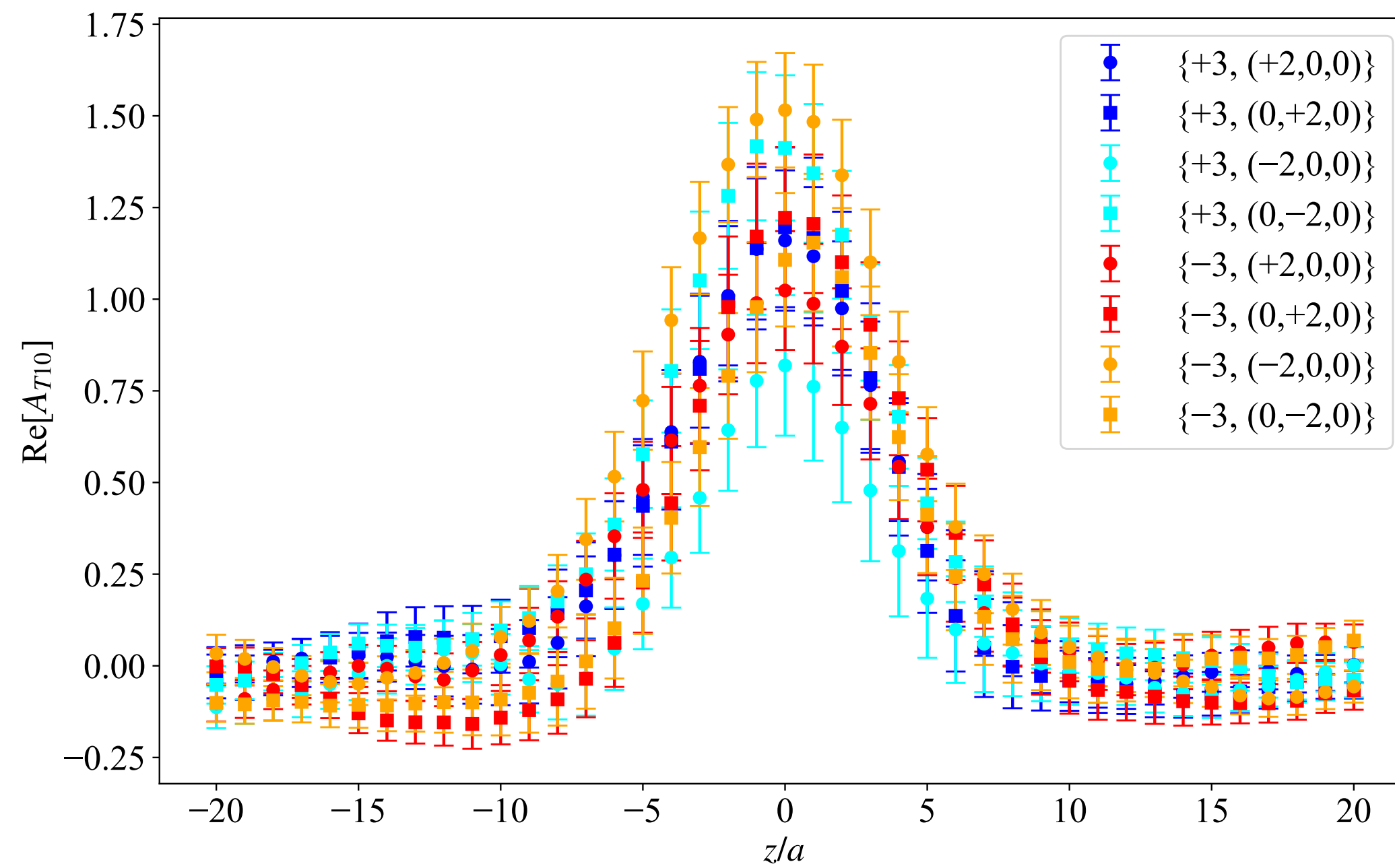
Transversity Amplitudes

Ratio  *Plateau*

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Ratio $\longrightarrow$ *Plateau* $\longrightarrow$ *Amplitudes*

Example: A_{T10}

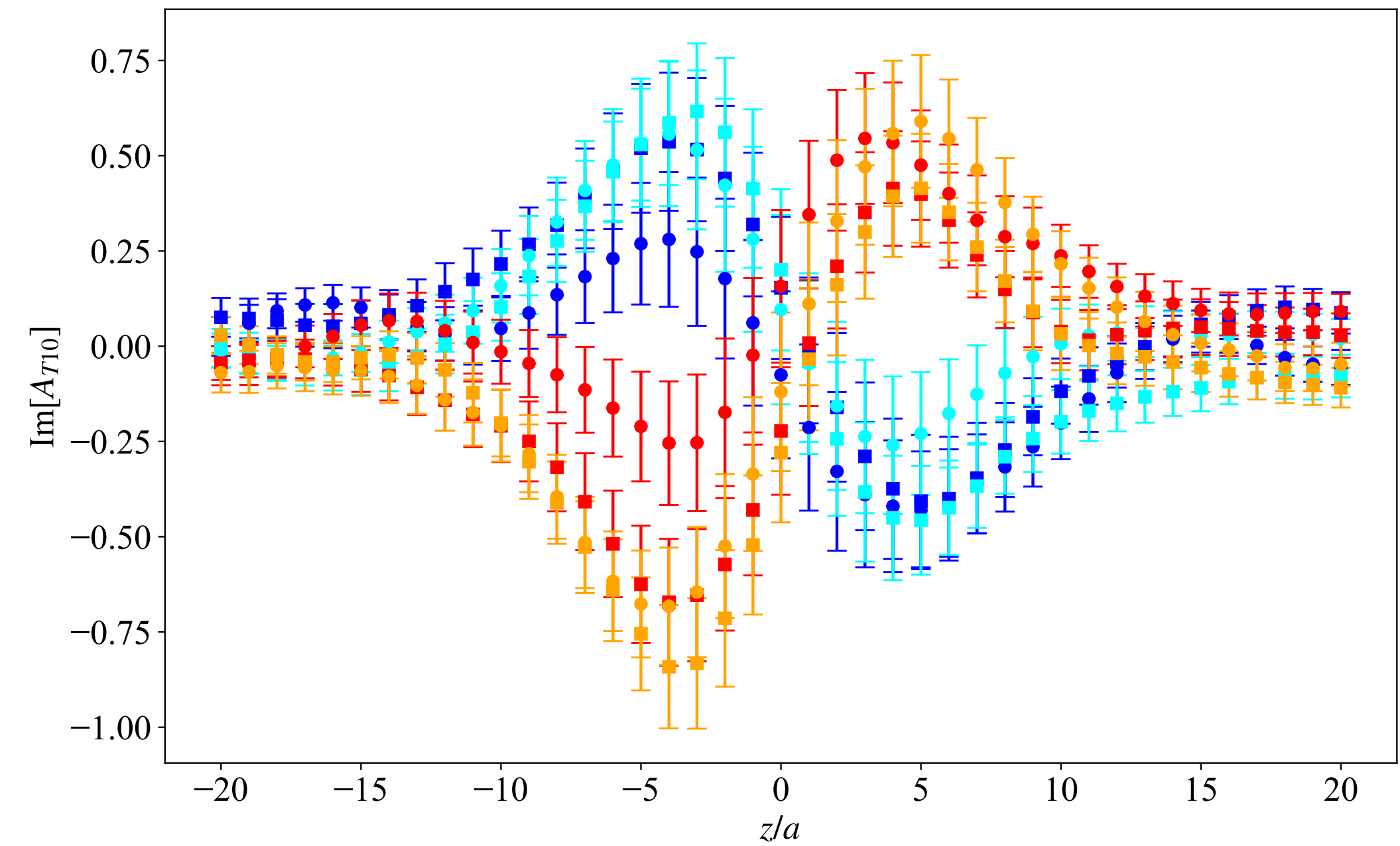
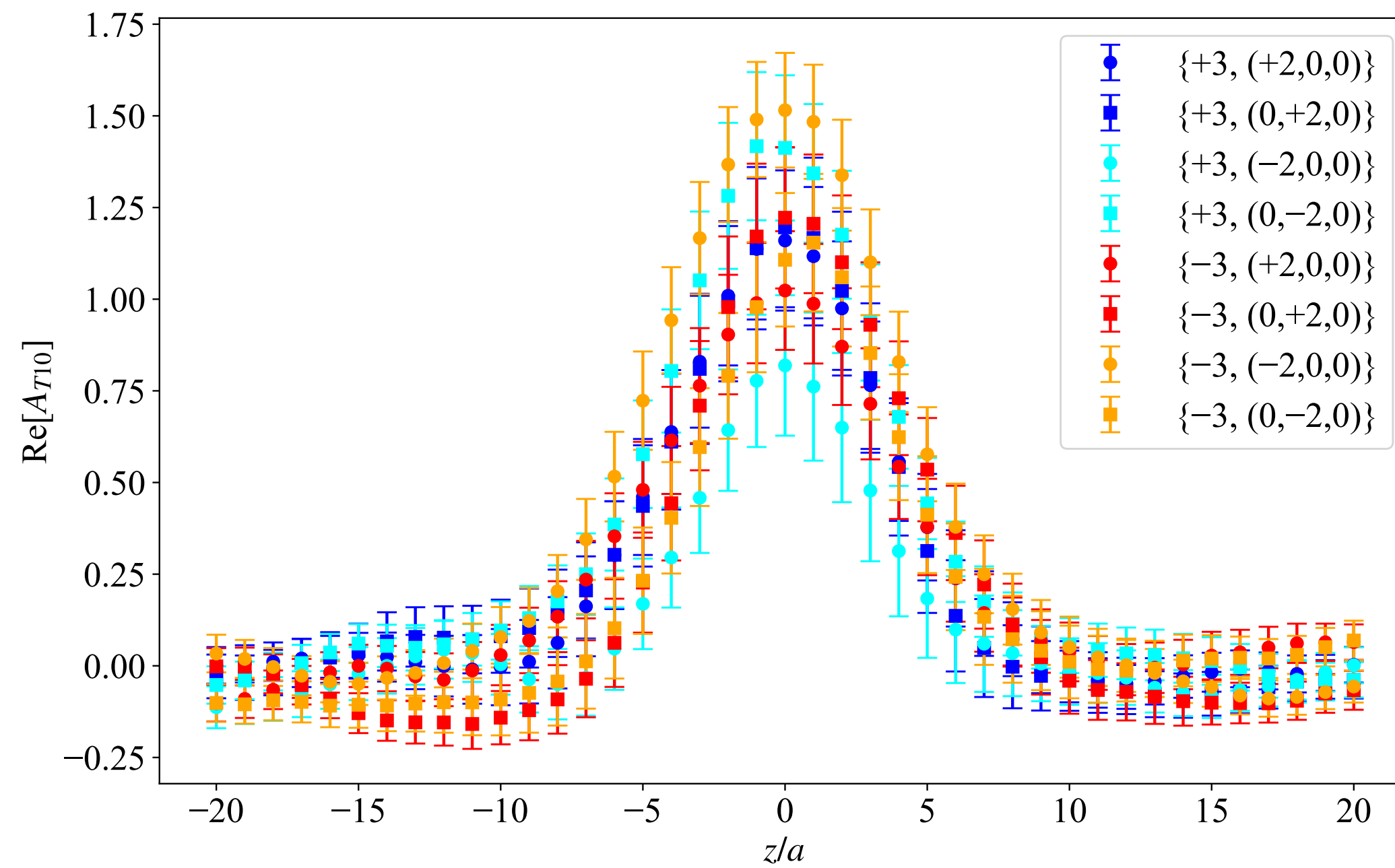


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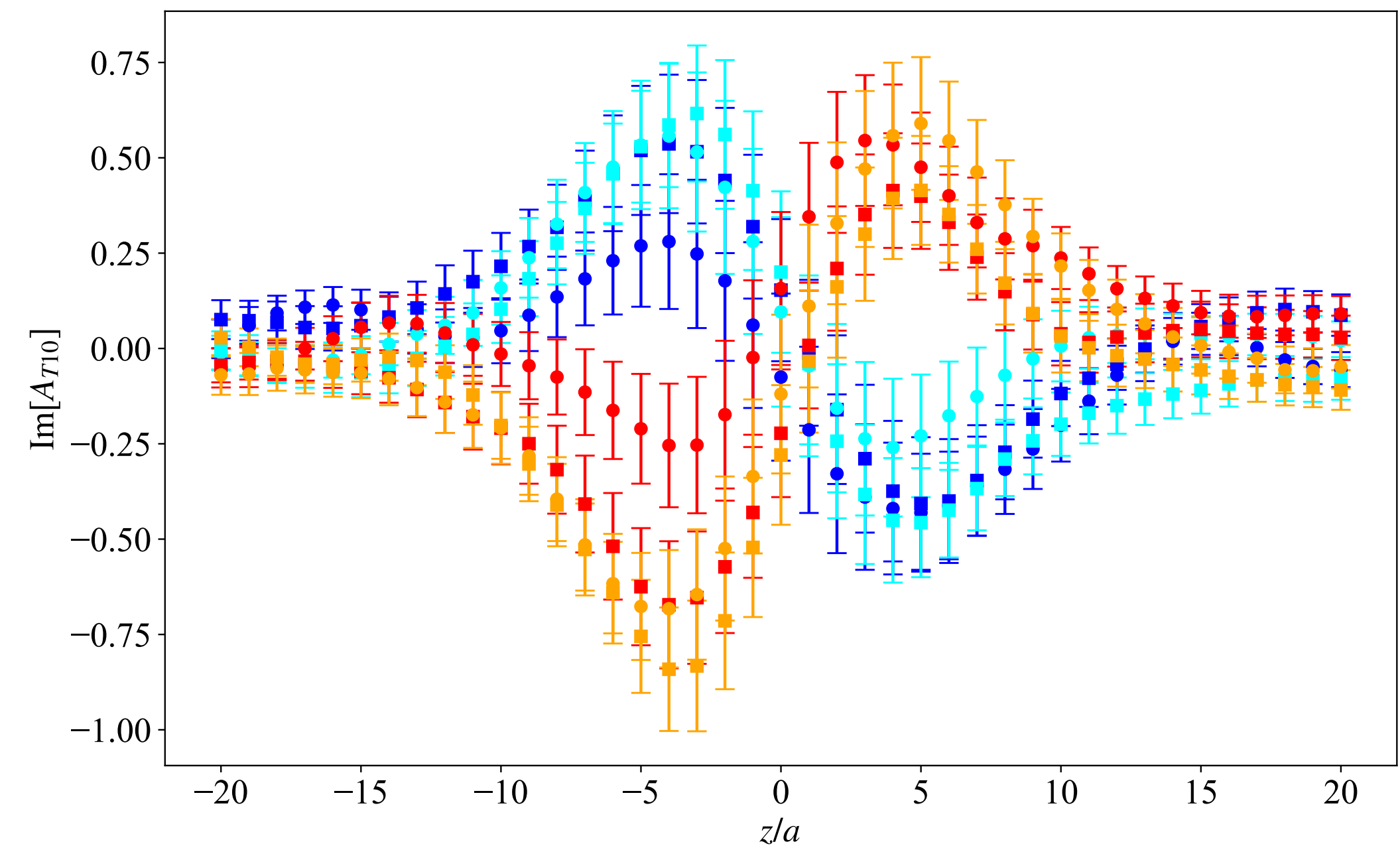
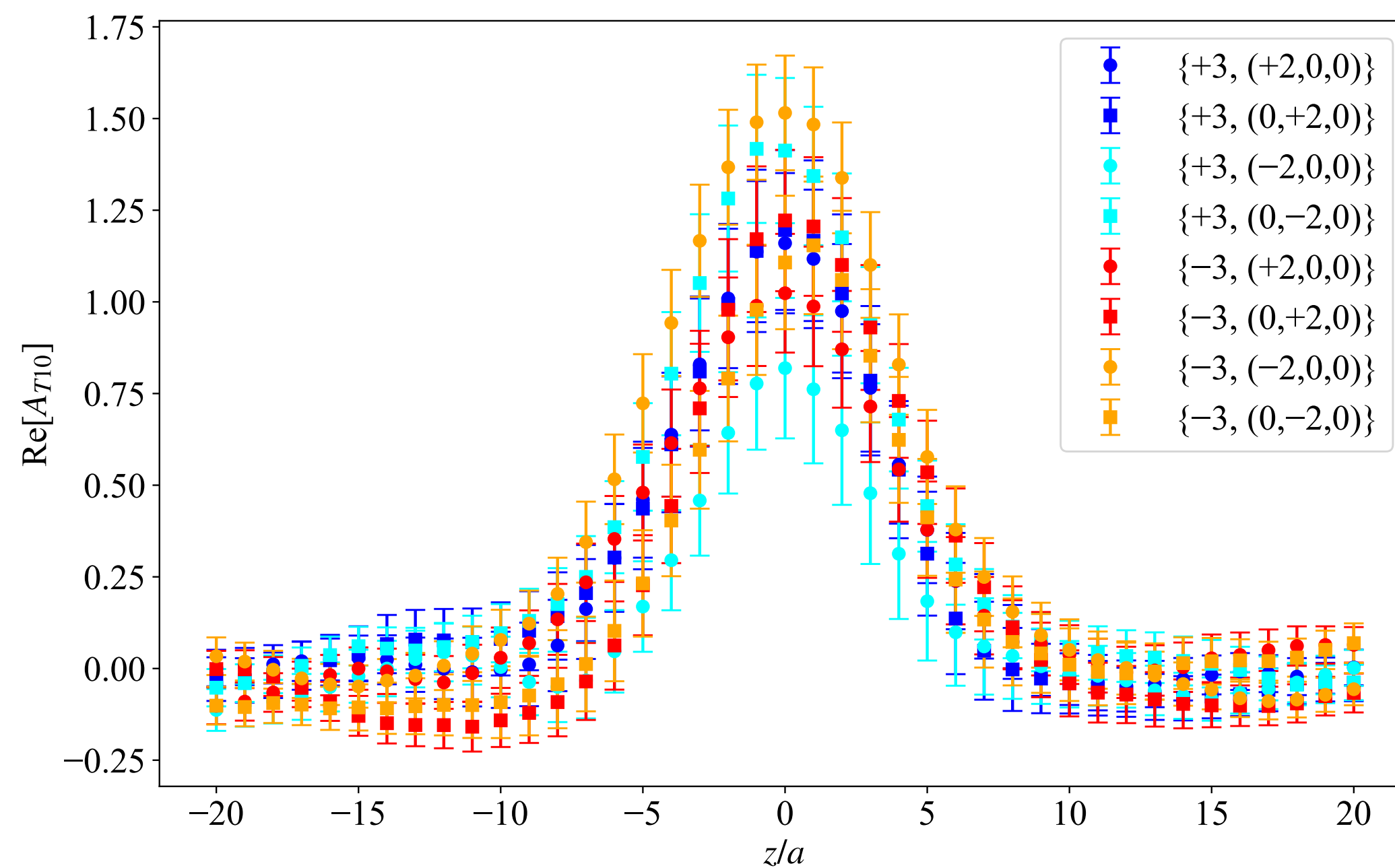
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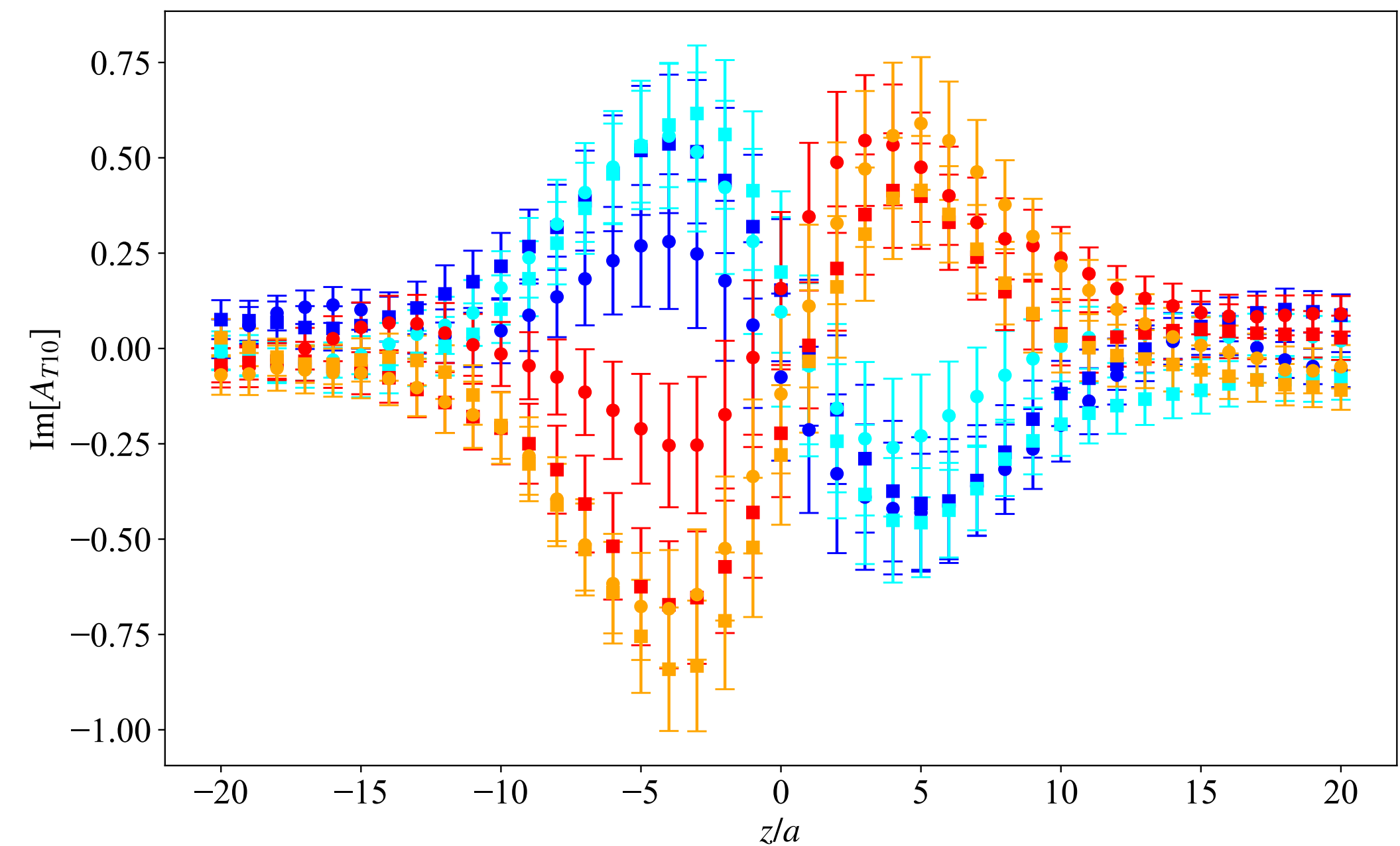
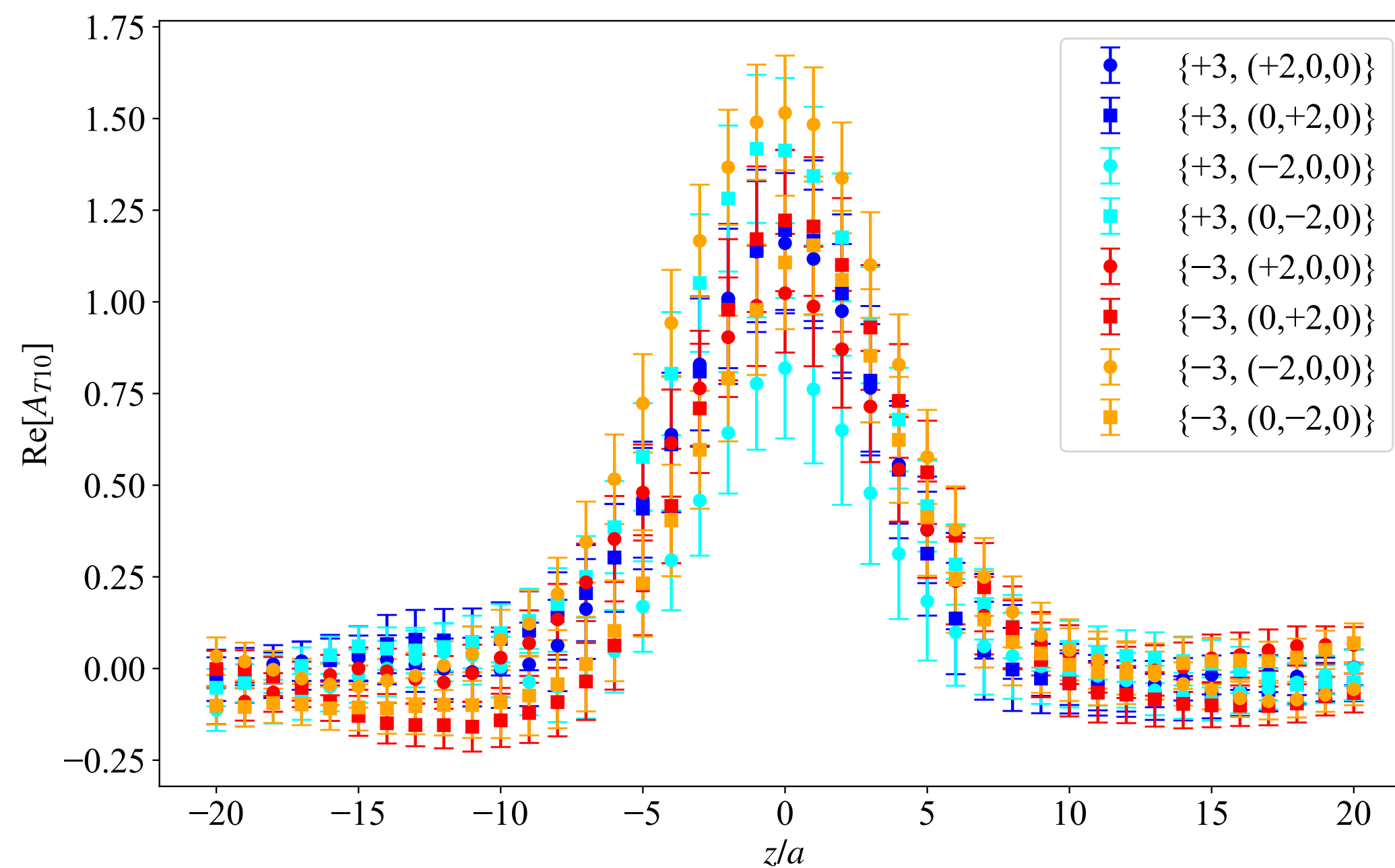
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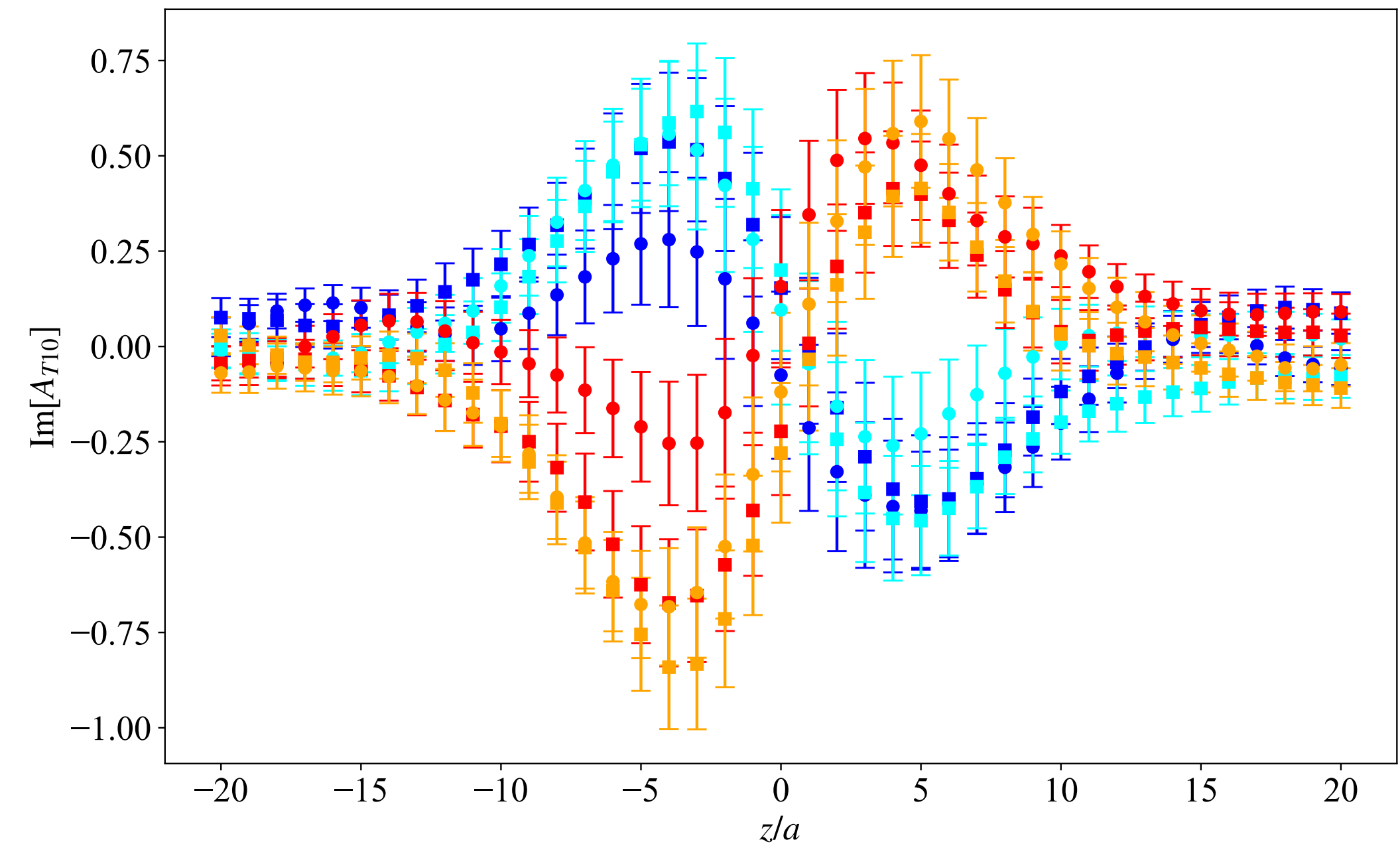
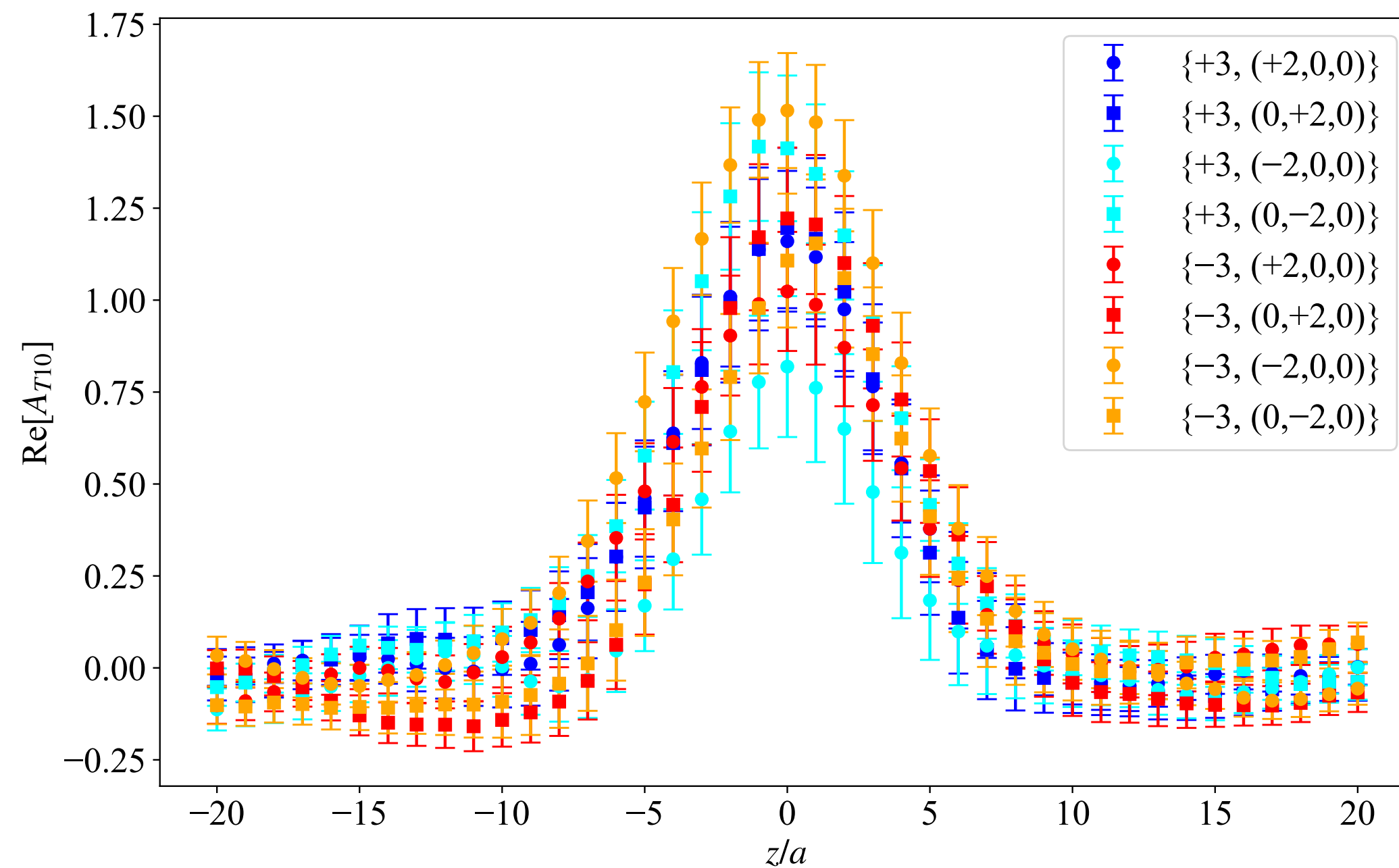
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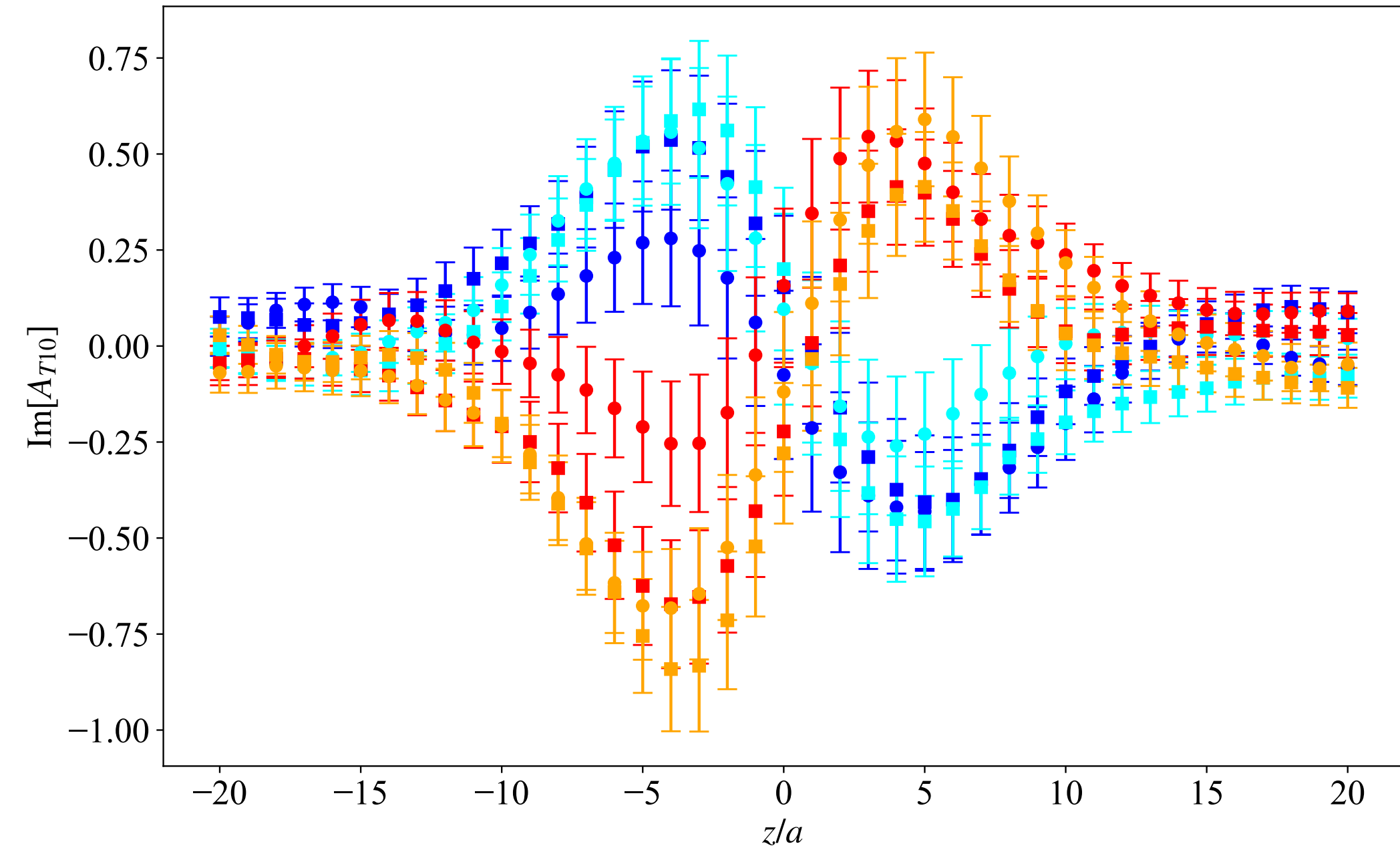
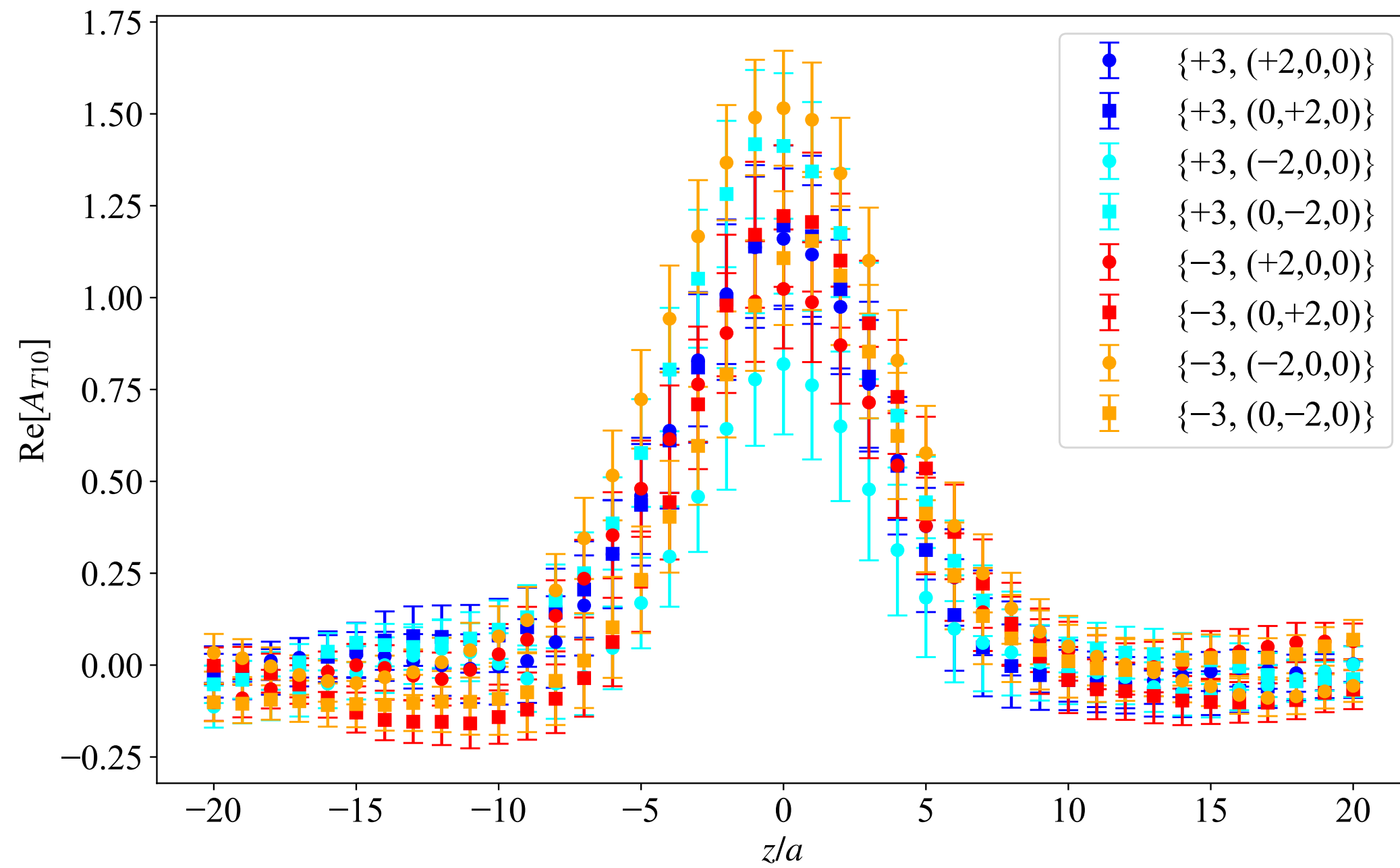
$\longrightarrow$ *Average* $\longrightarrow$ *Quasi-GPDs* $\longrightarrow$ *Backus-Gilbert* $\longrightarrow$ *Match*

Amplitude Symmetry

❖ Symmetry properties of amplitudes

$$A_{Ti}^*(-z \cdot P, z \cdot \Delta, \Delta^2, z^2) = A_{Ti}(z \cdot P, z \cdot \Delta, \Delta^2, z^2) \quad i = 1, 2, 4, 7, 8, 10, 11$$

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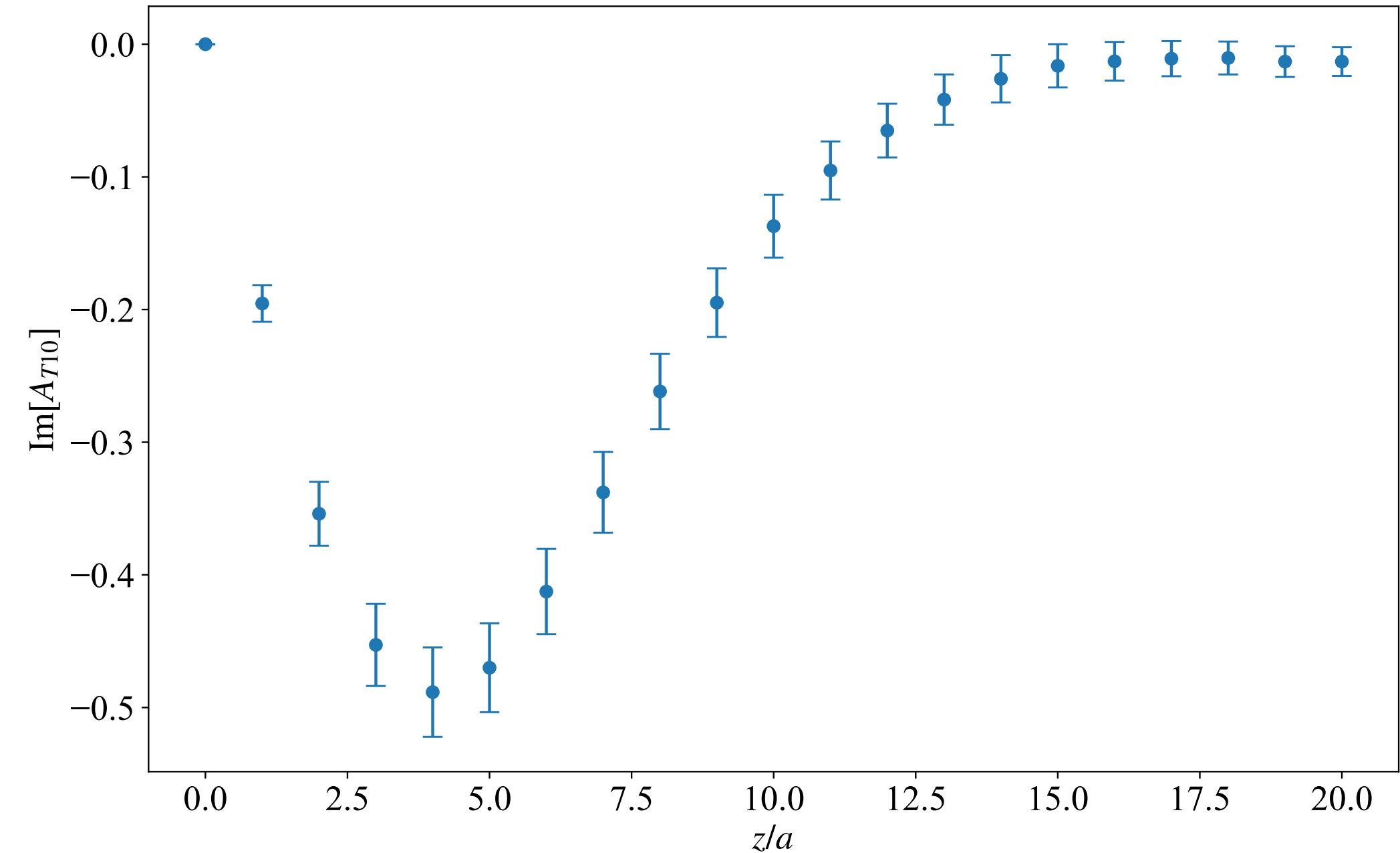
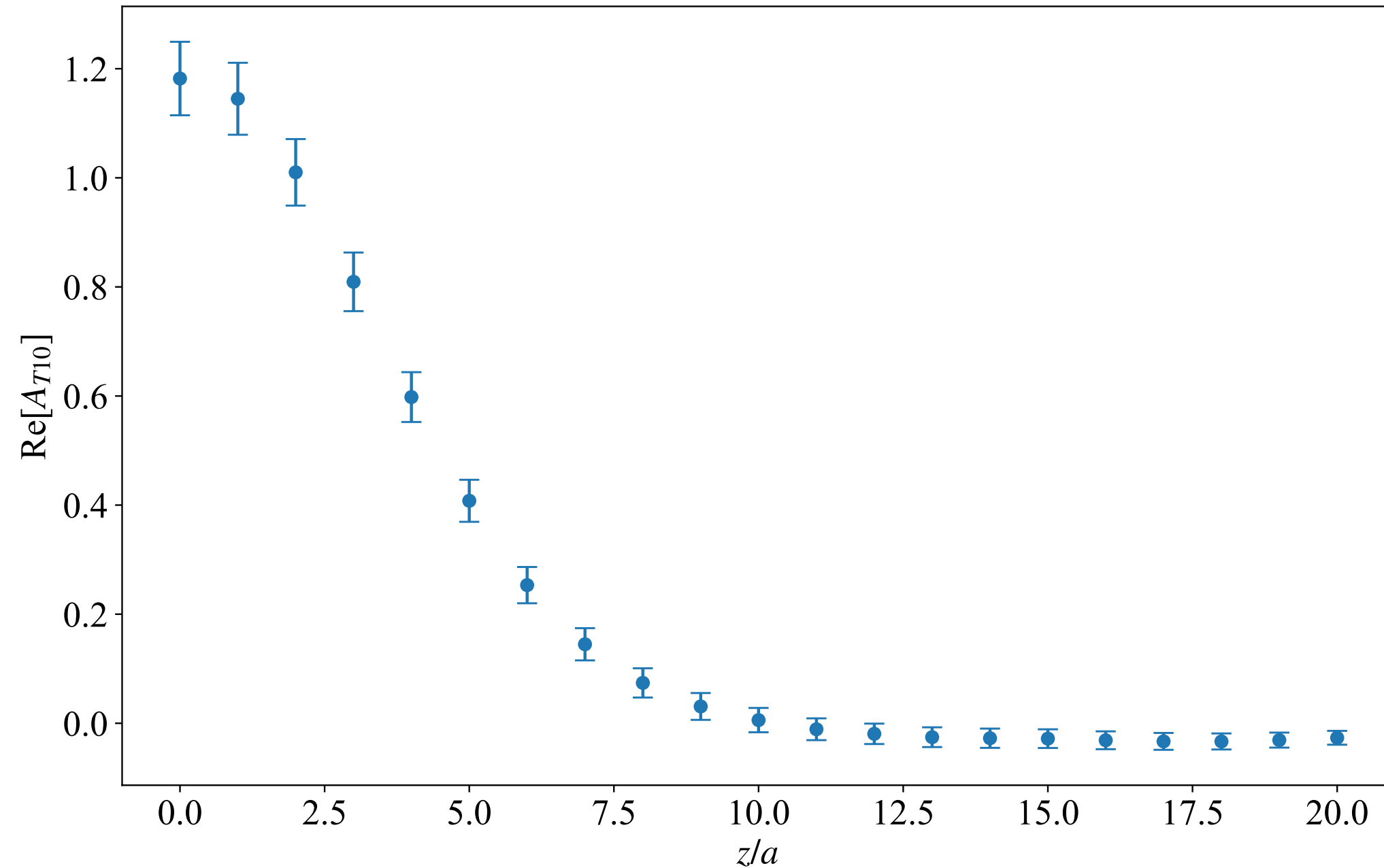


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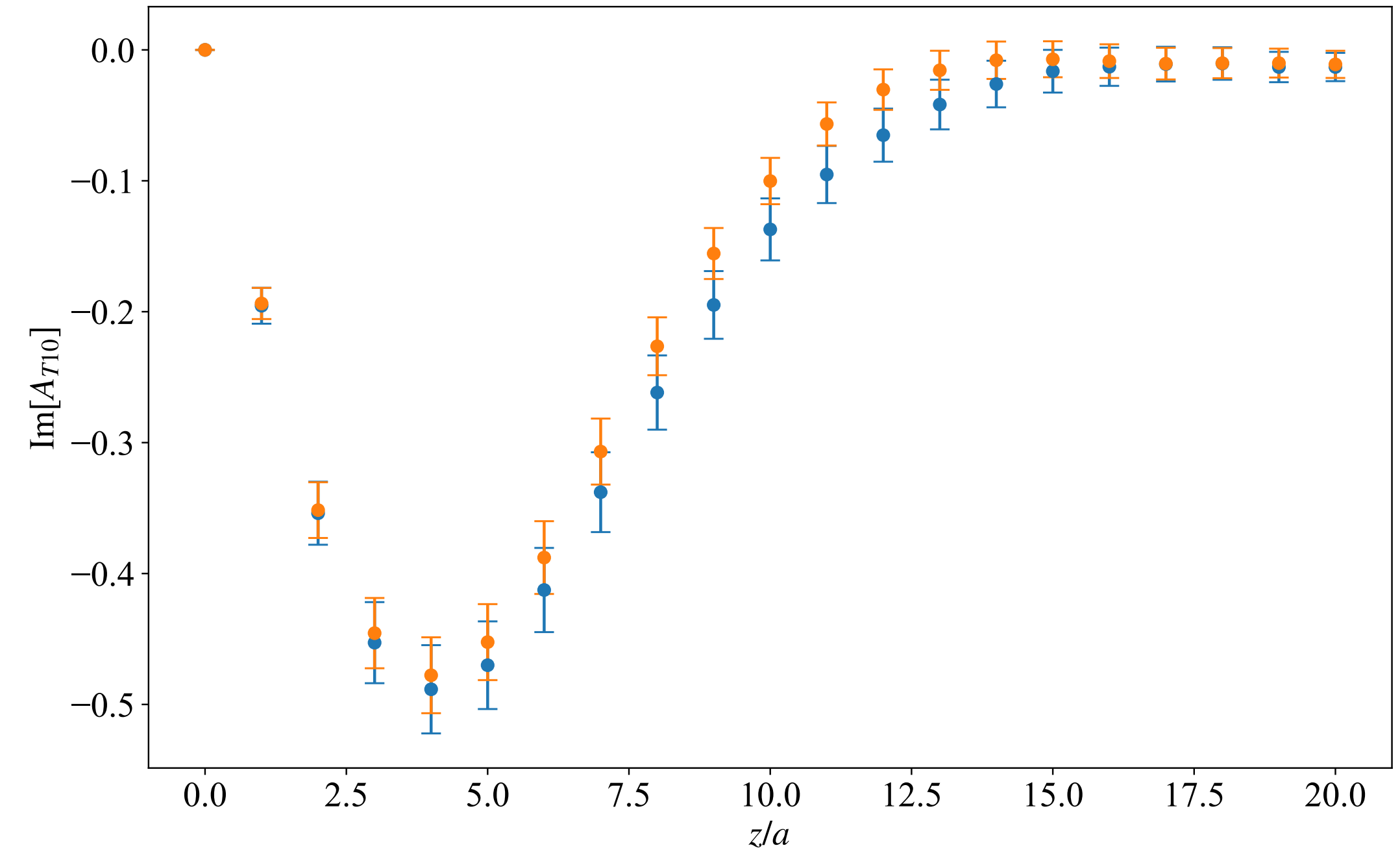
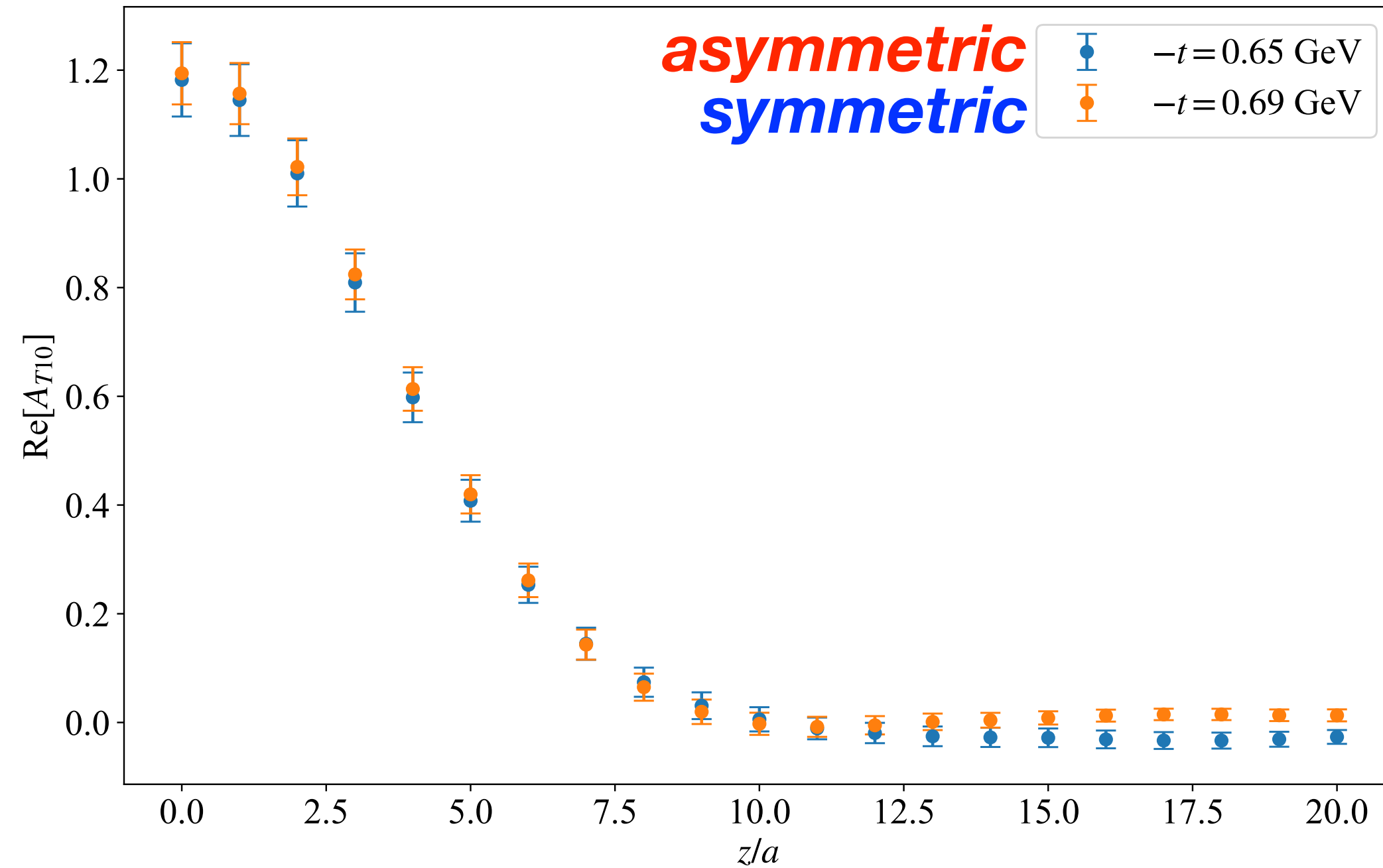
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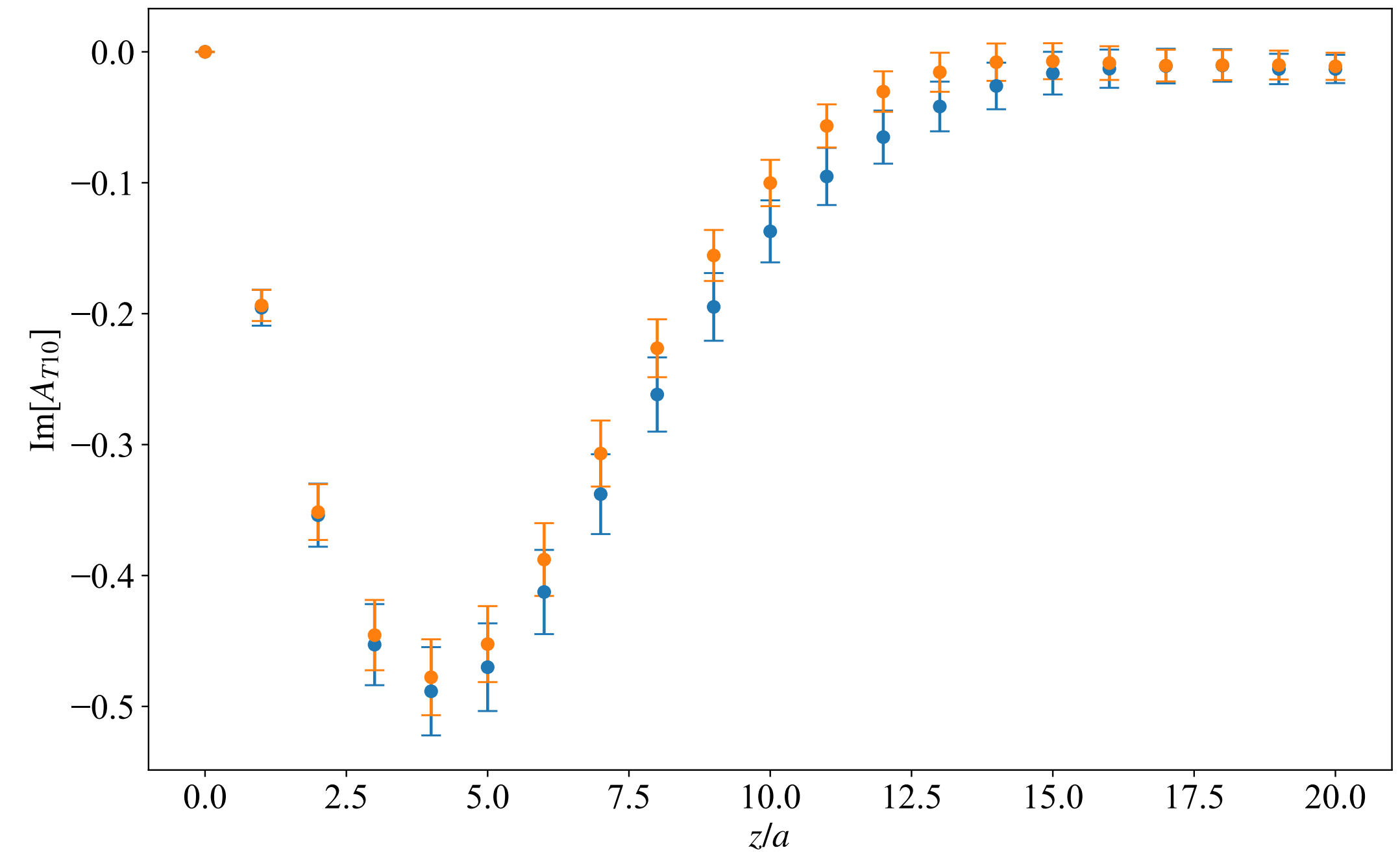
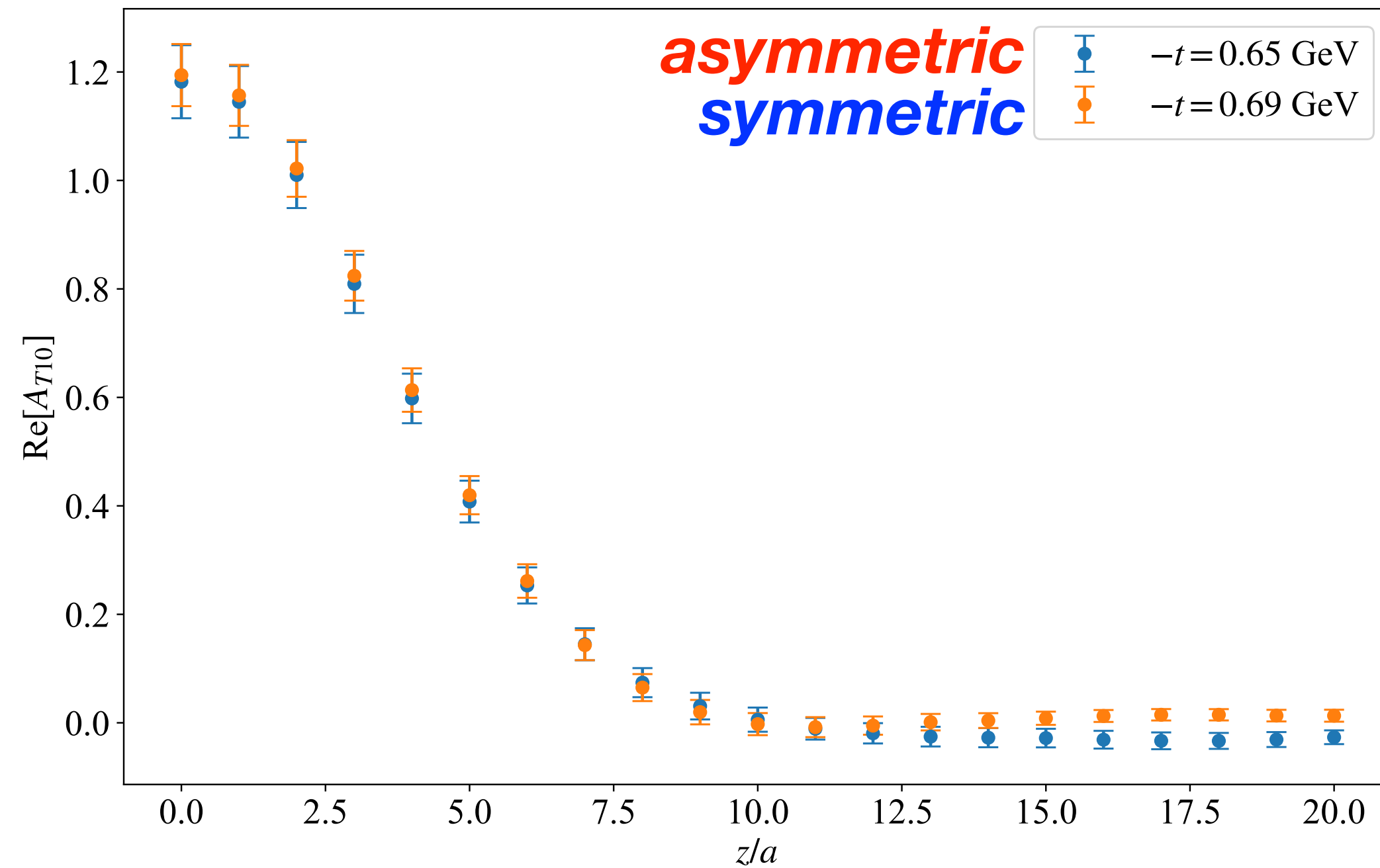
Averaging across 8 kinematic cases reduces the error by $1/\sqrt{8}$

Amplitude Comparison for different frames



- ❖ $-t = 0.65 \text{ GeV}^2$ corresponds to $|P_3| = 3$ and $\vec{\Delta} = (2,0,0)$ +permutations in **asymmetric** frame
- ❖ $-t = 0.69 \text{ GeV}^2$ corresponds to $|P_3| = 3$ and $\vec{\Delta} = (2,0,0)$ +permutations in **symmetric** frame
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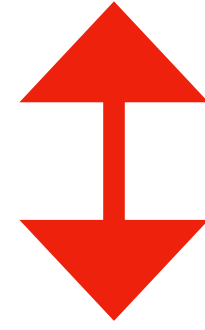
Amplitudes are frame independent!

Quasi-GPDs

$$F_{\lambda,\lambda'}^{[i\sigma^{j+}\gamma_5]}(z, \Delta, P) = -i\epsilon^{-+ij}\bar{u}(p', \lambda') \left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta_\perp^i - \Delta^+\gamma_\perp^i}{2M}E_T + \frac{P^+\Delta_\perp^i - P_\perp^i\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P_\perp^i - P^+\gamma_\perp^i}{M}\widetilde{E}_T \right] u(p, \lambda)$$

Quasi-GPDs

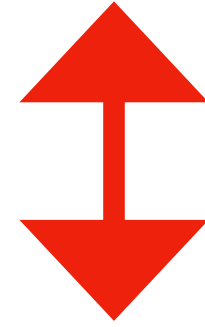
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Quasi-GPDs

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$$\mathcal{H}_T = -2A_{T2}\left(1 + \frac{P^2}{M^2}\right) + A_{T4} + A_{T10}$$

$$\mathcal{E}_T = 2A_{T2} - A_{T4}$$

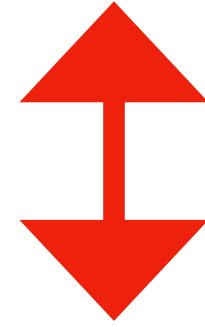
$$\tilde{\mathcal{H}}_T = -A_{T2}$$

$$\tilde{\mathcal{E}}_T = -2A_{T6} - 2P_3zA_{T8}$$

- ❖ Focus only on LI definitions
- ❖ Can substitute P^a , Δ^a directly
- ❖ Differences between LI and standard definitions
 - ❖ $\mathcal{H}_T^s$, $\mathcal{E}_T^s$ have a factor of A_{T8}
 - ❖ $\tilde{\mathcal{H}}_T^s$ has a factor of A_{T12}
 - ❖ $\tilde{\mathcal{E}}_T^s$ has a different coefficient for A_{T8}

Quasi-GPDs

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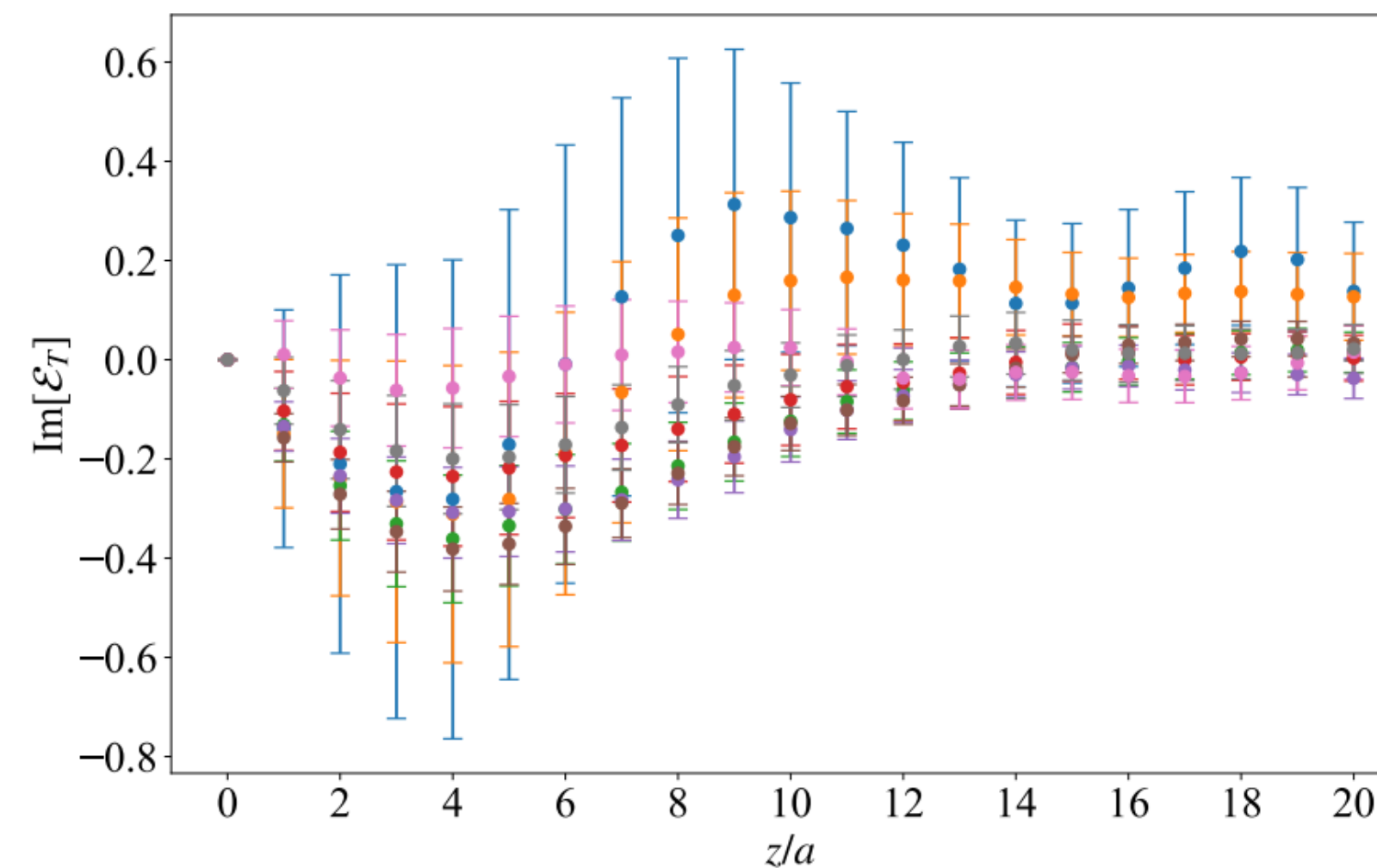
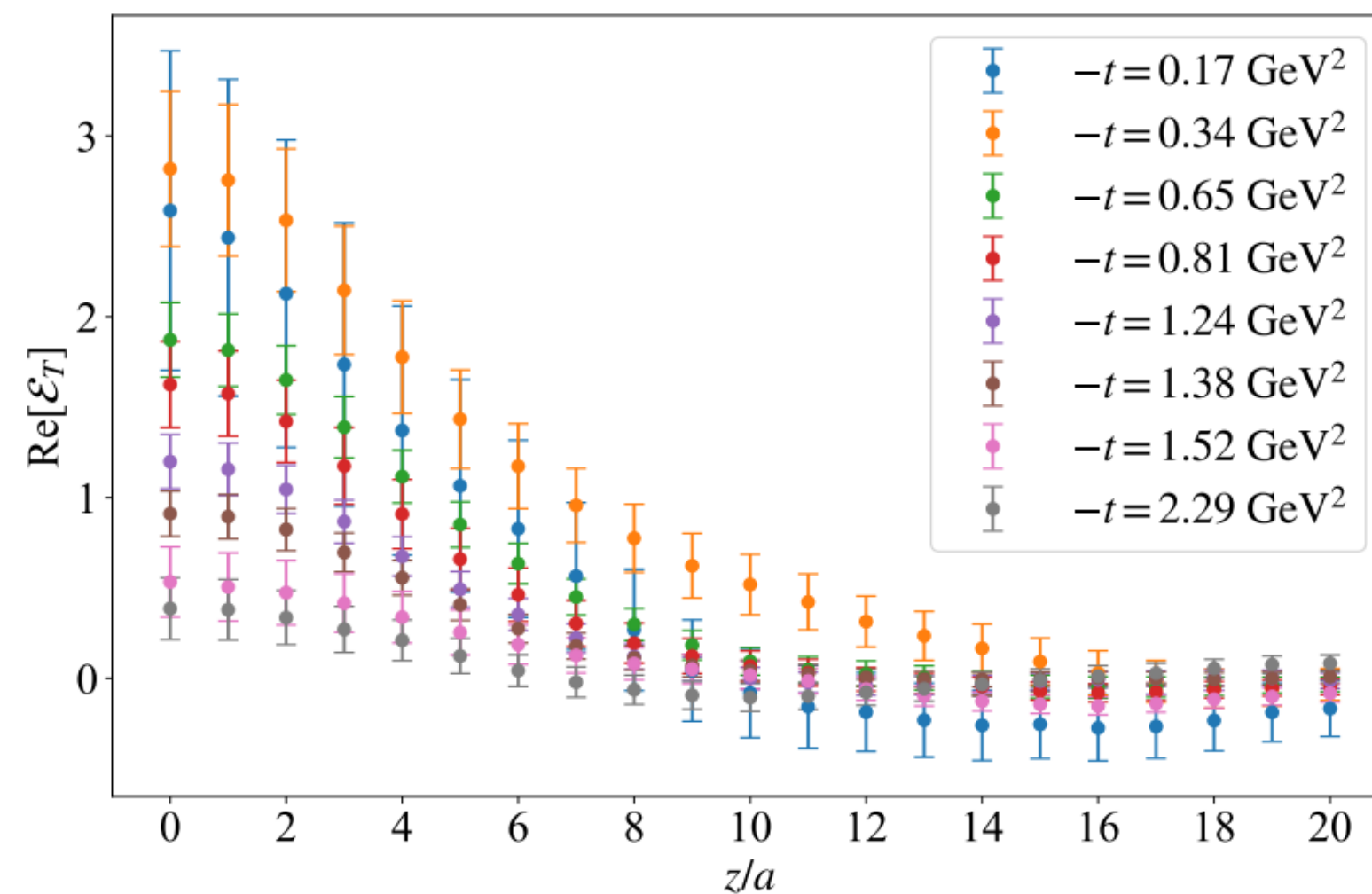
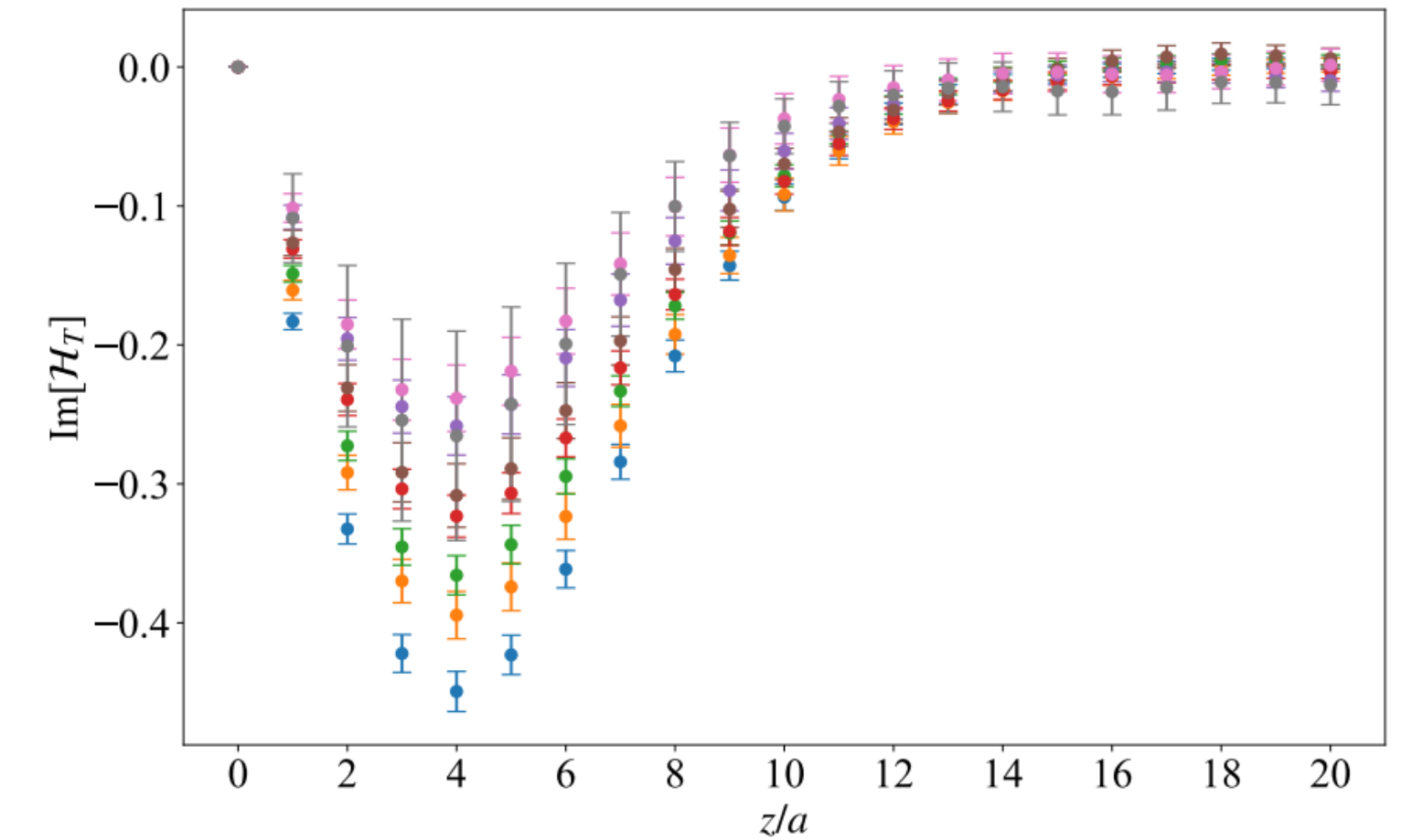
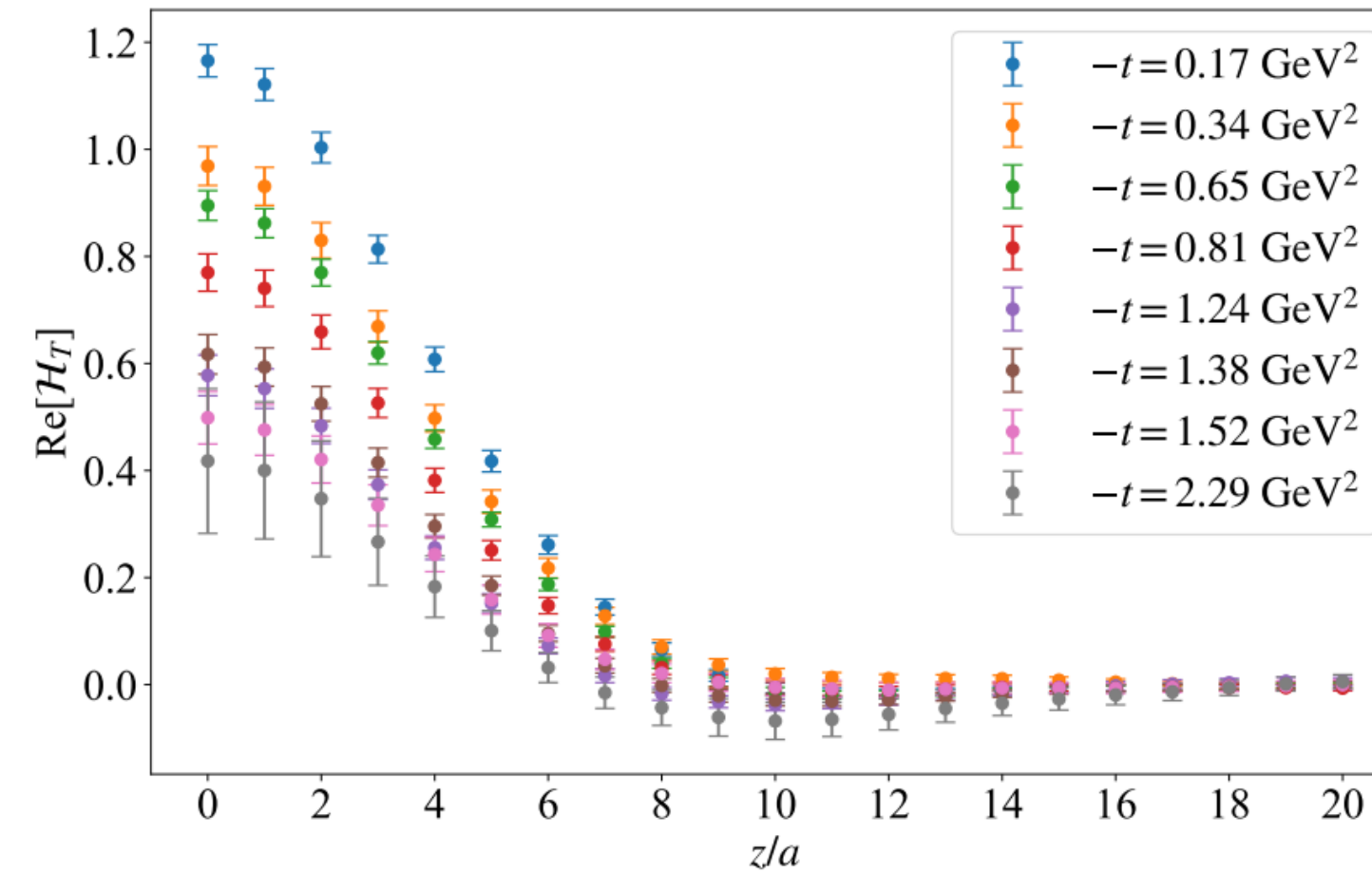
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Quasi-GPDs

- ❖ Results in agreement between asymmetric and symmetric frames!
- ❖ Asymmetric frame is **computationally much more efficient** to produce data for a dense range of $-t$

$$\mathcal{H}_T = -2A_{T2} \left(1 - \frac{(E_i + E_f)^2 - \vec{\Delta}^2 - 4P_3^2}{4m^2} \right) + A_{T4} + A_{T10}$$

- ❖ Large contribution from A_{T10}



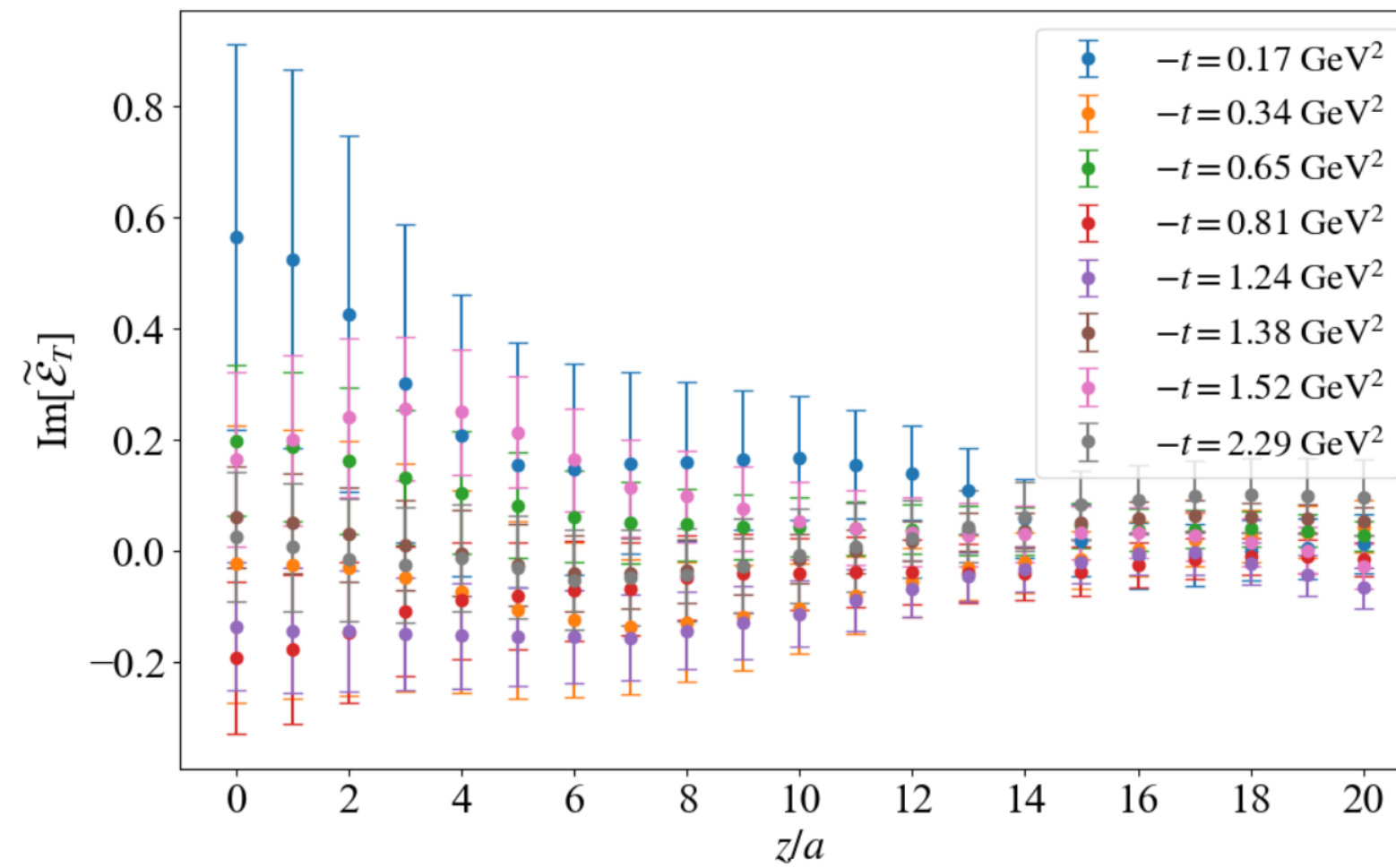
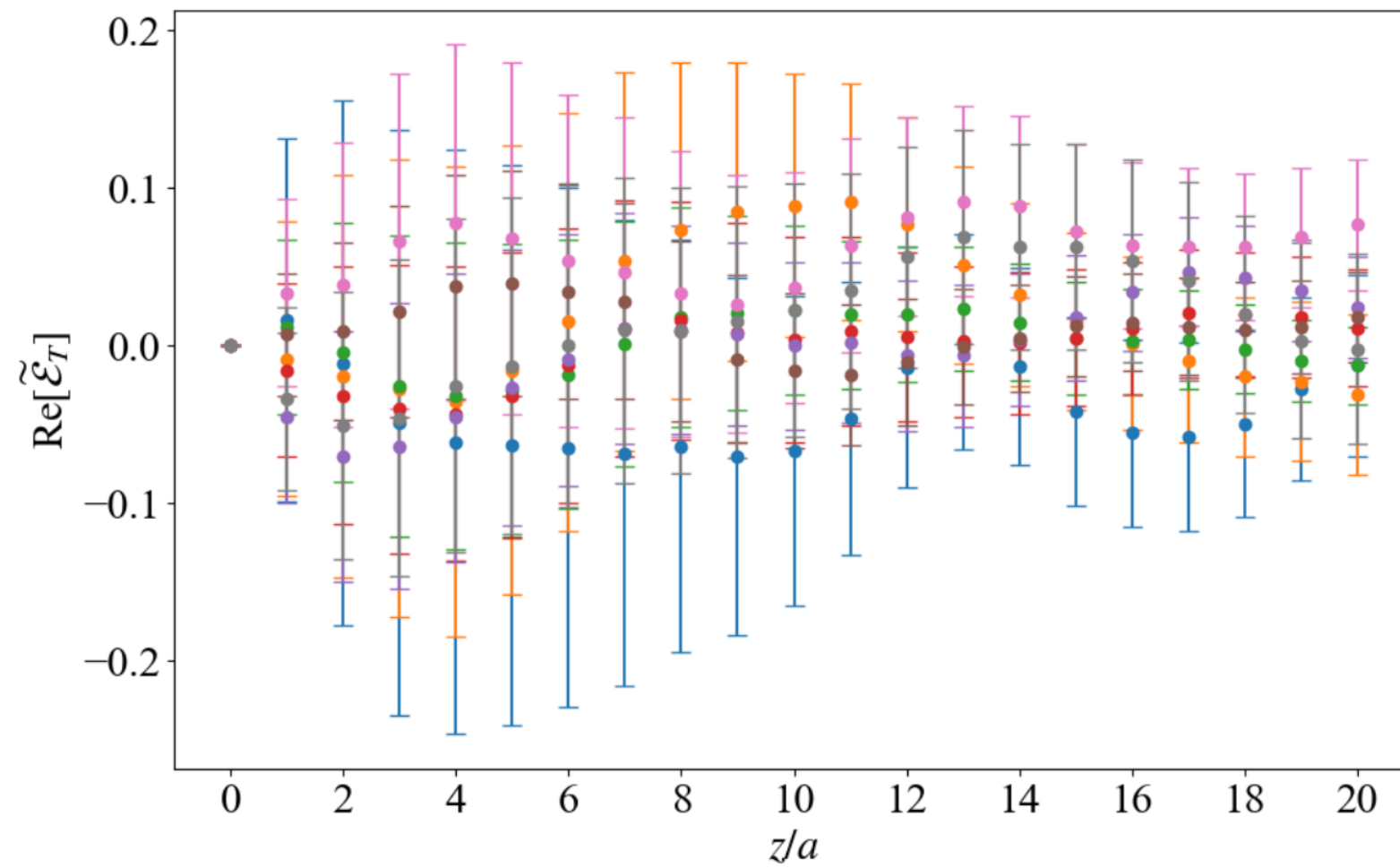
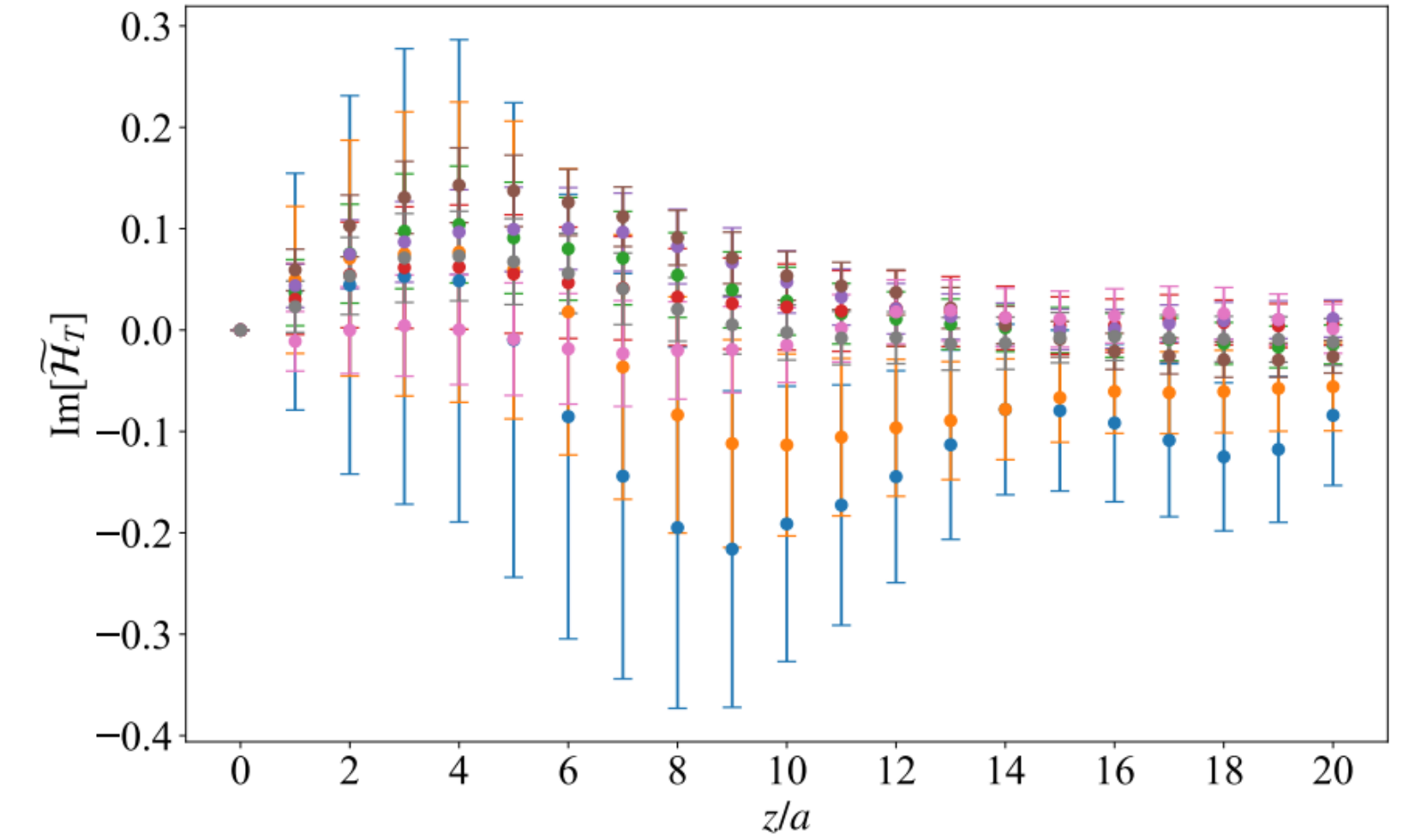
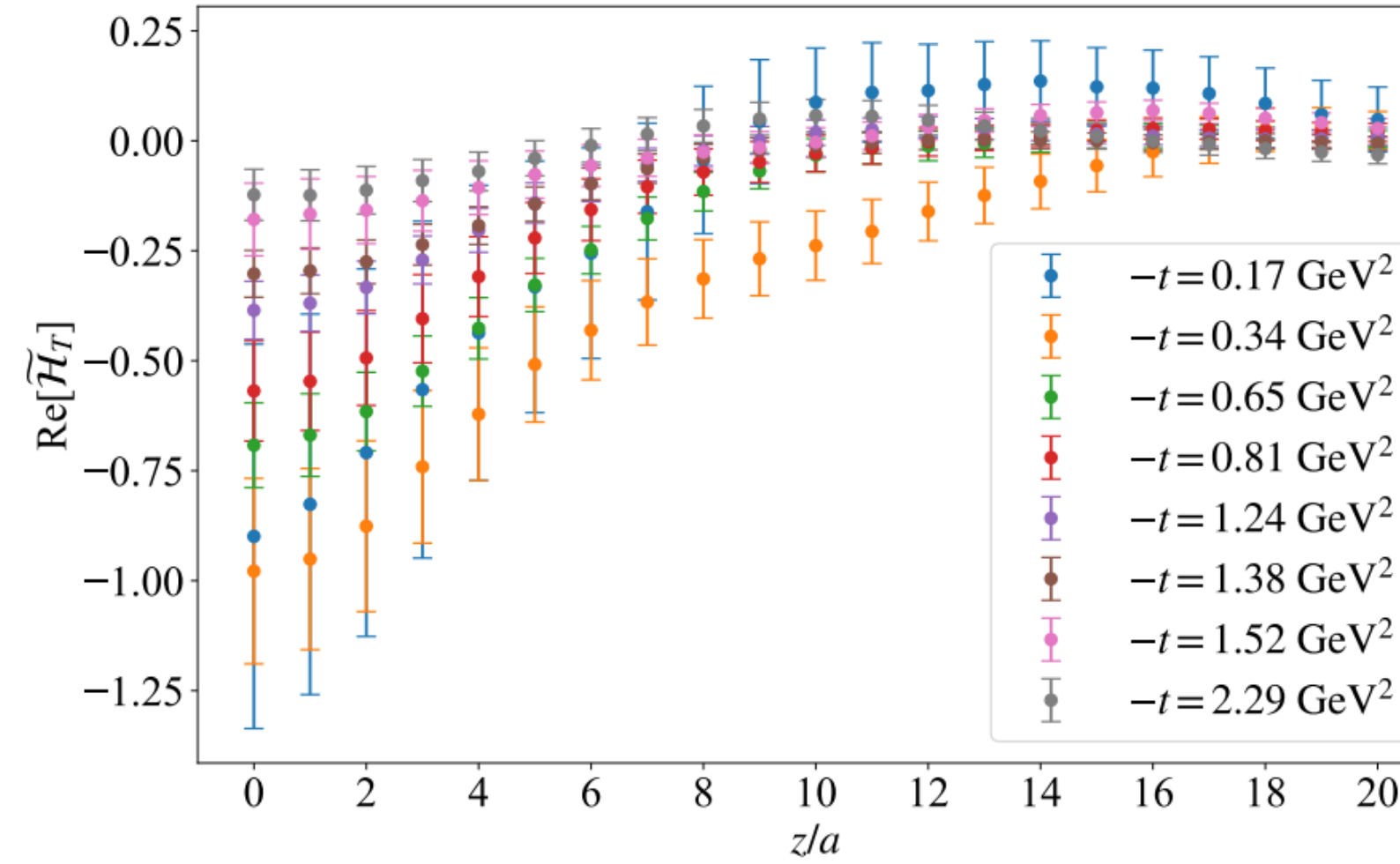
$$E_T = 2A_{T2} - A_{T4}$$

- ❖ Magnitude decreases as $-t$ increases

Quasi-GPDs

$$\tilde{\mathcal{H}}_T = -A_{T2}$$

❖ Magnitude decreases as $-t$ increases



$$\tilde{\mathcal{E}}_T = -2A_{T6} - P_3 z A_{T8}$$

❖ Odd in $\xi \rightarrow -\xi$

❖ Zero in $\xi = 0$

[Meissner, et al., JHEP 08, 056 (2009)]

Position space to Momentum space

- ❖ Multiple methods to consider
 - ❖ Standard Fourier Transform

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[Backus & Gilbert, Geophysical Journal International 16, 169 (1968)]

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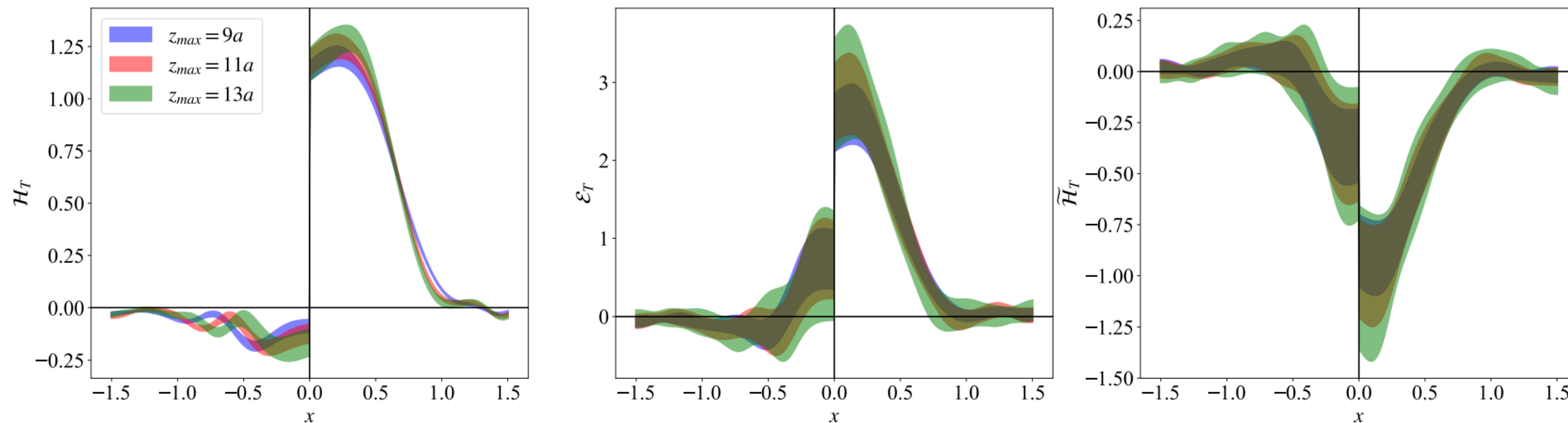
❖ Backus-Gilbert method $\longrightarrow$ *Why?* [Backus & Gilbert, Geophysical Journal International 16, 169 (1968)]

❖ Backus-Gilbert:

❖ Model independent

❖ Criterion: variance of solution with respect to statistical variation of input data is minimal

❖ Test the dependence on z_{max} in reconstruction



All calculations use $z_{max} = 11a$ to match to light-cone

Matching to the Light-Cone

[Liu, et al., Phys. Rev. D 100, 034006 (2019)]

$$GPD(x, \xi, t) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C_{\Gamma} \left(\frac{x}{y}, \frac{\xi}{y}, r, \frac{yP^z}{\mu}, \frac{yP^z}{p_R^z} \right) qGPD(y, \xi, t)$$

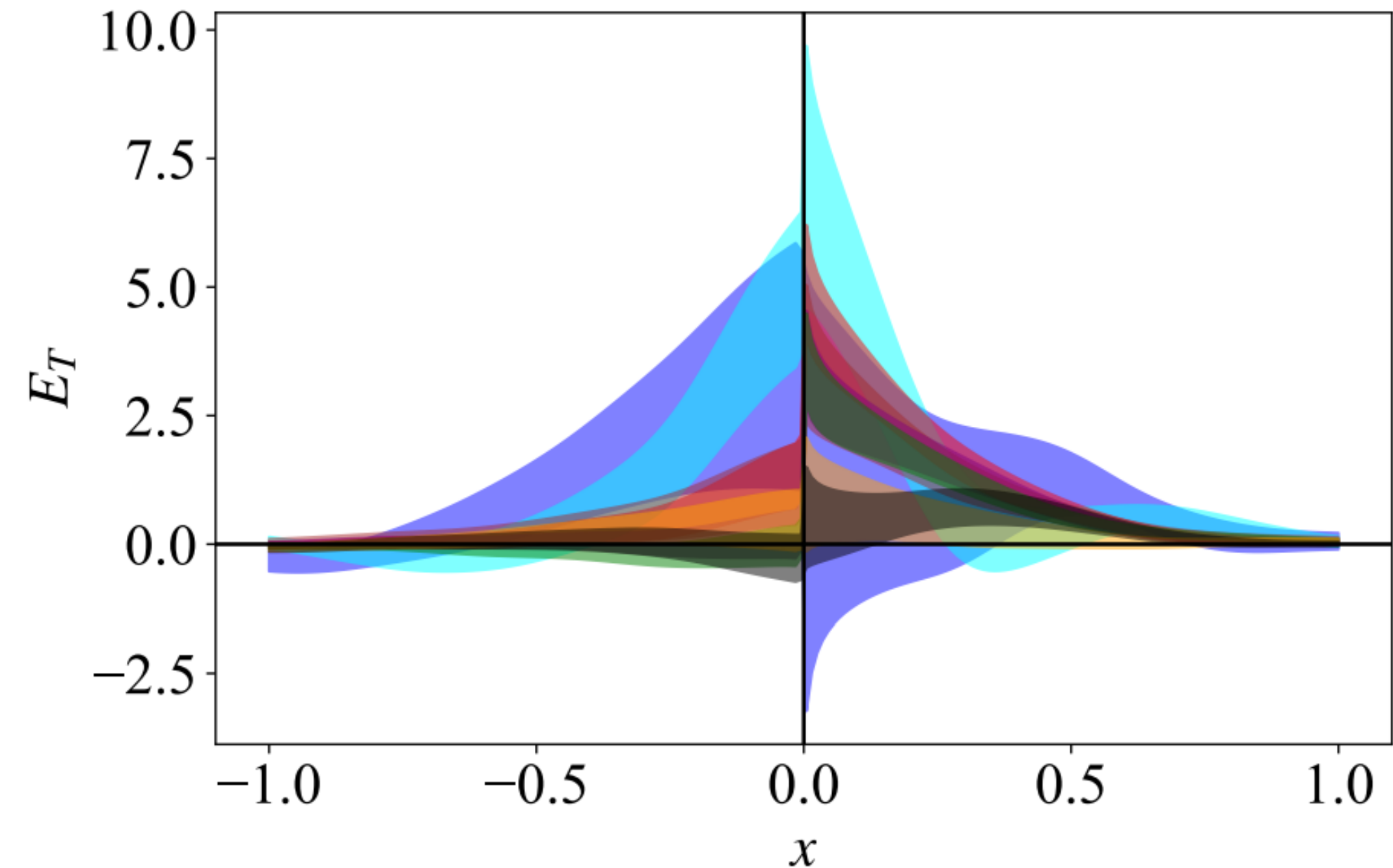
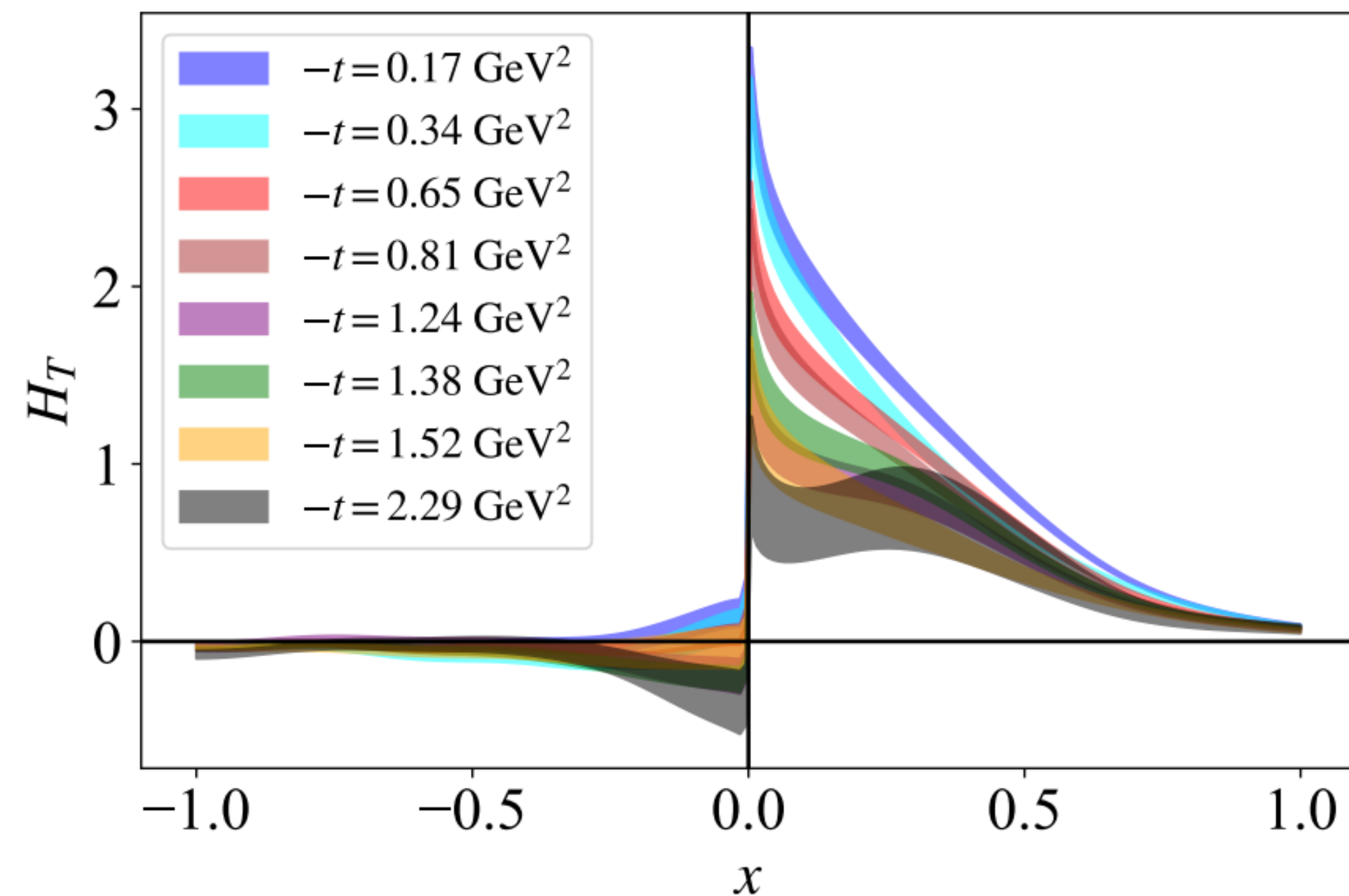
$$C_{i\sigma^z\perp} \left(x, \xi = 0, \frac{p^a}{\mu} \right) = \frac{\alpha_s C_F}{2\pi} f_1 \left(x, \xi = 0, \frac{p^z}{\mu} \right)_+ + 2\delta(1-x) \frac{\alpha_s C_F}{4\pi} \left[\frac{-1}{\epsilon_{UV}} + \ln \left(\frac{\mu^2}{\mu'^2} \right) \right] + \left[\left| \frac{p^z}{p_R^z} \right| \frac{\alpha_s C_F}{2\pi} f_2 \left(\frac{p^z}{p_R^z} (x-1) + 1, r \right) \right]_+$$

$$x < 0 : \quad f_1(x, \xi = 0) = \frac{-2x}{x-1} \ln \left(\frac{x-1}{x} \right) \quad f_2(x, \xi = 0) = \frac{-3}{2(1-x)} - \frac{r-2x}{(r-1)(r-4x+4x^2)} - \frac{-r+2x-rx}{(r-1)^{3/2}(x-1)} \tan^{-1} \left(\frac{\sqrt{r-1}}{2x-1} \right)$$

$$0 < x < 1 : \quad f_1(x, \xi = 0) = \frac{2x}{1-x} \left[\ln \left(\frac{4x(1-x)(p^z)^2}{\mu^2} \right) - 1 \right] \quad f_2(x, \xi = 0) = \frac{1-3r+2x}{2(r-1)(1-x)} + \frac{r-2x+rx}{(r-1)^{3/2}(1-x)} \tan^{-1} \left(\sqrt{r-1} \right)$$

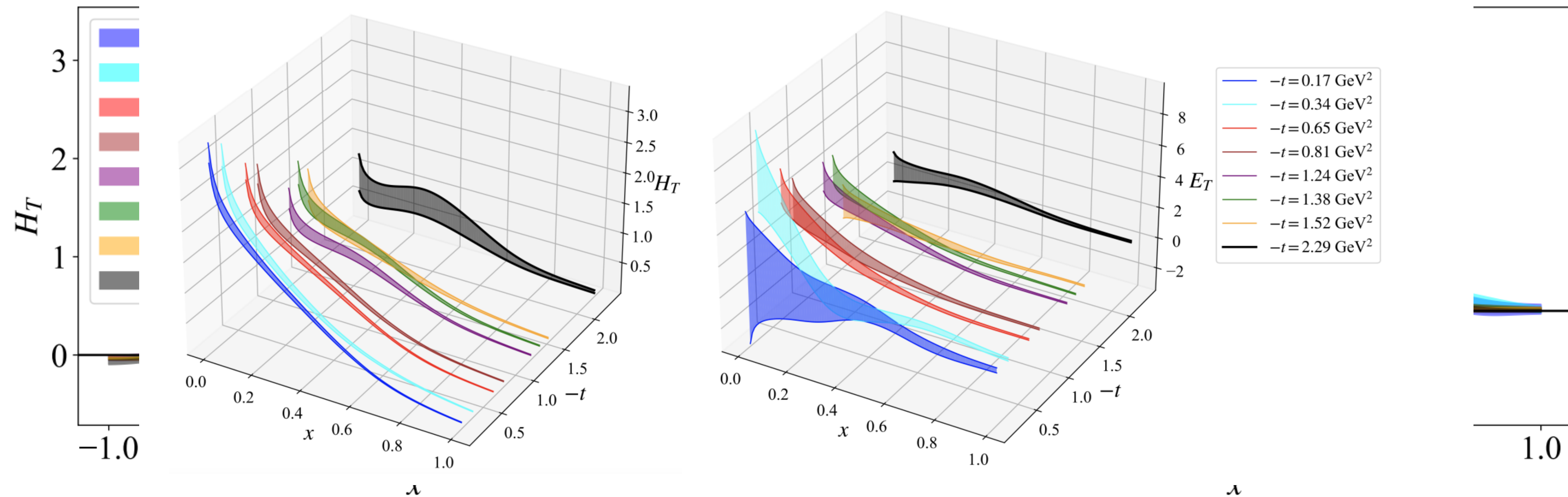
$$x > 1 : \quad f_1(x, \xi = 0) = \frac{2x}{x-1} \ln \left(\frac{x-1}{x} \right) \quad f_2(x, \xi = 0) = \frac{3}{2(1-x)} + \frac{r-2x}{(r-1)(r-4x+4x^2)} + \frac{-r+2x-rx}{(r-1)^{3/2}(x-1)} \tan^{-1} \left(\frac{\sqrt{r-1}}{2x-1} \right)$$

Light-Cone Results



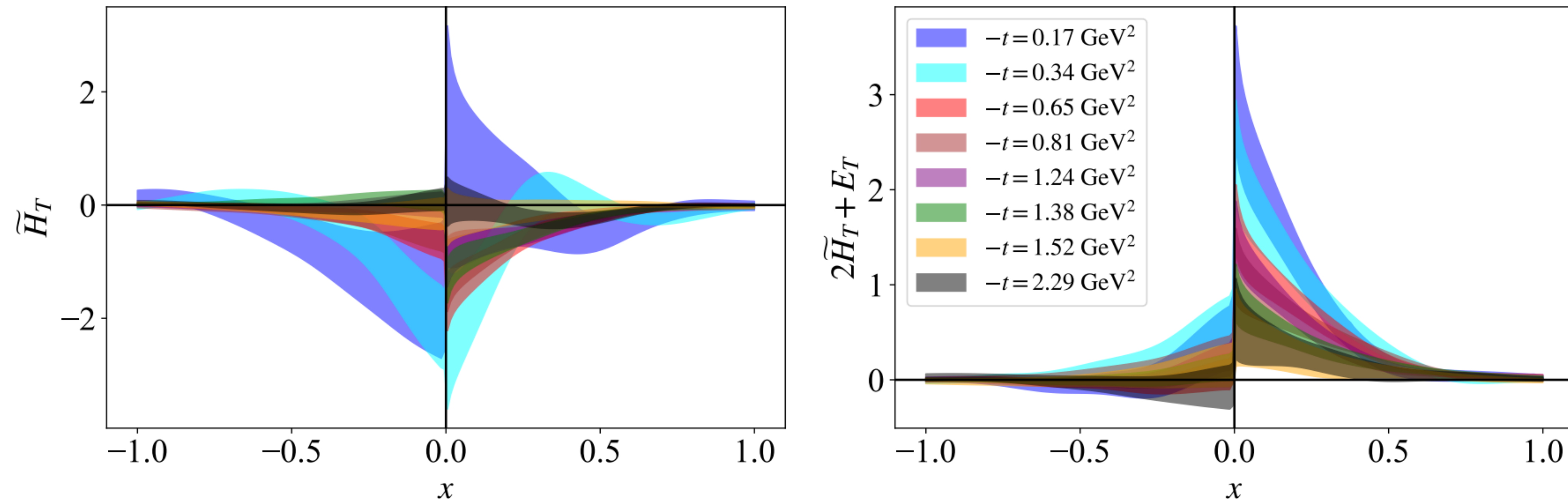
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Light-Cone Results



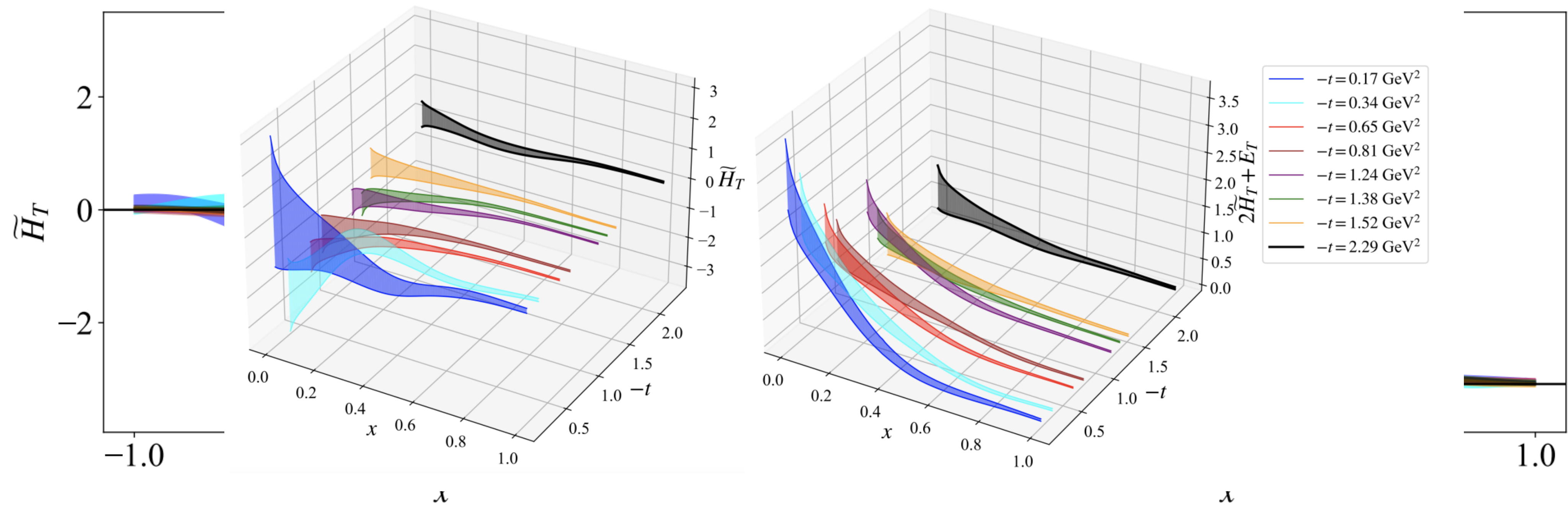
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 - ❖ Related to transverse spin structure
 - ❖ Interpretation is lateral deformation of transversely polarized quark in an unpolarized proton
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Light-Cone Results



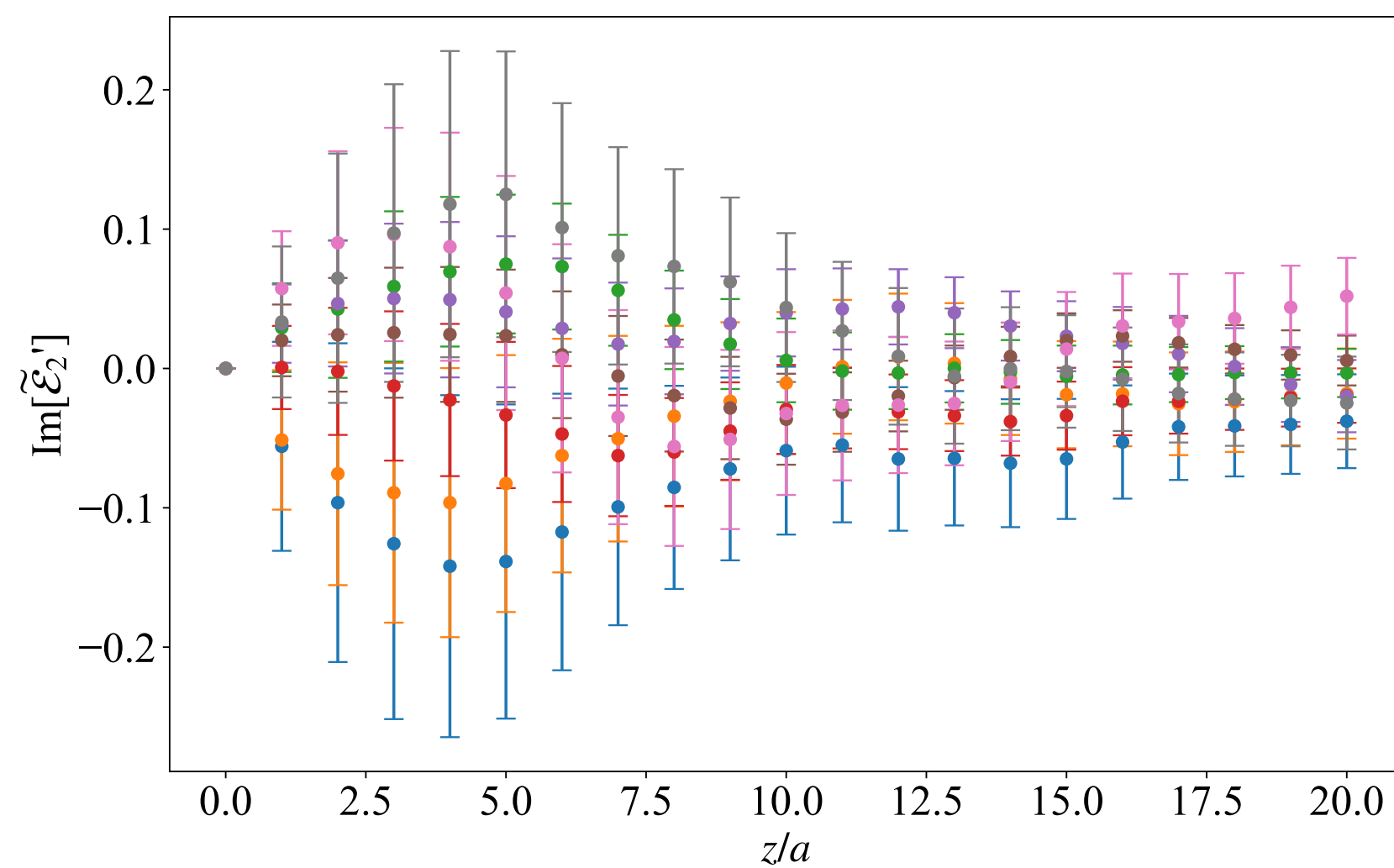
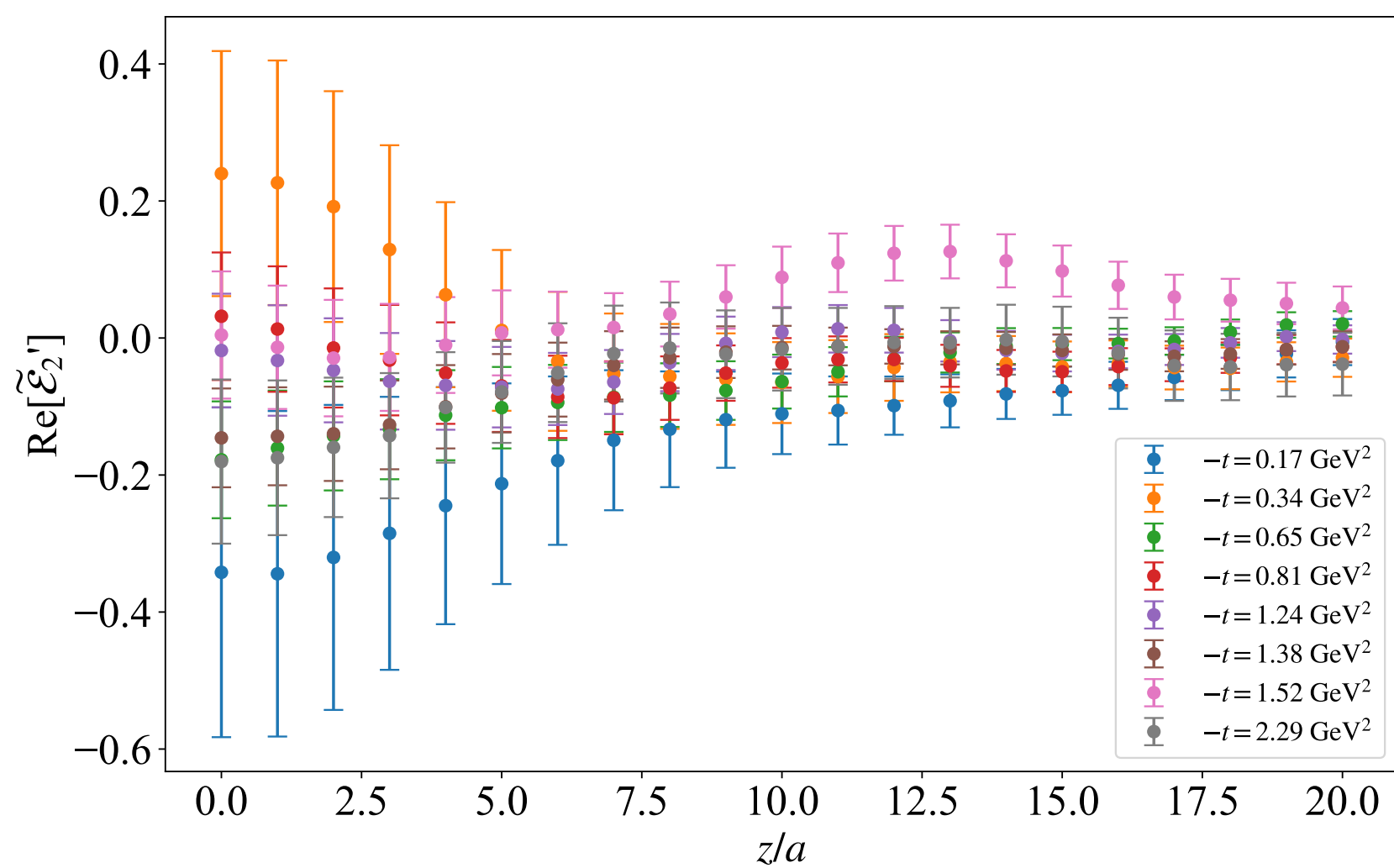
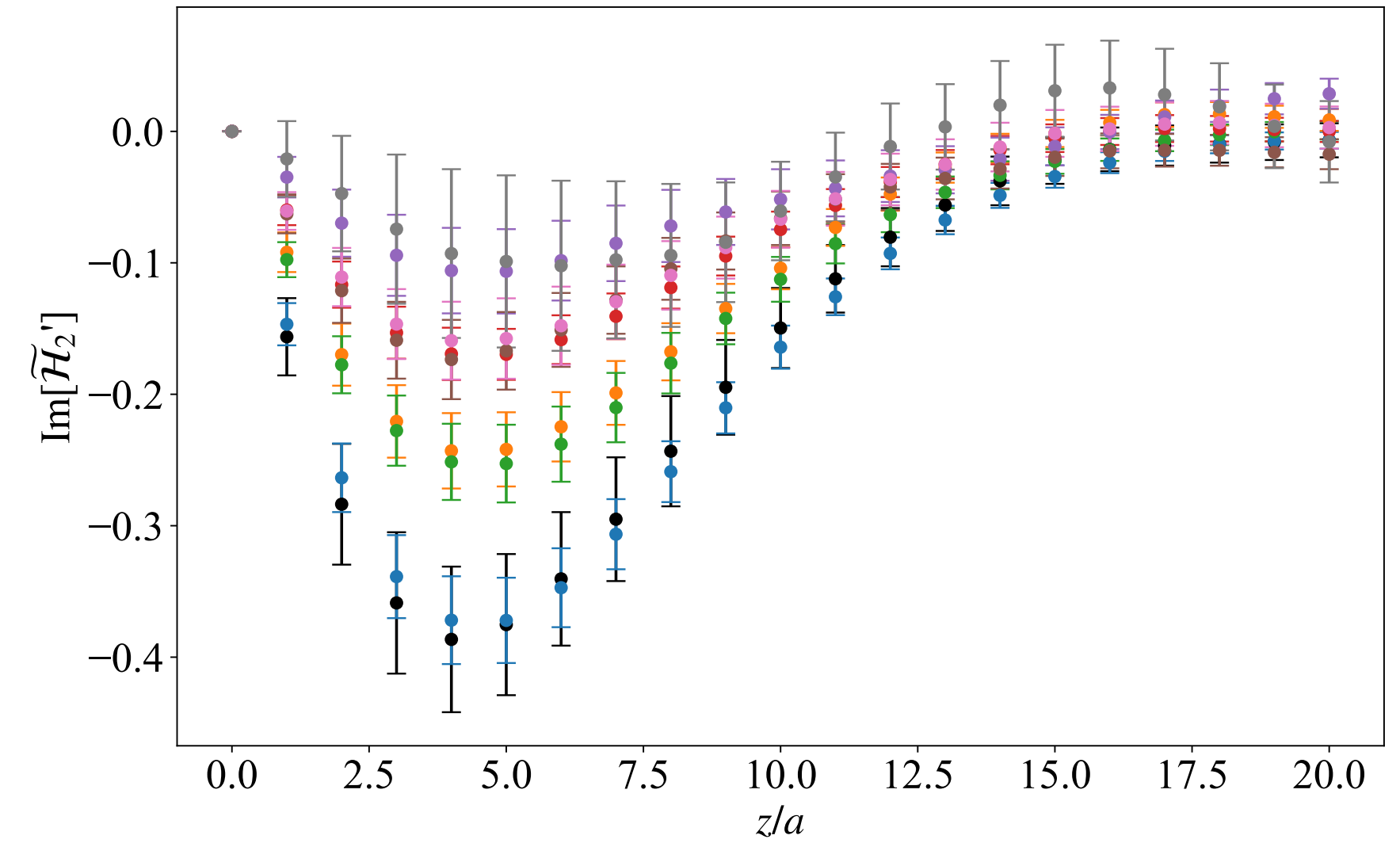
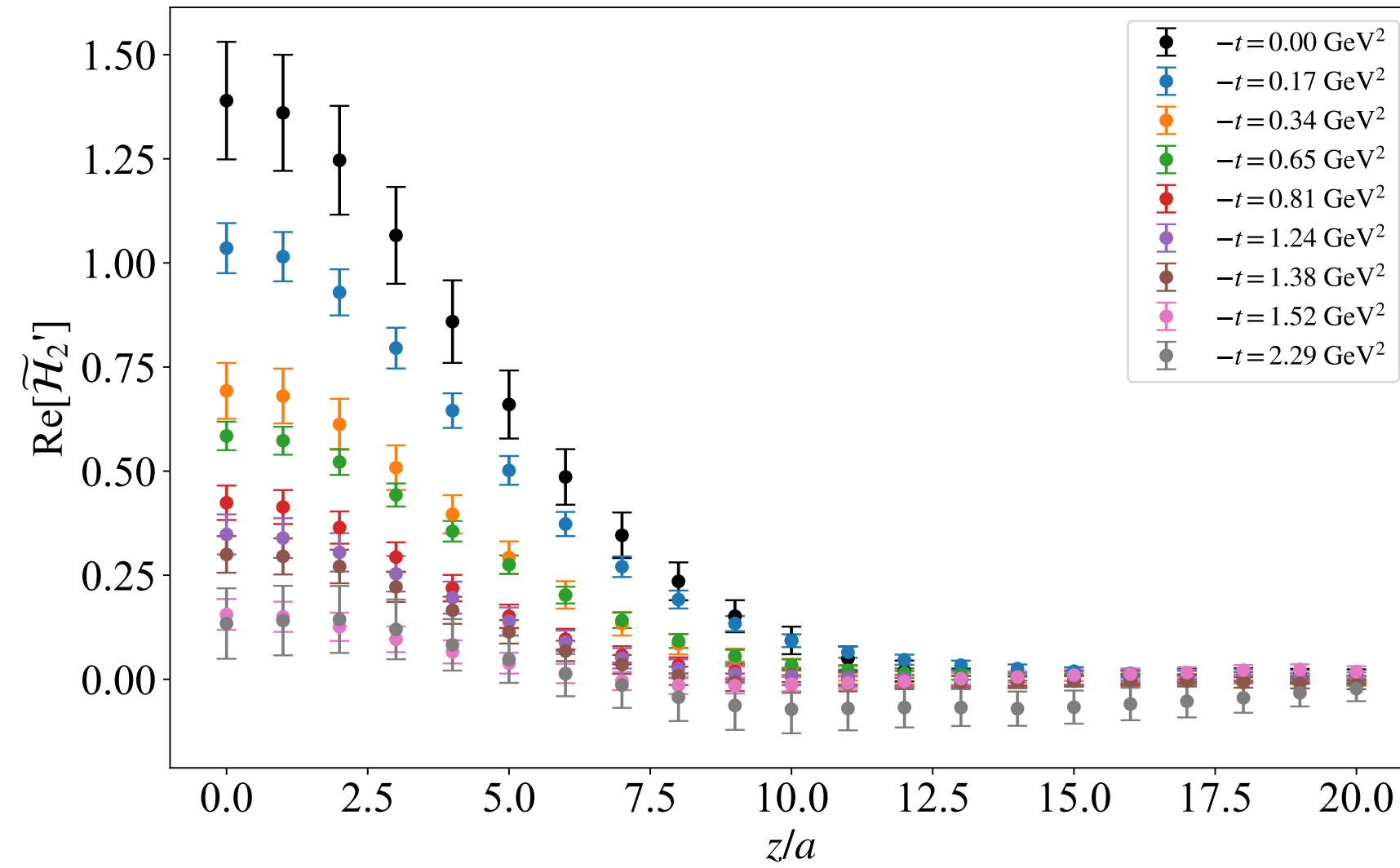
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Twist-3 Quasi-GPDs

$$F^{[\sigma^{+-}\gamma_5]} = \bar{u}(p_f, \lambda') \left[\frac{M}{P^+} \gamma^+ \gamma_5 \tilde{H}'_2 + \gamma_5 \tilde{E}'_2 \right] u(p_i, \lambda)$$

$$\tilde{\mathcal{H}}'_2 = \frac{E_f(E_f + E_i) - 2P_3^2}{2m^2} A_{T4} + P_3 z A_{T5} + \frac{E_f(E_f + E_i)}{2} z^2 A_{T7} + A_{T10}$$

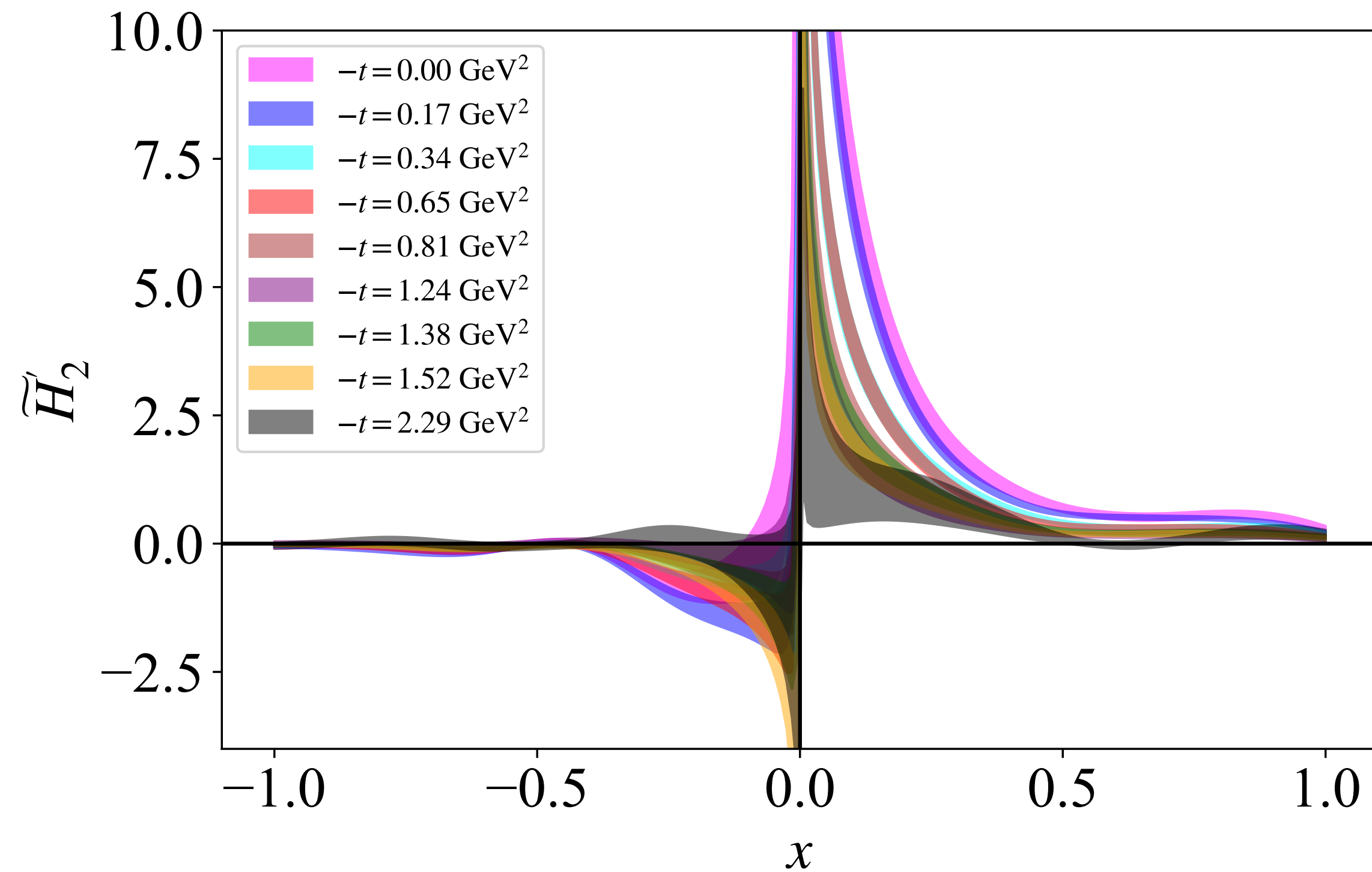
- ❖ Good signal to noise ratio
- ❖ Dominant contribution from A_{T10}



$$\tilde{\mathcal{E}}'_2 = \sqrt{\frac{E_f(E_f + E_i)}{2}} z A_{T1}$$

- ❖ Zero when $\xi = 0$

Light-Cone GPDs



- ❖ Good signal to noise ratio
- ❖ Dense range of $-t$ accessible!
- ❖ $-t = 2.29 \text{ GeV}^2$ not reliable
- ❖ Free in calculations

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Acknowledgements

- ❖ U.S. Department of Energy, Office of Nuclear Physics, Early Career Award under Grant No. DE-SC0020405
- ❖ This research used resources of the National Energy Research Scientific Computing Center, a DOE Office of Science User Facility supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231 using NERSC award ALCC-ERCAP0025953.
- ❖ PLGrid Infrastructure by Prometheus in Cracow
- ❖ Poznan Supercomputing and Networking Center by Eagle
- ❖ Interdisciplinary Centre for Mathematical and Computational Modeling of the Warsaw University by Okeanos
- ❖ Academic Computer Center in Gdańsk by Tryton

Thank You!!!