

Electromagnetic Form Factors of the Nucleon with Dispersively Improved Chiral Effective Field Theory

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Universidad
de Alcalá

Works done in collaboration with C. Weiss, D. Higinbotham,
and F. Gil-Domínguez

Introduction

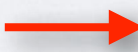
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- Electromagnetic FF:
 - Encodes the response of the nucleon under electromagnetic probes.
 - Related to the “Proton Radius Puzzle”.

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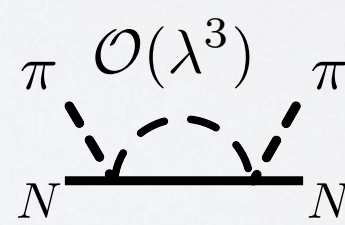
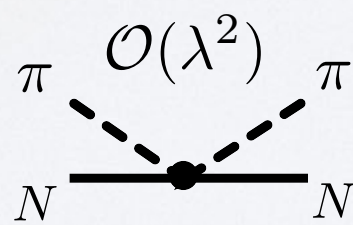
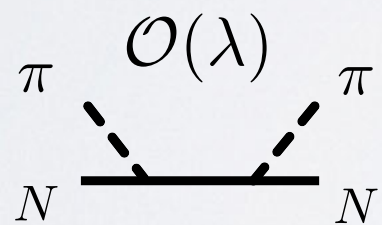
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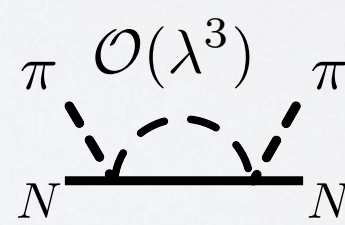
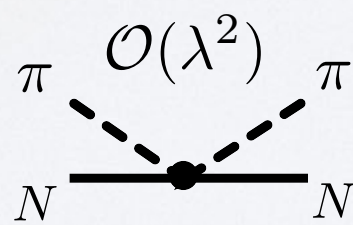
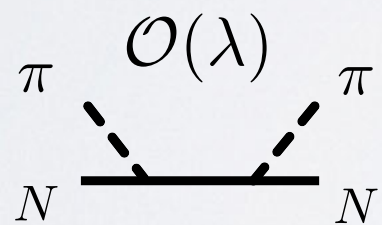
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- Systematic way of calculating theoretical uncertainties.



$$\lambda = \frac{M_\pi}{\Lambda_{\chi SB}}, \frac{p}{\Lambda_{\chi SB}}$$

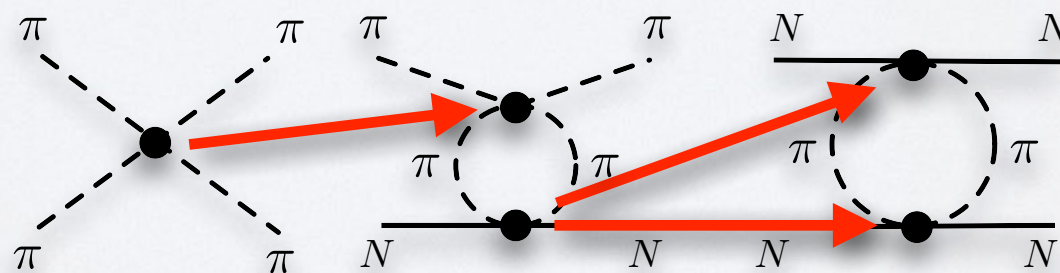
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- Unified approach



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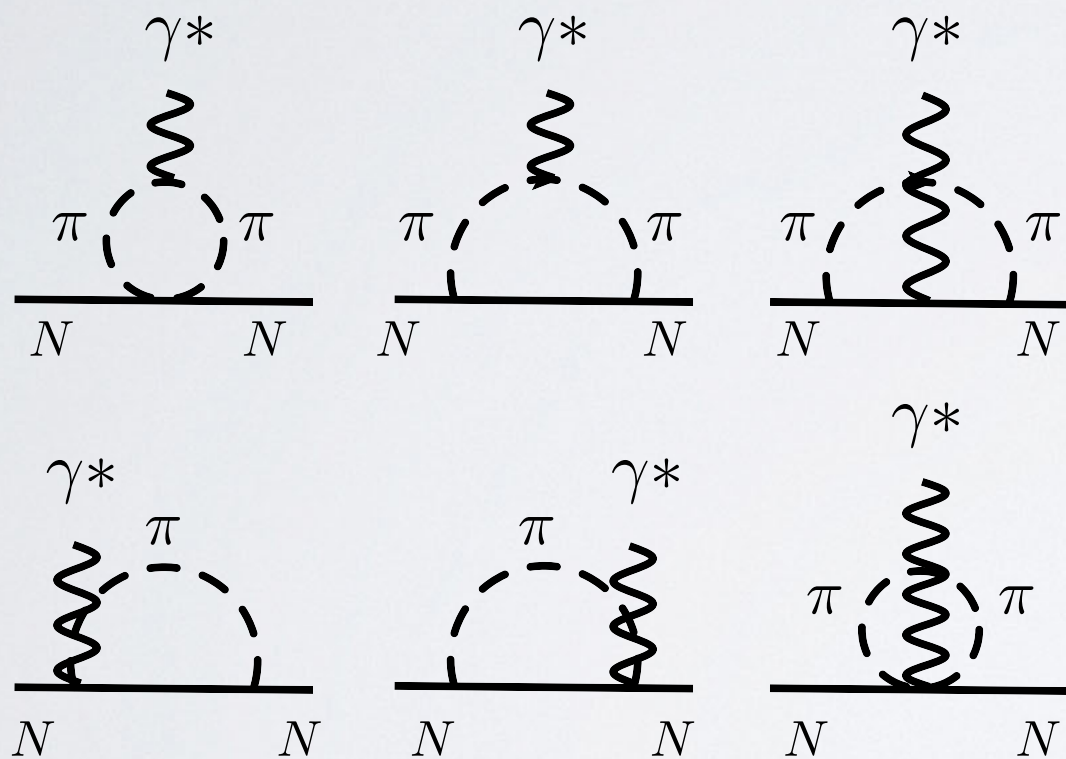
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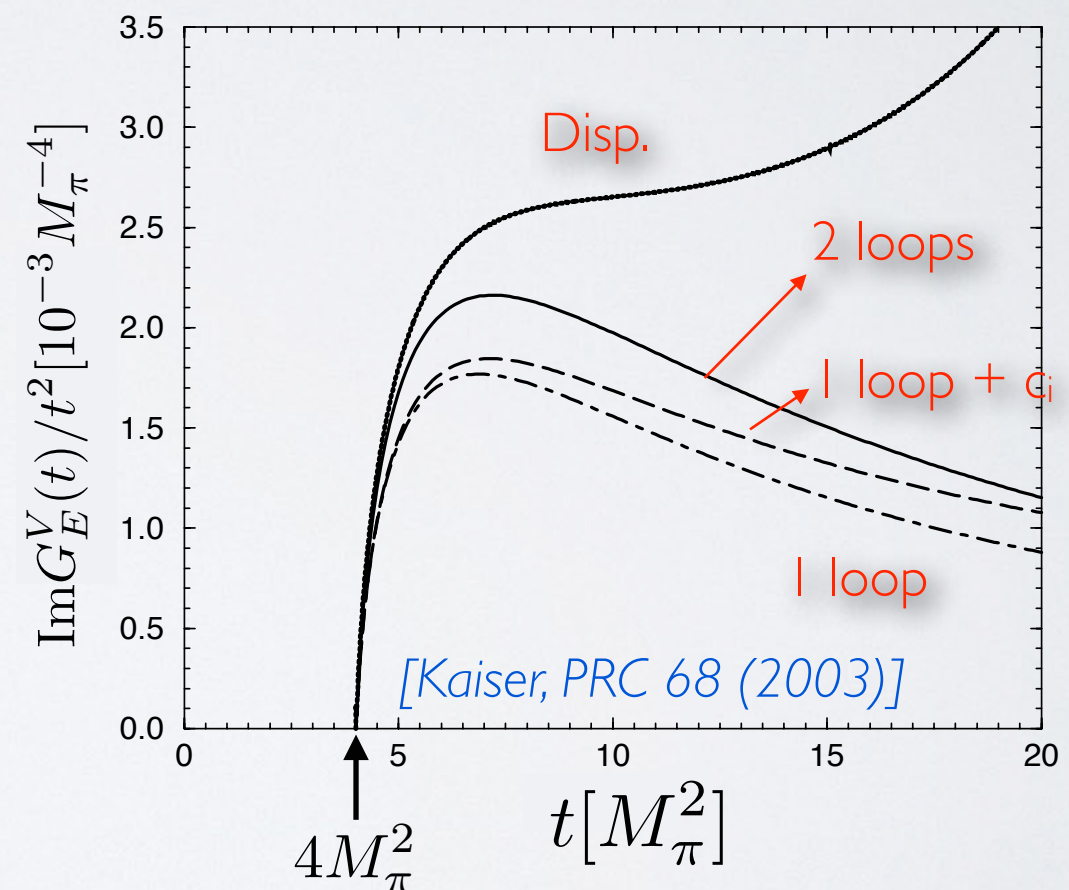
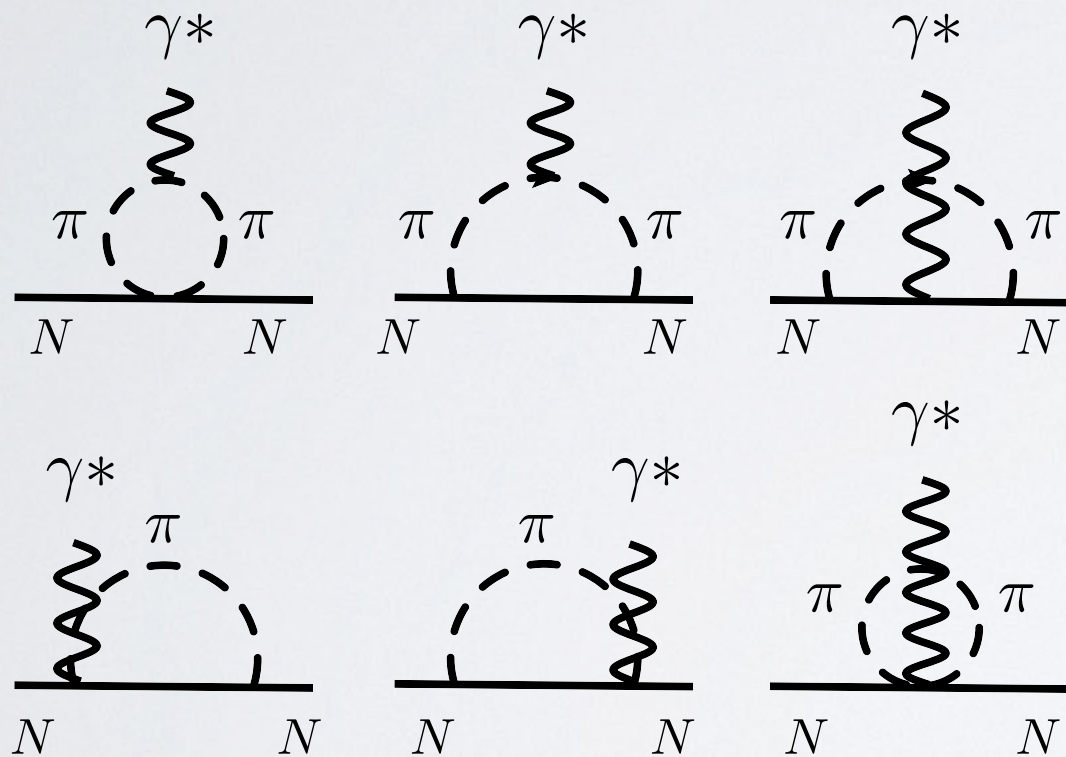
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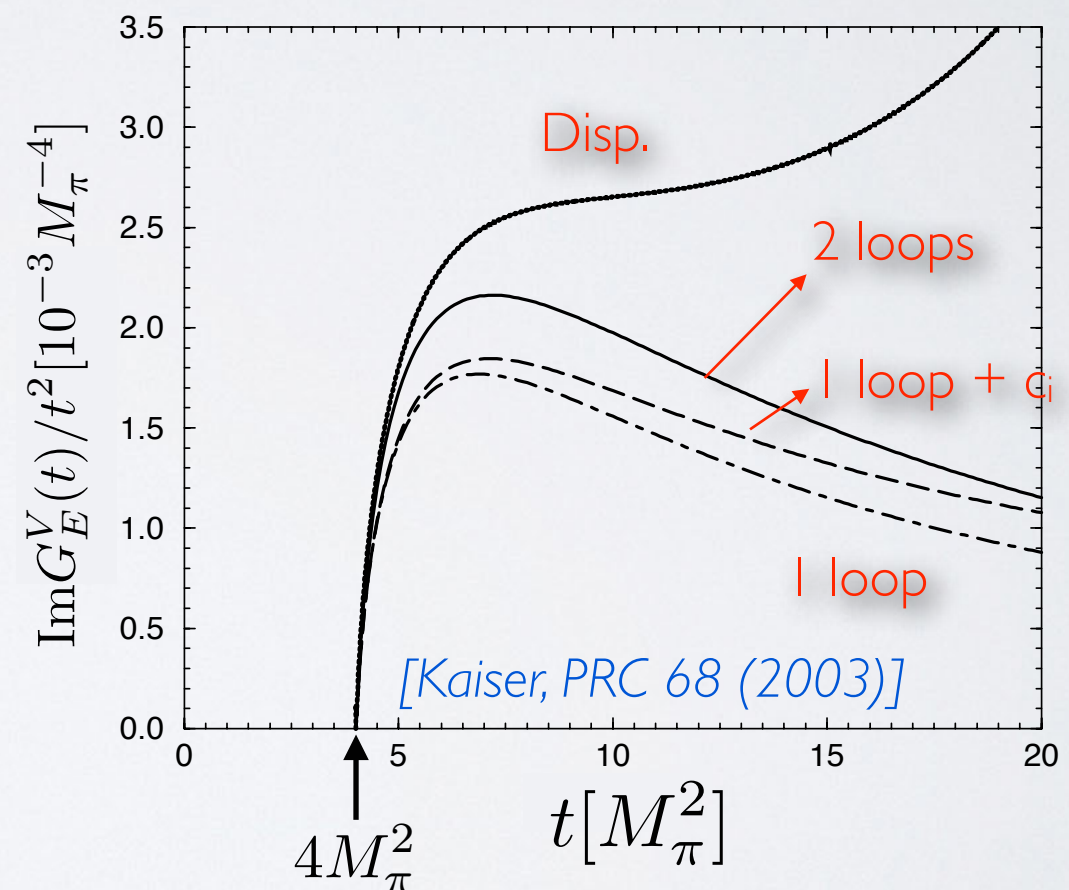
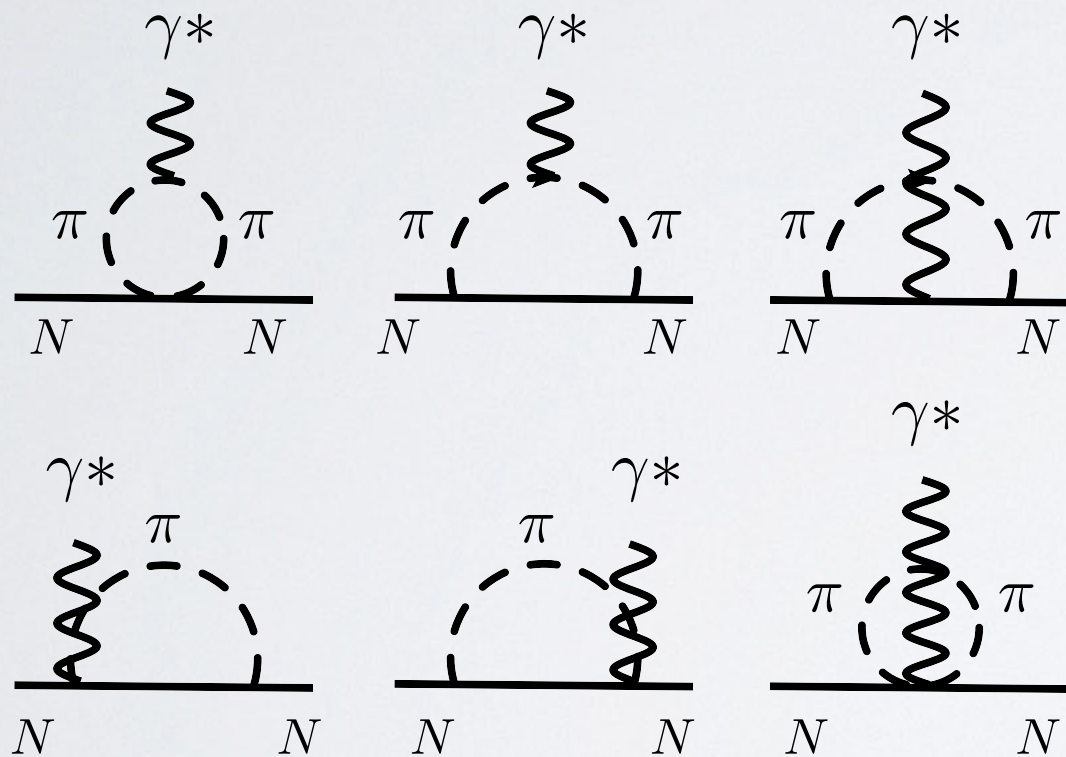
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- Higher order calculations become necessary $\longrightarrow$ Unpractical

Form factors and their analytic structure

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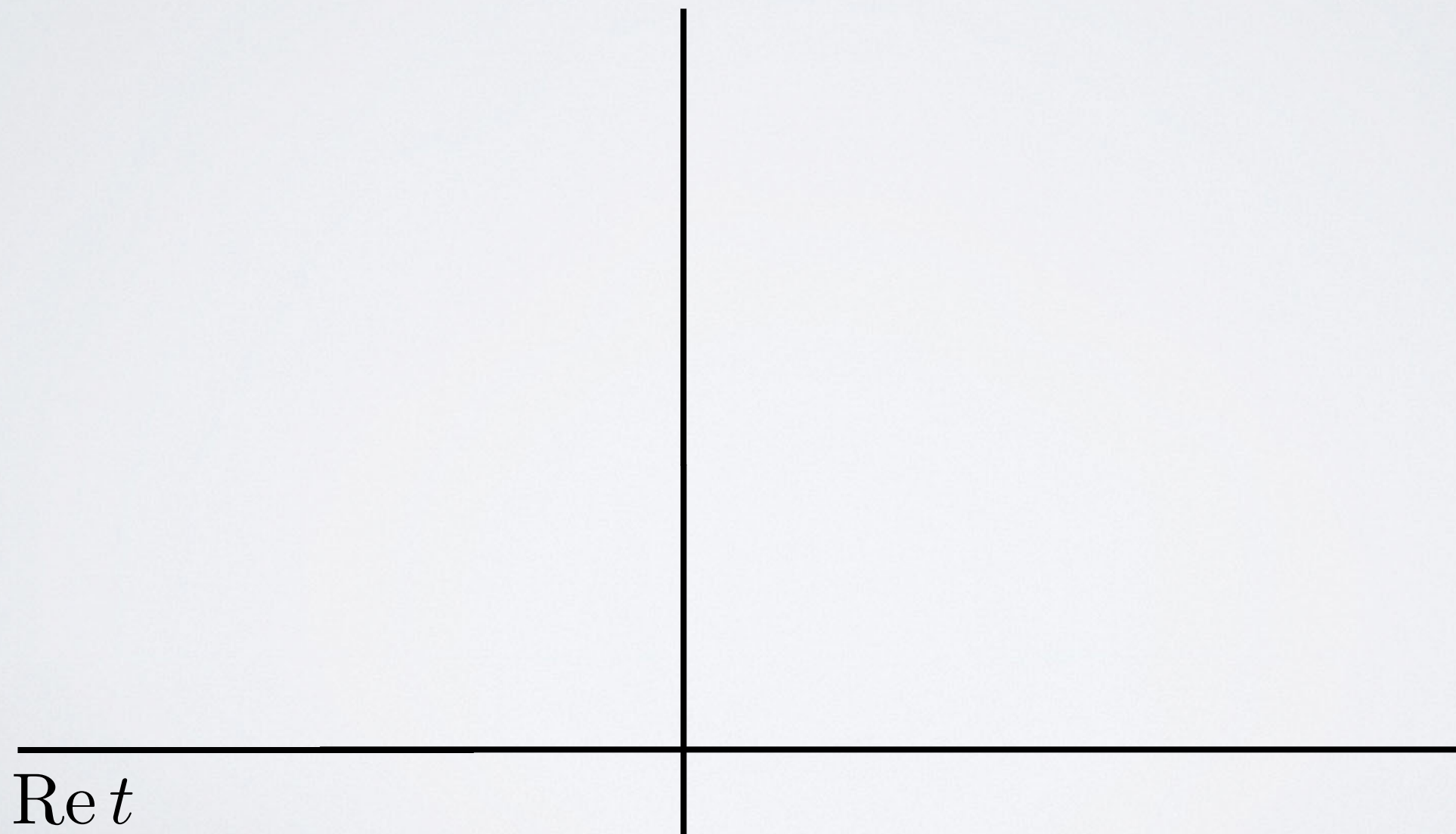
$$\langle N(p', s') | J_\mu(0) | N(p, s) \rangle = \bar{u}(p', s') \left[\gamma_\mu F_1(t) + \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} F_2(t) \right] u(p, s) \quad J_\mu(x) \equiv \sum_{q=u,d,\dots} e_q \bar{q}(x) \gamma_\mu q(x)$$

$$G_E(t) = F_1(t) + \frac{t}{4m_N^2} F_2(t) \quad G_M(t) = F_1(t) + F_2(t) \quad G_{E,M}^{V,S} \equiv \frac{1}{2} (G_{E,M}^p \mp G_{E,M}^n)$$

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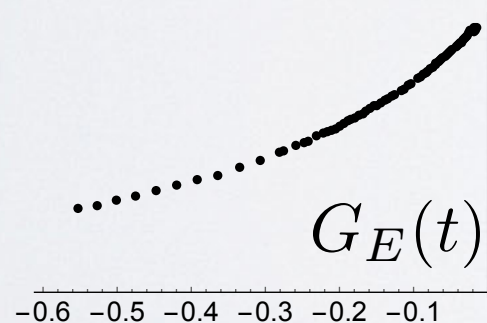
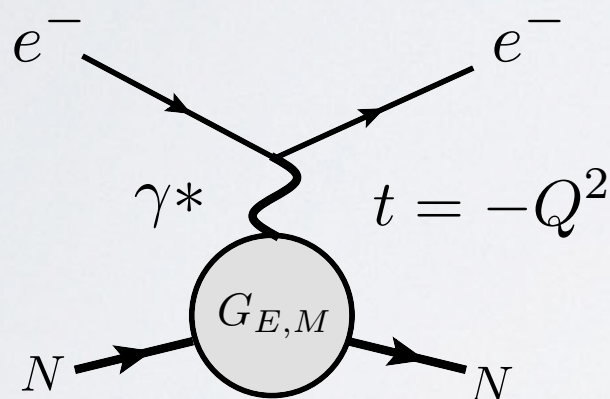


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Space-like region ($t < 0$)



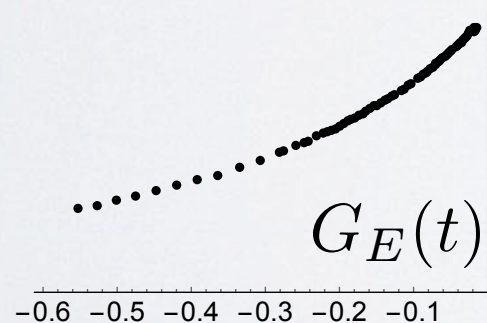
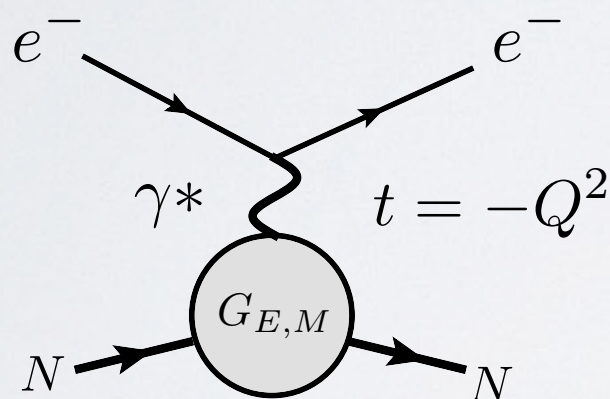
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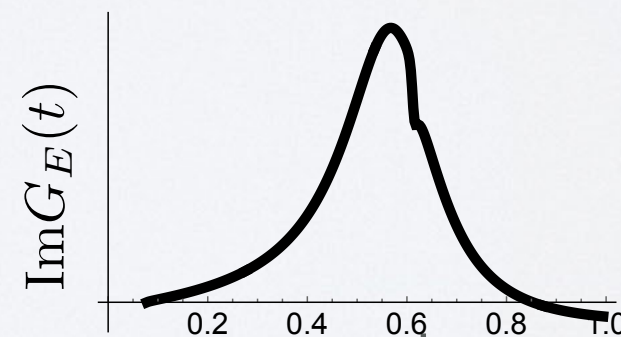
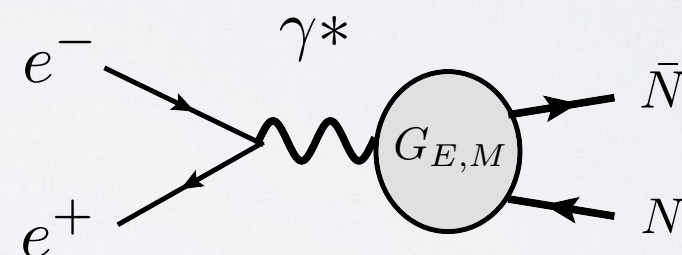
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Time-like region ($t > 0$)



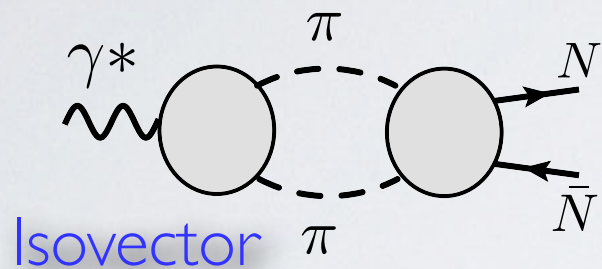
$4M_\pi^2 \quad 9M_\pi^2 \quad 16M_\pi^2$

- From unitarity + analyticity

$$\text{Im}G_{E,M} \propto \sum_h \int d\Pi_h M(\gamma^* \rightarrow h) M(h \rightarrow \bar{N}N)$$

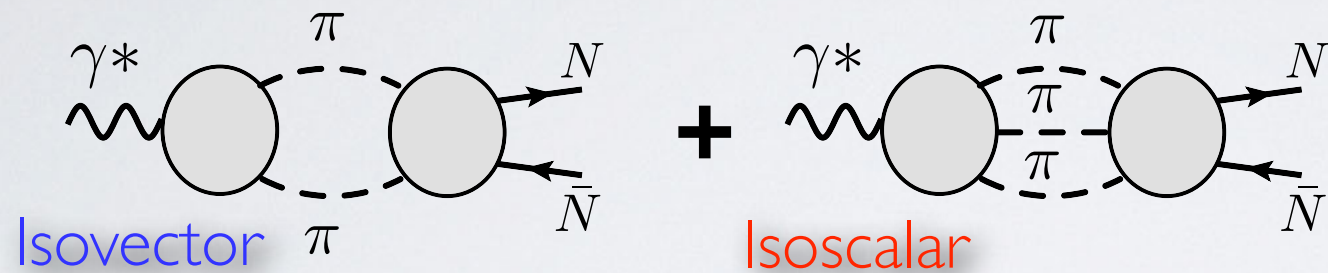
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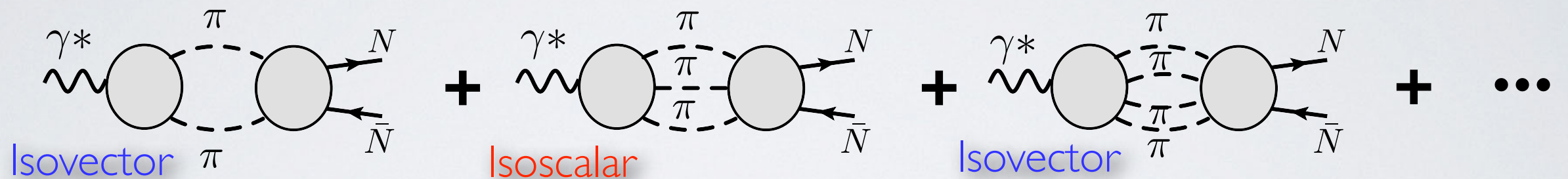
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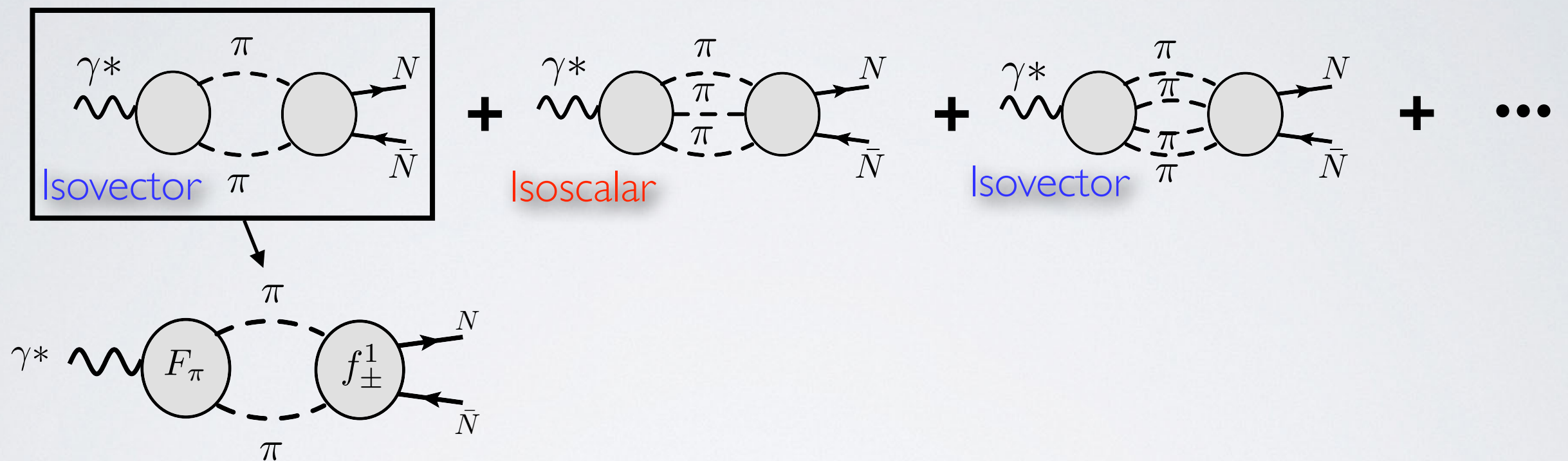
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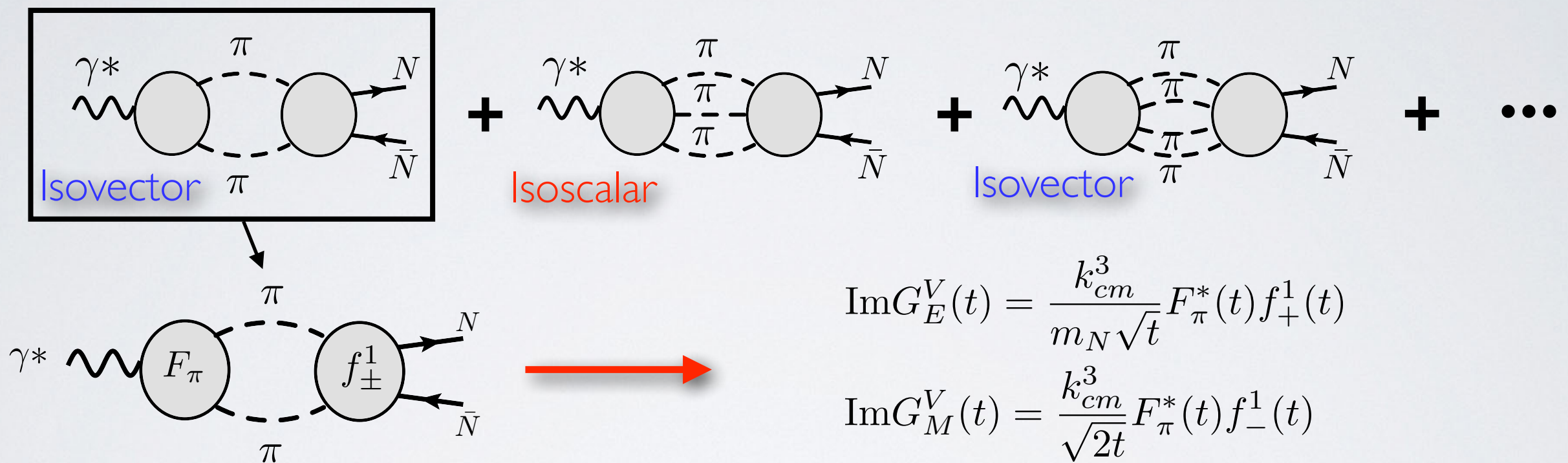
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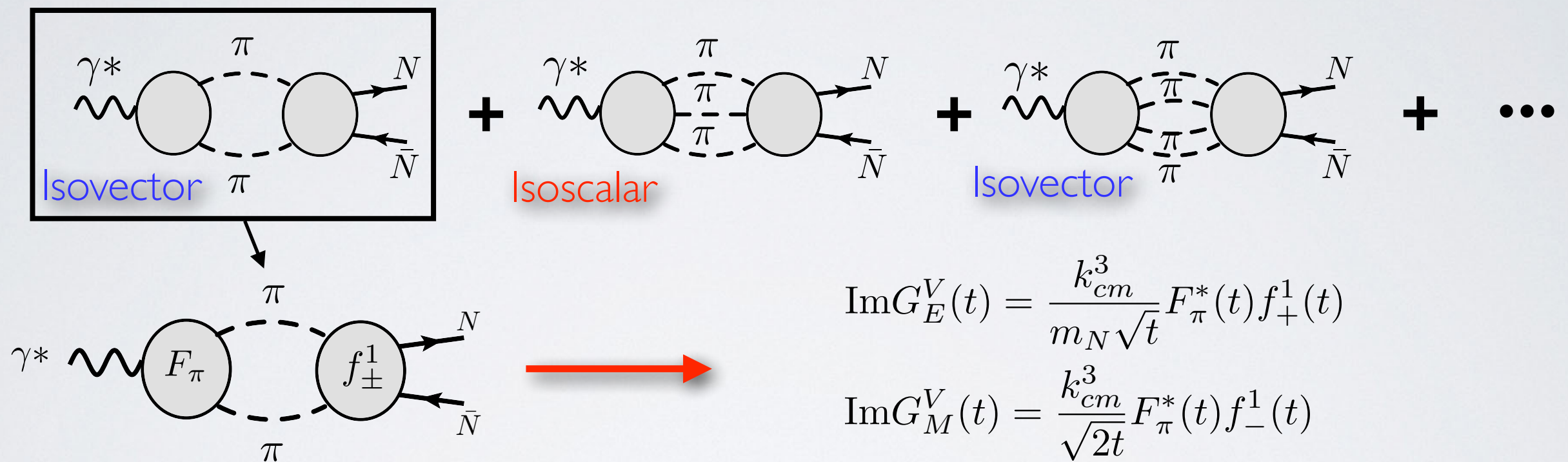


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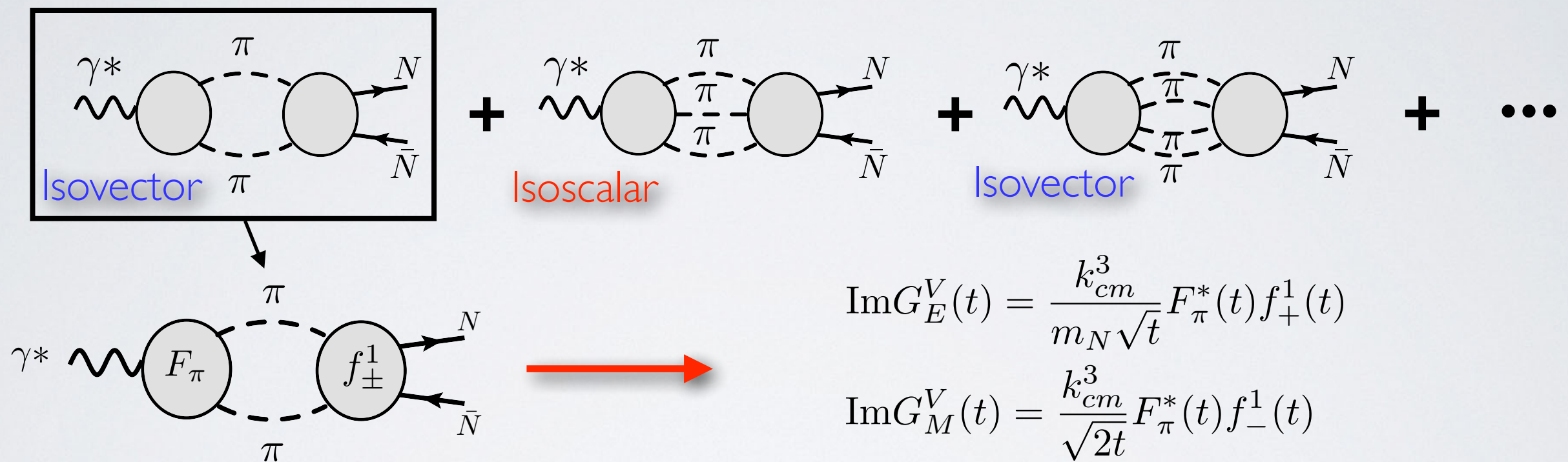
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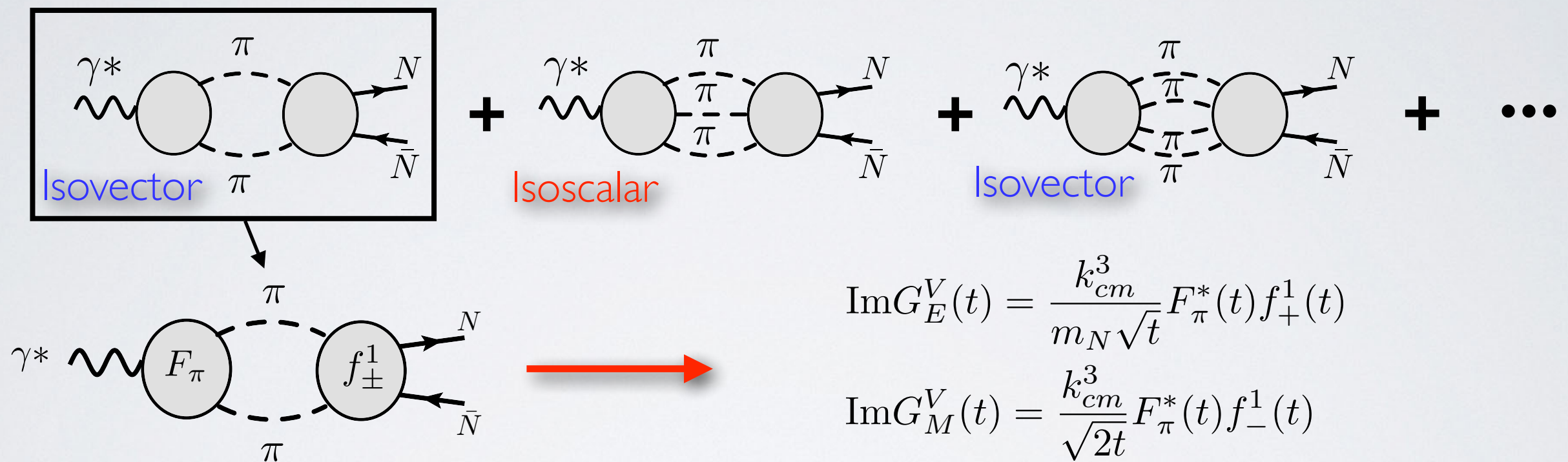
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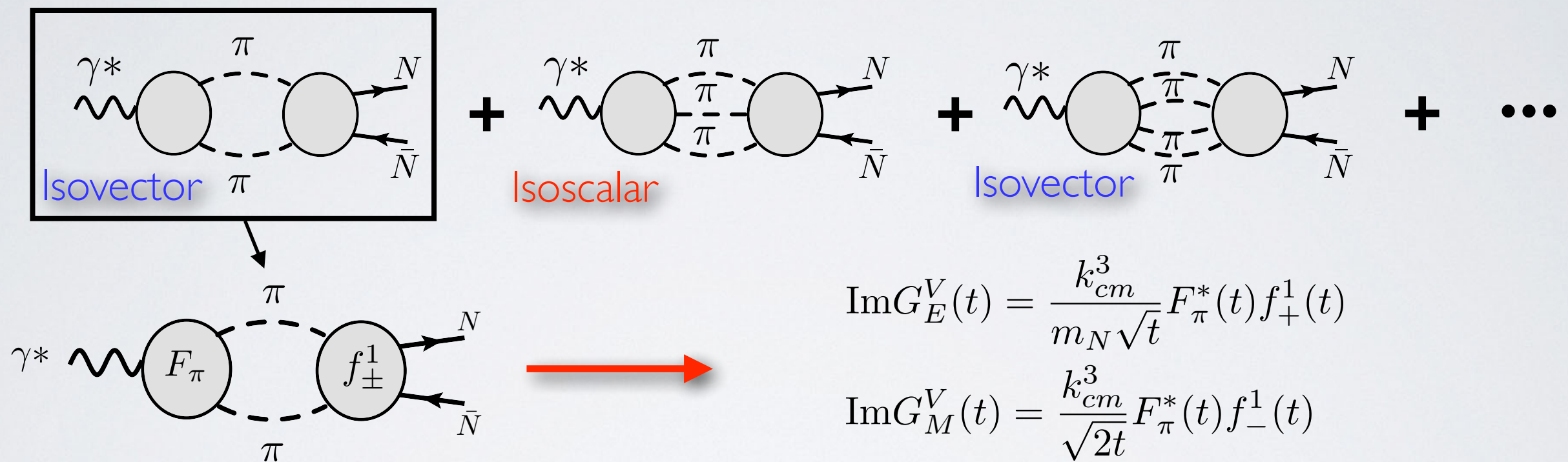
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DIχEFT

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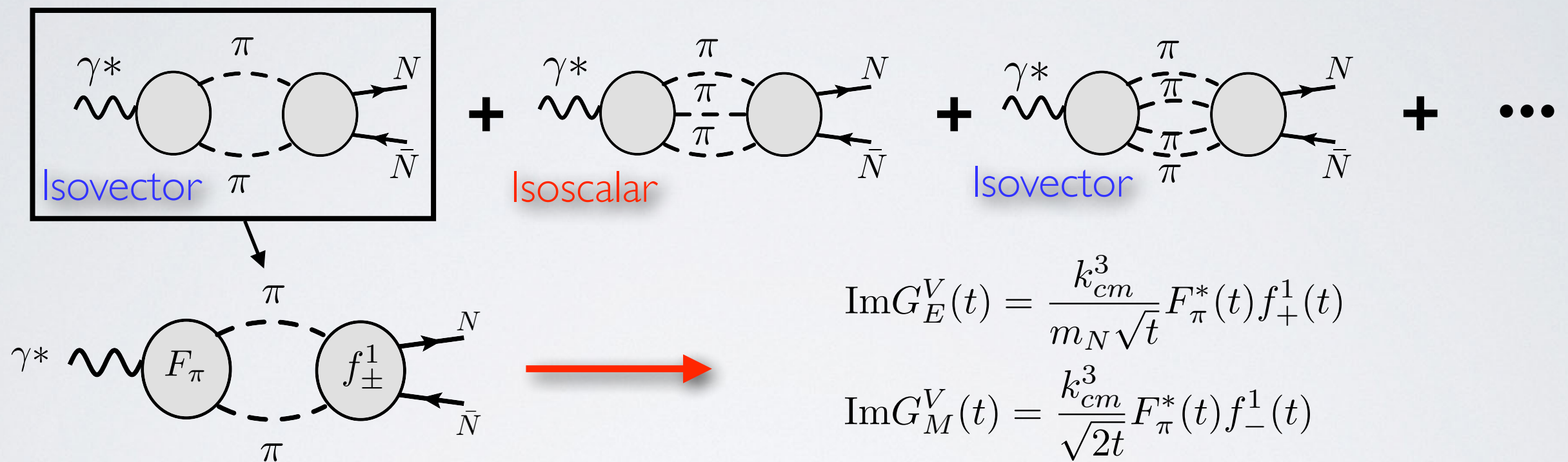
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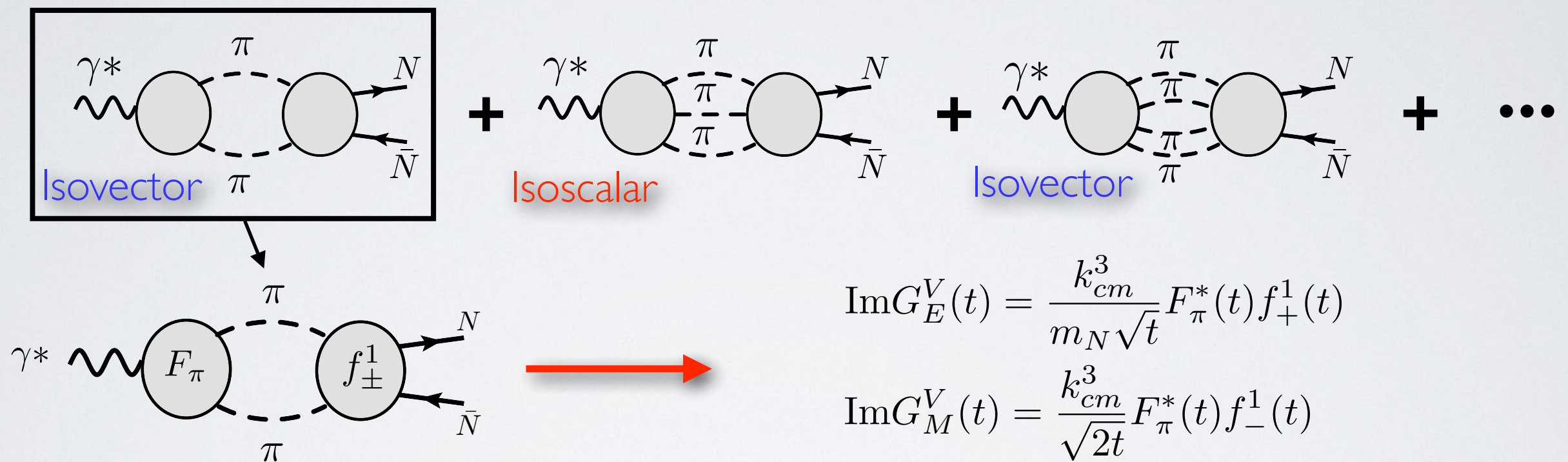
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- For $t < 1 \text{ GeV}^2$:
 - $J_{\pm}^1$ is calculated in ChEFT and used to reconstruct $\text{Im}G_{E,M}^V$.
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 - We parametrize the contribution from this region with effective poles

$$\text{Im}F_1^V = \pi a_1^{(V,0)} \delta(t - t_1^{(V,0)}) + \pi a_1^{(V,1)} \delta'(t - t_1^{(V,1)})$$

$$\text{Im}F_2^V = \pi a_2^{(V,0)} \delta(t - t_2^{(V,0)}) + \pi a_2^{(V,1)} \delta'(t - t_2^{(V,1)}) + \pi a_2^{(V,2)} \delta''(t - t_2^{(V,2)})$$

With nominal values: $t_1^{(V,0)} = t_1^{(V,1)} = 3M_\rho^2 = 1.8 \text{ GeV}^2$ } based on $e^+e^- \rightarrow \text{hadrons}$ and
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- We fix the strength of the deltas ($a_i^{(V,j)}$) by imposing:

$$\frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \frac{\text{Im}F_1^V}{t} = Q^V \quad \frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \frac{\text{Im}F_1^V}{t^2} = \frac{1}{6} \langle r^2 \rangle_1^V \quad \frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \text{Im}F_1^V = 0$$

$$\frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \frac{\text{Im}F_2^V}{t} = \kappa^V \quad \frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \frac{\text{Im}F_2^V}{t^2} = \frac{1}{6} \langle r^2 \rangle_2^V \quad \frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \text{Im}F_2^V = 0 \quad \frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt t \text{Im}F_2^V = 0$$

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- We fix the couplings following the same method that we used in the isovector case:

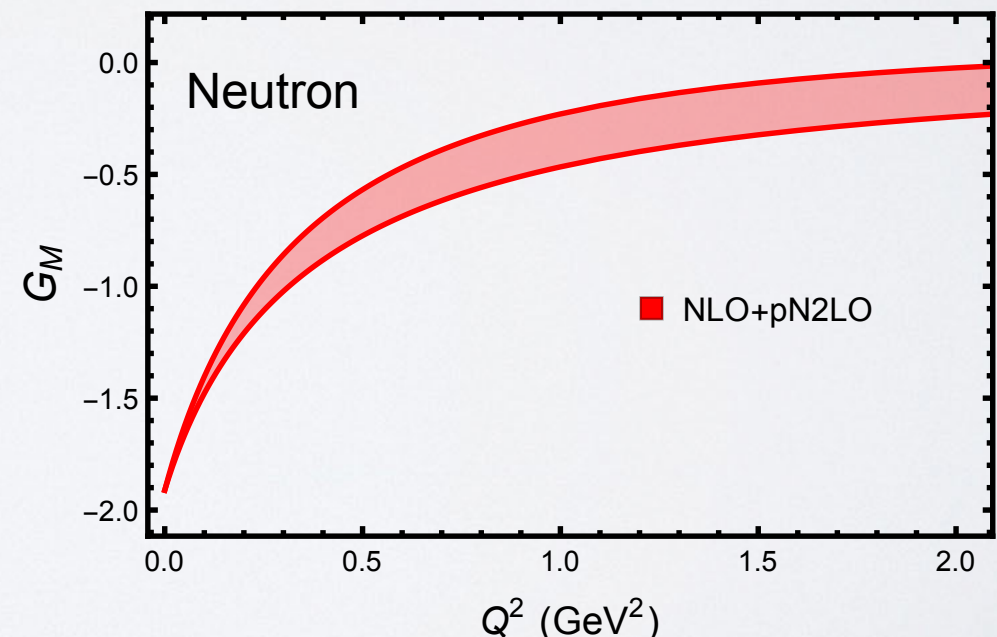
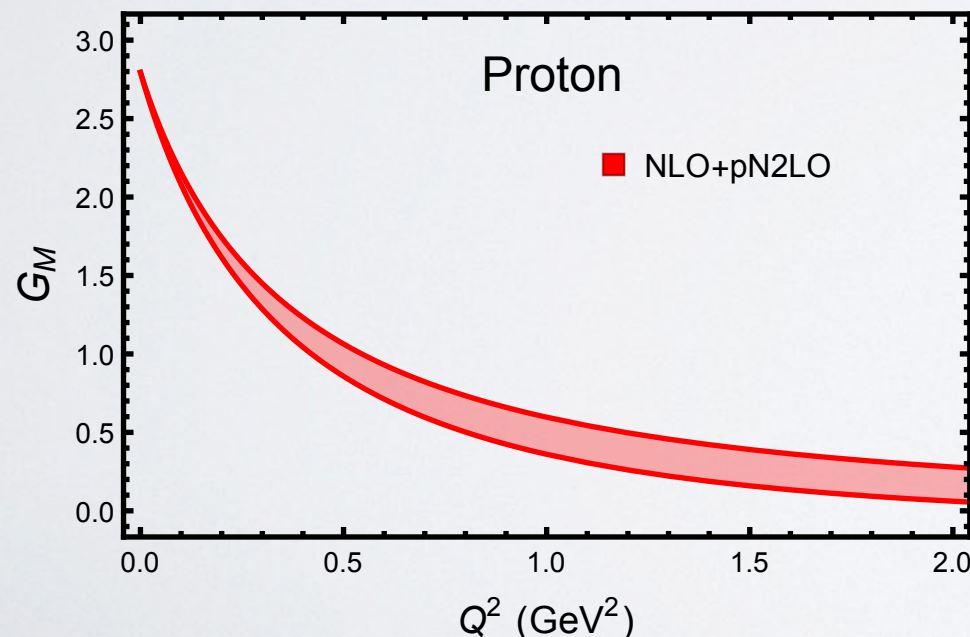
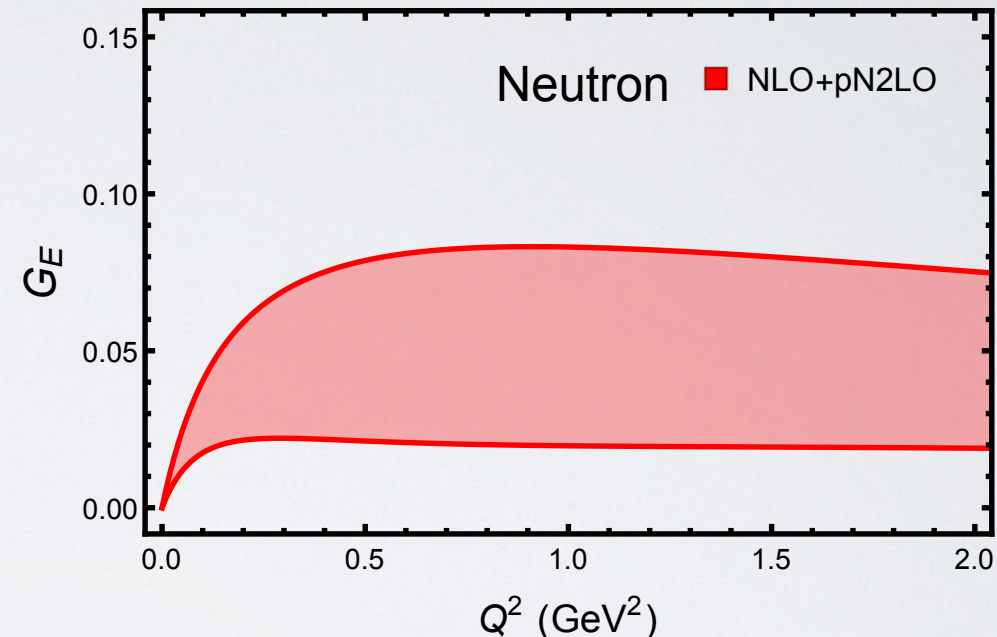
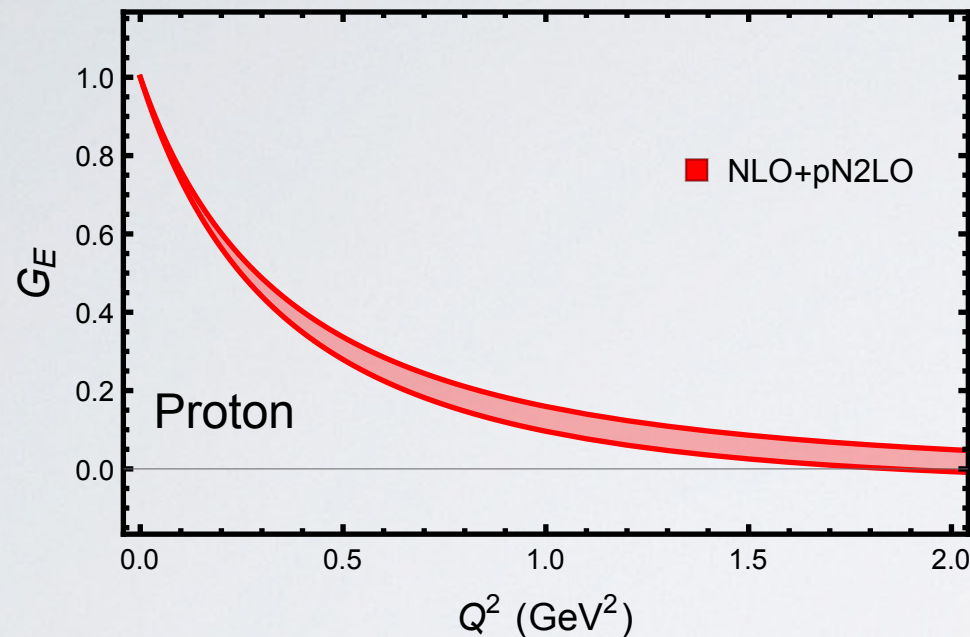
$$\frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \frac{\text{Im}F_1^S}{t} = Q^S \quad \frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \frac{\text{Im}F_1^S}{t^2} = \frac{1}{6} \langle r^2 \rangle_1^S \quad \frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \text{Im}F_1^S = 0$$

$$\frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \frac{\text{Im}F_2^S}{t} = \kappa^S \quad \frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \frac{\text{Im}F_2^S}{t^2} = \frac{1}{6} \langle r^2 \rangle_2^S \quad \frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt \text{Im}F_2^S = 0 \quad \frac{1}{\pi} \int_{t_{\text{thr}}}^{\infty} dt t \text{Im}F_2^S = 0$$

Results

- Reconstructing the form factors with $G_{E,M}^{p,n}(t) = \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} dt' \frac{\text{Im} G_{E,M}^{p,n}(t')}{t' - t - i0^+}$

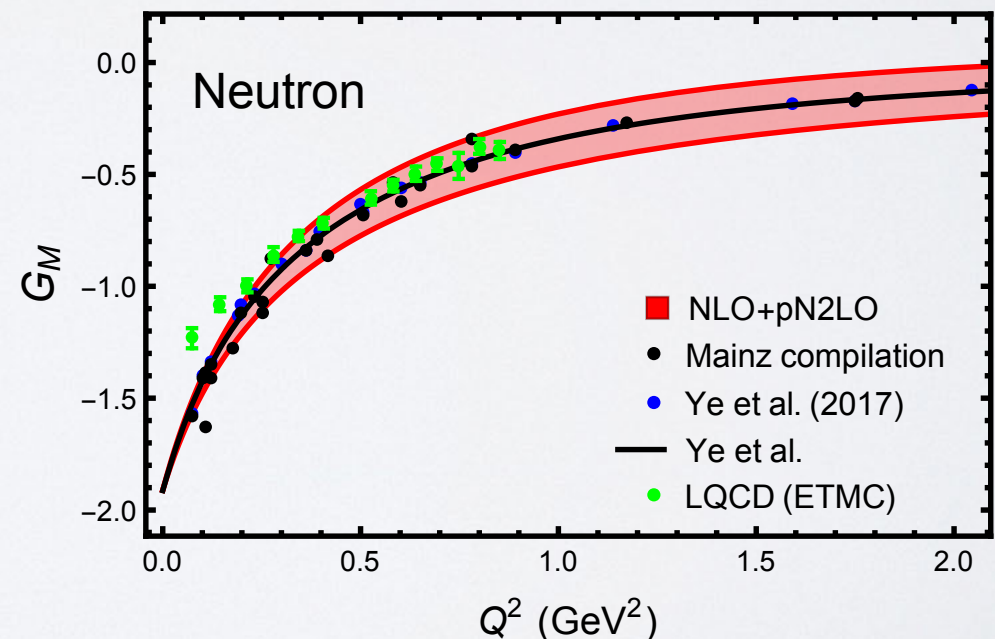
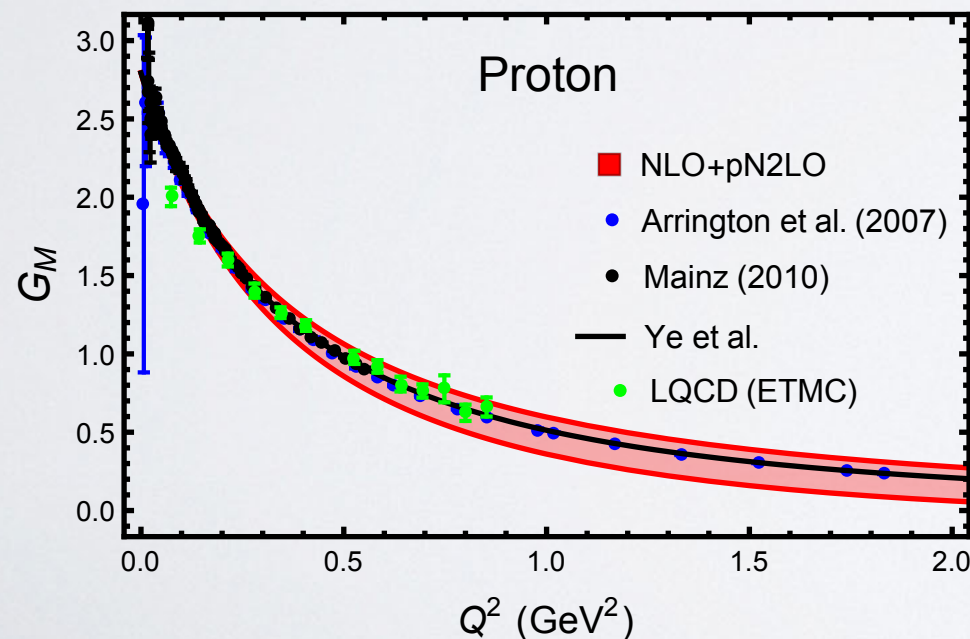
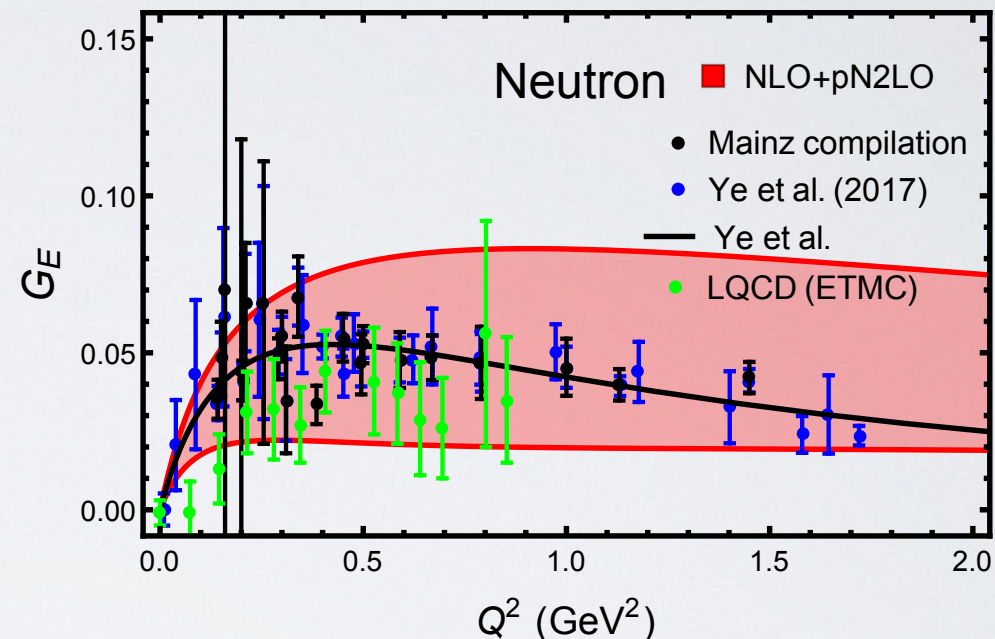
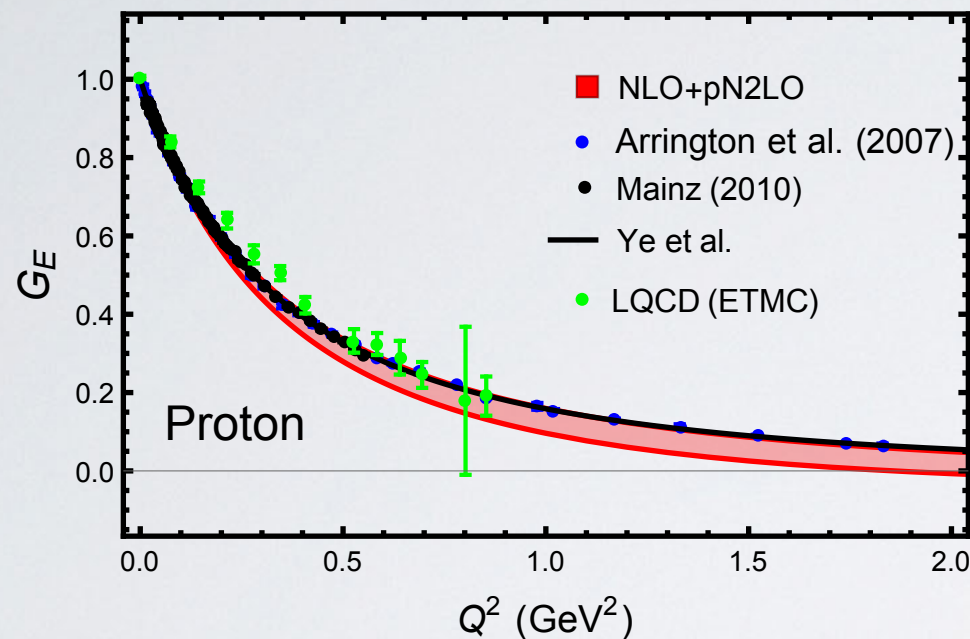
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[J. M. Alarcón, C. Weiss, *Phys. Lett. B* 784 (2018)]

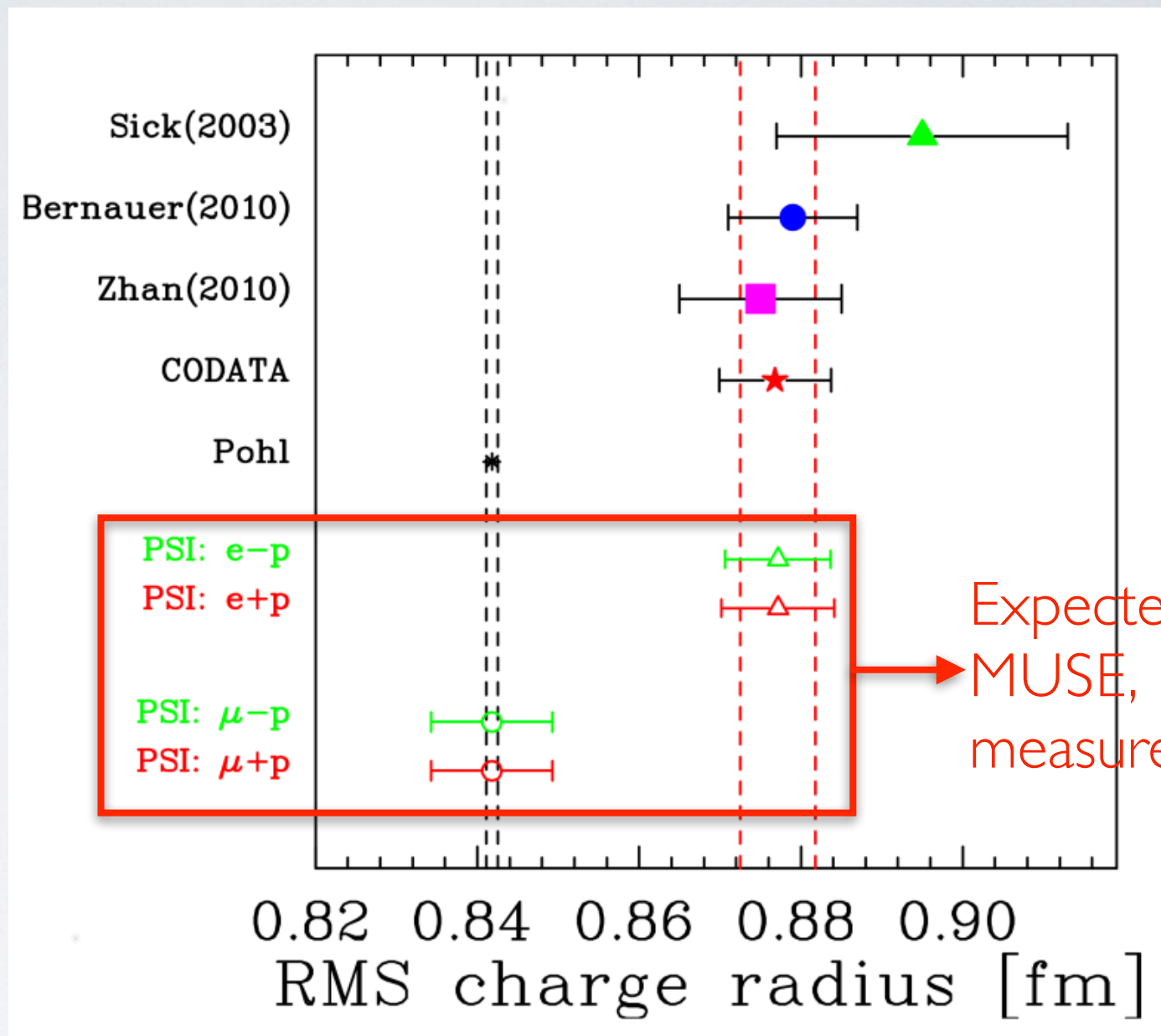
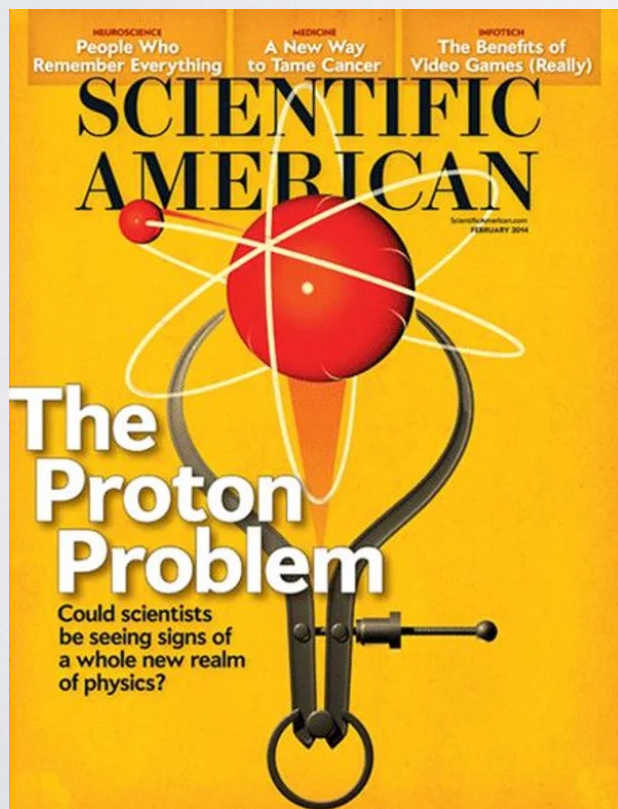
DI χ EFT

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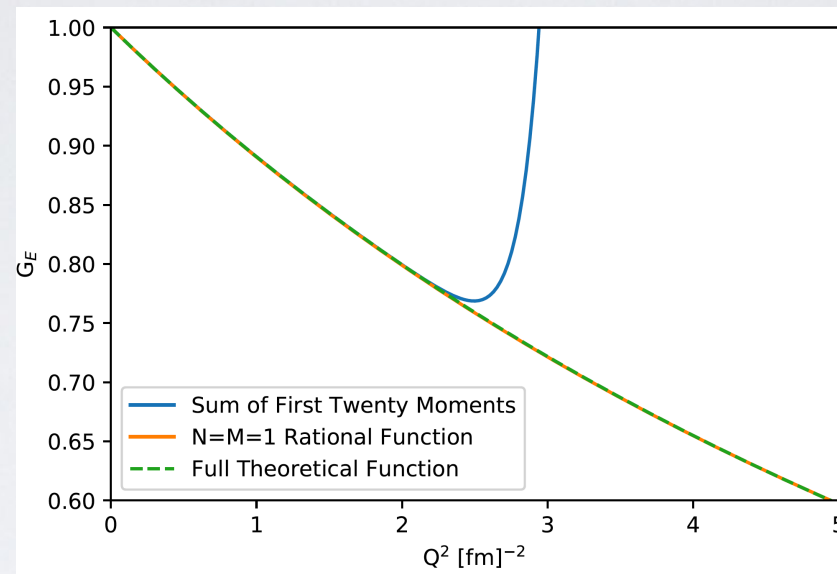
[J. M. Alarcón, C. Weiss, *Phys. Lett. B* 784 (2018)]

$DI\chi EFT$



[E.J. Downie, EPJ Web Conf. 113 (2016)]

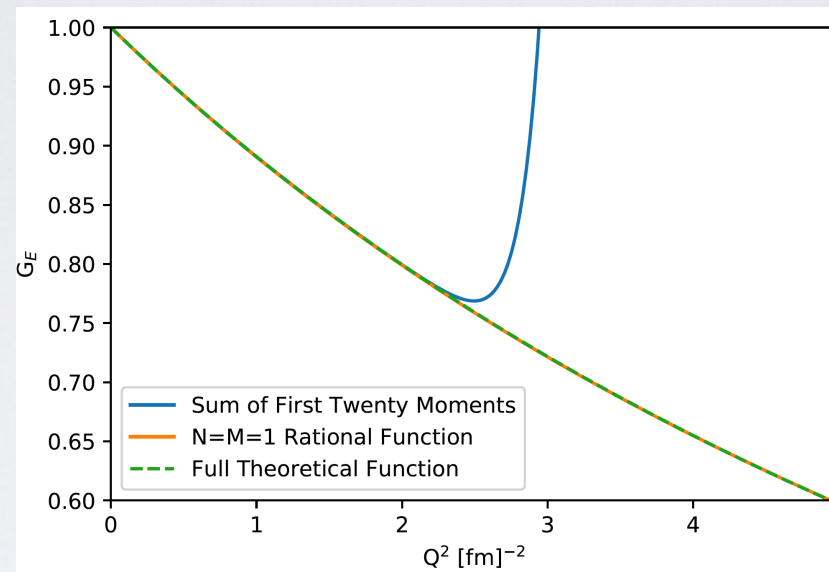
- Common polynomial fits have a small convergence radius.



[Figure courtesy of D. Higinbotham]

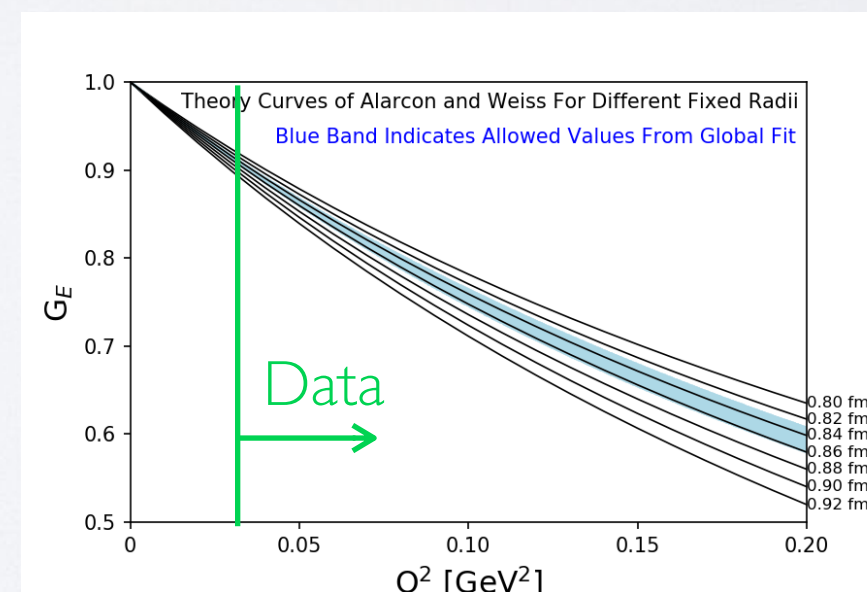
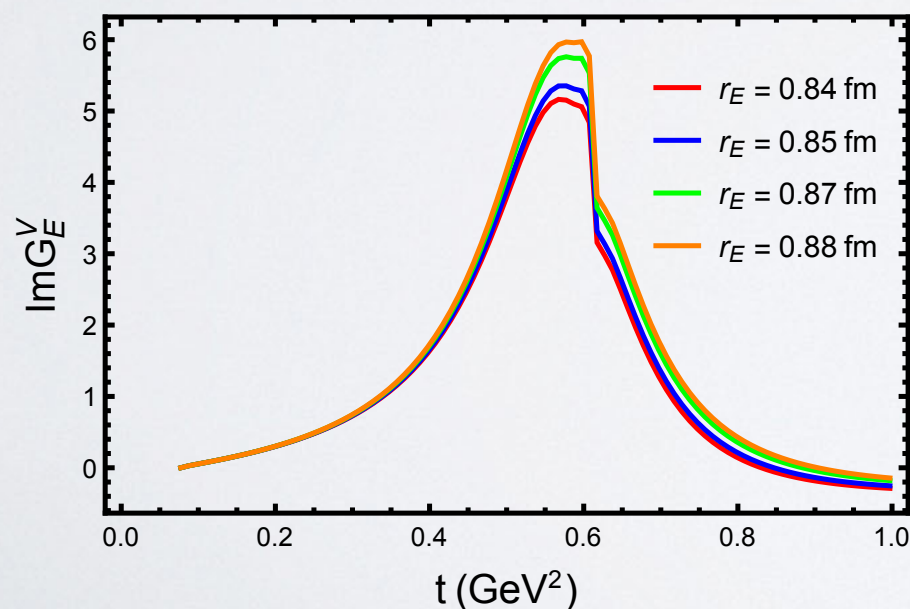
DI χ EFT

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[Figure courtesy of D. Higinbotham]

- DIChEFT correlates the radius and the Q^2 dependence of the FF.



- We used ep scattering data (subtracting TPE contributions) to determine r_E^p and r_M^p .

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \frac{\epsilon G_E^2 + \tau G_M^2}{\epsilon(1 + \tau)}$$

$$\tau = \frac{Q^2}{4m^2}$$

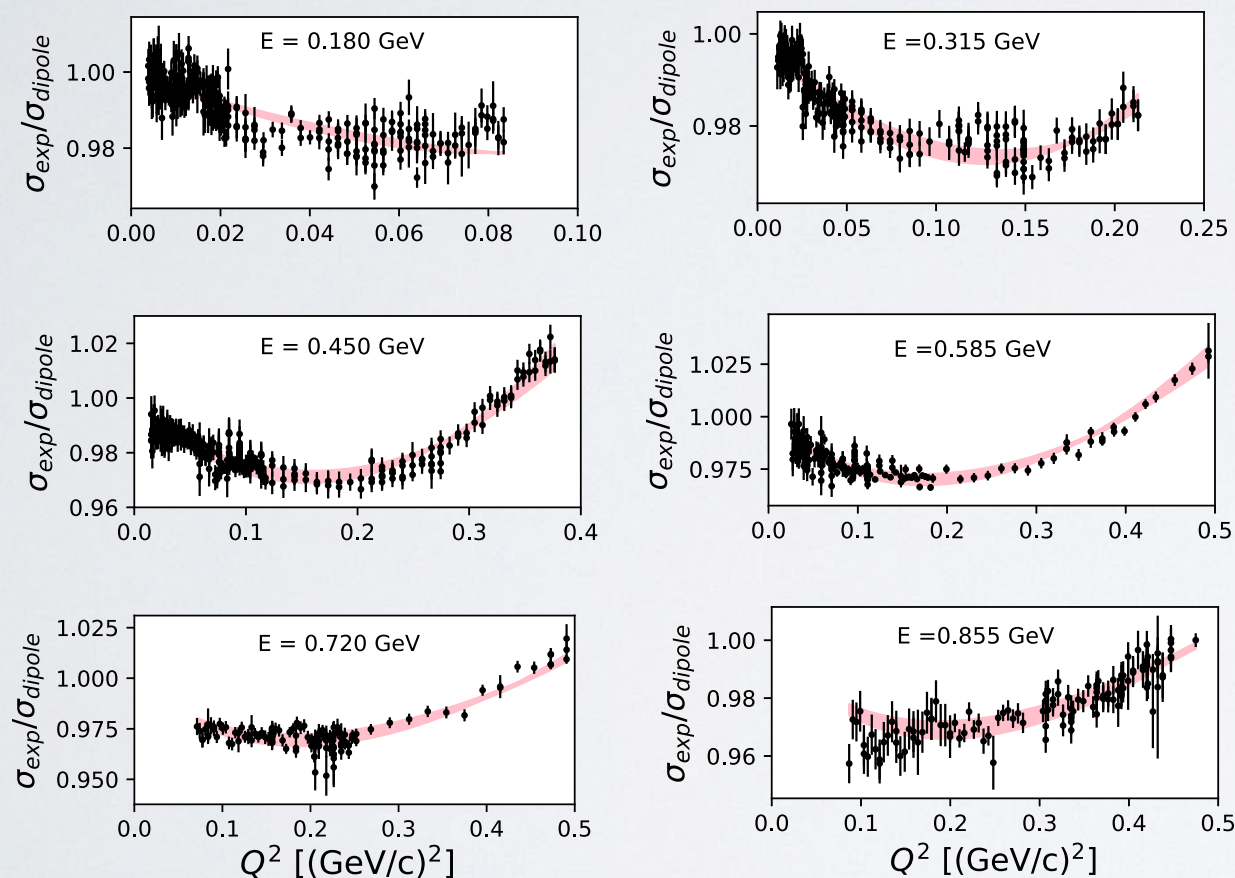
$$\epsilon = \frac{1 - \frac{Q^2}{2mE} \left(1 + \frac{m}{2E}\right)}{1 - \frac{Q^2}{2mE} \left(1 - \frac{m}{2E}\right) + \frac{1}{2} \left(\frac{Q^2}{2mE}\right)^2}$$

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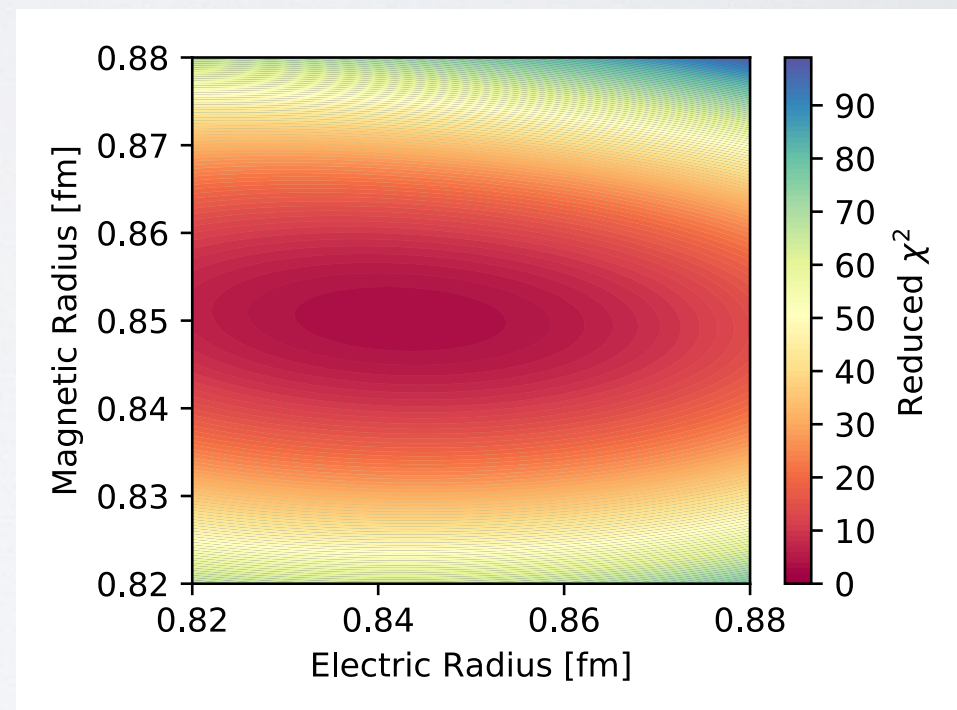
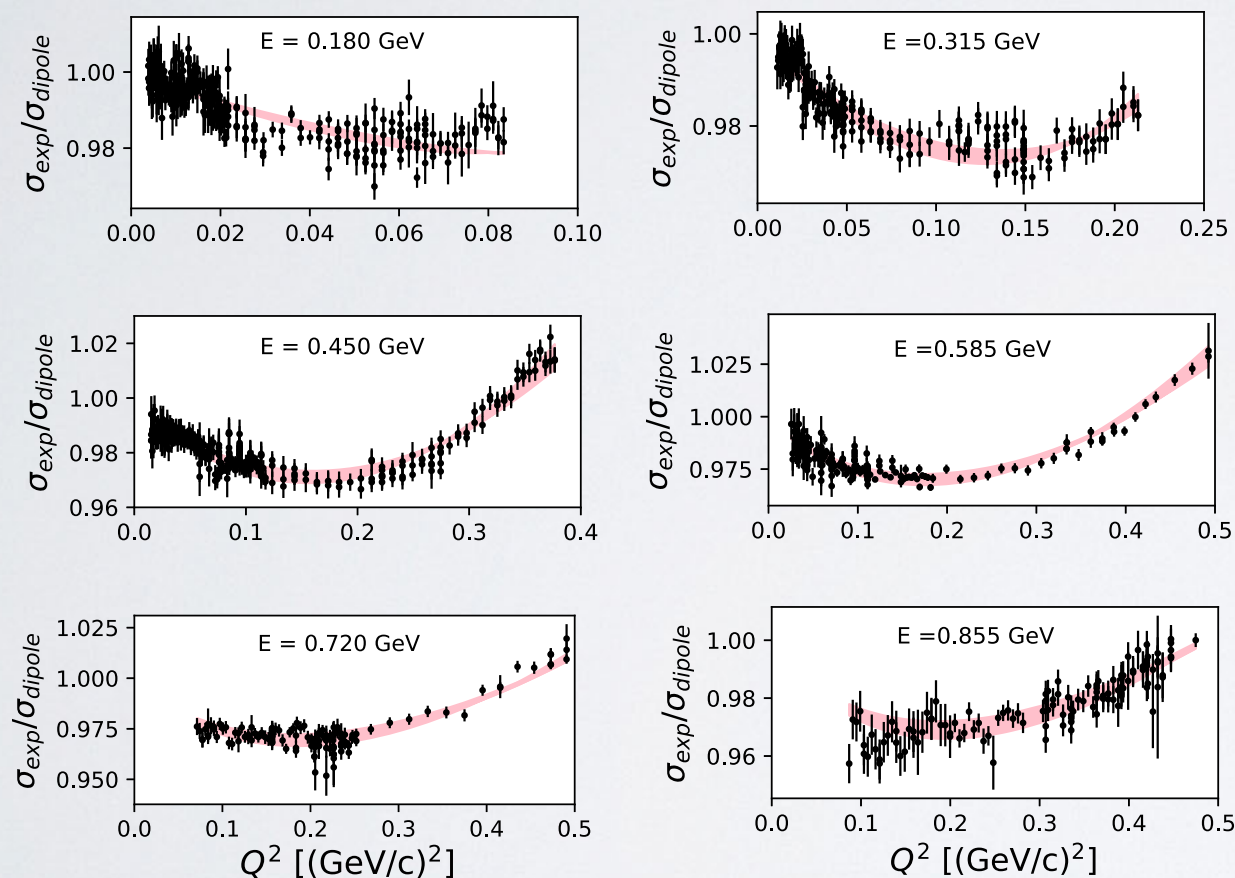
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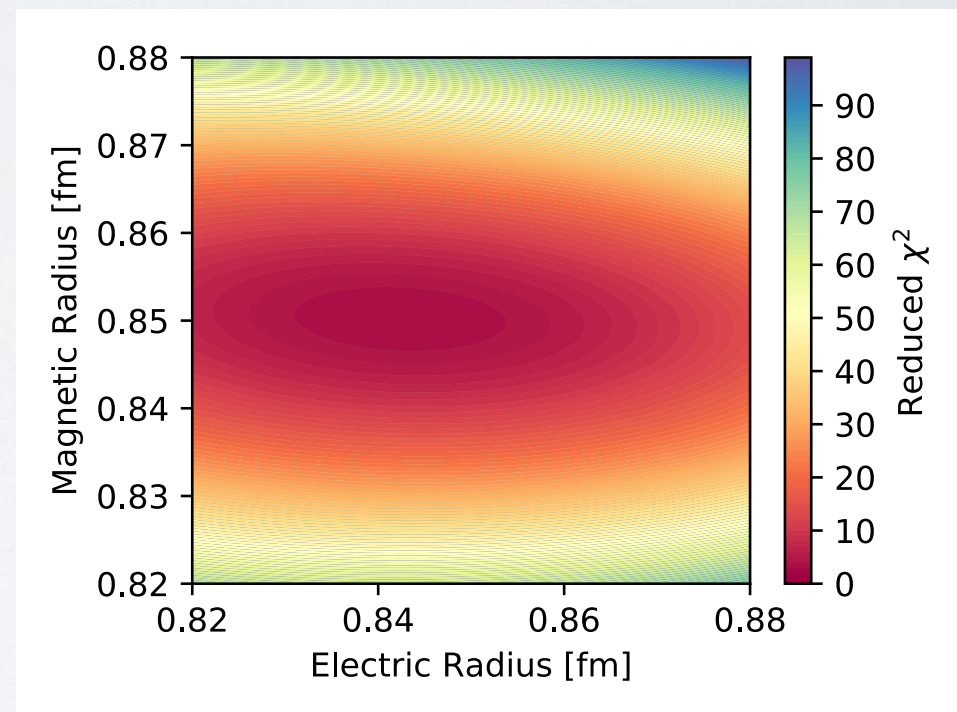
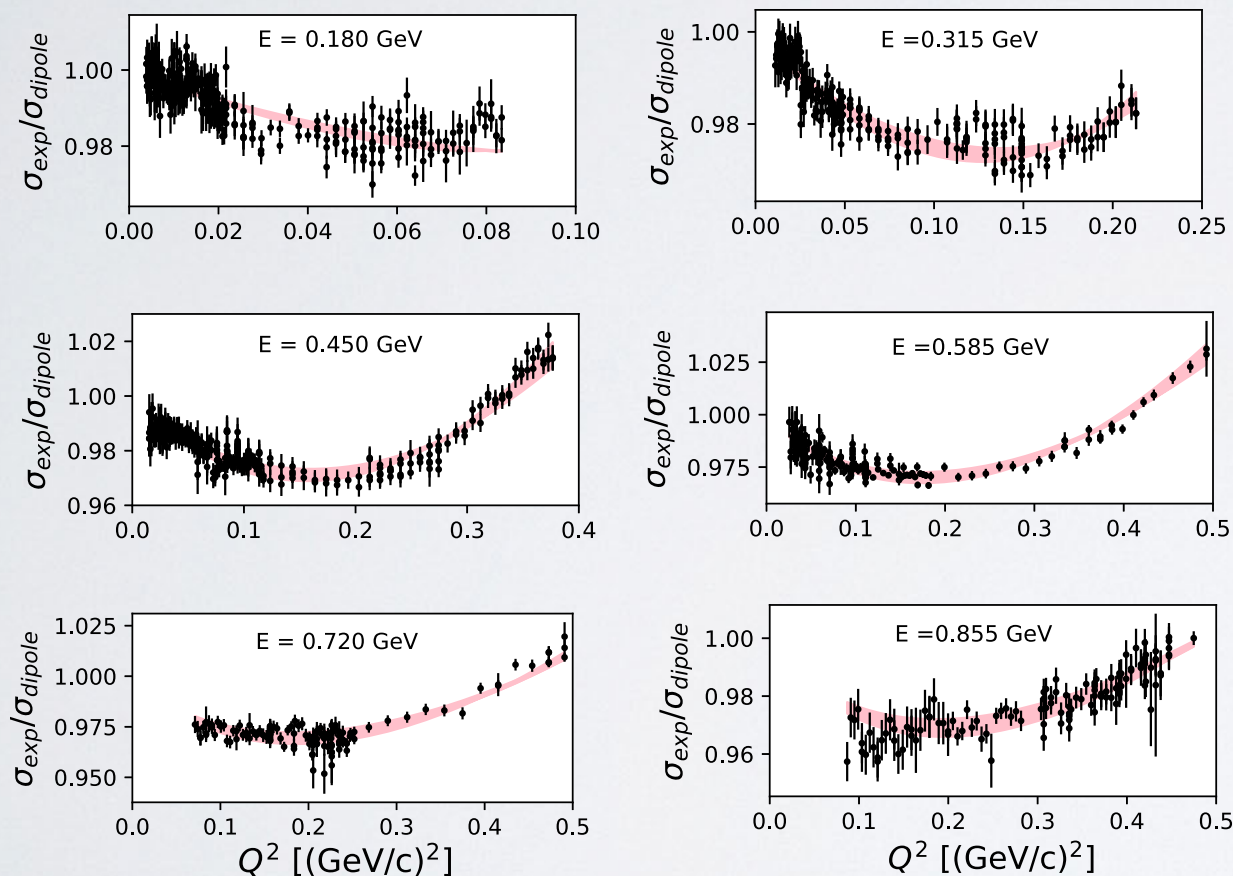
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$$r_E^p = 0.842 \pm 0.002(\text{stat})_{-0.002}^{+0.005}(\text{th}) \text{ fm}$$

$$r_M^p = 0.850 \pm 0.001(\text{stat})_{-0.004}^{+0.009}(\text{th}) \text{ fm}$$

[J. M. Alarcón, D. W. Higinbotham, and C. Weiss, *Phys. Rev. C* 102 (2020)]

- The results for the form factors can be used to estimate the cross sections measured at MUSE.

$$\frac{d\sigma_{1\gamma}}{dQ^2} = \frac{\pi\alpha^2}{2M^2|\mathbf{k}|^2} \frac{(\epsilon/\tau_P) G_E^2 + G_M^2}{1 - \epsilon_T} \quad \frac{d\sigma}{dQ^2} \approx \frac{d\sigma_{1\gamma}}{dQ^2} (1 + \delta_{2\gamma}) \quad \tau_P \equiv Q^2/(4M^2) \quad \epsilon_T = \frac{Q_{\max}^2 - Q^2}{Q_{\max}^2 - Q^2 + \frac{Q^2}{2s}(4M^2 + Q^2)} \quad \epsilon = \frac{Q_{\max}^2 - Q^2 + \frac{m^2}{s}(4M^2 + Q^2)}{Q_{\max}^2 - Q^2 + \frac{Q^2}{2s}(4M^2 + Q^2)}$$

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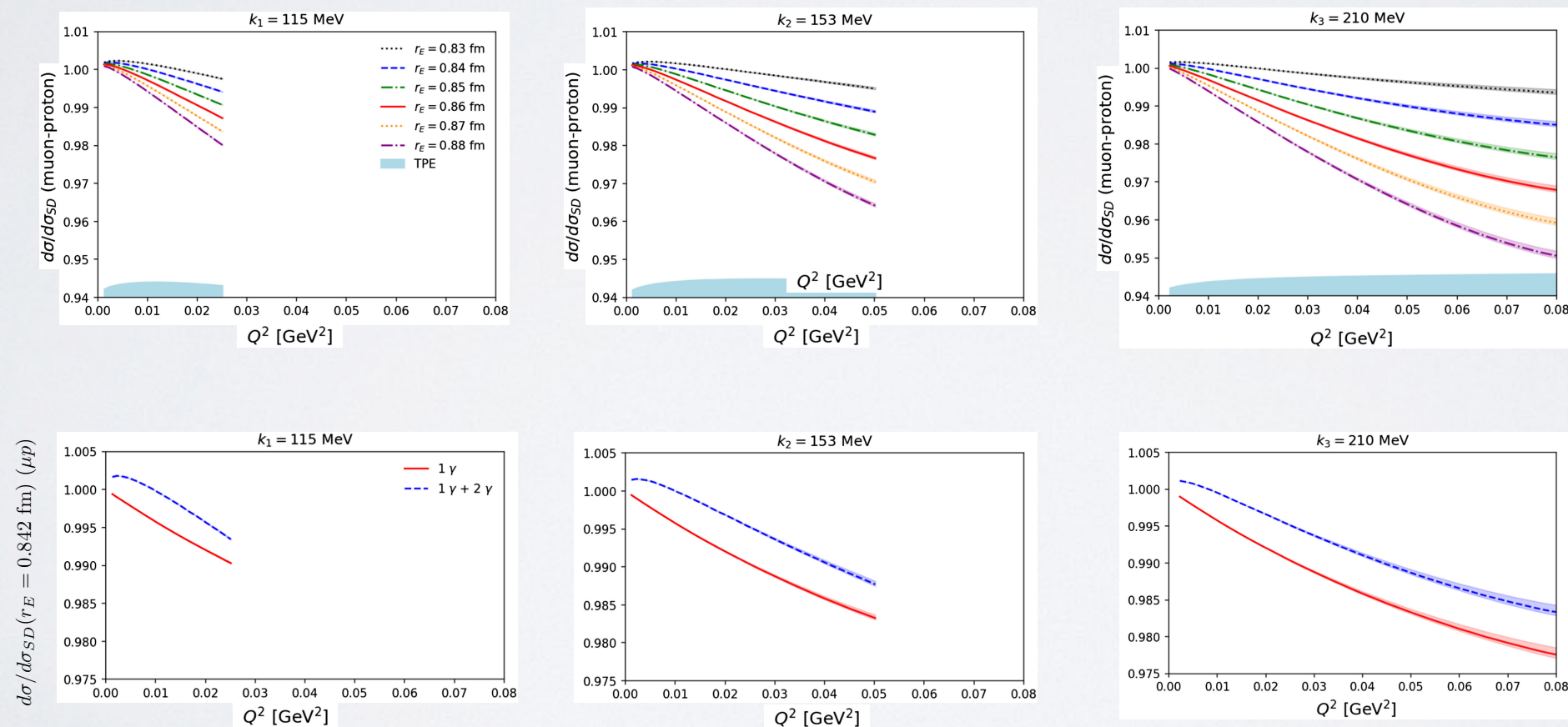
- $\delta_{2\gamma}$ taken from [\[O.Tomalak and M.Vanderhaeghen, Eur. Phys. J. C 76, 125 \(2016\)\]](#)

DIχEFT

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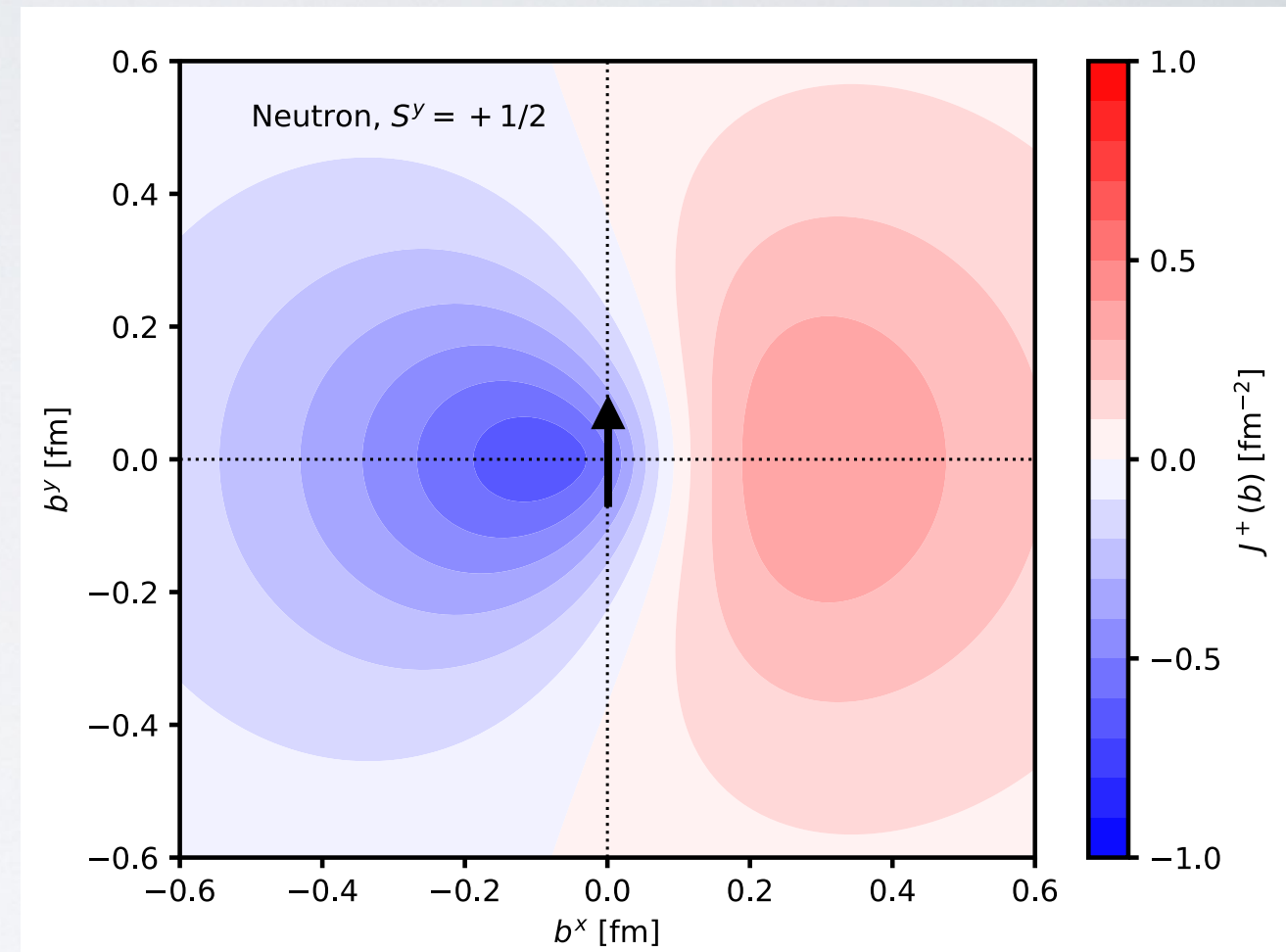
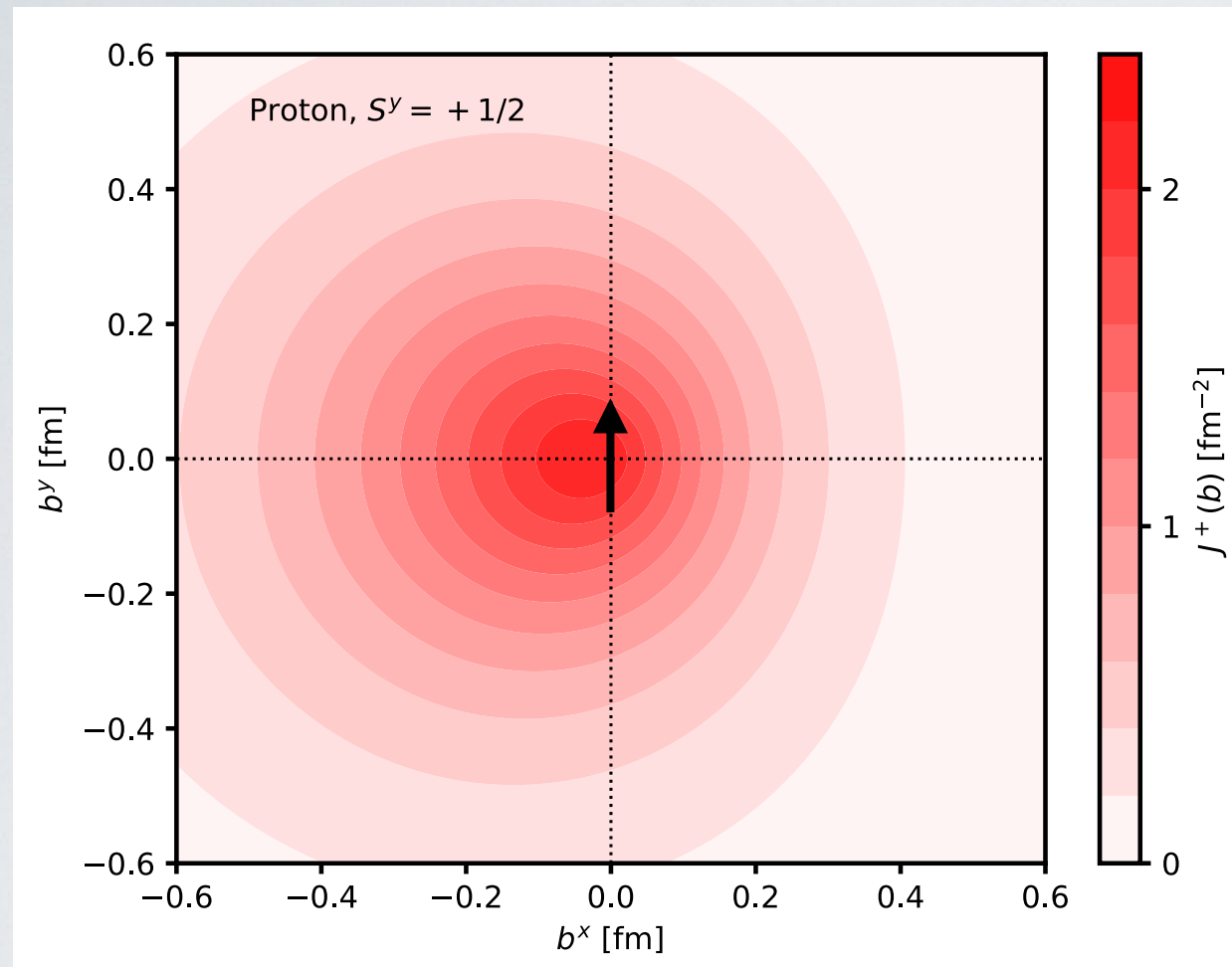
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[\[F. Gil-Domínguez, J. M. Alarcón and C. Weiss, Phys. Rev. D 108 \(2023\)\]](#)

Results



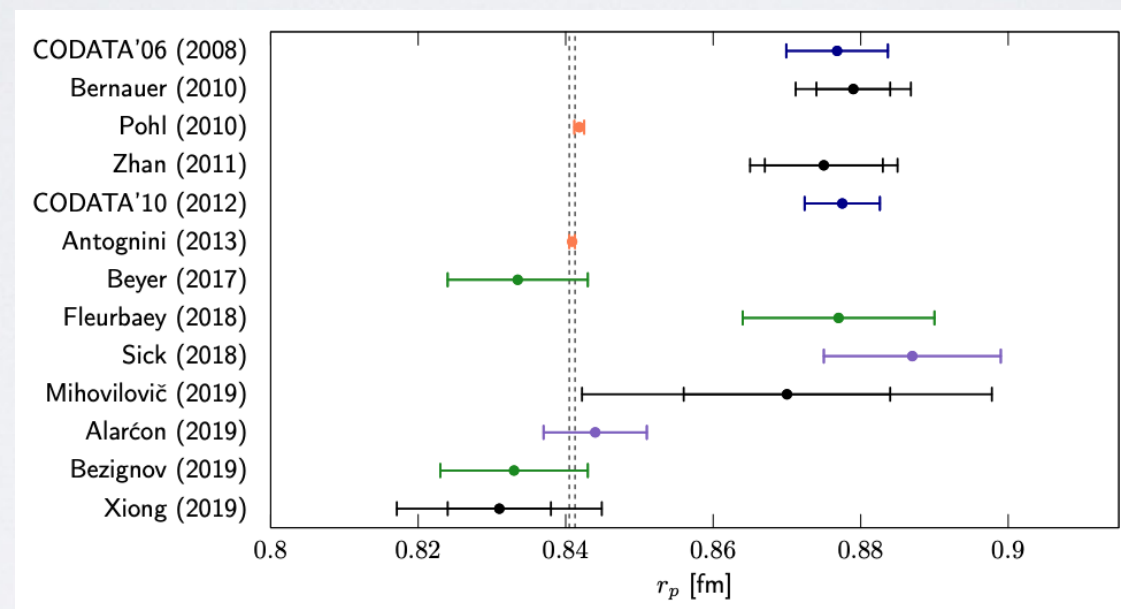
$$\langle J^+(\mathbf{b}) \rangle = (\dots) [\rho_1(b) + (2S^y) \cos \phi \tilde{\rho}_2(b)]$$

[J. M. Alarcón and C. Weiss, PRD 106 (2022)]

Summary and Conclusions

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- Through unitarity, it is possible to find a representation suited for ChEFT $\longrightarrow$ Predictions of the Nucleon Form factors.
- The results improve previous ChEFT calculations.
- Results used to understand “Proton Radius Puzzle”

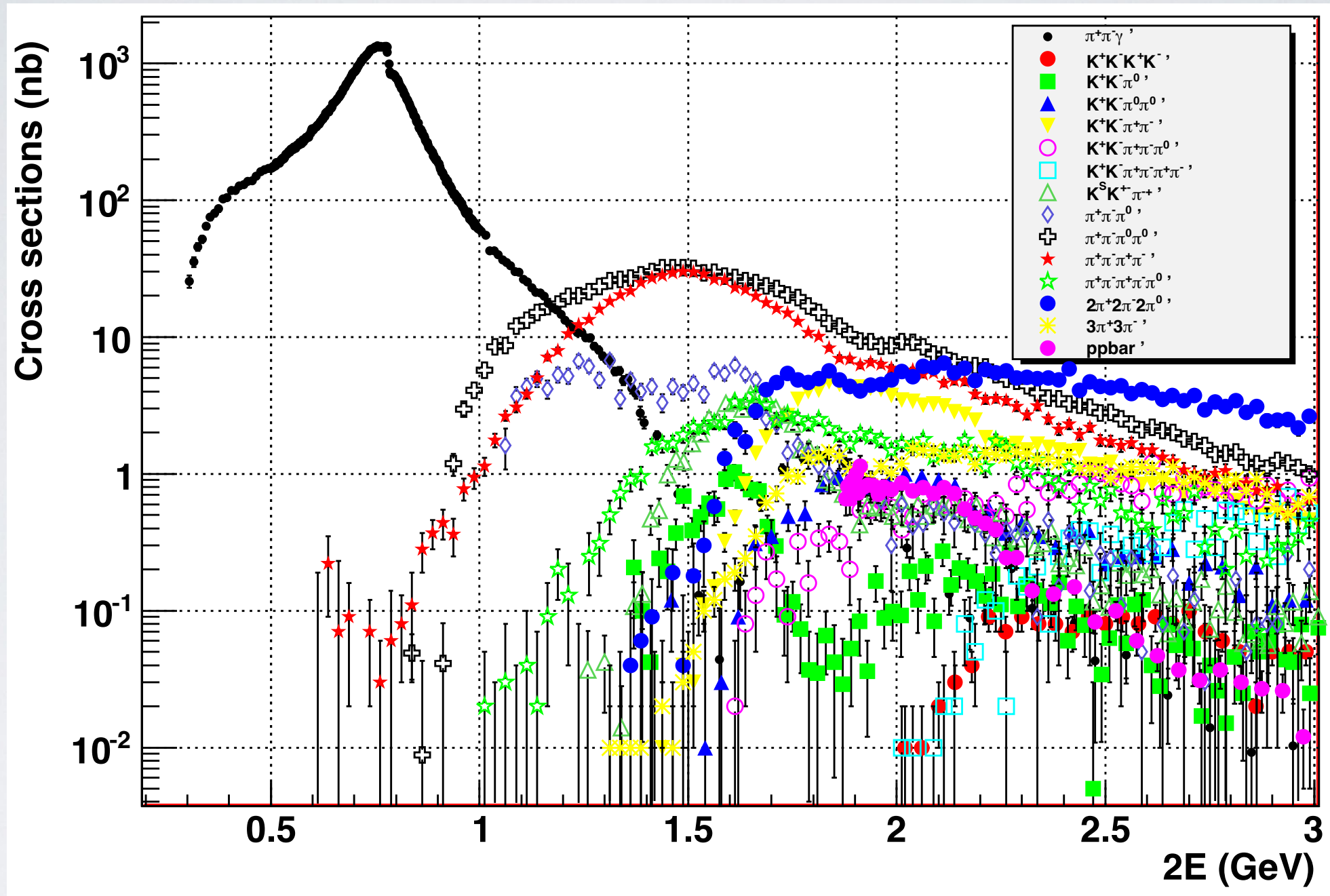


[J. C. Bernauer, EPJ Web Conf. 234 (2020)]

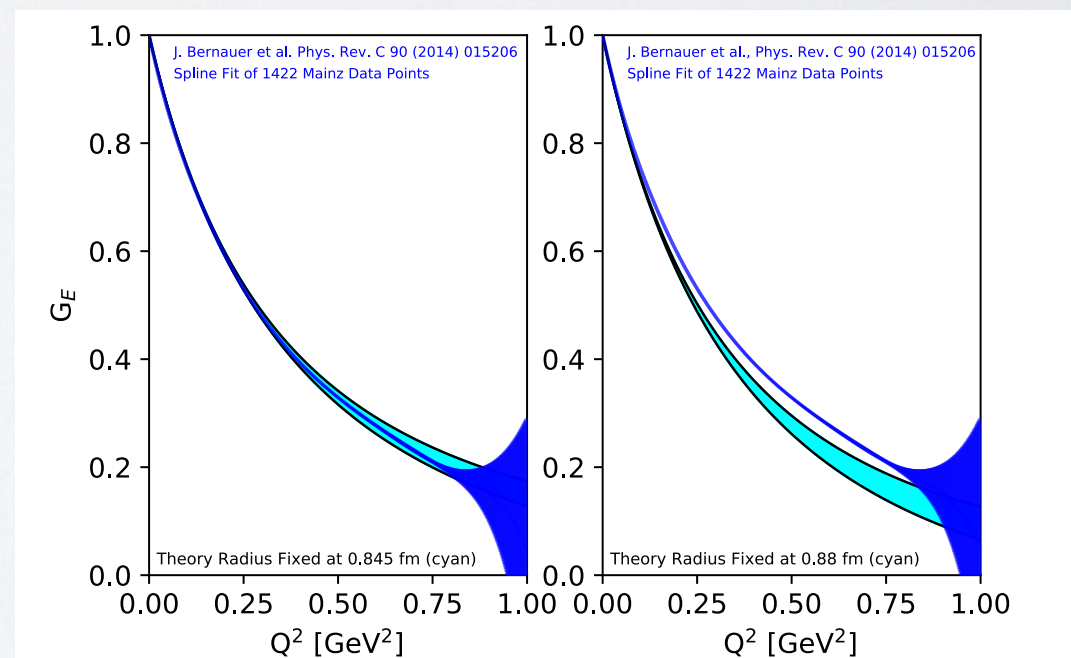
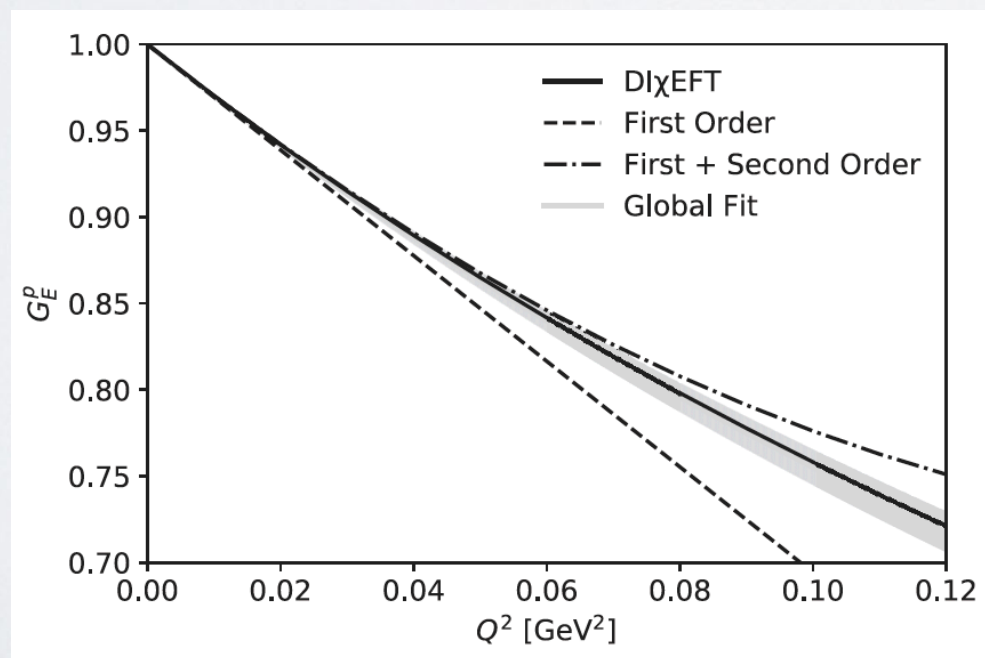
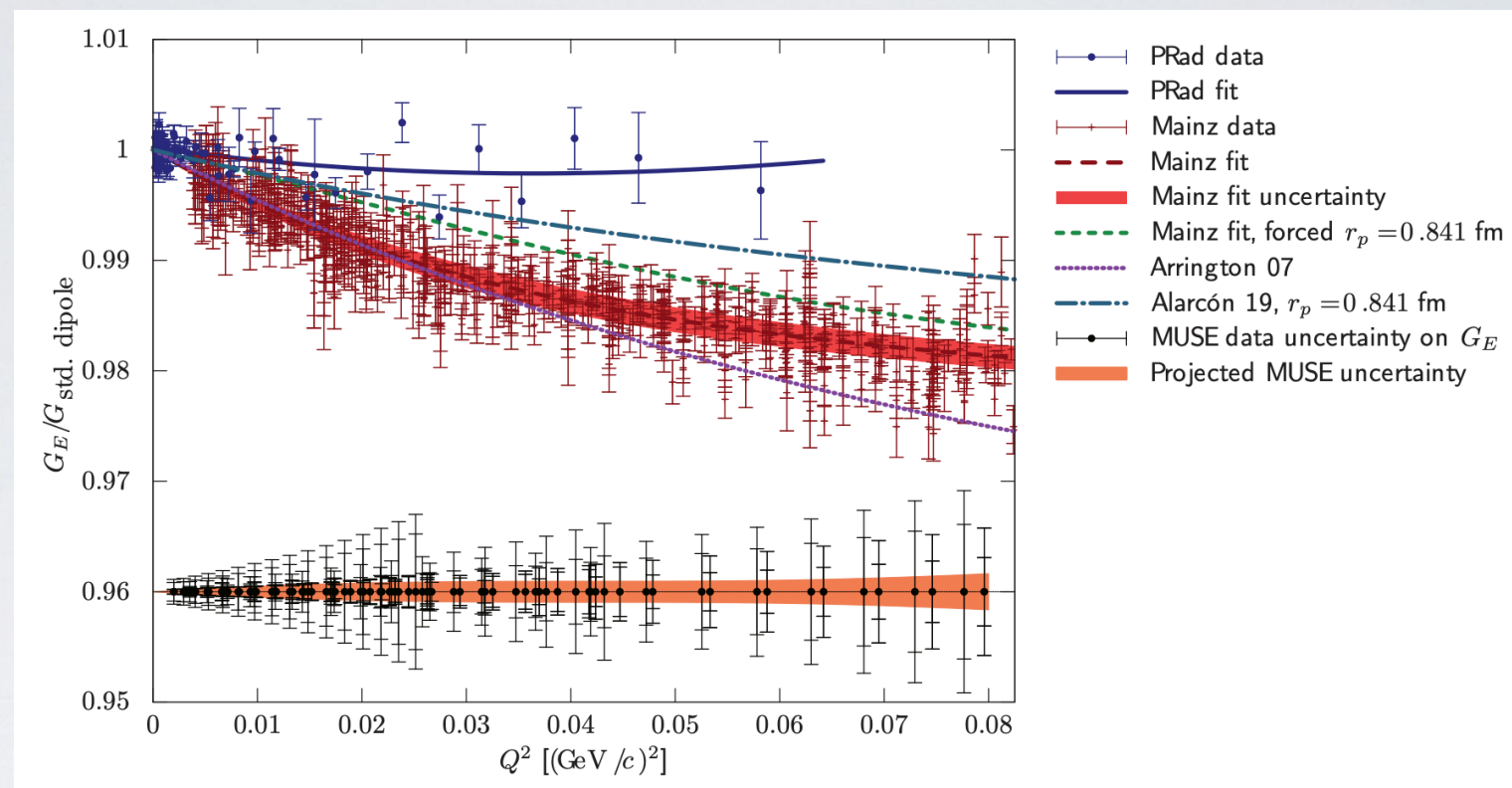
- Predictions for MUSE.
- New promising method to compute nucleon matrix elements from first principles (EM tensor, D-term, extension to G-parity odd, ...).

FIN

Spares



[Druzhinin, Eidelman, Serednyakov and Solodov, Rev. Mod. Phys. 83 (2011), 1545]



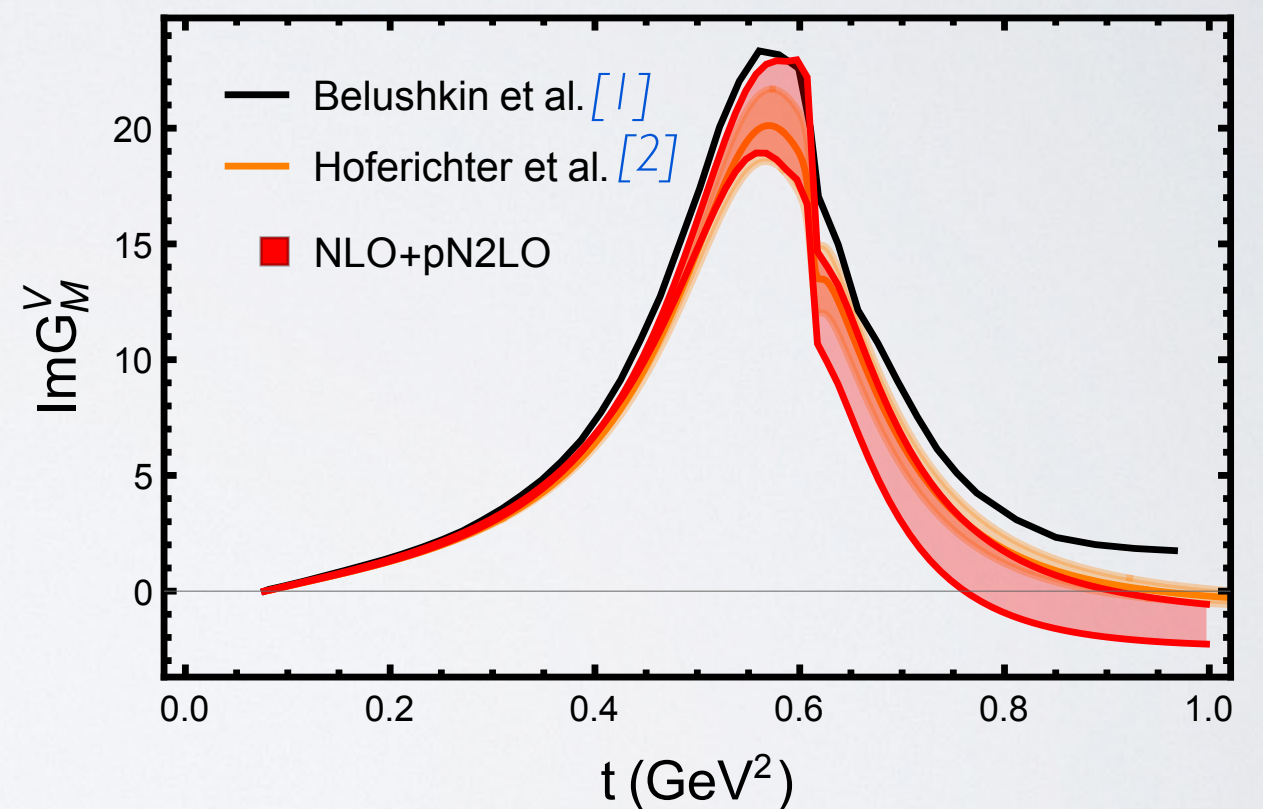
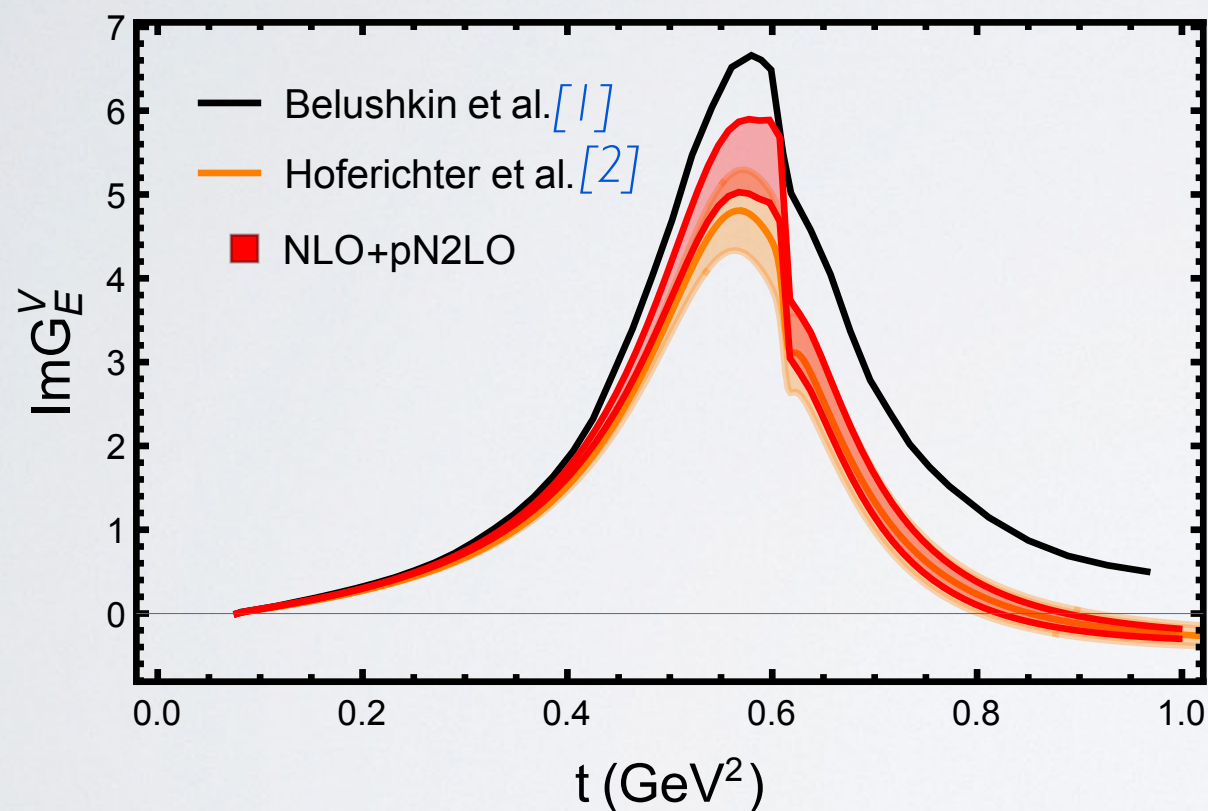
Spectral Functions

DI χ EFT

$$\text{Im}G_E^V(t) = \frac{k_{cm}^3}{m_N \sqrt{t}} \overset{\text{Experiment}}{\underset{\text{ChEFT}}{|F_\pi(t)|^2 J_+^1(t)}}$$

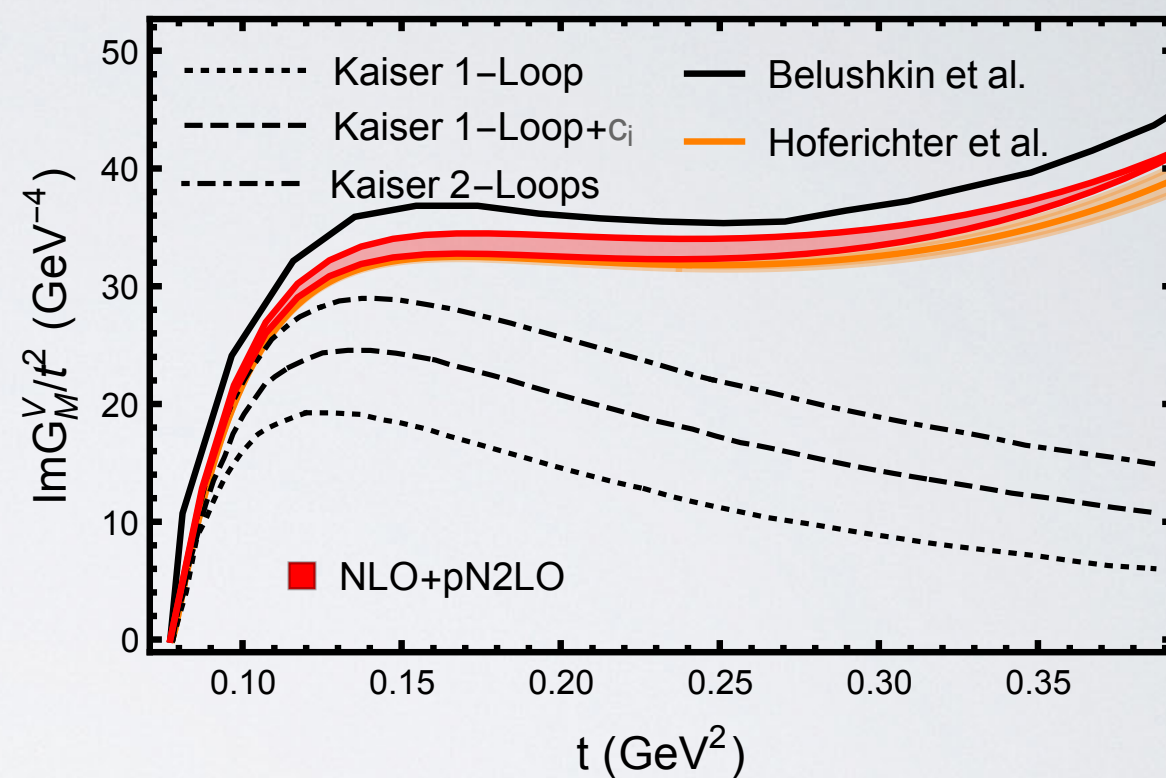
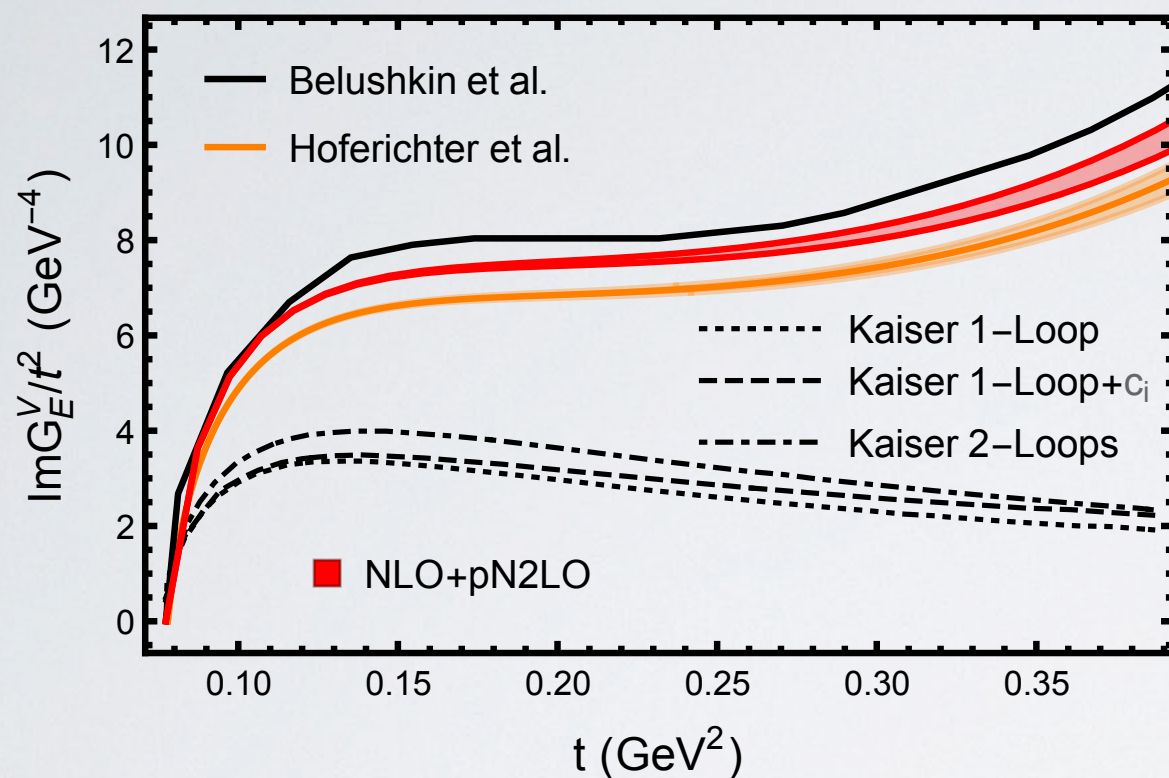
$$\text{Im}G_M^V(t) = \frac{k_{cm}^3}{\sqrt{2}t} \overset{\text{Experiment}}{\underset{\text{ChEFT}}{|F_\pi(t)|^2 J_-^1(t)}}$$

[J. M. Alarcón, C. Weiss, arXiv:1803.09748]



[1] Belushkin, Hammer and Meißner, PRC 75 (2007) [2] Hoferichter, Kubis, Ruiz de Elvira, Hammer, Meißner EPJA 52 (2016)

- Comparison with respect to the old results



- Conclusions:
 - Brute force calculations are hopeless.
 - Non-perturbative effects are visible in the near-threshold region.
 - Based on unitarity one achieves a factorization suitable for perturbative calculations.

$D|\chi EFT$

- We have two sources of uncertainty:
 - Isovector and isoscalar (Dirac and Pauli) radii:
 - We use the range given by the experimental based determinations.
 - Parametrization of the high energy region of the isovector spectral function:
 - According to the dual resonance model $M_n^2 = (2n + 1)M_\rho^2 \rightarrow$ supported by e^+e^- annihilation data [Druzhinin, Eidelman, Serednyakov and Solodov, Rev. Mod. Phys. 83 (2011), 1545].
 - We place the pole and derivatives at $t_{1,2}^{(V,i)} = 3M_\rho^2 \quad (i = 0, 1, 2)$
 - We allow a variation of these poles in the range $(2 - 4)M_\rho^2$
 - We reject values that produce a variation of the space-like FF larger than 10% with respect to the nominal value.

