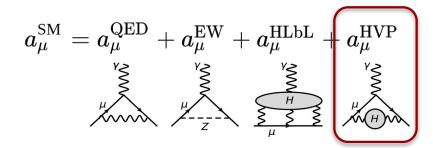
Muon g-2: from Anomaly to Agreement

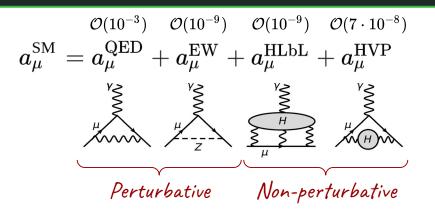
The HVP contribution from Lattice QCD

Simone Bacchio

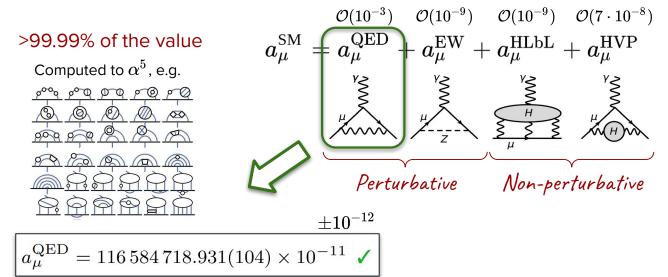
CaSToRC, The Cyprus Institute Extended Twisted Mass Collaboration (ETMC)



[WP25: arXiv:2505.21476]

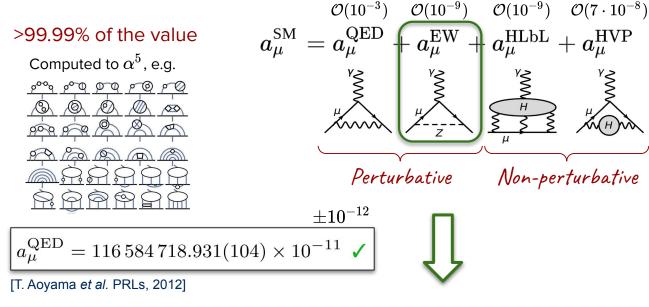


[WP25: arXiv:2505.21476]



[T. Aoyama et al. PRLs, 2012]

[WP25: arXiv:2505.21476]

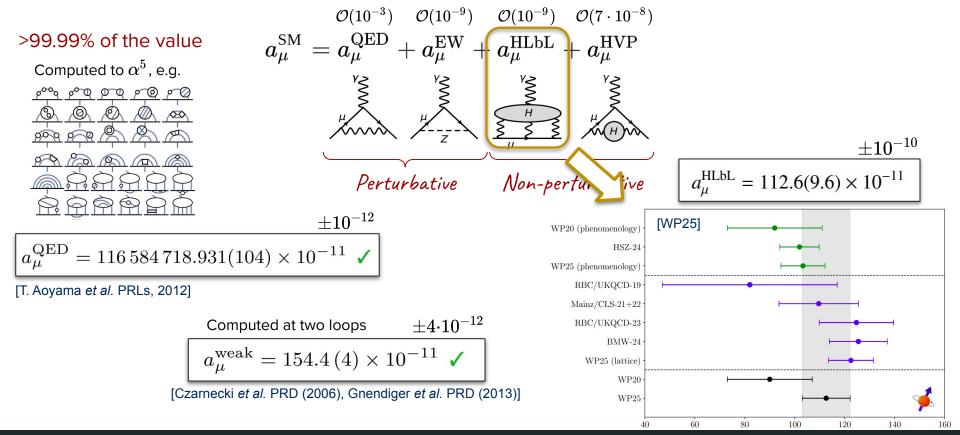


Computed at two loops
$$\pm 4\cdot 10^{-12}$$

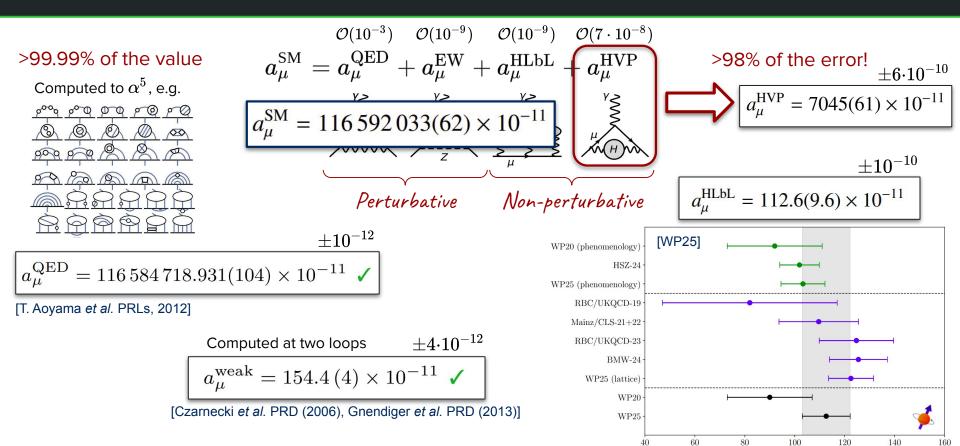
$$a_{\mu}^{\mathrm{weak}} = 154.4 \, (4) \times 10^{-11} \, \checkmark$$

[Czarnecki et al. PRD (2006), Gnendiger et al. PRD (2013)]

[WP25: arXiv:2505.21476]



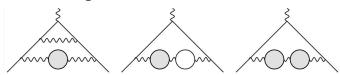
[WP25: arXiv:2505.21476]



The Hadronic Vacuum Polarization

$$a_{\mu}^{
m HVP} = a_{\mu}^{
m HVP,LO} + a_{\mu}^{
m HVP,NLO} + a_{\mu}^{
m HVP,NLO}$$

Relevant diagrams at NLO and NNLO:



The key ingredient is the spectral density of the HVP:



$$ho^{ ext{HVP}}(\omega)$$
 ho

$$a_{\mu}^{ ext{HVP}} = \left(rac{lpha}{\pi}
ight)$$

$$\int_0^\infty d\omega$$

$$ho^{ ext{HVP}}
ho^{ ext{HVP}}(\omega) \hspace{0.2cm} \Longrightarrow \hspace{0.2cm} a_{\mu}^{ ext{HVP}} \hspace{0.2cm} = \left(rac{lpha}{\pi}
ight)^{2} \int_{0}^{\infty} d\omega \hspace{0.2cm}
ho^{ ext{HVP}}(\omega) \hspace{0.2cm} \hat{K}_{a_{\mu}}(\omega) \hspace{0.2cm} = \left(rac{lpha}{\pi}
ight)^{2} \int_{0}^{\infty} dt \hspace{0.2cm} G^{ ext{HVP}}(t) \hspace{0.2cm} ilde{K}_{a_{\mu}}(t)$$

$$\int_{0}^{\infty} dt \, G^{ ext{HVP}}(t) \, ilde{K}_{a_{\mu}}(t)$$

Currently, two theoretical approaches to compute it:



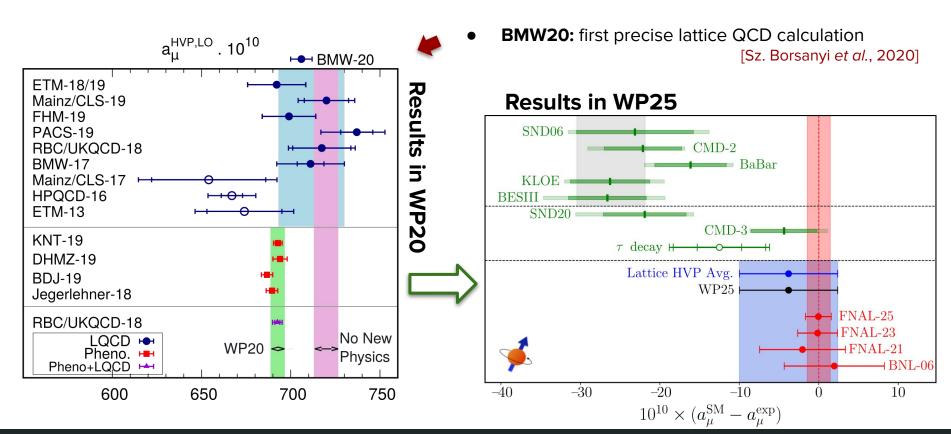
Relates a_{μ}^{HVP} to $e^{+}e^{-} \rightarrow \mathrm{hadrons}$ cross-sections via optical theorem (or to hadronic τ decay)

Lattice TMR approach:

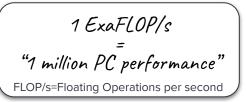
Relates $a_{\mu}^{ ext{HVP}}$ to vector-vector Euclidean correlation functions

with
$$G^{
m HVP}(t)=\int_0^\infty d\omega\,e^{-\omega t}\,
ho^{
m HVP}(\omega)$$
 $ilde{K}_{a_u}(t)=\int_0^\infty d\omega\,e^{-\omega t}\,\hat{K}_{a_u}(\omega)$

From Anomaly to Agreement



• (Pre-)Exascale computing: EuroHPC-JU is the largest pan-European initiative dedicated to funding HPC resources.











- (Pre-)Exascale computing: EuroHPC-JU is the largest pan-European initiative dedicated to funding HPC resources.
- Reach sub-percent precision: through the combination of cutting-edge HPC infrastructure and novel algorithms.

Gain 1: Multigrid methods employed for computing quark propagators achieved up to 100× speed-up for light quarks.

SHAN IN 101 Aμs

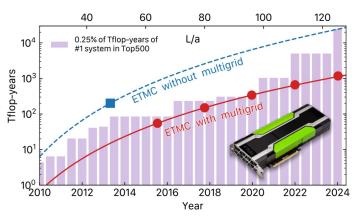
O Mixed-precision CG

Multigrid

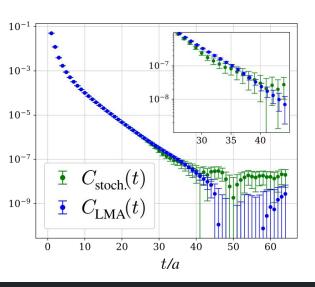
10-3 10-2 10-1

aμ

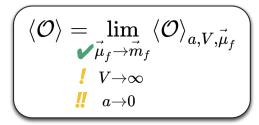
Gain 2: Increased computational resources allow to simulate larger ensembles and higher statistics



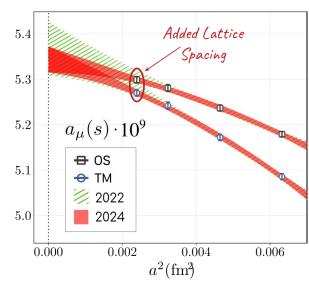
Gain 3: Deflation of low-modes allows to reach larger distance

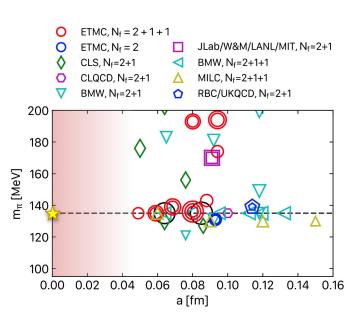


- (Pre-)Exascale computing: EuroHPC-JU is the largest pan-European initiative dedicated to funding HPC resources.
- Reach sub-percent precision: through the combination of cutting-edge HPC infrastructure and novel algorithms.
- Reach sub-percent accuracy: comprehensive set of ensembles enables full control of systematic effects.

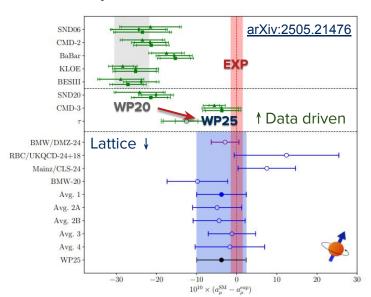


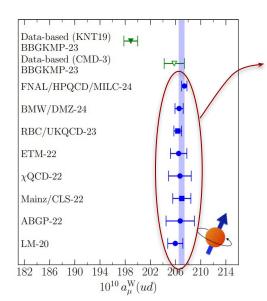
- $u \vec{\mu}_f \rightarrow \vec{m}_f$: Simulations directly at physical parameters
- $V{
 ightarrow}\infty$: Finite-size effects exponentially suppressed
- $m{"} a{
 ightarrow} 0$: <u>Inevitable extrapolation!</u>



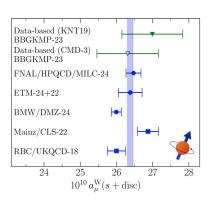


- (Pre-)Exascale computing: EuroHPC-JU is the largest pan-European initiative dedicated to funding HPC resources.
- Reach sub-percent precision: through the combination of cutting-edge HPC infrastructure and novel algorithms.
- Reach sub-percent accuracy: comprehensive set of ensembles enables full control of systematic effects.
- Independent calculations for robust final results: thanks to the collective effort of many collaborations.

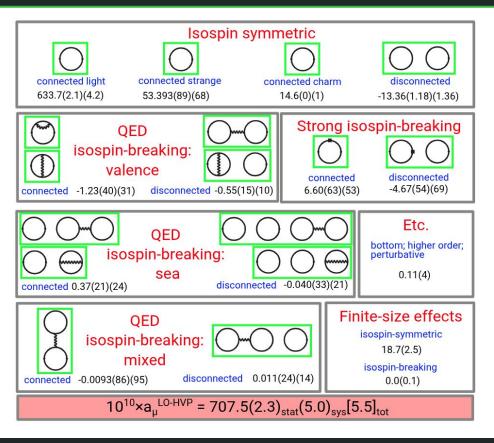




All <u>statistically independent</u> results, but with <u>over-estimated / conservative</u> <u>systematic</u> uncertainties!



The anatomy for the Lattice QCD calculation



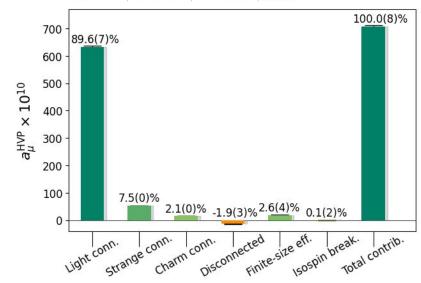
Article Published: 07 April 2021

Leading hadronic contribution to the muon magnetic moment from lattice QCD

Sz. Borsanyi, Z. Fodor ☑, J. N. Guenther, C. Hoelbling, S. D. Katz, L. Lellouch, T. Lippert,
K. Miura, L. Parato, K. K. Szabo, F. Stokes, B. C. Toth, Cs. Torok & L. Varnhorst

Nature 593, 51–55 (2021) Cite this article

24k Accesses 684 Citations 957 Altmetric Metrics



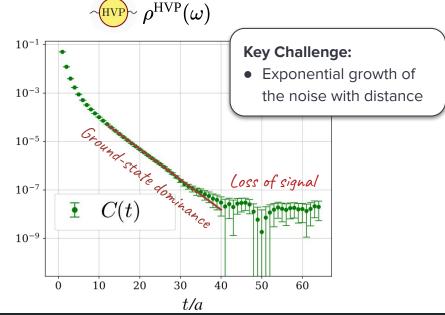
Key tool: Euclidean correlation functions

The correlation function between two electromagnetic currents:

$$C_{2pt}(t) \equiv \langle J_{em}(t)J_{em}(0) \rangle = \langle J_{em}|T^t|J_{em} \rangle = \sum_n \langle J_{em}|n \rangle \langle n|J_{em} \rangle e^{-E_n t}$$
 \longrightarrow Exponential suppression! e^{-H} \longrightarrow Exponential suppression! e^{-H} \longrightarrow Exponential growth of the noise with distance $J_{em}^i = \frac{2}{3} \bar{u} \gamma^i u - \frac{1}{3} \bar{d} \gamma^i d - \frac{1}{3} \bar{s} \gamma^i s + \frac{2}{3} \bar{c} \gamma^i c$



$$J_{em}(t)$$
 f $J_{em}(0)$ Disconnected



Key technique: Low-Mode Averaging (LMA)

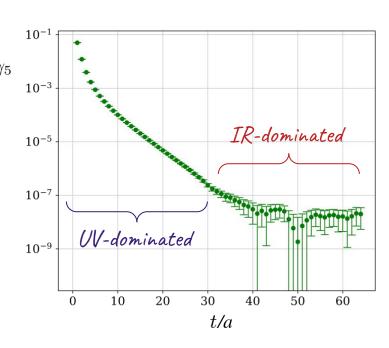
Concept: Low-modes dominate at large distance in correlation functions!

$$IR\ contribution \qquad \qquad UV\ contribution \\ S_r(x,y) = |P_{\rm IR}Q_r^{-1}P_{\rm IR}\eta(x)\rangle\,\langle\eta(y)|\,\gamma_5 + |P_{\rm UV}Q_r^{-1}P_{\rm UV}\eta(x)\rangle\,\langle\eta(y)|\,\gamma_5 \\ Q_{uark} \\ Propagator = \sum_{j=1}^K \frac{|v_j(x)\rangle\,\langle v_j(y)|\,\gamma_5}{\lambda_j + ir\mu} + \frac{1}{N}\sum_{\eta} |\widetilde{\phi}_r^{\eta}(x)\rangle\,\langle\eta(y)|\,\gamma_5 \bigg|_{N\gg 1} \,,$$

$$Computed\ \textit{Exactly} \qquad \textit{Computed\ \textit{Stochastically}}$$

In the correlation function computed stochastically,
 we replace the IR part with an exact knowledge of it

$$C_{\text{LMA}}(t) = C_{\text{stoch.}}(t) - C_{\text{stoch.}}^{\text{IR}}(t) + C_{\text{exact.}}^{\text{IR}}(t)$$



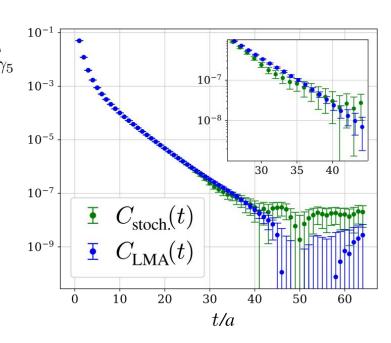
Key technique: Low-Mode Averaging (LMA)

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$$IR \ contribution \ \ UV \ contribution \ \ S_r(x,y) = |P_{\rm IR}Q_r^{-1}P_{\rm IR}\eta(x)\rangle \left\langle \eta(y)|\, \gamma_5 + |P_{\rm UV}Q_r^{-1}P_{\rm UV}\eta(x)\rangle \left\langle \eta(y)|\, \gamma_5 \right. \\ \left. \begin{array}{l} Q_{\rm uark} \\ P_{\rm ropagator} \end{array} \right. = \sum_{j=1}^K \frac{|v_j(x)\rangle \left\langle v_j(y)|\, \gamma_5}{\lambda_j + ir\mu} + \frac{1}{N} \sum_{\eta} |\widetilde{\phi}_r^{\eta}(x)\rangle \left\langle \eta(y)|\, \gamma_5 \right|_{N\gg 1}, \\ \\ \left. \begin{array}{l} Computed \ Exactly \end{array} \right. \\ \left. \begin{array}{l} Computed \ Stochastically \end{array} \right.$$

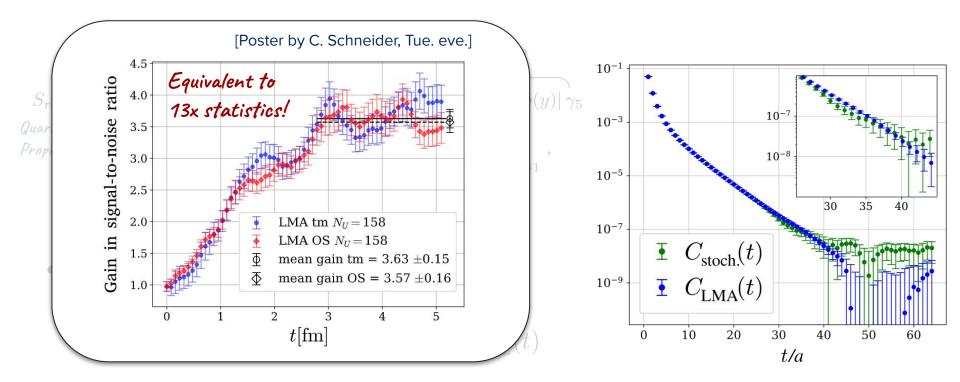
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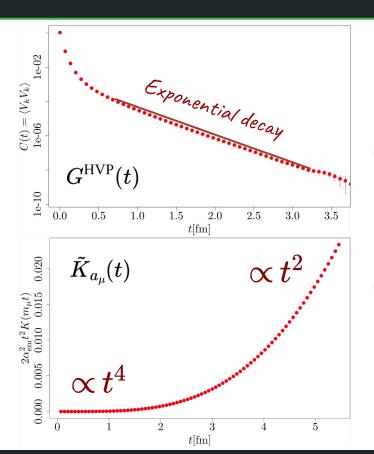


Key technique: Low-Mode Averaging (LMA)

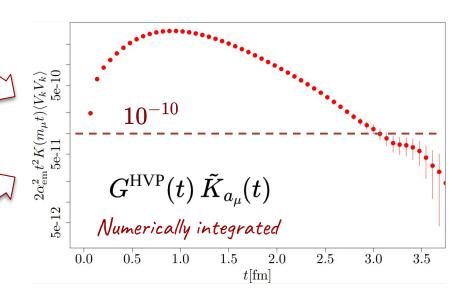
Concept: Low-modes dominate at large distance in correlation functions!



From correlator to HVP contribution

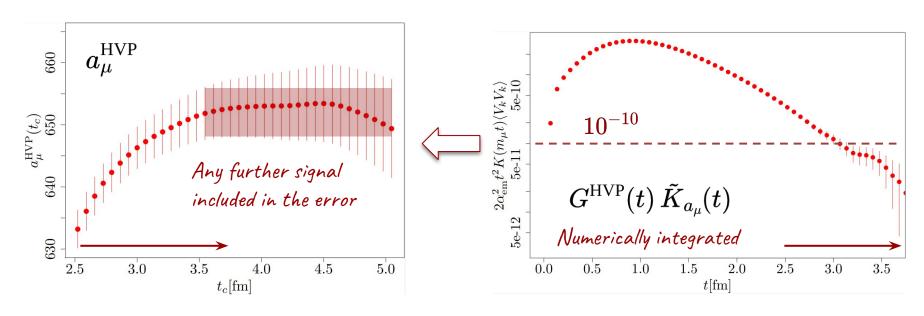


$$igg| \, a_{\mu}^{ ext{HVP}} = \left(rac{lpha}{\pi}
ight)^2 \int_0^{\infty} dt \, G^{ ext{HVP}}(t) \, ilde{K}_{a_{\mu}}\!(t) \, .$$



From correlator to HVP contribution

$$\left(a_{\mu}^{ ext{HVP}} = \left(rac{lpha}{\pi}
ight)^2 \int_0^{\infty} dt \, G^{ ext{HVP}}(t) \, ilde{K}_{a_{\mu}}\!(t)
ight)$$

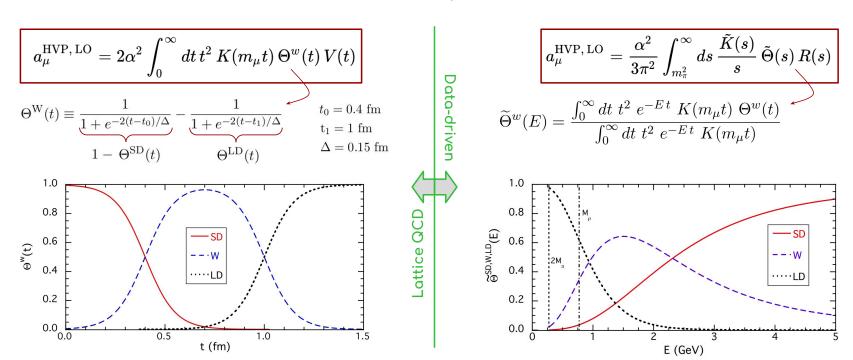


Taking the continuum limit reduces the parametrization systematic below other uncertainties.

Beyond Muon g-2: Windows observables [RBC/UKQCD]

$$a_{\mu}^{\mathrm{HVP}} = a_{\mu}^{\mathrm{SD}} + a_{\mu}^{\mathrm{W}} + a_{\mu}^{\mathrm{LD}}$$

Each window observable can be compared directly between lattice QCD and data-driven

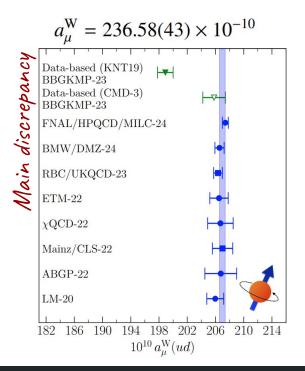


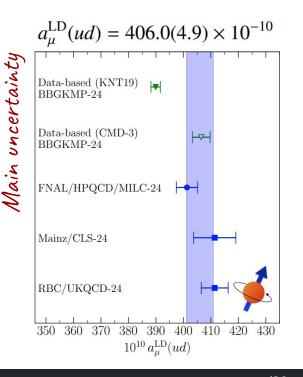
Beyond Muon g-2: Windows observables [RBC/UKQCD]

$$a_{\mu}^{\mathrm{HVP}} = a_{\mu}^{\mathrm{SD}} + a_{\mu}^{\mathrm{W}} + a_{\mu}^{\mathrm{LD}}$$

 $a_u^{\text{SD}} = 69.10(26) \times 10^{-10}$ Data-based (KNT19) BBGKMP-24 Data-based (CMD-3) BBGKMP-24 FNAL/HPQCD/MILC-24 SL-24 BMW/DMZ-24 Mainz/CLS-24 RBC/UKQCD-23 ETM-22 χ QCD-22 45 46 47 48 49 50 51 $10^{10} \, a_u^{\rm SD}(ud)$

Each window observable can be compared directly between lattice QCD and data-driven





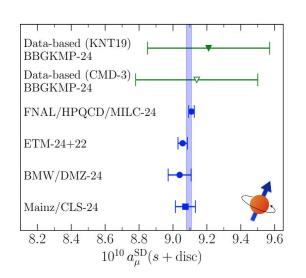
Beyond Muon g-2: ... with isoscalar (I=0) and isovector (I=1)

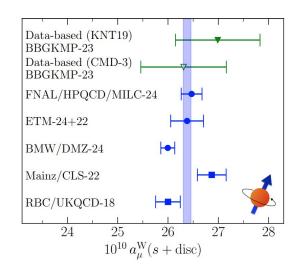


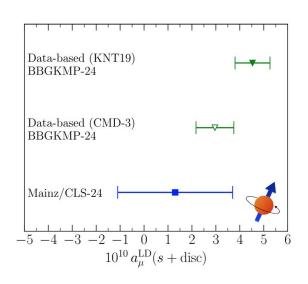
$$a_{\mu}(ud)=rac{10}{9}a_{\mu}^{I=1}$$

$$a_{\mu}(s+\mathrm{disc})=a_{\mu}^{I=1}-rac{1}{9}a_{\mu}^{I=1}$$

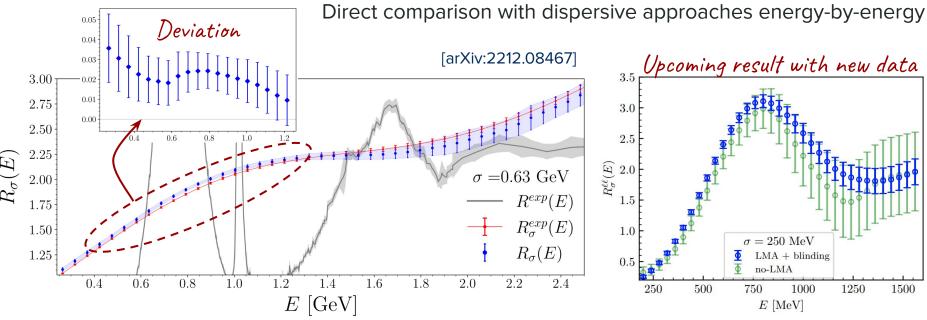
Confirmation that the two-pion channel is the main source of discrepancy

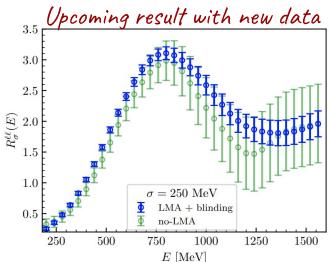




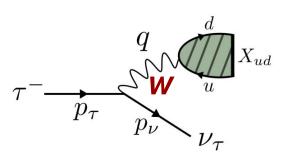


Beyond Muon g-2: Energy-smeared R-ratio



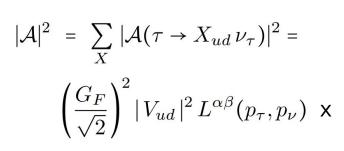


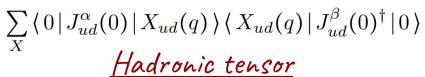
Other opportunities with current-current correlators

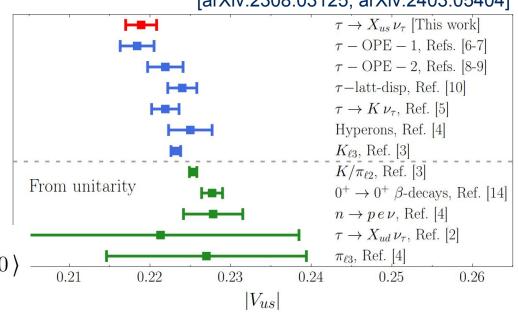


Inclusive semileptonic
$$\tau$$
 decay and determination of $|V_{us}|$

[arXiv:2308.03125, arXiv:2403.05404]







Conclusions

Resolving the $(g-2)_{\mu}$ puzzle was on of the recent greatest achievement of Lattice QCD, coming with many opportunities.

Credits:

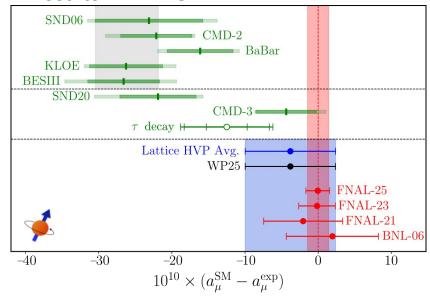
- BMW: First high-precision calculation
- RBC/UKQCD: Windows observables

From $(g-2)_{\mu}$ to dispersive vs lattice results:

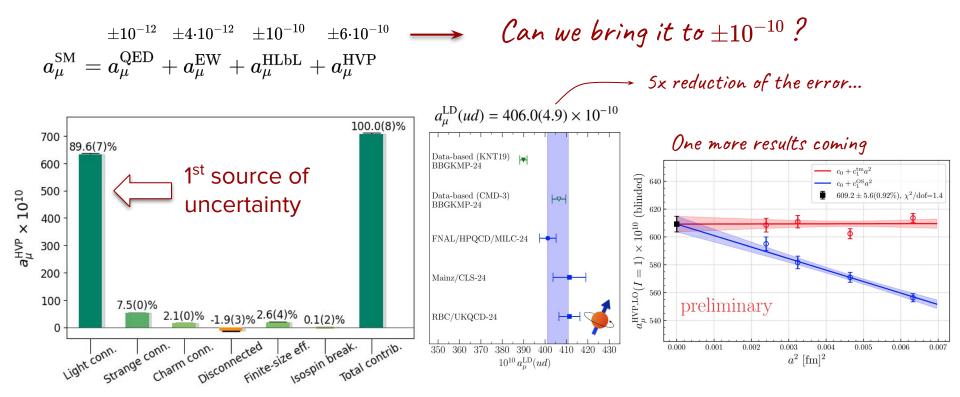
- Windows comparisons
- Isovector vs isoscalar
- Energy smeared R-ratio

And many other opportunities on the same data

Results in WP25

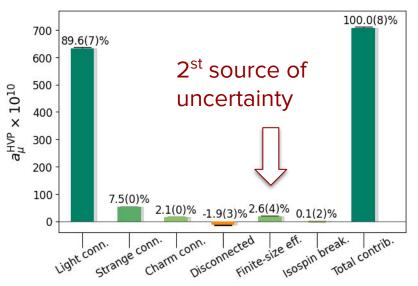


Question: Can we do better?



Question: Can we do better?

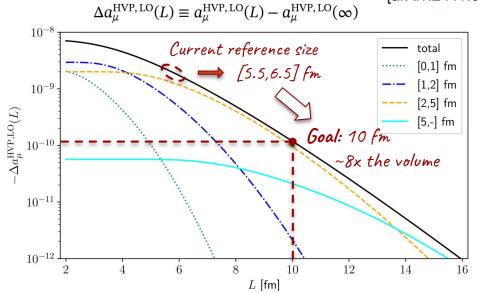
$$a_{\mu}^{ ext{SM}} = a_{\mu}^{ ext{QED}} + a_{\mu}^{ ext{EW}} + a_{\mu}^{ ext{HLbL}} + a_{\mu}^{ ext{HVP}}$$



Anatomy of finite-volume effect on hadronic vacuum polarization contribution to muon q-2

Sakura Itatani $^{\fbox{\scriptsize 0}}$,1,* Hidenori Fukaya $^{\fbox{\scriptsize 0}}$,2,† and Shoji Hashimoto $^{\fbox{\scriptsize 0}}$ 1,3,‡

[arXiv:2411.05413]



Question: Can we do better?

$$a_{\mu}^{ ext{SM}} = a_{\mu}^{ ext{QED}} + a_{\mu}^{ ext{EW}} + a_{\mu}^{ ext{HLbL}} + a_{\mu}^{ ext{HVP}}$$

700

600

500

400

200

100

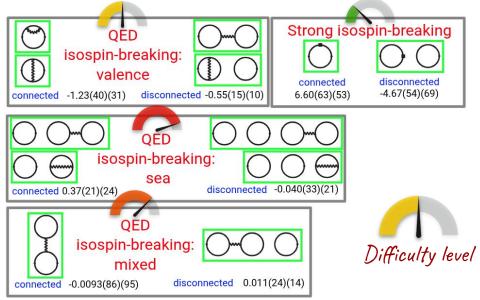
7.5(0)%

Strange conn.

Charm conn.

100.0(8)% 89.6(7)% Main source of systematics?

BMW is currently the only collaboration to have computed all contributions within the same framework. Remarkably, they cancel—but what if they didn't?



Disconnected

2.1(0)% -1.9(3)% 2.6(4)% 0.1(2)%

Finite-size eff.

Isospin break.

Answer: Yes, we can!

$$egin{aligned} & \pm 10^{-12} & \pm 4 \cdot 10^{-12} & \pm 10^{-10} & \pm 6 \cdot 10^{-10} \ a_{\mu}^{ ext{SM}} &= a_{\mu}^{ ext{QED}} + a_{\mu}^{ ext{EW}} + a_{\mu}^{ ext{HLbL}} + a_{\mu}^{ ext{HVP}} \end{aligned}$$

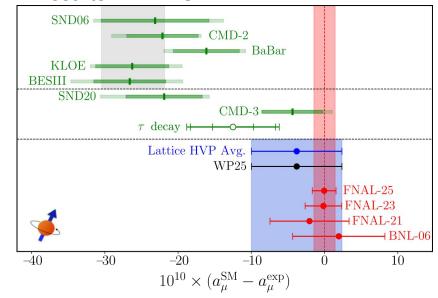
Nothing prevents Lattice QCD from reaching experimental precision.

It's only a matter of time, resources, and interest.

<u>Lattice QCD is a systematically</u> <u>improvable framework!</u>

Can we bring it to $\pm 10^{-10}$?

Results in WP25



Answer: Yes, we can!

$$a_{\mu}^{ ext{SM}} = a_{\mu}^{ ext{QED}} + a_{\mu}^{ ext{EW}} + a_{\mu}^{ ext{HLbL}} + a_{\mu}^{ ext{HVP}}$$

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<u>Lattice QCD is a systematically</u> <u>improvable framework!</u>

Can we bring it to $\pm 10^{-10}$?

Thank you for your attention!







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Backup

