Overview of Recent Developments for Transverse Momentum Dependent Parton Distribution Functions and Fragmentation Functions (TMDs)

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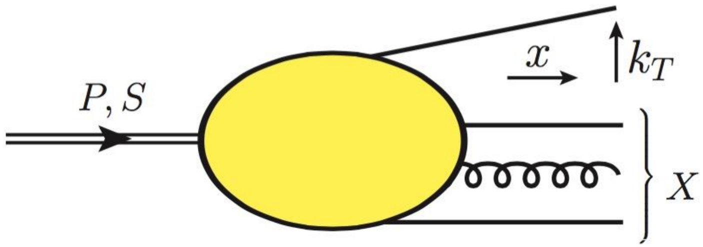
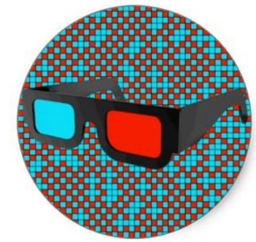
Outline

- TMD definition, factorization, and evolution
  - Leading power TMD PDFs and FFs
  - Collins-Soper-Sterman (CSS), ζ-prescription, SCET
  - Hadron Structure Oriented (HSO) approach
- Phenomenology and extraction of TMDs
  - Unpolarized
  - Polarized: Sivers, transversity, …
- Comparisons with Lattice QCD

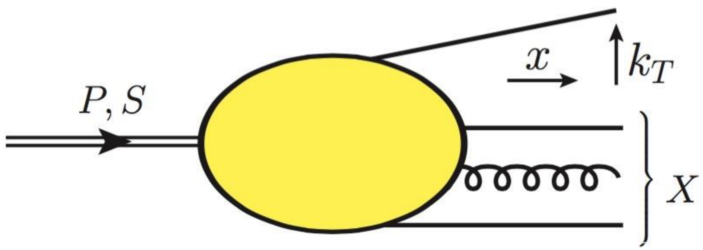
  More detailed talk on TMDs and lattice QCD next by Jianhui Zhang
- Other important topics and outlook

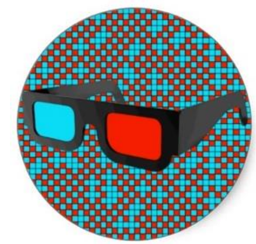
TMD Definition, Factorization, and Evolution



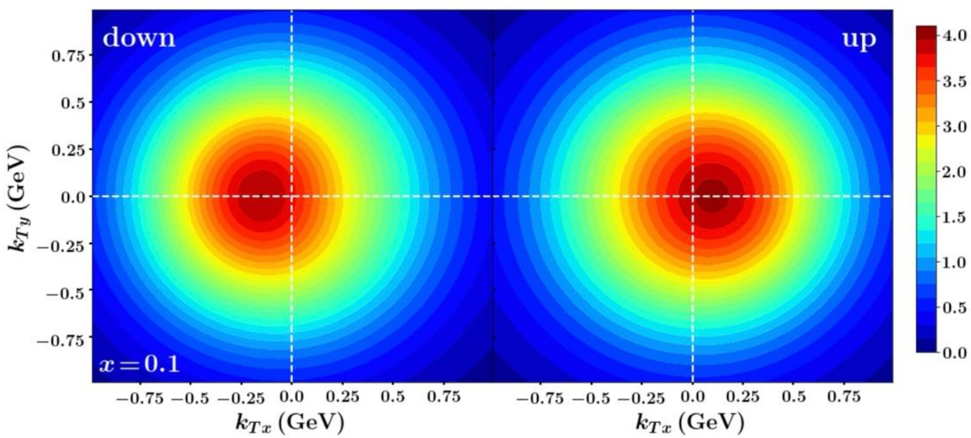


$$\Phi^q_{ij}(x;P,S|n) = \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P,S|\bar{\psi}^q_j(0)\mathcal{W}(0,\xi^-|n)\psi^q_i(\xi^-)|P,S\rangle$$





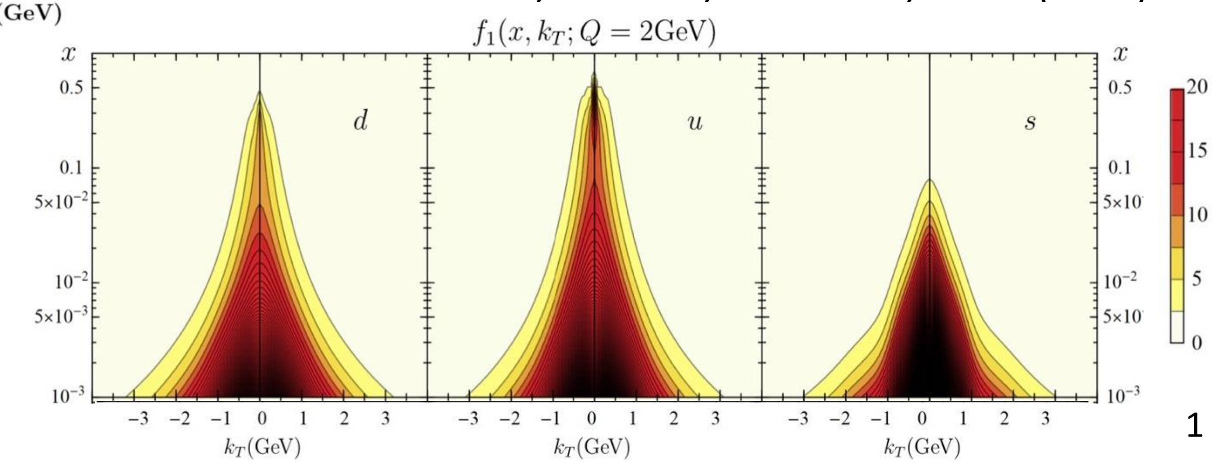
$$\Phi^q_{ij}(x; P, S|n) = \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P, S|\bar{\psi}^q_j(0)\mathcal{W}(0, \xi^-|n)\psi^q_i(\xi^-)|P, S\rangle$$



Based on Cammarota, et al. (2020)

3D nucleon tomography in momentum space, including the effect of spin-orbit correlations on the distribution of partons

Moos, Scimemi, Vladimirov, Zurita (2025)



Leading Power Quark TMDPDFs

Nucleon Spin   Quark Spin

		Quark Polarization		
		Un-Polarized (U)	**Longitudinally Polarized (L)**	**Transversely Polarized (T)**
Nucleon Polarization	**U**	$f_1 =$ Unpolarized		$h_1^{\perp} = \; - \;$ Boer-Mulders
	L		$g_{1L} = \; - \;$ Helicity	$h_{1L}^{\perp} = \; - \;$
	T	$f_{1T}^{\perp} = \; - \;$ Sivers	$g_{1T}^{\perp} = \; - \;$ Worm-gear	$h_1 = \; - \;$ Transversity $h_{1T}^{\perp} = \; - \;$ Pretzelosity

TMD Handbook (2304.03302)

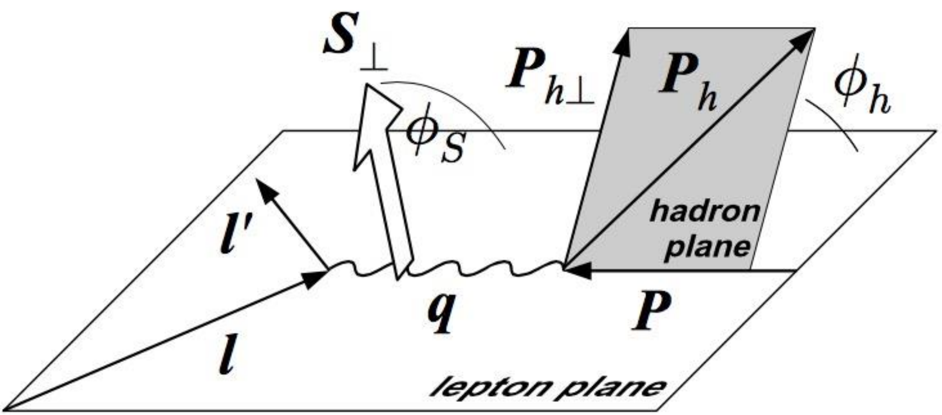
Leading Power Quark TMDFFs

Hadron Spin   Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		D_1 = Unpolarized		$H_1^\perp$ = − Collins
Polarized Hadrons	L		G_{1L} = − Helicity	$H_{1L}^\perp$ = −
	T	$D_{1T}^\perp$ = − Polarizing FF	$G_{1T}^\perp$ = −	H_1 = − Transversity $H_{1T}^\perp$ = −

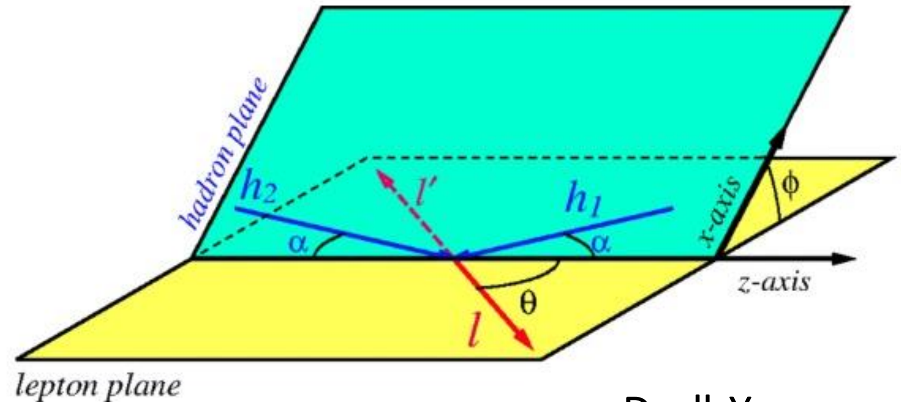
TMD Handbook (2304.03302)

$$\ell p^{\uparrow} \to \ell h X$$



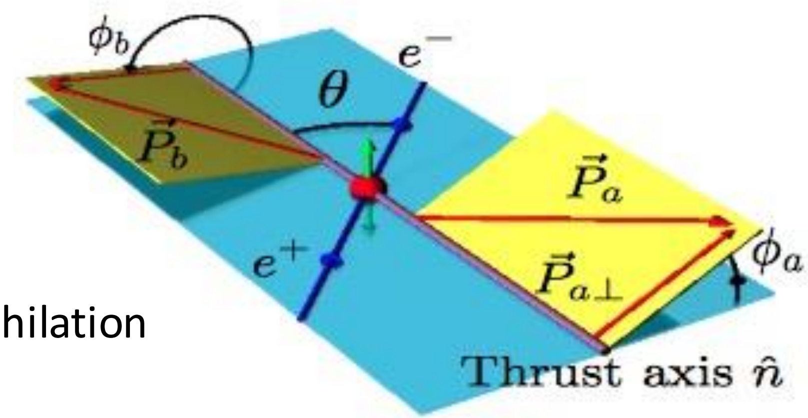
SIDIS

$$\{\pi, p\} p^{\uparrow} \to \{\ell^+\ell^-, W^{\pm}, Z\} X$$



Drell-Yan

$$e^+e^- \to h_1 h_2 X$$



e^+e^- annihilation

$$\Gamma(q_T, Q) \equiv \frac{d\sigma}{d^2 q_T dQ \cdots} = W(q_T, Q) + Y(q_T, Q) + O((m/Q)^c)\,\Gamma(q_T, Q)$$

$q_T << Q$ region

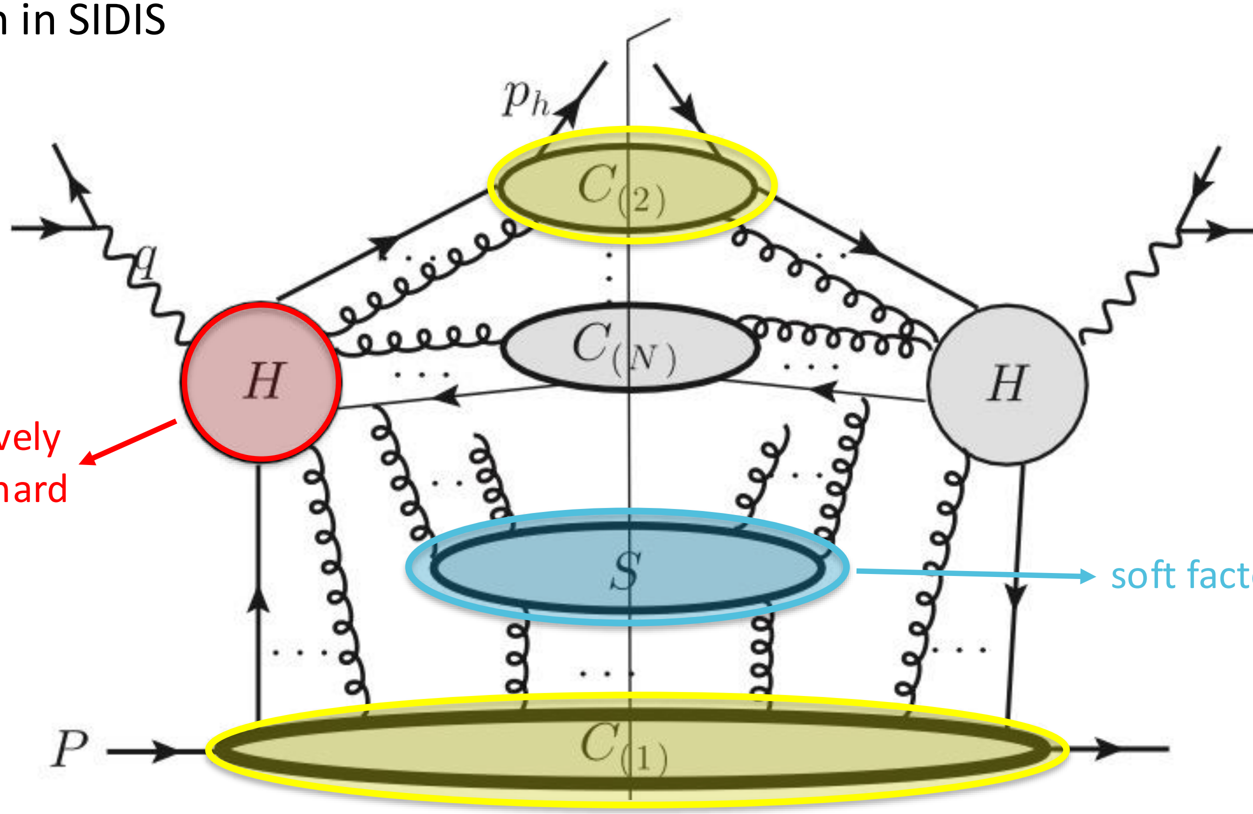
$q_T \sim Q$ region

(Collins, Soper Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011),…)

Measurements DO NOT give direct access to TMDs → factorization theorems → evolution

Factorization in SIDIS

perturbatively calculable hard factors



soft factor

Figure from Collins (2011)

non-perturbative functions that encode hadron structure

$$\frac{d\sigma}{dx\,dy\,d\phi_s\,dz\,d\phi_h\,(z^2 dq_T^2)} = W_{UU}(q_T,Q) + |\vec{S}_T|\sin(\phi_h-\phi_S)W_{UT}^{\rm siv}(q_T,Q)+\ldots$$

SIDIS measurements with $q_T << Q$

work in b space

$$W_{UU}(q_T,Q) = \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{i\vec{q}_T\cdot\vec{b}_T}\,\tilde{W}_{UU}(b_T,Q)$$

$$\frac{d\sigma}{dx\,dy\,d\phi_s\,dz\,d\phi_h\,(z^2 dq_T^2)} = W_{UU}(q_T,Q) + |\vec{S}_T|\sin(\phi_h-\phi_S)W_{UT}^{\text{siv}}(q_T,Q) + \ldots$$

SIDIS measurements with $q_T << Q$

work in b space

$$W_{UU}(q_T,Q) = \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\tilde{W}_{UU}(b_T,Q)$$

CS scale

renorm. scale

$$\tilde{W}_{UU}(b_T,Q) = \sum_j \underbrace{H_j(\mu,Q)}_{\text{Hard factor}}\underbrace{\tilde{f}_1^j(x,b_T;\zeta_A,\mu)\tilde{D}_1^{h/j}(z,b_T;\zeta_b,\mu)}_{b\text{-space TMDs}}$$

Hard factor

b-space TMDs

$$\tilde{f}_1(x, b_T; Q^2, \mu_Q) \quad \sim \quad \Big(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1(\hat{x}; \mu_{b_*})}\Big)$$
$$\times \exp\Big[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\Big]$$

(Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\tilde{f}_1(x, b_T; Q^2, \mu_Q) \sim \boxed{\left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1(\hat{x}; \mu_{b_*})}\right)}$$
$$\times \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

Operator Product Expansion

$$\tilde{f}_1(x, b_T; Q^2, \mu_Q) \sim \boxed{\left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1(\hat{x}; \mu_{b_*})}\right)}$$

$$\times \exp\left[\boxed{-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q)} - \boxed{S_{NP}^{f_1}(b_T, Q)}\right]$$

Operator Product Expansion

perturbative Sudakov factor

$$-\ln(Q/\mu_{b_*})\tilde{K}(b_*, \mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[\gamma(\alpha_s(\mu'); 1) - \gamma_K(\alpha_s(\mu')) \ln(Q/\mu')\right]$$

same for unpol. and pol.

non-perturbative Sudakov factor

$$g_{f_1}(x, b_T) + g_K(b_T) \ln(Q/Q_0)$$

different for each TMD

universal: extract from exp. or calculate on the lattice

$$\tilde{f}_1(x,b_T;Q^2,\mu_Q) \sim \boxed{\left(\tilde{C}^{f_1}(x/\hat{x},b_*(b_T);\mu_{b_*}^2,\mu_{b_*},\alpha_s(\mu_{b_*}))\otimes \boldsymbol{f_1(\hat{x};\mu_{b_*})}\right)}$$
$$\times\ \exp\left[\boxed{-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)}-\boxed{S_{NP}^{f_1}(b_T,Q)}\right]$$

Operator Product Expansion

perturbative Sudakov factor

non-perturbative Sudakov factor

$$-\ln(Q/\mu_{b_*})\tilde{K}(b_*,\mu_{b_*})-\int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1)-\gamma_K(\alpha_s(\mu'))\ln(Q/\mu')\right]$$

$$g_{f_1}(x,b_T)+g_K(b_T)\ln(Q/Q_0)$$

same for unpol. and pol.

different for each TMD

universal: extract from exp. or calculate on the lattice

$$b_*(b_T)\equiv\sqrt{\frac{b_T^2}{1+b_T^2/b_{\max}^2}} \qquad \mu_{b_*}=C_1/b_*(b_T)$$

*b** prescription – ensures that perturbatively calculated quantities are evaluated at small b_T

Evolution using ζ-prescription

(Scimemi and Vladimirov (2018))

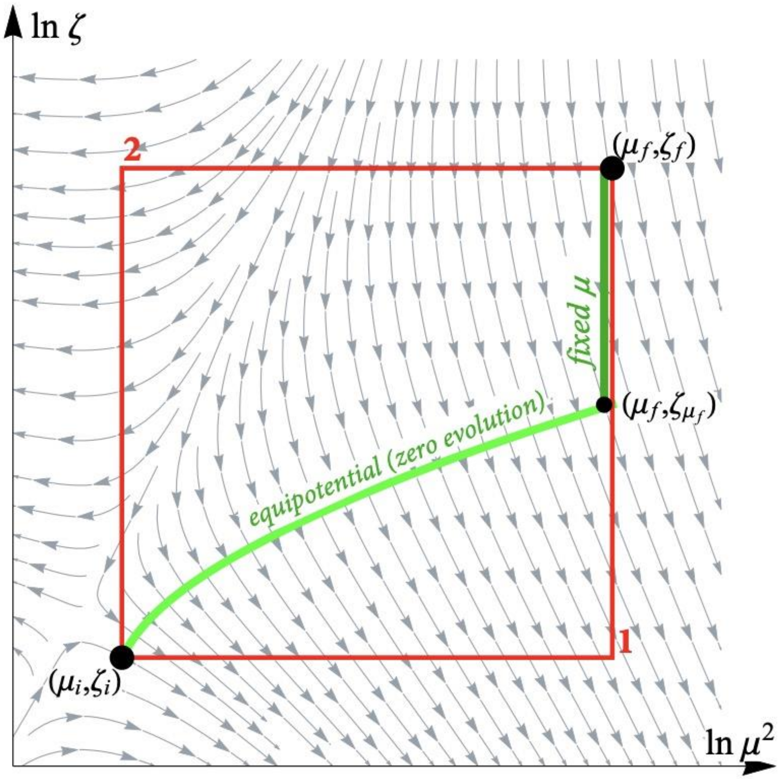


Figure from TMD Handbook, modified from Scimemi, Vladimirov (2018)

Evolution using SCET

(T. Becher and M. Neubert (2011); J.-Y. Chiu, A. Jain, D. Neill and I. Z. Rothstein (2012); M. G. Echevarria, A. Idilbi and I. Scimemi (2012, 2013, 2014),…)

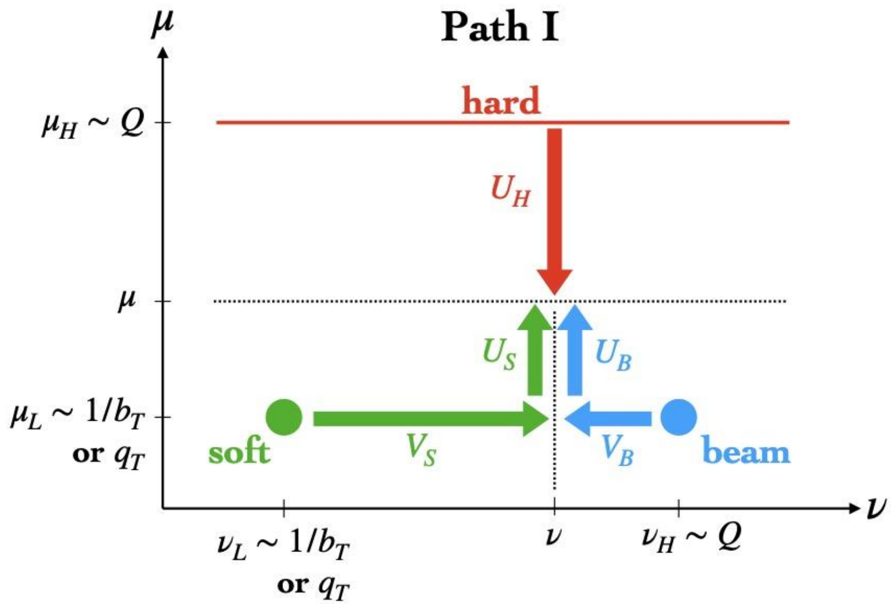


Figure from TMD Handbook

NB: Evolution can also be formulated directly in momentum space (P. F. Monni, E. Re and P. Torrielli (2016); M. A. Ebert and F. J. Tackmann (2017); D. Kang, C. Lee and V. Vaidya (2018))

Hadron-Structure Oriented (HSO) approach

(Gonzalez-Hernandez, Rogers, Sato (2022); Gonzalez-Hernandez, Rainaldi, Rogers (2023); Aslan, et al. (2024))

TMD integrated to a cutoff scale matched onto collinear PDF (see also Del Rio, et al. (2024))

$$f^{c}_{i/p}(x;\mu_Q;\mu_c) \equiv 2\pi \int_0^{\mu_c} \mathrm{d}k_\mathrm{T}\, k_\mathrm{T} f_{i/p}(x,\boldsymbol{k}_\mathrm{T};\mu_Q,Q^2) = f^{\overline{\mathrm{MS}}}_{i/p}(x;\mu_Q) + \Delta_{i/p}(\alpha_s(\mu_Q),\mu_c/\mu_Q) + O\left(\frac{m^2}{\mu_Q^2}\right)$$

Smooth/continuous interpolation between perturbative and non-perturbative b_T regions (no b^* or g's)

$$\tilde{f}_{i/p}(x,b_\mathrm{T};\mu_{Q_0},Q_0^2) = \tilde{f}_{\mathrm{inpt},i/p}(x,b_\mathrm{T};\mu_{\overline{Q}_0},\overline{Q}_0^2)\exp\left\{\int_{\mu_{\overline{Q}_0}}^{\mu_{Q_0}} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1) - \ln\left(\frac{Q_0}{\mu'}\right)\gamma_K(\alpha_s(\mu'))\right] + \ln\left(\frac{Q_0}{\overline{Q}_0}\right)\tilde{K}(b_\mathrm{T};\mu_{\overline{Q}_0})\right\},$$

$$\tilde{f}_{\mathrm{inpt},i/p}(x,b_\mathrm{T};\mu_{Q_0},Q_0^2) = \int d^2\mathbf{k}_T e^{-i\mathbf{k}_T\cdot\mathbf{b}_T} f_{\mathrm{inpt},i/p}(x,k_\mathrm{T};\mu_{Q_0},Q_0^2)$$

$$\begin{aligned} f_{\mathrm{inpt},i/p}(x,k_\mathrm{T};\mu_{Q_0},Q_0^2) = &\frac{1}{2\pi}\frac{1}{k_\mathrm{T}^2+m_{i,p,A}^2}A_{i/p}(x;\mu_{Q_0}) + \frac{1}{2\pi}\frac{1}{k_\mathrm{T}^2+m_{i,p,B}^2}B_{i/p}(x;\mu_{Q_0})\ln\left(\frac{Q_0^2}{k_\mathrm{T}^2+m_{i,p,L}^2}\right) \\ &+ \frac{1}{2\pi}\frac{1}{k_\mathrm{T}^2+m_{g,p}^2}A^g_{i/p}(x;\mu_{Q_0}) \\ &+ C_{i/p}\, f_{\mathrm{core},i/p}(x,k_\mathrm{T};Q_0^2)\,. \end{aligned}$$

"core" TMD function parameterization motivated by model calculations

$$f^{\mathrm{Gauss}}_{\mathrm{core},i/p}(x,\boldsymbol{k}_\mathrm{T};Q_0^2) = \frac{e^{-k_\mathrm{T}^2/M_\mathrm{F}^2}}{\pi M_\mathrm{F}^2}$$

$$f^{\mathrm{Spect}}_{\mathrm{core},i/p}(x,\boldsymbol{k}_\mathrm{T};Q_0^2) = \frac{1}{\pi}\frac{6L^6}{L^2+2(m_q+x\,M_p)^2}\frac{k_\mathrm{T}^2+(m_q+x\,M_p)^2}{(k_\mathrm{T}^2+L^2)^4}$$

Phenomenology and Extraction of TMDs

f_1 =  D_1 = 

Main challenge is to describe data across vastly different (x,Q^2) (Drell-Yan and SIDIS) that are sensitive to different aspects of the TMD functions

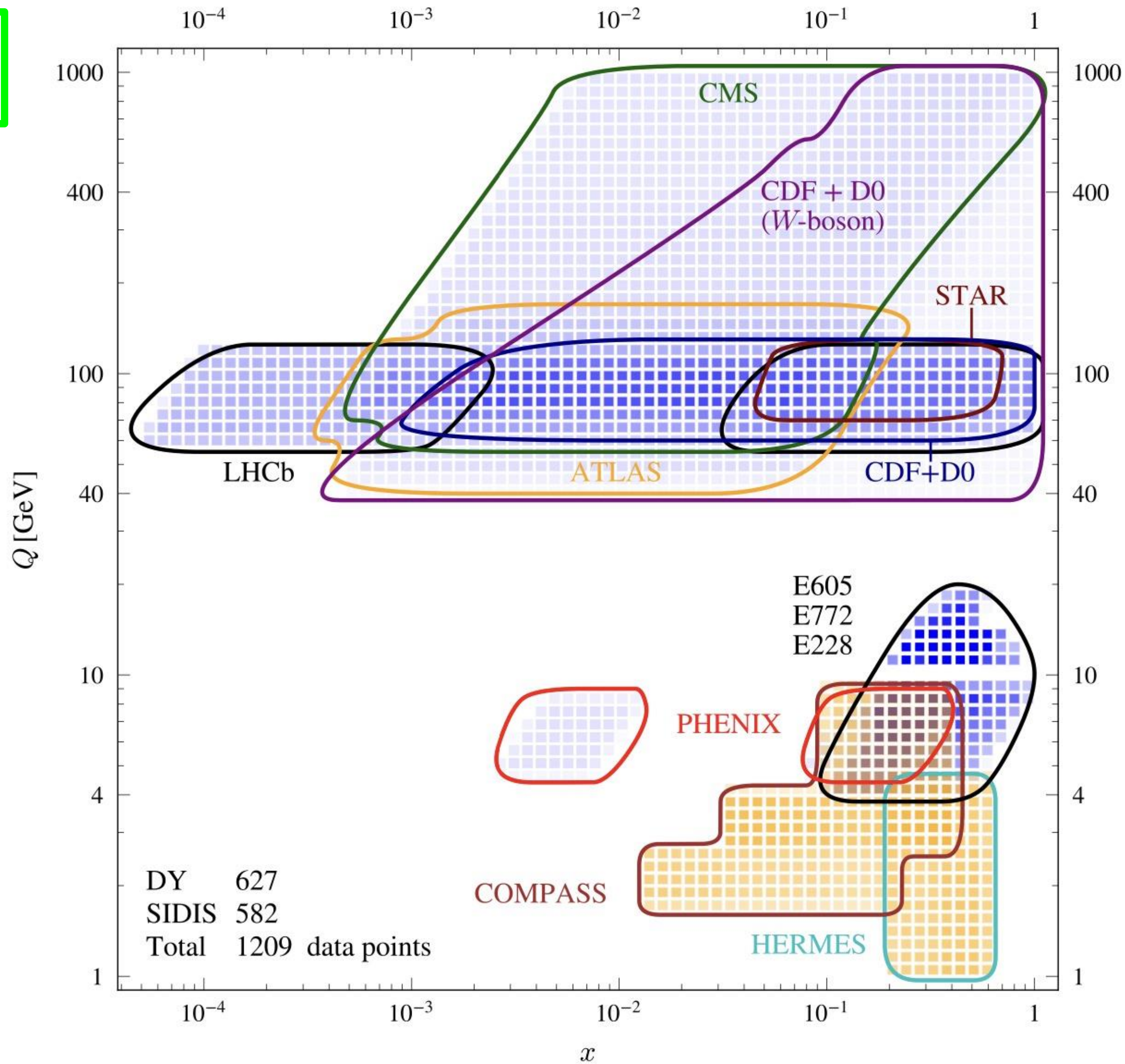
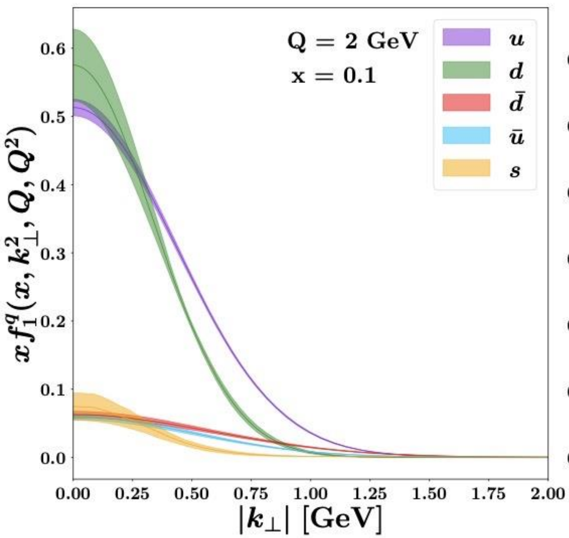
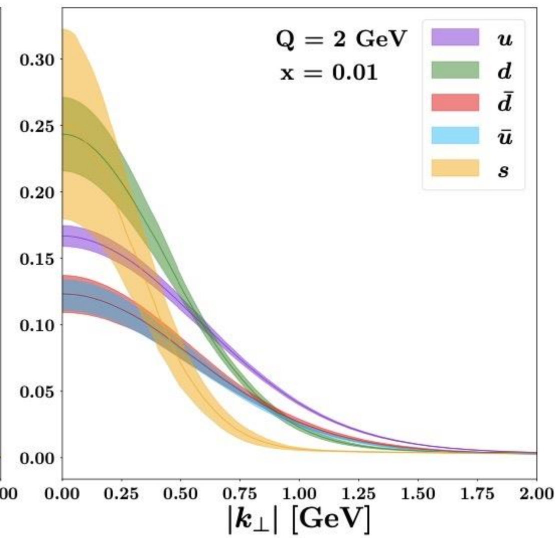
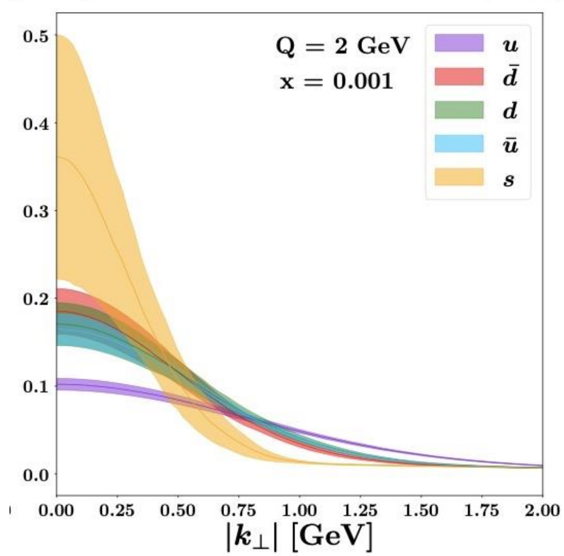


Figure from Moos, et al. (2025)

Map Collaboration (2024) (Bacchetta, Bertone, Bissolotti, Bozzi, Cerutti, Delcarro, Radici, Rossi, Signori)
CSS w/ b*, Drell-Yan+SIDIS, evolution@N^{3}LL, matching TMD to collinear@NNLO,
flavor-dependent NP TMD



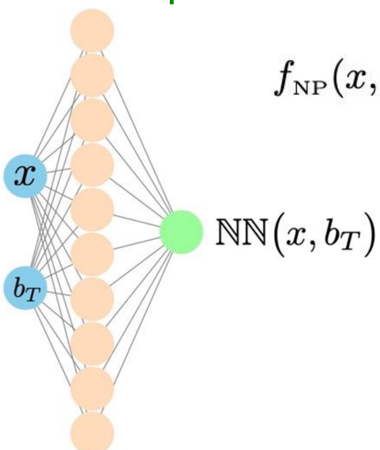




		N^{3}LL		
Data set	$N_{\rm dat}$	χ_D^2	χ_λ^2	χ_0^2
Tevatron total	71	1.10	0.07	1.17
LHCb total	21	3.56	0.96	4.52
ATLAS total	72	3.54	0.82	4.36
CMS total	78	0.38	0.05	0.43
PHENIX 200	2	2.76	1.04	3.80
STAR 510	7	1.12	0.26	1.38
DY collider total	251	1.37	0.28	1.65
E288 200 GeV	30	0.13	0.40	0.53
E288 300 GeV	39	0.16	0.26	0.42
E288 400 GeV	61	0.11	0.08	0.19
E772	53	0.88	0.20	1.08
E605	50	0.70	0.22	0.92
DY fixed-target total	233	0.63	0.31	0.94
DY total	484	1.02	0.29	1.31
HERMES total	344	0.81	0.24	1.05
COMPASS total	1203	0.67	0.27	0.94
SIDIS total	1547	0.70	0.26	0.96
Total	**2031**	**0.81**	**0.27**	**1.08**

Map Collaboration (2025) (Bacchetta, Bertone, Bissolotti, Cerutti, Radici, Rodini, Rossi)

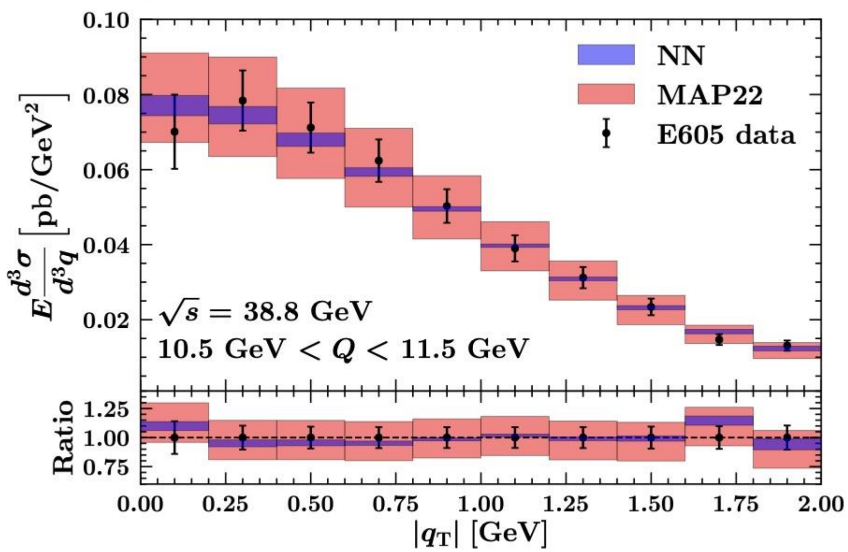
Neural Network, CSS w/ b*, Drell-Yan only, evolution@$\mathrm{N^3LL}$, matching TMD to collinear@NNLO, flavor-independent NP TMD

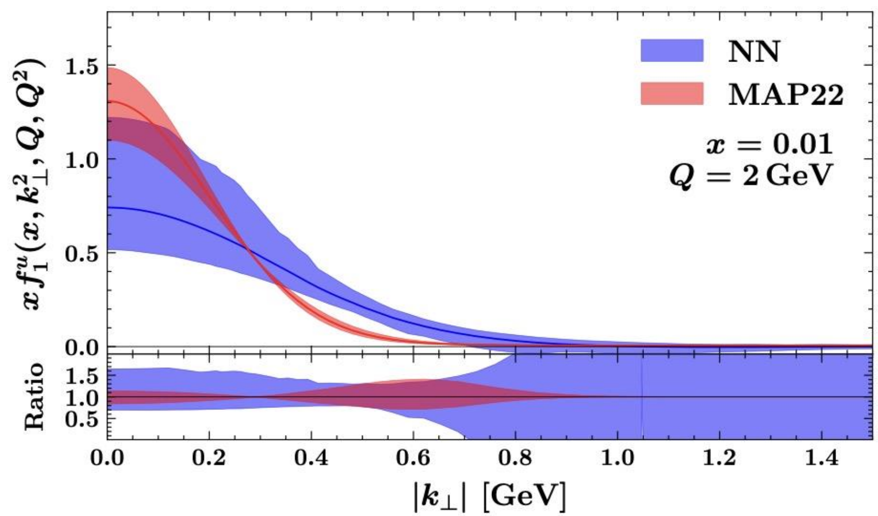


$$f_{\mathrm{NP}}(x, b_T; \zeta) = \frac{\mathbb{NN}(x, b_T)}{\mathbb{NN}(x, 0)} \exp\left[-g_2^2 b_T^2 \log\left(\frac{\zeta}{Q_0^2}\right)\right]$$

neural net for "intrinsic" part of NP TMD: (x, b_T) as input, 10 nodes in hidden layer

Experiment	N_{dat}	$\bar{\chi}^2$ $(\bar{\chi}_D^2 + \bar{\chi}_\lambda^2)$	
		NN	MAP22
Fixed-target	233	1.08 (0.98 + 0.10)	0.91 (0.70 + 0.21)
RHIC	7	1.11 (1.03 + 0.07)	1.45 (1.37 + 0.08)
Tevatron	71	0.80 (0.73 + 0.06)	1.20 (1.17 + 0.04)
LHCb	21	0.98 (0.88 + 0.10)	1.25 (1.05 + 0.20)
CMS	78	0.40 (0.38 + 0.02)	0.41 (0.35 + 0.06)
ATLAS	72	1.38 (1.09 + 0.29)	3.51 (3.03 + 0.49)
Total	482	0.97 (0.86 + 0.11)	1.28 (1.09 + 0.20)





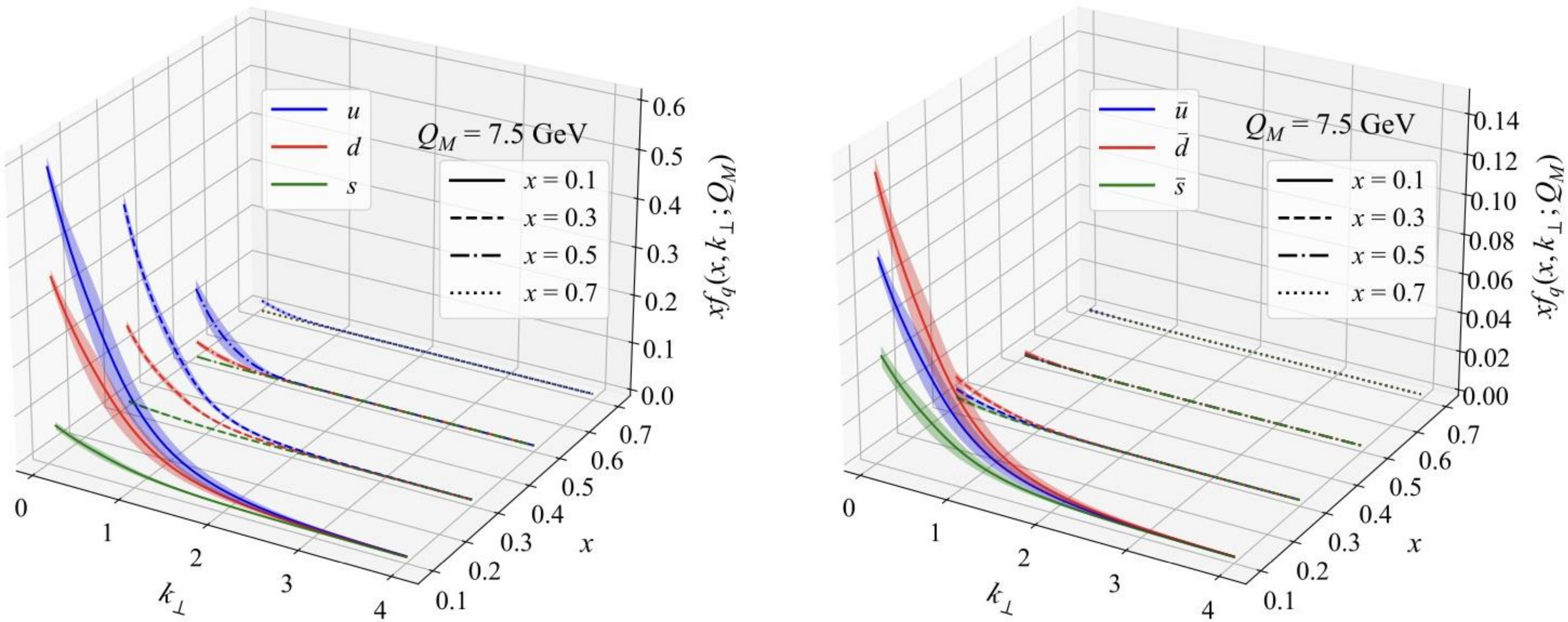
Larger uncertainties in the TMD with NN fit, but reduced uncertainties at the cross-section level: NN better adapts to data and relies much less on correlated shifts to minimize χ^2

Fernando and Keller (2025)

Neural Network, Drell-Yan (E288, E605) only, k_T –space with tree-level hard factor and no explicit logarithmic resummation, flavor-dependent profile

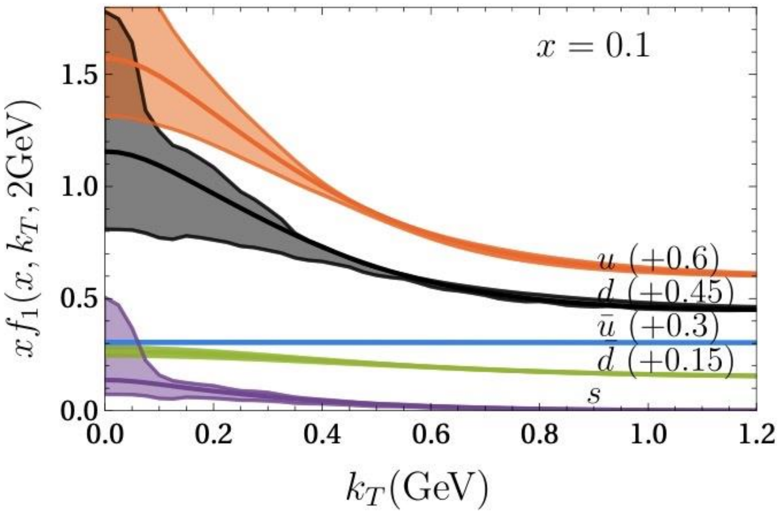
$$f_{q/h}(x, k_\perp; Q, Q^2) = f_{q/h}(x; Q^2)\, \boxed{s_h^q(x, k_\perp; Q)} \quad \text{where} \int d^2k_\perp\, s_h^q(x, k_\perp; Q) = 1,\ \forall (x, Q) \in \mathbb{R} \times \mathbb{R}_{>0}$$

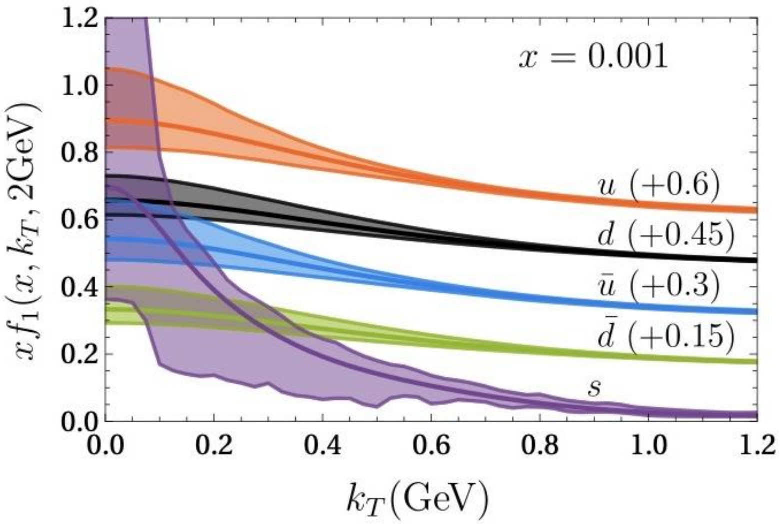
neural net with (x, k_T,Q) as input



Moos, Scimemi, Vladimirov, Zurita (2025), ζ-prescription w/ b*

Drell-Yan+SIDIS, evolution@N[4]LL, matching TMD to collinear@N[3]LO, flavor-dependent NP TMD

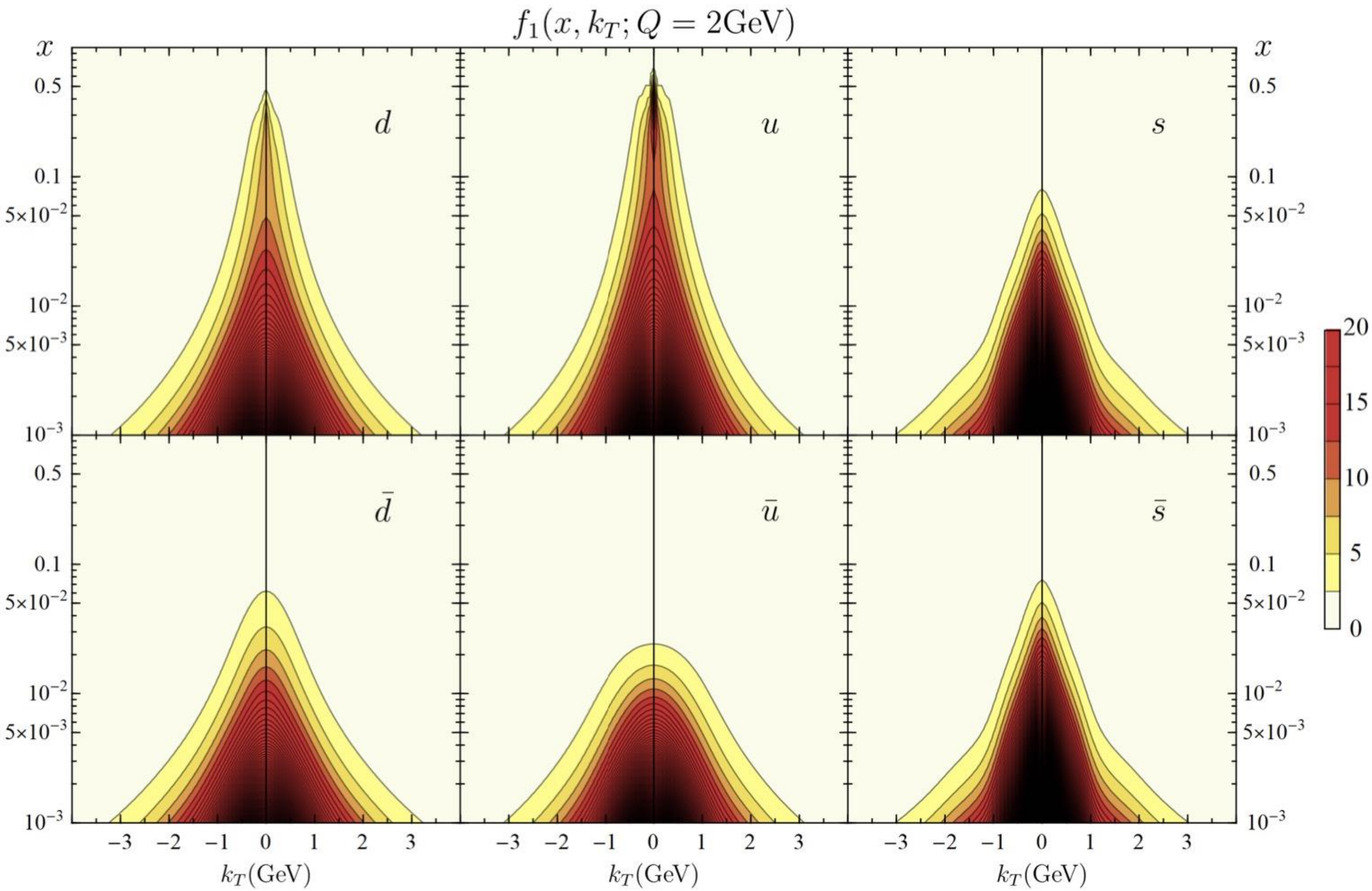




Data set	N_{pt}	χ^2_D/N_{pt}	χ^2_λ/N_{pt}	χ^2/N_{pt}
CDF	84	2.06	0.07	2.13
D0	36	2.20	0.08	2.28
ATLAS	49	1.43	0.25	1.68
CMS	113	0.69	0.13	0.82
LHCb	68	1.06	0.26	1.32
PHENIX	3	0.41	0.08	0.48
STAR	11	1.15	0.16	1.32
DY collider total:	364	1.34	0.15	1.49
E228	175	0.67	0.02	0.69
E772	35	1.25	0.12	1.37
E605	53	0.31	0.12	0.42
DY fixed-target total:	263	0.67	0.05	0.73
DY total:	627	1.06	0.11	1.17
HERMES π^+	48	1.62	0.08	1.70
HERMES π^-	48	1.17	0.12	1.29
HERMES K^+	48	0.47	0.00	0.47
HERMES K^-	48	1.31	0.03	1.34
COMPASS h^+	195	0.65	0.02	0.67
COMPASS h^-	195	0.91	0.00	0.91
SIDIS total:	582	0.90	0.02	0.92
Total:	1209	0.98	0.07	1.05

Moos, Scimemi, Vladimirov, Zurita (2025), ζ-prescription w/ b*

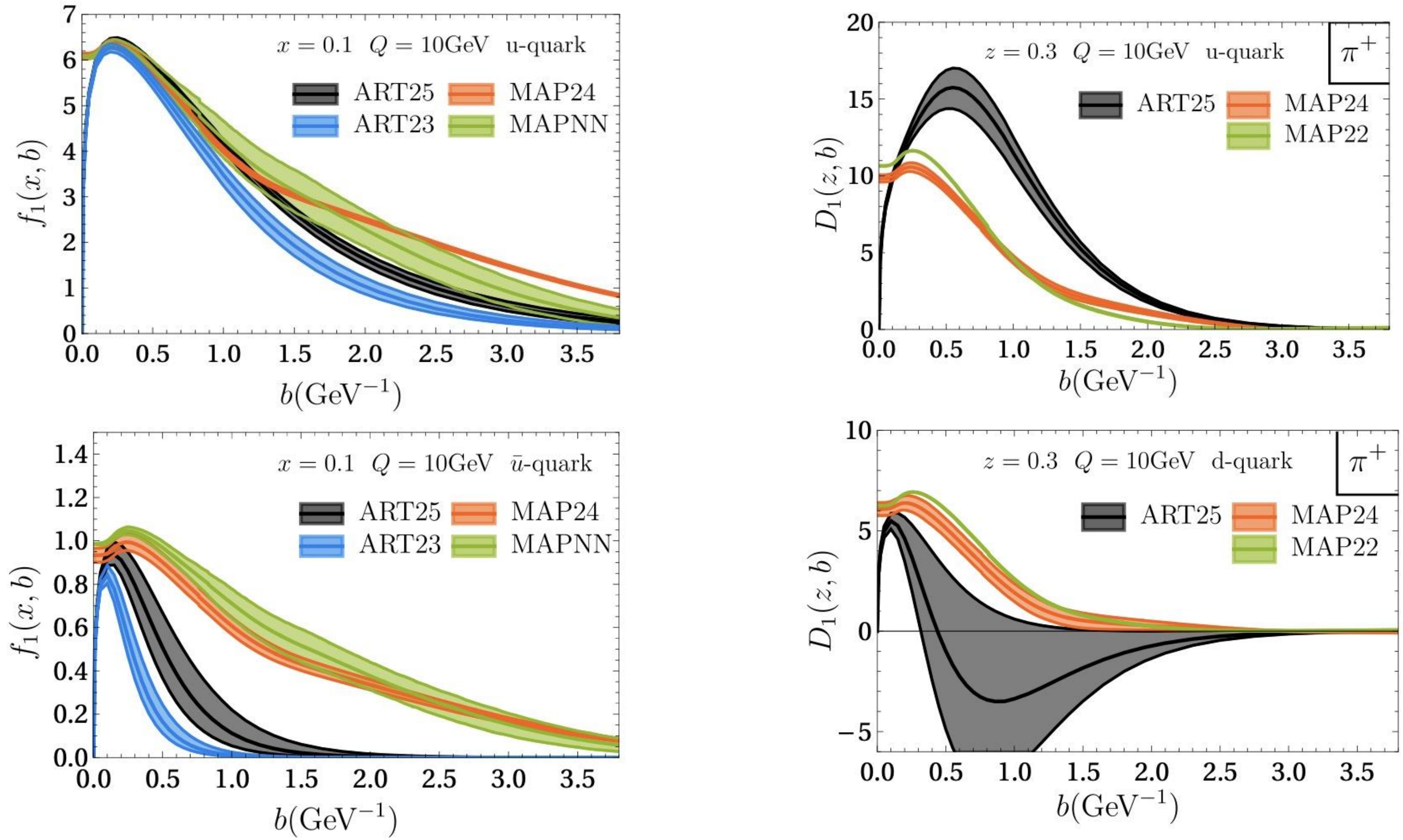
Drell-Yan+SIDIS, evolution@N^4LL, matching TMD to collinear@N^3LO, flavor-dependent NP TMD



3D imaging!

Moos, Scimemi, Vladimirov, Zurita (2025), ζ-prescription w/ b*

Drell-Yan+SIDIS, evolution@N^{4}LL, matching TMD to collinear@N^{3}LO, flavor-dependent NP TMD

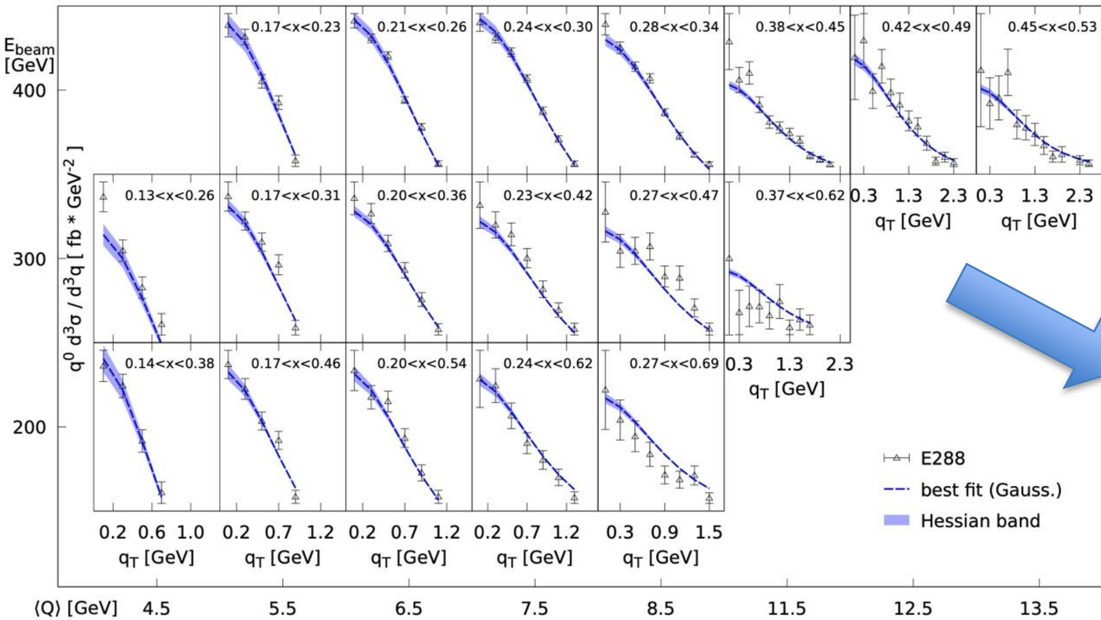


TMD PDFs and TMD FFs have noticeable discrepancies between groups

Can different approaches be reconciled to yield a more uniform picture of unpolarized TMDs?

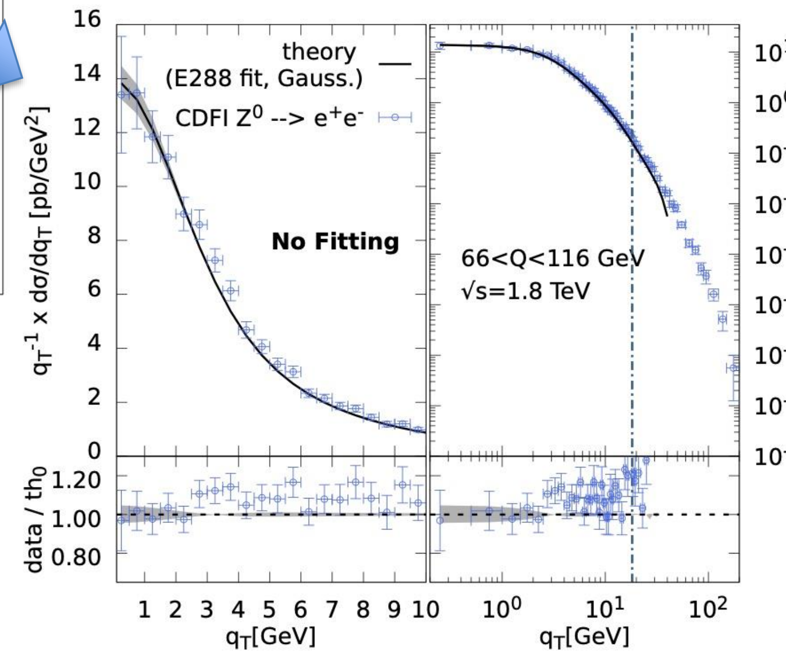
Aslan, Boglione, Gonzalez-Hernandez, Rainaldi, Rogers, Simonelli (2024)

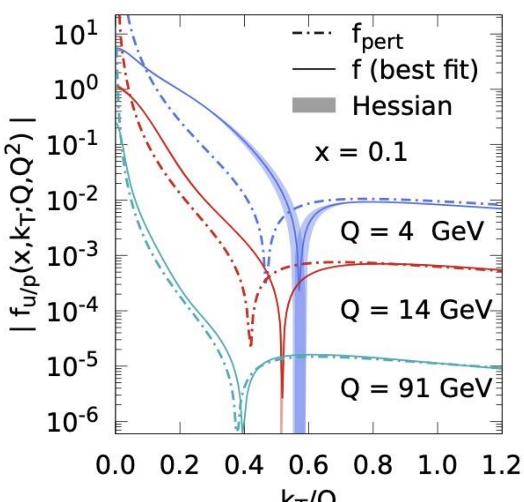
HSO approach (CSS, no b^*), Drell-Yan (E288, E605, CDF, D0) only, Proof-of-principle analysis

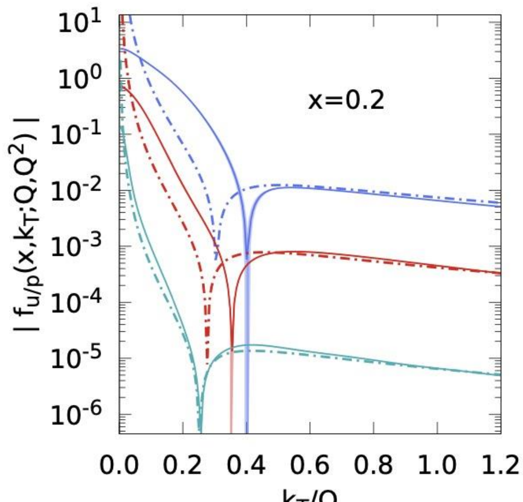


Gaussian fits

	E288 (130 pts.)	E605 (52 pts.)
$\chi^2_{\rm dof}$	1.04	1.68
M_0 (GeV)	0.0576	0.404
M_1 (GeV)	0.403	0.290
b_K	2.12	0.744
N(nuisance)	1.29	1.28







$$\tilde{f}_1(x,b_T;Q^2,\mu_Q) \sim \boxed{\left(\tilde{C}^{f_1}(x/\hat{x},b_*(b_T);\mu_{b_*}^2,\mu_{b_*},\alpha_s(\mu_{b_*}))\otimes \boldsymbol{f_1(\hat{x};\mu_{b_*})}\right)}$$
$$\times\ \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)-S_{NP}^{f_1}(b_T,Q)\right]$$

Operator Product Expansion

Need input for the collinear PDFs

$$\tilde{f}_1(x, b_T; Q^2, \mu_Q) \sim \boxed{\left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1(\hat{x}; \mu_{b_*})}\right)}$$
$$\times \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

Operator Product Expansion

Need input for the collinear PDFs

Use $f_1(x)$ from PDF global analysis groups (e.g., CT, NNPDF, MSHT, JAM) and propagate the error into the TMD extraction

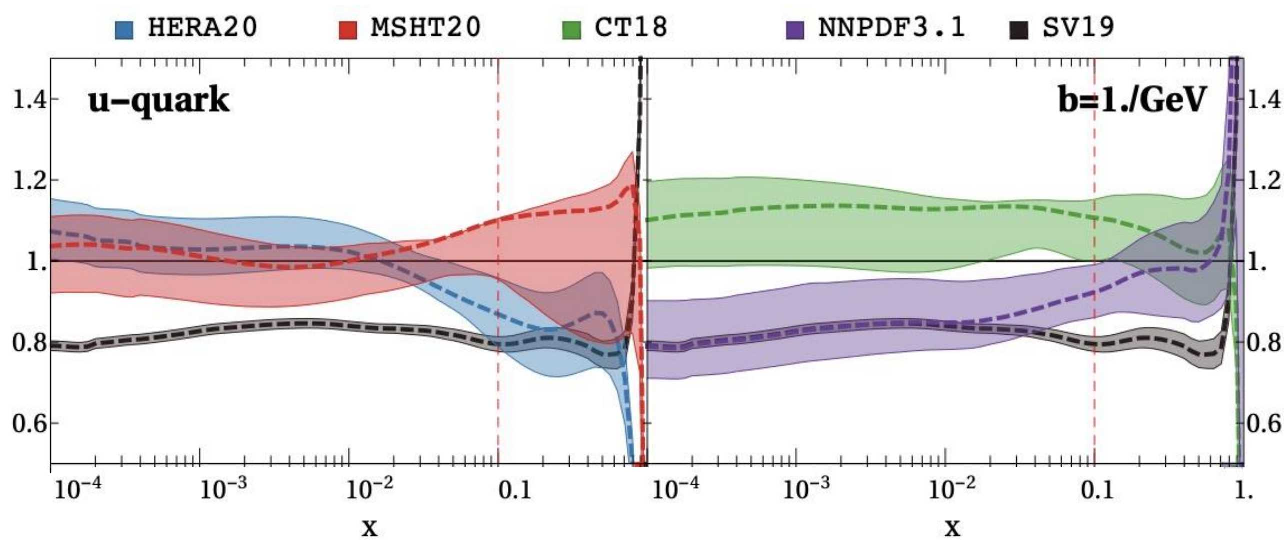
$$\tilde{f}_1(x, b_T; Q^2, \mu_Q) \sim \boxed{\left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1(\hat{x}; \mu_{b_*})}\right)} \times \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

Operator Product Expansion

Need input for the collinear PDFs

Use $f_1(x)$ from PDF global analysis groups (e.g., CT, NNPDF, MSHT, JAM) and propagate the error into the TMD extraction

Bury, Hautmann, Leal-Gomez, Scimemi, Vladimirov, Zurita (2022)



TMDs (and CS kernel) extracted with different PDF sets still display significant differences

$$\tilde{f}_1(x, b_T; Q^2, \mu_Q) \sim \boxed{\left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1(\hat{x}; \mu_{b_*})}\right)}$$
$$\times \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

Operator Product Expansion

Need input for the collinear PDFs

Use $f_1(x)$ from PDF global analysis groups (e.g., CT, NNPDF, MSHT, JAM) and propagate the error into the TMD extraction

Extract PDFs and TMDs *simultaneously* in the same global analysis

JAM Collaboration (2025), ζ-prescription w/ b*

(Barry, Prokudin, Anderson, Cocuzza, Gamberg, Melnitchouk, Moffat, Pitonyak, Qiu, Sato, Vladimirov, Whitehill)

TMD: Drell-Yan only **AND** COLLINEAR: DIS, Drell-Yan, W-boson, W+charm, jet, evolution@N^2LL, matching TMD to collinear@NLO, flavor-dependent NP TMD

Process	$N_{\rm pts}$	$\chi^2/N_{\rm pts}$ (Z-score)	
Collinear		PDF only	TMD+PDF
DIS – fixed target	2501	1.02 (+0.82)	1.02 (+0.59)
– HERA	1185	1.27 (+6.01)	1.27 (+6.11)
DY	205	1.27 (+2.54)	1.26 (+2.50)
W-ℓ asymmetry	70	0.80 (−1.20)	0.78 (−1.35)
W asymmetry	27	1.12 (+0.51)	1.13 (+0.53)
Z rapidity	56	1.09 (+0.55)	1.11 (+0.60)
jets	198	1.00 (+0.00)	0.98 (−0.13)
$W+c$	37	0.65 (−1.66)	0.62 (−1.84)
Total collinear	4279	1.10 (+4.31)	1.09 (+4.14)
TMD		TMD only	TMD+PDF
DY – E288, E605, E772	224	1.36 (+3.44)	1.31 (+3.02)
– CDF, D0	80	1.06 (+0.45)	1.11 (+0.71)
– STAR, PHENIX	12	1.15 (+0.47)	1.20 (+0.60)
– ATLAS	30	2.07 (+3.29)	1.78 (+2.55)
– CMS	64	1.18 (+1.03)	0.92 (−0.39)
– LHCb	26	0.53 (−1.97)	0.50 (−2.12)
Total TMD	436	1.27 (+3.72)	1.20 (+2.76)
Total	**4715**		**1.10 (+4.79)**

JAM Collaboration (2025), ζ-prescription w/ b*

(Barry, Prokudin, Anderson, Cocuzza, Gamberg, Melnitchouk, Moffat, Pitonyak, Qiu, Sato, Vladimirov, Whitehill)

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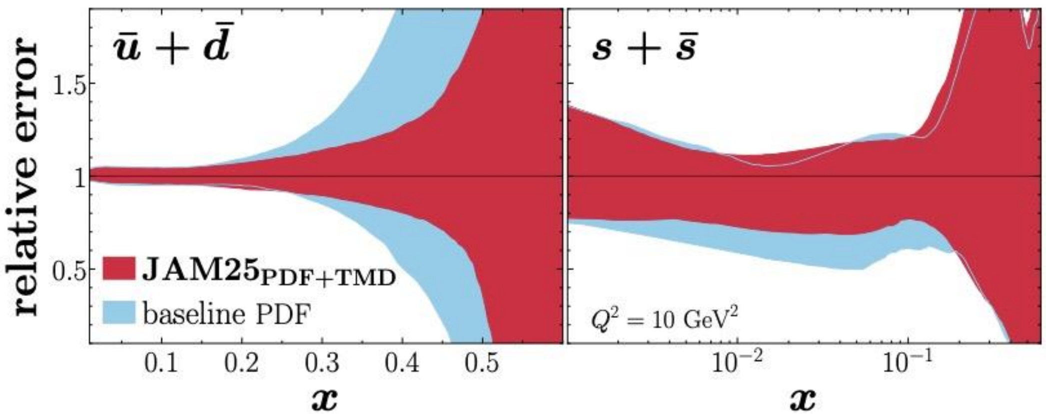
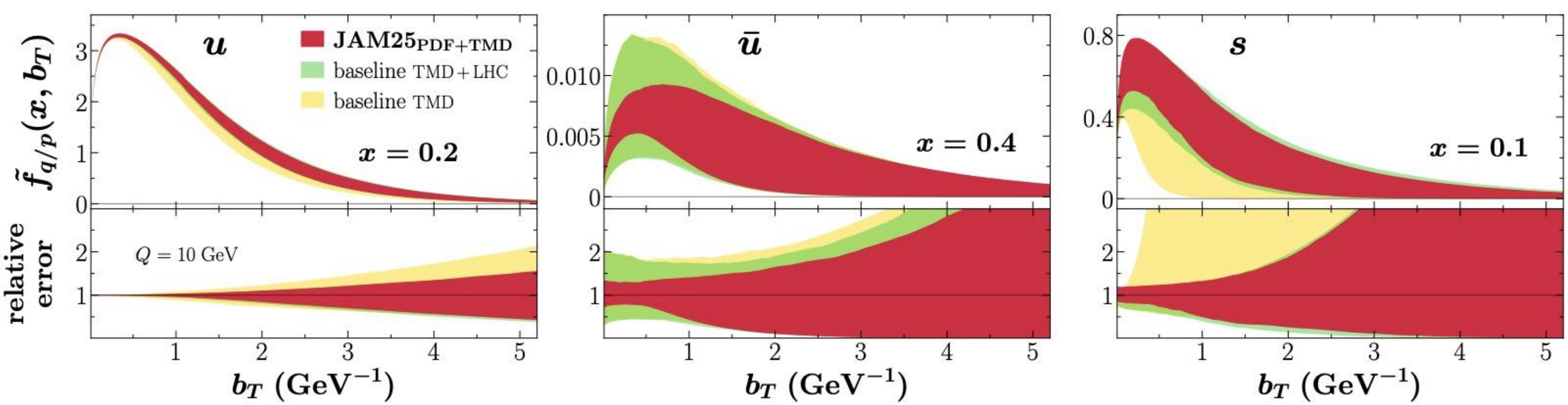
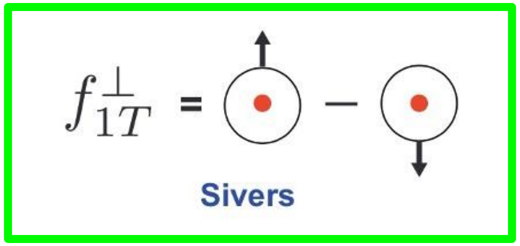
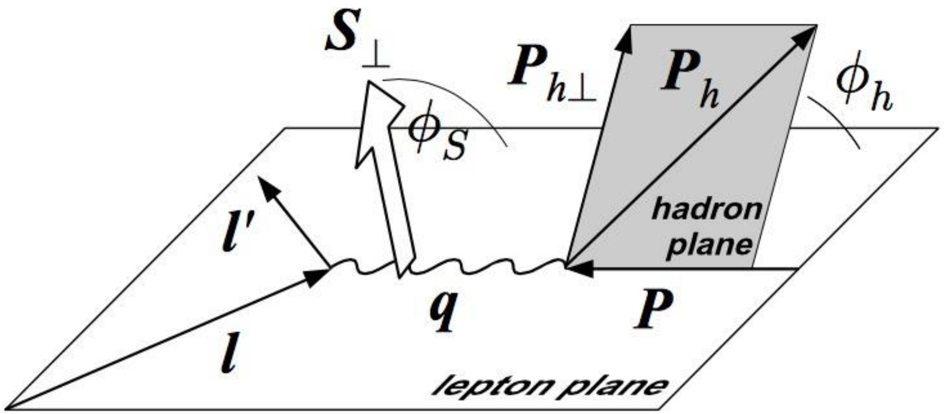


Simultaneous global analysis of TMD **and** collinear observables improves our knowledge of both TMDs and PDFs, especially in the sea quark sector, as well as the CS kernel

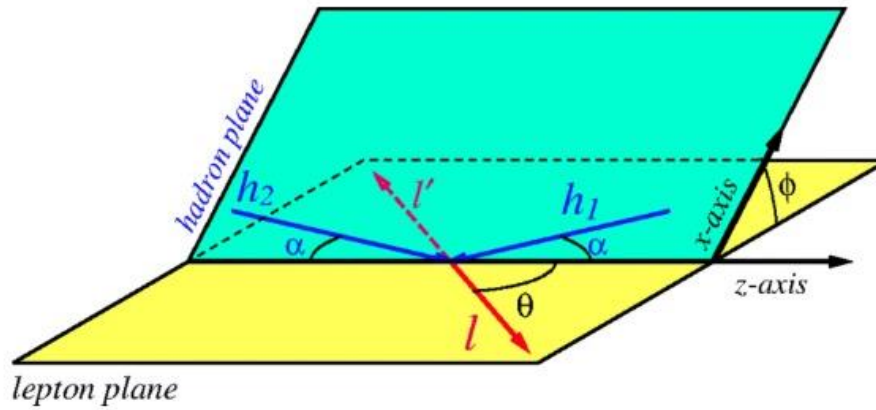




$$\ell p^{\uparrow} \to \ell h X$$



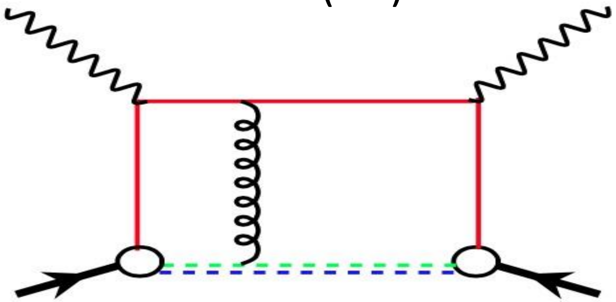
$$\{\pi, p\} p^{\uparrow} \to \{\ell^+\ell^-, W^{\pm}, Z\} X$$



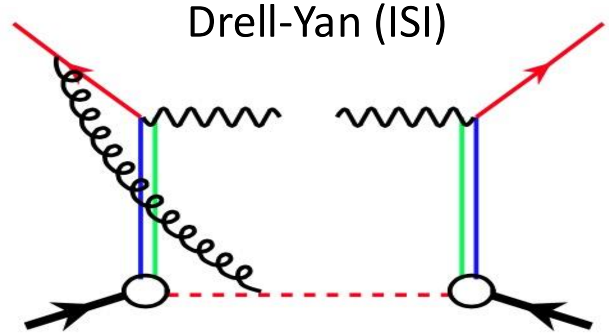
$$F_{UT}^{\sin(\phi_h - \phi_S)} = \mathcal{C}\left[-\frac{\hat{h} \cdot \vec{k}_T}{M} \boldsymbol{f_{1T}^{\perp}} D_1 \right]$$

$$F_{TU}^{\sin\phi} = \mathcal{C}\left[-\frac{\hat{h} \cdot \vec{k}_{aT}}{M_a} \boldsymbol{f_{1T}^{\perp}}\, \bar{f}_1 \right]$$

SIDIS (FSI)

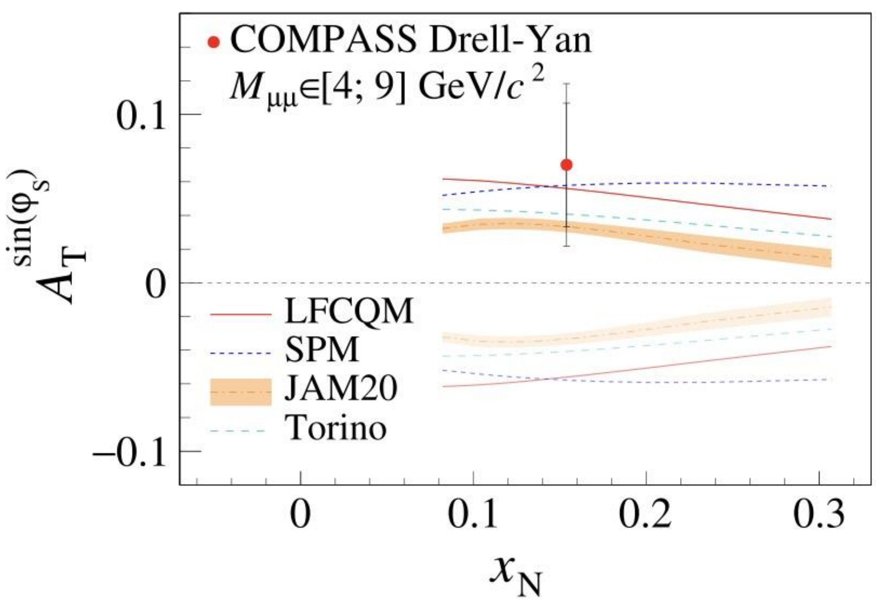


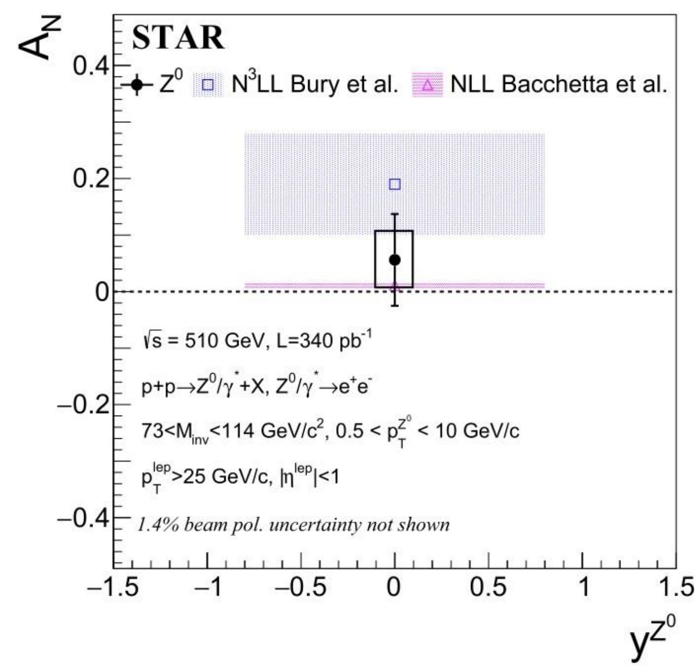
Drell-Yan (ISI)



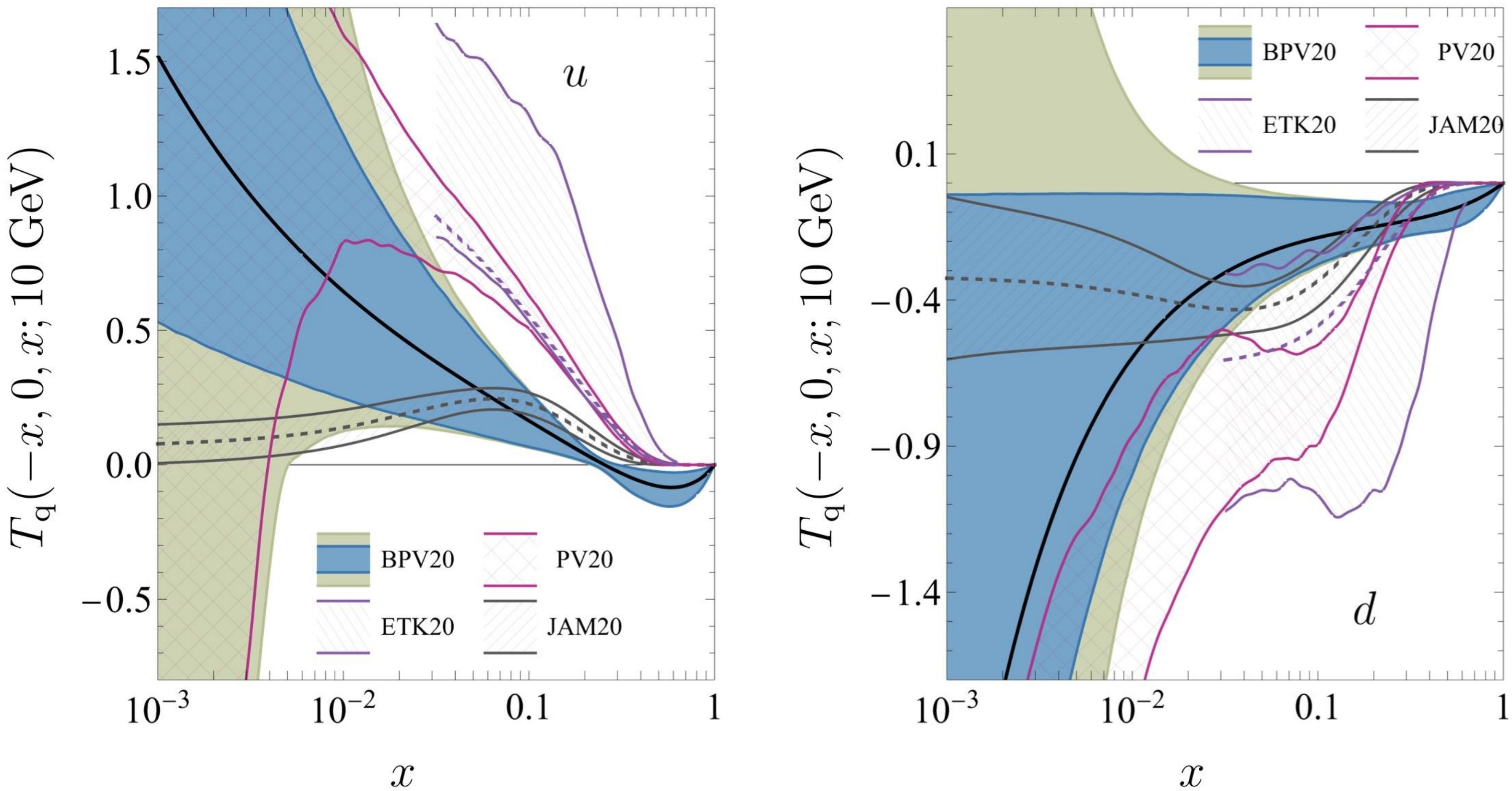
$$f_{1T}^{\perp}(x,\vec{k}_T^2)\big|_{SIDIS} = -f_{1T}^{\perp}(x,\vec{k}_T^2)\big|_{DY}$$ (Collins (2002))

The "sign change" of the Sivers function is an important *prediction* from our current understanding of QCD and TMD factorization





Bury, Prokudin, Vladimirov (2021)



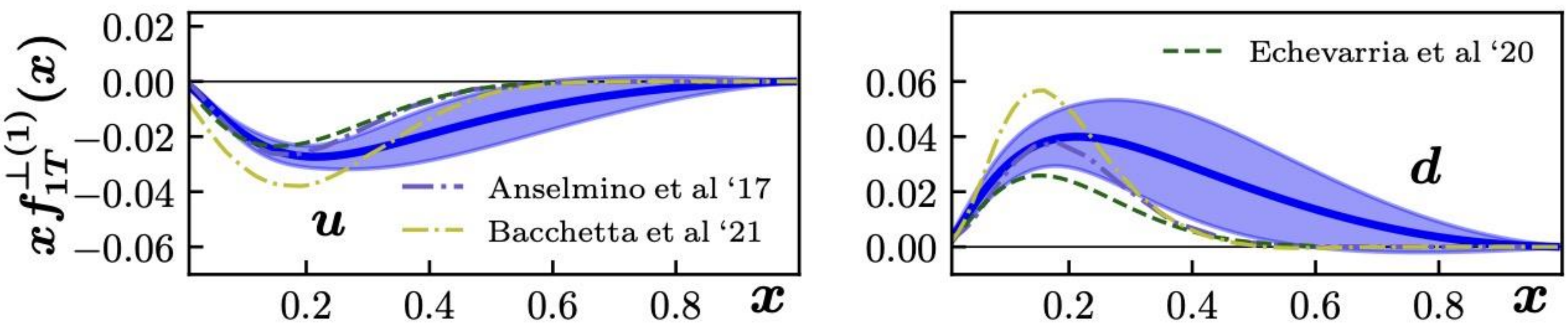
BPV20 – Bury, Prokudin, Vladimirov (2021)

PV20 – Bacchetta, et al. (2022)

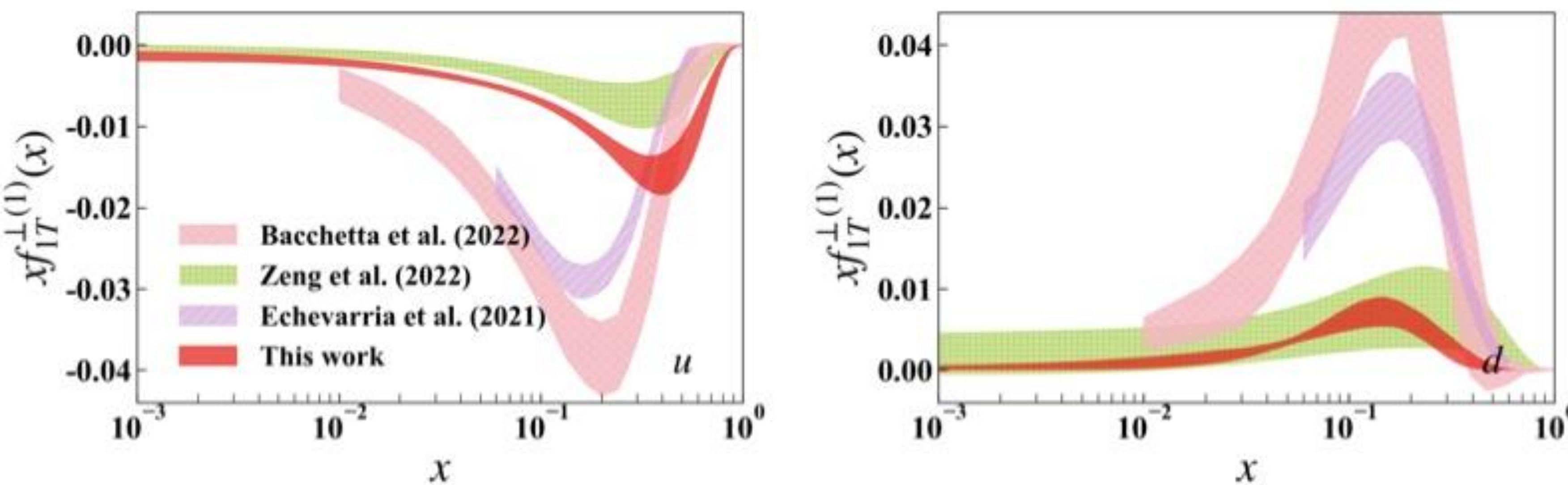
ETK20 – Echevarria, Terry, Kang (2020)

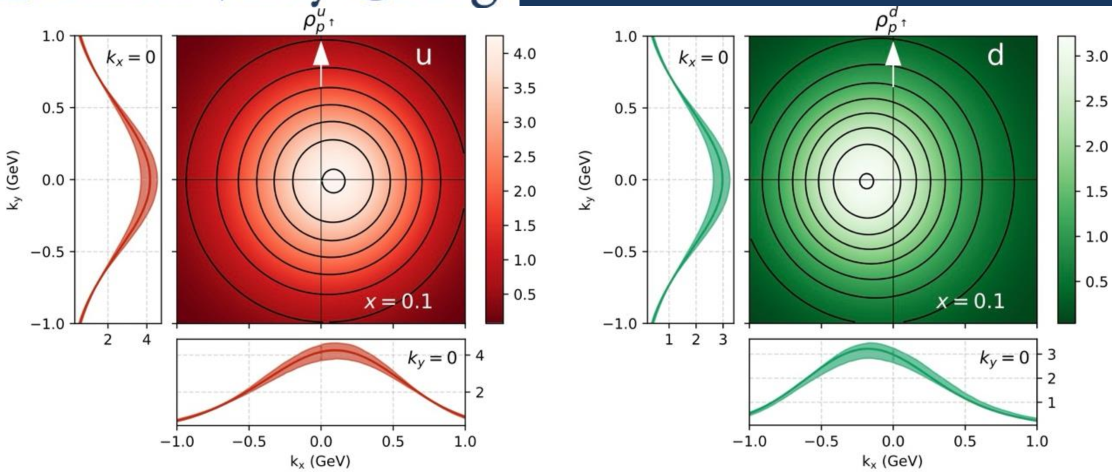
JAM20 – Cammarota, et al. (2020)

Gamberg, et al. (2022) – JAM

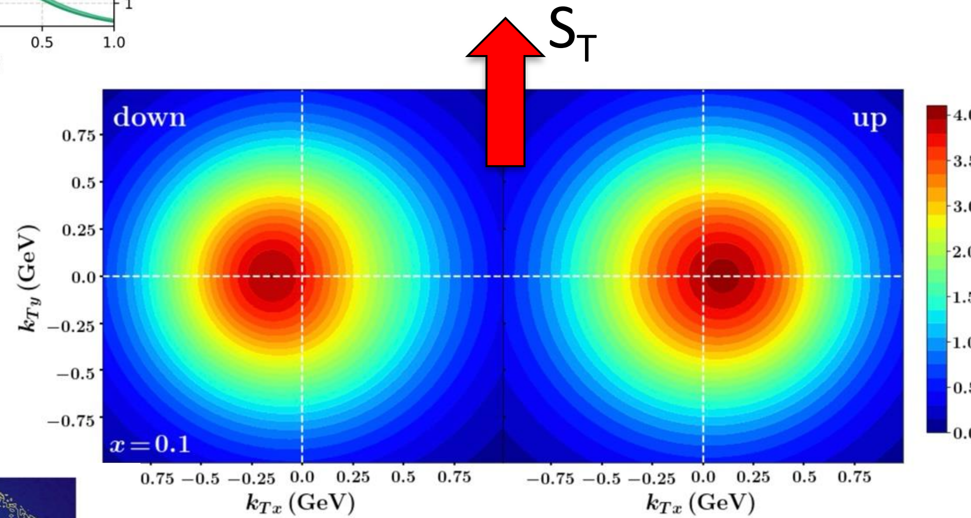


Zheng, Dong, Liu, Sun, Zhao (2024) – TNTC



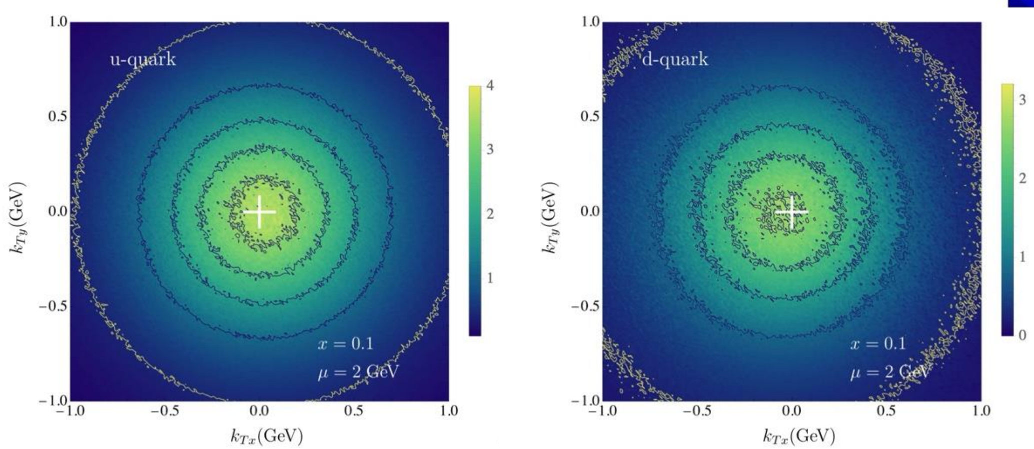


Bacchetta, et al. (2020)



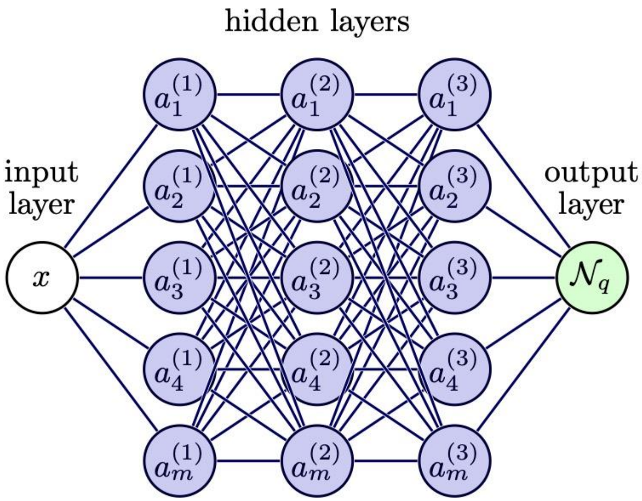
Based on Cammarota, et al. (2020)

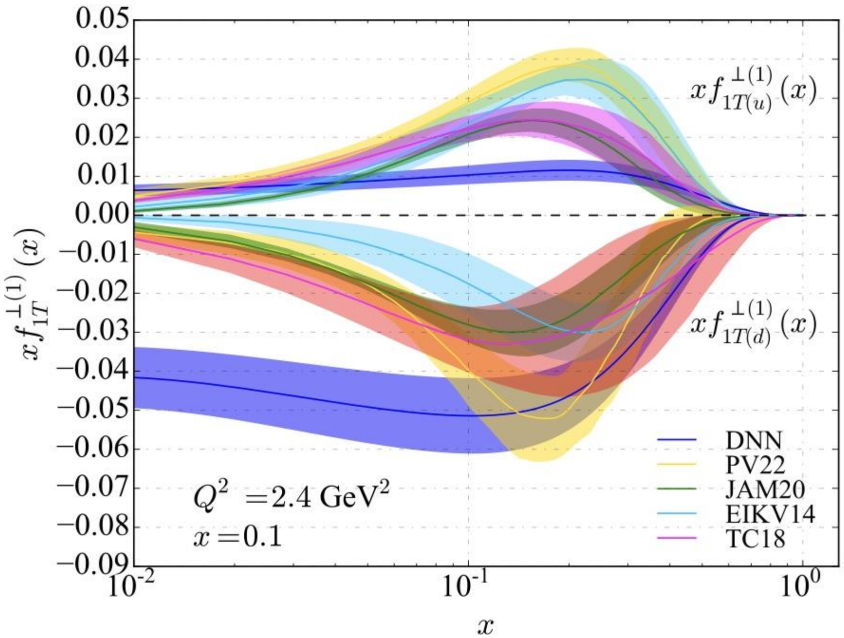
3D imaging!



Bury, Prokudin, Vladimirov (2021)

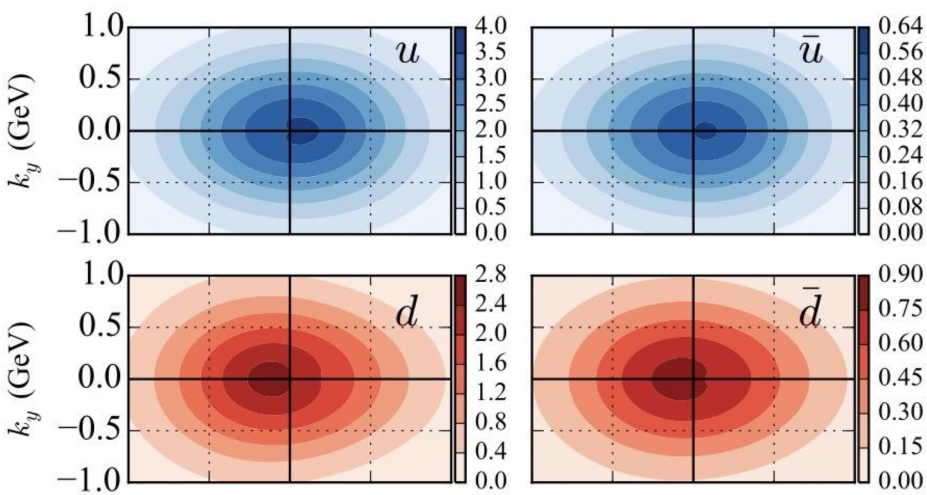
Fernando and Keller (2023)





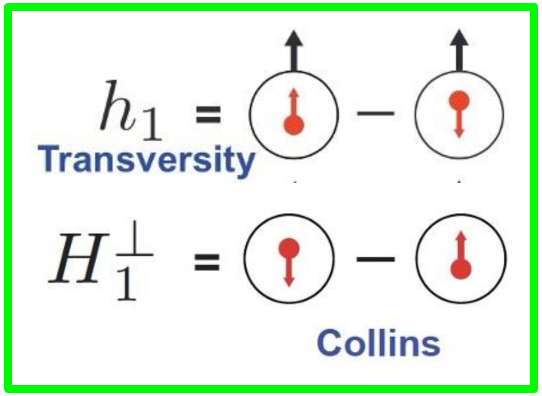
DNN – Fernando and Keller (2023)

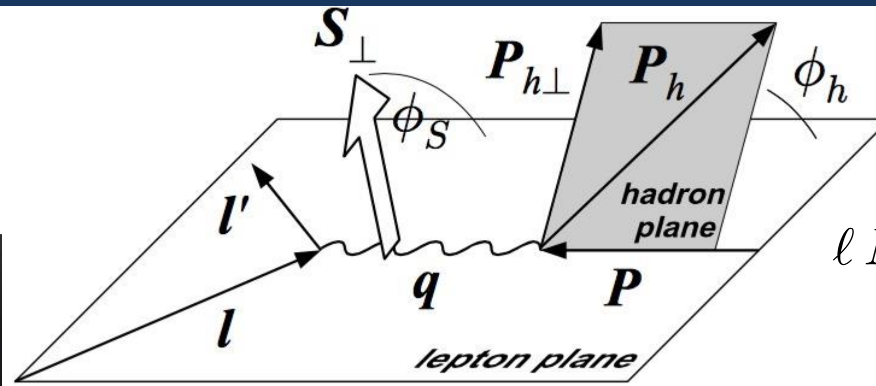
TC18 – Boglione, et al. (2018)



Extractions of the Sivers function are mostly consistent between groups

Need to better understand the sea quarks. Also, evolution is nontrivial (twist-3).

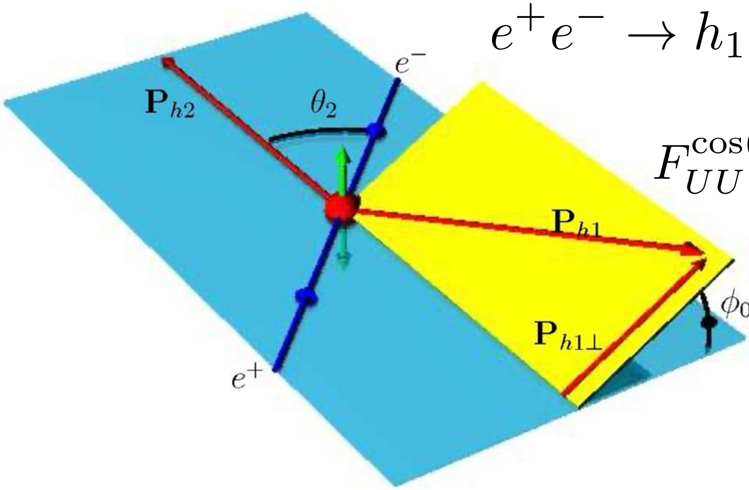




$\ell\, N^\uparrow \to \ell\, h\, X$

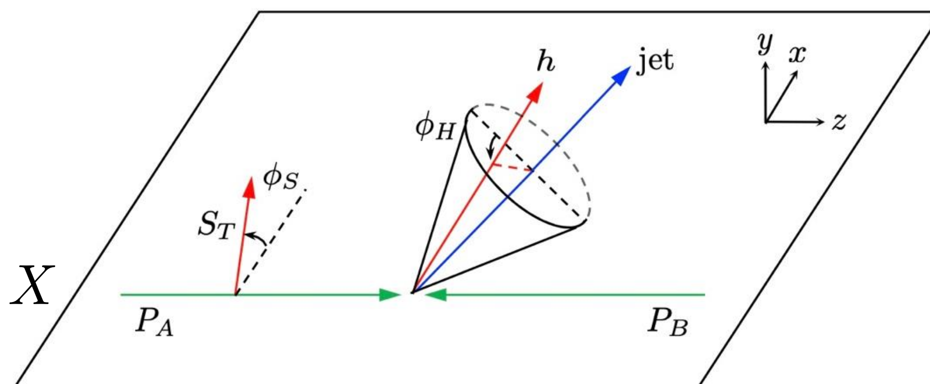
$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{p}_\perp}{M_h}\boldsymbol{h_1}\boldsymbol{H_1^\perp}\right]$$

$e^+e^- \to h_1 h_2 X$



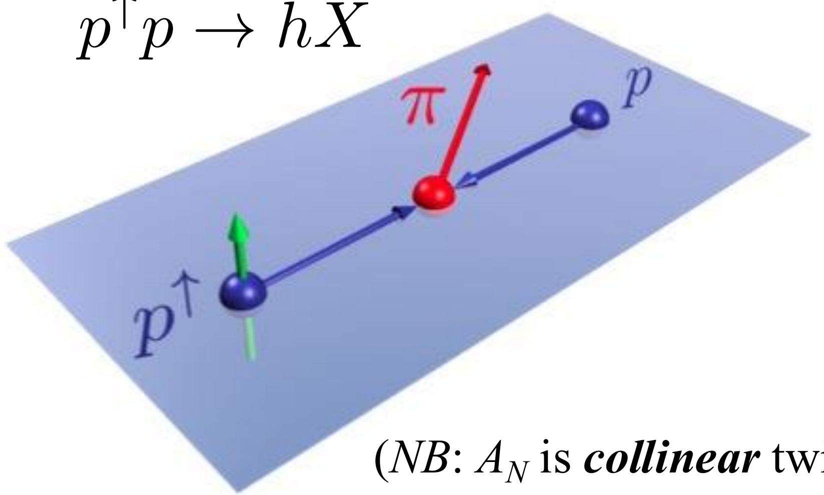
$$F_{UU}^{\cos(2\phi_0)} = \mathcal{C}\left[\frac{2\hat{h}\cdot\vec{p}_{a\perp}\,\hat{h}\cdot\vec{p}_{b\perp} - \vec{p}_{a\perp}\cdot\vec{p}_{b\perp}}{M_a M_b}\,\boldsymbol{H_1^\perp}\,\boldsymbol{\bar{H}_1^\perp}\right]$$

$p^\uparrow p \to (h\,jet)\,X$



$$F_{UT}^{\sin(\phi_S-\phi_H)} \sim H_{ab\to c}^{\text{Collins}}(\hat{s},\hat{t},\hat{u}) \otimes \boldsymbol{h_1^a(x_1)} \otimes f_1^b(x_2) \otimes (j_\perp/(z_h M_h))\boldsymbol{H_1^{\perp h/c}(z_h, j_\perp^2)}$$

$p^\uparrow p \to hX$

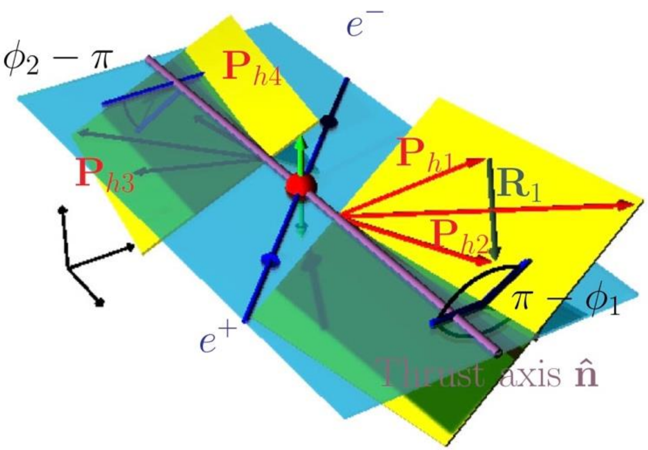


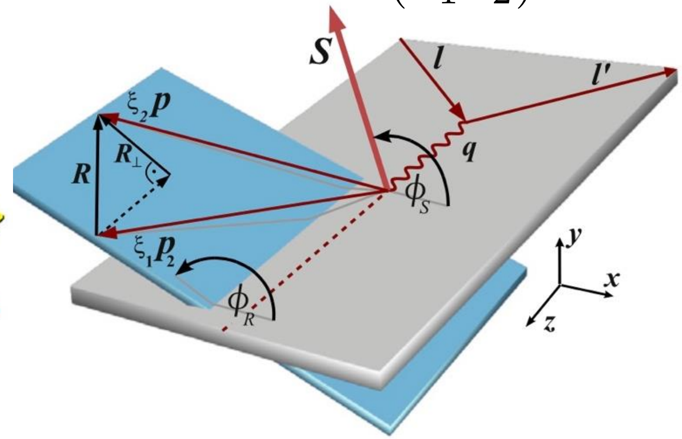
$$d\Delta\sigma(S_T) \sim \underbrace{H_{QS} \otimes f_1 \otimes \boldsymbol{F_{FT}} \otimes D_1}_{\text{Qiu-Sterman term}}$$

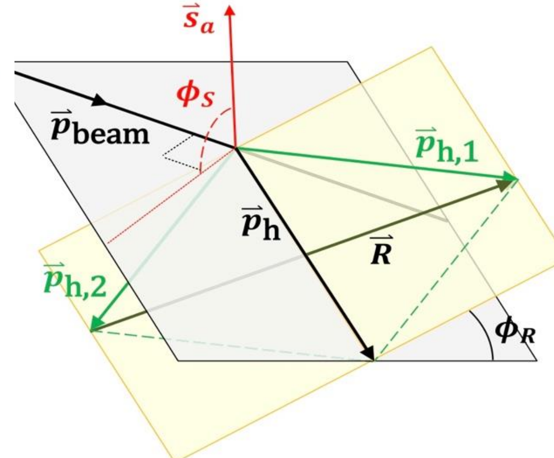
$$+ \boxed{\underbrace{H_F \otimes f_1 \otimes \boldsymbol{h_1} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}\right)}_{\text{Fragmentation term}}}$$

(NB: A_N is ***collinear*** twist-3)

$e^+e^- \to (h_1 h_2)(\bar{h}_1 \bar{h}_2)\, X$ $\qquad$ $\ell\, N^\uparrow \to \ell\,(h_1 h_2)\, X$ $\qquad$ $p^\uparrow p \to (h_1 h_2)\, X$







(*NB*: Dihadron observables are ***collinear*** twist-2)

$$a_{12R} = \frac{\sin^2\theta \sum_q e_q^2\, \boldsymbol{H_1^{\sphericalangle,q}(z, M_h^2) H_1^{\sphericalangle,\bar{q}}(\bar{z}, \overline{M}_h^2)}}{(1+\cos^2\theta)\sum_q e_q^2\, D_1^q(z, M_h^2) D_1^{\bar{q}}(\bar{z}, \overline{M}_h^2)}$$

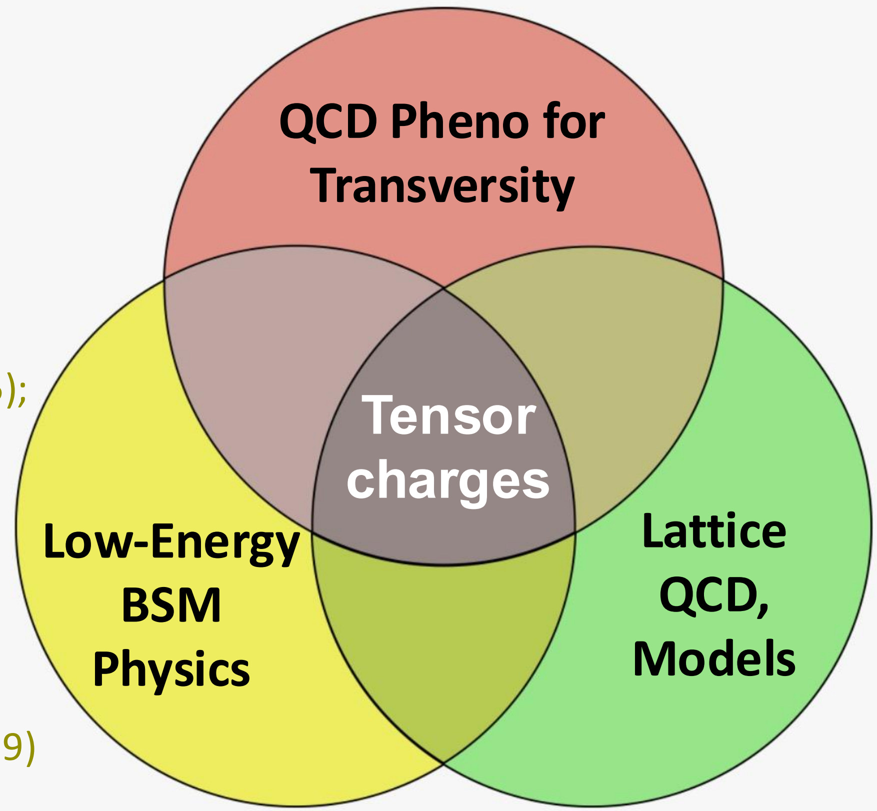
Artru-Collins asymmetry

$$A_{UT}^{\sin(\phi_R+\phi_S)} = \frac{\sum_q e_q^2\, \boldsymbol{h_1^q(x)}\, \boldsymbol{H_1^{\sphericalangle,q}(z, M_h^2)}}{\sum_q e_q^2\, f_1^q(x)\, D_1^q(z, M_h^2)}$$

$$A_{UT}^{\sin(\phi_R-\phi_S)} \sim \frac{\frac{d\Delta\hat{\sigma}_{ab^\uparrow\to c^\uparrow d}}{d\hat{t}} \otimes f_1^a(x_a) \otimes \boldsymbol{h_1^b(x_b)} \otimes \boldsymbol{H_1^{\sphericalangle,c}(z, M_h^2)}}{\frac{d\hat{\sigma}_{ab\to cd}}{d\hat{t}} \otimes f_1^a(x_a) \otimes f_1^b(x_b) \otimes D_1^c(z, M_h^2)}$$

Anselmino, et al. (2007, 2009, 2013, 2015);
Goldstein, et al. (2014); Kang, et al. (2016); Radici, et al. (2013, 2015, 2018);
Benel, et al. (2020); D'Alesio, et al. (2020); Cammarota, et al. (2020); Gamberg, et al. (2022);
Cocuzza, et al. (2024); Zheng, et al. (2024); Boglione, et al. (2024)

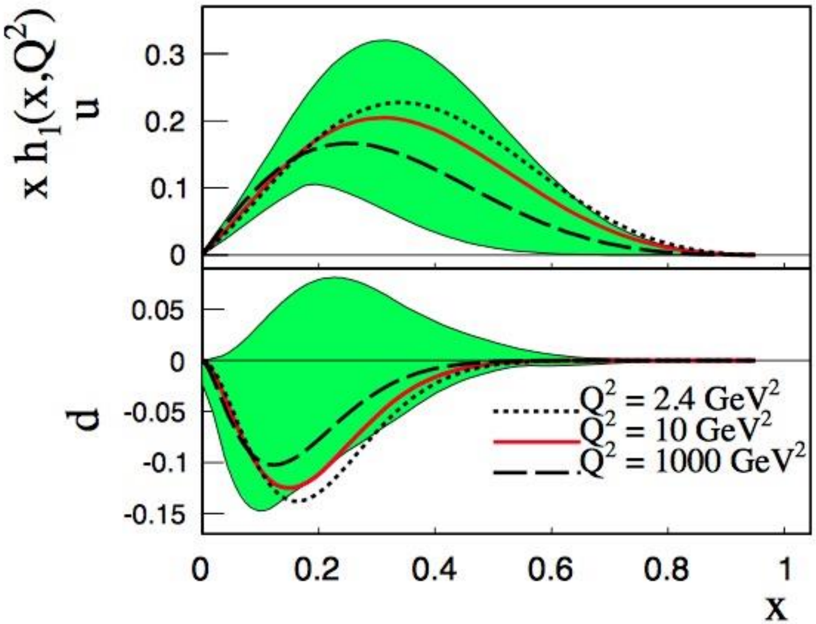
Herczeg (2001);
Erler, Ramsey-Musolf (2005);
Pospelov, Ritz (2005);
Severijns, et al. (2006);
Cirigliano, et al. (2013);
Courtoy, et al. (2015);
Yamanaka, et al. (2017);
Liu, et al. (2018);
Gonzalez-Alonso, et al. (2019)



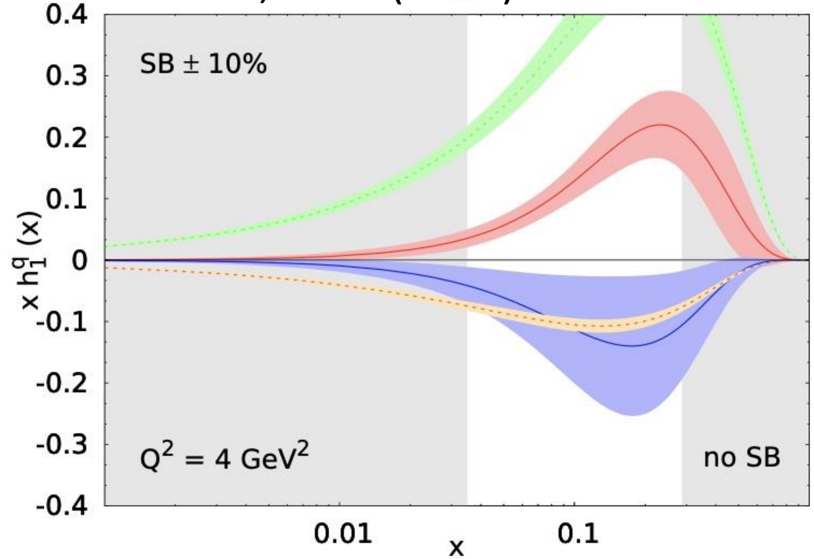
He, Ji (1995);
Barone, et al. (1997);
Schweitzer, et al. (2001);
Gamberg, Goldstein (2001);
Pasquini, et al. (2005);
Wakamatsu (2007);
Cloet, et al. (2007);
Lorce (2009);
Gupta, et al. (2018);
Yamanaka, et al. (2018);
Hasan, et al. (2019);
Alexandrou, et al. (2019, 2023);
Yamanaka, et al. (2013);
Pitschmann, et al. (2015);
Xu, et al. (2015);
Wang, et al. (2018);
Liu, et al. (2019);
Gao, et al. (2023)

$$\delta q \equiv \int_0^1 dx\,[h_1^q(x) - h_1^{\bar{q}}(x)] \qquad g_T \equiv \delta u - \delta d$$

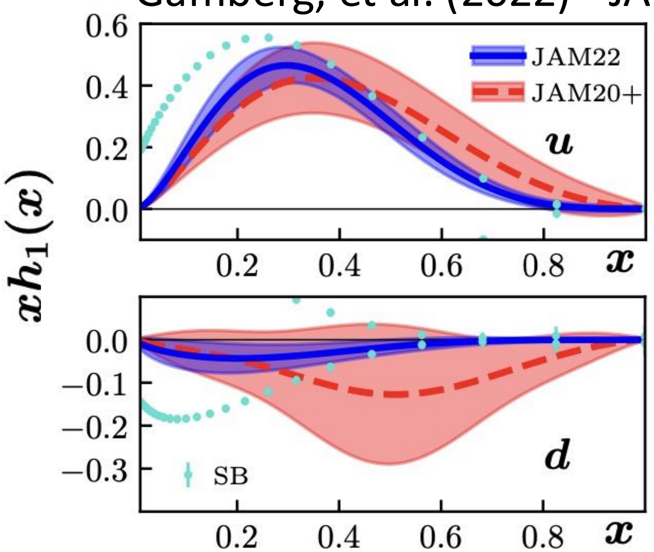
Kang, et al. (2016)



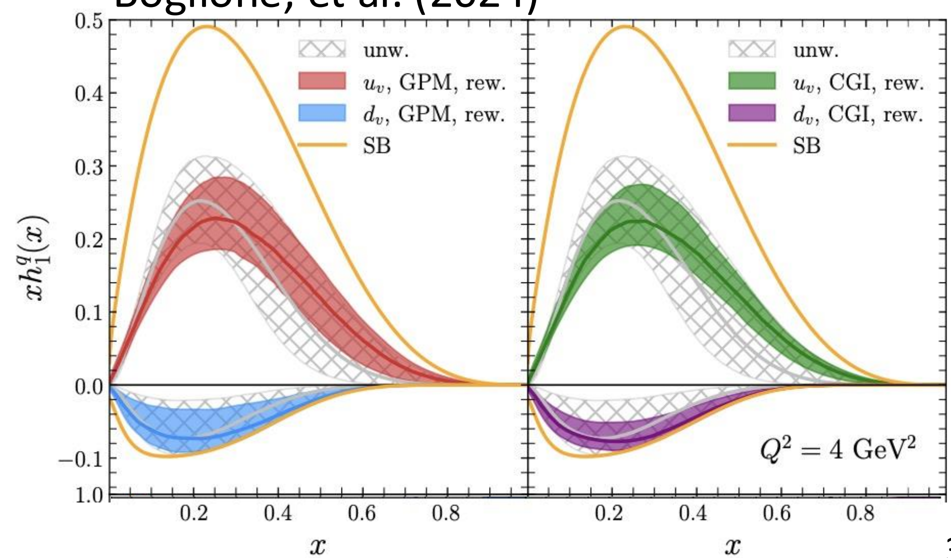
D'Alesio, et al. (2020)



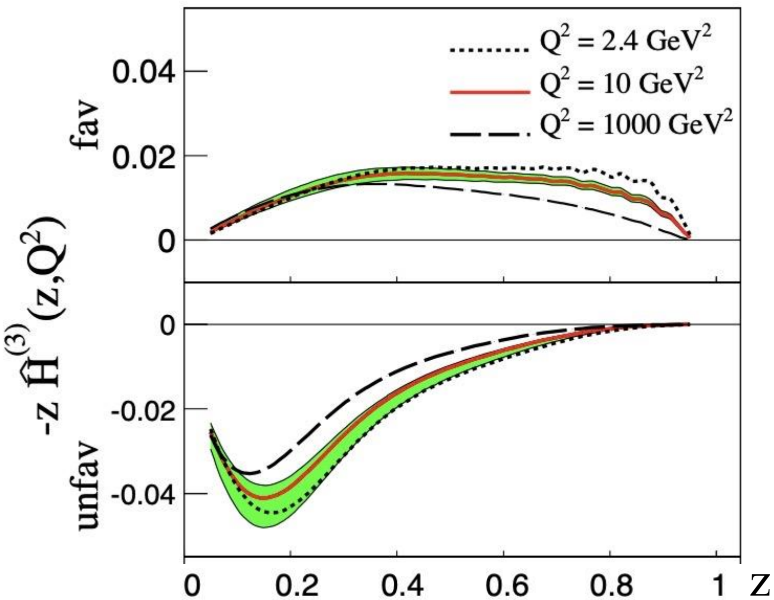
Gamberg, et al. (2022) - JAM



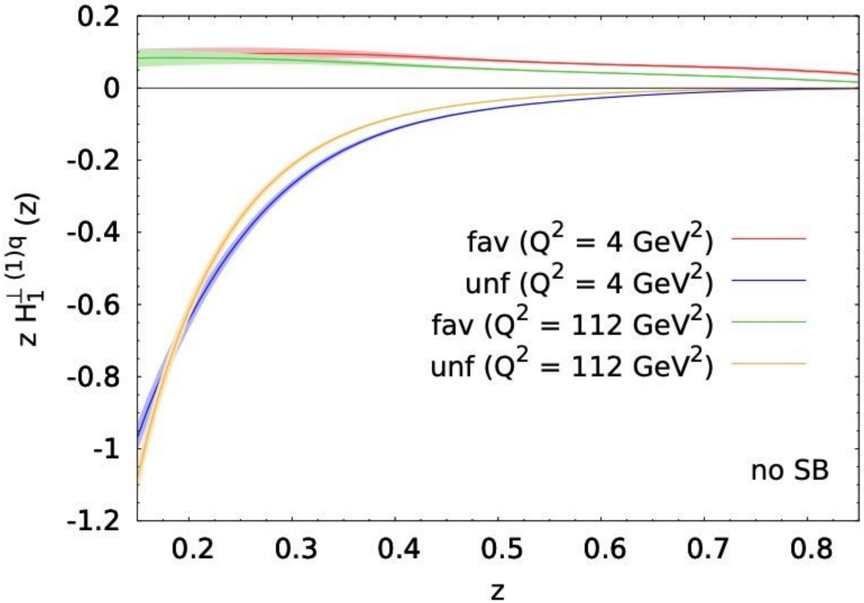
Boglione, et al. (2024)



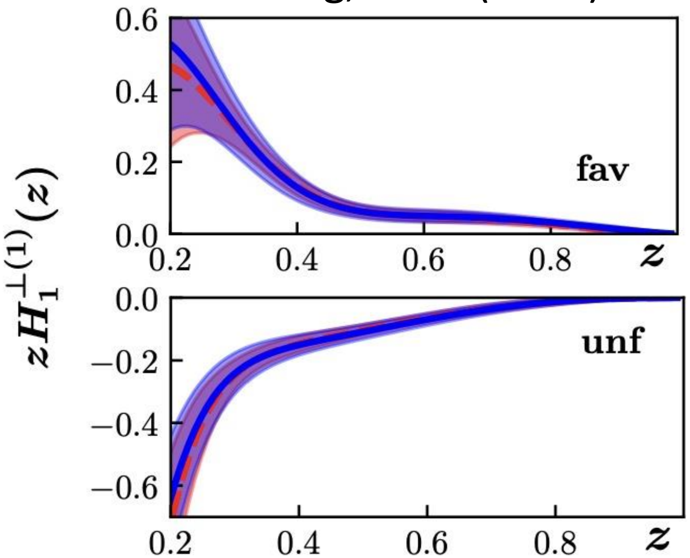
Kang, et al. (2016)



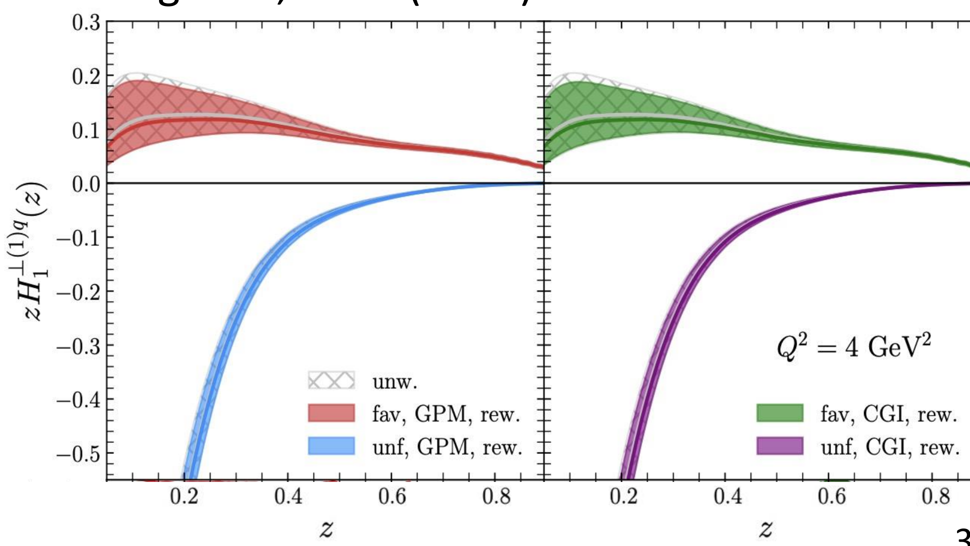
D'Alesio, et al. (2020)



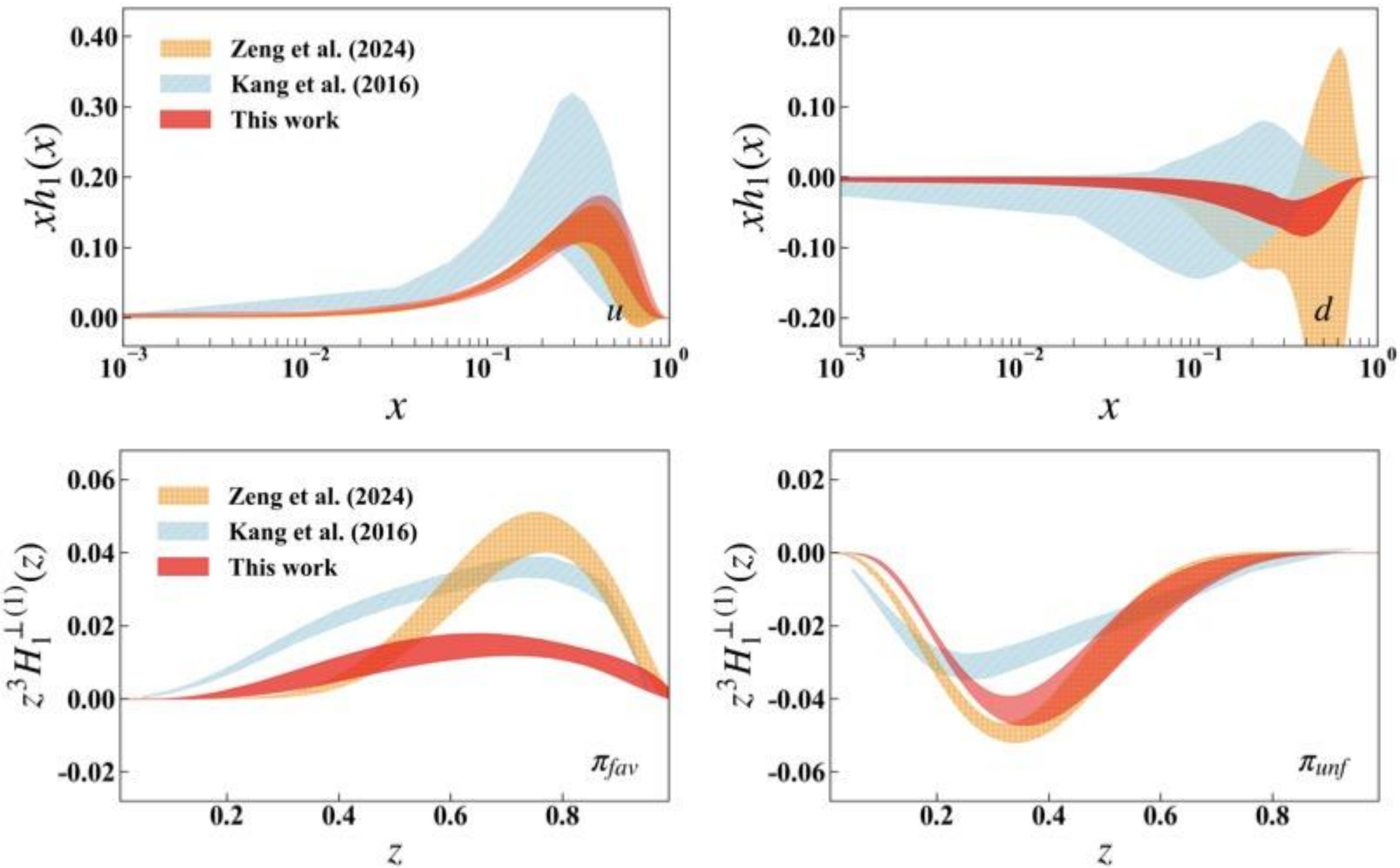
Gamberg, et al. (2022) - JAM



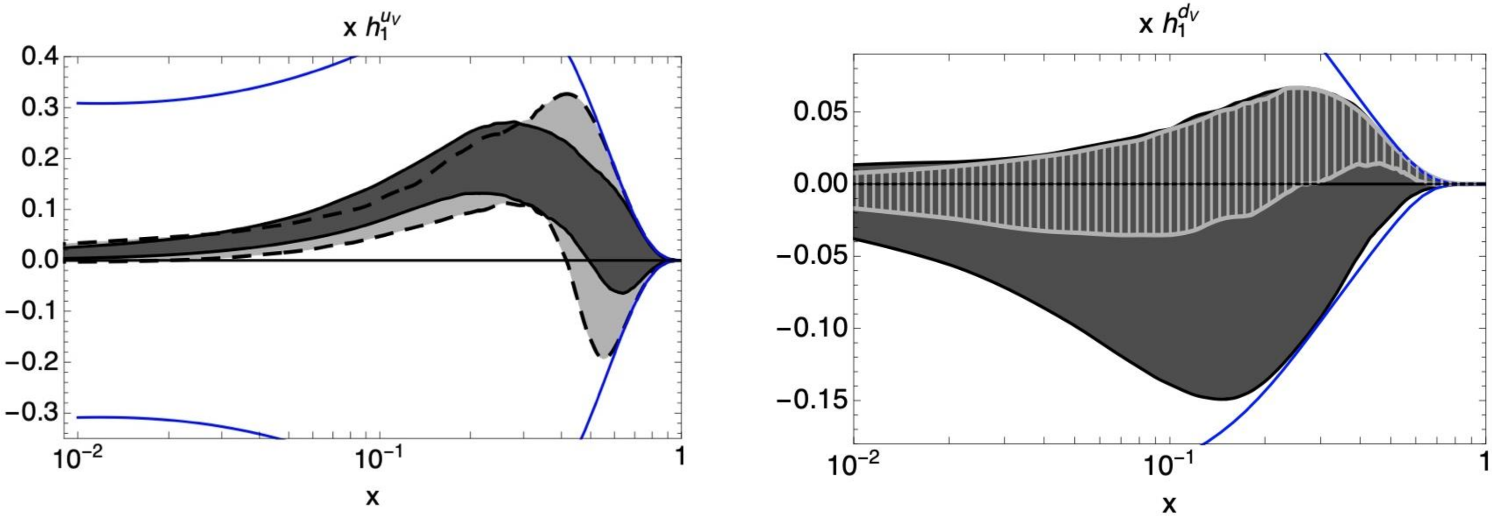
Boglione, et al. (2024)



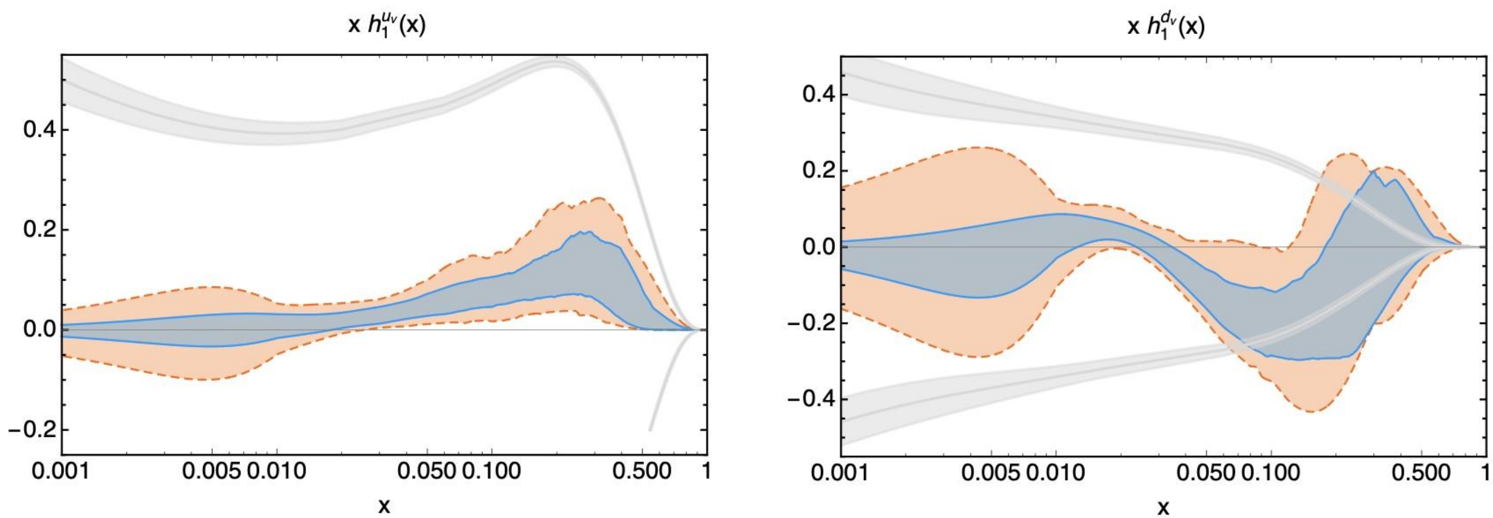
Zheng, Dong, Liu, Sun, Zhao (2024) – TNTC



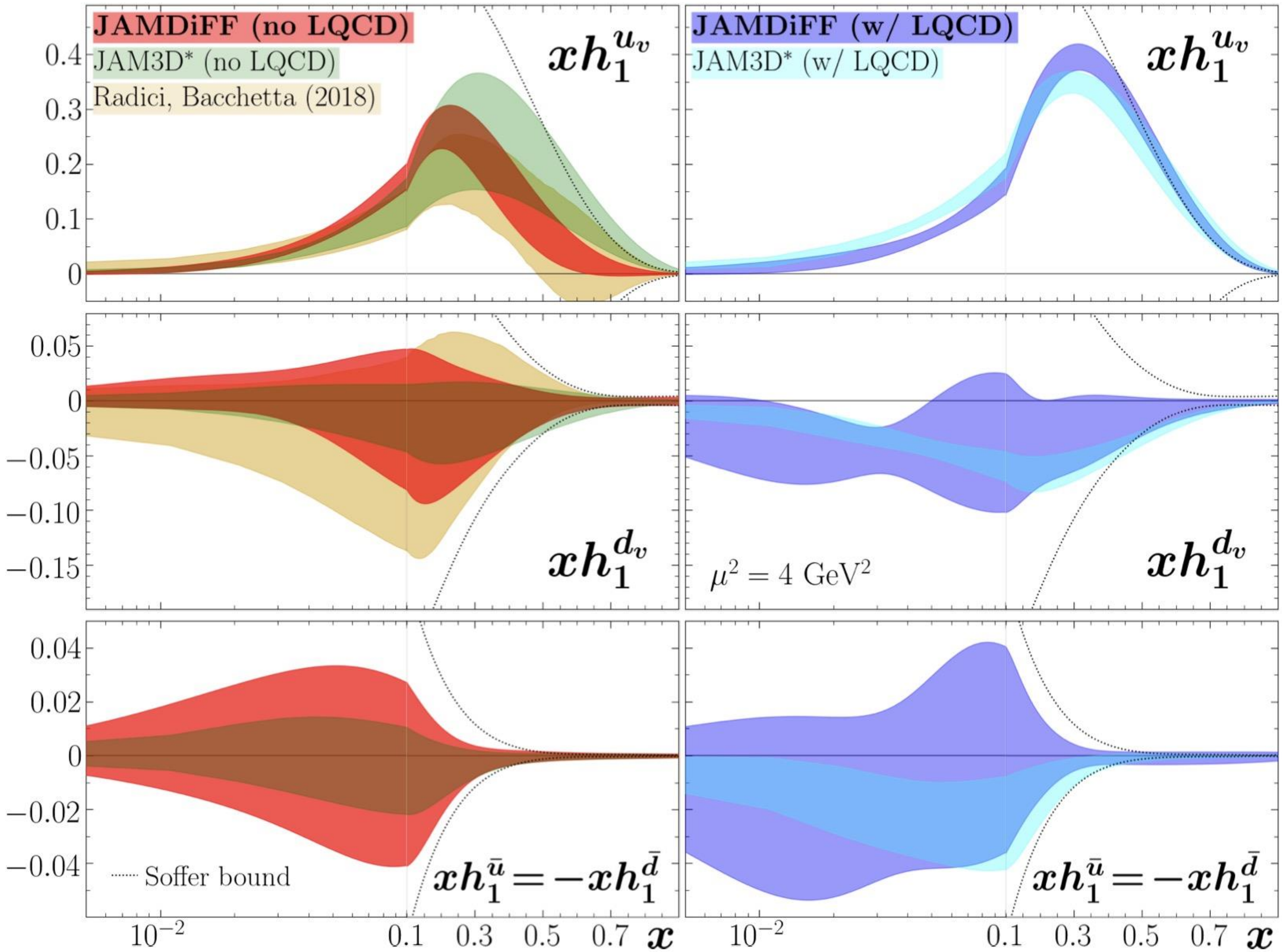
Radici, Bacchetta (2018)



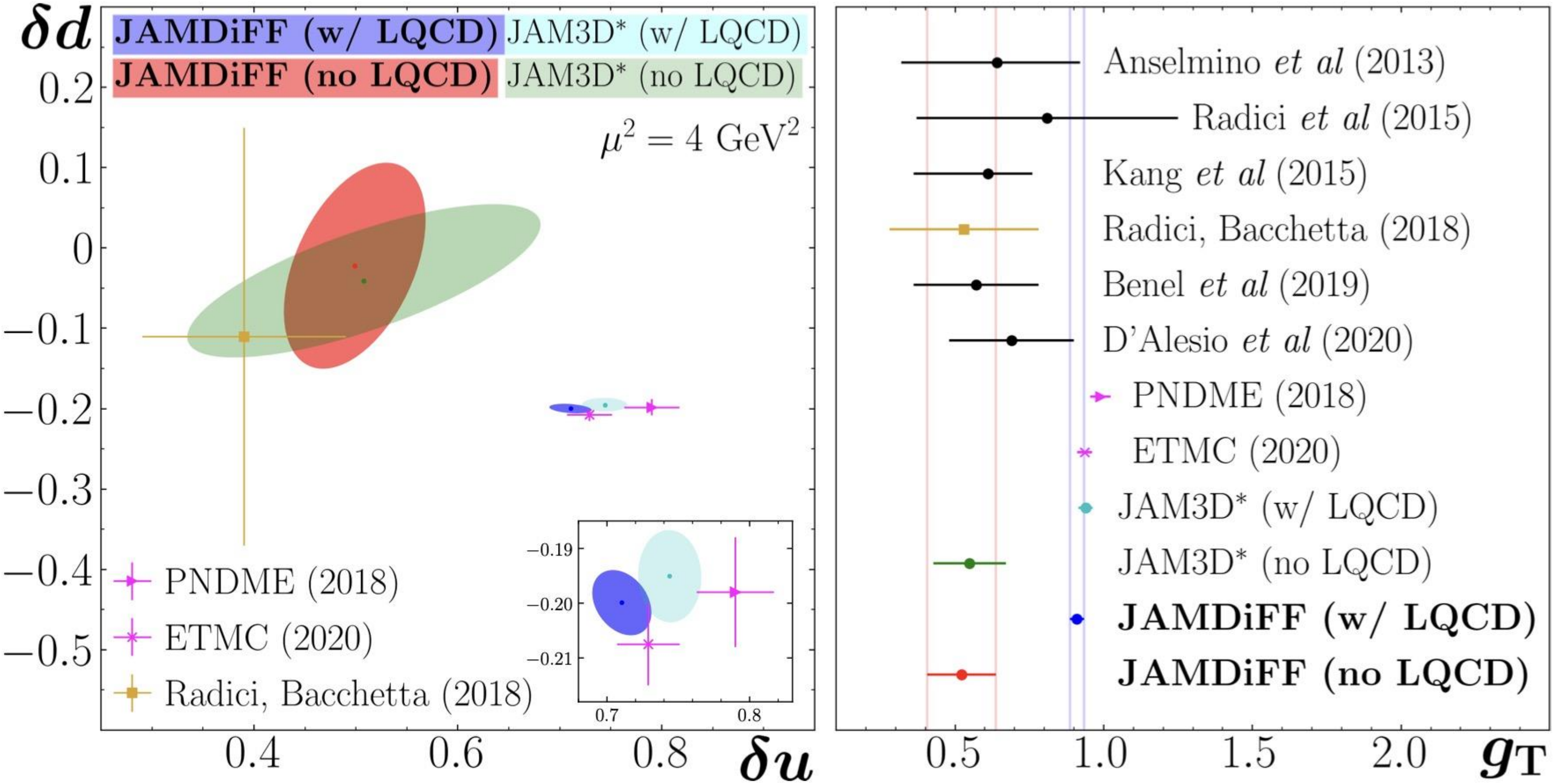
Benel, Courtoy, Ferro-Hernandez (2020)



Cocuzza, Metz, Pitonyak, Prokudin, Sato, Seidl (2024) - JAM

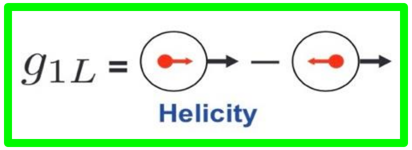


Cocuzza, Metz, Pitonyak, Prokudin, Sato, Seidl (2024) - JAM

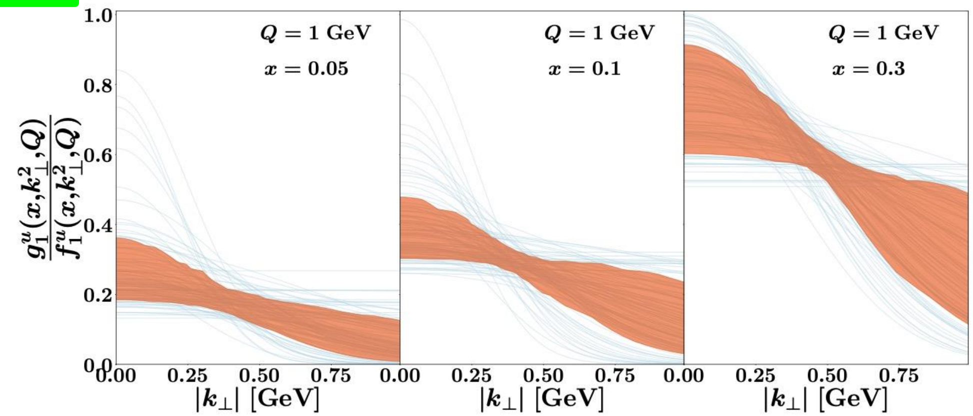


Recent analyses by the JAM Collaboration (Gamberg, et al. (2022), Cocuzza, et al. (2024)) show that lattice QCD tensor charge data can be accommodated within phenomenology, demonstrating a universality to all this available information.

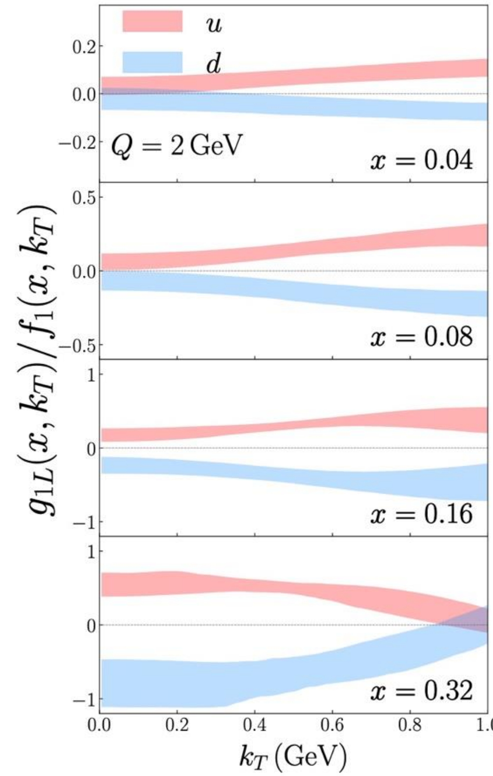
Need to include Collins hadron-in-jet data from STAR (2018, 2022, 2025) and better understand systematic biases in transversity extraction from experimental data only.

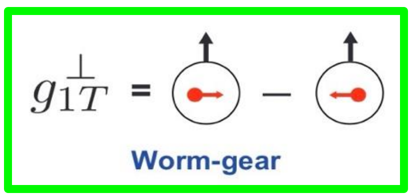


Bacchetta , Bongallino, Cerutti, Radici, Rossi (2024) - MAP

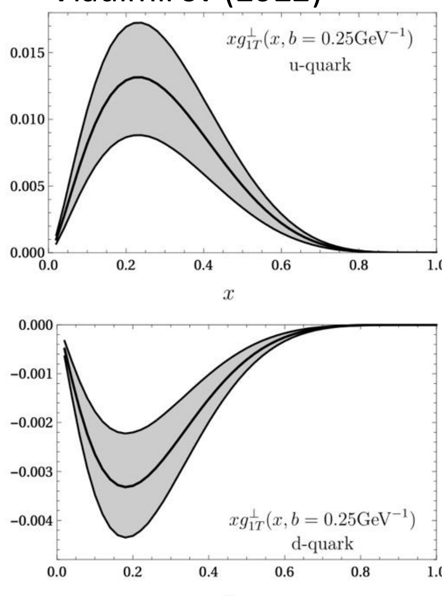


Yang, Liu, Sun, Zhao, Ma (2024)

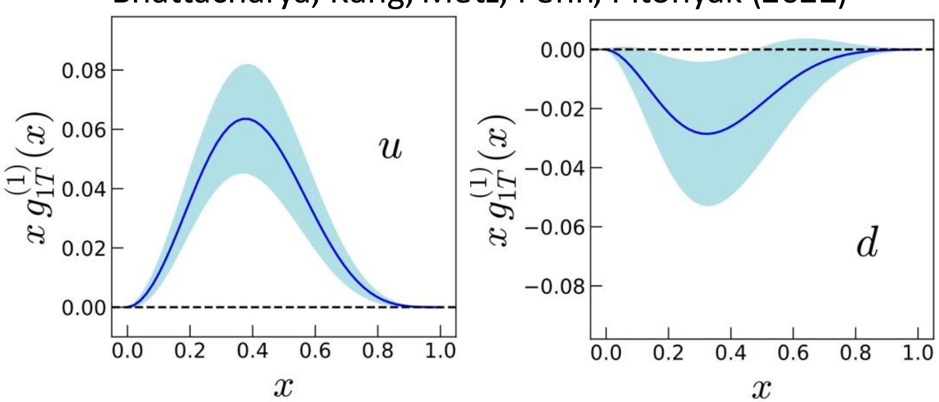


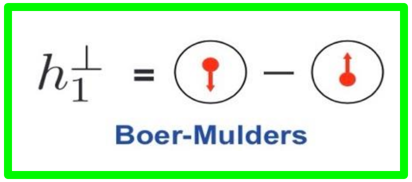


Horstmann, Schäfer, Vladimirov (2022)

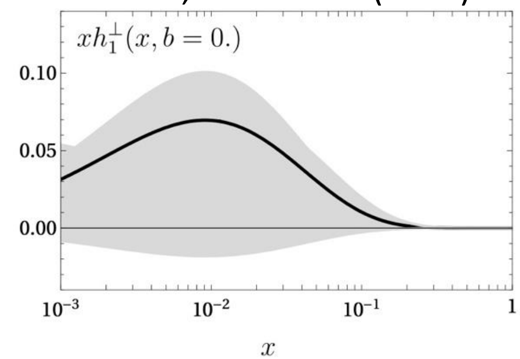


Bhattacharya, Kang, Metz, Penn, Pitonyak (2022)

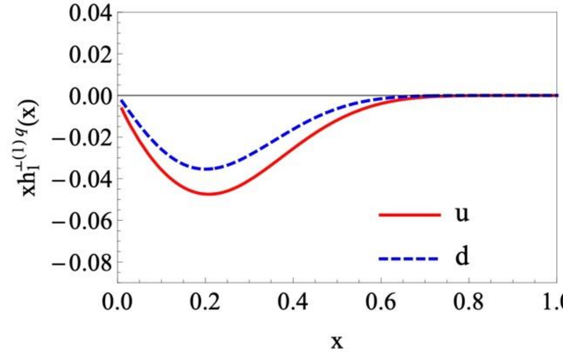


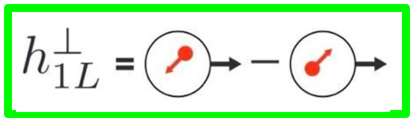


Piloñeta, Vladimirov (2024)

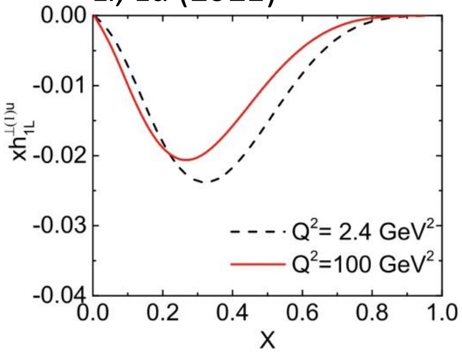


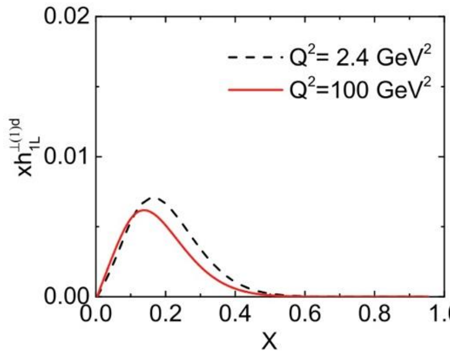
Barone, Melis, Prokudin (2010)

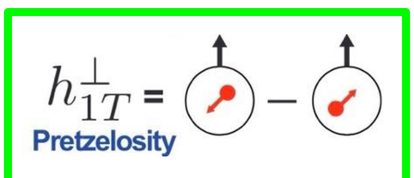




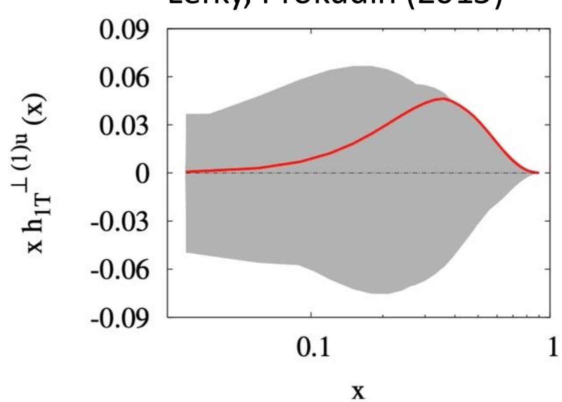
Li, Lu (2021)

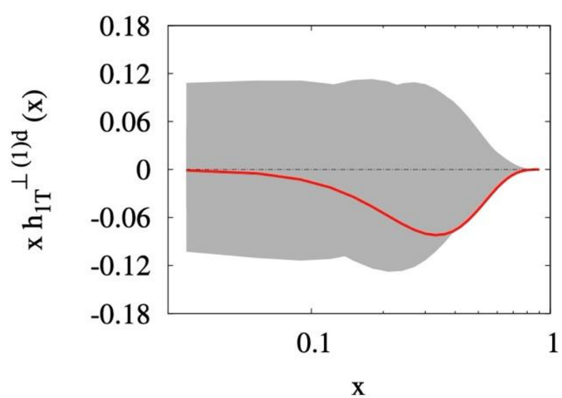


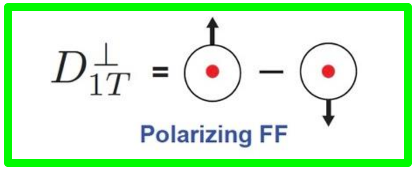




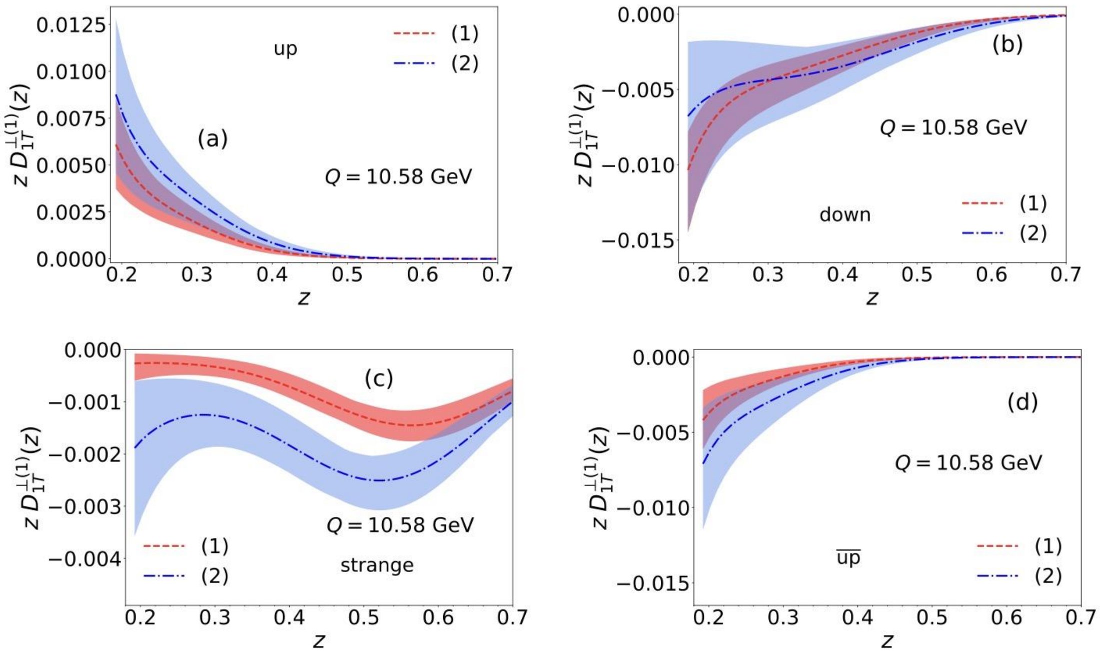
Lefky, Prokudin (2015)



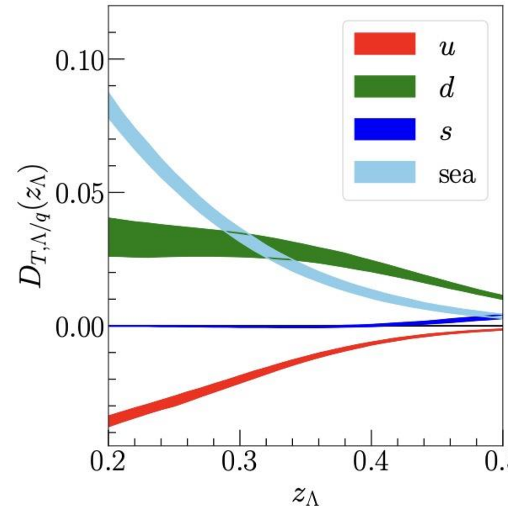


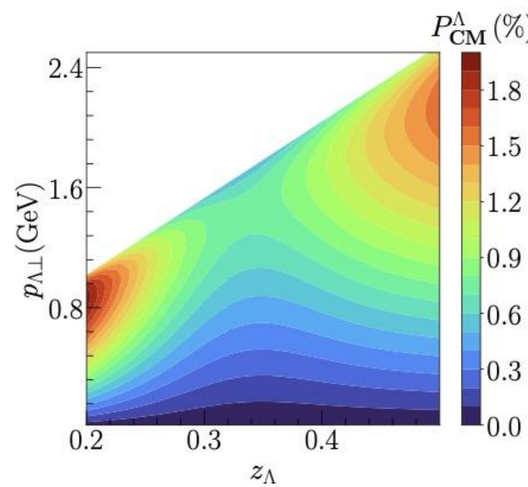


D'Alesio, Gamberg, Murgia, Zaccheddu (2023)



Gamberg, Kang, Shao, Terry, Zhao (2021)

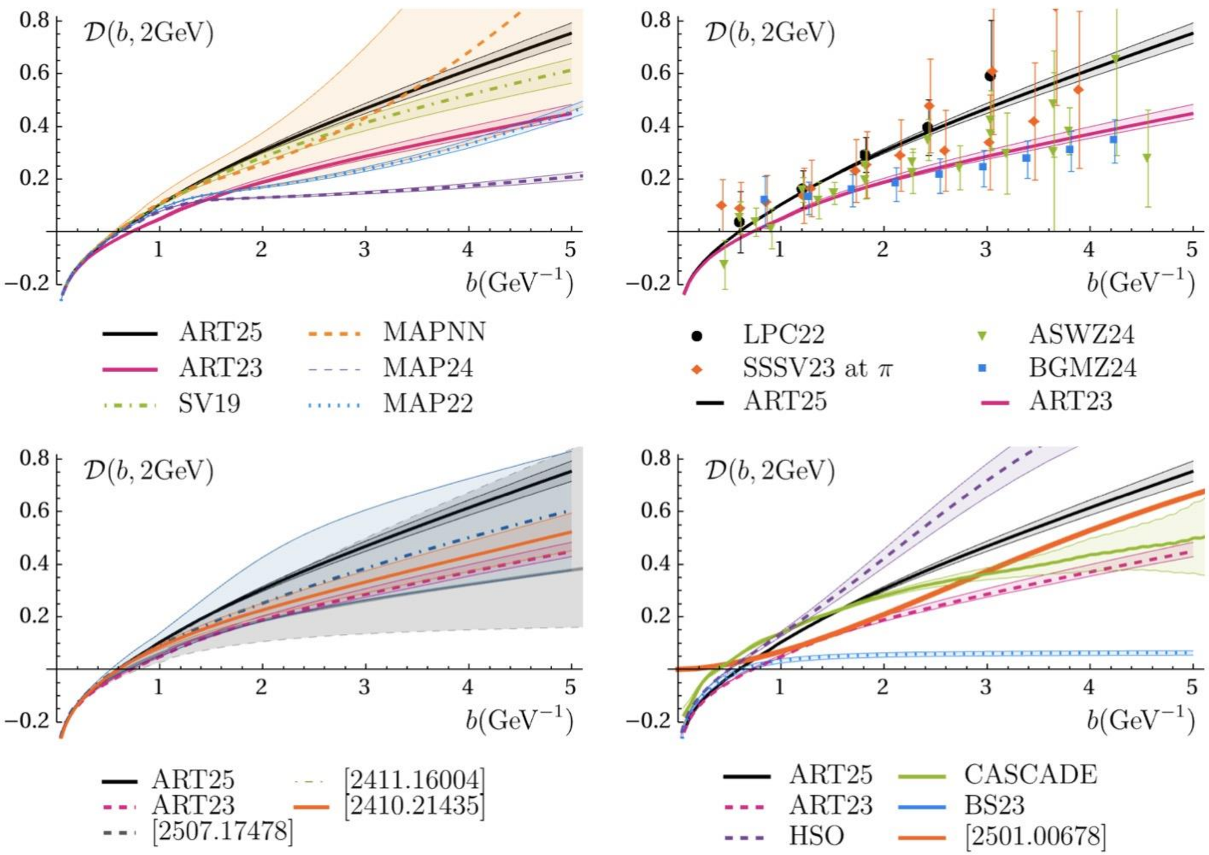




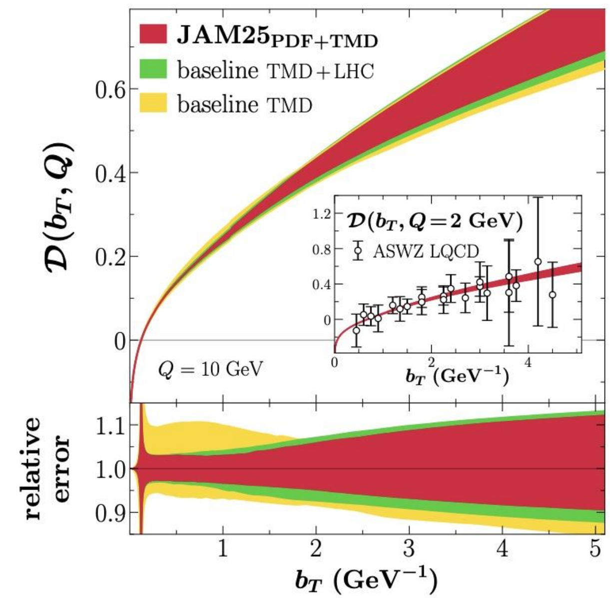
Comparisons with Lattice QCD

Collins-Soper Kernel

Moos, et al. (2025)



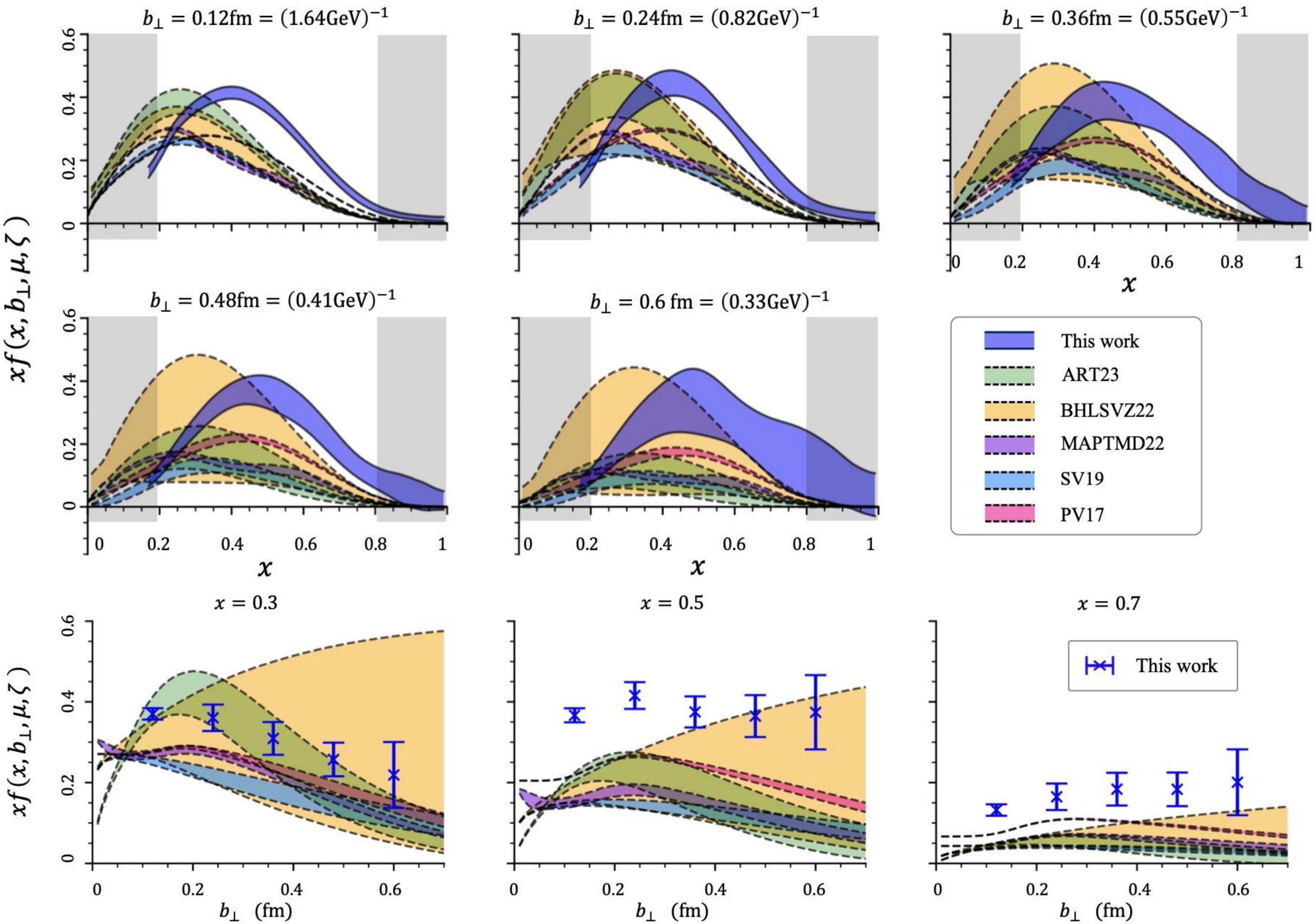
JAM25$_{\text{PDF+TMD}}$



Extractions from DY+SIDIS prefer a larger CS kernel at larger *b*, while DY only prefers a smaller CS kernel. Lattice data on the CS kernel is consistent with current phenomenology.

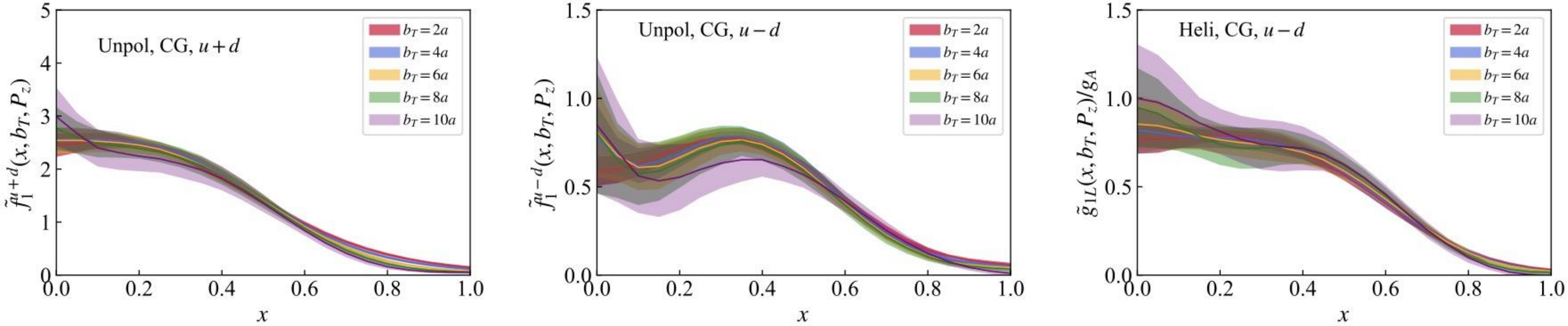
Uncertainties in the CS kernel are small (except for MAP-NN), possibly indicating a model bias from the constraints at small *b* *b*-space TMD

LPC Collaboration (2024) (He, Chu, Hua, Ji, Schäfer, Su, Wang, Yang, Zhang, Zhang)



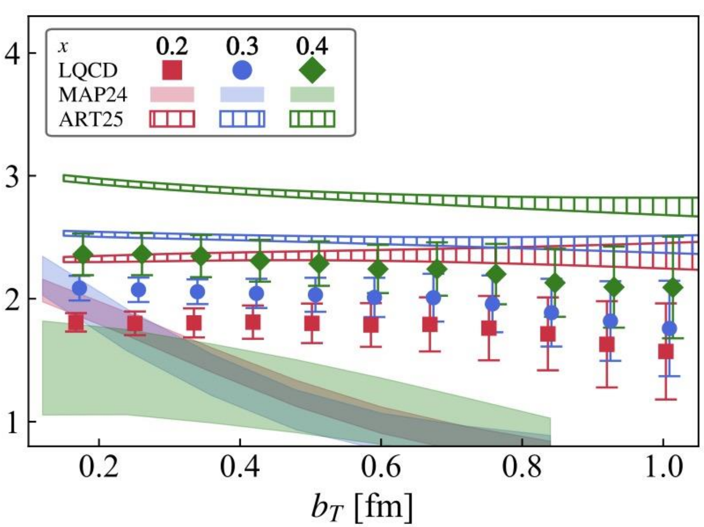
b-space TMD

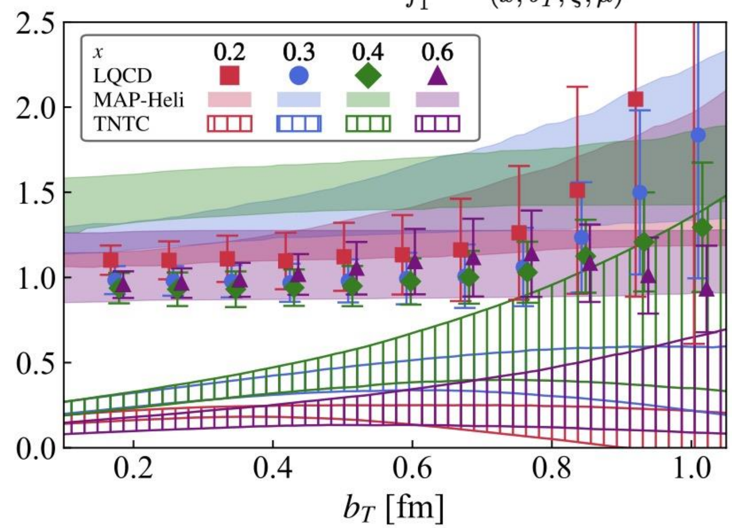
Bollweg, Gao, Mukherjee, Zhao (2025)



$$R_{f_1}^{u/d}(x,b_T) \equiv \frac{f_1^{u_v}(x,b_T,\zeta,\mu)}{f_1^{d_v}(x,b_T,\zeta,\mu)}$$

$$R_{g_{1L}/f_1}^{u-d}(x,b_T)\cdot g_A \equiv \frac{g_{1L}^{\Delta u_+-\Delta d_+}(x,b_T,\zeta,\mu)}{f_1^{u_v-d_v}(x,b_T,\zeta,\mu)}$$





Lattice computations of b-space TMDs show a promising agreement with the overall trend of the phenomenological extractions

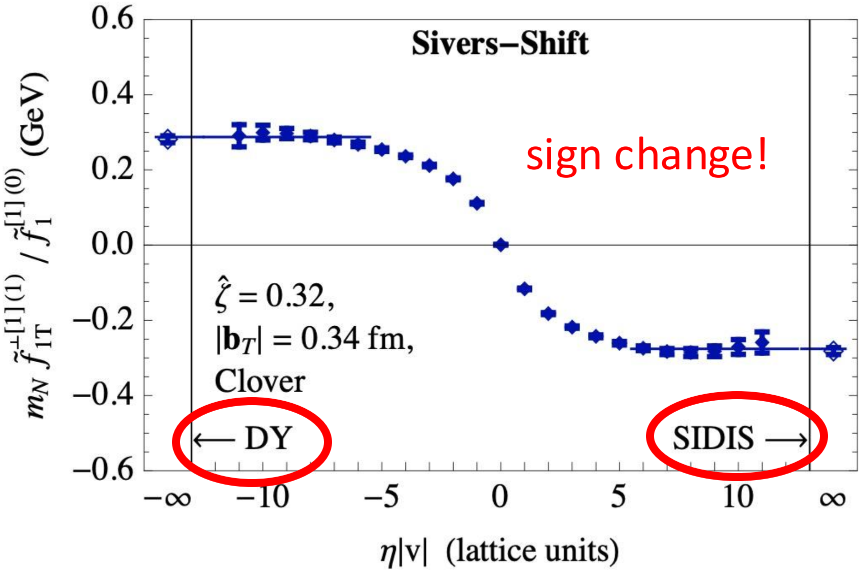
Lebanon Valley College

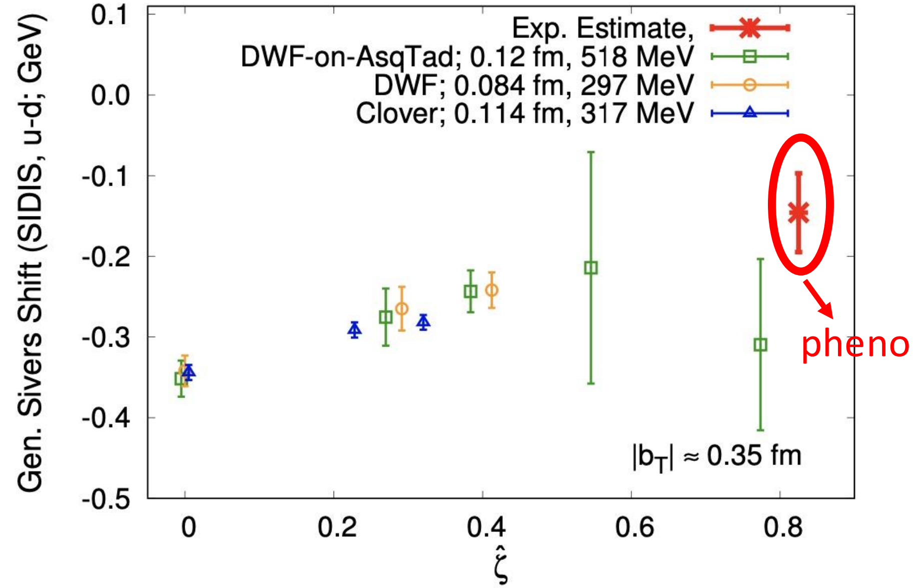
D. Pitonyak

NSF

k_T-space TMD "shifts"

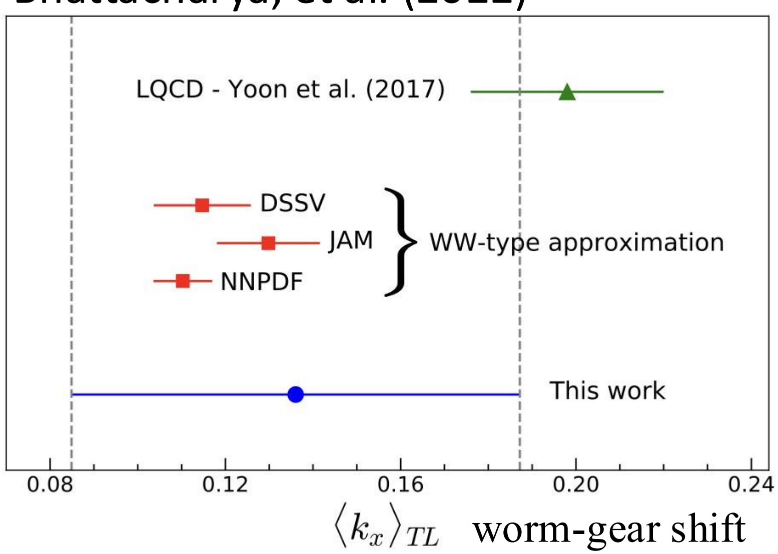
Yoon, et al. (2017)





NB: $\hat{\zeta} \to \infty$ is the TMD limit

Bhattacharya, et al. (2022)



Compatibility between lattice and phenomenology for polarized TMDs (Sivers, worm-gear, helicity)

Other Important Topics and Outlook

➢ TMDs at small-x

❑ Small-x asymptotics of TMDs

Adamiak, Santiago, Tawabutr (2024)

Leading Twist Quark TMDs				
		Quark Polarization		
		U	L	T
Nucleon Polarization	U	$f_1^{\mathrm{S}} \sim x^{-\frac{4\alpha_s N_c}{\pi}\ln(2)}$		$h_1^{\perp\mathrm{S}} \sim x$
	L		$g_1^{\mathrm{S}} \sim x^{-3.66\sqrt{\alpha_s N_c/2\pi}}$	$h_{1L}^{\perp\mathrm{S}} \sim x$
	T	$f_{1T}^{\perp\mathrm{S}} \sim x^{-2.9\sqrt{\alpha_s N_c/4\pi}}$	$g_{1T}^{\mathrm{S}} \sim x^{-2.9\sqrt{\alpha_s N_c/4\pi}}$	$h_1^{\mathrm{S}} \sim h_{1T}^{\perp\mathrm{S}} \sim x^{1-2\sqrt{\frac{\alpha_s N_c}{2\pi}}}$

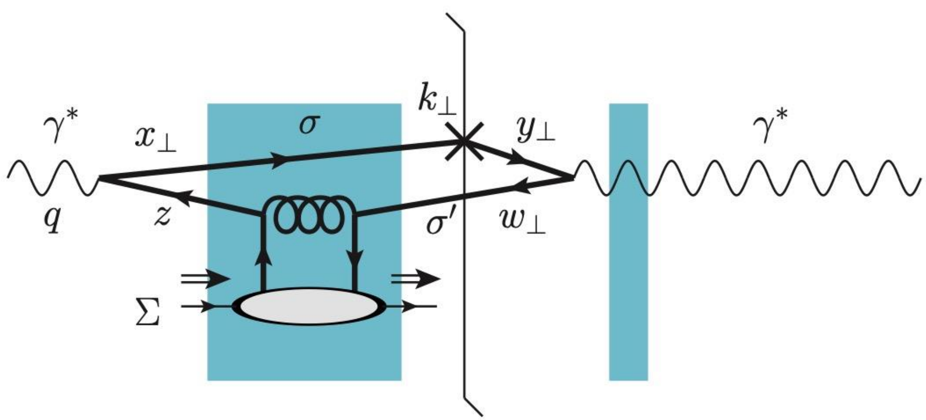
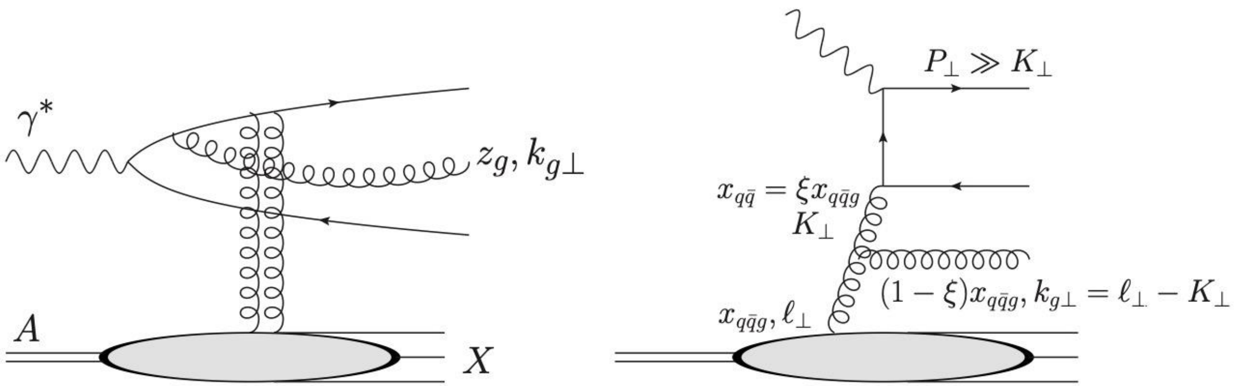


Figure from Kovchegov, Sivert, Pitonyak (2016)

➢ TMDs at small-x

❑ Connections between JIMWLK, DGLAP, and CSS

Caucal, Iancu, Salazar, Yuan (2025)

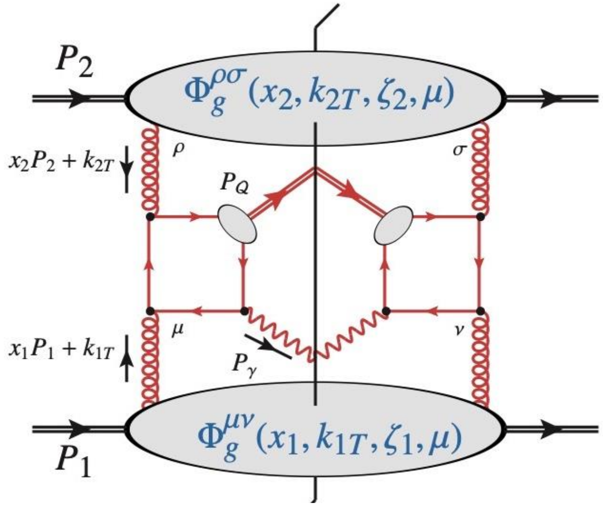


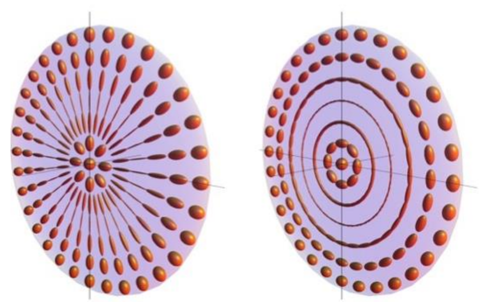
$$\frac{\mathrm{d}\sigma_{\mathrm{LO}}^{\gamma_T^* A \to q\bar{q}X}}{\mathrm{d}z_1 \mathrm{d}z_2 \mathrm{d}^2\boldsymbol{P}\mathrm{d}^2\boldsymbol{K}} = \left[\mathcal{H}_{\mathrm{LO}}^{mn} + \alpha_s \mathcal{H}_{\mathrm{NLO}}^{mn}\right] \mathcal{W}^{mn}(x_{q\bar{q}}, \boldsymbol{K}, P_\perp) + \mathcal{H}_{\mathrm{LO}}^{mn} \Delta \mathcal{W}_{\mathcal{R}}^{mn}(x_{q\bar{q}}.\boldsymbol{K})$$

See also Taels, Altinoluk, Beuf, Marquet (2022); Caucal, Salazar, Schenke, Venugopalan (2022); Caucal, Iancu (2024); Caucal, Iancu, Mueller, Yuan (2025); Duan, Kovner, Lublinsky (2025); Duan, Kovner, Lublinsky (2025), …

➢ Gluon TMDs

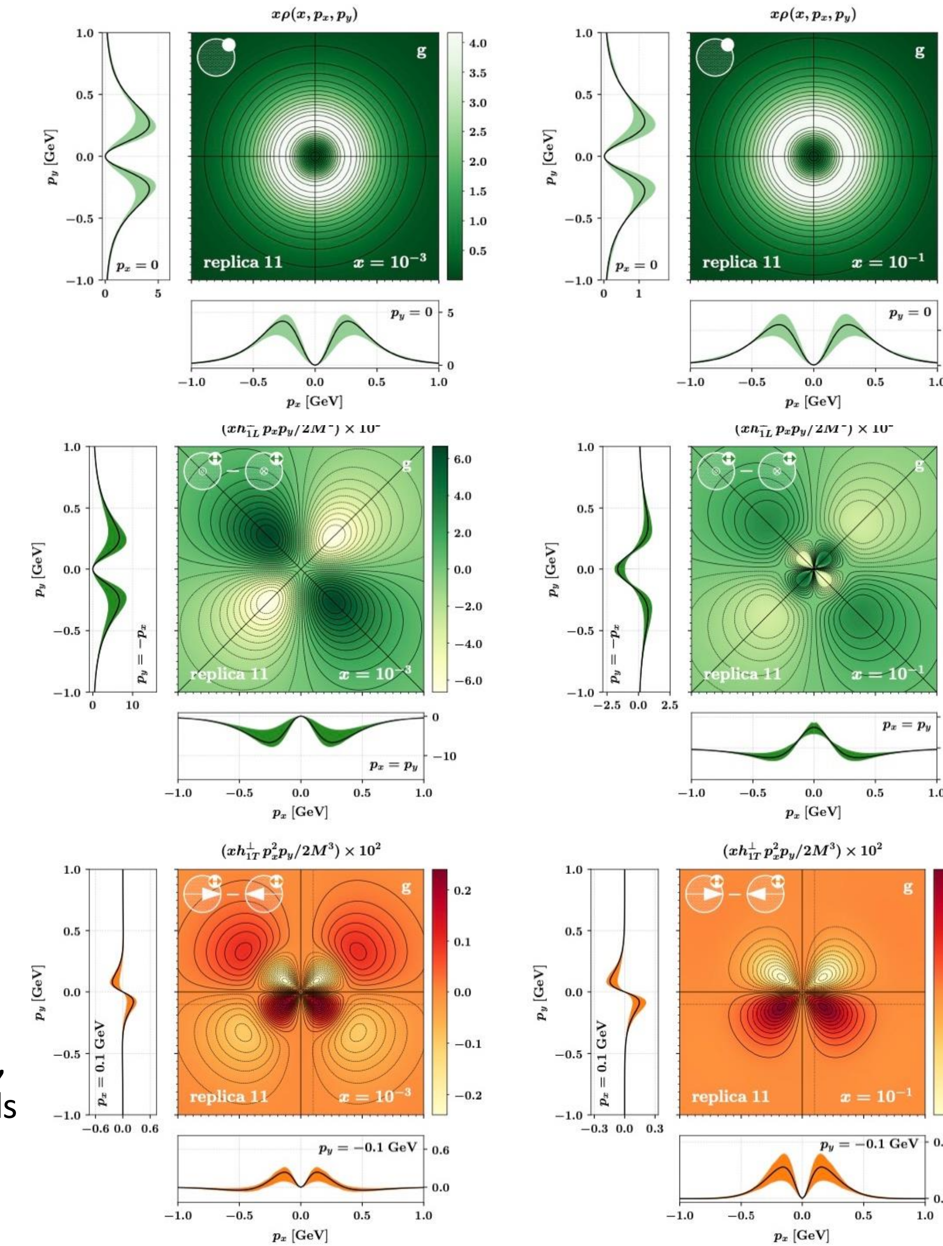
den Dunnen, Lansberg, Pisano, Schlegel (2014)



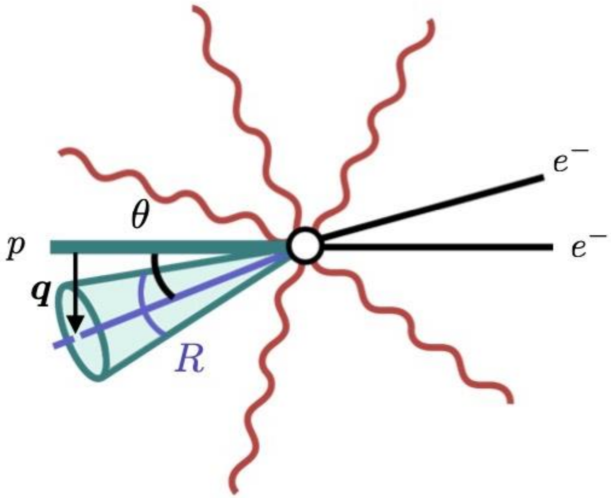


See also Godbole, Kaushik, Misra, Rawoot (2015); Boer, Mulders, Pisano, Zhou (2016); Lansberg, Pisano, Scarpa, Schlegel (2018); D'Alesio, Murgia, Pisano, Taels (2017); D'Alesio, Flore, Murgia, Pisano, Taels (2019); D'Alesio, Maxia, Murgia, Pisano, Rajesh (2020), …

Bacchetta, Celiberto, Radici (2024)



➢ Jet observables



Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi (2018, 2019)

$$\frac{\mathrm{d}\sigma_{ep\to eJX}}{\mathrm{d}Q^2\,\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}\boldsymbol{q}} = \sum \sigma_{0,q}^{\mathrm{DIS}}(x,Q^2)\,H_{\mathrm{DIS}}(Q^2,\mu) \int \frac{\mathrm{d}\boldsymbol{b}}{(2\pi)^2}\, e^{-\mathrm{i}\boldsymbol{b}\cdot\boldsymbol{q}} F_q(x,\boldsymbol{b},\mu,\zeta)\, J_q\Big(z,\boldsymbol{b},\frac{Q\mathcal{R}}{2},\mu,\zeta\Big)$$

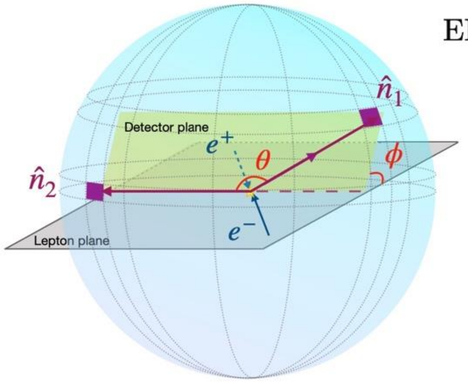
See also Neill, Scimemi, Waalewijn (2017); Kang, Liu, Ringer, Xing (2017); Gutierrez-Reyes, Makris, Vaidya, Scimemi, Zoppi (2019); Arratia, Kang, Prokudin, Ringer (2020); Liu, Ringer, Vogelsang, Yuan (2020); Arratia, Makris, Neill, Ringer, Sato (2021); Lai, Liu,Wang, Xin (2022); Caucal, Iancu, Mueller, Yuan (2025); Jaarsmaa, del Rio, Scimemi, Waalewijn (2025), …

Liu, Ringer, Vogelsang, Yuan (2018)

$$\frac{d\Delta\sigma(S_\perp)}{dy_\ell d^2k_{\ell\perp} d^2q_\perp} = \sigma_0 \epsilon^{\alpha\beta} S_\perp^\alpha \int \frac{d^2b_\perp}{(2\pi)^2} e^{iq_\perp\cdot b_\perp} \widetilde{W}_{Tq}^\beta$$

$$\widetilde{W}_{Tq}^\beta = x\tilde{f}_{1T}^{\perp\beta}(x,b_\perp)\, S_J(b_\perp,\mu_F)\, H_{\mathrm{TMD}}^\perp(Q,\mu_F)$$

➢ Energy-energy correlators



Kang, Lee, Shao, Zhao (2023)

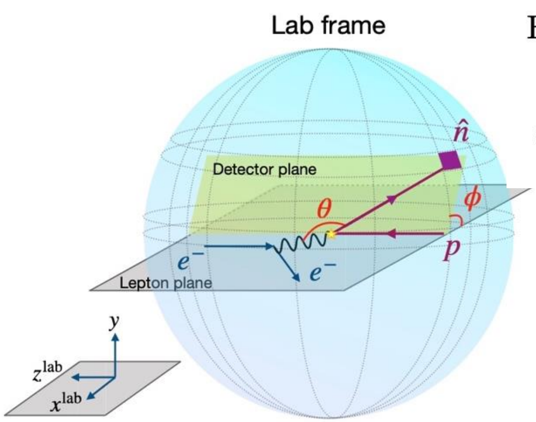
$$\mathrm{EEC}_{e^+e^-}(\tau,\phi) = \frac{d\Sigma_{e^+e^-}}{d\tau d\phi}$$

$$= \frac{1}{2}\sigma_0\, H(Q,\mu) \sum_q e_q^2 \int \frac{b\,db}{2\pi} \Big[J_0(b\sqrt{\tau}Q) J_q(b,\mu,\zeta) J_{\bar{q}}(b,\mu,\zeta) + \cos(2\phi)\, \frac{b^2}{8} J_2(b\sqrt{\tau}Q)\; J_q^{\perp}(b,\mu,\zeta) J_{\bar{q}}^{\perp}(b,\mu,\zeta) \Big]$$

$$J_q(b,\mu,\zeta) \equiv \sum_h \int_0^1 dz\, z\, \tilde{D}_{1,h/q}(z,b,\mu,\zeta)$$

$$J_q^{\perp}(b,\mu,\zeta) \equiv \sum_h \int_0^1 dz\, z\, \tilde{H}^{\perp}_{1,h/q}(z,b,\mu,\zeta)$$

See also Moult, Zhu (2018); Ebert, Mistlberger, Vita (2021); Kang, Penttala, Zhang (2024); Mäntysaari, Tawabutr, Tong (2025); Bhattacharya, Kang, Padilla, Penttala (2025); Cuerpo, Scimemi, Vladimirov (2025); Cao, Yu, Yuan, Zhang, Zhu (2025), …



$$\mathrm{EEC}_{\mathrm{DIS}}(\tau,\phi) = \frac{d\Sigma_{\mathrm{DIS}}}{dx_B dy d\tau d\phi}$$

$$= \sigma_0 \int d^2\boldsymbol{q}_T \delta(\tau - \frac{\boldsymbol{q}_T^2}{Q^2}) \int \frac{db\, b}{2\pi} \Big\{ \mathcal{F}_{UU} + \sin(\phi_h+\phi_s) \frac{2(1-y)}{1+(1-y)^2} \mathcal{F}_{UT}^{\sin(\phi_h+\phi_s)} + \cdots \Big\}$$

$$\mathcal{F}_{UU} = \int d^2\boldsymbol{q}_T \delta(\tau - \frac{\boldsymbol{q}_T^2}{Q^2}) \int \frac{bdb}{2\pi} J_0(bq_T)\, \mathcal{C}\left[\tilde{f}_1 J\right],$$

$$\mathcal{F}_{UT}^{\sin(\phi_h+\phi_s)} = \int d^2\boldsymbol{q}_T \delta(\tau - \frac{\boldsymbol{q}_T^2}{Q^2}) \int \frac{bdb}{2\pi} J_1(bq_T)\, \mathcal{C}^{\perp}\left[\tilde{h}_1 J^{\perp}\right]$$

- TMDs at small x
- Gluon TMDs
- Jet observables
- Energy-energy correlators

- TMDs at small *x*
- Gluon TMDs
- Jet observables
- Energy-energy correlators
- TMDs for pions
- TMDs for nuclei
- Twist-3/Next-to-leading power TMD factorization

  Talk by Ignazio Scimemi (Workshop 1, Oct. 29)

- QED radiative corrections in extractions of TMDs
- Model calculations
- GTMDs and Wigner functions

and many more…

- TMD physics has made great progress in understanding the 3D structure of hadrons (parton model → high-order TMD evolution)
- The field has synergies with many other areas: AI/ML, lattice QCD, small x, SCET, jets, energy correlators, …

Talk by Felix Ringer (Workshop 2, Oct. 28)

- Future high precision experimental measurements and lattice computations will allow us to further test and refine our knowledge of 3D hadronic structure

TMD Collaboration

TMD handbook

A modern introduction to the physics of Transverse Momentum Dependent distributions

[arXiv: 2304.03302](https://arxiv.org/abs/2304.03302)

TMD Winter School 2022

https://www.youtube.com/channel/UClWP7fyB4xNRYbGLtvUXVvw/featured

https://indico.mit.edu/event/300/timetable/#all.detailed